Homework3

Kenan Sooklall

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Problem set 1

(1) What is the rank of the matrix A?

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{bmatrix}$$

```r
library(Matrix)
m <- matrix(c(c(1,-1,0,5), c(2,0,1,4), c(3,1,-2,-2), c(4,3,1,-3)), nrow=4)
rankMatrix(m)
```

```
## [1] 4
## attr(,"method")
## [1] "tolNorm2"
## attr(,"useGrad")
## [1] FALSE
## attr(,"tol")
## [1] 8.881784e-16
```

The rank of matrix A is 4, each row is linearly independent

(2) Given an mxn matrix where m > n, what can be the maximum rank? The mini- mum rank, assuming that the matrix is non-zero?

The maximum rank a matrix of the shape mxn where m>n can be is m, the max(m, 1). The minimum rank a non-zero matrix can have is 1 where all rows are a linear combination of one row.

(3) What is the rank of matrix B?

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$$

```r
m <- matrix(c(c(1,3,2), c(2,6,4), c(1,3,2)), nrow=3)
rankMatrix(m)
```

```
## [1] 1
## attr(,"method")
## [1] "tolNorm2"
```

```
## attr(,"useGrad")
## [1] FALSE
## attr(,"tol")
## [1] 6.661338e-16
```

Rows 2 and 3 are a linear combination of row 1,

$row2 = row1 * 3$
$row3 = row1 * 2$

Therefore the rank of matrix B = 1

Problem set 2

Compute the eigenvalues and eigenvectors of matrix A. You'll need to show your work. You'll need to write out the characteristic polynomial and show your solution.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

Please show your work using an R-markdown document. Please name your assignment submission with your first initial and last name.

eigenvalues Starting with $det(A - \lambda * I) = 0$

Then

$$X = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} \lambda - 1 & -2 & -3 \\ 0 & \lambda - 4 & -5 \\ 0 & 0 & \lambda - 6 \end{bmatrix}$$

Next find the determinant

$det(X) = 0$

$(\lambda - 1)(\lambda - 4)(\lambda - 6) - (-2)(0) - 3(0) = (\lambda - 1)(\lambda^2 - 10\lambda + 24) = \lambda^3 - 11\lambda^2 + 3\lambda - 24 = 0$

Solving that cubic equation and assuming integer solutions. Factors of 24 are [1, 2, 3, 4, 6, 8, 12, 24], 12 and 24 will blow up with the cube so will check those last

$1 : 1^3 - 11 * 1^2 + 34 * 1 - 24 = -10 + 34 - 24 = 0 \quad \checkmark$
$2 : 2^3 - 11 * 2^2 + 34 * 2 - 24 = 8 + 24 - 24 = 8 \quad X$
$3 : 3^3 - 11 * 3^2 + 34 * 3 - 24 = 27 + 3 - 24 = 6 \quad X$
$4 : 4^3 - 11 * 4^2 + 34 * 4 - 24 = 64 - 40 - 24 = 0 \quad \checkmark$
$6 : 6^3 - 11 * 6^2 + 34 * 6 - 24 = 216 - 192 - 24 = 0 \quad \checkmark$

3 eigenvalues

$\lambda_1 = 1$
$\lambda_2 = 4$
$\lambda_3 = 6$

eigenvectors For $\lambda_1 = 1$

$A - \lambda I =$

$$\begin{bmatrix} 0 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

Simplifying

$2y + 3z = 0$
$3y + 5z = 0$
$5z = 0$

$z = 0 \rightarrow y = 0 \rightarrow x = x$

Getting the vector

$$x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

- $\lambda_1 = 1 \quad \overrightarrow{V}_1 = (1, 0, 0)$

For $\lambda_1 = 4$

$A - \lambda I =$

$$\begin{bmatrix} -3 & 2 & 3 \\ 0 & 0 & 5 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

Simplifying $-3x + 2y + 3z = 0$
$5z = 0$
$2z = 0$

$z = 0 \to 2y = 3x \to x = 2, y = 3$

Getting the vector

$$\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

- $\lambda_1 = 4 \quad \overrightarrow{V}_2 = (2, 3, 0)$

For $\lambda_1 = 6$

$A - \lambda I =$

$$\begin{bmatrix} -5 & 2 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \text{[RREF]} \begin{bmatrix} -5 & 0 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

Simplifying $-5xy + 8z = 0$
$-2y + 5z = 0$

$8z = 5x$
$5z = 2y$

$x = 8, z = 5, y = 12.5$

Multiply by 2 for integer values

$$\begin{bmatrix} 16 \\ 25 \\ 10 \end{bmatrix}$$

- $\lambda = 6 \quad \overrightarrow{V}_3 = (16, 25, 10)$