Homework7 - DATA 605

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Question 1

1. Let X1, X2, . . . , Xn be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k. Let Y denote the minimum of the Xi's. Find the distribution of Y.

Each Xi has k possibilities [1,2,3....k] for i in K therefore for all possibilities of X's = kˆn The number of way to get Y= 1 -> kˆn ways to get 1 and (k-1)ˆn to not get 1 So

$P(X = 1) = k^n * (k-1)^n/k^n$
$\rightarrow P(X = 2) = (k-2+1)^n * (k-2)^n/k^n$
$\rightarrow P(X = 3) = (k-3+1)^n * (k-3)^n/k^n$

$\rightarrow P(X = j) = (k-j+1)^n * (k-j)^n/k^n$

Question 2

2. Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part.).

```
p = 1/10
```

Geometric

a. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a geometric. (Hint: the probability is equivalent to not failing during the first 8 years..)

$ P(X = k) = (1-p) ˆ {k - 1}*p$

P(X>8) = 1 - P(X<=8)

Expected Value = 1/ 0.1 = 10

Standard deviation = sqrt((1-p)/pˆ2) = 9.48

```
1 - pgeom(8-1,p)
```

```
## [1] 0.4304672
```

```
(1-p)^(8-1)*p
```

```
## [1] 0.04782969
```

Exponential

b. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential.

$P(X >= k) = e^{-k/\mu}$

Expected Value = 10

Standard deviation = 10

P(X>=8)

```
1 - pexp(8, p)
```

```
## [1] 0.449329
```

```
exp(-8/10)
```

```
## [1] 0.449329
```

Binomial

c. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8 years)

$P(X > k) = p^k * (1-p)^{n-k}$

Expected Value = 8 * 0.1 = 0.8

Standard deviation = sqrt(8 * 0.1 * 0.9) = 0.85

```
k=0
n=8
pbinom(0, size=n, prob=p)
```

```
## [1] 0.4304672
```

```
0.1^0 * (0.9) ^ (8-0)
```

```
## [1] 0.4304672
```

Poisson

d. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson.

$P(X = 8) = \lambda^x e^{-\lambda}/(x!)$

Expected Value = 8 * 0.1 = 0.8

Standard deviation = sqrt(0.8) = 0.89

```
ppois(0, 0.8)
```

```
## [1] 0.449329
```

```
0.8 ^ 0 * exp(-0.8) / (factorial(0))
```

```
## [1] 0.449329
```