

15-210 Assignment 07

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1: Task 3.1

In Kruskal's algorithm, we greedily add minimum weight edges sequentially so long as the tree invariant is maintained. We know Kruskal's is correct, so if we show that Kruskal's algorithm adds the 2nd lightest edge, then we've proved that it must be in the MST.

Kruskal's first adds the lightest edge. Then it adds the second lightest edge. This does not create a cycle because the graph is simple so there are no parallel edges. Therefore the second lightest edge must be in the MST.

2: Task 3.2

The probability of a house winning a prize package is $\frac{1}{3}$ since of the house, the house to the left, and the house to the right, one of them will win, and each with equal probability.

Let X_i be an indicator variable for whether or not the i^{th} house wins a prize package. Let X be a random variable representing the number of houses which win prize packages. We know $X = \sum_{i=1}^n X_i$. Then by linearity of expectation:

$$\begin{aligned}\mathbb{E}[X] &= \\ \mathbb{E}\left[\sum_{i=1}^n X_i\right] &= \\ \sum_{i=1}^n \mathbb{E}[X_i] &= \\ \sum_{i=1}^n P(X_i) &= \frac{n}{3}\end{aligned}$$

3: Task 3.3

By markov's inequality,

$$\begin{aligned}Pr(X_n \geq \frac{2n}{3}) &\leq \frac{E[X_n]}{\frac{2n}{3}} \\ &= \frac{n/3}{2n/3} = \frac{1}{2}\end{aligned}$$

$$E[f(n)] \leq E[f(X_n)] + \Theta(n)$$

$$\leq E[f(X_n)|X_n \geq \frac{2n}{3}] + E[f(X_n)|X_n < \frac{2n}{3}] + \Theta(n)$$

The first term is upper bounded by $1/2$ by markovs inequality.

$$\leq 1/2 f(n) \dots$$

something something