Laplacce

Enrique Calderon & Elizabeth Portilla

March 3, 2024

1 By definition

$$\mathcal{L}(f(t)) = F(S) = \int_0^\infty e^{-St} f(t)dt$$

or

$$F(S) = \lim_{b\to\infty} \int_0^b e^{-St} f(t)dt$$

And the inverse

$$\mathcal{L}^{-1}(F(S)) = f(t) = \frac{1}{2\pi} \int_{-i\infty}^{i\infty} e^{St} F(S)$$

2 For differential equations

$$\mathcal{L}\{f^n(\mathcal{T})\} = S^n F(S) - S^{n-1} f(0) - ... - f^{n-1}(0)$$

To solve differential equations first convert with Laplace operator, solve it and then apply inverse Laplace operator.

3 Translation theorem

$$u(\mathcal{T}-a) = \begin{cases} 0 & 0 \leq t \leq a \\ 1 & 1 \geq a \end{cases}$$

$$\mathcal{L}\{u(\mathcal{T}-a)\} = \frac{e^{-aS}}{S}$$

$$\mathcal{L}\{f(\mathcal{T}-a)u(\mathcal{T}-a)\} = e^{-aS}F(S)$$

4 Dirac's delta

$$\delta(\mathcal{T}-\mathcal{T}_0) = \begin{cases} 0 & 0 \leq \mathcal{T} < \mathcal{T}_0 - a \\ \frac{1}{2}a & \mathcal{T}_0 - a \leq \mathcal{T} < \mathcal{T}_0 + a \\ 0 & \mathcal{T}_0 + a \leq \mathcal{T} \end{cases}$$

$$\mathcal{L}\{\delta(\mathcal{T}-\mathcal{T}_0)\} = e^{-\mathcal{T}_0 S}$$

5 Convolution

$$f * g = \int_0^{\mathcal{T}} f(x)g(\mathcal{T}-x)dx$$

$$\mathcal{L}\{f * g\} = \mathcal{L}\{f(\mathcal{T})\}\mathcal{L}\{g(\mathcal{T})\}$$

6 Differential equations system

1. Solve differential equations separately 2. Arrange the solutions of each differential equation as a system

$$\begin{cases} AX(s) + BY(S) = f_1(S) \\ CX(s) + DY(S) = f_2(S) \end{cases}$$

3. Solve system by Cramer's method to find values of X(s), Y(S)...

$$Y(s) = \frac{\begin{vmatrix} A & f_1(S) \\ C & f_2(S) \end{vmatrix}}{\begin{vmatrix} A & B \\ C & D \end{vmatrix}}$$

$$X(s) = \frac{\begin{vmatrix} f_1(S) & B \\ f_2(S) & D \end{vmatrix}}{\begin{vmatrix} A & B \\ C & D \end{vmatrix}}$$

4. Obtaining:

$$L^{-1}(X(S)) = X(t)$$

$$L^{-1}(Y(S)) = Y(t)$$