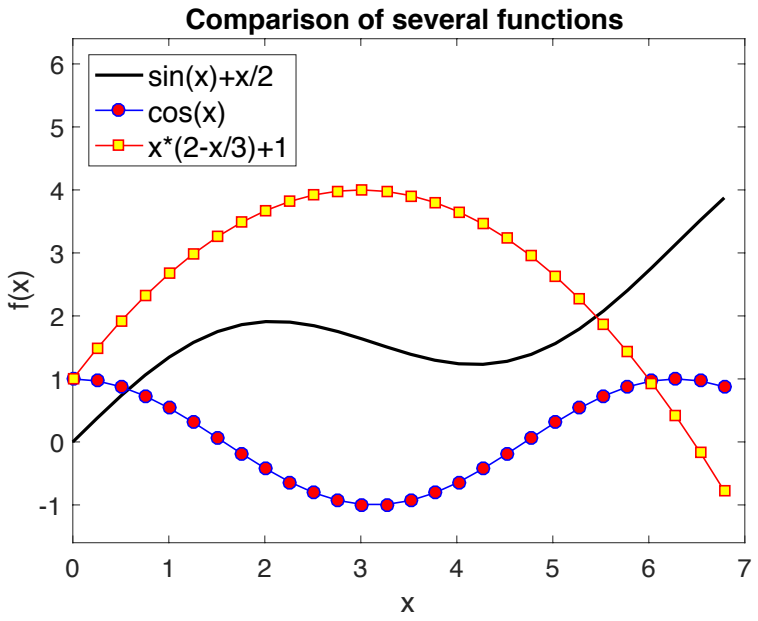
AM213B Assignment #0

Problem 1

In Matlab, learn how to plot multiple curves in one figure and how to add a legend to denote individual curves.

Write a Matlab code to produce a figure like the one below. Make sure that

- you use fonts of suitable size for title, labels, and legend, and
- you use lines of suitable line-width to plots curves.



Problem 2

Review the summation technique. Work out

- $\sum_{n=0}^{N-1} r^n = \frac{r^N - 1}{r - 1}$
- $\sum_{n=0}^{N-1} r^{n+m} = r^m \sum_{n=0}^{N-1} r^n = r^m \cdot \frac{r^N - 1}{r - 1}$

Problem 3

Review the technique of Taylor expansion.

Consider the second order numerical differentiation for approximating $f''(x)$

$$\underbrace{\frac{f(x+h)-2f(x)+f(x-h)}{h^2}}_{\text{Numerical approximation}} = \underbrace{f''(x)}_{\text{Exact}} + \underbrace{e(h)}_{\text{Discretization error}}$$

Use Taylor expansion to show that

$$e(h) = f^{(4)}(x)\frac{h^2}{12} + \cdots = C \cdot h^2 + \cdots = O(h^2)$$

Problem 4

Use the first and the second order methods to differentiate $f(x) = \sin(x)$ at $x = 1$. In this simple problem, the exact solution is known: $f'(x) = \cos(x)$.

The exact errors of the two methods are

$$er_1(h) = \left| \frac{f(x+h)-f(x)}{h} - f'(x) \right|, \qquad x = 1$$

$$er_2(h) = \left| \frac{f(x+h)-f(x-h)}{2h} - f'(x) \right|, \qquad x = 1$$

Calculate $\mathrm{er}_1(h)$ and $\mathrm{er}_2(h)$ for h=2.^(−[0:0.5:10]). Plot h vs $\mathrm{er}_1(h)$ and h vs $\mathrm{er}_2(h)$ in one figure. Use log-log plot to accommodate the wide ranges of h and errors. Add a legend. You should get a figure like the one below.