15-451 Assignment 05

Karan Sikka
ksikka@cmu.edu
Collaborated with Dave Cummings and Sandeep Rao.
Recitation: A
October 9, 2014

1: Cell Towers

2: Eliminating Negative Edges

(a) (b) (c)

3: Color Me Red, Color Me Blue

For convenience, we will call a $(k, n-k)$ partition which minimizes split edges a “minimizing k-partition”.

We will consider a function kPart as follows:

Given a node n, value $i (1 \leq i \leq k)$ and bit b,
The function returns NONE, or SOME (A, B, S).

$\texttt{kPart}(n, k, b) = \texttt{NONE}$ signifies that no k-partition of vertices in the subtree rooted at n exists, which happens when $(k > |n|)$.
$\texttt{kPart}(n, k, b) = \texttt{SOME}(A, B, S)$ signifies that A, B is some minimizing k-partition the nodes of the subtree rooted at n such that $n \in A \iff b = 1$, S is the set of split edges

We create a memo table for the computation which we assume is available to the function.
The table can be accessed by $T[n][k][b]$ where n is a node, k is an int from 0 to k, b is a bit.
We initialize the table with NULLs, and compute the values in the following recursive function.
Note that the table has $n * (k+1) * 2$ cells.

The procedure to compute kPart(r,k,b) is as follows: If $T[r][k][i] \neq$ NULL, return $T[r][k][i]$ If $k = 0$, $T[r][k][i] = (, \texttt{all}_{\texttt{n}}\texttt{odes}(b),)$ If $k > n$, $T[r][k][i] = \texttt{DNE}$

If both r-¿left and r-¿right are NULL, If $k = 1$ $T[r][k][i] = (r, ,)$ Else

If only r-¿left is NULL, $T[r][k][i] =$ The (A, B, S) with the minimum $|S|$ over the following answers computed for all possible colorings of r and r-¿right that might yield a minimizing k-partition: $r \in A, r- > right \in A =$¿ $A_r, B_r, S_r = \texttt{kPart}(r- > right, k-1, 1)$ answer is $(r \cup A_r, B_r, S_r)$ $r \in A, r- > right \in B =$¿ $A_r, B_r, S_r = \texttt{kPark}(r- > right, k-1, 0)$ answer is $(r \cup A_r, B_r, S_r \cup (r, r- > right))$ $r \in B, r- > right \in A =$¿ $A_r, B_r, S_r = \texttt{kPark}(r- > right, k, 1)$ answer is $(A_r, r \cup B_r, S_r \cup (r, r- > right))$ $r \in B, r- > right \in B =$¿ $1 + T[r- > right][k][0]$ $A_r, B_r, S_r = \texttt{kPark}(r- > right, k, 0)$ answer is $(A_r, r \cup B_r, S_r)$

If only r-¿right is NULL, Logic is symmetric to above case.

If r-¿left and r-¿right are not NULL, $T[r][k][i]$ = The (A, B, S) with the minimum $|S|$ over the following answers computed for all possible colorings of r, r-¿right, and r-¿left that might yield a minimizing k-partition: Left and right subtrees can be partitioned in at most k+1 ways such that the sum of their A sets should be $k-1$ if r in A or k otherwise. We must minimize the number of split edges accross all variations of A-set-size.

$r \in A, r->left \in A, r->right \in A$
$minimize|S|overallintegrali, j \geq 0sti + j = k - 1$
$A_l, B_l, S_l = \texttt{kPart}(r->left, i, 1)$
$A_r, B_r, S_r = \texttt{kPart}(r->right, j, 1)$ $answeris(A_l \cup A_r \cup r, B_l \cup B_r, S_l \cup S_r)$

$r \in A, r->left \in A, r->right \in Bminimize|S|overallintegrali, j \geq 0sti+j = k-1\texttt{kPart}(r->left, i, 1) + 1 + \texttt{kPart}(r->right, j, 0)$ $answeris(A_l \cup A_r \cup r, B_l \cup B_r, S_l \cup S_r \cup (r, r->right))$

$r \in A, r->left \in B, r->right \in Aminimize|S|overallintegrali, j \geq 0sti + j = k - 11 + \texttt{kPart}(r->left, i, 0) + \texttt{kPart}(r->right, j, 1)$ $answeris(A_l \cup A_r \cup r, B_l \cup B_r, S_l \cup S_r \cup (r, r->left))$

$r \in A, r->left \in B, r->right \in Bminimize|S|overallintegrali, j \geq 0sti + j = k - 11 + \texttt{kPart}(r->left, i, 0) + 1 + \texttt{kPart}(r->right, j, 0)$ $answeris(A_l \cup A_r \cup r, B_l \cup B_r, S_l \cup S_r \cup (r, r->left), (r, r->right))$

$r \in B, r->left \in A, r->right \in Aminimize|S|overallintegrali, j \geq 0sti + j = k1 + \texttt{kPart}(r->left, i, 1) + 1 + \texttt{kPart}(r->right, j, 1)$ $answeris(A_l \cup A_r, B_l \cup B_r \cup r, S_l \cup S_r \cup (r, r->left), (r, r->right))$

$r \in B, r->left \in A, r->right \in Bminimize|S|overallintegrali, j \geq 0sti + j = k1 + \texttt{kPart}(r->left, i, 1) + \texttt{kPart}(r->right, j, 0)$ $answeris(A_l \cup A_r, B_l \cup B_r \cup r, S_l \cup S_r \cup (r, r->left))$

$r \in B, r->left \in B, r->right \in Aminimize|S|overallintegrali, j \geq 0sti + j = k\texttt{kPart}(r->left, i, 0) + 1 + \texttt{kPart}(r->right, j, 1)$ $answeris(A_l \cup A_r, B_l \cup B_r \cup r, S_l \cup S_r \cup (r, r->right))$

$r \in B, r->left \in B, r->right \in Bminimize|S|overallintegrali, j \geq 0sti + j = k\texttt{kPart}(r->left, i, 0) + \texttt{kPart}(r->right, j, 0)$ $answeris(A_l \cup A_r, B_l \cup B_r \cup r, S_l \cup S_r)$

return $T[r][k][i]$