Investigation of Multi user signal detection in large scale MU-MIMO systems using real constellations

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Abstract--The aim of this paper is to find the required receiver bits by using one of the best techniques available .We check different types of algorithm for detection in large MIMO systems based on the complexity and performance. The key idea in our work is to generate multiple possible solutions or outputs from which we select the best one .We propose 4 different techniques namely ZF, MMSE, ML and LAS in which we compare all the results and select the best one .Complexity and BER are the factors we take into the consideration for picking up the best technique. The likelihood ascent search (LAS) achieves near-optimal BER performance in fully loaded large MIMO systems.

Index Terms- Multiple-input multiple-output (MIMO), Likelihood ascent search (LAS), low-complexity detection, Bit Error Rate, Maximum Likelihood.

I INTRODUCTION

IT is notable that the capacity of multiple input multiple output (MIMO) channels develops linearly with the minimum number of antennas wires at the transmitter and the receiving ends [1]. Accordingly one approach to accomplish high spectral efficiency is to exploit large number of antennas at both the transmitter and receiver. The principle bottleneck of such frameworks is the complexity of the receivers. A family of low complexity detectors termed Likelihood Ascent Search (LAS) detectors have been proposed in [2] for large MIMO systems. The power of the LAS detector lies in the linear average per bit complexity and the excellent BER performance in large MIMO systems (It has been proven [3] that the asymptotic BER performance of LAS detectors converges to that of maximum likelihood (ML) detector for example). The primary detriment of LAS indicators, which is the inspiration of our work, is that they require huge quantities of antennas to accomplish the ideal BER per performance, particularly in high order modulation [3]. We consider a MIMO system with Nt transmit antennas and Nr receive antennas ($Nr \geq Nt$). The baseband system model is given by

$$y = Hx + n \quad (1)$$

Where y = [y1, y2, · · · yNr] T is a Nr×1 receive signal vector, x = [x1, x2, · · · xNr] T is a Nt × 1 transmit signal vector, n is the complex white Gaussian noise. The elements of the Nr×Nt channel matrix H are assumed i.e. the complex Gaussian random variables with zero mean and variance of unity. We assume ideal channel estimation and synchronization at the receiver end.

II MUD TECHNIQUES

Multiuser detection is one of the receiver design technology that detects the desired user signal by eliminating noise and interference from neighbourhood user's signal. Generally, in SDMA system, the BS receiver often suffers from the multi user interference due to the influence of a strong user signal source on the reception of weak user signal [4]. Several MUD techniques are used to overcome this problem over the last fifteen years [4–10]. Using the multiuser detection process, the estimated signal vector can be expressed as

$$\hat{\boldsymbol{x}} = \boldsymbol{w}^H \boldsymbol{y}$$

Where w is the r × t dimension weight matrix. The broad classification of MUD schemes is presented in Figure 1.1.

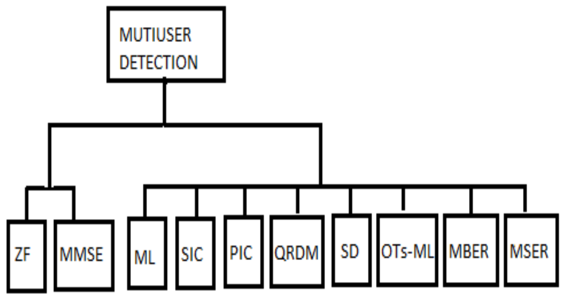
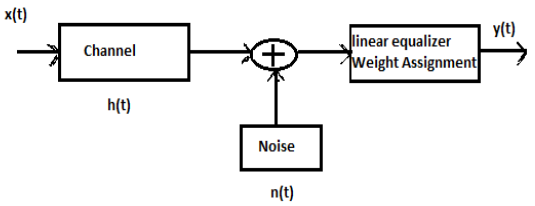


Figure 1.1: Classification of MUD schemes

Descriptions of some of the classical MUDs are given as follows. Among various classical MUD schemes, the linear Zero Forcing (ZF), Minimum Mean Square Error (MMSE), Maximum likelihood (ML) and Likelihood ascent search (LAS) MUDs exhibit low complexity at the cost of limited performance. Hence our aim is to know which technique is best for the detection taking complexity and performance into consideration. Here we use ZF-LAS, MF-LAS, and MMSE-LAS to denote the conventional LAS algorithms with initial vectors generated by Zero Forcing and Minimum Mean Square Error detectors respectively. Here in the below section we firstly discuss about ZF and MMSE

A) Zero Forcing (ZF)

Zero Forcing refers to a form of linear algorithm used in communication systems which applies the inverse of the frequency response of the channel. The Zero-Forcing Equalizer applies the inverse of the channel frequency response to the received signal, to restore the signal after the channel. It has many useful applications. For example, it is studied heavily for MIMO where knowing the channel allows recovery of the two or more streams which will be received on top of each other on each antenna. The name Zero Forcing corresponds to bringing down the intersymbol interference (ISI) to zero in a noise free case. If the channel response (or channel transfer function) for a particular channel is H(s) then the input signal is multiplied by the reciprocal of it. This is intended to remove the effect of channel from the received signal, in particular the intersymbol interference (ISI).



The ZF MUD scheme involves a linear transformation between the output signal and estimated channel.

Y→Received-signal X→Transmitted-signal

H→Channel Matrix N→Noise

$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \\ \vdots \\ \boldsymbol{x_t} \end{pmatrix} \qquad \boldsymbol{y} = \begin{pmatrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \\ \vdots \\ \boldsymbol{y_r} \end{pmatrix}$$

$$\mathbf{H}= \begin{pmatrix} \boldsymbol{h_{11}} & \boldsymbol{h_{12}} & \cdots & \boldsymbol{h_{1t}} \\ \boldsymbol{h_{21}} & \boldsymbol{h_{22}} & \cdots & \boldsymbol{h_{2t}} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{h_{r1}} & \boldsymbol{h_{r2}} & \cdots & \boldsymbol{h_{rt}} \end{pmatrix}$$

The transmitted signal is detected form the least square error $\|\boldsymbol{y}-\boldsymbol{Hx}\|^2$ as

$$\|\boldsymbol{y}-\boldsymbol{Hx}\|^2 = (\boldsymbol{y}-\boldsymbol{Hx})^H(\boldsymbol{y}-\boldsymbol{Hx})$$

$$= -2\boldsymbol{H}^H\boldsymbol{y} + 2\boldsymbol{H}^H\boldsymbol{Hx}$$

The optimal minima of $\boldsymbol{x}$ can be obtained from $\partial\|\boldsymbol{y}-\boldsymbol{Hx}\|^2 / \partial x = 0$. Hence

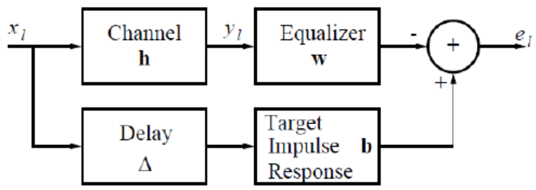
$$-2\boldsymbol{H}^H\boldsymbol{y} + 2\boldsymbol{H}^H\boldsymbol{Hx} = 0$$

$$\hat{\boldsymbol{x}} = \left(\boldsymbol{H}^H\boldsymbol{H}\right)^{-1}\boldsymbol{H}^H\boldsymbol{y}$$

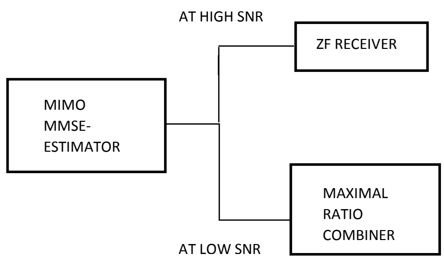
In the above equation $\left(\boldsymbol{H}^H\boldsymbol{H}\right)^{-1}\boldsymbol{H}^H = \boldsymbol{H}^\dagger$, where $\hat{\boldsymbol{H}}^\dagger$ is the pseudo inverse of $\boldsymbol{H}$.

B) MMSE

In telecommunication, a Minimum Mean Square Error (MMSE) estimator is a estimator which follows an estimation method, through which it minimizes the mean square error for the fitted values of various dependent variables. The method MMSE more closely refers to the estimation in a quadratic cost function in Bayesian setting. The thinking procedure behind this Bayesian approach is to estimate from various practical conditions we sometimes have some major information about the parameters which are required to be estimated.



MMSE receiver holds back both interference as well as noise components, but as far as the ZF receiver is concern, it only eliminates the interference or the noise. From this we can conclude that the Mean Square Error (MSE) is minimized. To overcome the drawback of noise enhancement of ZF, the concept of MMSE is introduced. So, we can say that, MMSE is pretentious to ZF in the presence of noise and interference.



The linear MMSE MUD scheme assumes a priori knowledge of noise variance and channel covariance. In this MMSE MUD, the weight matrix '$\boldsymbol{w}$' can be expressed by minimizing the mean square error,

$MSE = E[|\hat{x} - x|^2]$ Where $\hat{x}$ is the estimate of $\boldsymbol{x}$.

$$E\left[|\hat{\boldsymbol{x}}-\boldsymbol{x}|^2\right] = E\left[\left(\boldsymbol{w}^H\boldsymbol{y}-\boldsymbol{x}\right)^H\left(\boldsymbol{w}^H\boldsymbol{y}-\boldsymbol{x}\right)\right]$$

The optimal value of $\boldsymbol{w}^H$ can be obtained from $\partial E\left[|\hat{\boldsymbol{x}}-\boldsymbol{x}|^2\right]/\partial\boldsymbol{w} = 0$. This yield:

$\boldsymbol{w}^H = \boldsymbol{R}_{yy}^{-1}\boldsymbol{R}_{yx}$ Where $R_{yy} = E[yy^H]$ is the auto covariance of $\boldsymbol{y}$ and $R_{yx} = E[yx^H]$ is the cross covariance of $\boldsymbol{y}$ and $\boldsymbol{x}$**,** those are given by [16]:

$$R_{yy} = \left(\boldsymbol{H}^H\boldsymbol{H} + \sigma_n^2\boldsymbol{I}_P\right),$$

$$R_{yx} = H^H$$

Replacing R_{yy} and R_{yx} in the above equation

$$\boldsymbol{w}^H = (\boldsymbol{H}^H\boldsymbol{H} + 2\sigma_n^2\boldsymbol{I}_P)^{-1}\boldsymbol{H}^H$$

$$\hat{\boldsymbol{x}} = (\boldsymbol{H}^H\boldsymbol{H} + 2\sigma_n^2\boldsymbol{I}_P)^{-1}\boldsymbol{H}^H\boldsymbol{y}$$

Where $(.)^H$ indicates Hermitian transpose and $\boldsymbol{I}_P$ is P-dimensional identity matrix. In the above equation, if SNR is high then ${\sigma_n}^2$ will become negligible. Hence, at higher SNR values the performance of ZF and MMSE MUDs are almost equal. In general, the received signal contains residual interference which is not Gaussian distributed due to multiuser interference. But these linear detectors assume that the received signal is corrupted by AWGN only. In addition to that, the linear detectors fail to mitigate the nonlinear degradation caused by the wireless radio environment. Hence, the requirement of a non-linear detector is essential to detect users appropriately.

C) Maximum Likelihood (ML)

The ML detector uses the Maximum a Posteriori (MAP) detection when all the users are equally likely to transmit. The ML detector supporting L simultaneous transmitting users, invokes a total of 2^{mL} metric evaluations in order to detect the possible transmitted symbol vector $\hat{x}$, where m denotes the number of bits per symbol. This detector calculates the Euclidean distance for all possible transmitted signal vectors and estimates the signals as expressed here

$$\hat{\boldsymbol{x}} = \arg\left\{\min_u \|\boldsymbol{y} - \boldsymbol{H}\tilde{\boldsymbol{x}}_u\|^2\right\},\ u = 1, 2, \ldots, 2^{mL}$$

Where u is the set of total metric evaluations associated with the specific modulation order and $\tilde{\boldsymbol{x}}_u = \left[\tilde{x}_u^1, \ldots, \tilde{x}_u^L\right]^T$, $u = 1, 2, \ldots, 2^{mL}$ is a possible transmitted symbol. This optimal detector uses an exhaustive search for finding the most likely transmitted user's signal.

D) Likelihood Ascent Search (LAS Algorithm):

It is well known that the capacity of multiple-input multiple-output (MIMO) channels grows linearly with the minimum of the number of antennas at the transmitter and the receiver sides [11]. Therefore one way to achieve very high spectral efficiency is to exploit large numbers of antennas at both the transmitter and receiver. The main bottleneck of such systems is the complexity of the receivers. A family of low complexity detectors termed Likelihood Ascent Search (LAS) detectors have been proposed in [12] for large MIMO systems. The power of the LAS detector lies in the linear average per bit complexity and the excellent BER performance in large MIMO systems (It has been proven [13] that the asymptotic BER performance of LAS detectors converges to that of maximum likelihood (ML) detector for example). The main disadvantage of LAS detectors, which is the motivation of our work, is that they need very large numbers of antennas to achieve the optimal BER performance, especially in high order modulation [13].A conventional LAS detector [12] starts from an initial solution vector x which can be the output from any known detector such as zero-forcing for example. It then searches through a sequence of solution vectors to refine the solution with monotonic likelihood ascent. At step n, the update algorithm for BPSK modulation using the LAS algorithm can be explained as follows. Given the initial vector x(0) ∈ {+1,−1}L and the search candidate sets (SCS) L(n) ⊆ {1, 2, · · ·L}, ∀ $n \geq 0$, the jth bit of x(n + 1) is given by

$$x_j(n+1) = \begin{cases} +1, & \text{if } x_j(n) = -1 \text{ and } M(x(n+1)) < M(x(n)) \\ -1, & \text{if } x_j(n) = +1 \text{ and } M(x(n+1)) < M(x(n)) \\ x_j(n), & \text{otherwise} \end{cases}$$

where M(x) = ||y − Hx||2 is the likelihood metric for x. Note that the update rule (2) can be simplified and implemented more efficiently as in [12]. The LAS detector checks the candidate bits defined in the sequence of sets L(n) and updates x(n) according to (2). The LAS algorithm reaches a fixed point and terminates when there is no bit flipped in a certain period. Since the update algorithm (2) ensures monotonic likelihood ascent, it is guaranteed to converge to a local maximum likelihood (LML) point which will be the final output of the detector and will occur in a finite number of steps [14][15]. Obviously, the output LML point depends on the initial vector and the sequence of SCS. Specifying a sequence of L (n) for n ≥ 0 and an initial vector x, one determines a particular LAS detector. L (n) should be designed such that all the bits are regularly checked in the sequence of SCS. One straightforward SCS is that each L(n) contains only one element, with element value modulo(n, L), so the detector checks all the L bits one by one in L steps. This sequence is then repeated until no bit is flipped in the last cycle. We should mention that checking (or updating) the bits in different orders may lead to different outputs. This approach to selecting the SCS has been shown to have good BER performance in the family of LAS algorithms.

III Numerical results

In this section, we present the BER performance of the proposed MOS-LAS algorithms, and compare them with the conventional LAS algorithms. Here we use ZF-LAS, MF-LAS, and MMSE-LAS to denote the conventional LAS algorithms with initial vectors generated by Zero Forcing and Minimum Mean Square Error detectors respectively.

Table 3.1: Basic simulation parameters of the SDMA with classical MUDs

Parameters	**Value**
Number of Sub-carrier	128
Length of Guard Band	32
Number of OFDM Frames	1000
Number of Receiving Antennas (P)	4
Number of Users (L)	4
Conjugate Gradient algorithm	
Learning Rate (η)	0.08
Error Precision (β)	0.0001
Initial condition	MMSE solution
FEC Code	
FEC Scheme	Convolutional code
Code rate	1/2
polynomial	(133, 171)

Table 3.2: Basic simulation parameters of the SDMA–OFDM with classical MUDs

Parameters	**Value**
Data frame size (N_F)	1000
Number of data frames (N_D)	1000
Modulation technique	BPSK
Number of Receiving Antennas (P)	128
Number of Users (L)	128
Channel	Rayleigh Flat Fading

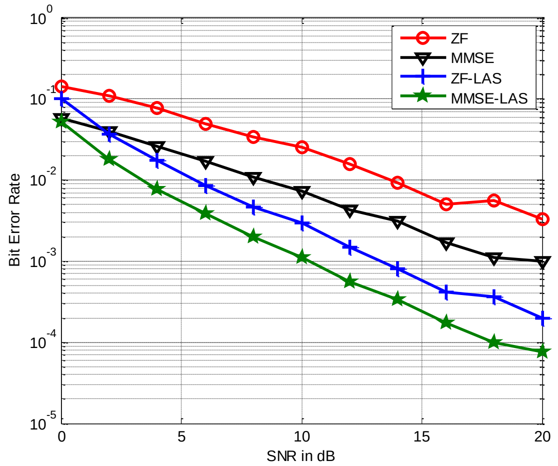


Figure 3.3: Average BER performance of all 64 users using various MUDs for a MU-MIMO system with 128 receiving antennas

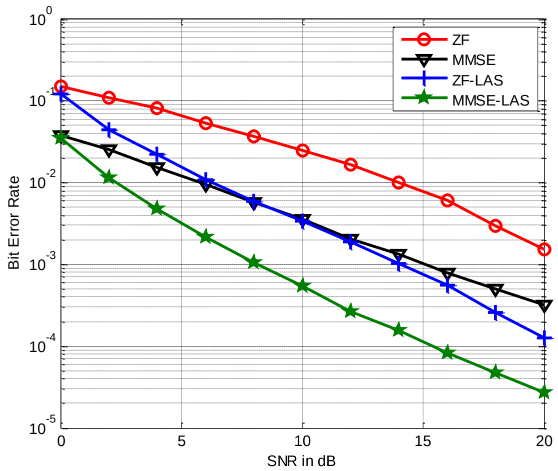
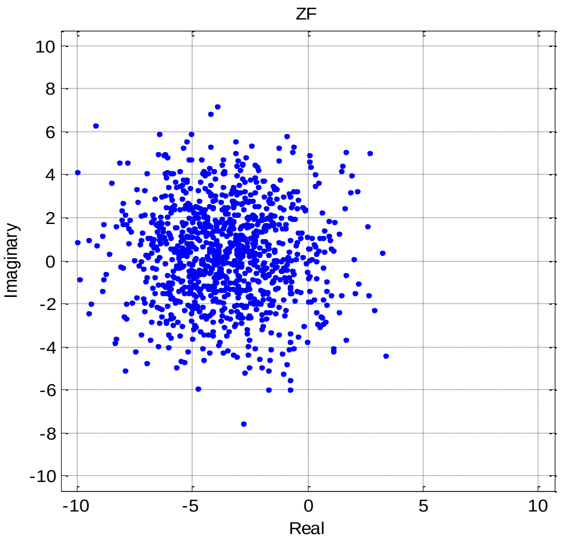
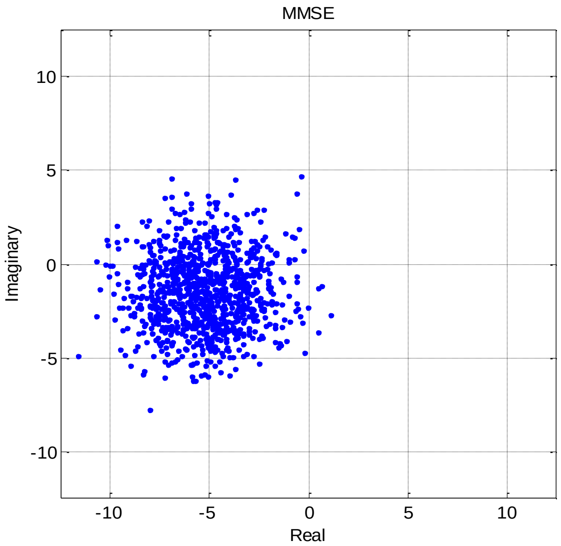


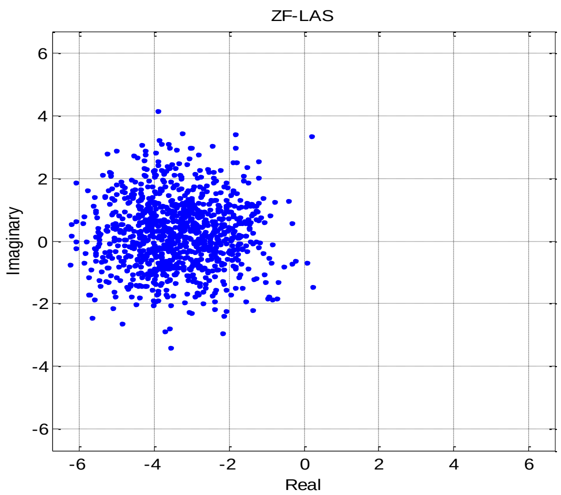
Figure 3.4: Average BER performance of all 128 users using various MUDs for a MU-MIMO system with 128 receiving antennas

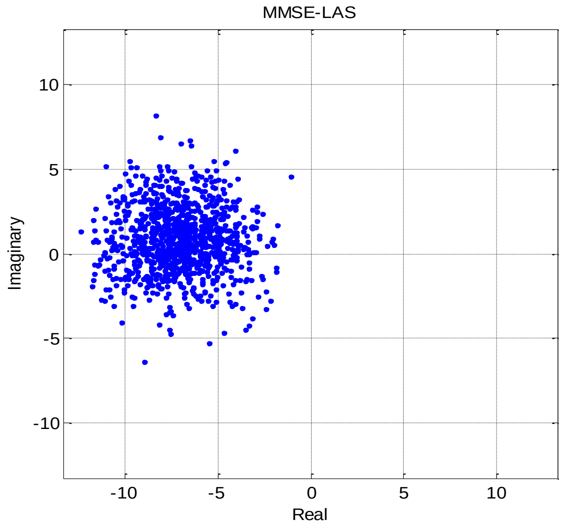


(a)



(b)





(d)

Figure 3.5: Estimated symbol distribution of User-1 using various MUDs when User-1 is always transmitting '–1' at 10 dB SNR value (a) ZF (b) MMSE (c) ZF-LAS (d) MMSE-LAS

Table 3.6: Complexity comparisons

Multiuser Detector	**Complexity order**
ZF	$N_F \times N_D$
MMSE	$N_F \times N_D$
ZF-LAS	$N_F \times N_D \times L$
MMSE-LAS	$N_F \times N_D \times 2^{mL}$

In the above fig 3.3, 3.4 the average BER performance for different proposed techniques have been plotted for two scenarios say 64 and 128 users. By the above plot we can clearly observe that LAS-MMSE has outperformed all the other techniques. Similarly in the boundary conditions scenario fig 3.5, the number of bits falling into the corresponding region has also been good in the case of LAS-MMSE than other techniques. So LAS when applied with initial vectors generated by ZF and MMSE has shown less complexity and optimal BER.

IV CONCLUSION

The basic background of this research work including MIMO system SDM and SDMA system model is presented. Detection schemes may be invoked for the sake of separating different users at the Base Station in an uplink SDMA system. Different classical multiuser detection techniques have been introduced. The performance evaluation of all MUD techniques based on simulation study has been carried out over three typical wireless channel environments in order to shown their adaptability and robustness. The advantages and drawbacks of the linear detection techniques like ZF and MMSE along with some nonlinear detection techniques like ML schemes have been explained. It is observed that, the performance the ML detector is optimal at the cost of additional complexity, especially in the context of a high number of users and for higher order modulation schemes. Also, the ZF and MMSE detectors exhibit low complexity at a cost of performance. The nonlinear successive detection technique outperforms the liner techniques, but still its performance is sub-optimal due to error propagation problem. As we have come across ML technique in the above discussion we faced much complexity issues so in order to reduce the complexity and to gain better optimal performance say higher capacity and BER we considered the LAS technique. Here to the LAS technique we applied multiple inputs and have drawn conclusions from output bits estimated say we have applied different techniques to ascent search namely LAS-ZF and LAS-MMSE and have drawn mathematical results by simulating with the help of MATLAB. Here we have compared different techniques by taking boundary conditions in which the transmitted bits fall into and also the BER performance into consideration and have drawn conclusions saying that LAS-ZF and LAS-MMSE has outperformed the conventional ZF,MMSE and ML techniques. However, all these MUD schemes fail to differentiate users in the critical overload scenario, when the number of users exceed number of BS receiving antenna. Here in this work we have discussed various scenarios followed by advantages and some disadvantages of each and every techniques and have drawn conclusions saying that LAS has better spectral efficiency and lesser complexity in detection of the received signal bits.

References

[1] G. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," Wireless Personal Commun., vol. 6, no. 3, pp. 311–335, 1998.

[2] K. Vardhan, S. Mohammed, A. Chockalingam, and B. Rajan, "A lowcomplexity detector for large mimo systems and multicarrier CDMA systems," IEEE J. Sel. Areas Commun., vol. 26, no. 3, p. 473, 2008.

[3] S. Mohammed, A. Chockalingam, and B. Rajan, "Asymptotic analysis of the performance of LAS algorithm

[4]. S. Verdu, "Multiuser Detection", Camb. Univ. Press, Cambridge, U.K, 1998.or large MIMO detection," online arXiv, vol. 806.

[5]. P.W. Wolniansky, G.J.Foschini, G.D. Golden and R.A. Valenzuela, "V-BLAST: an architecture for realizing very high data rates over the rich-scattering wireless channel," Inter. Symp. on Sig., Sys., and Electron., pp. 295–300, 1998.

[6]. M. Damen, K. A. Meraim and J. C. Belfiore, "Generalized sphere decoder for asymmetrical space-time communication architecture," IEEE Electron. Lett., vol. 36, pp. 166–167, Jan. 2000.

[7]. B. Hochwald and S. ten Brink, "Achieving near-capacity on a multiple-antenna channel," IEEE Trans. On Commun., vol. 51, pp. 389–399, Mar. 2003.

[8]. B. Hassibi and H. Vikalo, "On the sphere decoding algorithm: I expected complexity", IEEE Trans. On Signal Process., vol. 53, pp. 2806–2818, Aug. 2005.

[9]. K. J. Kim, J. Yue, R. A. Iltis and J. D. Gibson, "A QRD-M/Kalman filter-based detection and channel estimation algorithm for MIMO–OFDM systems," IEEE Trans. Wireless Commun., vol. 4, pp. 710–721, Mar. 2005.

[10]. M. Arar and A. Yongacoglu, "Efficient detection algorithm for 2N×2N MIMO systems using alamouti code and QR decomposition", IEEE Commun. Lett., vol. 10, pp. 819–821, Dec. 2006.

[11] G. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, no. 3, pp. 311–335, 1998.

[12] K. Vardhan, S. Mohammed, A. Chockalingam, and B. Rajan, "A low complexity detector for large mimo systems and multicarrier CDMA systems," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 3, p. 473, 2008.

[13] S. Mohammed, A. Chockalingam, and B. Rajan, "Asymptotic analysis of the performance of LAS algorithm for large MIMO detection," online arXiv, vol. 806.

[14] Y. Sun, "A family of linear complexity likelihood ascent search detectors for CDMA multiuser detection," in *Proc. IEEE 6th Intl. Symp. on Spread Spectrum Tech. & App*, 2000.

[15] ——, "A family of likelihood ascent search multiuser detectors: an upper bound of bit error rate and a lower bound of asymptotic multiuser efficiency," *IEEE Trans. Commun.*, vol. 57, no. 6, pp. 1743–1752, June 2009.

[16] J. S. Kim, S. H. Moon, and I. Lee, "A New Reduced Complexity ML Detection Scheme for MIMO Systems," IEEE Trans. on Commun., vol. 58, pp. 1302–1310, Apr. 2010.