# Electrical field and potential

Enrique Calderon

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## 1 Electric charge and distributions

Linear electric charge density:

$$\lambda = \frac{Q}{l}\left[\frac{C}{m}\right]$$

Superficial electric charge density:

$$\sigma = \frac{Q}{A}\left[\frac{C}{m^2}\right]$$

Q : Total electric charge

## 2 Coulomb law. Electric force in vector form

Electric force:

$$F = k\frac{|q_1||q_2|}{r^2}[N]$$

$$k = \frac{1}{4\pi\epsilon}\left[\frac{N \cdot m^2}{C^2}\right]$$

$\epsilon_0$ : Permittivity of air $8.8542 \times 10^{-12}\left[\frac{C^2}{N \cdot m^2}\right]$

$k$ : Coulomb's constant $9 \times 10^9\left[\frac{N \cdot m^2}{C^2}\right]$

In vector form:

$$\overline{F} = \left|\frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2}\right|\hat{u}[N]$$

## 3 Electric field as a vector field

Electric field:

$$\overline{E} = \frac{\overline{F}_{q0}}{q_0}\left[\frac{N}{C}\right]$$

- Positive charge has arrows outside and negative charge has arrows entering.
- Electric field vector in a point is tangent to the line.
- The number of lines is proportional to the value of electric charge.

## 4 Obtaining electric fields in vector form for discrete distributions

### 4.1 Originated by a punctual charge

$$\overline{E_A} = k\frac{|q|}{r^2}\hat{u}\left[\frac{N}{C}\right]$$

### 4.2 Originated by a linear distribution

$$\overline{E_P} = k|\lambda|\int\frac{dl}{r^2}\hat{u}\left[\frac{N}{C}\right]$$

### 4.3 Originated by a superficial distribution

$$\overline{E_P} = k|\sigma|\iint\frac{dA}{r^2}\hat{u}\left[\frac{N}{C}\right]$$

### 4.4 Originated by a infinite line

$$\overline{E_{P\lambda}} = \frac{2k|\lambda|}{a}\hat{u}\left[\frac{N}{C}\right]$$

### 4.5 Originated by a ring

$$\overline{E_P} = \frac{kb|q|}{(b^2+R^2)^{\frac{3}{2}}}\hat{u}\left[\frac{N}{C}\right]$$

### 4.6 Originated by a big or infinite surface

$$\overline{E_P} = \frac{|\sigma|}{2\epsilon_0}\left(1 - \frac{b}{(b^2+R^2)^{\frac{1}{2}}}\right)\hat{u}\left[\frac{N}{C}\right]$$

$$\overline{E_P} = \frac{|\sigma|}{2\epsilon_0}\hat{u}\left[\frac{N}{C}\right]$$

Where $a$ is the minimal distance between line and point.

# 5 Electrical flow

In a open surface:

$$\phi_E = \iint \overline{E} \cdot \hat{n} dA \left[\frac{N \cdot m^2}{C}\right]$$

If a surface is closed and the source of electric field is outside the electric flow is 0.

In a closed surface:

If a surface is closed and the source of electric field is outside the electric flow is 0.

If the source is inside:

$$\phi_E = \oint \oint \overline{E} \cdot \hat{n} dA \left[\frac{N \cdot m^2}{C}\right]$$

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# 6 Gauss law

Defined as:

$$\phi_E = \frac{Q_{\text{locked}}}{\epsilon_0} \left[\frac{N \cdot m^2}{C}\right]$$

And as a differential form:

$$\overrightarrow{\nabla} \cdot \overrightarrow{E} = \frac{\rho}{\epsilon_0}$$

Where $\rho$ is the density of volumetric electrical charge.

## 6.1 Electric field originated from a sphere with electric charge

$$\overrightarrow{E_p} = \frac{1}{4\pi\epsilon_0} \frac{|Q_{\text{esf}}|}{r^2} \hat{u} \left[\frac{N}{C}\right]$$

Where $Q_{\text{esf}} = \rho_{\text{esf}} A_{\text{esf}}$

## 6.2 Electric field originated from a infinite line

$$\overrightarrow{E_p} = \frac{2k|\lambda|}{a} \hat{u} \left[\frac{N}{C}\right]$$

## 6.3 Electric field originated by a infinite surface

$$\overrightarrow{E_p} = \frac{|\rho|}{2\epsilon_0} \hat{u} \left[\frac{N}{C}\right]$$

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# 7 Electrostatic field circulation and rotational

$$c = \oint \overline{E} \cdot d\overline{l} = 0$$

$$rot\overline{E} = \overline{\nabla} \times \overline{E} = \overline{0}$$

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# 8 Electric potential energy

$$V_a = \frac{{}_{\infty}W_A}{q} \left[\frac{J}{C}\right] = -\int_{\infty}^{A} \overline{E} \cdot d\overline{l}\, [V]$$

$$V_{AB} = \frac{{}_{B}W_A}{q}$$

$$V_{AB} = -V_{BA}$$

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# 9 Potential differences

## 9.1 Originated by a punctual charge

$$V_{AB} = kq\left(\frac{1}{r_A} - \frac{1}{r_B}\right) [V]$$

## 9.2 Originated by a infinite line

$$V_{AB} = 2k\lambda \ln\left(\frac{r_B}{r_A}\right) [v]$$

## 9.3 Originated by a big surface

$$V_{AB} = \frac{\sigma}{2\epsilon_0}(r_B - r_A)[V]$$

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# 10 Electric potential gradient

$$\overline{\nabla} V = \frac{\delta V}{\delta x}\hat{i} + \frac{\delta V}{\delta y}\hat{j} + \frac{\delta V}{\delta z}\hat{k} \left[\frac{V}{m}\right]$$

In other coordinates systems:

$$\overline{\nabla} V = \frac{\delta V}{\delta r}\hat{r} + \frac{1}{r \sin\phi}\frac{\delta V}{\delta \theta}\hat{\theta} + \frac{1}{r}\frac{\delta V}{\delta \phi}\hat{\phi} \left[\frac{V}{m}\right]$$

$$\overline{\nabla} V = \frac{\delta V}{\delta r}\hat{r} + \frac{1}{r}\frac{\delta V}{\delta \theta}\hat{\theta} + \frac{\delta V}{\delta z}\hat{z} \left[\frac{V}{m}\right]$$

Electric field as the potential gradient

$$\overline{E} = -\overline{\nabla} V = -\left(\frac{\delta V}{\delta x}\hat{i} + \frac{\delta V}{\delta y}\hat{j} + \frac{\delta V}{\delta z}\hat{k}\right) \left[\frac{V}{m}\right]$$

$$\overline{E} = \overline{E_x} + \overline{E_y} + \overline{E_z}$$