Problem 1

(1) Show that $A^T A \neq AA^T$ in general. (Proof and demonstration.)
(2) For a special type of square matrix A, we get A T A = AA T . Under what conditions could this be true? (Hint: The Identity matrix I is an example of such a matrix).

Proof by contradiction

Let: A be a matrix of shape mxn where m and n are in $\mathbb{N}$ and $m \neq n$

Suppose: $A^T A = AA^T$

Then: $nxm \times mxn = mxn \times nxm$ -> $nxn = mxm$

That equation is only true when $n = m$ which contradicts our assumption that $m \neq n$

Therefore: $A^T A \neq AA^T$

Exception: $m = n$ then A is a square matrix and consequently $A^T A = AA^T$ will hold

Problem 2

Matrix factorization is a very important problem. There are supercomputers built just to do matrix factorizations. Every second you are on an airplane, matrices are being factorized. Radars that track flights use a technique called Kalman filtering. At the heart of Kalman Filtering is a Matrix Factorization operation. Kalman Filters are solving linear systems of equations when they track your flight using radars.

Write an R function to factorize a square matrix A into LU or LDU, whichever you prefer. Please submit your response in an R Markdown document using our class naming convention, E.g. LFulton_Assignment2_PS2.png

You don't have to worry about permuting rows of A and you can assume that A is less than 5x5, if you need to hard-code any variables in your code. If you doing the entire assignment in R, then please submit only one markdown document for both the problems.

```
factorize <- function(X){
  n <- NROW(X)
  U <- X
  L <- diag(n)

  for (j in c(1:n)){
    for(i in c(2:n)){
      if(i > j){
        r <- U[j, ]
        v <- U[i, j] / r[j]
        U[i,] <- U[i,] - (v * r)
        L[i, j] <- v
      }
    }
  }
```

```r
  return (list(L=L, U=U))
}
```

```r
A <- matrix(c(1,2,3,1,1,1,2,0,1), nrow=3)
f <- factorize(A)
B <- f$L %*% f$U

sum(B == A) == length(A)
```

```
## [1] TRUE
```