Physics of Musical Sound– Homework 5

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Problem 1

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a) E_1 is a fifth above A_0.
$f(E_1) = (2^{7/12}) * 2^{-4}(440 \text{ Hz}) = 41.2 \text{ Hz}$

b) $F_3^\sharp$ is a major sixth above A_2
$f(F_3^\sharp) = (2^{9/12}) * 2^{-2}(440 \text{ Hz}) = 185.0 \text{ Hz}$

c) G_7 is a minor seventh above A_6
$f(G_7) = (2^{10/12}) * 2^{2}(440 \text{ Hz}) = 3136.0 \text{ Hz}$

d) $D_4^\flat$ is a major third above A_3
$f(D_4^\flat) = (2^{4/12}) * 2^{-1}(440 \text{ Hz}) = 277.2 \text{ Hz}$

Problem 2

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a) This is a minor third since it spans E,F,G and it's 3 semitones.
Just ratio: $6/5 = 1.2$
Just cents: 316
Equal temp ratio: $2^{1/4} = 1.189$
Equal temp cents: 300

b) This is a major sixth since it spans G,A,B,C,D,E and it's 9 semitones.
Just ratio: $5/3 = 1.667$
Just cents: 884
Equal temp ratio: $2^{9/12} = 1.682$
Equal temp cents: 900

c) This is a major third since it spans D,E,F and it's 4 semitones.
Just ratio: $15/8 = 1.875$
Just cents: 1088
Equal temp ratio: $2^{11/12} = 1.888$
Equal temp cents: 1100

d) This is a major seventh since it spans D,E,F,G,A,B,C and it's 11 semitones.
Just ratio: $5/4 = 1.250$
Just cents: 386
Equal temp ratio: $2^{1/3}$
Equal temp cents: 400

Problem 3

An octave above E_4 is E_5.
In just intonation, the ratio is 2 and the interval is 1200 cents.
In equal temperament, the ratio is 2 and the interval is 1200 cents.

A perfect fifth above B_3 is $F^\sharp_4$
In just intonation, the ratio is $3/2 = 1.5$ and the interval is 702 cents.
In equal temperament, the ratio is $2^{7/12} = 1.498$ and the interval is 700 cents.

A minor seventh below $B^\flat_4$ is C_4.
In just intonation, the ratio is $9/5 = 1.8$ and the interval is 1018 cents.
In equal temperament, the ratio is $2^{10/12} = 1.782$ and the interval is 1000 cents.

Problem 5

D_2, D_3, D_4, D_5, because the 1st, 2nd, 4th, and 8th harmonics are octaves apart. The 3rd harmonic is A_3 because $\frac{3}{2}D_3$ is a fifth above D_3. Therefore the 6th and 12th harmonics are A_4 and A_5. The 5th harmonic is $F^\sharp_4$ because $\frac{5}{4}D_3$ is a major third above D_4, the fourth harmonic. The 10th harmonic is easily derived from this fact to be $F^\sharp_5$. The 9th harmonic is 9/8 times the 8th harmonic, and 9/8 is a major second. Therefore the 9th harmonic is a major second above D_5, which is E_5.

Thus we have all of he harmonics except the 7th and the 11th. Looking at the table in the back of the book, we can see that D_2 has a frequency of 73.42 Hz. The 7th harmonic has a frequency of $7 * 73.42 = 513.9$ Hz and the 11th harmonic has a frequency of $11 * 73.42 = 807.6$ Hz. Matching this up with the notes in the back of the book, we see that the 7th harmonic is approximately C_5 and the 11th harmonic is approximately G_5.

The notes of the harmonic series are as follows:
D_2, D_3, A_3, D_4, $F^\sharp_4$, A_4, C_5, D_5, E_5, $F^\sharp_5$, G_5, A_5

Problem 5

To get from the fourth harmonic to the fifth, you multiply by 5/4ths, which tells us that the fifth harmonic is a major third above the fourth harmonic. Therefore, the fourth harmonic is a major third below A_5, which is F_5. The first harmonic is 2 octaves lower, which is F_3.

The ninth harmonic is 9/8 times the eigth harmonic, so the eigth harmonic is a major second below the ninth. A major second below $B^\flat_6$ is $A^\flat_6$. The first harmonic is 3 octaves below the eigth, so it is $A^\flat_3$.

Problem 6

	Construction	Semitones	Interval	Note in C	Ratio	Cents
	$2{\cdot}\mathrm{M3} + \mathrm{P5}{-}8^{ve}$	3	m3	D♯	75/64	275
a)	$2 \cdot 8^{ve} - \mathrm{P5} - 2 \cdot \mathrm{M3}$	9	M6	A	125/75	925
	$2 \cdot \mathrm{M3} - \mathrm{P5}$	1	m2	C♯	25/24	71
	$8^{ve} + \mathrm{P5} - 2 \cdot \mathrm{M3}$	11	M7	B	48/25	1129

b) m3:Just = $75/64 : 6/5 = 125/128$
M6:Just = $125/75 : 5/3 = 128/125$
m2:Just = $25/24 : 16/15 = 125/128$
M7:Just = $48/25 : 15/8 = 128/125$

c) Three Just major thirds don't make exactly an octave.

Problem 7

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a. Cents above f_1 for the harmonics of f_1 and $f_1 + \mathrm{M3}$:

Note	1	2	3	4	5	6	7	8	9	10
f_1	0	1200	1902	2400	2786	3102	3369	3600	3804	3986
$f_1 + \mathrm{M3}$	400	1600	2302	2800	3186	3502	3769	4000	4204	4386

This data was collected with the aid of Excel, using the cents to Hz conversion formula. Also, a major third is 400 cents in 12-tet.

b. Beat frequency will be lowest when the cents difference is lowest and the harmonic number is lowest. Therefore, the lowest beat frequency will occur between the 5th harmonic of f_1 and the 4th harmonic of $f_1 + \mathrm{M3}$ (difference of 14 cents.) The next lowest beat frequency is between the 10th harmonic of f_1 and the 8th harmonic of $f_1 + \mathrm{M3}$ (difference of 14 cents). Again, you know that one has a lower beat frequency than the other because it also depends on the harmonic number.

c. Assume the 1st harmonic for f_1 is 440 Hz. The 5th harmonic of f_1 is $5 * 400\mathrm{Hz} = 2200\mathrm{Hz}$. The 4th harmonic of $f_1 + \mathrm{M3}$ is $4 * 2^{4/12} * 440\mathrm{Hz} = 2217.6\mathrm{Hz}$. The difference of 17.6 Hz is the beat frequency.

   The 10th harmonic of f_1 is $10{*}440\mathrm{Hz} = 4400\mathrm{Hz}$. The 8th harmonic of $f_1{+}\mathrm{M3}$ is $8{*}2^{4/12}{*}440\mathrm{Hz} = 4434.9\mathrm{Hz}$. The difference of 34.9 Hz is the beat frequency.

Problem 8

The chord $\mathrm{A}_3\mathrm{C}^\sharp_4\mathrm{E}_4$ in just intonation.

To find the beat frequency, we find the frequency of the notes and give the difference.

A_3 is $440/2$ Hz $= 220$ Hz. $\mathrm{C}_4^\sharp$ is a M3 above A_3, so it's frequency is $\frac{5}{4}220$ Hz $= 275$ Hz. E_4 is a P5 above A_3, so it's frequency is $\frac{3}{2}220$ Hz $= 330$ Hz. The difference between the frequencies is 55 Hz between the first two notes, and 55 Hz between the second two notes, and 110 Hz between the outer two notes. You will notice the 55 Hz beating the most. $T = \frac{1}{f}$ so the period is $\frac{1}{55\mathrm{Hz}} = 0.1818\mathrm{s}$.