

*A hundred times every day I remind myself that my inner and outer life are based on the labors of other men, living and dead, and that I must exert myself in order to give in the same measure as I have received and am still receiving...*



Kumaraguru Sivasankaran

MS in Aeronautics and Astronautics Engineering  
Candidate for Controls Engineer

K I T T Y H A W K



**PURDUE**  
UNIVERSITY



# Journey so far..



# Grounding work

## Modern Control theory

- Linear systems
- Non linear Systems
- Distributed Network Control
- Optimal Control and Estimation
- Linear Algebra

## Computational Science

- Optimization
- Computer Vision
- Parallel Computing
- Machine learning
- Deep learning

## Applied Statistics

- Statistical methods
- Probability
- Design of Experiments
- Regression Analysis
- Computational Statistics I

### Control design:

- PID controller
- LQR controller
- Model predictive Controller
- Sliding mode controllers
- High Gain observers
- Hybrid system control

### Trajectory Optimization:

- Dynamic Programming
- Calculus of Variations

### Sensor fusion / Estimation:

- Kalman filtering, EKF, UKF
- Least Squares Minimization
- Levenberg Marquardt optimization
- Sampling methods
- Monte Carlo simulations
- 3D reconstruction from 2D images
- Object detection and localization
- Camera calibration

### System modeling:

- Physics based modeling
- Markov Decision Process
- Partially Observable Markov Decision Process
- Deep Neural Networks
- Gaussian Process models



# My mentors @Purdue



*With Dr. Martin Corless, Professor, Dynamics and Controls,  
Aeronautics and Astronautics Engineering  
Work: Consensus algorithms*



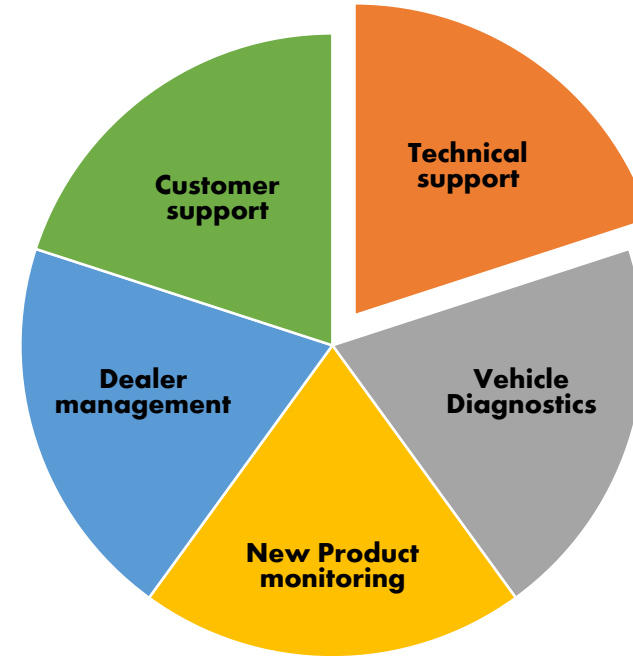
*With Dr. Xiao Wang, Professor, Dept. of Statistics  
Work: Reinforcement learning*



*With Dr. Jan E Mansson, Distinguished Professor of Materials and Chemical  
Engineering and AAE (by Courtesy)  
Work: Technical Cost Modeling*

# Life at Daimler (2014-16)

- Experienced with the lifecycle of a truck from production to full operation and finally to resale / scrap.
- Familiar with Maintenance cycle, Product failures, Failure analysis and Total Cost of Ownership calculations (TCO).
- Offered technical support when a trained technician was unable to troubleshoot a vehicle off road.
- Ensured customer satisfaction in terms of overall Aftersales service support to West Gujarat, India
- Collaborated with people at different levels
  - Technician, Dealer manager, Dealer Principal, Customer
  - Regional manager, Technical services, Warranty, PMG, VP
- Led field trials for establishing performance and responsible for New Product Monitoring
- Appreciated for quick learning, taking up responsibility and delivering with consistently high KPI in the region



**Aftersales Service Manager**





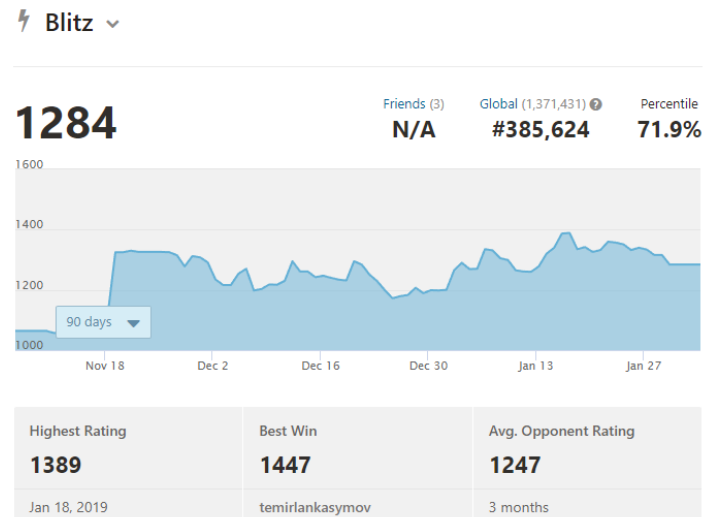
# Coop at Volvo (2017)

- Verification and Validation of control systems for Production Vehicle Evaluation (PVE)
- Over 500 miles of road testing per week
- Involved Data collection, Analysis, Failure reporting and documentation
- More than 50 tests performed in a span of 4 months (Aug- Dec)
- Familiar with use of functional documentation for evaluation and troubleshooting





# Hobbies



And more..

# Decentralized controller for Multiagent systems – A motivation



Wanis Kabbaj: What a driverless world could look like



Where do we stand now?



# Safety barrier Certificate: A review

Non-linear system:  $\dot{x} = f(x) + g(x)u$

Control invariance set:  $C = \{x \in R^n : h(x) \geq 0\}$

$$\partial C = \{x \in R^n : h(x) = 0\}$$

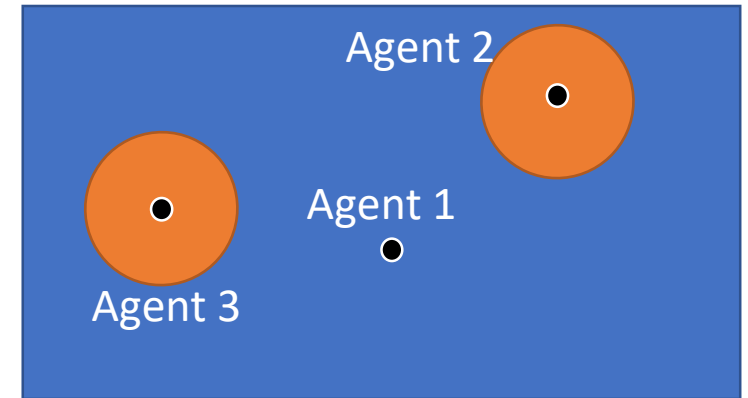
$$\text{Int}(C) = \{x \in R^n : h(x) > 0\}$$

Barrier function:  $\inf_{x \in \text{Int}(C)} B(x) \geq 0 \quad \lim_{x \rightarrow \partial C} B(x) = \infty$

Barrier function dynamics:  $\dot{B} \leq \frac{\gamma}{B}$  Stronger condition:  $\dot{B} \leq 0$

Control Barrier function:  $\inf_{u \in U} \left[ L_f B(x) + L_g B(x)u - \frac{\gamma}{B(x)} \right] \leq 0$

$$K_{cbf}(x) = \{u \in U : L_f B(x) + L_g B(x)u - \frac{\gamma}{B(x)} \leq 0\}$$



# Review (contd.)

Solution: controller based on Quadratic programming:

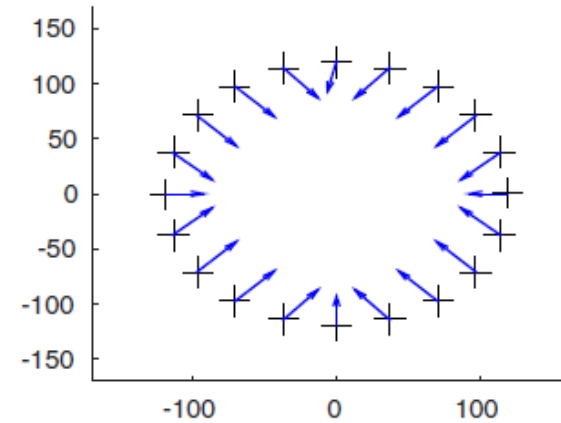
$$u^* = \operatorname{argmin}_u J(u) = \sum_i ||u_i - \hat{u}_i||^2$$

$$s.t. \quad A_{ij}u \leq b_{ij}, \quad \forall i \neq j$$

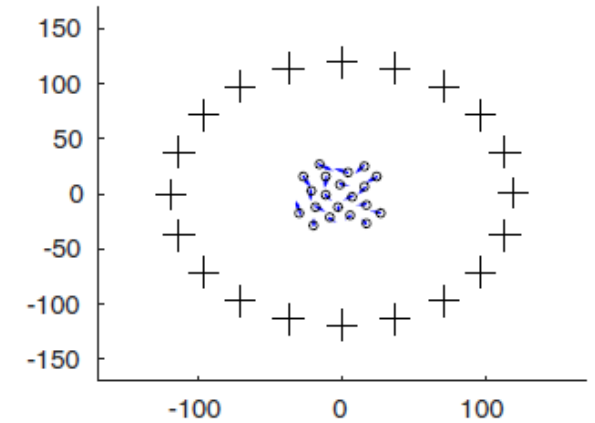
$$||u_i||_{\infty} \leq a_{max}, \quad \forall i \in M$$

Problems with this approach:

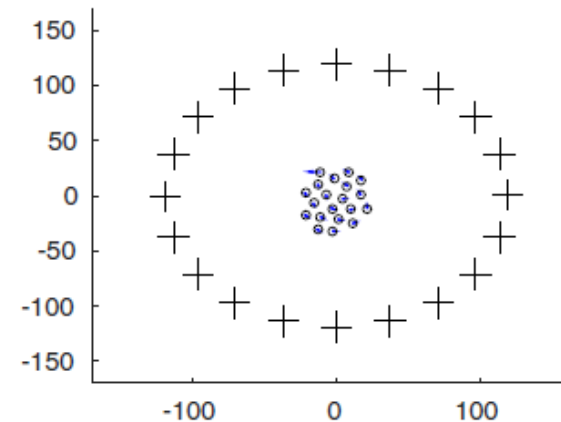
- Sluggish overall
- Conservative behavior in decentralized case
- Deadlocks due to infeasible optimization solutions



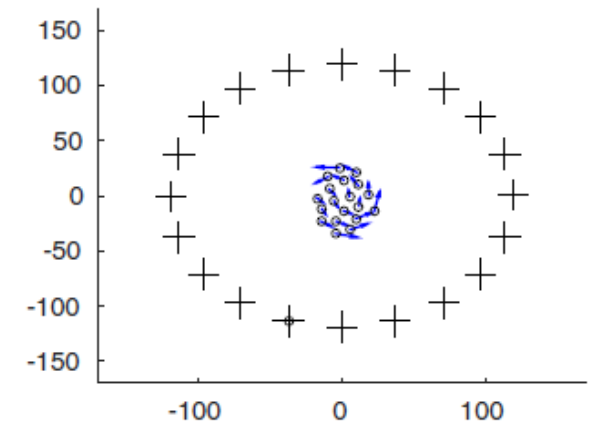
(a) Time step 3



(b) Time step 200



(c) Time step 600



(d) Time step 850



# Problem setup and approach

Double integrator dynamics:

$$\begin{bmatrix} \dot{p}_i \\ \dot{v}_i \end{bmatrix} = \begin{bmatrix} 0 & I_{2 \times 2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_i \\ v_i \end{bmatrix} + \begin{bmatrix} 0 \\ I_{2 \times 2} \end{bmatrix} u_i$$

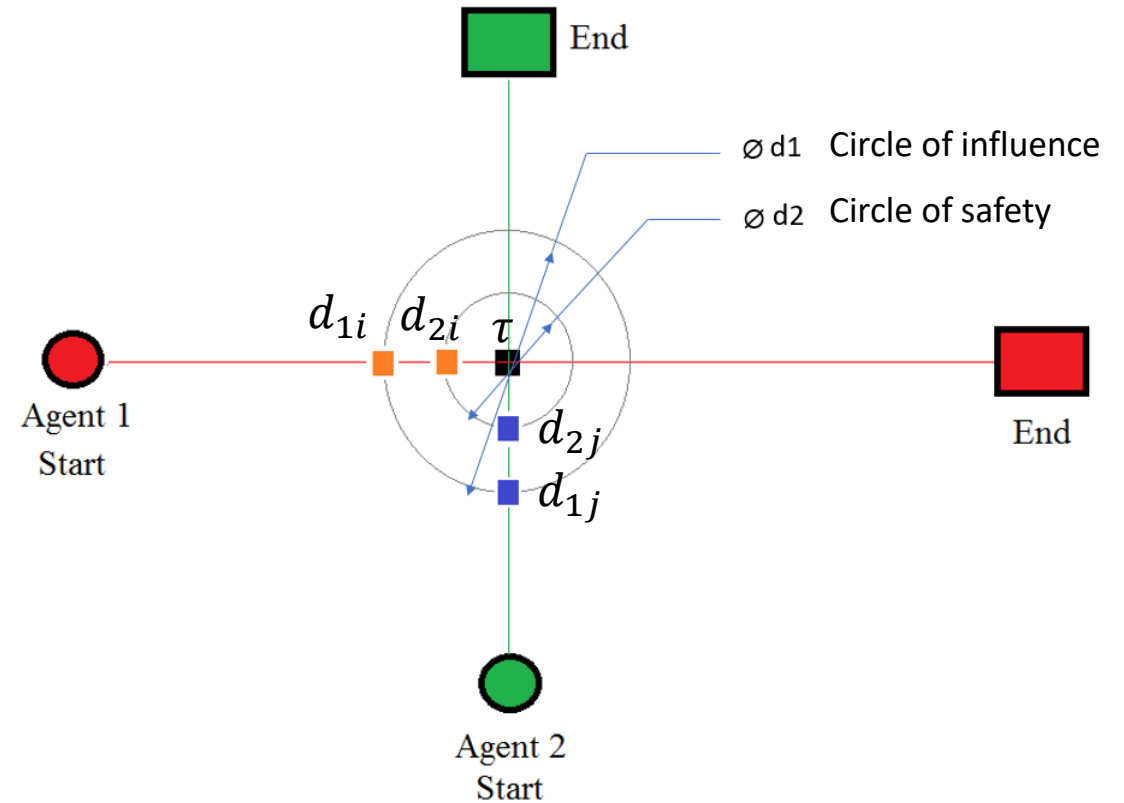
$p_i \in R^2, v_i \in R^2, u_i \in R^2$  are position, velocity and acceleration of agent  $i$

Reference point (final):

$$r_i = \begin{bmatrix} r_{ix} \\ r_{iy} \end{bmatrix}$$

Nominal controller to achieve end goal with zero velocity:

$$u_i = -k_1(p_i - r_i) - k_2 v_i$$



# Target point control

Position vectors:

$$A_i = \begin{bmatrix} p_{xi} \\ p_{yi} \end{bmatrix} + \mu \begin{bmatrix} p_{xi} - r_{xi} \\ p_{yi} - r_{yi} \end{bmatrix} \quad A_j = \begin{bmatrix} p_{xj} \\ p_{yj} \end{bmatrix} + t \begin{bmatrix} p_{xj} - r_{xj} \\ p_{yj} - r_{yj} \end{bmatrix}$$

At point of intersection:

$$A_i = A_j$$

Solve for intersection point:

$$\begin{bmatrix} \mu \\ t \end{bmatrix} = \begin{bmatrix} p_{xi} - r_{xi} & r_{xj} - p_{xj} \\ p_{yi} - r_{yi} & r_{yj} - p_{yj} \end{bmatrix}^{-1} \begin{bmatrix} p_{xj} - p_{xi} \\ p_{yj} - p_{yi} \end{bmatrix}$$

$$\tau = \begin{bmatrix} p_{xi} \\ p_{yi} \end{bmatrix} + \mu^* \begin{bmatrix} p_{xi} - r_{xi} \\ p_{yi} - r_{yi} \end{bmatrix} = \begin{bmatrix} p_{xj} \\ p_{yj} \end{bmatrix} + t^* \begin{bmatrix} p_{xj} - r_{xj} \\ p_{yj} - r_{yj} \end{bmatrix}$$

Target Points:

$$d_{1i} = p_i - \frac{||\tau - p_i|| - d_1}{||\tau - p_i||} (\tau - p_i) \quad d_{2i} = p_i - \frac{||\tau - p_i|| - d_2}{||\tau - p_i||} (\tau - p_i)$$

$$d_{1j} = p_j - \frac{||\tau - p_j|| - d_1}{||\tau - p_j||} (\tau - p_j) \quad d_{2j} = p_j - \frac{||\tau - p_j|| - d_2}{||\tau - p_j||} (\tau - p_j)$$



# Hybrid distributed control design

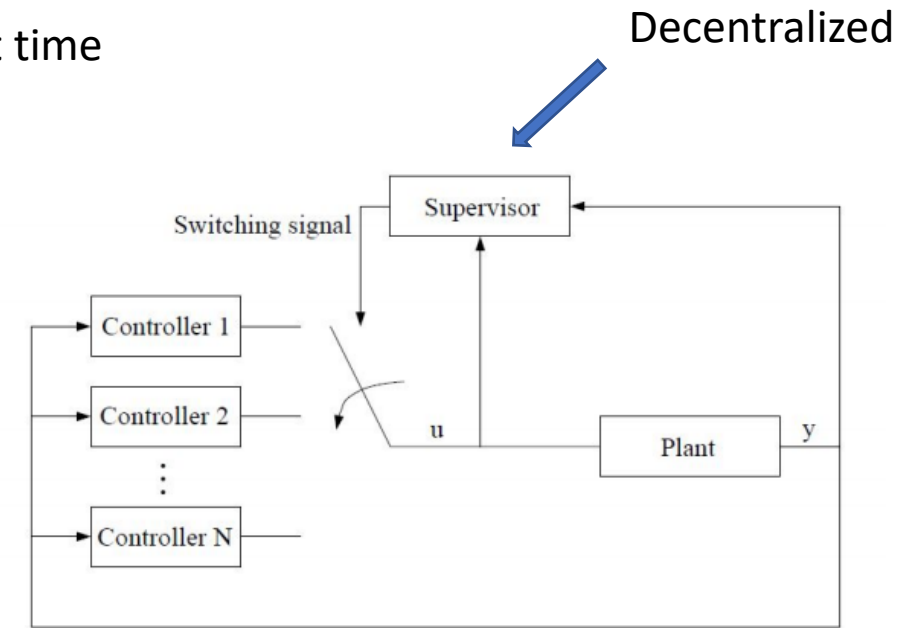
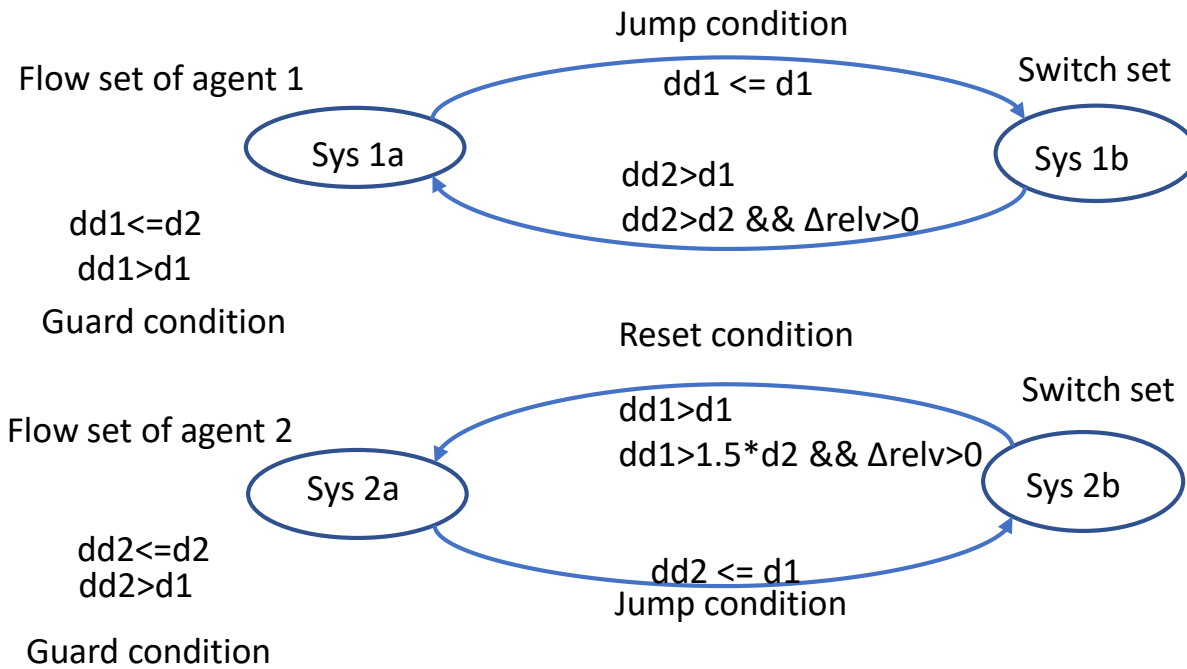
Flow set : Continuous ODE dynamic system where agent spends most time

Switch set: Alternate system dynamics

Guard condition: Keeps system in flow set

Jump condition: Discrete transitions from flow set to switch set

Reset condition: Returns to flow set



Nominal

$$u_i = -k_1(p_i - r_i) - k_2 v_i$$

Switched

$$u_i = -k_1(p_i - d_{2i}) - k_2 v_i$$

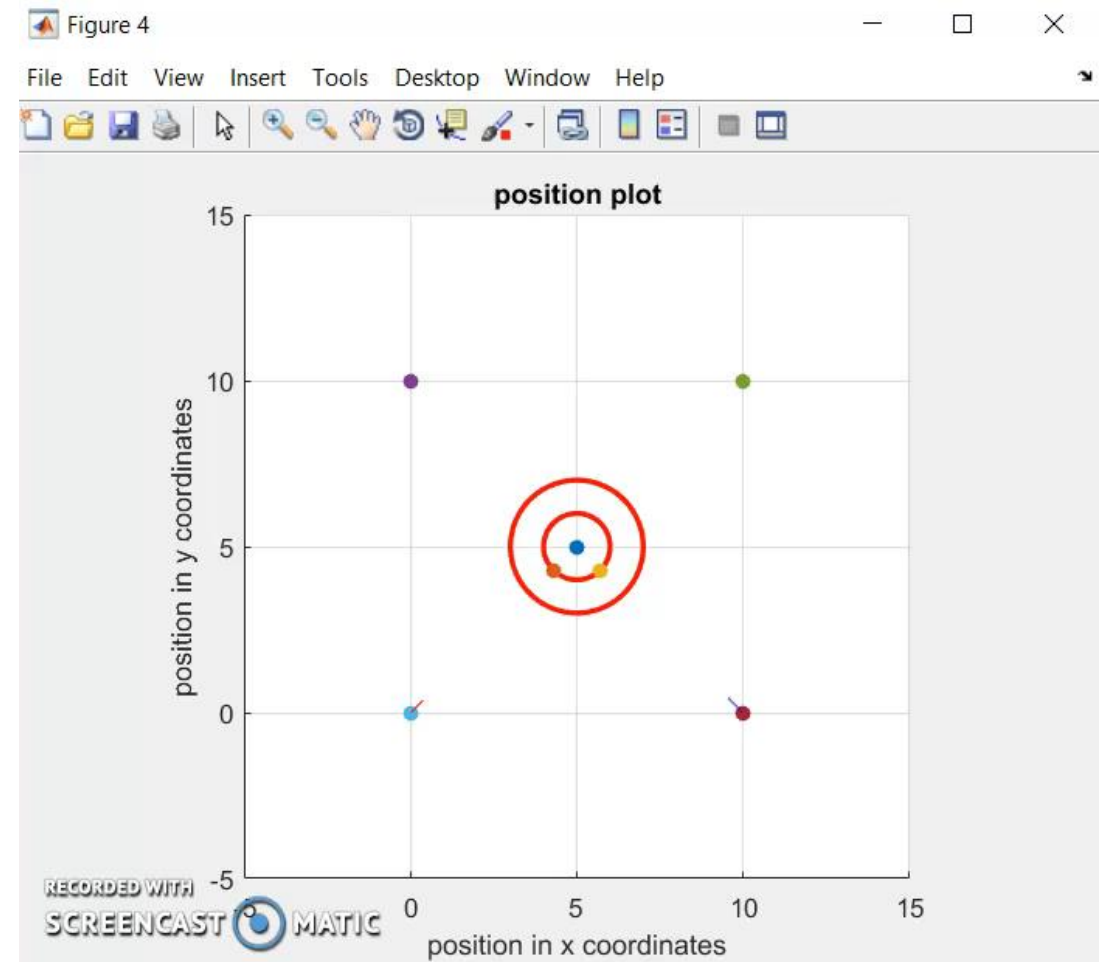
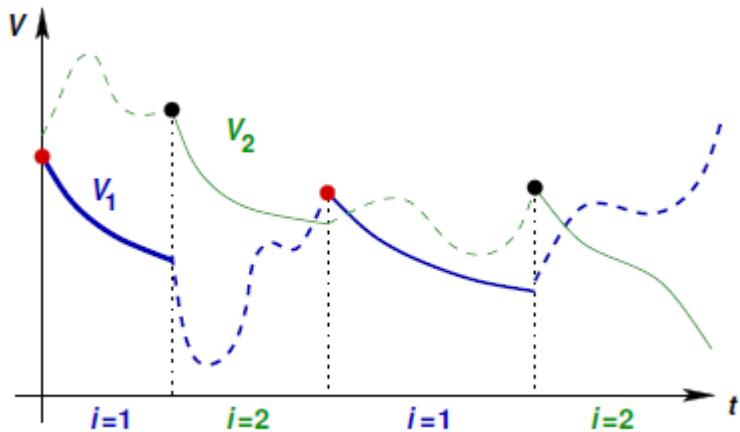
# Analysis and Results

Theorem:

Consider switched system with all sub models  $\dot{x} = f_i(x)$  as Globally asymptotically stable with corresponding Lyapunov function  $V_i$ .

Suppose for every pair of switching times  $(t_k, t_l), t < l$  with  $\sigma(t_k) = \sigma(t_l) = i$  and  $\sigma(t_m) \neq i$  for  $t_k < t_m < t_l$ , we have

$V_i(x(t_l)) - V_i(x(t_k)) \leq -\rho(||x(t_k)||) < 0$ , then the switched system is GAS

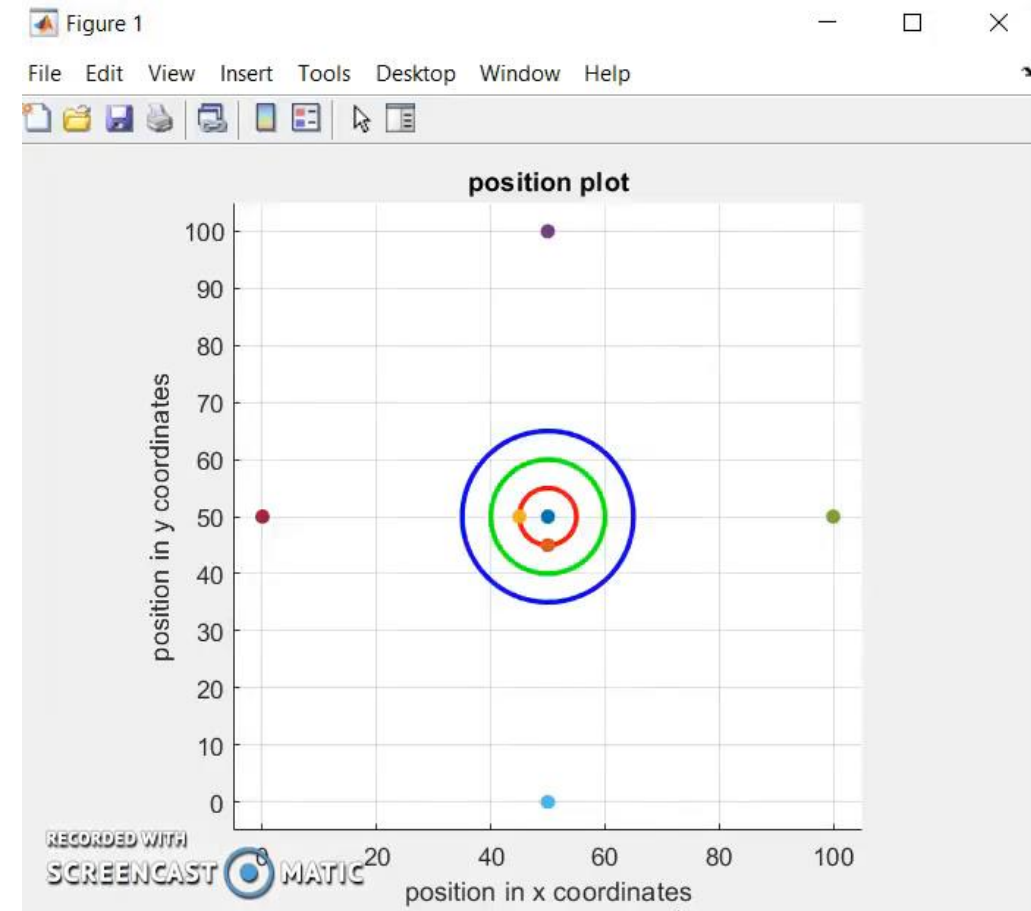
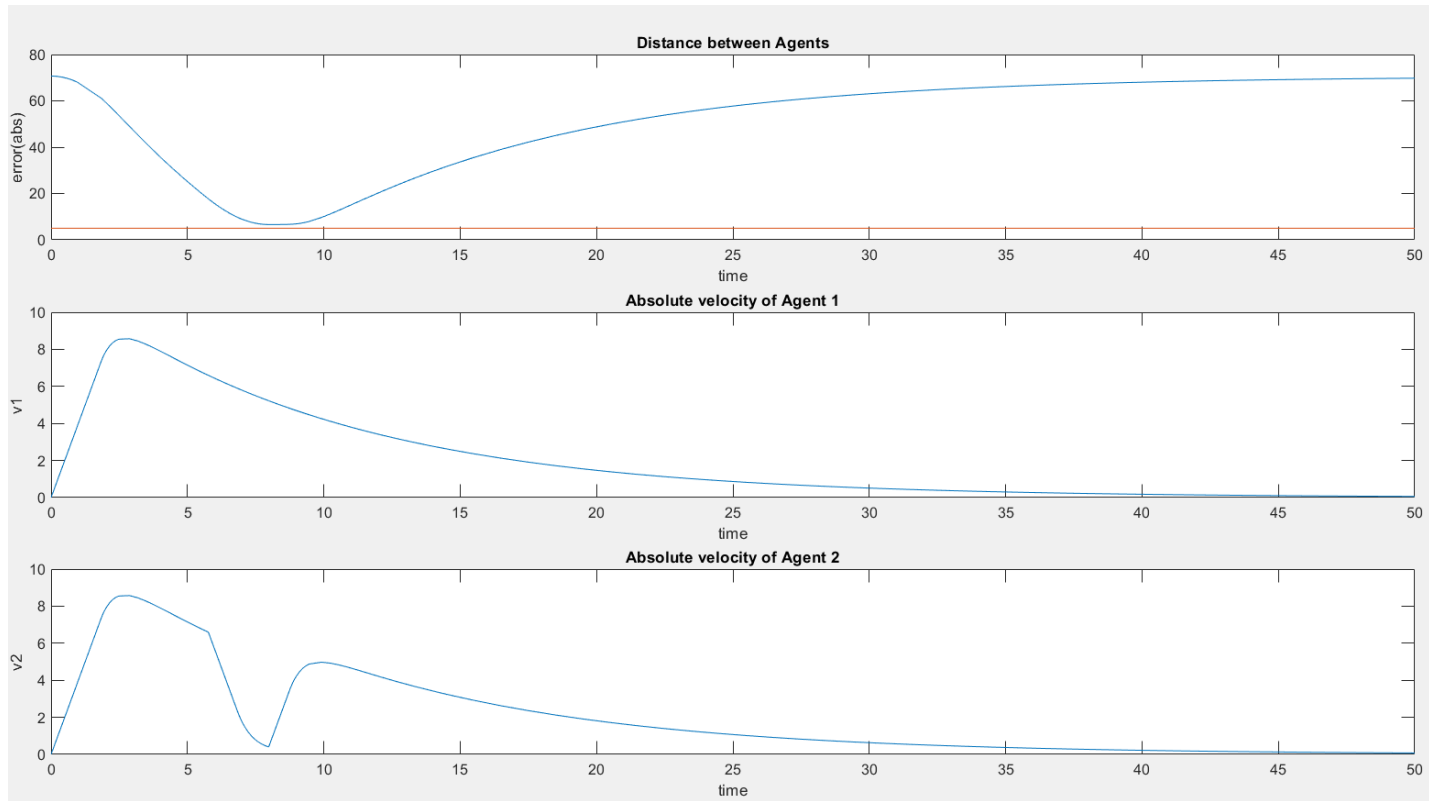


Agents having different max speed

Reference: Branicky, M. S. (1998). Multiple Lyapunov functions and other analysis tools for switched and hybrid systems. IEEE Transactions on automatic control, 43(4), 475-482.



# Results:



Agents having same max speed

# Advantages

- Combination of two ideas – target point control and control barrier certificates (switching).
- Simpler than Quadratic programming controller.
- Eliminates unwanted deviation from path.
- Fast response and less use of computation.

## Disadvantages:

- Assumes that the path travelled by both agents are always in straight lines.
- Requires additional control algorithm for scaling to multiple agents (e.g. consensus based ranking for resolving deadlocks)
- Symmetric cases are not handled.



# High-gain observers

Non-linear system  $\dot{x}_1 = x_2$

$$\dot{x}_2 = \phi(x, u, w)$$

Measurement  $y = x_1$

Control Objective *To stabilize the origin  $x=0$*   
 $u = \gamma(x, w)$

Observer  $\dot{\hat{x}}_1 = \hat{x}_2 + h_1(y - \hat{x}_1)$   
 $\dot{\hat{x}}_2 = \phi_0(\hat{x}, u) + h_2(y - \hat{x}_1)$

Error Dynamics  $\dot{\tilde{x}}_1 = -h_1\tilde{x}_1 + \tilde{x}_2$   
 $\dot{\tilde{x}}_2 = -h_2\tilde{x}_1 + \delta(x, \tilde{x}, w)$

$$\tilde{x}_1 = x_1 - \hat{x}_1$$

$$\tilde{x}_2 = x_2 - \hat{x}_2$$

$$\delta = \phi(x, \gamma(\hat{x}), w) - \phi_0(\hat{x}, \gamma(\hat{x}))$$

# Contd.

Change of variables

$$h_1 = \frac{\alpha_1}{\epsilon} \quad h_2 = \frac{\alpha_2}{\epsilon^2}$$

$$\eta_1 = \frac{\tilde{x}_1}{\epsilon} \quad \eta_2 = \tilde{x}_2$$

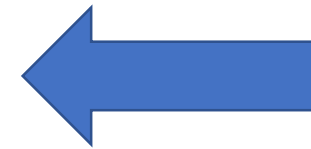
New Dynamics

$$\epsilon \dot{\eta}_1 = -\alpha_1 \eta_1 + \eta_2$$

$$\epsilon \dot{\eta}_2 = -\alpha_2 \eta_2 + \epsilon \delta$$

Peaking phenomenon

$$\eta_1(0) = \frac{\tilde{x}_1(0)}{\epsilon} = \frac{x_1(0) - \hat{x}_1(0)}{\epsilon}$$



$$\eta_1(0) = O\left(\frac{1}{\epsilon}\right)$$

Transient response could be impulsive in short time and could destabilize the system

Solution: Control saturation

# Advantages

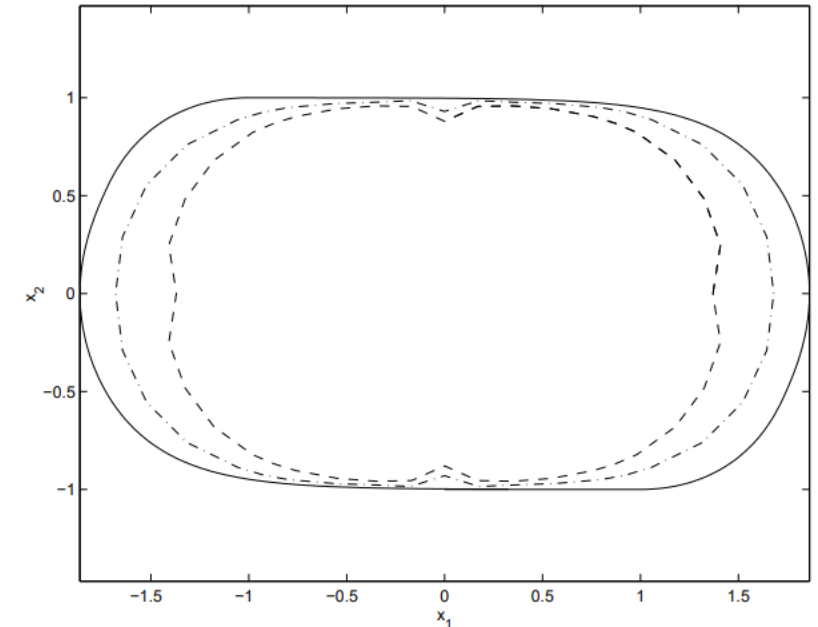
- Asymptotic stability of origin as  $\epsilon$  tends to zero
- Recovers region of attraction
- Recovers state trajectories

## Disadvantages

- High gain increases error due to noisy measurements
- Faster response at transient but poor steady-state performance
- No guarantee of asymptotic stability to origin. Converges within an error of  $\epsilon$

Solution: Non-linear gain scheduling – high gains during transient phase and lower gain at steady state

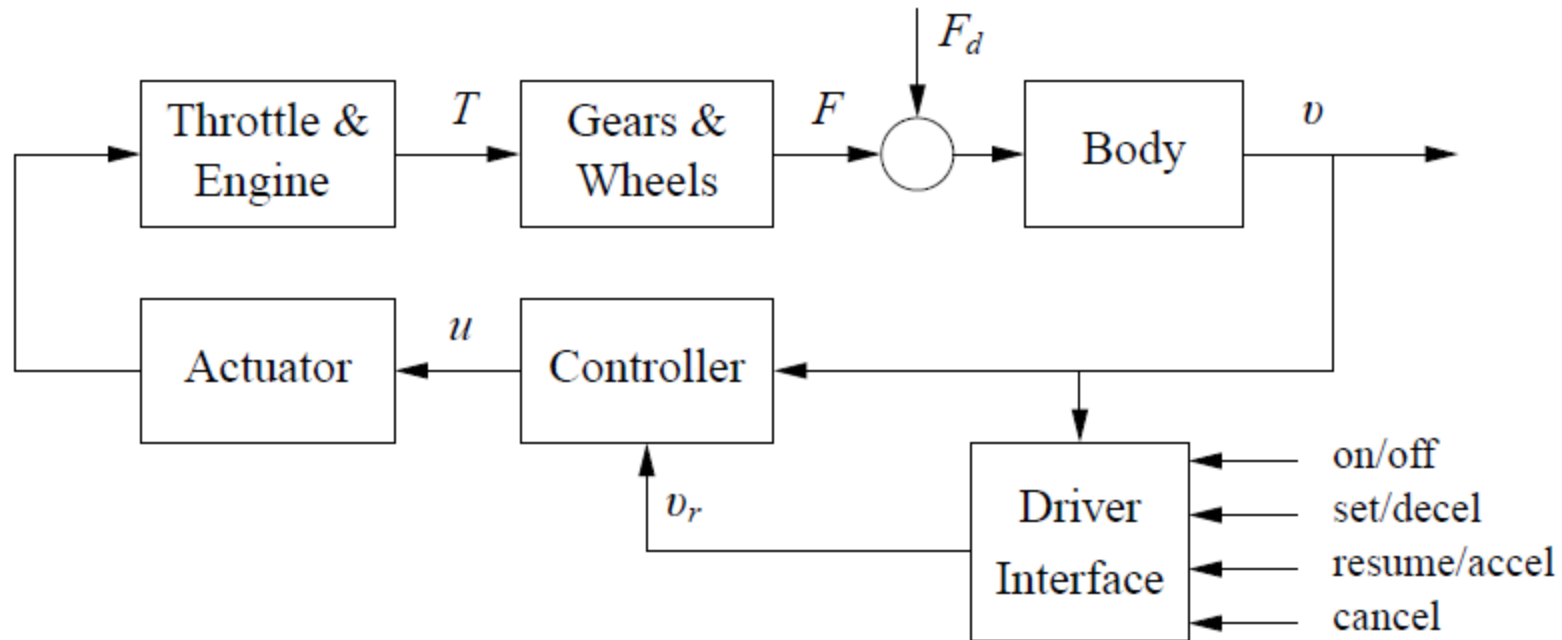
Alternate: Sliding mode controller



*Region of attraction under state feedback  
and output feedback for different values of  $\epsilon$*



# Extended High Gain observers for output feedback control of Non-linear systems



Motivating example: Adaptive Cruise control

# System dynamics

Newton's law:  $m \frac{dv}{dt} = F_{engine} - F_{drag}$

Engine rpm:  $\omega = \frac{n}{r} v =: \alpha_n v$

Force due to engine:  $F_{engine} = \frac{nu}{r} T(\omega) = \alpha_n u T(\alpha_n v)$

$$T(\omega) = T_m (1 - \beta \left( \frac{\omega}{\omega_m} - 1 \right)^2)$$

Force due to friction:  $F_r = mg C_r \text{sign}(v)$

Aerodynamic drag:  $F_a = \frac{1}{2} \rho C_d A v^2$

$r$  = wheel radius,  $n$  = gear ratio,  
 $v$  = linear velocity

$u$  = Control input (rate of fuel injection)

$T_m$  = Max torque = 190 Nm  
 $\omega_m$  = Max Engine rpm = 420  $\text{rads}^{-1}$   
 $\beta$  = 0.4

$m$  = mass of car,  $g$  = gravity,  
 $C_r$  = 0.01 = Coefficient of Rolling friction

$\rho$  = Density of air =  $1.3 \frac{\text{kg}}{\text{m}^3}$ ,  $C_d$  = Drag coefficient = 0.32  
 $A$  = Frontal area of car =  $2.4 \text{m}^2$

Typical values

$\alpha_1 = 40$   
 $\alpha_2 = 25$   
 $\alpha_3 = 16$   
 $\alpha_4 = 12$   
 $\alpha_5 = 10$

# Contd.

Weight  
component:

$$F_w = mg \sin(\theta)$$

$\theta = \text{slope of road}$

Total forces:

$$m \frac{dv}{dt} = \alpha_n u T(\alpha_n v) - mg C_r \text{sign}(v) - \frac{1}{2} \rho C_d A v^2 - mg \sin(\theta)$$

PID Controller:  $u(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau + k_d \dot{e}(t)$

*Perfect state  
information available*



*State feedback  
controller works*

Unmodelled dynamics/ non-linearities always exists.  
Finding perfect model is a challenging task than designing controller



# Extended High-Gain Observer

Non-linear system

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = a(x, w) + b(x, w)u$$

Measurement

$$y = x_1$$

Extended Dynamics

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = a_0(x) + b_0(x)u + \sigma$$

$$\text{where } \sigma = a(x, w) - a_0(x) + (b(x, w) - b_0(x))u$$

$$\dot{\sigma} = \phi(x, w, \dot{w}, u, \dot{u})$$

$$y = x_1$$

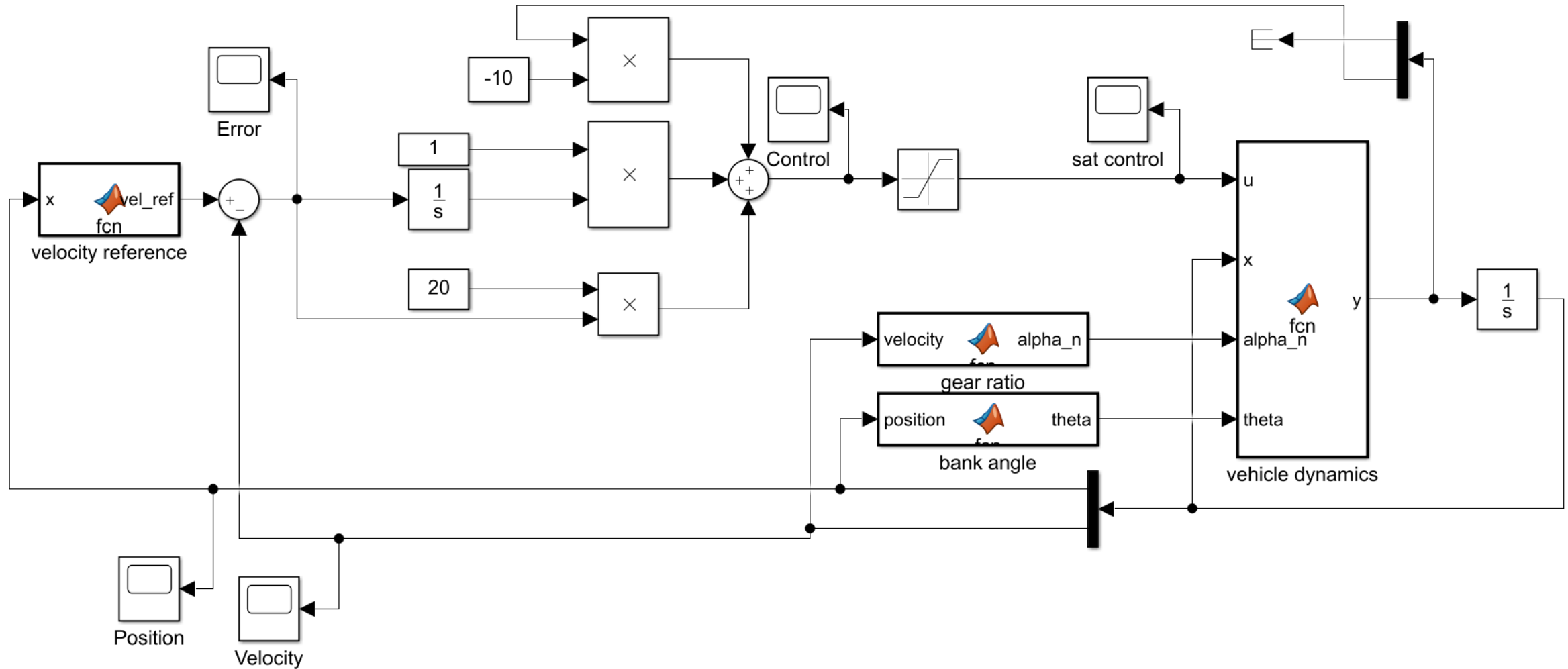
High-gain observer

$$\dot{\hat{x}}_1 = \hat{x}_2 + \frac{\alpha_1}{\epsilon}(y - \hat{x}_1)$$

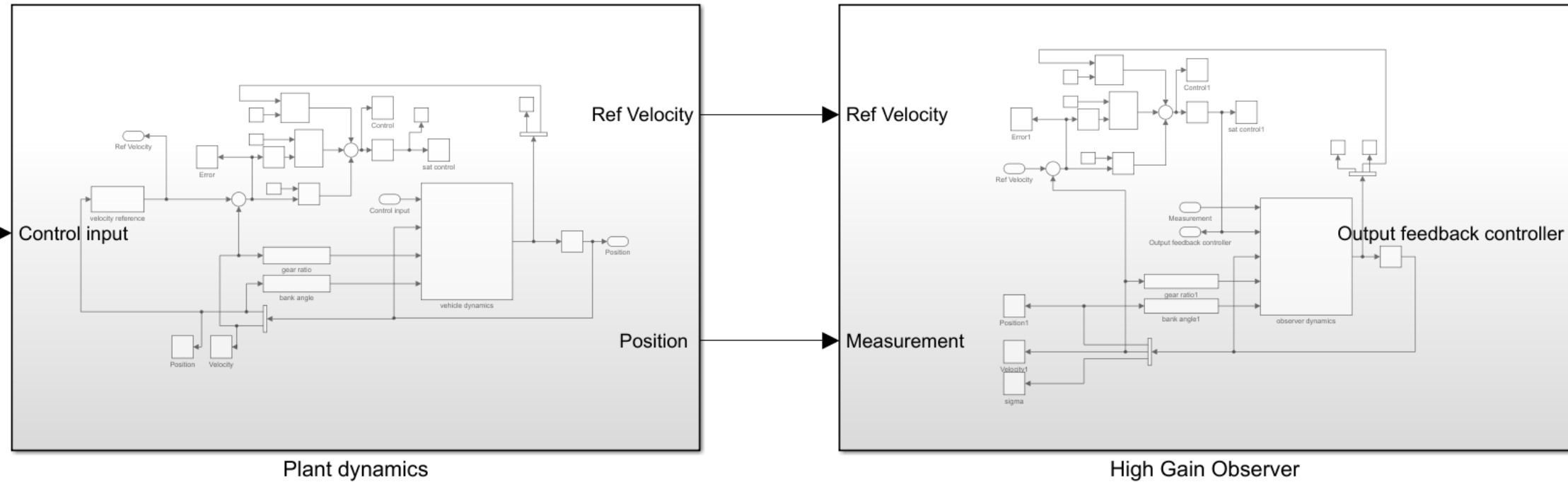
$$\dot{\hat{x}}_2 = a_0(\hat{x}) + b_0(\hat{x})u + \hat{\sigma} + \frac{\alpha_2}{\epsilon^2}(y - \hat{x}_1)$$

$$\dot{\hat{\sigma}} = \frac{\alpha_3}{\epsilon^3}(y - \hat{x}_1)$$

# State feedback controller

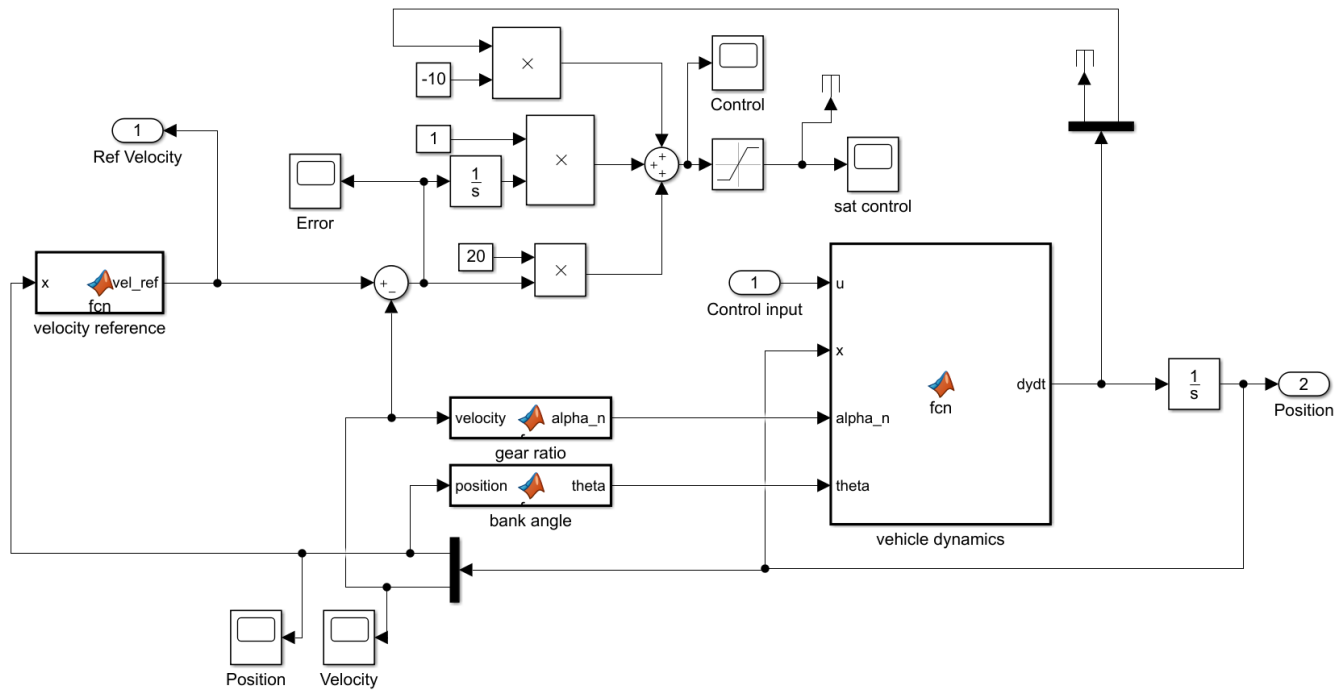


# Output feedback controller

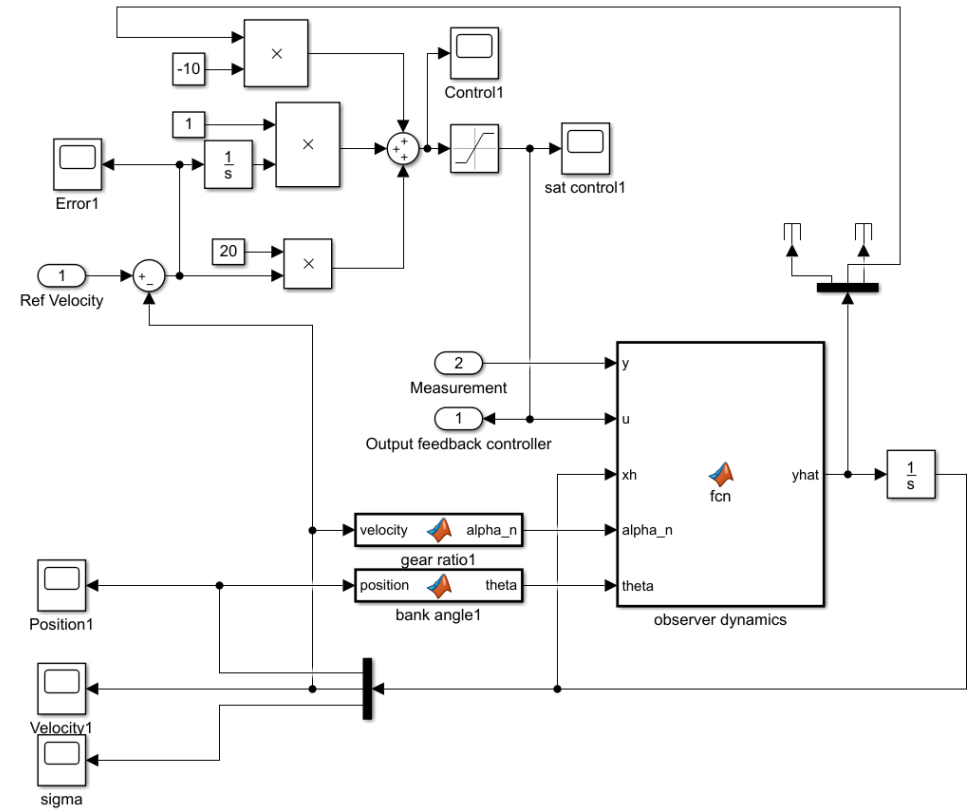




# Simulation

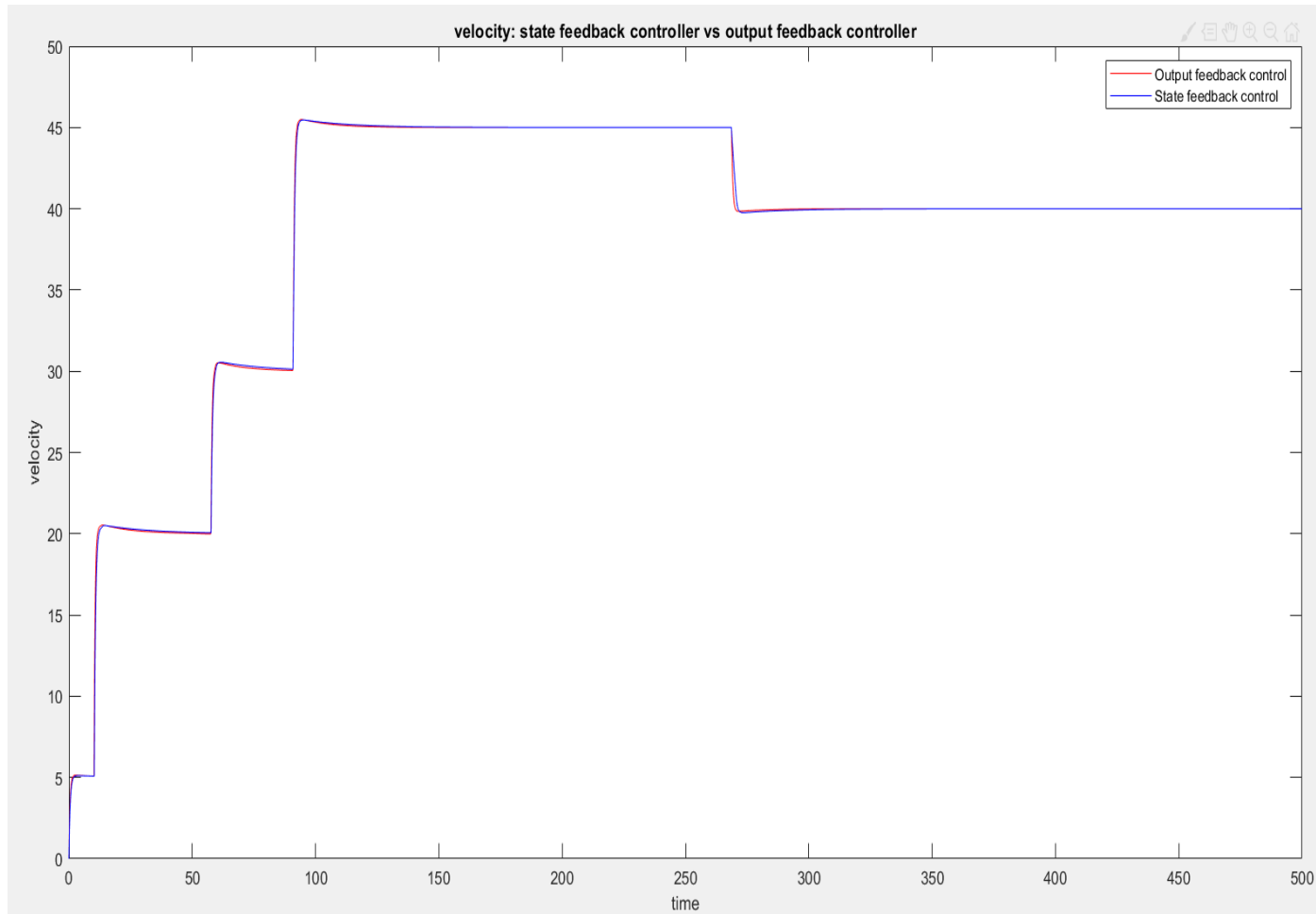


Plant dynamics



Observer dynamics

# Results



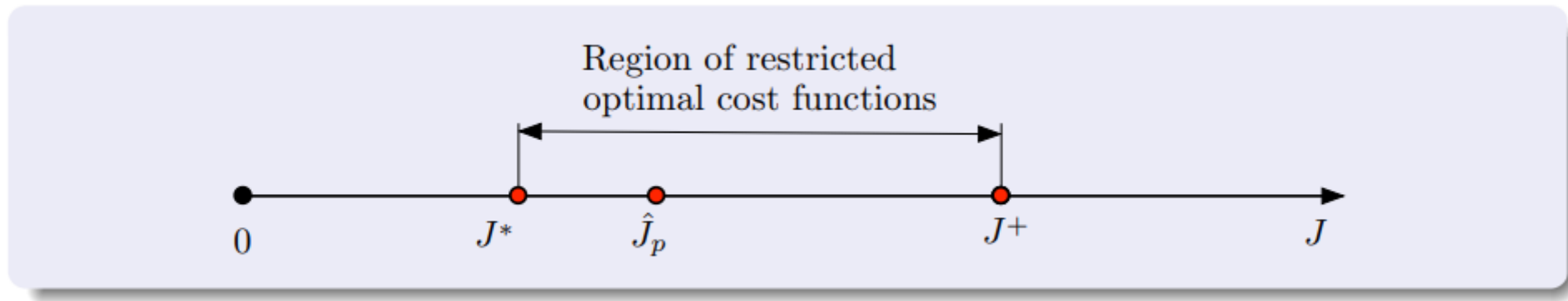
Parameters in Observer dynamics are modified to induce modeling inaccuracy.

For example: mass of the car was varied  $\pm 30\%$ , frictional force set to zero, drag coefficient varied.

Yet, output feedback controller (using high gain observers) perform\* at par with state feedback controller as seen here.

\*Requires tuning gain, epsilon for higher modeling errors

# Thank you



Optimality



Stability

$$J(x) = \inf_{u \in U(x)} H(x, u, J)$$