

Electrical field and potential

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1 Electric charge and distributions

Linear electric charge density:

$$\lambda = \frac{Q}{l} \left[\frac{C}{m} \right]$$

Superficial electric charge density:

$$\sigma = \frac{Q}{A} \left[\frac{C}{m^2} \right]$$

Q : Total electric charge

2 Coulomb law. Electric force in vector form

Electric force:

$$F = k \frac{|q_1||q_2|}{r^2} [N]$$

$$k = \frac{1}{4\pi\epsilon} \left[\frac{N \cdot m^2}{C^2} \right]$$

ϵ_0 : Permittivity of air $8.8542 \times 10^{-12} \left[\frac{C^2}{N \cdot m^2} \right]$

k : Coulomb's constant $9 \times 10^9 \left[\frac{N \cdot m^2}{C^2} \right]$

In vector form:

$$\overline{F} = \left| \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2} \right| \hat{u} [N]$$

3 Electric field as a vector field

Electric field:

$$\overline{E} = \frac{\overline{F}_{q0}}{q_0} \left[\frac{N}{C} \right]$$

- Positive charge has arrows outside and negative charge has arrows entering.
- Electric field vector in a point is tangent to the line.
- The number of lines is proportional to the value of electric charge.

4 Obtaining electric fields in vector form for discrete distributions

4.1 Originated by a punctual charge

$$\overline{E}_A = k \frac{|q|}{r^2} \hat{u} \left[\frac{N}{C} \right]$$

4.2 Originated by a linear distribution

$$\overline{E}_P = k|\lambda| \int \frac{dl}{r^2} \hat{u} \left[\frac{N}{C} \right]$$

4.3 Originated by a superficial distribution

$$\overline{E}_P = k|\sigma| \iint \frac{dA}{r^2} \hat{u} \left[\frac{N}{C} \right]$$

4.4 Originated by a infinite line

$$\overline{E}_{P\lambda} = \frac{2k|\lambda|}{a} \hat{u} \left[\frac{N}{C} \right]$$

4.5 Originated by a ring

$$\overline{E}_P = \frac{kb|q|}{(b^2 + R^2)^{\frac{3}{2}}} \hat{u} \left[\frac{N}{C} \right]$$

4.6 Originated by a big or infinite surface

$$\overline{E}_P = \frac{|\sigma|}{2\epsilon_0} \left(1 - \frac{b}{(b^2 + R^2)^{\frac{1}{2}}} \right) \hat{u} \left[\frac{N}{C} \right]$$

$$\overline{E}_P = \frac{|\sigma|}{2\epsilon_0} \hat{u} \left[\frac{N}{C} \right]$$

Where a is the minimal distance between line and point.

5 Electrical flow

In a open surface:

$$\phi_E = \iint \overline{E} \cdot \hat{n} dA \left[\frac{N \cdot m^2}{C} \right]$$

If a surface is closed and the source of electric field is outside the electric flow is 0.

In a closed surface:

If a surface is closed and the source of electric field is outside the electric flow is 0.

If the source is inside:

$$\phi_E = \oint \oint \overline{E} \cdot \hat{n} dA \left[\frac{N \cdot m^2}{C} \right]$$

6 Gauss law

Defined as:

$$\phi_E = \frac{Q_{\text{locked}}}{\epsilon_0} \left[\frac{N \cdot m^2}{C} \right]$$

And as a differential form:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Where ρ is the density of volumetric electrical charge.

6.1 Electric field originated from a sphere with electric charge

$$\vec{E}_p = \frac{1}{4\pi\epsilon_0} \frac{|Q_{\text{esf}}|}{r^2} \hat{u} \left[\frac{N}{C} \right]$$

Where $Q_{\text{esf}} = \rho_{\text{esf}} A_{\text{esf}}$

6.2 Electric field originated from a infinite line

$$\vec{E}_p = \frac{2k|\lambda|}{a} \hat{u} \left[\frac{N}{C} \right]$$

6.3 Electric field originated by a infinite surface

$$\vec{E}_p = \frac{|\rho|}{2\epsilon_0} \hat{u} \left[\frac{N}{C} \right]$$

7 Electrostatic field circulation and rotational

$$c = \oint \overline{E} \cdot d\vec{l} = 0$$

$$\text{rot} \overline{E} = \vec{\nabla} \times \overline{E} = \vec{0}$$

8 Electric potential energy

$$V_a = \frac{\infty W_A}{q} \left[\frac{J}{C} \right] = - \int_{\infty}^A \overline{E} \cdot d\vec{l} [V]$$

$$V_{AB} = -V_{BA}$$

9 Potential differences

9.1 Originated by a punctual charge

$$V_{AB} = kq \left(\frac{1}{r_A} - \frac{1}{r_B} \right) [V]$$

9.2 Originated by a infinite line

$$V_{AB} = 2k\lambda \left(\frac{r_B}{r_A} \right) [v]$$

9.3 Originated by a big surface

$$V_{AB} = \frac{\sigma}{2\epsilon_0} (r_B - r_A) [V]$$

10 Electric potential gradient

$$\vec{\nabla} V = \frac{\delta V}{\delta x} \hat{i} + \frac{\delta V}{\delta y} \hat{j} + \frac{\delta V}{\delta z} \hat{k} \left[\frac{V}{m} \right]$$

In other coordinates systems:

$$\vec{\nabla} V = \frac{\delta V}{\delta r} \hat{r} + \frac{1}{r \sin \phi} \frac{\delta V}{\delta \theta} \hat{\theta} + \frac{1}{r} \frac{\delta V}{\delta \phi} \hat{\phi} \left[\frac{V}{m} \right]$$

$$\vec{\nabla} V = \frac{\delta V}{\delta r} \hat{r} + \frac{1}{r} \frac{\delta V}{\delta \theta} \hat{\theta} + \frac{\delta V}{\delta z} \hat{z} \left[\frac{V}{m} \right]$$

Electric field as the potential gradient

$$\overline{E} = -\vec{\nabla} V = - \left(\frac{\delta V}{\delta x} \hat{i} + \frac{\delta V}{\delta y} \hat{j} + \frac{\delta V}{\delta z} \hat{k} \right) \left[\frac{V}{m} \right]$$

$$\overline{E} = \overline{E}_x + \overline{E}_y + \overline{E}_z$$