

# Modeling Biological Systems With Cybernetic Control Laws and Steady State Flux Distributions

Master of Science Exam

Department of Chemical and Biomolecular Engineering

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## Overview

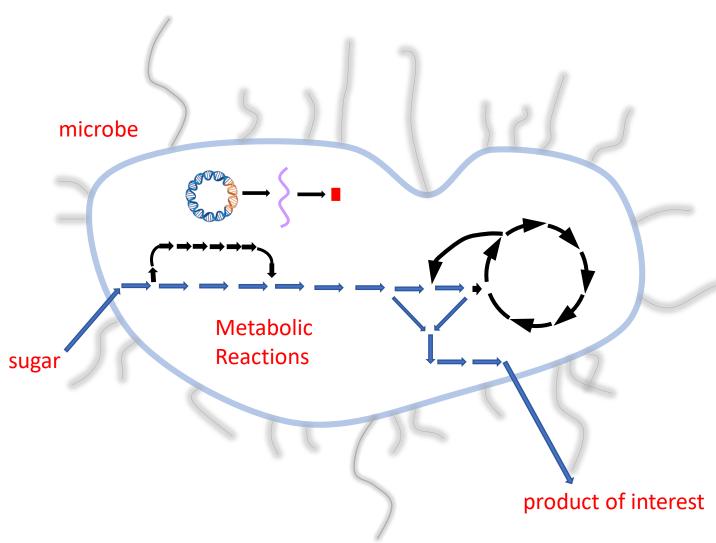
Review Basic Concepts

Model of Escherichia coli (E. coli) cells Model of *Chinese Hamster Ovary*(*CHO*) cells



# Motivation: Metabolic Engineering of Organisms

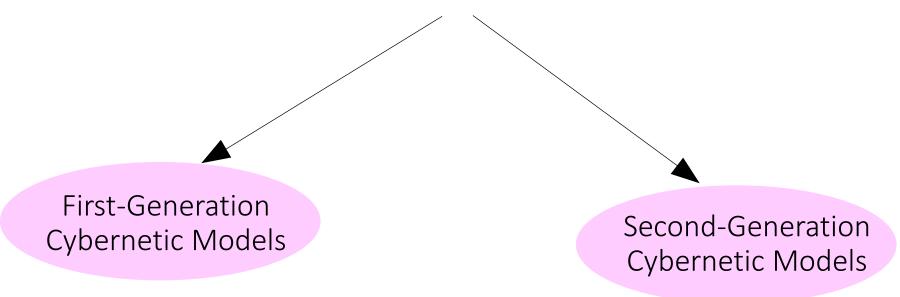






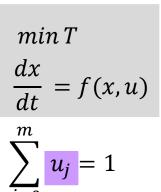
# Cybernetic Models

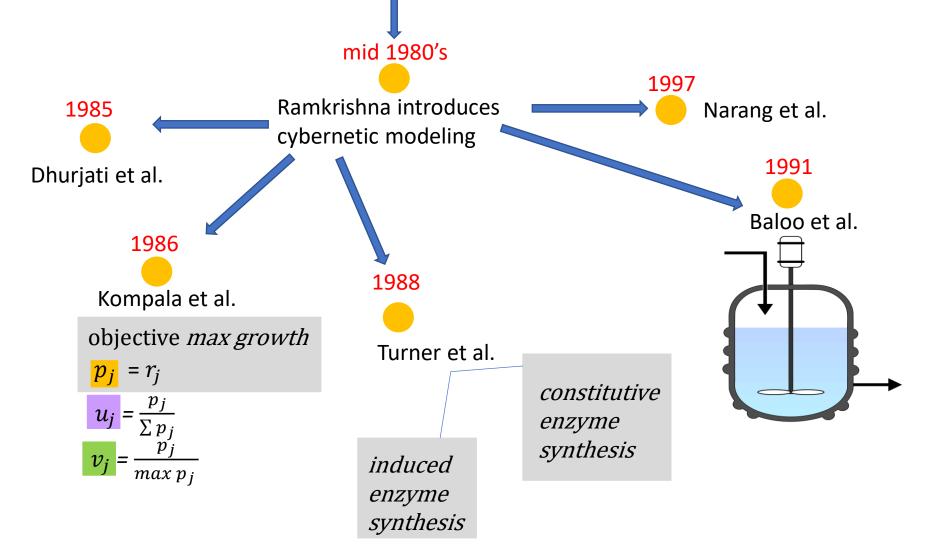
- 1. Assumption: The ultimate goal of an organism is to grow and survive
- 2. A cell will act, sense, compare itself to its goal and self correct its inner functions until it can successfully achieve its goal

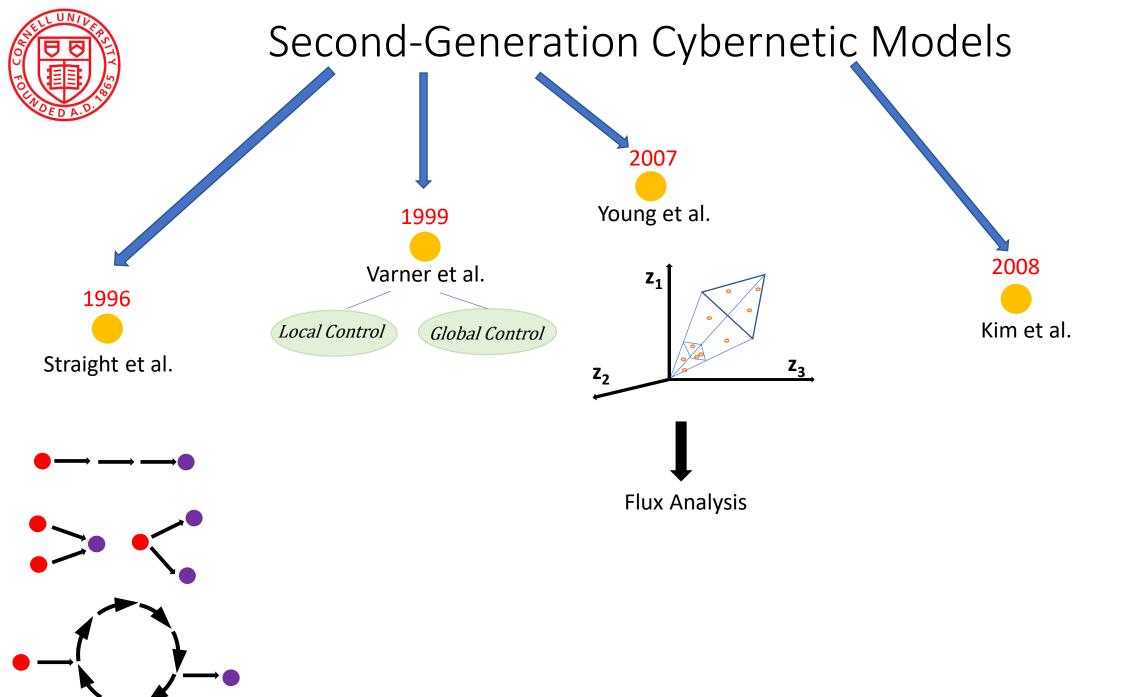




# First-Generation Cybernetic Models





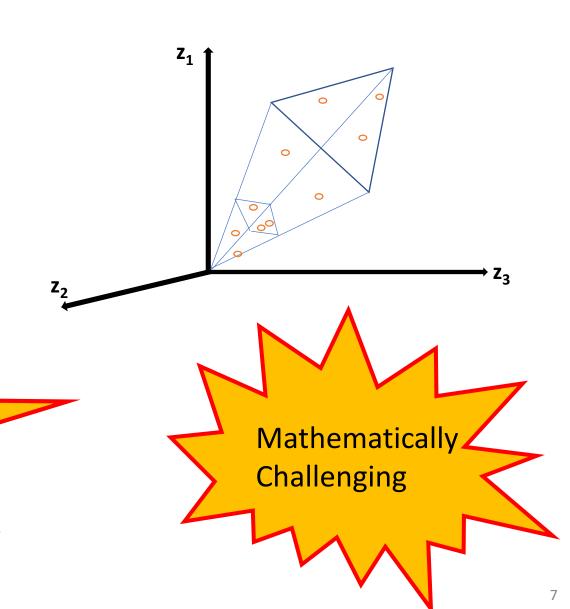




# What are elementary flux modes?

$$\sum_{j=1}^{R} S_{ij} z_j = 0$$
 for all  $i = 1,...,M$  j = 1,...,R

(Steady state assumption) constraints  $lb_i \le z_i \le ub_i$ 



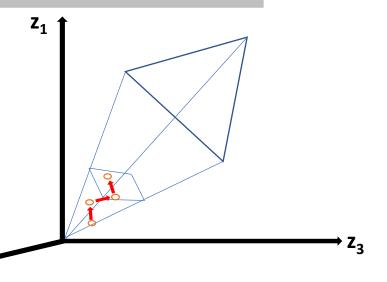
**Computationally Expensive** 



# Steady State Flux Analysis

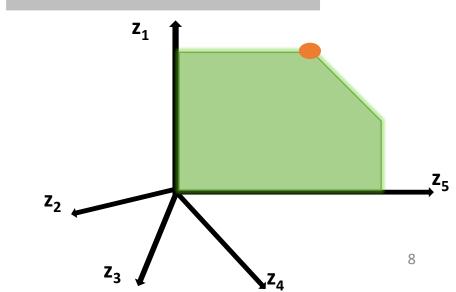
Markov-Chain Monte-Carlo

**UnBiased Flux Analysis** 



Flux Balance Analysis

Biased Flux Analysis





# Flux Balance Analysis

Intracellular fluxes are difficult to measure

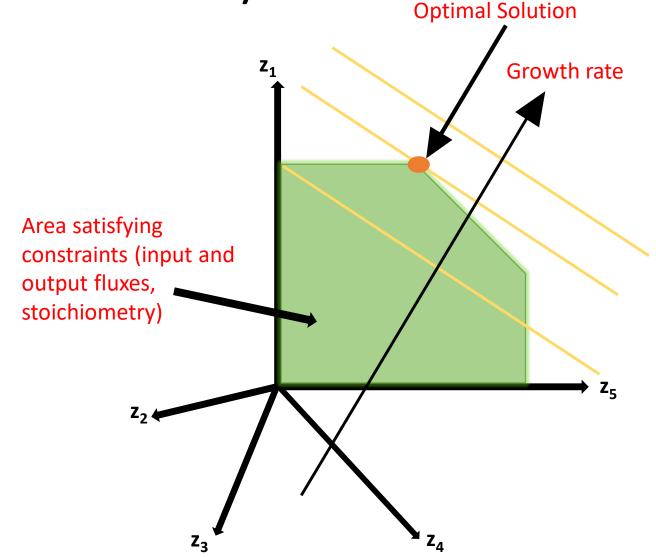
Extracellular fluxes are relatively easier to measure and are used to constrain the solution space

Mathematical Formulation

 $max c^T z$ 

$$\sum_{j=1}^{R} S_{ij} z_{j} = 0$$
 for all  $i = 1,...,M$  j = 1,...,R

(Steady state assumption) constraints  $lb_i \le z_i \le ub_i$ 





# Markov-Chain Monte-Carlo Sampling

1. Implement following constraints to get the solution space :

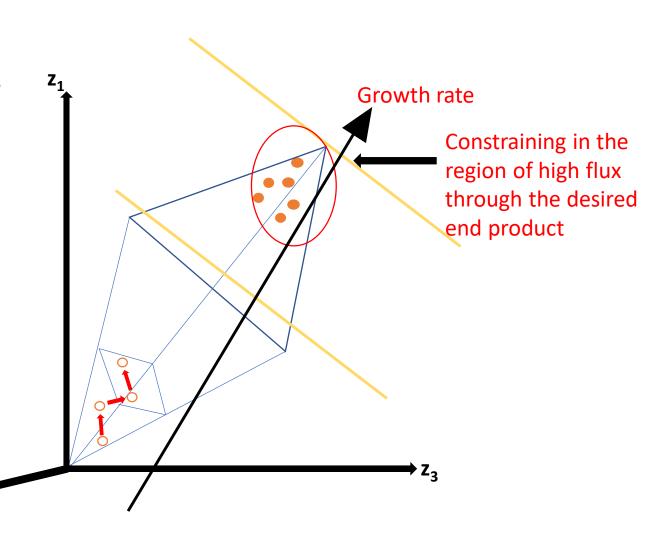
$$\sum_{j=1}^{R} S_{ij} z_{j} = 0 \text{ for all } i = 1,...,M$$

$$j = 1,...,R$$

(Steady state assumption) constraints  $|b_i \le z_i \le ub_i$ 

2. Select an initial point

3. Choose a random direction and step size to reach next point in the solution space





#### FBA vs Markov-Chain Monte-Carlo

#### Flux Balance Analysis

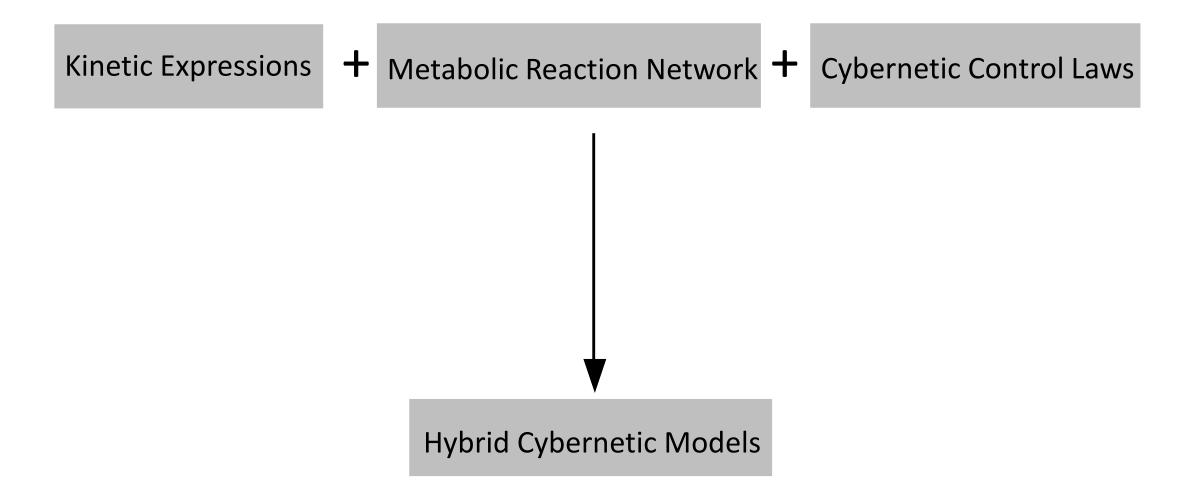
- 1. Assumes an objective function like growth maximization
- 2. Artificial objective functions are used to generate multiple flux distributions
- 3. Solution time is within seconds

#### Markov-Chain Monte-Carlo

- 1. No growth maximization assumption
- 2. A sample of feasible flux distribution is obtained
- 3. Solution time ranges in minutes to an hour



# Hybrid Cybernetic Models





#### Contribution

Central Carbon Constraints on Flux Balance Cybernetic Kinetic Rate Hybrid Cybernetic Model stoichiometry + extracellular + Analysis + Control Laws + Expressions = with FBA (E. coli) fluxes

Genome scale stoichiometry (CHO-K1) Constraints on fluxes Flux Balance Analysis - Cybernetic Kinetic Rate Hybrid Cybernetic Model + Control Laws + Expressions = with FBA

Genome scale | Constraints on stoichiometry (CHO-K1) | Cybernetic | Cy



## Overview

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Model of Escherichia coli (E. coli) cells Model of *Chinese*Hamster Ovary
(CHO) cells



# Hybrid Cybernetic Models

#### **Rate Expressions**

$$r_{M,l} = \frac{k_{M,l}}{k_l} e_l \frac{s_l}{k_l + s_l}$$

$$r_{E,l} = \frac{s_l}{K_l + s_l}$$

#### **Cybernetic Control Laws**

$$u_{l} = \frac{z_{s,l} r_{M,l}}{\sum_{l=1}^{L} z_{s,l} r_{M,l}}$$

$$z_{1}$$

$$z_{2}$$

$$v_l = \frac{z_{s,l} r_{M,l}}{\max_{l=1,\dots,L} z_{s,l} r_{M,l}}$$



# The E. coli System

#### Model Equations and Assumptions

Batch fermentation of E. coli has been simulated. A reaction network of 118 reactions and 62 metabolites was used. Dynamics within the central carbon metabolism were the focus. Cell death has been ignored.

Specific growth rate for each mode is given as

$$r_{G,l} = z_{biomass,l} r_{M,l}$$

The abundance of extracellular species is given as

$$\frac{ds_i}{dt} = \sum_{i=1}^{R} \sum_{l=1}^{L} S_{i,j} z_j r_{M,l} v_l c$$

Total growth rate of the system is written as

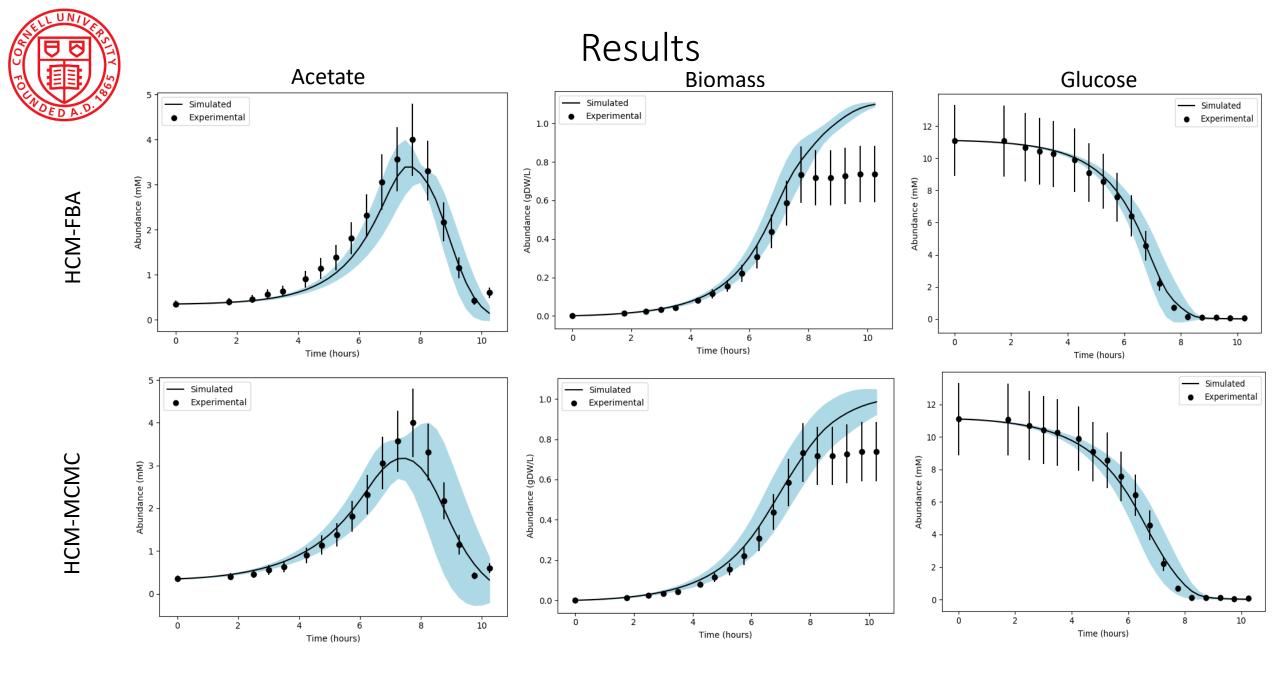
$$\mu = \sum_{l=1}^{L} r_{G,l} v_l$$

Pseudoenzyme levels are governed by

$$\frac{de_l}{dt} = \alpha_l + r_{E,l} u_l - (\beta_l + \mu) e_l$$

Cell mass is governed by

$$\frac{dc}{dt} = \mu c$$





# Summary

- 1. 29 flux modes were generated using FBA and MCMC sampling
- 2. The framework selects glucose as the preferred substrate
- 3. There is a clear switch in substrate utilization when glucose is exhausted. Acetate uptake begins at this time
- 4. HCM-MCMC has no performance penalty compared to HCM-FBA
- 5. HCM-MCMC offers a better solution strategy than HCM-FBA by eliminating the need to define an objective function that might not be biologically relevant



## Overview

Review Basic Concepts

Model of

Escherichia coli

(E. coli) cells

Model of *Chinese Hamster Ovary*(*CHO*) cells



# Chinese Hamster Ovary System

#### Model Equations and Assumption

Fed-batch based kinetics are performed for *CHO-K1* cells has been simulated. A reaction network of 4723 reactions and 2773 metabolites was used. Cell death has been ignored. Volumetric flow rate will affect dynamics of this system.

Feed stream enters the reactor at a volumetric flow rate F

$$F = \frac{dV}{dt}$$

The abundance of extracellular species is given as

$$\frac{ds_i}{dt} = D(s_f - s) + \sum_{j=1}^{R} \sum_{l=1}^{L} S_{i,j} z_{i,j} r_{M,l} v_l c$$

Rate of dilution can be written as

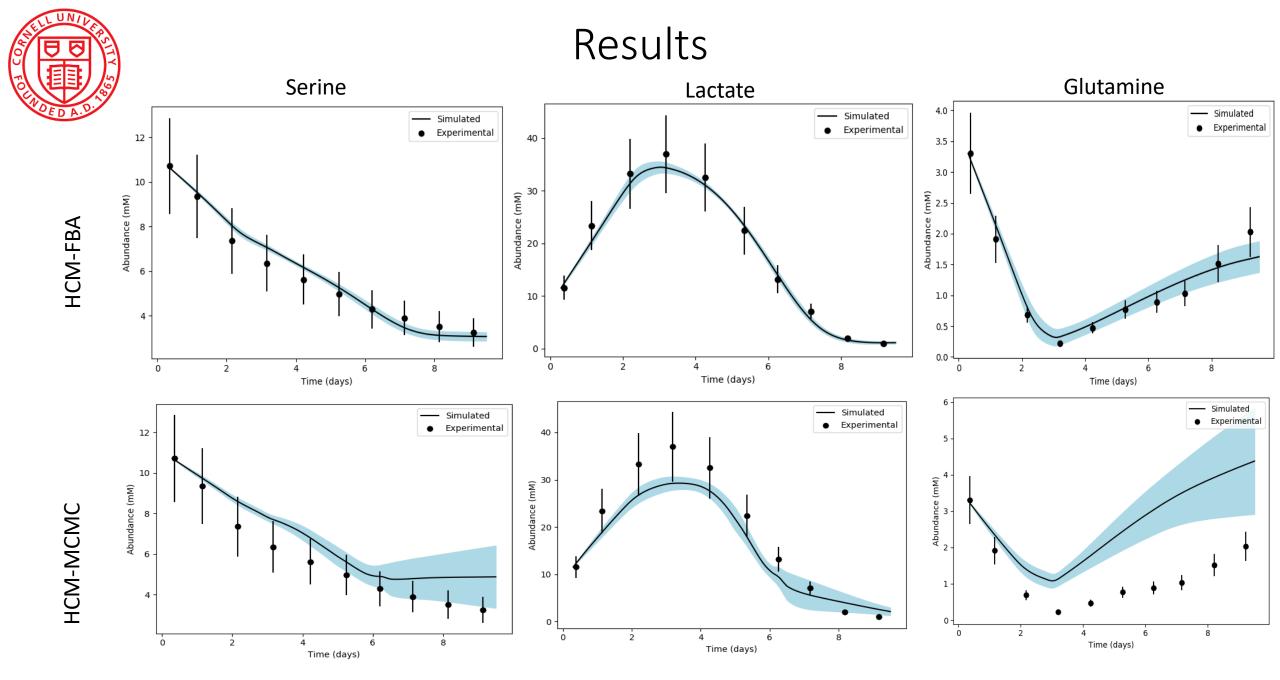
$$D = \frac{F}{V_o + F}$$

Pseudoenzyme levels are governed by

$$\frac{de_l}{dt} = \alpha_l + r_{E,l} u_l - (\beta_l + \mu) e_l$$

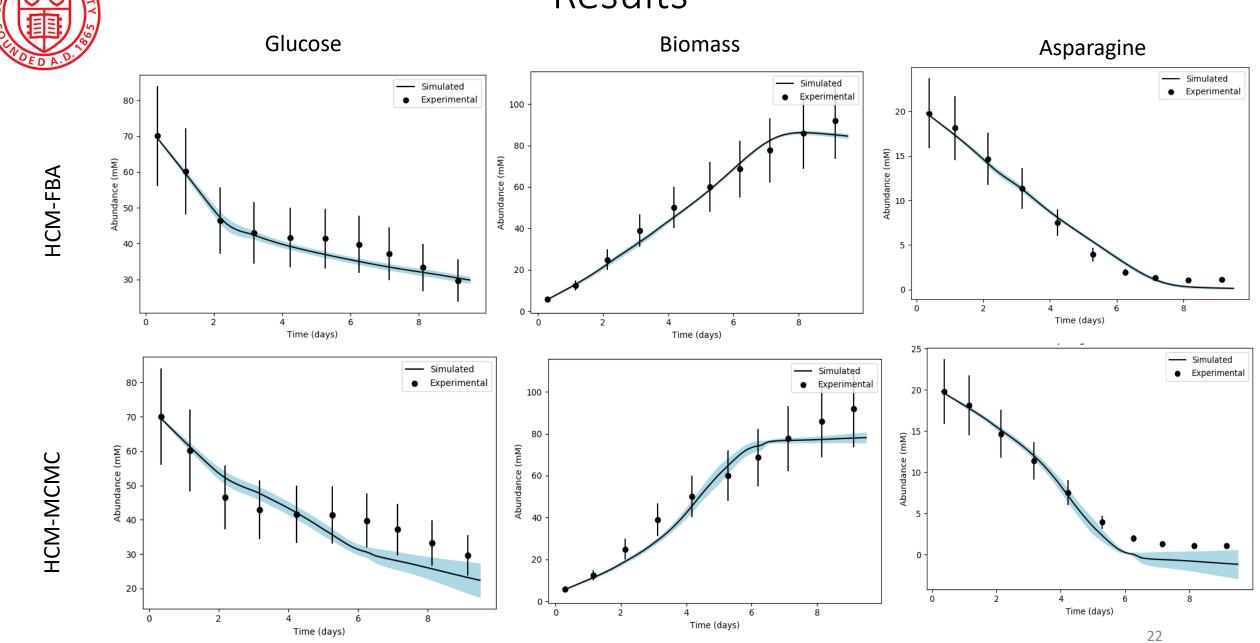
Cell mass is governed by

$$\frac{dc}{dt} = (\mu - D) c$$



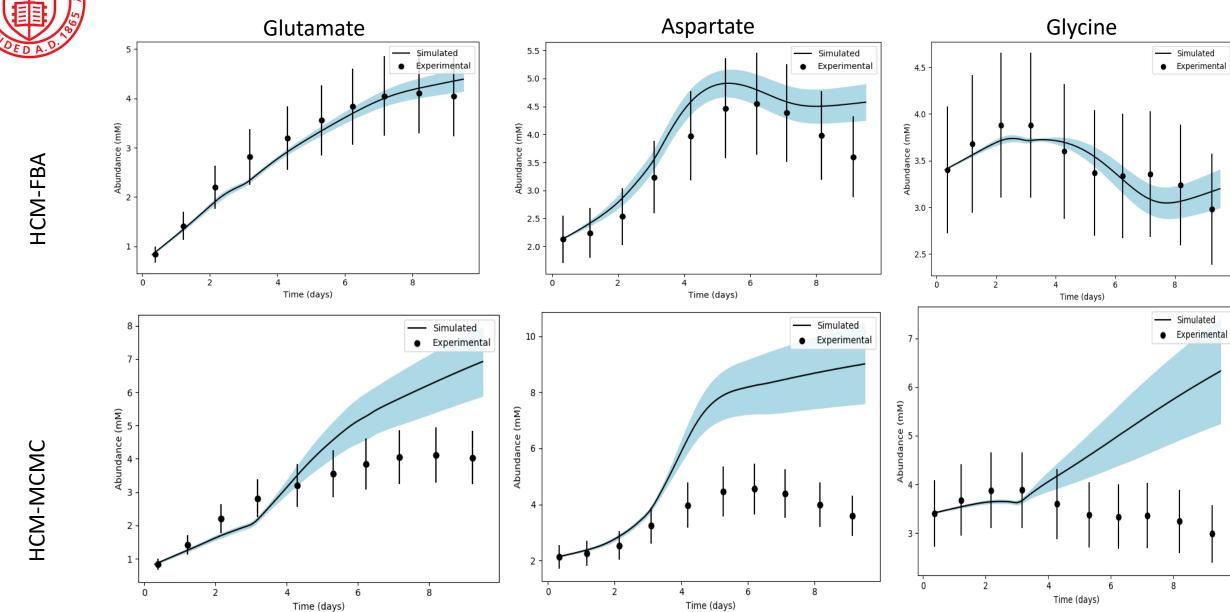
# HCM-FBA

# Results



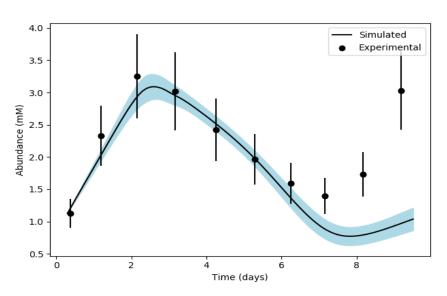


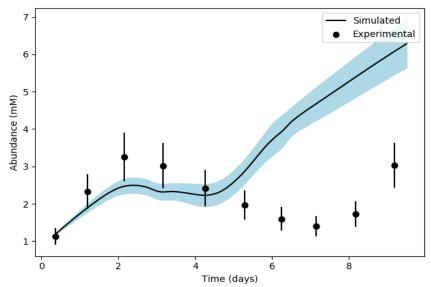
# Results

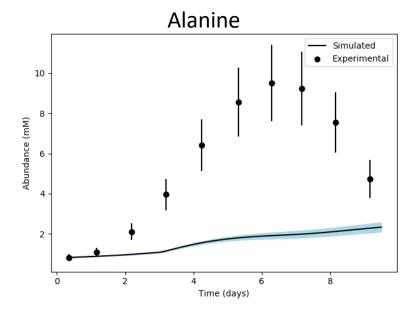


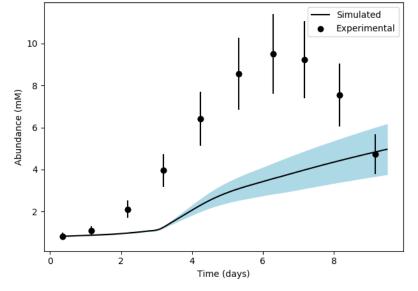
# Results

#### Ammonia









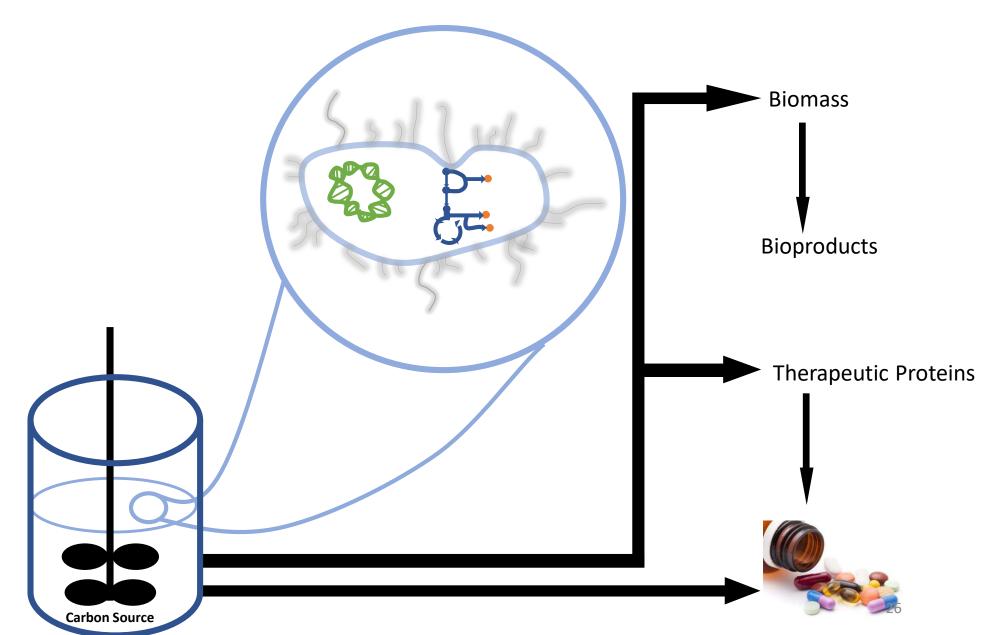


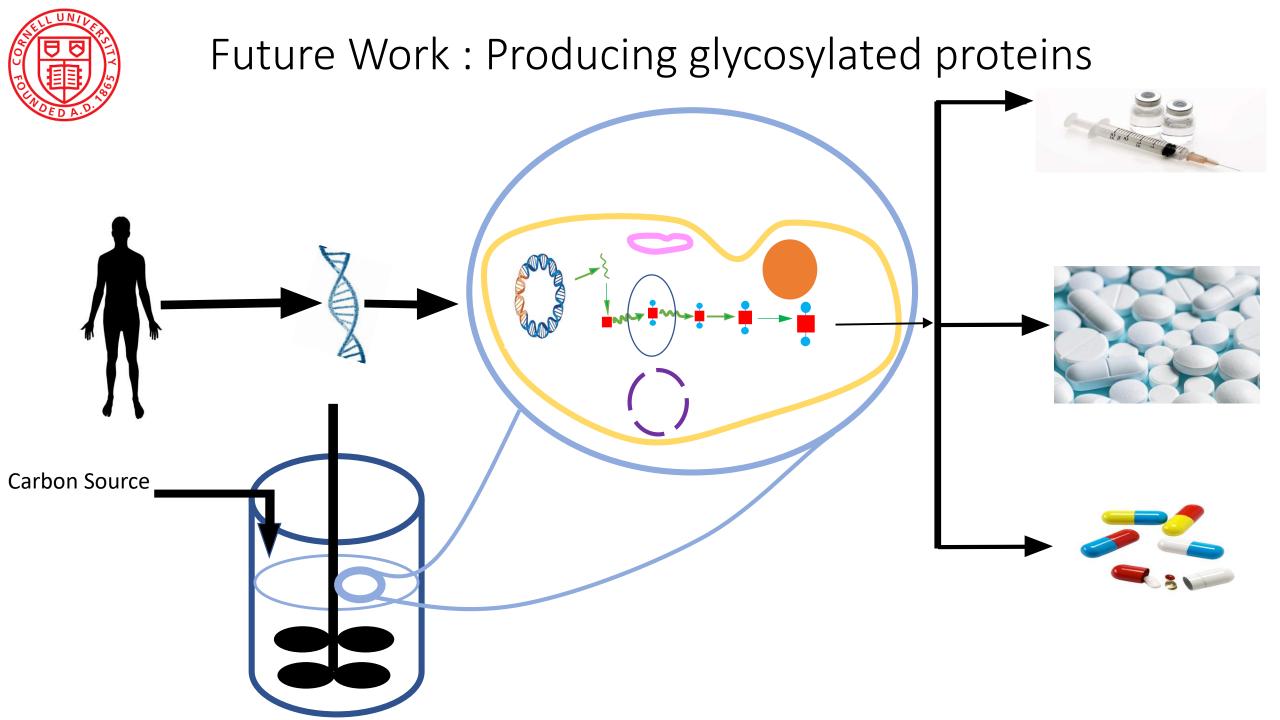
# Summary

- 1. 3 flux modes have been used to describe dynamics
- 2. E. coli shows single-substrate kinetics. CHO-K1 demonstrates multisubstrate kinetics
- 3. E. coli shows sequential uptake pattern. CHO-K1 demonstrates simultaneous uptake pattern
- 4. Glutamine, glucose, lactate, asparagine and serine are important for biomass production



# Future Work: Implementing New Age Biology







# Acknowledgement

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