Exact and heuristic methods for trading-off makespan and stability in stochastic project scheduling

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Project scheduling literature: mostly RCPSP

- activities, resources, precedence and resource constraints
- durations = single-point estimates
- NP-Hard (Blazewicz et al. 1983)

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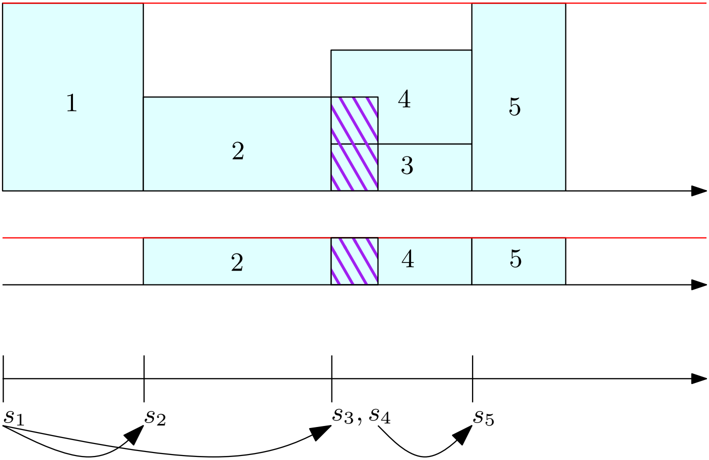
How useful for real-world projects?

Example RCPSP & solution



Reactive stochastic project scheduling: S-RCPSP

- stochastic durations $\boldsymbol{D} = (D_1, \ldots, D_n)$ with known $\mathbb{P}[D_i = t]$
- solution = ~~fixed schedule~~ scheduling policy Π
- find Π that minimizes $\mathbb{E}[\max_{i=1,\ldots,n}(S_i(\Pi, \boldsymbol{D}) + D_i)]$
- RCPSP generalization (at least as hard)

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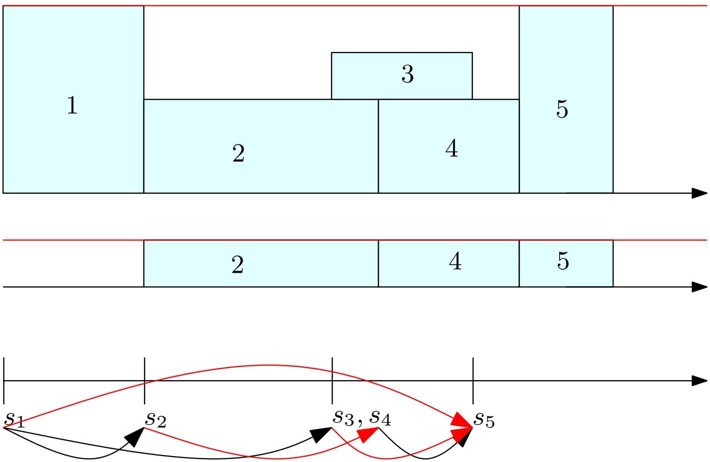
Various classes of Π

- list-based policies, earliest-start policies, etc. (Möhring et al. 1984,1985)
- exact procedures (Stork 2000)
- meta-heuristics (e.g. Ballestín 2007, Ashtiani et al. (2011))

Example S-RCPSP & solution



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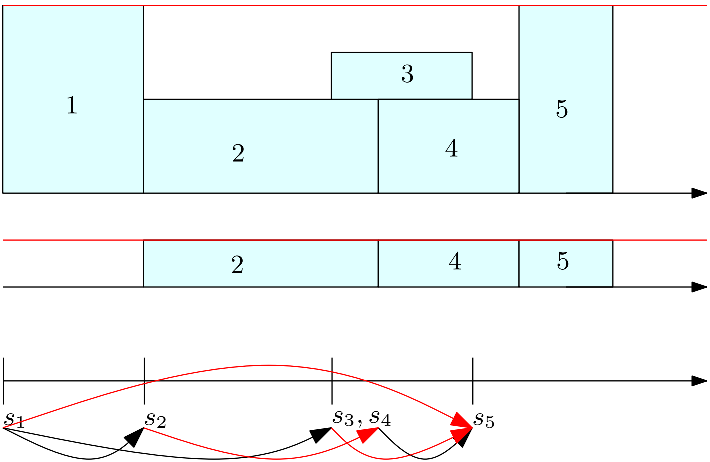

Disadvantage: no fixed-time schedule at all

Proactive-reactive stochastic project scheduling

- stochastic durations $\boldsymbol{D} = (D_1, \ldots, D_n)$ with known $\mathbb{P}[D_i = t]$
- solution = (proactive schedule $\boldsymbol{t}$, reactive policy Π)
  - $\boldsymbol{t} = (t_1, \ldots, t_n)$: buffered and can more-or-less be trusted
  - Π: what to do in case of buffer overruns
- **railway mode**: $S_i(\Pi, \boldsymbol{t}) \geq t_i$
- find $(\boldsymbol{t}, \Pi)$ that balances *expected instability* and *expected makespan*

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Minimize both

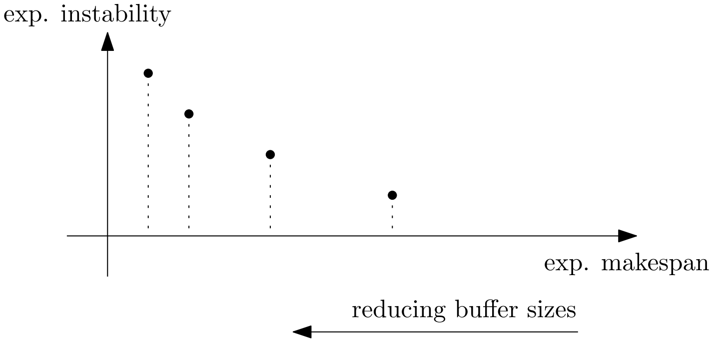
1. expected instability

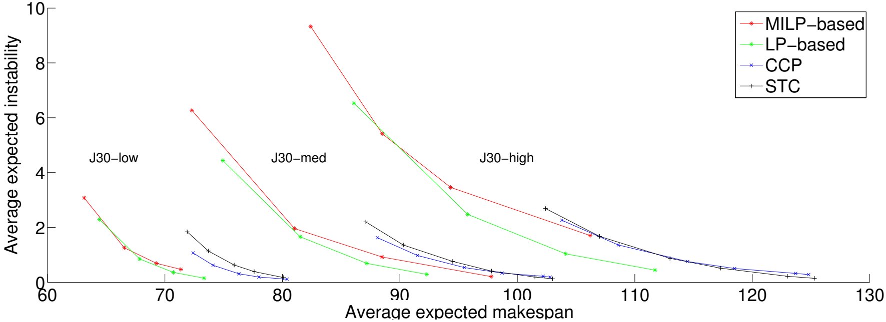
$$\mathbb{E}[\sum_i (S_i((\Pi, \boldsymbol{t}), \boldsymbol{D}) - t_i)]$$

2. expected makespan

$$\mathbb{E}[\max_i (S_i((\Pi, \boldsymbol{t}), \boldsymbol{D}) + D_i)]$$

Tuning some tradeoff parameter

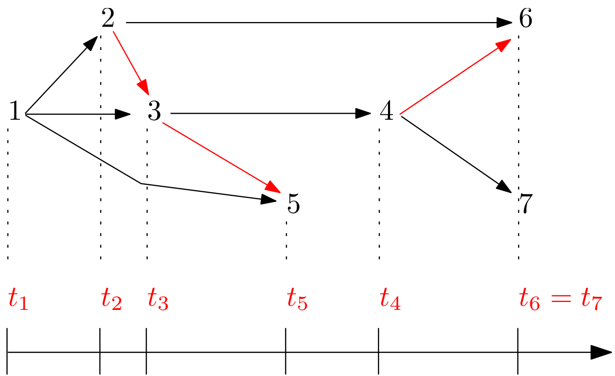




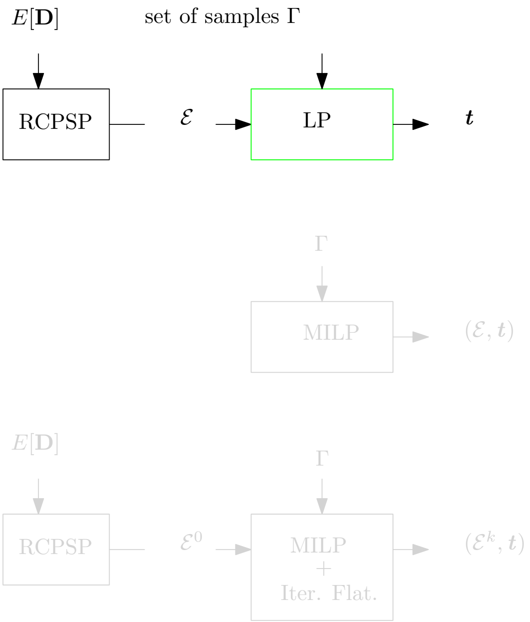
- CCP model (Lamas and Demeulemeester, Journal of Scheduling, 2015)
- STC heuristic (Van de Vonder et al., EJOR, 2008)
- MILP-based heuristic ($\mathcal{O}(2^m)$ with $m < n$)
- LP-based heuristic ($\mathcal{O}(n^4)$)

Define the Proactive Stochastic (PS-)RCPSP

- focus specifically on earliest-start (es-)policies
- find $(\mathcal{E}, \boldsymbol{t})$ together as a pair
- to minimize $\alpha \cdot \texttt{Exp.Makespan}(\mathcal{E}, \boldsymbol{t}) + (1-\alpha)\texttt{Exp.Instability}(\mathcal{E}, \boldsymbol{t})$



Stochastic optimization problem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set of samples Γ
RCPSP
$\mathcal{E}$
LP
$\boldsymbol{t}$
Γ
MILP
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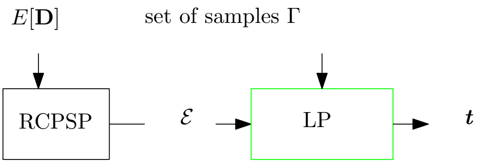
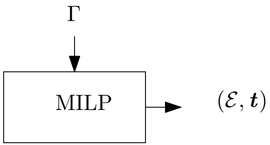
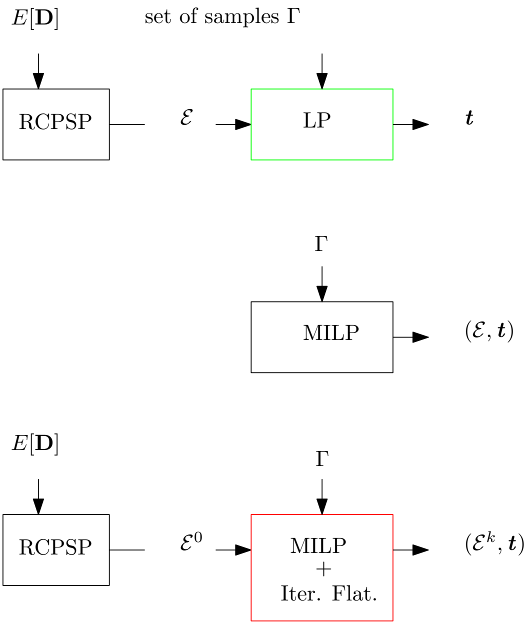
$\mathcal{E}^0$
MILP + Iter. Flat.
$(\mathcal{E}^k, \boldsymbol{t})$

Our Approach: Solving PS-RCPSP



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Thank you; Questions?