

## Problem 1

- (1) Show that  $A^T A \neq A A^T$  in general. (Proof and demonstration.)
- (2) For a special type of square matrix  $A$ , we get  $A^T A = A A^T$ . Under what conditions could this be true? (Hint: The Identity matrix  $I$  is an example of such a matrix).

### Proof by contradiction

**Let:**  $A$  be a matrix of shape  $m \times n$  where  $m$  and  $n$  are in  $\mathbb{N}$  and  $m \neq n$

**Suppose:**  $A^T A = A A^T$

**Then:**  $n \times m \times m \times n = m \times n \times n \times m \rightarrow n \times n = m \times m$

That equation is only true when  $n = m$  which contradicts our assumption that  $m \neq n$

**Therefore:**  $A^T A \neq A A^T$

**Exception:**  $m = n$  then  $A$  is a square matrix and consequently  $A^T A = A A^T$  will hold

## Problem 2

Matrix factorization is a very important problem. There are supercomputers built just to do matrix factorizations. Every second you are on an airplane, matrices are being factorized. Radars that track flights use a technique called Kalman filtering. At the heart of Kalman Filtering is a Matrix Factorization operation. Kalman Filters are solving linear systems of equations when they track your flight using radars.

Write an R function to factorize a square matrix  $A$  into LU or LDU, whichever you prefer. Please submit your response in an R Markdown document using our class naming convention, E.g. LFullton\_Assignment2\_PS2.png

You don't have to worry about permuting rows of  $A$  and you can assume that  $A$  is less than  $5 \times 5$ , if you need to hard-code any variables in your code. If you doing the entire assignment in R, then please submit only one markdown document for both the problems.

```
factorize <- function(X){
  n <- NROW(X)
  U <- X
  L <- diag(n)

  for (j in c(1:n)){
    for(i in c(2:n)){
      if(i > j){
        r <- U[j, ]
        v <- U[i, j] / r[j]
        U[i,] <- U[i,] - (v * r)
        L[i, j] <- v
      }
    }
  }
}
```

```
    return (list(L=L, U=U))  
  }
```

```
A <- matrix(c(1,2,3,1,1,1,2,0,1), nrow=3)  
f <- factorize(A)  
B <- f$L %*% f$U  
  
sum(B == A) == length(A)
```

```
## [1] TRUE
```