

Homework 13

Kenan Sooklall

4/27/2021

Exercise 1 Use integration by substitution to solve the integral below.

$$\int 4e^{-7x} dx \rightarrow u = -7x \quad du = -7dx \quad -du/7 = dx \rightarrow -4/7 \int e^u du = -4e^u/7 = -4e^{-7x}/7 + c$$

Exercise 2 Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of $dN/dt = -3150/t^4 - 220$ bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a $N(t)$ to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter.

$$N(t) = \int -3150/t^4 - 220 \rightarrow -3150/(-3t^3) - 220t + c \quad N(1) = 6530 = -3150/(-3t^3) - 220t + c \rightarrow c = 5700 \quad N(t) = 1050/t^3 - 220t + 5700$$

Exercise 3 Find the total area of the red rectangles in the figure below, where the equation of the line is $f(x) = 2x - 9$.

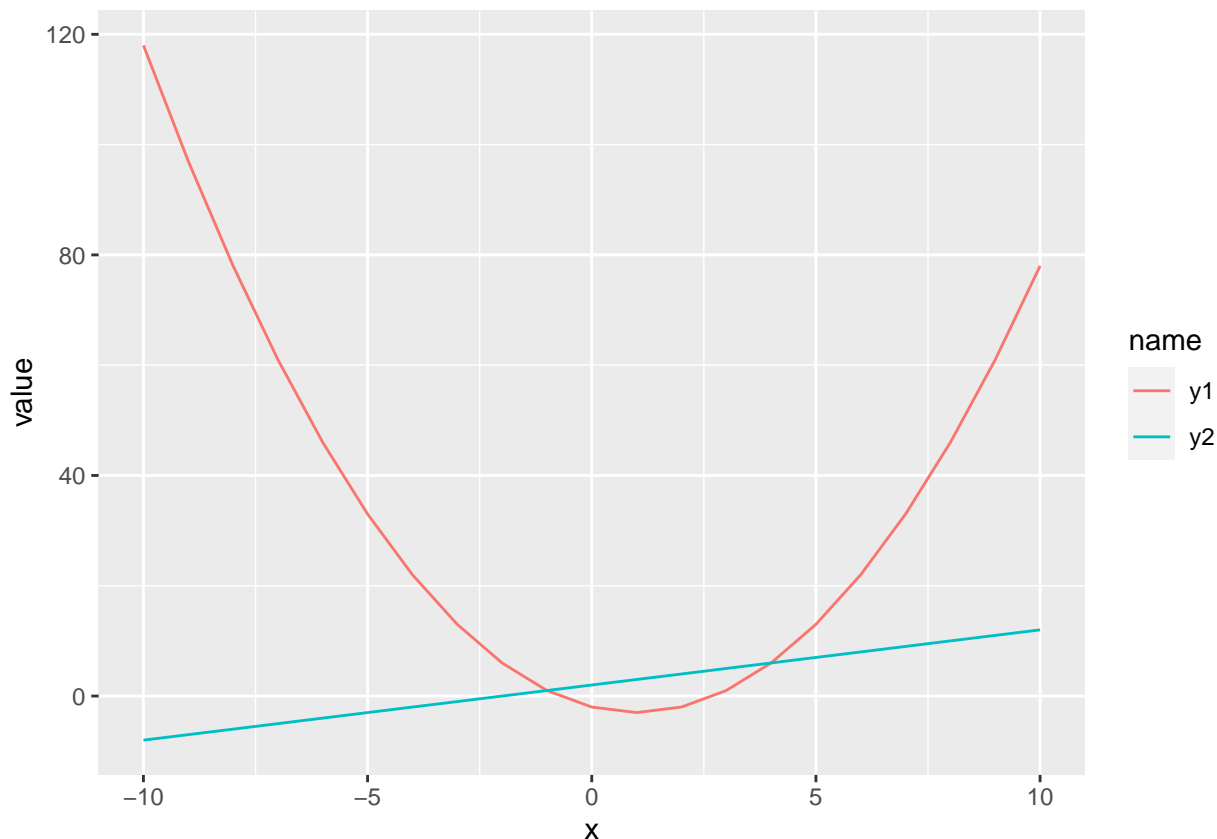
```
data.frame(x=seq(from=5, to=8)) %>% mutate(y=2 * x - 9) %>% pull(y) %>% sum
```

```
## [1] 16
```

The base is always 1 so the sum of the rectangles will be the sum of their heights

Exercise 4 Find the area of the region bounded by the graphs of the given equations. $y = x^2 - 2x - 2$, $y = x + 2$

```
data.frame(x=seq(from=-10, to=10, by=1)) %>% mutate(y1 = x^2 - 2*x - 2, y2 = x + 2) %>%  
  pivot_longer(!x) %>%  
  ggplot(aes(x=x, y=value)) + geom_line(aes(colour = name))
```



$$x^2 - 2x - 2 = x + 2 \rightarrow x^2 - x - 4 = 0 \rightarrow x = 4, x = -1 \int_{-1}^4 x + 2 - (x^2 - 2x - 2) = \int_{-1}^4 -x^2 + 3x + 4 = -x^3/3 + 3x^2/2 + 4x \Big|_{-1}^4 = 20.83$$

Exercise 5 A beauty supply store expects to sell 110 flat irons during the next year. It costs 3.75 to store one flat iron for one year. There is a fixed cost of \$8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

x = number of iron n = lot size $nx = 110$

$$C = 8.25x + 3.75 \cdot (110/x)/2 = 8.25x + 206.25/x \quad dC/dx = 0 = 8.25 - 206.25/x^2 \rightarrow x = 5 \quad nx = 110 = n \cdot 5 \rightarrow n = 22$$

A lot size of 22 and 5 orders per year will minimize inventory costs

Exercise 6 Use integration by parts to solve the integral below.

$$\int \ln(9x) x^6 dx \quad u = \ln(9x), dv = x^6 \rightarrow du = 1/x, v = x^7/7 \quad \ln(9x)/7 - \int x^7/7 \cdot (1/x) = x^7 \ln(9x)/7 - x^7/49 = x^7(7 \ln(9x) - 1)/49$$

Exercise 7 Determine whether $f(x) = 1/(6x)$ is a probability density function on the interval $1, e$. If not, determine the value of the definite integral.

$$\int_1^{e^6} 1/6x = \ln(x)/6 \Big|_1^{e^6} = \ln(e^6)/6 - \ln(1)/6 = 1 - 0 = 1$$

Therefore $f(x)$ is a probability density function