15-451 Algorithms, Fall 2014
Gupta and Sleator

Homework #9 **Oral Presentations: Nov 20-24, 2014 (Thu-Mon)**

- This is an oral presentation assignment. You should work in groups of three. Soon we will post a signup sheet linked off the course web page. At some point before **Monday, November 17 at 11:59pm** your group should sign up for a 1-hour time slot on this signup sheet.
- Each person in the group must be able to present every problem. The TA/Professor will select who presents which problem. The other group members may assist the presenter.
- You are not required to hand anything in at your presentation, but you may if you choose.
- You should solve Problems #1-3.

(100/3 pts) 1. **Maximum Overlap**

You're given a set S of n axis-aligned rectangles $S = \{r_1, r_2, \ldots, r_n\}$. A rectangle is said to cover a point p if p is in the interior of the rectangle (not on the boundary). The covering number of a point p, denoted $C_S(p)$, is the number of rectangles in S that cover it. Give an $O(n \log n)$ algorithm to compute

$$\max_{p \in \mathbb{R}^2} C_S(p).$$

Note that this problem makes use of the usual convention in computational geometry, and allows algorithms can make use of real numbers: storing, comparing, doing arithmetic, etc. Of course the algorithm may not abuse this privilege.

(100/3 pts) 2. **Lasers and Furniture**

(a) There is an empty house with n lasers mounted in the walls, shining out from the walls. There are k pieces of furniture that are to be placed into the house, in precise locations specified by the architect. The moving company, which employs many computer science dropouts, has been hired to put the furniture into the house. The only choice the movers can make is what order to place the pieces into the house.

When a piece is placed into in its position in the house some of the lasers may be impinging on it. How many lasers hit it depends on which other pieces have been placed before it, because once a laser beam hits a piece of furniture it stops there. So a piece p may be hit by some number L_p if it is placed first into the house. If placed later it may be hit by fewer laser beams.

The movers are charged a penalty of \$$x$ when a piece of furniture that they place is struck by x laser beams. The order in which the pieces are placed can have a dramatic effect on the penalty. For example, suppose that there's one laser beam shining from left to right. And suppose there are k couches in line with the laser.

If the couches are placed in order from right to left, the total penalty will be k. If however they are placed in left to right order the penalty will be 1.

Since the movers dropped out of school, they have no idea how to compute an efficient ordering in which to place the k pieces of furniture, so they decide to generate a random permutation of the pieces and place them in that order. State and prove a good (e.g. nk is not good) upper bound on the expected penalty the movers will have to pay.

(b) Let's modify the above scenario. In this case the lasers are not mounted on the walls of the house, but rather are mounted on the pieces of furniture. Each piece has two lasers. When they place a piece the penalty is \$1 for each beam that hits the new piece, and \$1 for each of the new beams that hit another piece. Again, the k pieces are placed in a random order. As above, state and prove a good upper bound on the expected penalty incurred in this scenario.

(100/3 pts) 3. **Where to put the safe?**

An office building has one safe where valuables are kept. There are n rooms in the building numbered $1, 2, \ldots, n$. The distance between the rooms is defined by a metric $D_{i,j}$. There is a sequence of requests where an employee in some room needs to access the safe. We model the cost of this operation as the employee goes to the safe uses it, then returns to her room. So if the employee is in room i and the safe is in room j the cost of this operation is $2D_{i,j}$. Management is monitoring these activities, and has the option to move the safe from time to time to a different room. The cost of moving the safe from i to j is $mD_{i,j}$.

The requests are adversarially generated, and future requests are unknown by management. After each request management has the option of moving the safe from one place to another. Management's goal is to obtain a deterministic algorithm with low total cost. (The total cost includes the employee movement costs plus the costs incurred by moving the safe.) The criterion of any management algorithm is the competitiveness of the algorithm, as defined in class.

Actually, let's simplify the problem to the case of $n = 2$. There are two rooms, 1, and 2, and the distance between them is $1/2$, and the cost of moving the safe is p. In other words if the safe is in room 1 and the employee in room 1 uses it, the cost is 0. If the employee in room 2 uses it the cost is 1, and if management decides to move the safe to room 2, the cost is p. Note that if management decides to move the safe after it sees a request r, the employee must first fulfill the request, after which management is allowed to move the safe.

(a) What is the competitive factor of the following algorithm? Each time the employee in the room not holding the safe needs the safe, she uses the safe at a cost of 1, then management moves the safe into her room at a cost of p.

(b) Now give an algorithm with the best competitive factor you can. Prove your result. Partial credit will be given for algorithms that do not achieve the smallest possible competitive factor.

Hints: (1) Consider algorithms that count the number of cost-1 accesses, and when the count reaches some value, move the safe and reset the counter. (2) Potential functions are useful.