

80-311 Assignment 11

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April 29, 2014

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Claim:

Let $f: a \rightarrow b$ and x and y be subsets of a , then:

$$f[x \cup y] = f[x] \cup f[y]$$

To prove this we show that the LHS is a subset of the RHS, and the RHS is a subset of the LHS.

Part 1: $LHS \subseteq RHS$

First, let an arbitrary element t be a member of $f[x \cup y]$. Then by the defn of image, we know that there exists u such that $f(u) = t$ and $u \in (x \cup y)$.

By the defn of union, u must be in either x or y .

Then t is in either $f[x]$ or $f[y]$, so it is in $f[x] \cup f[y]$, which is the RHS.

Therefore any element t which is a member of the LHS is also a member of the RHS.

Then $LHS \subseteq RHS$

Part 2: $RHS \subseteq LHS$

Consider element t in $f[x] \cup f[y]$.

Then there is an element u either in x or y such that $f(u) = t$.

Then u is in $x \cup y$. Then t is in $f[x \cup y]$, so every element in the RHS is also in the LHS.