Capacitance

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1 Capacitance

By definitions:

$$C = \frac{Q}{V_{\text{ab}}}$$

With the units:

$$1[F] = 1\frac{[C]}{[V]}$$

Expanding:

$$C = \epsilon_0 \frac{A}{d}[F]$$

2 Energy inside a capacitor

$$U_c = \frac{1}{2}\frac{Q^2}{C}[J]$$

$$U_c = \frac{1}{2}CV_{\text{ab}}^2[J]$$

3 Matter and electricity

Vector of dipole electrical moment:

$$\overline{p} = q_i\bar{l}[C \cdot m]$$

Permittivity:

$$\begin{aligned} \epsilon_{\text{material}} &= k_e\epsilon_0 && \left[\frac{C^2}{N \cdot m^2}\right] \\ &= \epsilon_r\epsilon_0 && \left[\frac{C^2}{N \cdot m^2}\right] \end{aligned}$$

Relative permittivity:

$$k_e = 1 + x_e[1]$$

Rupture electric field:

$$E_r = \frac{V_{\text{max}}}{d}\left[\frac{V}{m}\right]$$

4 Electric vectors

Polarization vector:

$$\begin{aligned} \overline{P} &= \frac{Q_i}{A}\hat{u} && \left[\frac{C}{m^2}\right] \\ &= \sigma_i\hat{u} && \left[\frac{C}{m^2}\right] \\ &= \epsilon_0 x_e\overline{E} && \left[\frac{C}{m^2}\right] \\ &= \epsilon_0(k_e - 1)\overline{E} && \left[\frac{C}{m^2}\right] \end{aligned}$$

Displacement vector:

$$\begin{aligned} \overline{D} &= k_e\epsilon_0\overline{E} && \left[\frac{C}{m^2}\right] \\ &= \epsilon_0\overline{E} + \overline{P} && \left[\frac{C}{m^2}\right] \end{aligned}$$

5 Capacitor connections

5.1 Series

$$Q_{\text{ad}} = Q_1 = Q_2 = ...$$

$$V_{\text{ab}} = V_1 + V_2 + ...$$

$$C_{\text{eq}} = \left(\sum_{i=1}^{n}\frac{1}{C_i}\right)^{-1}[F]$$

5.2 Parallel

$$Q_{\text{ad}} = Q_1 + Q_2 + ...$$

$$V_{\text{ab}} = V_1 = V_2 = ...$$

$$C_{\text{eq}} = \sum_{i=1}^{n} C_i[F]$$

6 Capacitor with dielectrics configuration

6.1 Series

$$C_1 = k_1 \epsilon_0 \frac{A_1}{d_1} [F]$$

$$C_2 = k_2 \epsilon_0 \frac{A_2}{d_1} [F]$$

$$A = A_1 = A_2$$

6.2 Parallel

$$C_1 = k_1 \epsilon_0 \frac{A_1}{d_1} [F]$$

$$C_2 = k_2 \epsilon_0 \frac{A_2}{d_1} [F]$$

$$\frac{1}{2} A = A_1 = A_2$$