Homework 13

Kenan Sooklall

4/27/2021

Exercise 1 Use integration by substitution to solve the integral below.

$$\int 4e^{-7x}dx \to u = -7x \qquad du = -7dx \qquad -du/7 = dx \to -4/7 \int e^u du = -4e^u/7 = -4e^{-7x}/7 + c$$

Exercise 2 Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of $dN1/dt = -3150/t^4 - 220$ bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a N(t) to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter.

$$N(t) = \int -3150/t^4 - 220 \to -3150/(-3t^3) - 220t + cN(1) = 6530 = -3150/(-3t^3) - 220 + c \to c = 5700 N(t) = 1050/t^3 - 220t +$$

Exercise 3 Find the total area of the red rectangles in the figure below, where the equation of the line is $f(x) = 2x - 9$.

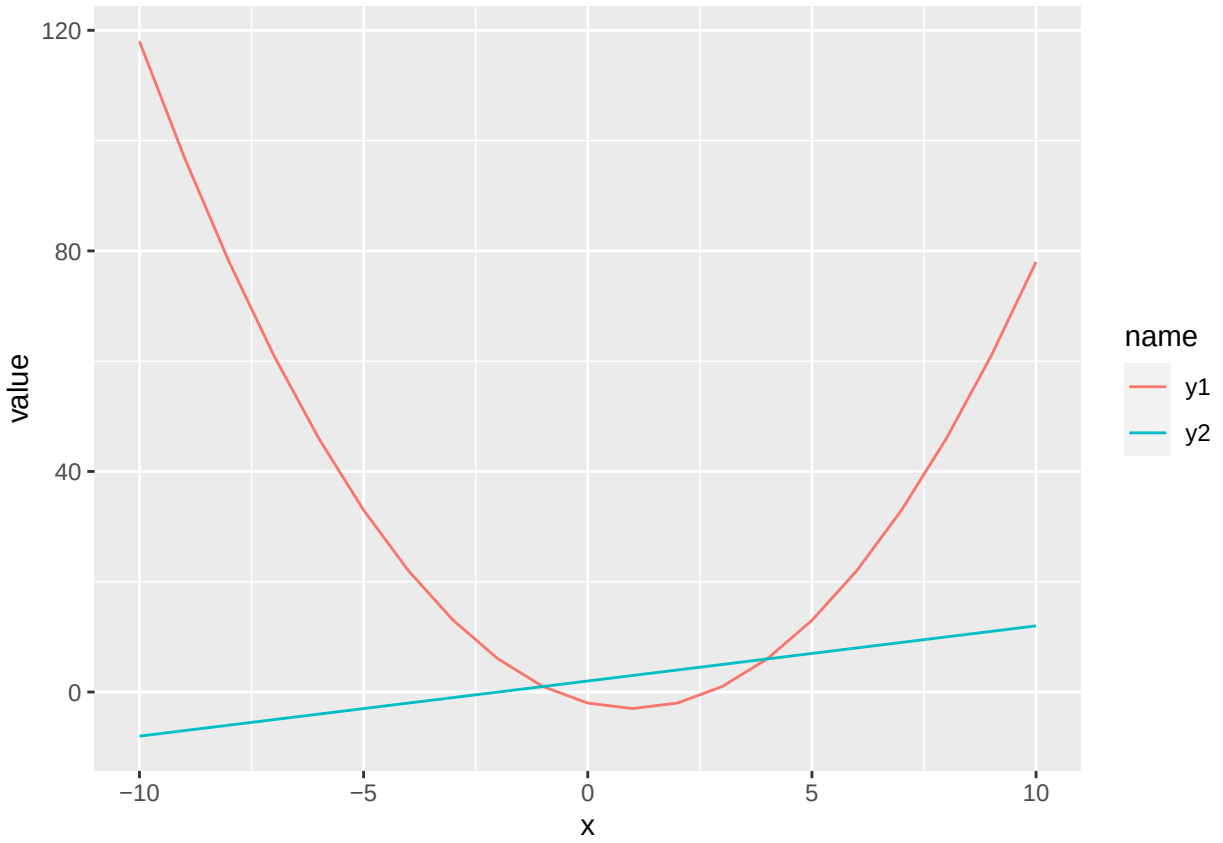
```
data.frame(x=seq(from=5, to=8)) %>% mutate(y=2 * x - 9) %>% pull(y) %>% sum
```

```
## [1] 16
```

The base is always 1 so the sum of the rectangles will be the sum of their heights

Exercise 4 Find the area of the region bounded by the graphs of the given equations. $y = x^2 - 2x - 2, y = x + 2$

```
data.frame(x=seq(from=-10, to=10, by=1)) %>%  mutate(y1 = x^2 - 2*x - 2, y2 = x + 2)  %>%
                pivot_longer(!x)  %>%
                ggplot(aes(x=x, y=value)) + geom_line(aes(colour = name))
```



$$x^2-2x-2=x+2 \rightarrow x^2-x-4=0 \rightarrow x=4, x=-1 \int_{-1}^{4} x+2-(x^2-2x-2)=\int_{-1}^{4} -x^2+3x+4=-x^3/3+3x^2/2+4x\Big|_{-1}^{4}=20.83$$

Exercise 5 A beauty supply store expects to sell 110 flat irons during the next year. It costs 3.75 to store one flat iron for one year. There is a fixed cost of $8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

x = number of iron n = lot size nx = 110

$$C=8.25x+3.75*(110/x)/2=8.25x+206.25/xdC/dx=0=8.25-206.25/x^2 \rightarrow x=5nx=110=n*5 \rightarrow n=22$$

A lot size of 22 and 5 orders per year will minimize inventory costs

Exercise 6 Use integration by parts to solve the integral below.

$$\int ln(9x)x^6dx \quad u=ln(9x), dv=x^6 \rightarrow du=1/x, v=x^7/7x^7ln(9x)/7-\int x^7/7*(1/x)=x^7ln(9x)/7-x^7/49=x^7(7ln(9x)-1$$

Exercise 7 Determine whether f(x)=1/(6x) is a probability density function on the interval 1, e 6 . If not, determine the value of the definite integral.

$$\int_1^{e^6} 1/6x = ln(x)/6\Big|_1^{e^6} = ln(e^6)/6 - ln(1)/6 = 1 - 0 = 1$$

Therefore f(x) is a probability density function