

## Research Article

# An Unconditionally Stable Positivity-Preserving Scheme for the One-Dimensional Fisher–Kolmogorov–Petrovsky–Piskunov Equation

Sangkwon Kim,<sup>1</sup> Chaeyoung Lee,<sup>1</sup> Hyun Geun Lee,<sup>2</sup> Hyundong Kim,<sup>1</sup> Soobin Kwak,<sup>1</sup> Youngjin Hwang,<sup>1</sup> Seungyoon Kang,<sup>1</sup> Seokjun Ham,<sup>1</sup> and Junseok Kim<sup>1</sup>

<sup>1</sup>Department of Mathematics, Korea University, Seoul 02841, Republic of Korea

<sup>2</sup>Department of Mathematics, Kwangwoon University, Seoul 01897, Republic of Korea

Correspondence should be addressed to Junseok Kim; cfdkim@korea.ac.kr

DWUHW \$ EVWU\_ TVd \$" \$ #- 3UWfW # A UfaTVd \$" \$ #- BgT 7eZW # + A UfaTVd \$" \$ #

Academic Editor: Youssef N. Raoul

Copyright © 2021 Sangkwon Kim et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In this study, we present an unconditionally stable positivity-preserving numerical method for the Fisher–Kolmogorov–Petrovsky–Piskunov (Fisher–KPP) equation in the one-dimensional space. The Fisher–KPP equation is a reaction-diffusion system that can be used to model population growth and wave propagation. The proposed method is based on the operator splitting method and an interpolation method. We perform several characteristic numerical experiments. The computational results demonstrate the unconditional stability, boundedness, and positivity-preserving properties of the proposed scheme.

## 1. Introduction

In an infinitely long domain, the behavior of virile mutant propagation can be modeled by Fisher's equation [1]. It was also independently studied by Kolmogorov, Petrovsky, and Piskunov [2]. There are some numerical methods to solve the Fisher–Kolmogorov–Petrovsky–Piskunov (Fisher–KPP) or fractional Fisher–KPP equations such as the spectral collocation-type method [3, 4], Adams–Bashforth–Moulton method [5], and Mittag-Leffler kernel [6]. In addition, various finite difference methods [7–10] and the radial basis functions [11] have been developed.

In this paper, we present an unconditionally stable positivity-preserving numerical method for the Fisher–KPP equation:

$$\frac{\partial u}{\partial t} - D \frac{\partial^2 u}{\partial x^2} = u(1-u)^p, \quad x \in \mathbb{R}; \quad t > 0, \quad (1)$$

where  $u(x; t)$  is the population density at position  $x$  and time  $t$ . Here,  $D$  is the diffusion coefficient,  $K$  is the

coefficient of nonlinearity, and  $p$  is a positive integer.

There exist several other unique approaches to solve the Fisher–KPP equation. Saeed and Mustafa [12] used the reduced differential transformation method to solve the equation and showed that their result coincides with the exact solution. From a biological point of view, the population density should be nonnegative and bounded from above by one if the density is scaled by the carrying capacity. El-Hachem et al. [13] studied approaches to modelling biological invasion and recession using the Fisher–KPP model. In addition, they utilized the Fisher–Stefan model, another generalization of the Fisher–KPP-type model that simulates biological recession. Xu et al. [14] proposed a nonlinear finite volume scheme for two-dimensional diffusion equations which preserves the maximum principle. Fourth-order Fisher–KPP equation is one of the widely studied and extended variations, and various methods have been proposed to solve it. Kadri and Omrani [15] utilized the extrapolation technique, and Başhan et al. [16] applied the differential quadrature method. An explicit positivity-preserving numerical method for the Fisher–KPP equation has been





















