# 21-301 Assignment 06

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**1**

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We will construct a sequence of $kl$ by constructing $l$ increasing sequences each of length $k$.

Starting from some starting number, which we will call $a$, consider the increasing sequence $\langle a, a+1, ..., a+k \rangle$.
Consider another similarly constructed sequence, but instead start from $a-1-k$
Do it again, but instead start from $a-1-2k$
Repeat until you construct a sequence starting from $a-1-lk$. Note that we constructed $l$ sequences of length $k$.

The longest increasing subsequence is an individual increasing sequence of length $k$, and the longest decreasing subsequence consists of one from every such sequence of which there are $l$.

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**2**

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At most, a Hasse diagram of a poset can have $\alpha * \alpha$ edges between two levels, and the number of times this can occur is equal to the height $\omega$. This is because you can't have edges which cross levels, since that would break the property that edges must be predecessors.

Then we invoke the Erdos-Szekeres theorem to get the following intermediate result:

$$|E| \leq \omega\alpha\alpha \leq n\alpha$$

We see that the maximum number of edges depends on the $n$ and width of the poset. We know that a large poset is either tall or wide, and to maximize number of edges, this equation tells us it must be as wide as possible. Therefore, we know that a poset of maximum size should have height of 1.

Visually, this poset looks like a bipartite graph. To maximize the number of edges, we wish the bipartitions to be of equal size. Therefore, each bipartition is of size $\frac{n}{2}$, and the maximum number of edges occurs in the complete graph case where every node in $A$ is connected to every node in $B$. The maximum number of edges in this graph is

$$\frac{n}{2}\frac{n}{2} = \frac{n^2}{4}$$

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To see an example, we define the poset where $X$ is $n/2$ distinct sets of size 1, and $n/2$ distinct sets of size 2. We define the relation to be $xRy$ iff $|x| < |y| \vee x = y$. We see that it is reflexive, antisymmetric, and transitive. The Hasse diagram resembles a complete bipartite graph such as the one in the proof, since all the sets of size 1 are predecessors of all the sets of size 2.

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## 4

## 5

## 6