Online Appendix for "Measuring Risk Information"

March 2021

In this online appendix, we provide supplementary results and discussion for the paper "Measuring Risk Information." Section 1 assesses when information on market-level risk introduces a bias in our measure, and shows how to empirically test for such a bias. Section 2 provides a steady-state version of our equilibrium model. Section 3 provides simulations that demonstrate the efficacy of our measure to greater variation in the discount rate and its robustness to the steady-state equilibrium model. Section 4 develops an alternative version of our measure that estimates information on the risk of total assets and demonstrates that this version of the measure also passes our validation tests. Section 5 extends our tests of text-based proxies for risk to conference calls. Section 6 provides evidence on the relation between coverage and earnings-announcement volatility. Finally, Section 7 documents the relationship between $RiskInfo_t$ and announcement-date returns.

Contents

1. *RiskInfo*$_t$ and Information on Market-Level Risk

In this section, re-conduct the analysis in the main text allowing for the event to reveal information relevant for predicting the market-level return variance. Our goals are to: (i) determine when this information leads to a bias, (ii) assess how bias may be minimized, and (iii) derive a means to empirically assess the magnitude of the bias created by such information in a given setting. Stated in formal terms, we now allow for the possibility that:

$$\mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[\int_{\tau_D}^{\tau_D+t} \tilde{\sigma}^2_{D,s} ds\right] \neq \mathbb{E}_{\tau_D^-}\left[\int_{\tau_D}^{\tau_D+t} \tilde{\sigma}^2_{D,s} ds\right].$$

Unless the proportion of the risk information in the announcement that is systematic vs. idiosyncratic remains constant over time, this will also tend to lead to a violation of the condition used to derive $AnnVar_{t_1,t_2}$ in the main text,

$$\frac{t_2}{t_2-t_1}\mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[\int_{\tau_D}^{\tau_D+t_1} \tilde{\sigma}^2_{T,s} ds\right] = \frac{t_1}{t_2-t_1}\mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[\int_{\tau_D}^{\tau_D+t_2} \tilde{\sigma}^2_{T,s} ds\right].$$

Thus, we also relax this assumption. In particular, we assume now only that this condition holds for the exposures that are not revealed by the announcement, i.e.,

$$\frac{t_2}{t_2-t_1}\mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[\int_{\tau_D}^{\tau_D+t_1} \left(\tilde{\sigma}^2_{T,s} - \tilde{\sigma}^2_{D,s}\right) ds\right] = \frac{t_1}{t_2-t_1}\mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[\int_{\tau_D}^{\tau_D+t_2} \left(\tilde{\sigma}^2_{T,s} - \tilde{\sigma}^2_{D,s}\right) ds\right]. \quad (1)$$

Furthermore, we assume that, for the exposures revealed by the announcement, the expected variance is constant under the actual, as opposed to risk-neutral, return distribution:

$$\frac{t_2}{t_2-t_1}\mathbb{E}_{\tau_D^-}\left[\int_{\tau_D}^{\tau_D+t_1} \tilde{\sigma}^2_{T,s} ds\right] = \frac{t_1}{t_2-t_1}\mathbb{E}_{\tau_D^-}\left[\int_{\tau_D}^{\tau_D+t_2} \tilde{\sigma}^2_{T,s} ds\right]. \quad (2)$$

Intuitively, this requires only that the announcement reveals the same amount of total information on risk over time – the fraction of this information that is systematic may now vary over time. As in the main text, this assumption is likely violated in many settings, such as when the firm's risk is mean-reverting. However, this is likely to introduce measurement error, rather than bias, into the measure.

Given these relaxed assumptions, letting $\tilde{\pi}_t$ denote the SDF process, we now have:

$$\begin{aligned}
t * \Delta IV_t &= \mathbb{E}^{\mathbb{Q}}_{\tau_D}\left[RV_{\tau_D,\tau_D+t}\right] - \mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[RV_{\tau_D^-,\tau_D+t}\right] \\
&= \mathbb{E}^{\mathbb{Q}}_{\tau_D}\left[RV_{\tau_D,\tau_D+t}\right] - \mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[RV_{\tau_D,\tau_D+t} + \tilde{J}^2\right] \\
&= \mathbb{E}_{\tau_D}\left[\int_{\tau_D}^{\tau_D+t} \tilde{\sigma}^2_{D,s} ds\right] - \mathbb{E}_{\tau_D^-}\left[\int_{\tau_D}^{\tau_D+t} \tilde{\sigma}^2_{D,s} ds\right] - \mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[\tilde{J}^2\right] \\
&\quad + \int_{\tau_D}^{\tau_D+t} cov_{\tau_D}\left[\tilde{\sigma}^2_{D,s}, \tilde{\pi}_s\right] ds - \int_{\tau_D}^{\tau_D+t} cov_{\tau_D^-}\left[\tilde{\sigma}^2_{D,s}, \tilde{\pi}_s\right] ds \\
&= RiskInfo_t - \mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[\tilde{J}^2\right] - \underbrace{\int_{\tau_D}^{\tau_D+t} cov_{\tau_D^-}\left[\tilde{\sigma}^2_{D,s}, \tilde{\pi}_s\right] ds.}_{\text{Event-Date Drop in VRP}}
\end{aligned}$$

We see that there is now an additional deterministic component in the change in implied variance, which equals the covariance between the revealed risk exposures and the SDF, $\int_{\tau_D}^{\tau_D+t} cov_{\tau_D^-}\left[\tilde{\sigma}^2_{D,s}, \tilde{\pi}_s\right] ds$. Under essentially all standard equilibrium and empirical models, this covariance is positive, i.e., assets that are positively exposed to the variance earn negative returns (e.g., **?**, **?**, **?**). Moreover, note that:

$$\begin{aligned}
AnnVar_{t_1,t_2} &= \mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[\tilde{J}^2\right] + \frac{t_2}{t_2-t_1}\mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[\int_{\tau_D}^{\tau_D+t_1} \tilde{\sigma}^2_{T,s} ds\right] - \frac{t_1}{t_2-t_1}\mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[\int_{\tau_D}^{\tau_D+t_2} \tilde{\sigma}^2_{T,s} ds\right] \\
&= \mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[\tilde{J}^2\right] + \frac{t_2}{t_2-t_1}\mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[\int_{\tau_D}^{\tau_D+t_1} \tilde{\sigma}^2_{D,s} ds\right] - \frac{t_1}{t_2-t_1}\mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[\int_{\tau_D}^{\tau_D+t_2} \tilde{\sigma}^2_{D,s} ds\right] \\
&= \mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[\tilde{J}^2\right] + \frac{t_2}{t_2-t_1}\int_{\tau_D}^{\tau_D+t_1} cov_{\tau_D^-}\left[\tilde{\sigma}^2_{D,s}, \tilde{\pi}_s\right] ds \\
&\quad - \frac{t_1}{t_2-t_1}\left\{\int_{\tau_D}^{\tau_D+t_1} cov_{\tau_D^-}\left[\tilde{\sigma}^2_{D,s}, \tilde{\pi}_s\right] ds + \int_{t_1}^{t_2} cov_{\tau_D^-}\left[\tilde{\sigma}^2_{D,s}, \tilde{\pi}_s\right] ds\right\} \\
&= \mathbb{E}^{\mathbb{Q}}_{\tau_D^-}\left[\tilde{J}^2\right] + \int_{\tau_D}^{\tau_D+t_1} cov_{\tau_D^-}\left[\tilde{\sigma}^2_{D,s}, \tilde{\pi}_s\right] ds - \frac{t_1}{t_2-t_1}\int_{t_1}^{t_2} cov_{\tau_D^-}\left[\tilde{\sigma}^2_{D,s}, \tilde{\pi}_s\right] ds,
\end{aligned}$$

where the second line applies equation (1) and the third line applies equation (2) together with the fact that, for any random variable $\tilde{x}$, $\mathbb{E}^{\mathbb{Q}}_{\tau_D^-}[\tilde{x}] = \mathbb{E}_{\tau_D^-}[\tilde{x}] + cov_{\tau_D^-}[\tilde{x}, \tilde{\pi}]$. Now, let $\widehat{RiskInfo}_t = t * \Delta IV_t + AnnVar_{t_1,t_2}$ denote our estimator of $RiskInfo_t$. Assuming $t_1 = t$, we have:

$$\begin{aligned}\widehat{RiskInfo}_t &= RiskInfo_t - \int_{\tau_D}^{\tau_D+t} cov_{\tau_D^-}\left[\tilde{\sigma}^2_{D,s}, \tilde{\pi}_s\right] ds \\ &\quad + \int_{\tau_D}^{\tau_D+t_1} cov_{\tau_D^-}\left[\tilde{\sigma}^2_{D,s}, \tilde{\pi}_s\right] ds - \frac{t_1}{t_2-t_1}\int_{t_1}^{t_2} cov_{\tau_D^-}\left[\tilde{\sigma}^2_{D,s}, \tilde{\pi}_s\right] ds \\ &= RiskInfo_t - \frac{t}{t_2-t}\int_{t}^{t_2} cov_{\tau_D^-}\left[\tilde{\sigma}^2_{D,s}, \tilde{\pi}_s\right] ds,\end{aligned}$$

where, again, $\int_t^{t_2} cov_{\tau_D^-}\left[\tilde{\sigma}^2_{D,s}, \tilde{\pi}_s\right] ds > 0$ under standard equilibrium models. This expression reveals three important facts about the effect of information on market-level risk on our measure:

1. Any news on the market-level variance before the horizon of risk information one seeks to calculate does not create a bias in the measure. In this sense, information on the "short-term" market-level return variance does not influence the measure. The reason is that, while such information pushes down ΔIV_t, it has a precisely offsetting positive impact on $AnnVar_{t,t_2}$. Intuitively, $AnnVar_{t,t_2}$ is derived from the difference between the implied variances from options of maturity t and $t_2 > t$ prior to the announcement. Information on the return variance prior to date t causes the variance risk premium to be disproportionately large in an option with maturity t relative to one with maturity t_2, which leads to an increase in $AnnVar_{t,t_2}$.

2. News on the systematic variance between dates $\tau_D + t$ and $\tau_D + t_2$ negatively biases the measure. Thus, choosing two options with close maturities when calculating $AnnVar_{t_1,t_2}$ minimizes such bias.

3. Because the bias, should it exist, is always negative, one can empirically assess the magnitude of this bias. In particular, a small sample-wide mean of $\widehat{RiskInfo}_t$ indicates the bias is small, while a large negative sample-wide mean of $\widehat{RiskInfo}_t$ indicates the bias is significant.

2. A Steady-State Equilibrium Model

In this section, we develop a steady-state version of the equilibrium model that we present in the paper's Appendix A. Specifically, we consider a Lucas-tree style model in which a firm continuously pays dividends. Investors face uncertainty regarding both the covariance between these dividends and consumption and idiosyncratic dividend volatility. On an anticipated date, news arrives regarding the level of future dividends, their variance, and their covariance with consumption. Formally, consider a representative investor who trades in a firm's stock and options over a time interval $[0, \infty)$. This investor has time-additive CRRA utility:

$$U_t\left(\{\tilde{c}_s\}_{s\in[t,\infty)}\right) = \int_t^\infty \frac{e^{-\delta(s-t)}\tilde{c}_s^{1-\alpha}}{1-\alpha}ds,$$

and has consumption that evolves as:[1]

$$\frac{d\tilde{c}_t}{\tilde{c}_t} = \sum_{i=1}^{k-1}\sigma_{f,i}dW_{i,t}^c.$$

The Brownian motions $\left\{dW_{i,t}^c\right\}_{i\in\{1,\ldots,k-1\}}$ denote the priced risk factors in the economy, and the coefficients $\sigma_{f,i}$ determine their associated risk premia.

The firm pays a dividend $\tilde{x}_t$ at time t, where $\tilde{x}_t$ follows the following stochastic process:

$$d\tilde{x}_t = \tilde{x}_t\mu_x dt + \tilde{x}_t\sum_{i=1}^{k-1}\tilde{\beta}_i dW_{i,t}^c + \tilde{x}_t\tilde{\sigma}_{\varepsilon,t}dW_t^\varepsilon + dN_t^x; \tag{3}$$

$$d\tilde{\sigma}_{\varepsilon,t}^2 = \kappa_\varepsilon\left(\tilde{\sigma}_{\varepsilon,t}^2 - \theta_\varepsilon\right)dt + \xi_\varepsilon\tilde{\sigma}_{\varepsilon,t}dW_t^{\sigma\varepsilon}, \tag{4}$$

and where all Brownian motions in the model are independent. The investor observes W_t^ε at date t, which leads the equilibrium price to continuously evolve. Note the terms $\tilde{\beta}_i$ capture the firm's systematic risk-factor exposures, while $\tilde{\sigma}_{\varepsilon,t}$, which follows a CIR process, captures the firm's time-varying idiosyncratic volatility. The assumption that the firm's risk-factor exposures are constant allows for a simple closed-form expression for price, but is not essential for the results.

As in Appendix A, at time τ_D that is known in advance to the investor, an announcement occurs. The firm's dividend process exhibits a jump on the date of the information event, which is captured by dN_t^x, where:

$$N_t^x = \tilde{x}_{\tau_D^-}\left(\exp\left(\tilde{J}_x\right) - 1\right) * \mathbf{1}\left(t \geq \tau_D\right),$$

where $\tilde{J}_x$ is independent of other variables in the model. We again assume that this announcement perfectly resolves all distributional uncertainty, i.e., it perfectly reveals $\left\{\tilde{\beta}_i\right\}_{i\in\{1,\ldots,k-1\}}$ and $\{\tilde{\sigma}_{\varepsilon,t}\}_{t\in[\tau_D,T]}$. In order to ensure that there exists an equilibrium price, we assume that,

[1] Growth in consumption can be incorporated into the model with no change in the results, outside of a shift in the risk-free rate.

for any realization of $\left\{\tilde{\beta}_i\right\}_{i\in\{1,\ldots,k-1\}}$,

$$r-\mu_x+\alpha\sum_{i=1}^{k-1}\tilde{\beta}_i\sigma_{f,i}>0.$$

We next derive the firm's equilibrium price process.

Proposition 1. *Let* $r=\delta-\frac{\alpha(1+\alpha)}{2}\sum_{i=1}^{k-1}\sigma_{f,i}^2$. *The firm's stock price just prior to the announcement,* $\lim_{t\to\tau_D^-}P_t$, *satisfies:*

$$\lim_{t\to\tau_D^-}P_t=\tilde{x}_{\tau_D^-}\mathbb{E}_{\tau_D^-}\left[\exp\left(\tilde{J}_x\right)\left(r-\mu_x+\alpha\sum_{i=1}^{k-1}\tilde{\beta}_i\sigma_{f,i}\right)^{-1}\right], \tag{5}$$

and the firm's price at a date following the announcement, $t\geq\tau_D$ *satisfies:*

$$P_t=\tilde{x}_t\left(r-\mu_x+\alpha\sum_{i=1}^{k-1}\tilde{\beta}_i\sigma_{f,i}\right)^{-1}. \tag{6}$$

On date τ_D, *the firm's price jumps, such that:*

$$\frac{P_t}{\lim_{t\to\tau_D^-}P_t}=\frac{\exp\left(\tilde{J}_x\right)\left(r-\mu_x+\alpha\sum_{i=1}^{k-1}\tilde{\beta}_i\sigma_{f,i}\right)^{-1}}{\mathbb{E}_{\tau_D^-}\left[\exp\left(\tilde{J}_x\right)\left(r-\mu_x+\alpha\sum_{i=1}^{k-1}\tilde{\beta}_i\sigma_{f,i}\right)^{-1}\right]}.$$

Following date τ_D, *the firm's value evolves as:*

$$\frac{dP_t}{P_t} = \mu_x dt+\sum_{i=1}^{k-1}\tilde{\beta}_i dW_{i,t}^c+\tilde{\sigma}_{\varepsilon,t}dW_t^{\varepsilon}. \tag{7}$$

$$\text{where } d\tilde{\sigma}_{\varepsilon,t}^2 = \kappa_\varepsilon\left(\tilde{\sigma}_{\varepsilon,t}^2-\theta_\varepsilon\right)dt+\xi_\varepsilon\tilde{\sigma}_{\varepsilon,t}dW_t^{\sigma\varepsilon}. \tag{8}$$

Proof. Let $\tilde{C}_t\equiv\ln\left(\tilde{c}_t\right)$. Standard arguments imply that, in this economy, a necessary condition for the representative agent's first-order condition to be satisfied is that there exists a stochastic-discount factor process M_t, where (e.g., **?**):

$$\begin{aligned} M_t &= e^{-\delta t}\frac{\tilde{c}_t^{-\alpha}}{c_0^{-\alpha}} \\ &= \exp\left[-\delta t-\alpha\left(\tilde{C}_t-C_0\right)\right]. \end{aligned} \tag{9}$$

This implies the date t price of a bond paying off 1 dollar at date $s > t$ equals:

$$\mathbb{E}_t\left[\frac{M_s}{M_t}\right] = \mathbb{E}_t\left[\exp\left[-\delta(s-t) - \alpha\left(\tilde{C}_s - \tilde{C}_t\right)\right]\right]. \tag{10}$$

Note that, as of date t,

$$-\alpha\left(\tilde{C}_s - \tilde{C}_t\right) \sim N\left(\frac{\alpha\,(s-t)\sum_{i=1}^{k-1}\sigma_{f,i}^2}{2}, \alpha^2\,(s-t)\sum_{i=1}^{k-1}\sigma_{f,i}^2\right) \tag{11}$$

Therefore, $\mathbb{E}_t\left[\frac{M_s}{M_t}\right] = \exp\left[\left(-\delta + \frac{\alpha(1+\alpha)}{2}\sum_{i=1}^{k-1}\sigma_{f,i}^2\right)(s-t)\right]$, which implies that the equilibrium instantaneous return on the bond is $r \equiv \delta - \frac{\alpha(1+\alpha)}{2}\sum_{i=1}^{k-1}\sigma_{f,i}^2$. Moreover, the stock price satisfies:

$$\begin{aligned} P_t &= \int_t^\infty \mathbb{E}_t\left[\tilde{x}_s\frac{M_s}{M_t}\right]ds \qquad (12)\\ &= \int_t^\infty e^{-\delta(s-t)}\mathbb{E}_t\left[\tilde{x}_s\exp\left[-\alpha\left(\tilde{C}_s - \tilde{C}_t\right)\right]\right]ds. \end{aligned}$$

Now, note that, for $s > t$:

$$\begin{aligned} \tilde{x}_s &= \tilde{x}_t * \exp\left(\mathbf{1}\left(\tau_D \in (t,s]\right)\tilde{J}_x\right) \qquad (13)\\ &\quad * \exp\left(\mu_x\,(s-t) - \frac{\int_t^s\tilde{\sigma}_{\varepsilon,y}^2dy}{2} - \frac{(s-t)\sum_{i=1}^{k-1}\tilde{\beta}_i^2}{2} + \sum_{i=1}^{k-1}\int_t^s\tilde{\beta}_i dW_{i,y}^c + \int_t^s\tilde{\sigma}_{\varepsilon,y}dW_y^\varepsilon\right). \end{aligned}$$

Substituting, applying iterated expectations, and simplifying, we have that:

$$\begin{aligned} &\mathbb{E}_t\left\{\tilde{x}_s\exp\left[-\alpha\left(\tilde{C}_s - \tilde{C}_t\right)\right]\right\} \qquad (14)\\ =\ &\mathbb{E}_t\left\{\mathbb{E}_t\left[\tilde{x}_s\exp\left[-\alpha\left(\tilde{C}_s - \tilde{C}_t\right)\right] \mid \left\{\tilde{\beta}_i\right\}_{i=1}^{k-1}, \left\{\tilde{\sigma}_{\varepsilon,y}\right\}_{y\in[t,s]}\right]\right\}\\ =\ &\exp\left(\alpha\tilde{C}_t + \mu_x\,(s-t)\right)\tilde{x}_t\mathbb{E}_t\left\{\exp\left(\mathbf{1}\left(\tau_D \in (t,s]\right)\tilde{J}_x - \frac{\int_t^s\tilde{\sigma}_{\varepsilon,y}^2dy}{2} - \frac{(s-t)\sum_{i=1}^{k-1}\tilde{\beta}_i^2}{2}\right)\right.\\ &\left. * \mathbb{E}_t\left[\exp\left(\sum_{i=1}^{k-1}\int_t^s\tilde{\beta}_i dW_{i,y}^c - \alpha\tilde{C}_s + \int_t^s\tilde{\sigma}_{\varepsilon,y}dW_y^\varepsilon\right) \mid \left\{\tilde{\beta}_i\right\}_{i=1}^{k-1}, \left\{\tilde{\sigma}_{\varepsilon,y}\right\}_{y\in[t,s]}\right]\right\}. \end{aligned}$$

To evaluate the expectations in this expression, note that, as of date t, conditional on $\left\{\tilde{\beta}_i\right\}_{i=1}^{k-1}$ and $\left\{\tilde{\sigma}_{\varepsilon,y}\right\}_{y\in[t,s]}$,

$$\sum_{i=1}^{k-1}\int_t^s\tilde{\beta}_i dW_{i,y}^c - \alpha\tilde{C}_s \sim N\left(-\alpha\tilde{C}_t + \frac{\alpha\,(s-t)\sum_{i=1}^{k-1}\sigma_{f,i}^2}{2}, (s-t)\sum_{i=1}^{k-1}\left(\tilde{\beta}_i - \alpha\sigma_{f,i}\right)^2\right). \tag{15}$$

Furthermore, $\int_t^s \tilde{\sigma}_{\varepsilon,y} dW_y^{\varepsilon} | \left\{\tilde{\sigma}_{\varepsilon,y}\right\}_{y\in[t,s]} \sim N\left(0, \int_t^s \tilde{\sigma}_{\varepsilon,y}^2 dy\right)$. Thus, expression (14) reduces to:

$$\begin{aligned}
& \tilde{x}_t \mathbb{E}_t \left[\exp\left(\mu_x (s-t) + (s-t)\sum_{i=1}^{k-1} \frac{\left[\left(\tilde{\beta}_i - \alpha\sigma_{f,i}\right)^2 + \alpha\sigma_{f,i}^2 - \tilde{\beta}_i^2\right]}{2} + \mathbf{1}(\tau_D \in (t,s])\,\tilde{J}_x\right)\right] \\
= \ & \tilde{x}_t \mathbb{E}_t \left[\exp\left(\mu_x (s-t) + \alpha (s-t)\left(\frac{1+\alpha}{2}\sigma_{f,i}^2 - \sum_{i=1}^{k-1}\tilde{\beta}_i \sigma_{f,i}\right) + \mathbf{1}(\tau_D \in (t,s])\,\tilde{J}_x\right)\right] \qquad (16)
\end{aligned}$$

This implies that:

$$\begin{aligned}
\lim_{t\to\tau_D^-} P_t &= \lim_{t\to\tau_D^-} \int_t^\infty e^{-\delta(s-t)} \mathbb{E}_t \left[\tilde{x}_s \frac{\tilde{c}_s^{-\alpha}}{c_t^{-\alpha}}\right] ds \qquad (17) \\
&= \lim_{t\to\tau_D^-} \int_t^\infty \tilde{x}_t \mathbb{E}_t \left[\exp\left((\mu_x - r)(s-t) - \alpha (s-t)\sum_{i=1}^{k-1}\tilde{\beta}_i\sigma_{f,i} + \mathbf{1}(\tau_D \in (t,s])\,\tilde{J}_x\right)\right] ds \\
&= \tilde{x}_{\tau_D^-} \lim_{t\to\tau_D^-} \mathbb{E}_t \left[\int_t^\infty \exp\left((\mu_x - r)(s-t) - \alpha (s-t)\sum_{i=1}^{k-1}\tilde{\beta}_i\sigma_{f,i} + \mathbf{1}(\tau_D \in (t,s])\,\tilde{J}_x\right) ds\right].
\end{aligned}$$

Note that, as $t \to \tau_D^-$, the region of the above integral on which $\mathbf{1}(\tau_D \in (t,s]) = 0$ shrinks towards zero. Moreover, on this region, the integrand in the above expression is bounded. Consequently, applying the bounded convergence theorem, the above expression reduces to:

$$\begin{aligned}
& \tilde{x}_{\tau_D^-} \lim_{t\to\tau_D^-} \mathbb{E}_t \left[\exp\left(\tilde{J}_x\right)\int_t^\infty \exp\left((\mu_x - r)(s-t) - \alpha (s-t)\sum_{i=1}^{k-1}\tilde{\beta}_i\sigma_{f,i}\right) ds\right] \qquad (18) \\
= \ & \tilde{x}_{\tau_D^-} \mathbb{E}_t \left[\exp\left(\tilde{J}_x\right) \frac{1}{r - \mu_x + \alpha\sum_{i=1}^{k-1}\tilde{\beta}_i\sigma_{f,i}}\right],
\end{aligned}$$

which verifies equality (5). Next, for $t \geq \tau_D$, note that:

$$\begin{aligned}
P_t &= \int_t^\infty \tilde{x}_t \mathbb{E}_t \left[\exp\left((\mu_x - r)(s-t) + \mathbf{1}(\tau_D \in (t,s])\,\tilde{J}_x - \alpha (s-t)\sum_{i=1}^{k-1}\tilde{\beta}_i\sigma_{f,i}\right)\right] ds \\
&= \int_t^\infty \tilde{x}_t \exp\left((\mu_x - r)(s-t) - \alpha (s-t)\sum_{i=1}^{k-1}\tilde{\beta}_i\sigma_{f,i}\right) ds \qquad (19) \\
&= \tilde{x}_t \left(r - \mu_x + \alpha\sum_{i=1}^{k-1}\tilde{\beta}_i\sigma_{f,i}\right)^{-1},
\end{aligned}$$

which verifies equality (6). Next, let $R_p \equiv \left(r - \mu_x + \alpha\sum_{i=1}^{k-1}\tilde{\beta}_i\sigma_{f,i}\right)^{-1}$. Applying Ito's lemma to the above expression, for $t \geq \tau_D$,

$$
\begin{aligned}
dP_t &= \mu_x \tilde{x}_t R_p dt + \tilde{x}_t R_p \sum_{i=1}^{k-1} \tilde{\beta}_i dW_{i,t}^c + \tilde{x}_t R_p \tilde{\sigma}_{\varepsilon,t} dW_t^{\varepsilon} \\
&= P_t \left(\mu_x dt + \sum_{i=1}^{k-1} \tilde{\beta}_i dW_{i,t}^c + \tilde{\sigma}_{\varepsilon,t} dW_t^{\varepsilon} \right),
\end{aligned} \tag{20}
$$

which completes the derivation of equation (7). □

The price process in the model matches the one in the main text upon setting $\mu_t = \mu_x$ and:

$$
\begin{aligned}
\tilde{\sigma}_{i,t} &= \tilde{\beta}_i \text{ for } t \in [\tau_D, \infty),\ i = 1, ..., k-1; \\
\tilde{\sigma}_{k,t} &= \tilde{\sigma}_{\varepsilon,t} \text{ for } t \in [\tau_D, \infty); \\
\tilde{y} &= \tilde{J}_x - \ln \mathbb{E}_{\tau_D^-} \left[\exp\left(\tilde{J}_x \right) \right]; \\
g\left(\{\tilde{\sigma}_{i,t}\}_{i \in \{1,\ldots,k\}} \right) &= \ln \left[\frac{\left(r - \mu_x - \alpha \sum_{i=1}^{k-1} \tilde{\beta}_i \sigma_{f,i}^2 \right)^{-1}}{\mathbb{E}_{\tau_D^-}\left[\left(r - \mu_x - \alpha \sum_{i=1}^{k-1} \tilde{\beta}_i \sigma_{f,i}^2 \right)^{-1} \right]} \right].
\end{aligned} \tag{21}
$$

In the next section, we simulate this model and show that our measure is effective at capturing the risk information in the event.

3. Additional Simulations

In this section, we conduct two additional simulations, which we summarize below.

1. Simulating the Steady-State Model. We first simulate the model in the previous section. We assume that $\tilde{J}_x \sim N\left(-\frac{1}{2}\sigma_J^2, \sigma_J^2\right)$ and adjust θ_ε to match $\tilde{\sigma}_{\varepsilon,0}^2$ in order to ensure that the stock and variance price paths begin in steady state, and again focus on a single risk factor $W_{1,t}^c$, and thus a single risk-factor exposure, $\tilde{\beta}_1$ and assume that it is uniformly distributed on $[\beta_L, \beta_H]$. As in the simulations in the text, we calculate the measures using ATM option prices and examine 30-day and 182-day versions of our measure. We assess the average absolute deviation between the measure and the true change in investors' beliefs. The parameters and results are summarized in Figure 1. The figure shows that the average error in our measure remains below 10%.

2. Additional variation in the discount rate. We next consider how the error in our measure varies with the amount of variation in the discount rate on the earnings date. This lets us assess whether information on firms' betas creates an error in our measure. We conduct these simulations under the same specification applied in the simulations in the main text. As discussed in the text, information on firms' betas can create return

Figure 1. This figure displays the results of our simulations. In these simulations, we vary $\sigma_J^2, \xi_\varepsilon$, and $\tilde{\sigma}_{\varepsilon,0}^2 = \theta_\varepsilon$ on a 6^3 dimensional grid ranging uniformly over $[0, 0.3] \times [0, 0.3] \times [0.3, 1]$ and fix $\kappa = 2$. We then set δ such that the risk-free rate r is 4% and choose α, $\sigma_{f,1}$, β_H and β_L such that the stock's average discount rate in excess of r is 5%, and such that, following the announcement, the discount rate varies by $+/-$ 20 bps. For each set of parameters, we first generate variance paths and option prices. We then calculate our measure using these option prices and calculate the percentage error as the difference between the measure and the true change in the market's beliefs regarding the firm's risk as a fraction of the average level of firm risk over the relevant horizon. The plots depict the average percentage error as the parameters vary.

Figure 2. This figure displays the performance of our measure as a function of variation in the discount rate. In these simulations, we set $\sigma_J^2 = 0.15^2, \xi_\varepsilon = 0.15, \tilde{\sigma}_{\varepsilon,0}^2 = \theta_\varepsilon = 0.65, \kappa = 2$, and set δ such that the risk-free rate r is 4%. For each set of parameters, we first generate variance paths and option prices. We then calculate our measure using these option prices and calculate the percentage error as the difference between the measure and the true change in the market's beliefs regarding the firm's risk as a fraction of the average level of firm risk over the relevant horizon.

autocorrelation, which can cause $AnnVar$ to overshoot the true expected return variance on the announcement date. Figure 2 depicts the parameters and results. It shows that the error in our measure grows with the amount of discount rate variation. However, even when the firm's discount rate varies uniformly between $+/-2\%$, the error in the measure remains below 10%. To put this in perspective, recall that we find the difference between the firm's cost of capital in the upper and lower quintiles of $RiskInfo$ is 40 basis points in our quarterly earnings sample. Thus, even when the variation in discount rate is an order of magnitude larger than in our sample, the error remains small.

4. Leverage Effects: A Measure of Information on Asset Risk

Prior literature originating with **?** shows that a major determinant of a stock's risk arises from the implicit leverage embedded in equity when the firm is financed in part by debt. This "leverage effect," and its associated impact on equity risk, increases when a firm's expected equity value decreases. Given that our measure is derived from equity options, it captures an announcement's information regarding equity risk, and thus captures this leverage effect. Consistent with this, we show in the main text in the case of quarterly earnings announcements, positive news tends to decrease, and negative news tends to increase our measure.

When applying the measure to test hypotheses regarding equity risk, it is, in fact, desirable for our measure to capture this leverage effect. However, in some contexts, one might wish to capture information on the risk of total assets, i.e., the combined risk faced by equity- and debt-holders. For instance, when evaluating the effects of regulation that requires a novel disclosure on firm risk, one may seek to understand both how it influences information on the total risk faced by both equity- and debt-holders.

We next outline a simple approach to calculate a variant of our measure that estimates asset-level risk. This approach is based upon **?**, who develop and validate a simple formula for translating equity-level implied volatilities into asset-level implied volatilities.[2] Their formula is as follows:

$$\sigma_V \approx \frac{E}{E+F}\sigma_E + \frac{F}{E+F}\left(0.05 + 0.25\sigma_E\right), \tag{22}$$

where E is the market value of equity, F is the total face value of its debt, and σ_E is the implied volatility from equity options. To generate the asset-level version of our measure, we recalculate $RiskInfo_{30}$ by replacing implied volatilities with the ex ante and ex post versions

[2]We are grateful to an anonymous reviewer for suggesting this approach.

of σ_V in equation (22). In doing so, we account for changes in firms' equity prices on the event date, which effectively adjusts these volatilities for the leverage effect.

When applying the measure to quarterly earnings, we hold constant the total face value of the firm's debt at its level on the quarterly earnings date under consideration. This approach effectively assumes that investors are already aware of any material changes in the firm's capital structure that occurred during the quarter before earnings are made public. This is consistent with such changes being pre-empted via 8-K's. In more general applications, F should equal the best estimate of investors' beliefs regarding the market value of the firm's debt.

As an example of how the leverage-corrected measure can be applied, Table 1 of this document illustrates that the leverage-corrected measure continues to predict changes in volatility, abnormal volatility, liquidity, spreads, and firm fundamentals. These results suggest the leverage effect is not the exclusive driver of risk information in quarterly earnings announcements.

5. Textual Analyses of Conference Calls

To supplement our analysis of text-based proxies of risk information, we compare $RiskInfo_t$ calculated around firms' earnings announcements against the level of, and within-firm change in, the number of uncertainty-related words, as defined in **?**, used in firms' conference call transcripts. *Uncertain Words (%)* equals the percentage of uncertainty-related words in the conference call. *Uncertain Words (Level)* equals the log of one plus the number of uncertainty-related words in the conference call. Similarly, Δ*Uncertain Words (%)* and Δ*Uncertain Words (Level)* reflect the within-firm change in each measure relative to the same quarter in the prior year.

The results in Table 3 of this document shows that text-based proxies based on uncertain words do not portend increases in volatility. Consistent with the findings in Table 7 of the main paper, these findings suggest risk-word-based proxies are more likely to reflect managers' acknowledgement of or explanation for the *level* of firms' risks, than to capture the novel information investors glean about risks from firms' 10Ks.

6. Analyst Coverage and Announcement Volatility

Table 3 of the main paper documents a positive relation between COV and $AnnVar$, indicating investors anticipate heightened announcement volatility for firms with greater analyst coverage. In Table 3 of this document, we verify this relation using firms' absolute earnings announcement returns. The heightened volatility for firms with greater coverage is consistent with prior evidence that analysts tend to time their forecast and recommendation revisions around earnings announcements (e.g., **?**, **?**) and that competition spurs more timely analyst behavior (e.g., **?**)

7. Risk Information and Returns

Table 4 of this document examines the link between risk information and equity returns. The table contains averages of, and corresponding t-statistics for, firms' raw and market-adjusted returns relative to their earnings announcement date, sorted into quintile portfolios of $RiskInfo_{30}$. We find that risk information is negatively related to firms' earnings announcement returns consistent with investors' perceptions of risk increasing when firms report negative information.[3]

[3] We find no significant relation between risk information and *future* returns. The absence of a link with post-announcement returns can be squared with the finding that ICCs tend to rise with risk information due to the presence of post-earnings announcement drift (given that negative earnings are associated with an increase in $RiskInfo_{30}$) and the fact that realized returns are notoriously noisy as an indication of expected returns (**?**).

Table 1. Leverage Correction

This table contains results from re-estimating our main tests when using a leverage corrected version of our risk information measure as described in Section 3. $RiskInfo_{30}$ is our (leveraged corrected) proxy for the amount of information about risk contained in a firm's disclosure from 30-day options. ΔIV_{30} is the change in 30-day implied variance from standardized options in the three-day window centered on firms' announcement date. $\Delta VLTY_{30}$, is defined as the log of the standard deviation of firm's daily returns over the 30 days starting 5 days after the earnings announcement, scaled by the standard deviation of firm's daily returns over the 30 days ending 5 days prior to the earnings announcement. ΔICC is the post- versus pre-announcement change in firms' implied cost of capital (ICC) using the estimation approach in **?**. *RiskInfoTerm* captures the difference in risk information derived from 30-day and 182-day options. $\Delta AMIHUD$ is the within-firm changes in firms' Amihud illiquidity ratio, defined as absolute returns scaled by dollar trading volume in the months surrounding the announcement date. *RiskInfoTerm*, which captures the difference in risk information derived from 30-day and 182-day options and is defined as $RiskInfo_{30}$ minus $RiskInfo_{182}$. *VolTime* is the post-announcement volatility term-structure, defined as the difference in the standard deviation of firm's daily market-adjusted returns from 5 to 35 trading days after the earnings announcement, relative to the standard deviation of firm's daily returns from 36 to 187 trading days after the earnings announcement. ΔIV_{30} is the change in 30-day implied variance from standardized options in the three-day window centered on firms' announcement date and ΔIV_{182} is defined analogously. *IVTime* is the level difference in the two implied variance change measures. $\Delta AltmanZ - Score$ and *Campbell Proxy* reflect the changes in firms' distress risk as measured by firm's Altman Z score and the distress risk proxy from **?**. $\Delta R\&DSpending$ in the forward change in firms' research and development spending scaled by total assets. *SIZE* is the log of market capitalization, *LBM* is the log of firm's book-to-market ratio, and *SURP* is the firms' analyst-based surprise at their earnings announcement. Firm and year fixed effects are used throughout. The parentheses contain t-statistics based on standard errors clustered by firm and year.

Panel A: Leverage Correction when Forecasting Volatility

	(1)	(2)	(3)	(4)
	$\Delta VLTY$	ΔGLS	$\Delta AMIHUD$	Δ *VW-Spreads*
$RiskInfo_{30}$	0.323***	5.704**	0.826***	0.294***
	(5.06)	(2.86)	(2.97)	(3.58)
ΔIV_{30}	2.290***	15.987***	2.605***	1.586***
	(6.92)	(6.55)	(7.00)	(7.73)
SIZE	0.023***	0.408***	0.079***	0.035***
	(4.64)	(8.88)	(9.76)	(6.73)
LBM	-0.054***	-1.698***	-0.035	-0.018
	(-4.65)	(-4.69)	(-1.52)	(-1.33)
SURP	-0.349	-6.129	-2.368***	-1.046***
	(-1.58)	(-1.71)	(-5.14)	(-4.70)
N	85690	41436	85704	84570
adj. R^2	0.048	0.035	0.070	0.032

Panel B: Leverage Correction when Forecasting Fundamentals

	(1)	(2)	(3)	(4)
	VolTime	Δ*Altman Z-Score*	ΔCampbell Proxy	Δ*R&D Spending*
$RiskInfo_{30}$ (w/Leverage Correction)	–	-1.787***	-0.204***	0.002**
	–	(-3.32)	(-5.83)	(2.69)
ΔIV_{30}	–	-4.552***	-0.493***	0.003*
	–	(-3.76)	(-4.61)	(1.78)
RiskInfoTerm (w/Leverage Correction)	0.021***	–	–	–
	(5.18)	–	–	–
IVTime	0.007***	–	–	–
	(3.70)	–	–	–
SIZE	-0.023***	-0.470***	-0.046***	0.000***
	(-3.71)	(-4.45)	(-5.96)	(4.10)
LBM	0.014	-0.109	0.019**	0.001***
	(0.75)	(-1.64)	(2.68)	(5.83)
SURP	-0.305**	1.906*	0.369***	0.007**
	(-2.14)	(1.74)	(2.85)	(2.53)
N	85267	62680	62681	71997
adj. R^2	0.061	0.059	0.080	0.009

Table 2. Analysis of Conference Call Transcripts

Panel A contains Spearman correlations above (below) the main diagonal. ΔIV_{30} is change in 30-day implied variance from standardized options in the three-day window centered on firms' announcement date. *SIZE* is the log of market capitalization, $VLTY_{30}$ is the standard deviation of firm's daily returns over the 30 trading days ending 5 days prior to the earnings announcement, and $\Delta VLTY_{30}$ is the post-announcement change in return volatility, defined as the level difference in the standard deviation of firm's daily returns over the 30 trading days starting 5 days after the earnings announcement relative to $VLTY_{30}$. *Uncertain Words (%)* equals the percentage of uncertainty-related words in the conference call. *Uncertain Words (Level)* equals the log of one plus the number of uncertainty-related words in the conference call. Similarly, Δ*Uncertain Words (%)* and Δ*Uncertain Words (Level)* reflect the within-firm change in each measure relative to the same quarter in the prior year. Panel B contains results from regressions of $\Delta VLTY_{30}$ with firm- and time-fixed effects. The sample for this analysis spans 1996 through 2019 and consists of 52,274 quarterly earnings announcements.

Panel A: Spearman Correlations

	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*
(1) RiskInfo$_{30}$		0.987	0.496	0.055	0.061	0.006	0.021	0.005	0.006
(2) SRiskInfo$_{30}$	0.987		0.495	0.047	0.061	0.011	0.020	0.005	0.007
(3) ΔIV_{30}	0.496	0.495		0.124	0.082	0.017	0.033	-0.006	-0.010
(4) SIZE	0.055	0.047	0.124		0.056	-0.282	0.133	-0.016	-0.001
(5) $\Delta VLTY$	0.061	0.061	0.082	0.056		-0.025	-0.032	-0.015	-0.015
(6) UncertainWords (%)	0.006	0.011	0.017	-0.282	-0.025		0.337	0.433	0.256
(7) UncertainWords (Level)	0.021	0.020	0.033	0.133	-0.032	0.337		0.249	0.456
(8) Δ*UncertainWords (%)*	0.005	0.005	-0.006	-0.016	-0.015	0.433	0.249		0.532
(9) Δ*UncertainWords (Level)*	0.006	0.007	-0.010	-0.001	-0.015	0.256	0.456	0.532	

Panel B: Regressions of Changes in Volatility

	(1)	(2)	(3)	(4)
RiskInfo$_{30}$	4.756***	4.800***	4.901***	4.906***
	(5.33)	(5.34)	(4.88)	(4.89)
ΔIV_{30}	3.631***	3.572***	3.286***	3.282***
	(4.08)	(4.03)	(3.37)	(3.37)
SIZE	0.042**	0.046**	0.054**	0.054**
	(2.31)	(2.51)	(2.58)	(2.59)
LBM	-0.206**	-0.201**	-0.205**	-0.205**
	(-2.73)	(-2.68)	(-2.41)	(-2.41)
SURP	-3.087*	-3.135**	-4.219**	-4.229**
	(-2.09)	(-2.15)	(-2.54)	(-2.55)
UncertainWords (%)	-7.288	–	–	–
	(-1.74)	–	–	–
UncertainWords (Level)	–	-0.097***	–	–
	–	(-3.79)	–	–
Δ*UncertainWords (%)*	–	–	-3.869	–
	–	–	(-1.27)	–
Δ*UncertainWords (Level)*	–	–	–	-0.026
	–	–	–	(-1.09)
Obs	52282	52282	46102	46102
Adj. R^2	0.045	0.045	0.047	0.047
Firm FE?	Y	Y	Y	Y
Year FE?	Y	Y	Y	Y

Table 3. Link Between Analyst Coverage and Absolute Returns

This table contains regression results of the absolute value of three-day market-adjusted earnings announcement returns on firms' analyst-based earnings surprises ($SURP$) and the log of one plus the number of analysts covering a firm (COV). Firm and year-by-NYSE-size-decile fixed effects are used as indicated at the bottom of the table. The parentheses contain t-statistics based on standard errors clustered by firm and year. The sample for this analysis spans 1996 through 2019 and consists of 87,460 quarterly earnings announcements.

	(1)	(2)
COV	0.161*	0.168*
	(1.72)	(1.81)
SURP	–	-11.502***
	–	(-2.96)
Obs	83247	83247
adj. R^2	0.201	0.202
Firm FE?	Y	Y
Year x NYSD FE?	Y	Y

Table 4. Link Between Risk Information and Returns

This table contains averages of, and corresponding t-statistics for, firms' raw and market-adjusted returns relative to their earnings announcement date, sorted into quintile portfolios of $RiskInfo_{30}$. $RiskInfo_{30}$ is our proxy for the amount of information about risk contained in a firm's disclosure from 30-day options. Quintile portfolios are formed on a rolling basis, using the distributional breakpoints of $RiskInfo_t$ from the prior calendar quarter. High-Low denotes the difference between the first and fifth quintile. *RET(-1,+1)* corresponds to the three-day cumulative return around firms' announcement date.

		Quintiles of $RiskInfo_{30}$					
		Q1 (Low)	Q2	Q3	Q4	Q5 (High)	High-Low
RET(-1,+1)	*Raw*	2.961	1.430	0.547	-0.533	-3.271	-6.232
		(16.03)	(10.85)	(5.60)	-(3.60)	-(13.02)	-(20.10)
RET(-1,+1)	*Market-Adj*	2.388	1.069	0.270	-0.553	-2.945	-5.333
		(17.63)	(10.34)	(3.39)	-(4.40)	-(14.07)	-(19.68)