Homework 7 Submission – CBE 9413 Intro to Sustainable Energy Systems

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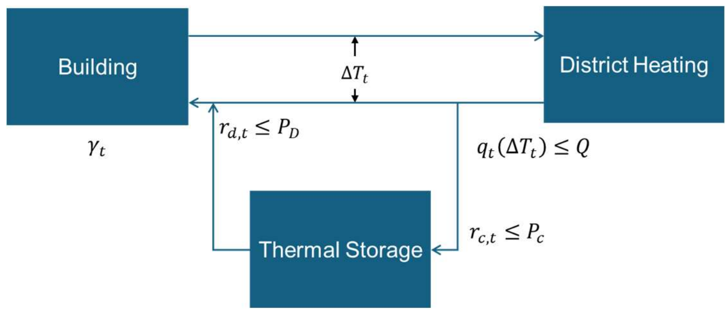
Problem 1

Part A



The formulation of optimization problem is

$$Min \sum_{t=1}^{T} aq_t + bq_t^2$$

Subject to

Supply-demand balance constraint:

$$q_t - r_{c,t} + r_{d,t} \geq \gamma_t$$

$$q_t = m_t c_p \Delta T_t$$

Capacity constraints:

$$q_t \leq Q$$

$$\alpha_L \leq m_t \leq \alpha_U$$

$$\tau_L \leq \Delta T_t \leq \tau_U$$

Thermal storage inventory balance:

(Assumption: Perfectly insulated thermal storage with zero self-discharge i.e. thermal loss)

$$S_t = S_{t-1} + \eta_c r_{c,t} - \frac{r_{d,t}}{\eta_d}$$

$$S_0 = 0$$

$$S_T = S_0$$

$$S_t \leq E$$

$$r_{c,t} y_t \leq P_C; y_t \in \{0,1\}$$

$$r_{d,t}(1 - y_t) \leq P_D$$

Ramping constraint:

$$m_{t+1} - m_t \leq \delta_U$$

Non-negativity constraints: $q_t \geq 0;\ r_{c,t} \geq 0;\ r_{d,t} \geq 0; \Delta \geq 0$

Part B

A) Piecewise linear approximation of the objective function $\sum_{t=1}^{T} aq_t + bq_t^2 = \Sigma_{t=1}^{T} f(q_t)$

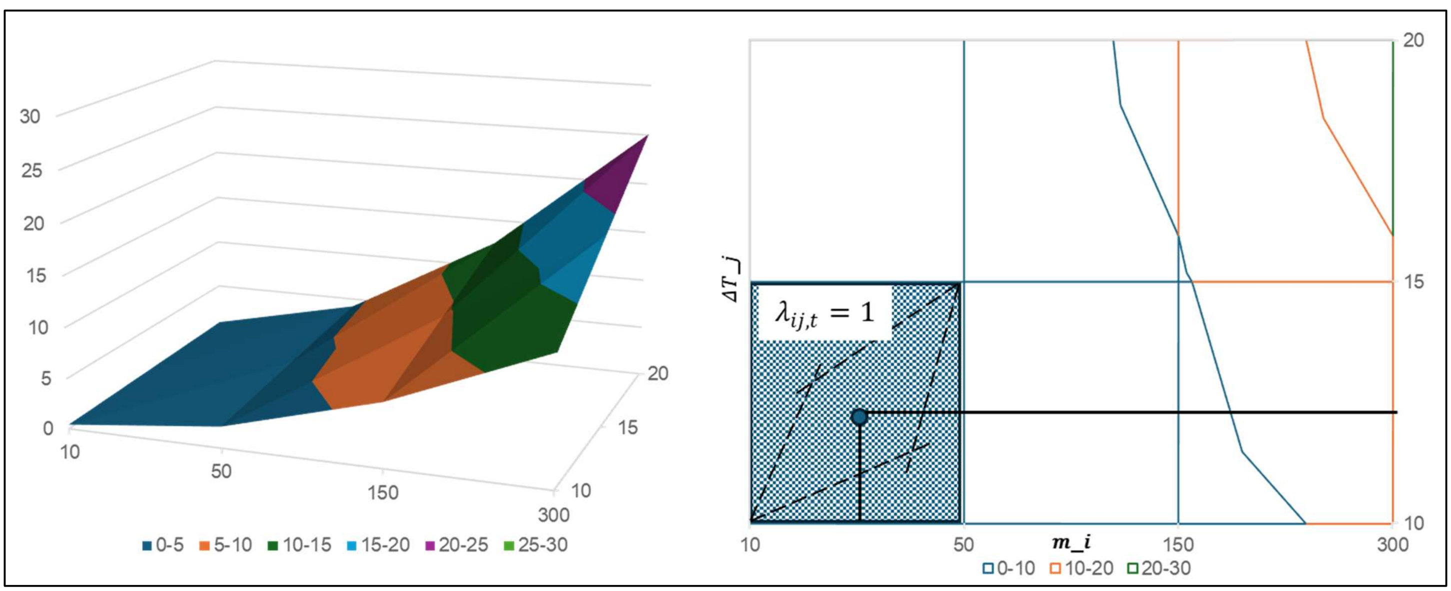
$$Obj = \sum_{n=1}^{N+1} \sum_{t=1}^{T} \alpha_{n,t} f(q_n)$$

$$q_t = \sum_{n=1}^{N+1} \alpha_{n,t} q_n$$

$$\left.\begin{array}{c} \alpha_{n,t} \leq h_{n-1,t} + h_{n,t} \\ \sum_{n=1}^{N+1} h_{n,t} = 1 \\ 0 \leq \ \alpha_{n,t} \leq 1 \\ h_{n,t} \in \{0,1\}; h_{0,t} = 0, h_{N+1,t} = 0 \end{array}\right\} \alpha_{n,t} = SoS2$$

B) Piecewise McCormick Envelopes for $q_t = m_t c_p \Delta T_t$

1. The function space is divided into K segments in m direction and L segments in ΔT direction. Using the actual break values in part C, we see that the function can be approximated as follows:
2. We will have K X L grids (6 in case of our example) out of which only one will be active at a time t through the binary variable $\lambda_{ij,t}$.
3. The dummy variables $r_{ij,t}$ and $s_{ij,t}$ will then approximate the actual q_t value in the chosen grid through McCormick Envelopes.



$$m_t = \sum_{i=1}^{K} \sum_{j=1}^{L} r_{ij,t}$$

$$\Delta T_t = \sum_{i=1}^{K} \sum_{j=1}^{L} s_{ij,t}$$

$$m_i \lambda_{ij,t} \le r_{ij,t} \le m_{i+1} \lambda_{ij,t}$$

$$\Delta T_j \lambda_{ij,t} \le s_{ij,t} \le \Delta T_{j+1} \lambda_{ij,t}$$

$$\sum_{i=1}^{K} \sum_{j=1}^{L} \lambda_{ij,t} = 1\,;\ \lambda_{ij,t} \in \{0,1\}$$

$$q_t \ge c_p \sum_{i=1}^{K} \sum_{j=1}^{L} (m_i s_{ij,t} + \Delta T_j r_{ij,t} - m_i \Delta T_j \lambda_{ij,t})$$

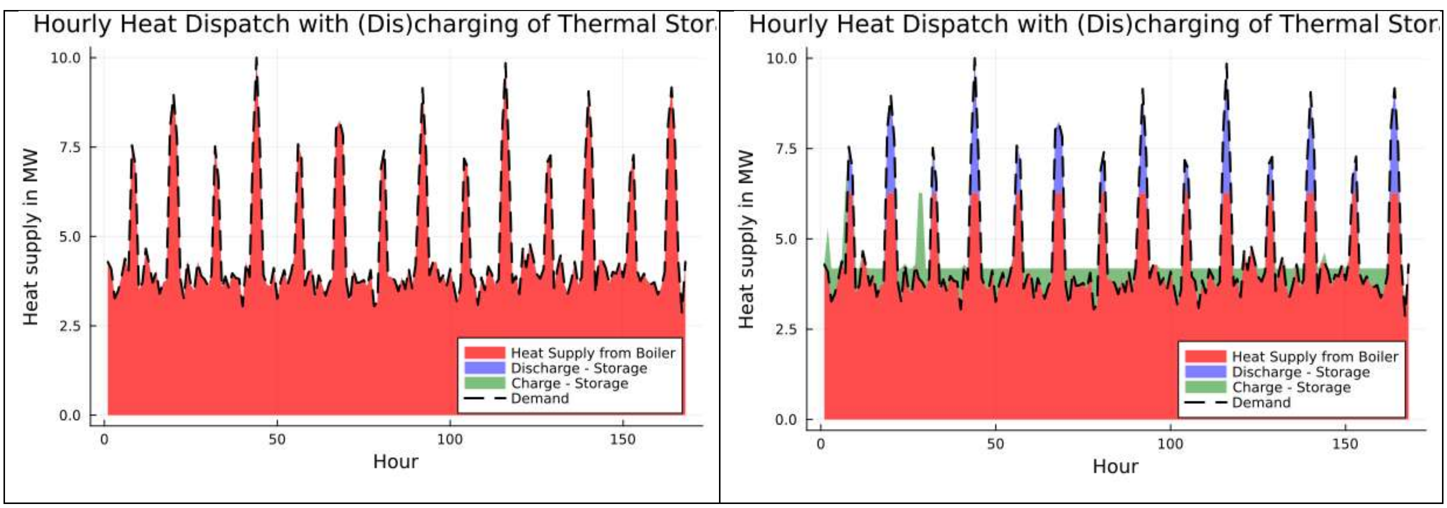
$$q_t \ge c_p \sum_{i=1}^{K} \sum_{j=1}^{L} (m_{i+1} s_{ij,t} + \Delta T_{j+1} r_{ij,t} - m_{i+1} \Delta T_{j+1} \lambda_{ij,t})$$

$$q_t \le c_p \sum_{i=1}^{K} \sum_{j=1}^{L} (m_{i+1} s_{ij,t} + \Delta T_j r_{ij,t} - m_{i+1} \Delta T_j \lambda_{ij,t})$$

$$q_t \le c_p \sum_{i=1}^{K} \sum_{j=1}^{L} (m_i s_{ij,t} + \Delta T_{j+1} r_{ij,t} - m_i \Delta T_{j+1} \lambda_{ij,t})$$

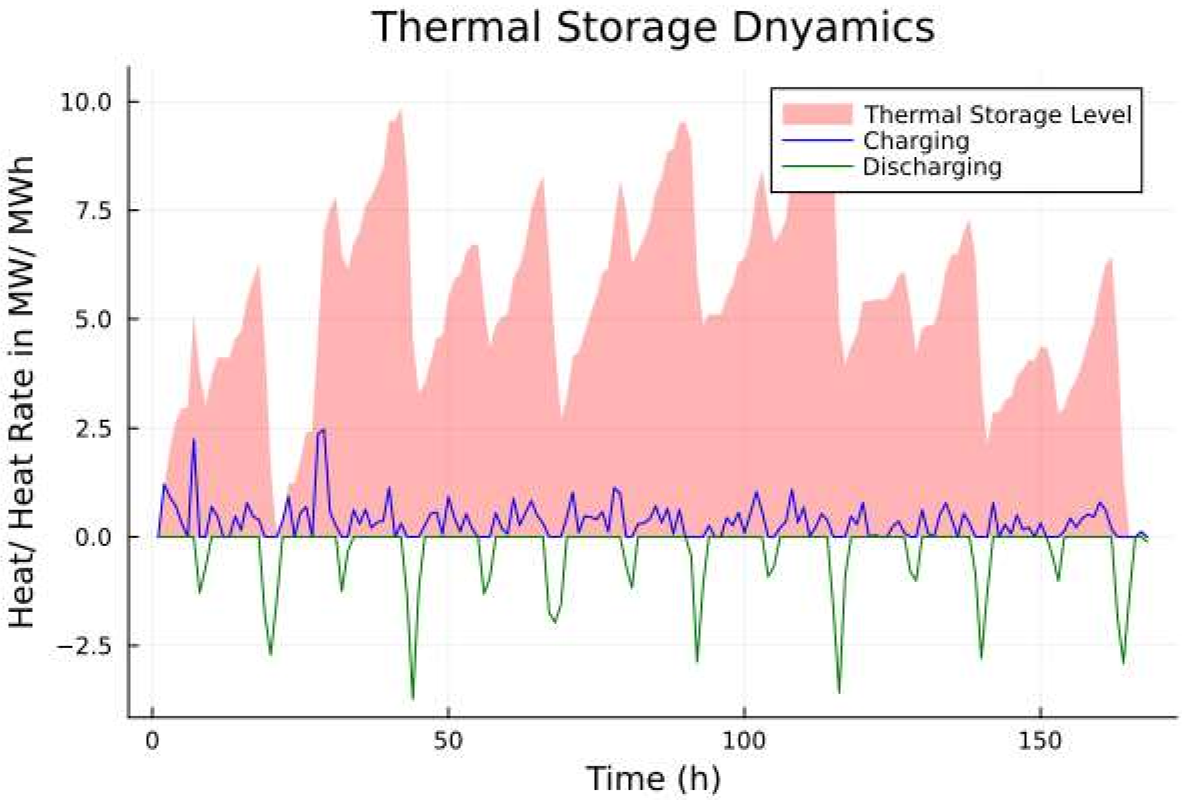
Part C (BONUS POINTS)

1. The operating cost difference between two configurations – with and without thermal storage is ~$250 per week which is < 1%.
2. There is benefit in installing thermal storage because it helps to cut peak supply and instead thermal storage discharge is used to meet excess demand. (*See RHS for ops with thermal storage*)



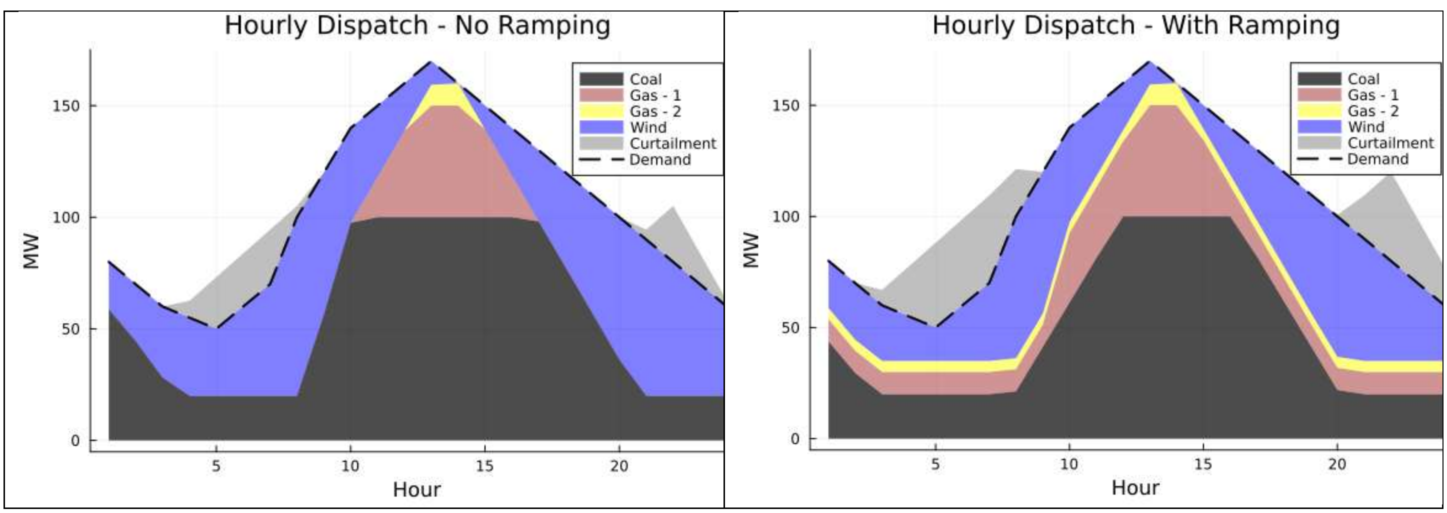
3. Cost saving is due to the quadratic nature of cost of operations with respect to amount of heat supplied (where **marginal cost of supply is not constant, but an increasing function of heat supply**).
4. This means it is better to supply 6 + 6 MW in two consecutive hours rather than 4 + 8 MW (higher marginal cost). We can check this through the mathematical property of cost function.

5. This yields very different results in the operation of district heating with thermal storage, which offers this flexibility to produce and store extra units of heat at low rates and then fulfill excess demand.
6. I would recommend installation of thermal storage on technical grounds but seeing the minor cost improvement relative to baseline, recommend further evaluation.



Problem 2 – Optimal Dispatch with and w/o Ramping

Part 1 – Impact of Ramping on System Operation

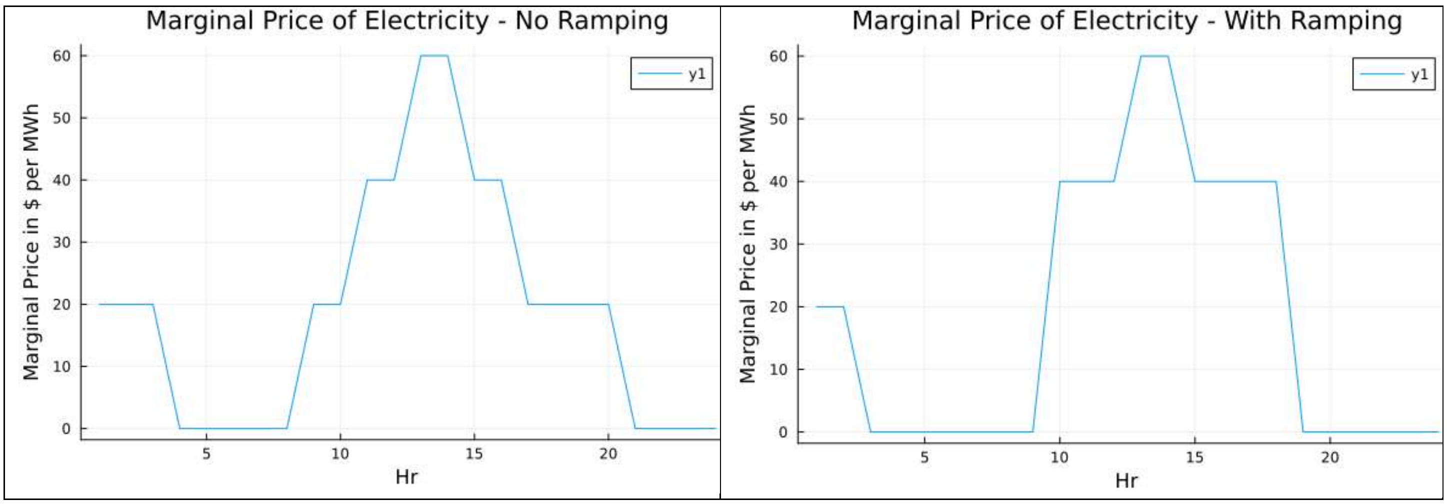


Interpretation:

1. The optimal dispatch follows the merit order based on marginal cost of production for each generator. Wind power is first dispatched to meet the given demand, followed by coal and then by gas plants.

2. Gas plants have higher ramping rates and lower startup costs which is useful for their “peaker” behavior (ability to ramp up production during peak demand hours).
3. In the case of no ramping constraints, coal plant can quickly ramp up/down between 9-10 hrs which means no gas plant is required to dispatch. This reduces total cost of power dispatch (~$37,940).
4. When ramping constraints are introduced, gas plants need to dispatch power during 9-10 hrs to fulfill demand, leading to higher total cost of power dispatch (~$47,750).
5. Both coal and gas plants must maintain their minimum operating capacity to prevent shutdown, which leads to wind power curtailment during off-peak hours.

Part 2 – Marginal Electricity Prices



Part 3 – Wind Curtailment

1. Wind curtailment is ~12% in case of no ramping and ~26% in case of ramping constraints.

Part 4 – Overall interpretation

1. Lack of flexibility (in terms of ramping) on fossil fuel generators leads to curtailment of renewable energy, which may become uneconomical for RE producers.
2. Marginal electricity price is also higher in the case of ramping constraints than without ramping constraints (since gas-based electricity is expensive to produce on marginal basis).
3. The power system will benefit from higher flexibility on fossil generators or by including energy storage to prevent RE curtailment.