21-301 Assignment 06

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November 15, 2013

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We will construct a sequence of kl by constructing l increasing sequences each of length k.

Starting from some starting number, which we will call a, consider the increasing sequence $\langle a, a+1, ..., a+k \rangle$.
Consider another similarly constructed sequence, but instead start from $a - 1 - k$
Do it again, but instead start from $a - 1 - 2k$
Repeat until you construct a sequence starting from $a - 1 - lk$. Note that we constructed l sequences of length k.

The longest increasing subsequence is an individual increasing sequence of length k, and the longest decreasing subsequence consists of one from every such sequence of which there are l.

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At most, a Hasse diagram of a poset can have $\alpha * \alpha$ edges between two levels, and the number of times this can occur is equal to the height γ. This is because you can't have edges which cross levels, since that would break the property that edges must be predecessors.

Then we invoke the Erdos-Szekeres theorem to get the following intermediate result:

$$|E| \leq \gamma\alpha\alpha \leq n\alpha$$

We see that the maximum number of edges depends on the n and width of the poset. We know that a large poset is either tall or wide, and to maximize number of edges, this equation tells us it must be as wide as possible. Therefore, we know that a poset of maximum size should have height of 1.

Visually, this poset looks like a bipartite graph. To maximize the number of edges, we wish the bipartitions to be of equal size. Therefore, each bipartition is of size $\frac{n}{2}$, and the maximum number of edges occurs in the complete graph case where every node in A is connected to every node in B. The maximum number of edges in this graph is

$$\frac{n}{2}\frac{n}{2} = \frac{n^2}{4}$$

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To see an example, we define the poset where X is $n/2$ distinct sets of size 1, and $n/2$ distinct sets of size 2. We define the relation to be xRy iff $|x| < |y| \vee x = y$. We see that it is reflexive, antisymmetric, and transitive. The Hasse diagram resembles a complete bipartite graph such as the one in the proof, since all the sets of size 1 are predecessors of all the sets of size 2.

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