Homework 6 Submission – CBE 9413 Intro to Sustainable Energy Systems

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Disclaimer: I have used ChatGPT to write Julia pseudo-code.

Problem 1

The objective function of the minimization problem is
Operational cost of grid operation = Operational cost of fossil generator + Operational cost of renewable generator + cost of load shedding

$$Obj = \sum_{t=1}^{t} P_f(t) \cdot \alpha_f + \sum_{t=1}^{t} P_R(t) \cdot 0 + \sum_{t=1}^{t} P_L(t) \cdot \alpha_L = \alpha_f \sum_{t=1}^{t} P_f(t) + \alpha_L \sum_{t=1}^{t} P_L(t)$$

Subject to following constraints:
Supply-demand balance constraint: $P_f(t) + P_R(t) + P_L(t) \geq D(t)$
Capacity constraint (fossil): $P_f(t) \leq C_f$
Capacity constraint (renewable): $P_R(t) \leq C_R \cdot \beta_{R,t}$
Non-negativity constraints: $P_f(t) \geq 0;\ P_R(t) \geq 0;\ P_L(t) \geq 0$
Let $\upsilon_f(t), \upsilon_R(t)$ be the multipliers for capacity constraints.
Let $\mu_f(t), \mu_R(t), \mu_L(t)$ be the multipliers for the nonnegativity constraints.
Then, the Lagrangian of this objective function is

$$L = \alpha_f \sum_{t=1}^{t} P_{f,t} + \alpha_L \sum_{t=1}^{t} P_{L,t} + \lambda_t\left(D_t - P_{f,t} - P_{R,t} - P_{L,t}\right) + \upsilon_f(t) \cdot \left(C_f - P_f(t)\right) + \upsilon_R(t) \cdot \left(C_R \beta_{R,t} - P_R(t)\right) - \mu_f(t) P_{f,t} - \mu_R(t) P_{R,t} - \mu_L(t) P_{L,t}$$

As per KKT conditions, for any t in the feasible space, the following conditions must hold

1) Stationarity Conditions:

$$\frac{\partial L}{\partial P_{f,t}} = \alpha_f - \lambda_t - \upsilon_f(t) - \mu_f(t) = 0 \ldots (1a)$$

$$\frac{\partial L}{\partial P_{R,t}} = -\lambda_t - \upsilon_R(t) - \mu_R(t) = 0 \ldots (1b)$$

$$\frac{\partial L}{\partial P_{L,t}} = \alpha_L - \lambda_t - \mu_L(t) = 0 \ldots (1c)$$

2) Dual feasibility:

$$\upsilon_f(t), \upsilon_R(t) \geq 0 \ldots (2a)$$

$$\mu_f(t), \mu_R(t), \mu_L(t) \geq 0 \ldots (2b)$$

3) Complementary slackness:

$$\lambda_t\left(D_t - P_{f,t} - P_{R,t} - P_{L,t}\right) = 0 \ldots (3a)$$

$$\upsilon_f(t) \cdot \left(C_f - P_f(t)\right) = 0 \ldots (3b)$$

$$\upsilon_R(t) \cdot \left(C_R\beta_{R,t} - P_R(t)\right) = 0 \ldots (3c)$$

$$\mu_f(t)P_{f,t} = 0 \ldots (3d)$$

$$\mu_R(t)P_{R,t} = 0 \ldots (3e)$$

$$\mu_L(t)P_{L,t} = 0 \ldots (3f)$$

Case (1) :

$$P_{L,t} > 0 \ in \ (2f) \implies \mu_L(t) = 0$$

Substituting in (1c) gives $\alpha_L = \lambda_t$

Hence, proved that marginal electricity price is equal to cost of load shedding.

Interpretation: For the market to clear (i.e. mathematically feasible solution), the grid operator must pay the cost of load shedding to the consumer. Otherwise, there will be imbalance in supply and demand.

Case (2) :

$$P_{L,t} > 0 \ in \ (3f) \implies \mu_L(t) = 0$$

Substituting in (1c) gives $\alpha_L = \lambda_t$

Substituting $\lambda_t = \alpha_L$ in (1a) gives $\alpha_f - \alpha_L - \upsilon_f(t) - \mu_f(t) = 0 \implies \alpha_f - \alpha_L = \upsilon_f(t) + \mu_f(t)$

Since we know $\alpha_f \ll \alpha_L \implies \upsilon_f(t) + \mu_f(t) < 0$

Since we know from $(2b)$ $\mu_f(t) \geq 0 \implies \upsilon_f(t) < 0$

For the above condition to be true, (3b) must have $C_f - P_f(t) = 0 \implies P_f(t) = C_f$

Hence, proved that fossil generator operates at maximum output.

Interpretation: If the fossil generator operates at less than its maximum output, grid operator can dispatch additional power at market clearing price $\alpha_f \ll \alpha_L$. A market clearing price of α_L must mean that all the dispatch capacity of fossil generator is exhausted.

Case (3) :

$$P_{R,t} > 0 \ in \ (2e) \implies \mu_R(t) = 0$$

Curtailment of RE implies $C_R\beta_{R,t} > P_R(t) \ in \ (2c) \implies \upsilon_R(t) = 0$

Substituting in (1b) gives $-\lambda_t - \mu_R(t) = 0$

Since we know from $(2b)$ $\mu_R(t) \geq 0 \implies \lambda_t = 0$ Hence, proved.

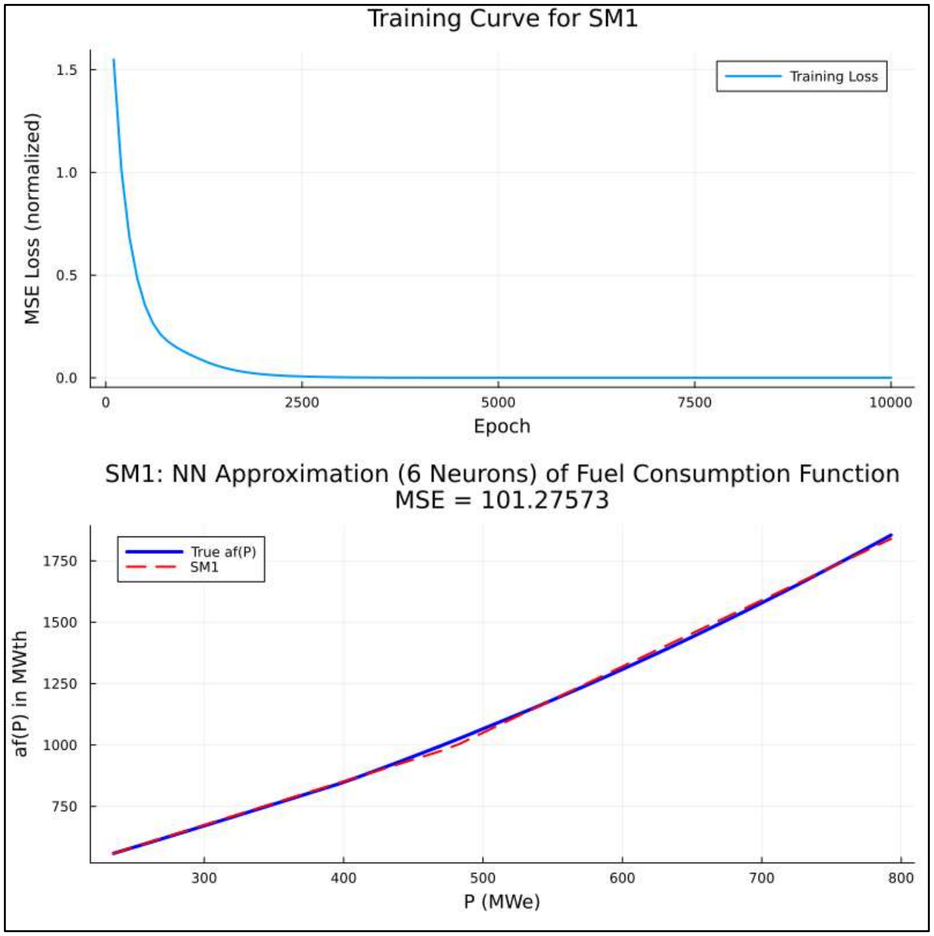
Interpretation: If the market clearing price λ_t were anything higher than zero, grid operator can dispatch the surplus production to fulfill that demand. Otherwise, curtailment indicates an oversupply in the market whereby prices must go down to zero (no willing buyers).

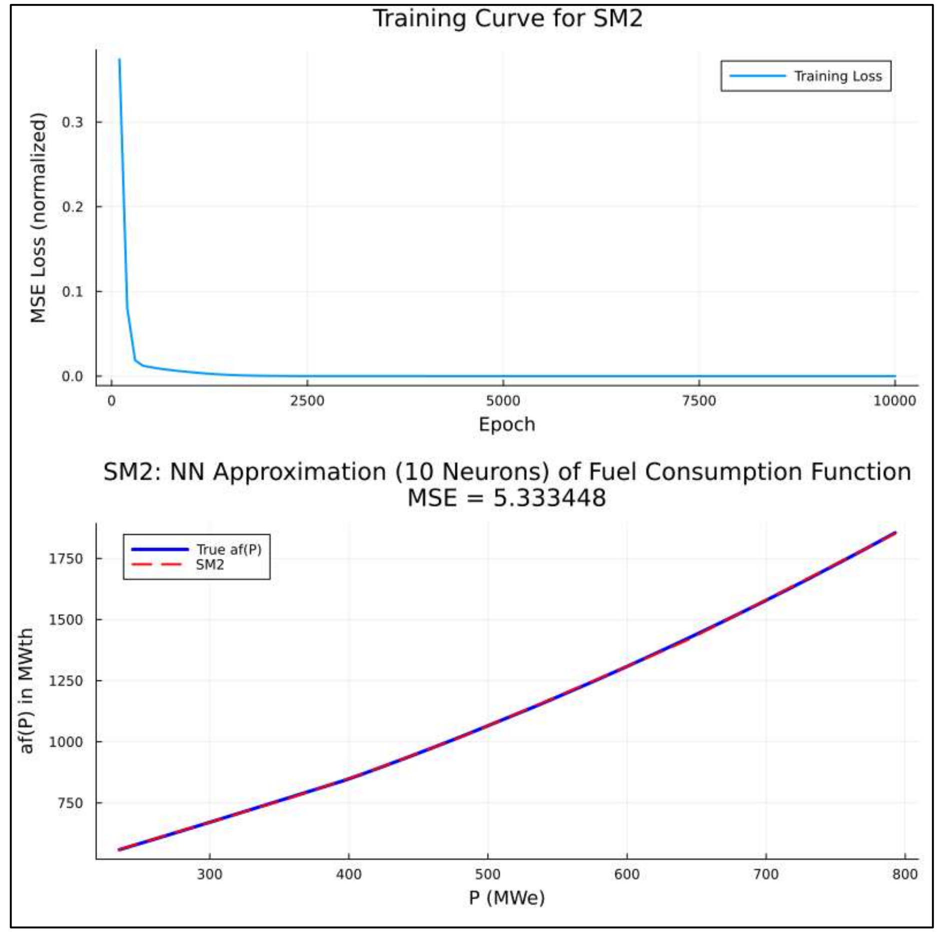
Problem 2

Part A

a) MSE for SM1 (6 neurons) = 101, MSE for SM2 (10 neurons) = 5

b) Comparative plot of SM1, SM2 and Original function $\alpha_f(P)$



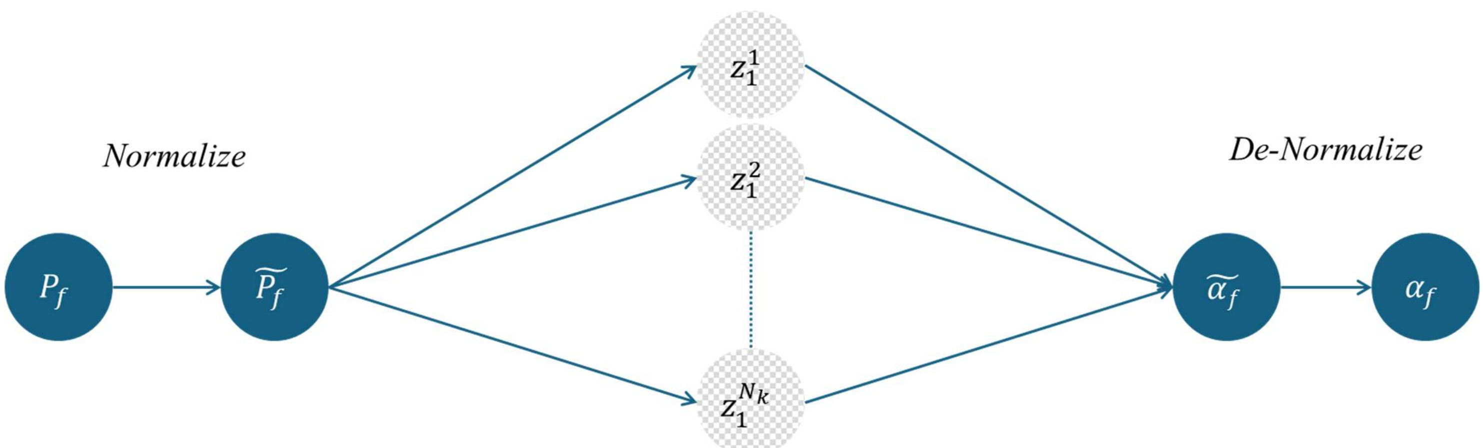


c) Trained parameter values for SM1 & SM2

SM1 (6 neurons)			**SM2 (10 neurons)**		
w1	**b1**	**w2**	**w1**	**b1**	**w2**
-0.835	1.288	-1.159	-0.437	-0.038	-0.064
0.322	-0.497	-0.071	1.038	0.075	0.877
0.272	-0.421	-0.321	0.335	-0.226	-0.025
-0.418	-0.048	0.964	-0.098	-0.172	0.579
-0.448	0.691	-0.473	-0.498	-0.036	-0.862
0.004	-0.018	0.670	-0.194	0.300	-0.827
b2 = 1.664			-0.511	-0.041	-0.618
			-0.230	-0.165	0.381
			-0.332	-0.143	0.289
			0.881	-0.642	0.243
			b2 = 0.059		

A simplified representation of training neural network with normalization is shown below:

$$\widehat{z_1} = w_1\widetilde{P_f} + b_1 \qquad z_1 = \max(\widehat{z_1}, 0) \qquad \widetilde{\alpha_f} = w_2 z_1 + b_2$$



Part B

Modified cost minimization problem from Problem 1 is as follows:

$$Min \sum_{t=1}^{24} \alpha_f(t) \cdot \rho_f \cdot F + \alpha_L \sum_{t=1}^{24} P_L(t)$$

Where α_f is the hourly fuel consumption in MWth and $\rho_f = 4$ is fuel cost for fossil generator in \$/mmbtu and $F = 3.412$ is the energy conversion factor (mmbtu/MWh), $\alpha_L = 1000$ is the cost of load shedding in \$/MWh.

Note that $\alpha_f(t)$ is a non-linear function of $P_f(t)$ which will be approximated through mixed-integer linear neural network constraints.

Subject to following constraints:

$$P_f(t) + P_S(t) + P_W(t) + P_L(t) \geq D(t)$$

$$C_{f,min} \leq P_f(t) \leq C_f$$

$$P_S(t) \leq C_S \cdot \beta_{S,t}$$

$$P_W(t) \leq C_W \cdot \beta_{W,t}$$

$$P_f(t) \geq 0;\ P_S(t) \geq 0; P_W(t) \geq 0;\ P_L(t) \geq 0$$

Where C_f = 800 MW, $C_{f,min}$ = 160 MW, C_S = 400 MW, C_W = 400 MW

$\beta_{S,t}$ and $\beta_{W,t}$ are the hourly solar and wind CF.

$\alpha_f(t)$ is approximated as NN using the method taught in the lecture, starting with normalization of $P_f(t)$ from sample of 200 points over $P_f(t) \in (160,800)$.

$$\widetilde{P_f(t)} = \frac{(P_f(t) - \overline{P_f(t)})}{\sigma_{P_f(t)}}$$

$$\widehat{z_1}(t) = W_1\widetilde{P_f(t)} + b_1$$

$$z_1(t) \geq \widehat{z_1}(t)$$

$$z_1(t) \leq \widehat{z_1}(t) - \widehat{z_1}^L(1 - d(t))$$

$$z_1(t) \leq \widehat{z_1}^U d(t)$$

$$\widetilde{\alpha_f}(t) = W_2 z_1(t) + b_2$$

$$\alpha_f(t) = \sigma_{\alpha_f(t)}\widetilde{\alpha_f}(t) + \overline{\alpha_f(t)}$$

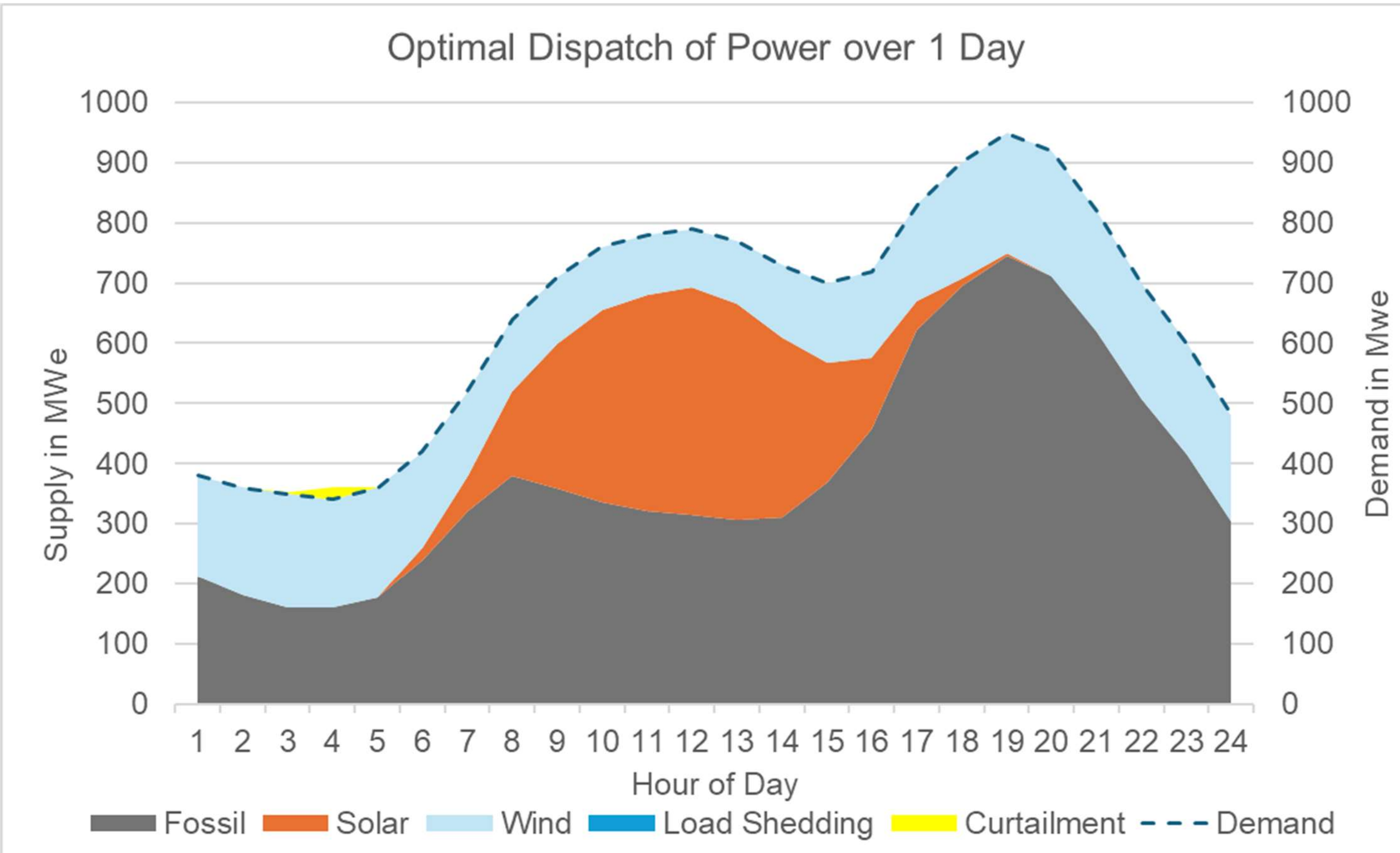
The values of matrices W1, W2, b1 and b2 are obtained in Q2 for both SM1 and SM2.

$$\widehat{z_1}^U = \max(W_1\widetilde{P_f(t)}) + b_1$$

$$\widehat{z_1}^L = \min(W_1\widetilde{P_f(t)}) + b_1$$

Part C

1. For piecewise linear approximation, we must include the space between $P_f \in (380,420)$ to account for a bump in the function value (2 segments).
2. The function is almost linear outside this range of fossil power production where remaining 4 segments can be situated.
3. The optimal power dispatch profile for all three approximations is the same as follows:



4. Note that Curtailment is calculated as $P_f(t) + P_S(t) + P_W(t) + P_L(t) - D(t)$ and occurs due to minimum load restriction of fossil power plant.
5. There is no load shedding and the fossil plant dispatches to meet the demand that cannot be fulfilled by RE generation in a given hour. This depicts the role of flexibility offered by fossil power plant in high VRE integrated grid (41% share on this day).
6. The objective value, runtime and optimal dispatch profile for all three approximations are compared in the table below.
7. Note that the objective value is only an approximation of true cost of dispatch. By substituting the optimized dispatch values, we calculate **the true cost of dispatch to be $293,843**.

SN	Approximation	Objective Value ($)	Solver runtime (s)	Accuracy[1]	Parametric Score[2]
1	SM1	282864	0.016	99.85%	62
2	SM2	282350	0.039	99.83%	26
3	Piecewise Linear	284545	0.019	99.89%	48

8. I have defined model **accuracy** as a parameter that measures the extent to which objective value obtained of the approximation is closer to the true cost of dispatch (see definition in the footnote).
9. SM2 has the lowest objective value, but it is not an accurate depiction of true cost dispatch.
10. **Piecewise Linear is the most accurate** approximation in terms of true cost of dispatch, while **SM1 is the quickest** approximation to solve**.**
11. I defined a **parametric score** combining both accuracy and solver runtime. Based on this, **I will choose SM1 approximation for this problem**, as it is closer to true dispatch cost and quicker to solve than other approximations.

[1] Accuracy = 1- $\left(\frac{Objective\ Value - True\ Cost\ of\ Dispatch}{True\ Cost\ of\ Dis}\right)^2$

[2] Parametric Score = $\left(\frac{\text{Accuracy}}{\text{Solver Runtime}}\right)$