Optimization 2 (RM 294)

Project 2 – Dynamic Programming for Airline Flight Booking

Prepared For:

Dr. Daniel Mitchell
McCombs School of Business
The University of Texas at Austin
2110 Speedway Austin, TX 78705

Prepared by:

Names
Cole Brown
Vishwa Patel
Kimberly Simmonds
Albert Nguyen

Sunday, April 6, 2025

Table of Contents

1. Executive Summary

1.1 Introduction

Airline overbooking is a common practice designed to maximize revenue by compensating for no-show passengers. However, this strategy can backfire, leading to increased costs and dissatisfaction among passengers when flights are overbooked. This report addresses the optimization problem of determining the optimal overbooking and dynamic pricing policy for an airline flight, aimed at maximizing expected discounted profit.

The project uses dynamic programming to solve the airline ticket pricing problem, specifically incorporating the decision to overbook coach seats. Given the constraints and the associated costs, we will explore various policies to identify the optimal balance between maximizing revenue and minimizing the costs and negative impacts of overbooking.

1.2 Overview of Project Objective

This project explores how airlines can maximize profits by strategically overbooking coach seats. We integrated simulations and a dynamic programming approach to model passenger demand (low-fare coach, high-fare coach, low-fare first class, high-fare first class) along with the costs of upgrading or removing passengers when coach capacity is exceeded. Also, looking at various scenarios with different price gaps between low-fare and high-fare coach tickets—and adjusting the probabilities of a passenger selecting each fare class—we sought to identify the optimal overbooking policy and measure its effect on profitability.

1.3 Key Findings & Recommendations

- **Overbooking Drives Profit but Raises Costs**
  Selling more tickets than physical coach capacity (e.g., 20 seats above capacity) generally increases revenue by accommodating higher demand. However, when demand exceeds capacity, the airline incurs upgrade costs ($50 each) and removal costs ($425 each). Balancing these factors is crucial for net profitability.
- **Impact of Increasing the Coach Price Gap**
  When the high-fare coach price increases from $350 to $450—and the probability of buying the high-fare coach simultaneously decreases (e.g., from 30% to 20%)—the percentage of overbooked flights drops, leading to fewer forced removals and a lower overbooking penalty. Consequently, overall profit rises, although not dramatically, suggesting that a wider price gap can slightly reduce overbooking frequency while still improving average revenue.
- **Passenger Removal is Costly**
  Most simulations show that a substantial portion of overbooking costs stems from

removing passengers entirely (at $425 each). Even a moderate reduction in the rate of forced removals can significantly improve net profit.

- **Recommendation**
  - Maintain a deliberate overbooking strategy (such as +20 seats in coach) to capture additional revenue from otherwise uncertain demand.
  - If feasible, increase the price difference between low and high coach fares while improving or maintaining the probability of buying each type of ticket, as this reduces the likelihood of hitting capacity in coach. It also channels a portion of potential coach passengers into either low-fare or alternative fare classes, lowering the probability and severity of overbooking events.
  - Closely monitor the trade-off: a very high price difference might reduce the portion of travelers choosing high-fare coach too much, offsetting the benefits. Careful demand modeling is recommended.

1.4 Summary of Optimal Overbooking Policy & Profitability

- **Optimal Overbooking Level:**
  Simulations indicate that overbooking by 20 seats in coach consistently yields strong financial returns compared to lower or higher overbooking levels, balancing extra ticket sales against penalty costs for upgrades and removals.
- **Profitability Observations:**
  - Under a narrower price gap ($50 difference), over 80% of flights were overbooked, leading to higher average overbooking costs ($41,700).
  - With a wider gap ($150–$200 difference) and a reduced probability of buying the high-fare coach, the overbooking rate dropped substantially (by over 15–20 percentage points), and fewer passengers needed to be removed. This pushed profits notably higher (~$45,700 average) while simultaneously cutting overbooking costs by more than half.
- **Bottom Line:**
  The dynamic programming solution and simulations both confirm that a well-calibrated policy of overbooking around 20 coach seats—together with a strategic spread between coach fare options—can maximize expected revenue while minimizing costs.

Overall, these findings underscore the importance of **pricing strategy** in managing overbooking risk. By choosing a coach price gap that nudges demand distribution away from extreme overcapacity while still leveraging high-fare purchases, the airline improves net profits and reduces costly disruptions.

2. Problem Statement

As analysts in the pricing department of a major airline, our objective is to optimize both ticket pricing and the overbooking strategy for an upcoming flight with two ticket classes: coach and first-class. The primary goal is to maximize the expected discounted profit, which is calculated as revenue from ticket sales minus any costs incurred due to overbooking. We first look at some of the values of the key parameters.

Key Considerations

- **Seating Capacity**: 100 coach seats (which may be overbooked) and 20 first-class seats (no overbooking allowed).
- **Show-Up Probabilities**:
  - Coach passenger: 95%
  - First-class passenger: 97%
- **Ticket Pricing & Probabilities**:
  - **Coach**:
    - Low price = \$300, sale probability = 65%
    - High price = \$350, sale probability = 30%
  - **First-class**:
    - Low price = \$425, sale probability = 8%
    - High price = \$500, sale probability = 4%
  - If first-class is sold out, the probability of selling a coach ticket increases by 3 percentage points (e.g., from 65% to 68% or from 30% to 33%).
- **Overbooking Costs**:
  - \$50 per passenger upgraded from coach to first-class
  - \$425 per passenger bumped (i.e., unable to fly)
- **Discounting**: 17% annual discount rate (daily factor =1/(1+0.17/365)
- **Time Horizon**: 365 days to sell tickets prior to flight departure.

The problem is to find the best sequence of pricing and overbooking decisions over this 365-day window, taking into account random variations in passenger demand and the likelihood of no-shows. The solution must balance the benefit of selling more tickets to increase revenue against the risk and cost of overbooking, all while accounting for the time value of money.

3. Dynamic Programming Methodology

A dynamic programming (DP) approach is well suited for solving this airline ticketing and overbooking problem because it systematically evaluates decisions over time, captures the stochastic nature of demand and no-shows, and applies discounting appropriately. Below is an outline of the core principles and steps used in the DP framework:

1. **State Definition -** Define the state on day t by:

- Number of **coach seats sold**, which may exceed 100 up to a certain overbooking limit.
- Number of **first-class seats sold**, capped at 20.
- The **day index** t itself, ranging from 1 to 365.

Each state thus encapsulates how many tickets in each class have been sold so far and how many days remain until departure.

2. **Decision Variables -** On each day t, the airline chooses:

   p_c = Coach ticket price

   p_f = First class ticket price

- **Coach ticket price**: Either “low” ($300) or “high” ($350). In some models (in Model 2), there may also be a “no-sale” option for coach if it is optimal to pause ticket sales.
- **First-class ticket price**: Either “low” ($425) or “high” ($500).

These decisions directly affect the probability that a ticket will be sold for each class on day t.

3. **Dynamics -** Probability of selling seats in coach and first class:

   $prob_c$ = probability a coach ticket is sold (if not at capacity),

   $prob_f$ = probability a first-class ticket is sold (if not at capacity).

- **No coach sale, no first-class sale**

  Indicator: $s_c = 0$, $s_f = 0$

Transition probability: $(1 - prob_c) * (1 - prob_f)$

New state: (t+1, c, f)

- **No coach sale, first-class sale**

  Indicator: $s_c = 0$, $s_f = 1$

  Transition probability: $(1 - prob_c) * prob_f$

  New state: (t+1, c, f+1)

- **Coach sale, no first-class sale**

  Indicator: $s_c = 1$, $s_f = 0$

  Transition probability: $prob_c * (1 - prob_f)$

  New state: (t+1, c+1, f)

- **Coach sale, first-class sale**

  Indicator: $s_c = 1$, $s_f = 1$

  Transition probability: $prob_c * prob_f$

  New state: (t+1, c+1, f+1)

4. **Value Function -** No cost incurred today; overbooking costs incurred at the end of the booking period

   $c' = c + 1$(up to C_{max})

   $f' = f + 1$(up to 20)

   $V(t, c, f) = R(t, c, f, p_c, p_f) + \gamma * \sum P(s_c, s_f) * V(t+1, c', f')$

5. **Bellman Equation**

   $$V(t, c, f) = \max_{p_c \in \{300,350\} p_f \in \{425,500\}} [E[Revenue(p_c, p_f)] + \gamma * E[V(t+1, c', f')]]$$

   a. **Rewards today**

      $E[Revenue] = prob_c(p_c, f) * p_c + prob_f(p_f) * p_f$ (Coach and First-class revenue)

   b. **Expected future value**

      $E[V(t+1, c', f')] = \sum_{s_c \in \{0,1\}} \sum_{s_f \in \{0,1\}} P(s_c, s_f) * V(t+1, c', f')$

Discount factor: $\gamma = [1 / (1+(0.17/365))]$

6. **Terminal Condition** - In an overbooked scenario, no further revenue is earned, but costs of $50 per coach passenger moved to first class and $425 per passenger removed from the flight are incurred, meaning the terminal condition reflects these penalties instead of being zero.
   a. **Binomial Probabilities:**
   - Coach show up – probability k out of c coach passengers show up (95%)
   - First Class show up – probability m out of f first class passengers show up (97%)

   b. **Overbooking costs:**

      Cost(k,m) = 50*min(max(k-100, 0), 20–m)+425*min(max(k-100, 0)–(20–m), 0)
      - k – number of coach passengers that show up
      - m – number of first class passengers that show up
      - max(k-100, 0) – excess coach passengers
      - 20-m – available first class seats after accounting for first class passengers who show up

Simulations

With the expected values obtained from dynamic programming, it would be worthwhile to simulate scenarios and validate the accuracy of these optimal values. By the Law of Large Numbers, if enough simulations are run, then it should approach the expected value, which is the value obtained from the dynamic programming problem.

4. Model 1: Fixed Overbooking Policy

Implementation

Under a fixed overbooking policy, the airline decides in advance how many extra coach seats—between 5 and 15—are allowed to be sold beyond the physical capacity of 100. Each day, the airline chooses between two possible coach ticket prices ($300 or $350) and two first-class prices ($425 or $500). If both coach and first-class seats are available, the DP evaluates four scenarios (low coach/low first, low coach/high first, high coach/low first, high coach/high first). Once either coach or first-class is sold out, the probabilities adjust accordingly.

The main state variables are:

1. t: days remaining until departure (from 365 down to 0).
2. c: number of coach seats already sold (up to 100 + overbooking).

3. f: number of first-class seats already sold (up to 20).

On each day, we decide whether to charge a low or high price for both coach and first class. Once first class is sold out, the probability of selling a coach ticket increases by 3%. At departure (day 0), we apply the overbooking costs if more passengers show up than there are physical seats.

Ticket pricing involved two distinct prices per class, each linked to specific probabilities of daily ticket sales. Coach tickets had low ($300, 65% probability) and high ($350, 30% probability) price points, while first-class tickets were set at either $425 with an 8% probability of sale or $500 with a 4% probability. When first-class was fully booked, we adjusted coach ticket probabilities upward by 3%.

The model accounted for overbooking by incorporating costs associated with exceeding the seating capacity, specifically the $50 upgrade fee per passenger moved to first-class and the $425 cost for passengers bumped from the flight entirely.

```
# Function that runs the overbooking policy based on three potential coach scenarios and two first class scenarios
def overbooking_dp(self, overbooking, prob_mp, no_coach_policy, seasonality):
    '''
    Inputs:
        overbooking: The number of seats you wish to overbook coach by
        prob_mp: The probability multiplier (as a function of t)
        no_coach_policy: 0 doesn't have choice of setting coach demand to 0 on given day, 1 has the choice of forcing coach demand to 0
        seasonality: 0 for no seasonality, 1 for seasonality
    Outputs:
        V: Updated value array
        Uc: Updated optimal coach choices
        Uf: Updated optimal fc choices
    '''
```

Question 1: No Seasonality and No Forcing Coach Demand to 0

```
V, _, _ = overbooking_session.overbooking_dp(overbooking=5, prob_mp=probability_multiplier, no_coach_policy=0, seasonality=0)

print(f"Value when overbooking 5 seats with no seasonality and no option to force coach demand to 0: ${V[0, 0, 0]:.4f}")
```

```
Value when overbooking 5 seats with no seasonality and no option to force coach demand to 0: $41886.16
```

Results

No Seasonality, Overbooking = 5

Under an initial scenario of overbooking coach by 5 seats (i.e., selling up to 105 coach tickets), the expected discounted profit from the DP model was approximately **$41,886.16**.

Best Fixed Overbooking Level (6 to 15 seats)

By systematically allowing 6, 7, 8, …, 15 seats, we found that overbooking by **9** seats yielded the highest expected discounted profit (about **$42,134.62**). Profits initially increase as moderate overbooking captures extra demand but then level off or decline when excessive overbooking costs dominate.

Forward Simulations

- Forward-simulation results confirm that moderate overbooking can improve revenue, but also increase the frequency of paying upgrade/bumping fees.
- Seasonality (increasing demand closer to departure) shifts the optimal decisions slightly, but in some cases the gains in late-season demand can be offset by higher overbooking costs if the airline becomes too aggressive.

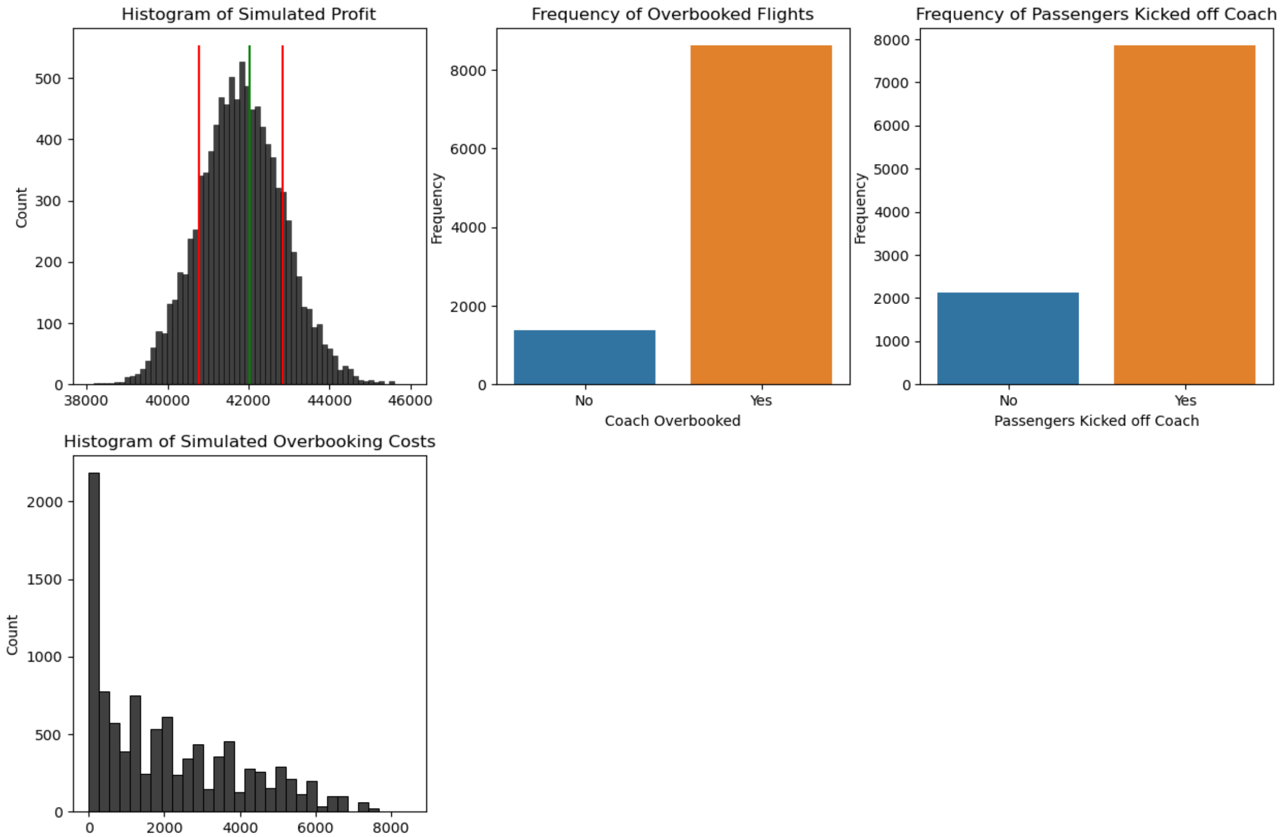
An illustrative simulation for **overbooking = 20 coach seats with no seasonality and no ability to force coach demand to 0** (to see more pronounced overbooking effects) without seasonality produced these statistics (10,000 simulations):

```
Overbooking 20 coach seats. No seasonality policy and no forcing coach seat demand to 0.
Price of Low Coach: 300. Price of High Coach: 350.
Price of Low First Class: 425. Price of High First Class: 500.
Probability of buying low coach: 65.0000%. Probability of buying high coach: 30.0000%
Probability of buying low first class: 8.0000%. Probability of buying high first class: 4.0000%
Running 10000 simulations...
----------------------------------------------------------------------------------------------------
Overbooked in 86.2600% of simulations.
Average Overbooking Cost: $2090.4975
Passengers kicked off coach in 78.6200% of simulations.
Average Profit: $41810.6908
Expected Profit from Dynamic Programming: $42028.7370
Standard Deviation of Profit: $1034.03
Skewness of Profit: 0.1116
Kurtosis of Profit: 0.0742
```

While the DP's expected profit for **overbooking=20** with no seasonality and no ability to force coach demand to 0 was **$42,028.737**, the average realized profit in simulation was closer to $41,810.69 due to random variation. Overbooking occurred in ~86.26% of runs, with ~78.62% of booking simulations requiring at least one passenger to be bumped. This highlights that while higher overbooking captures more sales, it also substantially raises bumping frequency and costs.

Visual Example

A histogram of the simulated profit distribution (for overbooking=20) shows a long central "peak" around $41,000–$43,000, which is within one standard deviation. There are some left-tail outcomes where overbooking costs significantly reduced profit. A bar chart of "overbooked" vs. "not overbooked" flights confirms high overbooking frequency in this more aggressive policy.



5. Model 2: Flexible Sale Policy with No Sale Option

Implementation
In the second model, we allowed three possible daily actions for coach tickets:

1. Sell coach at the **low price**.
2. Sell coach at the **high price**.
3. **Do not sell coach tickets** that day (i.e., force demand for coach to zero).

First-class is still priced daily at either $425 or $500. Additionally, the maximum number of coach tickets that can be sold increases to 120, thereby introducing more flexibility in managing potential demand spikes. By sometimes choosing "no-sale," the airline can proactively avoid overbooking too heavily when many days remain or when a surge in demand is likely. This flexible policy adapted dynamically based on how close the flight date was and the number of tickets already sold, optimizing ticket pricing to better balance revenue opportunities against potential overbooking costs.

Question 2: Best Overbooking Policy

```
overbooking_vals = list(range(6, 16))
vals = []
for overbooking in overbooking_vals:
    V, _, _ = overbooking_session.overbooking_dp(overbooking=overbooking, prob_mp=probability_multiplier, no_coach_policy=0, seasonality=0)
    vals.append(V[0,0,0])

print(f"Best overbooking policy: {overbooking_vals[np.argmax(vals)]} coach seats with a profit of ${np.max(vals):.4f}.")
```

```
Best overbooking policy: 9 coach seats with a profit of $42134.62.
```

Question 3: No Coach Policy

```
overbooking = 20

V, _, _ = overbooking_session.overbooking_dp(overbooking=overbooking, prob_mp=probability_multiplier, no_coach_policy=1, seasonality=0)
print(f'Overbooking Policy of {overbooking} coach seats with forcing coach demand to 0: ${V[0, 0, 0]:.4f}')
```

```
Overbooking Policy of 20 coach seats with forcing coach demand to 0: $42139.89
```

Results

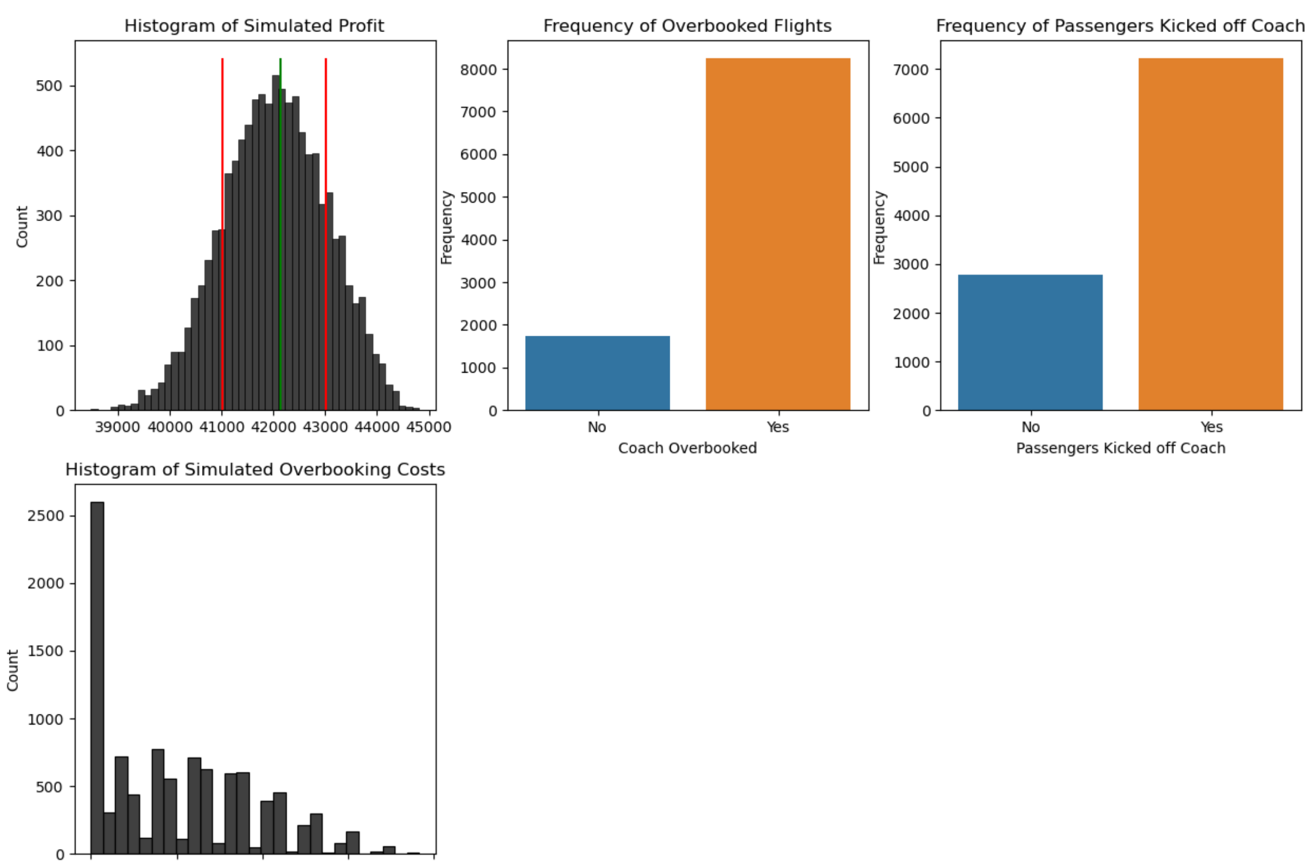
When we included the no-sale coach option (and tested it with an overbooking limit of 20 seats), the DP indicated an expected discounted profit of about **$42,139.89**, narrowly exceeding the best result ($42,134.62) from the fixed overbooking strategy with 9 seats. In fact, when checking all policies with overbooking ranges from 5-20 seats, it turned out that this was indeed the optimal policy. While the difference is relatively small, it highlights that dynamic control—especially late in the selling horizon—can help avoid extreme overbooking costs.

In forward simulations, this flexible approach tended to reduce the frequency of severe overbooking scenarios and thus modestly lowered overbooking cost volatility, suggesting a more stable financial performance under typical market conditions. Moreover, during periods of higher demand (e.g., under a seasonality adjustment), the model can revert to “no-sale” for coach on days when the probability of additional sales is high and an overbooking threshold is nearly reached. Seasonality adjustments emphasized the effectiveness of this policy. With demand increasing closer to departure, the flexible sales strategy allowed for better exploitation of rising ticket demand without unnecessarily increasing overbooking risks.

Overall, the flexible sales policy demonstrated benefits over the fixed overbooking approach, highlighting the value of adaptive decision-making in airline ticket pricing and capacity management.

Below is a simulation for **overbooking = 20 coach seats with no seasonality but with the ability to force coach demand to 0** (to see more pronounced overbooking effects) without seasonality produced these statistics (10,000 simulations):

```
Overbooking 20 coach seats. No seasonality policy and forcing coach seat demand to 0.
Price of Low Coach: 300. Price of High Coach: 350.
Price of Low First Class: 425. Price of High First Class: 500.
Probability of buying low coach: 65.0000%. Probability of buying high coach: 30.0000%
Probability of buying low first class: 8.0000%. Probability of buying high first class: 4.0000%
Running 10000 simulations...
--------------------------------------------------------------------------------
Overbooked in 82.5000% of simulations.
Average Overbooking Cost: $987.1625
Passengers kicked off coach in 72.2600% of simulations.
Average Profit: $42021.5599
Expected Profit from Dynamic Programming: $42139.8928
Standard Deviation of Profit: $996.93
Skewness of Profit: -0.1031
Kurtosis of Profit: -0.2887
```



6. Summary of Seasonality and Simulations

To account for seasonal effects, we modified the daily probability of purchase by the following function:

$$p_{\text{modified}} = p_{\text{original}} \times \left(0.75 + \frac{t}{730}\right)$$

In this function, t is the current day index ($0 \leq t < 365$). Generally, probabilities are lower in earlier days and rise closer to departure.

Question 4: Seasonality

```
overbooking = 20

V, _, _ = overbooking_session.overbooking_dp(overbooking=overbooking, prob_mp=probability_multiplier, no_coach_policy=1, seasonality=1)
print(f'Overbooking Policy of {overbooking} coach seats with seasonality and forcing coach demand to 0: ${V[0, 0, 0]:4f}')
print('---------------------------------------------------------------------------------------------------')

V, _, _ = overbooking_session.overbooking_dp(overbooking=overbooking, prob_mp=probability_multiplier, no_coach_policy=0, seasonality=1)
print(f'Overbooking Policy of {overbooking} coach seats with seasonality and not forcing coach demand to 0: ${V[0, 0, 0]:4f}')
```

```
Overbooking Policy of 20 coach seats with seasonality and forcing coach demand to 0: $41826.44687221107
---------------------------------------------------------------------------------------------------
Overbooking Policy of 20 coach seats with seasonality and not forcing coach demand to 0: $41718.65026398371
```

In many simulations, the seasonality factor increases overall demand later in the year. However, that rise in demand can push the airline into more frequent overbooking, so the net effect on profit can be mixed. For instance, one test with **overbooking=20 + seasonality** reported a DP profit around **$41,826.44** (forcing no coach) vs. **$41,718.65** (regular policy)—slightly below the no-seasonality best results.

The choice to include an option to force coach demand to 0 or not for a given number of coach seats overbooked did produce an interesting pattern in the data. Referring back to the program's output, for example, when overbooking 6-9 coach seats, it did not matter whether there was the option to force coach demand to 0 or not – the expected profit was exactly the same. This implied forcing demand to 0 was never the most profitable option in any state. However, if there was an option to force coach demand to 0, suddenly, the profit continued to increase until about 15 overbooked coach seats, which is when the profit appeared to plateau. The same cannot be said about the profit when there is no option to force demand to 0 – the profit appears to reach a maximum when the flight allows an additional 9 coach seats to be booked and the profit begins to decrease. This is likely due to overbooking no longer being profitable, with costs being too great and heavily offsetting the revenue made. Without an option to force demand to 0, most flights become fully overbooked and many people end up tossed from coach, as first class usually does not have enough room to accommodate the extra coach passengers. Adding in the option to force coach demand to 0 allows the airline to incur lower costs, and this visibly adds more benefit, as the profit increases as more seats are allowed to be overbooked in coach.

We also performed multiple forward simulations to assess how often overbooking actually happens, how frequently passengers are kicked off, and how sensitive profits are to ticket price changes or to the probability of first-class purchases.

1. **Overbooking Frequency**: Depending on the chosen overbooking limit, anywhere from ~50% to ~90% of simulated flights became overbooked (i.e., more total coach ticket holders show up than the coach cabin can seat).
2. **Bumping Costs**: With aggressive overbooking (e.g., allowing 20 extra coach seats), we observed bumping costs in the $2,000–$4,000 range on average, with around 75% to 90% of flights requiring at least one bump or coach-to-first-class upgrade.
3. **Average Profit & Volatility**:
    - Under moderate overbooking (e.g., +9 seats), average simulated profit clustered around $42,000 with a relatively moderate standard deviation (under $1,000).
    - More extreme policies (e.g., +20 seats) yield a slightly higher mean DP profit but also significantly higher volatility (standard deviation of $1,000–$1,500 or more) in the simulation outcomes.
4. **Seasonality**: In many runs, the rising demand near departure can boost profits if the airline manages capacity well. However, it can also magnify overbooking costs if the airline oversells too aggressively.

7. Comparison of Models 1 and 2 and Key Takeaways

---

1. **Fixed Overbooking vs. Flexible "No-Sale"**
    - The **best fixed overbooking** (allowing +9 coach seats) achieved about $42,134.62 in the DP model (no seasonality).
    - The **flexible policy** with a no-sale coach option can push the expected profit slightly higher, up to around $42,139.89, largely by letting the airline react dynamically to remaining days and seats sold. In practice, the improvement is small but does reduce risk and bumping frequency.

2. **Seasonality Considerations**
   - Seasonality (gradually higher demand closer to departure) can improve late-ticket revenues, but it also elevates the risk of exceeding capacity when selling continues late into the year.
   - Careful daily decisions—especially the "no-sale" option—help mitigate bumps.
3. **Risk vs. Reward**
   - More aggressive overbooking (15–20 extra seats) often increases expected revenue but also amplifies volatility and bumping penalties.
   - Moderate overbooking (5–10 seats) may be a better risk-reward balance in many scenarios, though the final choice depends on corporate tolerance for bumping costs and potential passenger dissatisfaction.
4. **Sensitivity to Pricing Gaps and First-Class Demand**
   - We also tested scenarios where the high coach fare was increased from $350 to $400, $450, or $500. Larger price gaps raised DP profits substantially (e.g., up to $57,000 in some tests) but also tended to increase bumping frequency since more passengers were willing to buy the cheaper fare.
   - Similarly, raising the probability of selling first-class tickets from 8% up to 13% or more modestly improved average profits (from about $42,000 to $43,000+). However, it also slightly increased the fraction of overbooked flights, as first-class seats sold out more frequently, bumping some coach demand up by 3 percentage points.

Below is a simulation for **overbooking = 20 coach seats with seasonality and the ability to force coach demand to 0** (to see more pronounced overbooking effects) without seasonality produced these statistics (10,000 simulations):

Question 5: Simulation

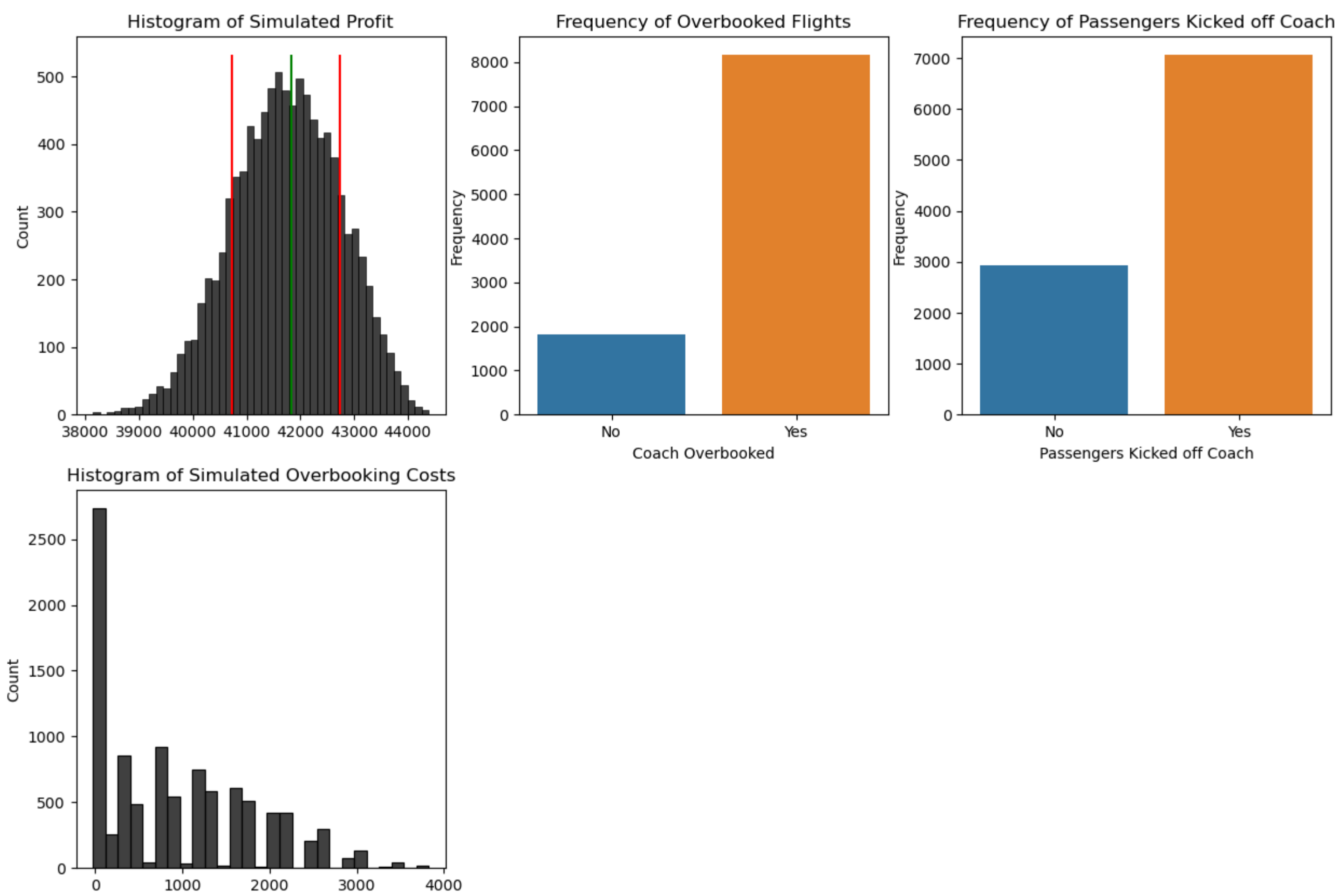
```
# Changable parameters
nsims = 10000
ob = 20
seasonality = 1
no_coach_policy = 1
print_graphs = 1

Uc, Uf = overbooking_session.run_simulations(nsims, ob, seasonality, no_coach_policy, probability_multiplier, print_graphs)
```

Overbooking 20 coach seats. Seasonality policy and forcing coach seat demand to 0.
Price of Low Coach: 300. Price of High Coach: 350.
Price of Low First Class: 425. Price of High First Class: 500.
Probability of buying low coach: 65.0000%. Probability of buying high coach: 30.0000%

Probability of buying low first class: 8.0000%. Probability of buying high first class: 4.0000%
Running 10000 simulations...

Overbooked in 81.6800% of simulations.
Average Overbooking Cost: $935.1325
Passengers kicked off coach in 70.6800% of simulations.
Average Profit: $41731.6757
Expected Profit from Dynamic Programming: $41826.4469
Standard Deviation of Profit: $1007.11
Skewness of Profit: -0.1580
Kurtosis of Profit: -0.2848



Overbooking Program

To determine the optimal expected profit alongside being able to approximate what the average profit could look like over simulations, it is necessary to have a program that can efficiently do this.

The code that we use to model simulations and solve the dynamic programming problem is within the class object called **`OverbookingProblem`**. The class will take in several important parameters and initial its attributes so that the variables can be used in functions that deal with dynamic programming and simulations.

To perform the dynamic programming problem, the main method is **`overbooking_dp(self, overbooking, prob_mp, no_coach_policy, seasonality)`**

The parameters are the number of coach seats you allow to be overbooked, the probability multiplier question for seasonality, whether there is an option to force coach demand to 0 on a given day and if there is seasonality involved.

Model 1, which dealt with no seasonality and no forcing coach demand to 0, would be calculated by setting **`overbooking_dp(self, overbooking, prob_mp, 0, 0).`**

Model 2, which dealt with no seasonality but an option to force coach demand to 0, would be calculated by setting **`overbooking_dp(self, overbooking, prob_mp, 1, 0).`**

Finally, if seasonality were required, simply set **`seasonality=1`**.

For simulations, the method used is **`run_simulations(self, nsims, overbooking, seasonality, no_coach_policy, probability_multiplier, print_graphs).`**

The parameters are the number of simulations you wish to run, the four parameters needed to run the dynamic programming problem, and an option to print graphs onto the screen.

8. Additional Observations

Question 1
Throughout the problem, coach has usually been consistently overbooked in most of the simulations. Thus, it would be interesting to know what would happen if the price difference between the low and high coach ticket were to change while the probability of buying a high coach ticket were to simultaneously decrease.

Additional Question 1:

What happens if we adjust the price difference in coach between low and high?

```
# Changable parameters
nsims = 2000
ob = 20
seasonality = 0
no_coach_policy = 0
print_graphs = 0
priceL_coach = 300
priceH_coach = [priceL_coach + 50*i for i in range(1, 5)]

for priceH in priceH_coach:
    overbooking_session.modify_parameters(priceH_coach=priceH) # First modify

    # Run simulations
    overbooking_session.run_simulations(nsims, ob, seasonality, no_coach_policy, probability_multiplier, print_graphs)

# Reset back
overbooking_session.reset_to_original(params)
```

```
======================================================================
Overbooking 20 coach seats. No seasonality policy and no forcing coach seat demand to 0.
Price of Low Coach: 300. Price of High Coach: 350.
Price of Low First Class: 425. Price of High First Class: 500.
Probability of buying low coach: 65.0000%. Probability of buying high coach: 30.0000%
Probability of buying low first class: 8.0000%. Probability of buying high first class: 4.0000%
Running 2000 simulations...
----------------------------------------------------------------------------------------------------
Overbooked in 87.5500% of simulations.
Average Overbooking Cost: $2100.7000
Passengers kicked off coach in 79.4500% of simulations.
Average Profit: $41762.4034
Expected Profit from Dynamic Programming: $42028.7370
Standard Deviation of Profit: $1002.70
Skewness of Profit: 0.1380
Kurtosis of Profit: 0.1493
=====================================================================
======================================================================
Overbooking 20 coach seats. No seasonality policy and no forcing coach seat demand to 0.
Price of Low Coach: 300. Price of High Coach: 450.
Price of Low First Class: 425. Price of High First Class: 500.
Probability of buying low coach: 65.0000%. Probability of buying high coach: 20.0000%
Probability of buying low first class: 8.0000%. Probability of buying high first class: 4.0000%
Running 2000 simulations...
----------------------------------------------------------------------------------------------------
Overbooked in 67.9500% of simulations.
```

Average Overbooking Cost: $938.9250
Passengers kicked off coach in 59.5000% of simulations.
Average Profit: $45675.9438
Expected Profit from Dynamic Programming: $45756.1261
Standard Deviation of Profit: $1454.45
Skewness of Profit: -0.0954
Kurtosis of Profit: 0.0371
==

Looking at what happens if the price difference goes from 50 to 200, while the probability of buying high coach decreases from 30% to 20%, the number of flights that were overbooked drops and thus, less people are being kicked off coach. Since buying a high ticket for coach is more lucrative when it is 200 dollars more expensive than a low coach, it makes sense there are more vacant seats in first class than normal. Profit also increases, but not as drastically.

Question 2

If the probability for selling a first class ticket for either low or coach were to increase, how would that affect the problem?

Additional Question 2:

What happens if the probability of selling a first class ticket is changed?

```
# Changable parameters
nsims = 2000
ob = 20
seasonality = 0
no_coach_policy = 0
print_graphs = 0
fc_probL, fc_probH = overbooking_session.pL_first, overbooking_session.pH_first

prob_mods = [0.03, 0.05, 0.07, 0.09]

for prob in prob_mods:

    overbooking_session.modify_parameters(pL_first=[fc_probL[0]-prob, fc_probL[1]+prob],
                                          pH_first=[fc_probH[0]-prob, fc_probH[1]+prob]) # First modify

    # Run simulations
    overbooking_session.run_simulations(nsims, ob, seasonality, no_coach_policy, probability_multiplier, print_graphs)

# Reset back
overbooking_session.reset_to_original(params)
```

==

Overbooking 20 coach seats. No seasonality policy and no forcing coach seat demand to 0.
Price of Low Coach: 300. Price of High Coach: 350.
Price of Low First Class: 425. Price of High First Class: 500.
Probability of buying low coach: 65.0000%. Probability of buying high coach: 30.0000%
Probability of buying low first class: 11.0000%. Probability of buying high first class: 7.0000%
Running 2000 simulations...

Overbooked in 88.9000% of simulations.

Average Overbooking Cost: $2727.0625
Passengers kicked off coach in 83.3500% of simulations.
Average Profit: $42541.5855
Expected Profit from Dynamic Programming: $42978.6067
Standard Deviation of Profit: $991.26
Skewness of Profit: 0.0889
Kurtosis of Profit: 0.0072

==

===

Overbooking 20 coach seats. No seasonality policy and no forcing coach seat demand to 0.
Price of Low Coach: 300. Price of High Coach: 350.
Price of Low First Class: 425. Price of High First Class: 500.
Probability of buying low coach: 65.0000%. Probability of buying high coach: 30.0000%
Probability of buying low first class: 17.0000%. Probability of buying high first class: 13.0000%
Running 2000 simulations...

Overbooked in 92.1500% of simulations.
Average Overbooking Cost: $3604.0750
Passengers kicked off coach in 90.3000% of simulations.
Average Profit: $42877.5428
Expected Profit from Dynamic Programming: $43396.5559
Standard Deviation of Profit: $1033.06
Skewness of Profit: 0.1014
Kurtosis of Profit: -0.0679

===

Interestingly, more flights are overbooked while passengers are being kicked off coach more often. More people having to be kicked off coach makes sense as first class is almost always full, meaning that when coach is overbooked, the excess passengers must be kicked off. However, it is interesting that overbooking also increases. This happens since first class is quickly filled up, meaning coach is the only type of seat available, and with increased probability of selling coach, it makes sense that coach gets overbooked much more as first class ticket selling probability rises.

Behavior of Simulations

The Central Limit Theorem is not in play here since each simulation is just the actual profit being made after a full year has passed and all costs are priced in. The values being plotted are not the sample means being drawn from the full population of simulated profits. However, it can be seen that the resulting distribution of profits still mimics a normal distribution very closely. This could be due to how the profit is made by determining whether a ticket is sold or not, which is a binary choice. Over each day, this choice is repeated, thus making the profits actually fall under a binomial distribution. With the binomial distribution being well approximated by the normal, it may make sense as to why the profits look normally distributed without having to apply the Central Limit Theorem.

9. Conclusion

The dynamic programming analysis and subsequent simulations highlight the trade-offs between additional revenue from selling extra seats and the costs (both financial and reputational) associated with overbooking. In general:

- **Fixed Overbooking** strategies of around 5–10 extra coach seats deliver strong expected profits without incurring excessively high bumping costs.
- **Flexible "No-Sale"** policies can further refine these decisions, modestly boosting expected profits and reducing the frequency of severe overbookings, especially under seasonality.
- **Aggressive Overbooking** (e.g., +20 seats) can yield the highest theoretical DP profit but does so at the cost of large bumping fees and high volatility of outcomes.

From an airline management perspective, a moderately overbooked coach cabin combined with a dynamic, flexible approach to daily pricing and sales shutdown appears to be the most balanced strategy. Seasonality amplifies the importance of responding dynamically to how many seats have been sold and how much time remains—suggesting that advanced revenue management systems are essential to achieving the best trade-off between overbooking profits and passenger satisfaction.