

## 02-512 Assignment 06

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### 1

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(a) The variables we are trying to find are the rate of infection,  $\lambda_1$  and the rate of recovery,  $\lambda_2$ .

We can run the CTMM as a simulation and compare it with real data. The output of the simulation will be sequence of states over time where each state will be  $(S_t, I_t, R_t)$ .

Let  $(Sr_t, Ir_t, Rr_t)$  be the real data. One possible objective function to minimize is

$$L(\lambda_1, \lambda_2) = \sum_{\text{realdatapoints}} (S_t - Sr_t)^2 + (I_t - Ir_t)^2 + (R_t - Rr_t)^2$$

You can use steepest/gradient descent, Newton-Raphson's method, or a similar algorithm to find parameters yielding a local minimum. Rather than analytically computing the gradient, you'd have to approximate it using finite difference methods. Performance may be a concern, depending on how long the simulation has to run for.

(b) Let  $G$  be the growth rate,  $x_1$  be the conc of nutrient 1  $x_2$  be the conc of nutrient 2.

$$G = \theta_1 x_1^2 + \theta_2 x_1 + \theta_3 x_2^2 + \theta_4 x_2 + \theta_5$$

Where  $\vec{\theta}$  are the parameters we're trying to estimate.

Let  $Gr(x_1, x_2)$  be the experimenally determined growth rate.

One possible objective function to minimize is

$$L(x_1, x_2) = \sum (Gr(x_1, x_2) - G(x_1, x_2))^2$$

You can use steepest descent once again, like in part a.

(c) Call parameters we are estimating,  $f_B$  and  $f_b$ , which are the frequencies of the B allele and b allele respectively.

Let  $B$  be the number of people observed with brown eyes, and  $b$  be the number of people observed with blue eyes.

The likelihood function is as follows:

$$Pr(B = B, b = b | p = p) =$$

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### 2

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(a) Say there are  $m$  biomarkers. Let  $\vec{\theta}$  be an  $m + 1$  dimensional vector of parameters.

Let  $\mu = \vec{\theta} \cdot \vec{x} + \theta_{m+1}$ .

$$\mathcal{L}(\mu, \sigma^2; \theta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(\sum_{i=1}^n (x_i - \mu)^2)}{2\sigma^2}\right]$$

(b) TODO GAUSSIAN<sup>2</sup> LIKELIHOOD

(c) Do both and take the one with the smaller error?

(d) Metropolis

(e) Sampling vs solving

### 3

Given two points, the eqn of a line is derived as follows:

(Points given as t2 x2 t1 x1)

$$t = (x_2 - x_1)/(t_2 - t_1) x + b$$

$$t_1 - (x_2 - x_1)/(t_2 - t_1) x_1 = b$$

or

$$t_2 - (x_2 - x_1)/(t_2 - t_1) x_2 = b$$

$$t - t_2 = (x_2 - x_1)/(t_2 - t_1) * (x - x_2)$$

$$\text{if } 0 \leq t < 2, t - 2 = (5/2)(x - 5)$$

$$\text{if } 2 \leq t < 5, t - 5 = ((6-5)/(5-2))(x-6)$$

$$\text{if } 5 \leq t < 8, t - 8 = ((10-6)/(8-5))(x-10)$$

$$\text{if } 8 \leq t \leq 10, t - 10 = ((20-10)/(8-10))(x-20)$$

(b) TODO interp formula

(c) TODO interp formula + some deriv stuff.

### 4

(a)

If  $b_i$  is 0, there are no boojum on island  $i$ . Based on this observation, we note the following:

$$Pr(b_i = 0|f) = (1 - f)^{s_i}$$

$$Pr(b_i = 1|f) = 1 - Pr(b_i = 0|f) = 1 - (1 - f)^{s_i}$$

$$Pr(b|f) = \prod_{i=1}^n Pr(b_i = b_i)$$

(b)

$$\hat{f} = \frac{\sum_{i=1}^n y_i}{\sum_{i=1}^n s_i}$$

(c)

$$E[y_i] = b_i * \hat{f} * s_i$$

(d)

Submitted online

(e)

TODO