

21-301 Assignment 06

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We will construct a sequence of kl by constructing l increasing sequences each of length k .

Starting from some starting number, which we will call a , consider the increasing sequence $\langle a, a + 1, \dots, a + k \rangle$.

Consider another similarly constructed sequence, but instead start from $a - 1 - k$

Do it again, but instead start from $a - 1 - 2k$

Repeat until you construct a sequence starting from $a - 1 - lk$. Note that we constructed l sequences of length k .

The longest increasing subsequence is an individual increasing sequence of length k , and the longest decreasing subsequence consists of one from every such sequence of which there are l .

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At most, a Hasse diagram of a poset can have $\alpha * \alpha$ edges between two levels, and the number of times this can occur is equal to the height γ . This is because you can't have edges which cross levels, since that would break the property that edges must be predecessors.

Then we invoke the Erdos-Szekeres theorem to get the following intermediate result:

$$|E| \leq \gamma \alpha \leq n \alpha$$

We see that the maximum number of edges depends on the n and width of the poset. We know that a large poset is either tall or wide, and to maximize number of edges, this equation tells us it must be as wide as possible. Therefore, we know that a poset of maximum size should have height of 1.

Visually, this poset looks like a bipartite graph. To maximize the number of edges, we wish the bipartitions to be of equal size. Therefore, each bipartition is of size $\frac{n}{2}$, and the maximum number of edges occurs in the complete graph case where every node in A is connected to every node in B . The maximum number of edges in this graph is

$$\frac{n}{2} \frac{n}{2} = \frac{n^2}{4}$$

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To see an example, we define the poset where X is $n/2$ distinct sets of size 1, and $n/2$ distinct sets of size 2. We define the relation to be xRy iff $|x| < |y| \vee x = y$. We see that it is reflexive, antisymmetric, and transitive. The Hasse diagram resembles a complete bipartite graph such as the one in the proof, since all the sets of size 1 are predecessors of all the sets of size 2.

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