Introduction to electric circuits

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1 Concepts and definitions

Electric current

$$I = \frac{\delta Q}{\delta t}[A]$$

Ampere unit

$$1[A] = \frac{1[C]}{1[S]}$$

Number of free charge carriers

$$n = \frac{\#e^-}{\mathbb{V}}\left[\frac{1}{m^3}\right]$$

Average speed of free charge carriers

$$v_p = \frac{I}{n\,|q_e|\,A}\left[\frac{m}{s}\right]$$

Electric current density

$$J = \frac{I}{A}\left[\frac{A}{m^2}\right] = nq_e v_p\left[\frac{A}{m^2}\right]$$

Electric current density in a vector form

$$\overline{J} = nq_e\overline{v}_p\left[\frac{A}{m^2}\right]$$

2 Ohm law

In vector form

$$\overline{J} = \sigma\overline{E}\left[\frac{A}{m^2}\right] = \frac{\overline{E}}{\rho}\left[\frac{A}{m^2}\right]$$

Resistivity

$$\rho = \frac{1}{\sigma}\,[\Omega \cdot m]$$

Resistivity according to temperature

$$\rho = \rho_0\,[1 + \alpha(T - T_0)]\,[\Omega \cdot m]$$

Where:

- ρ : Resistivity of the material at a temperature T.
- ρ_0 : Resistivity of the material at $20\,[^\circ C]$
- α : Temperature coefficient of the material $[^\circ C]^{-1}$

Electric resistance

$$R = \rho\frac{l}{A}\,[\Omega]$$

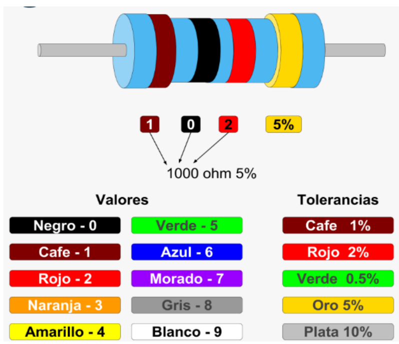
Ohm unit

$$1\,[\Omega] = \frac{1[V]}{1[A]}$$

Potential difference

$$V_{ab} = RI\,[V]$$

Resistors colors



3 Joule law

Electric power

$$P = (V_{ab} \cdot I)[w]$$

Watt unit

$$1[w] = \frac{1[J]}{1[s]}$$

Joule law

$$P = R \cdot I^2 = \frac{V^2}{R}[w]$$

Dissipated energy

$$\begin{aligned} U_R &= Pt && [J] \\ &= VIt && [J] \\ &= RI^2t && [J] \\ &= \frac{V^2}{R}t && [J] \end{aligned}$$

4 Resistance connections

In series

$$I = I_1 = I_2 = I_3 = ...$$
$$V_{ad} = V_1 + V_2 + ...$$
$$R_{eq} = \sum_{i=1}^{n} R_i [\Omega]$$

In parallel

$$V_{ab} = V_1 = V_2 = V_3 = ...$$
$$I = I_1 + I_2 + I_3 + ...$$
$$R_{eq} = \left(\sum_{i=1}^{n} \frac{1}{R_i} \right)^{-1} [\Omega]$$

5 Electromotive Force

It is defined as

$$\varepsilon = \frac{w}{q} = \int \overline{E} \cdot d\bar{l} [V]$$

Ideal electromotive force

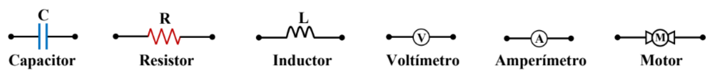
$$\begin{aligned} P_{\text{ideal}} &= \varepsilon I && [w] \\ U_{\text{ideal}} &= Pt && [J] \\ U_{\text{ideal}} &= \varepsilon I t && [J] \end{aligned}$$

Real electromotive force

$$\begin{aligned} P_{\text{real}} &= P_{\text{ideal}} - P_{ri} && [w] \\ P_{\text{real}} &= \varepsilon I - r_i I^2 && [w] \\ U_{\text{real}} &= P_{\text{real}} t && [J] \\ U_{\text{real}} &= (\varepsilon I - r_i I^2) t && [J] \end{aligned}$$

6 Nomenclature

- Branch: A a electric device with two terminals.



- Principal branch: The branch or branches that connect two adjacent principal nodes.
- Node: Union of two or more branches.
- Principal node: Union of three or more branches.
- Mesh: Branch set connected in a closed trajectory.

7 Kirchhoff Laws

- LCK: The sum of electric current in a node is equal to 0. $\sum_{k=1}^{n} I_k = 0$. If the current enters is positive and if it exits is negative.
- Independent equations in node (EIN): Its equal to the number of principal nodes minus one. $EIN = N_p - 1$
- LVK: The sum of potential differences in a mesh with closed trajectory is zero. $\sum_{k=1}^{n} V_k = 0$. $(- \implies +)$ is positive and $(+ \implies -)$ is negative.
- Independent equation in meshes (EIM): Its equal to the number of principal meshes minus the number of independent nodes. $EIM = R_p - EIN$.

8 Introduction to RC circuits

Initial electric current, maximum electric current

$$i_0 = \frac{E}{R} [A]$$

Total electric current

$$q_{\max} = C\varepsilon [C]$$

Getting the current according the time

$$i(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}} [A]$$

Tau constant: $\tau = RC[s]$
We can obtain time as:

$$t = -RC \ln \frac{i}{i_0} [s]$$

Determine electric charge on a capacitor

$$q(t) = C\varepsilon(1 - e^{-\frac{t}{RC}})[C]$$

Determine potential difference on a capacitor

$$V_c(t) = \varepsilon(1 - e^{-\frac{t}{RC}})[V]$$

Determine potential difference on a resistor

$$V_R(t) = \varepsilon e^{-\frac{t}{RC}} [V]$$

Determine electric charge on a capacitor when discharging

$$q(t) = C\varepsilon e^{-\frac{t}{RC}} [C]$$

Determine the electric current in the circuit
$i(t) = -\frac{\varepsilon}{R} e^{-\frac{t}{RC}} [A]$