

## 02-512 Assignment 02

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October 1, 2014

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(a)

For one combination of substitutions made, define the following variables:

Let  $\vec{x}$  be an  $n$ -dimensional vector of 1s and 0s where  $x_i$  is 1 if the  $i^{th}$  substitution was made, and 0 if not.

Let  $\vec{k}$  be an  $n$ -dimensional vector of weights, where  $k_i$  is the amount that making the  $i^{th}$  substitution contributes to the expression level.

Let  $c$  be the baseline expression level.

Let  $y$  be the expression level.

Then:

$$\vec{k} \cdot \vec{x} + c = y$$

If we have  $n+1$  such equations, one for each combination, we can represent the linear system in the following matrix: ( $x_i$  will now be  $x_{j,i}$  where  $j$  is the ordinal number of the combination/equation, and apply a similar transformation to the indexes of  $k$  and  $y$ )

$$\begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,n} & 1 \\ x_{2,1} & x_{2,2} & \cdots & x_{2,n} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{n+1,1} & x_{n+1,2} & \cdots & x_{n+1,n} & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \\ c \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n+1} \end{bmatrix}$$

We know the eqns are independent since the combinations of substitutions are distinct, and may not be scalar multiples of one another because all values are zero or one.

Therefore the matrix is full-rank, and you'll find exact solutions for  $\vec{k}$  using Gaussian Elimination.