80-311 Assignment 06

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April 8, 2014

1.1

$$\texttt{THM}(\varphi) \iff (\exists\triangle(\texttt{PRF}(\triangle, \varphi)))$$

1.2

Self-reference lemma:
If $\varphi(x)$ is any formula in the language of ZF with exactly the indicated free variable then there is a sentence ψ such that:

$$ZF \vdash (\psi \iff \varphi('\psi'))$$

Obtaining Godel's sentence G:
Let $\psi = G$ and $\varphi = \neg\texttt{THM}$. Then we substitute into the self reference lemma to obtain:

$$ZF \vdash (G \iff \neg\texttt{THM}('G'))$$

1.3

Assume that ZF is consistent, and assume for sake of contradiction that $ZF \vdash \texttt{THM}('G') \implies G$.

Then by the defn of G: $ZF \vdash \texttt{THM}('G') \implies \neg\texttt{THM}('G')$.

Which is basically saying for some formula φ where φ is G: $ZF \vdash \varphi \implies \neg\varphi$.

Which is saying that it's provable in ZF that ZF is inconsistent, and this is contradictory to our assumption that ZF is consistent. Therefore if ZF is consistent then our assumption that $ZF \vdash \texttt{THM}('G') \implies G$ is false.

2.1

2.2
