

Combinatorics

Leonardo Cervantes, Isaac Botello

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1 First counting

1. Given non-negative integers $k \leq n$, how many ordered lists of k different elements can be made if there are n different elements available?
2. Given a $n \times m$ grid, find the number of rectangles with their four sides in lines of the grid.

1.1 Delimiters

1. Suppose that we have 9 identical balls and 9 distinguishable boxes. Count the number of ways to distribute the balls in the boxes (there may be several or no balls in the same box).
2. There are 10 boxes and 30 balls, of which 10 are green, 10 are blue and 10 are red. The balls of the same color are identical. In how many different ways can we distribute the 30 balls in the 10 boxes?

2 Paths in boards

1. Suppose we have a board of size $m \times n$ (m rows and n columns) divided into unit squares. How many paths are there on the sides of the squares that move only up or to the right and go from the bottom left corner to the top right one?
2. Prove the following identities:
 - $\binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k}$
 - $\sum \binom{n}{i} \binom{n}{n-i} = \binom{2n}{n}$

3 Catalan numbers

1. Find the number of valid parenthesis expressions that consist of n left parentheses and n right parentheses.
2. Find the number of paths in the edges of an $n \times n$ board that start in the lower left corner, end in the top right corner and only move up or to the right and never go above the diagonal joining these two corners (they are allowed to touch it).

4 Inclusion-exclusion principle

1. Let n be a positive integer. Find the number of permutations of $(1, 2, \dots, n)$ such that no number remains in its original place.

5 Codeforces problems

- [1771 A](#)
- [1178 C](#)