Prepared by Julian García, based on material by
David Albrecht and María García de la Banda

Lecture 32
Priority Queues

FIT 1008
Introduction to Computer Science

MONASH University
Information Technology

Objectives

- To understand the **Priority Queue ADT**.
- Consider different implementations (advantages/ disadvantages)
- To understand the **Heaps**, and the Heap-based implementation of Priority Queues.

“Form an orderly queue to the left..”

“Form an orderly queue to the left..”

Uses of Priority Queues

- Hospital emergency rooms
- Job scheduling
- Discrete event simulations
- Graph algorithms
- Genetic algorithms

Priority Queue

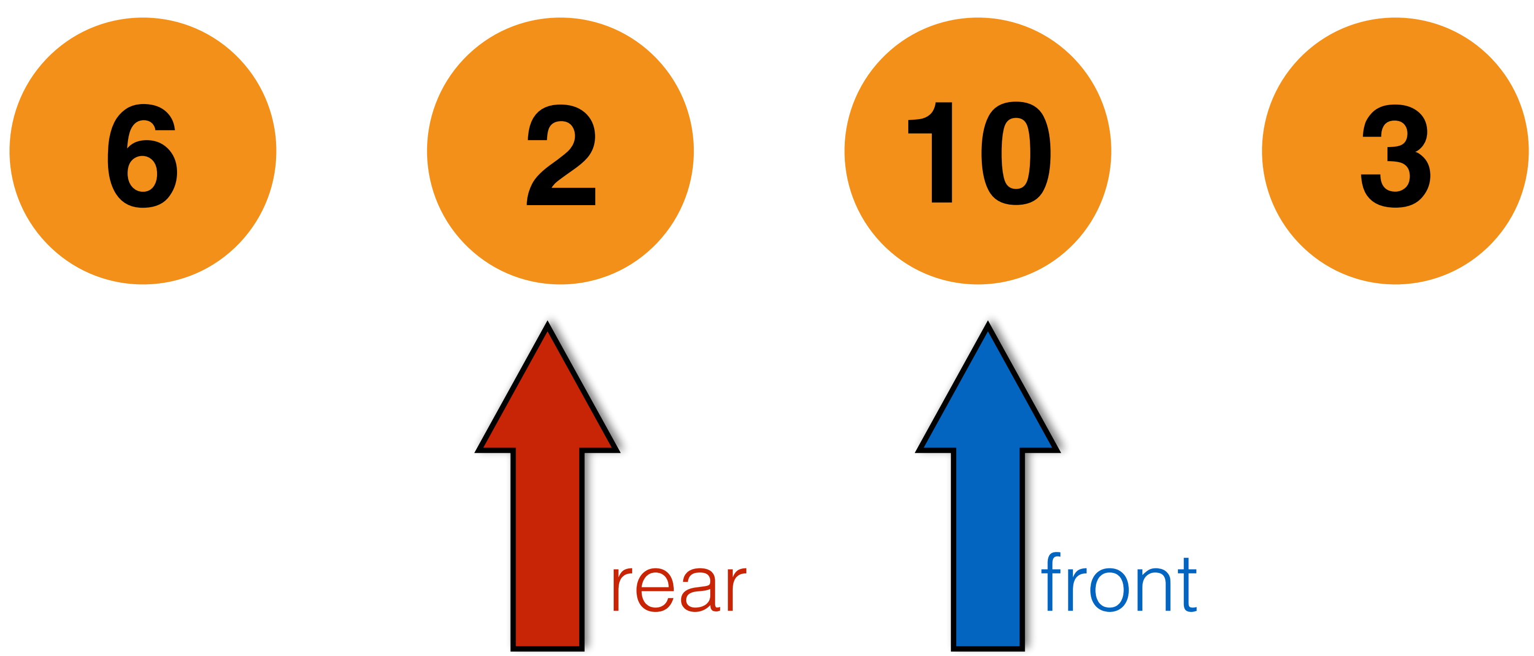
lowest in a dual implementation

FIFO if priority is assumed to be time spent in the queue.

- Each element has a numeric **priority**.
- Element with **highest priority** is processed **first**.

Operations:

```
add(key, element)
get_max()
```



The following data structures can be used to support the implementation of a Priority Queue ADT...

- Array-based List (sorted and unsorted)
- Linked List (sorted and unsorted)
- Binary Search Trees

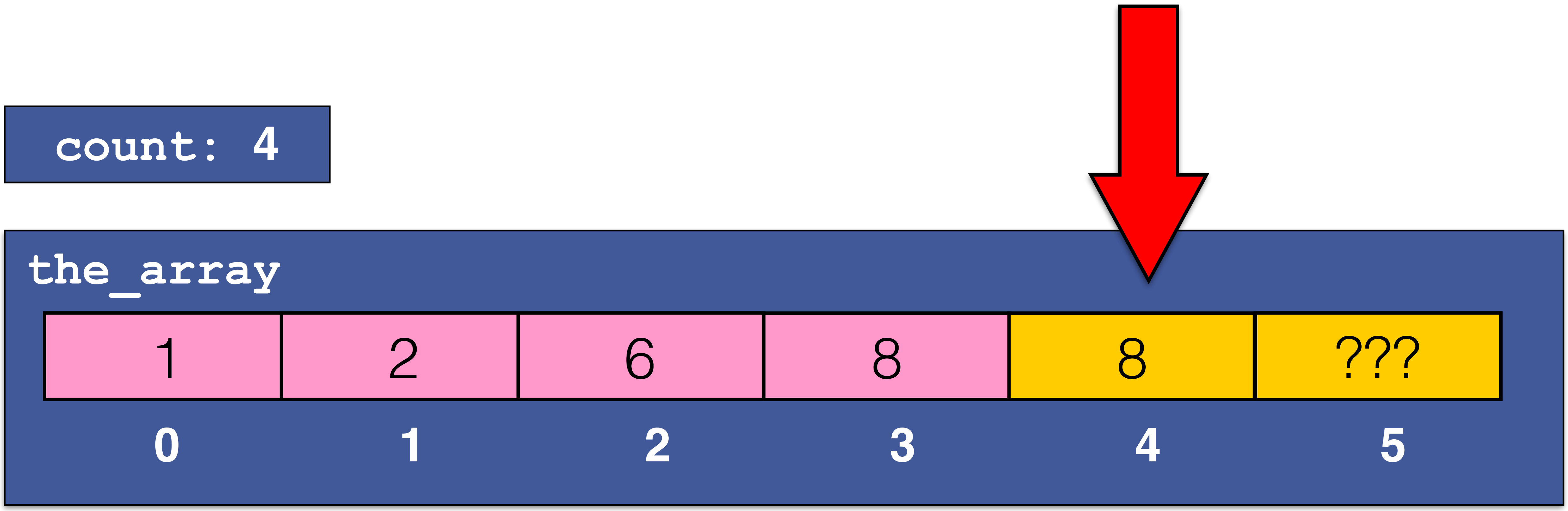
- Heaps (Binary Tree-based)
- Heaps (Array-based)

Implementing Priority Queues

- Standard operations:
  `is_empty, __len__, __init__`
- Core operations:
  - **`get_max():`**
    returns the max element (and removes it from the queue)
  - **`add(element)`**: adds element to the priority queue

Priority Queue

(Unsorted) Array-Based List



`get_max()`

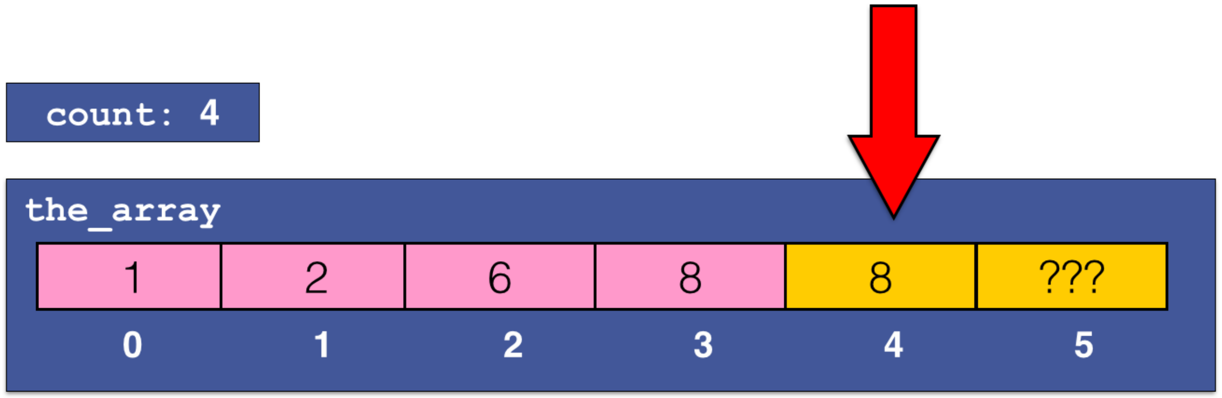
- Find max item.
- Remove and reshuffle.

`add()`

- Add item at the back.

Complexity

(Priority Queue using **Unsorted Lists**)



`get_max()`

- Find max item.
- Remove and reshuffle.

O(n)

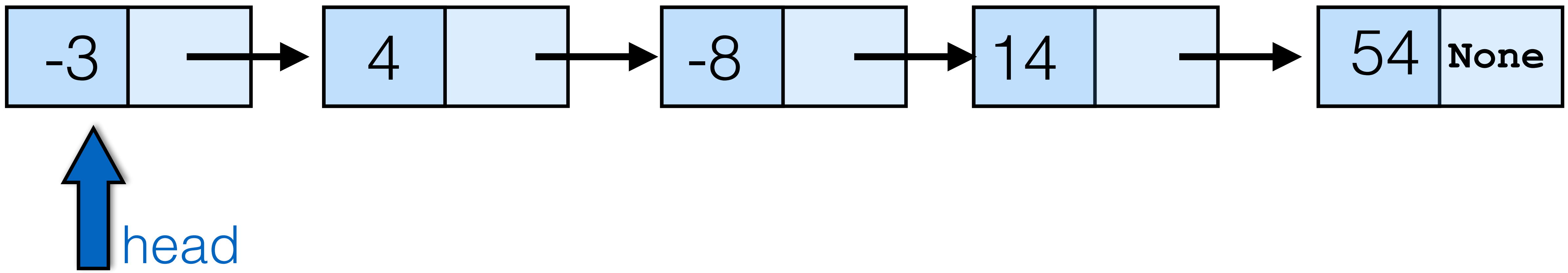
`add()`

- Add item at the back.

O(1)

Complexity

(Priority Queue using **Unsorted Linked Lists**)



`get_max()`

- Find max item.
- Remove.

O(n)

`add()`

O(1)

- Add item at the head.

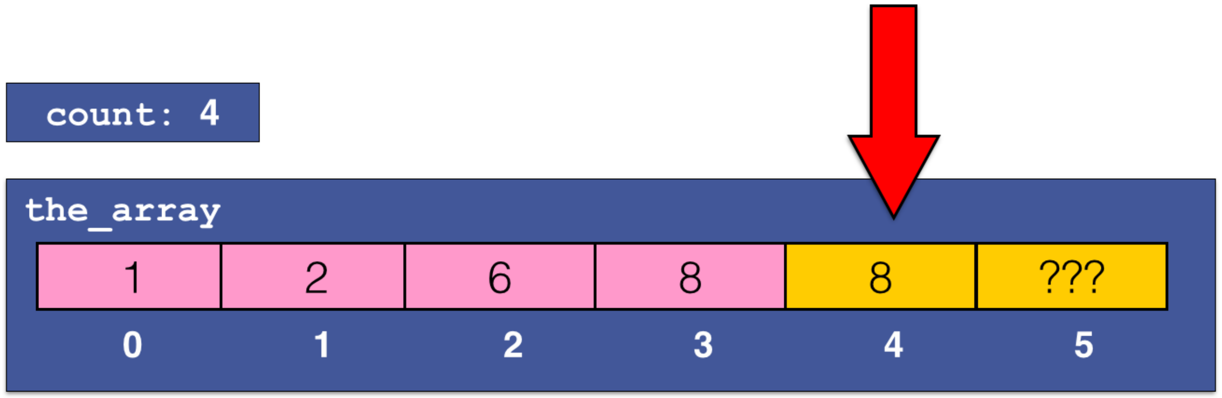
Adding to an array-based sorted list

```
def add(self, new_item):
    # easy if the list is empty
    if self.is_empty():
        self.the_array[self.count] = new_item
        self.count += 1
        return True
    # if the lis is not empty...
    has_place_left = not self.is_full()
    if has_place_left:
        # find correct position
        index = 0
        while index < self.count and new_item > self.the_array[index]:
            index+=1
        # now index has the correct position
        # we go backwards from count -1 up to index
        for i in range(self.count-1, index-1, -1):
            # "moving" the item in position i to position i+1
            self.the_array[i+1] = self.the_array[i]
        # insert new item
        self.the_array[index] = new_item
        # increment counter
        self.count+=1
    return has_place_left
```

Find correct position

Move things to make space

Complexity
(Priority Queue using **Sorted** Lists)



`get_max()`

- Find max item.
- Remove.

Position `count-1`

O(1)

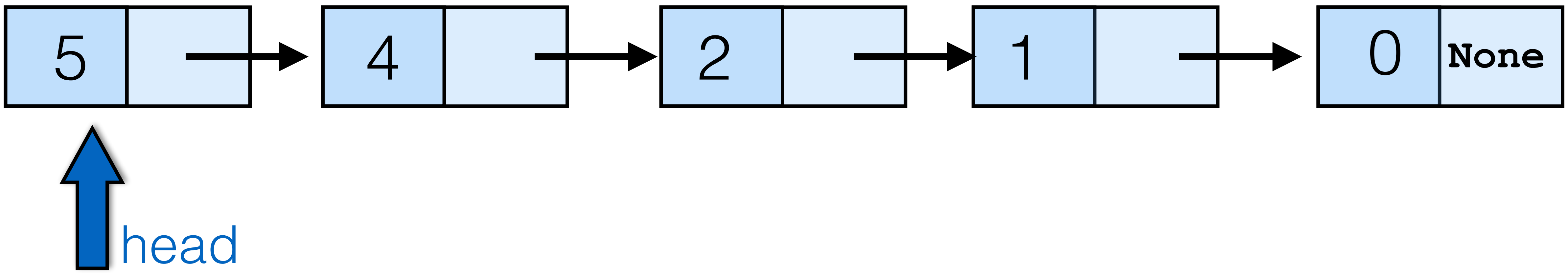
`add()`

- Add item.

O(n)

Complexity

(Priority Queue using **Sorted** Linked Lists)



`get_max()`

at head

- Find max item.
- Remove.

O(1)

`add()`

- Find correct position.

O(n)

Priority Queues using **linear** structures…

Implementation	`get_max()`	`add`
Unsorted array	O(n)	O(1)
Unsorted linked list	O(n)	O(1)
Sorted array	O(1)	O(n)
Sorted linked list	O(1)	O(n)

```python
class BinarySearchTreeNode:
    def __init__(self, key, item=None, left=None, right=None):
        self.key = key
        self.item = item
        self.left = left
        self.right = right
```

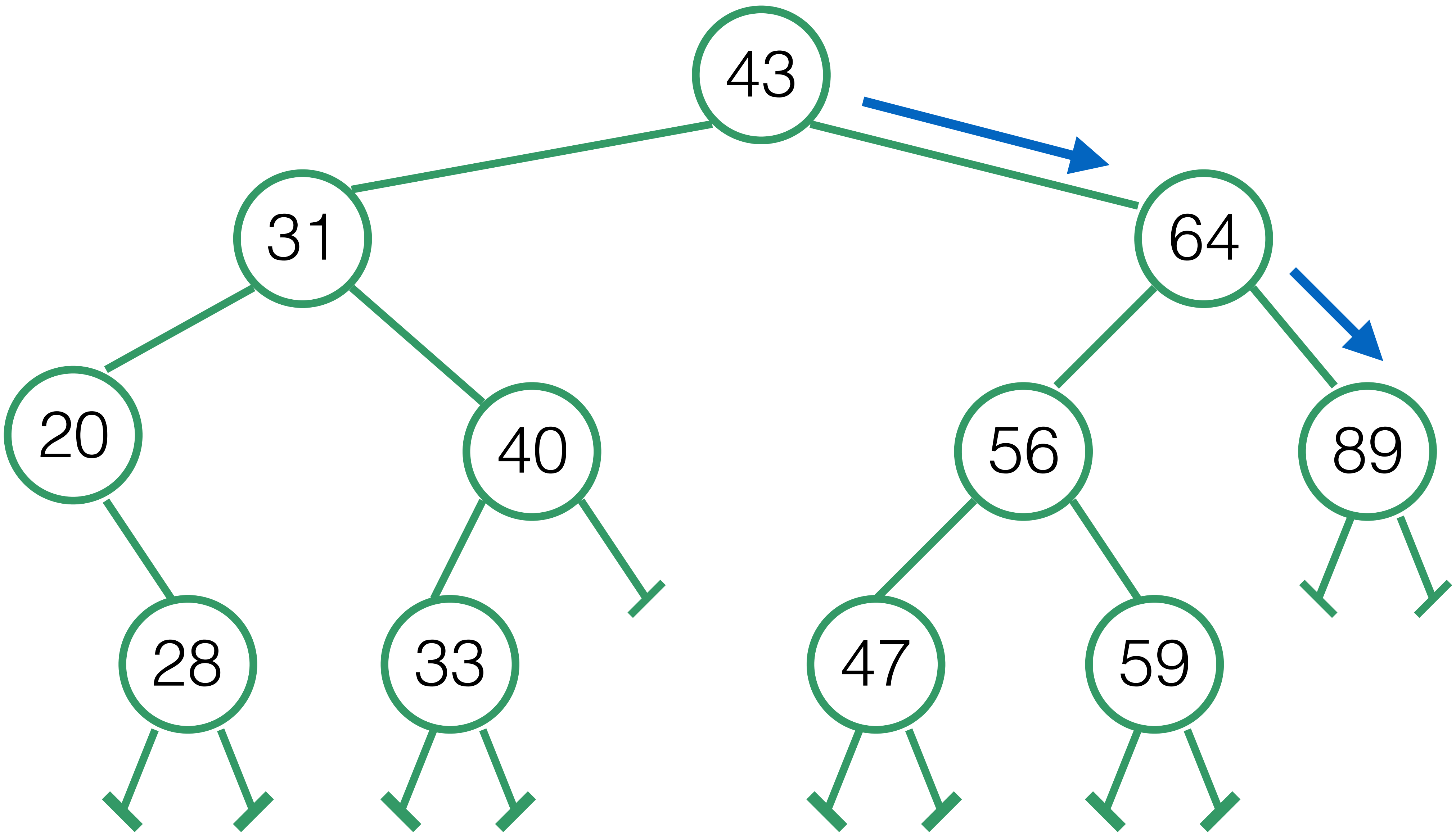
`add()` trivial.

```python
class BinarySearchTree:
    def __init__(self):
        self.root = None

    def is_empty(self):
        return self.root is None
```

`get_max()`?

BST:

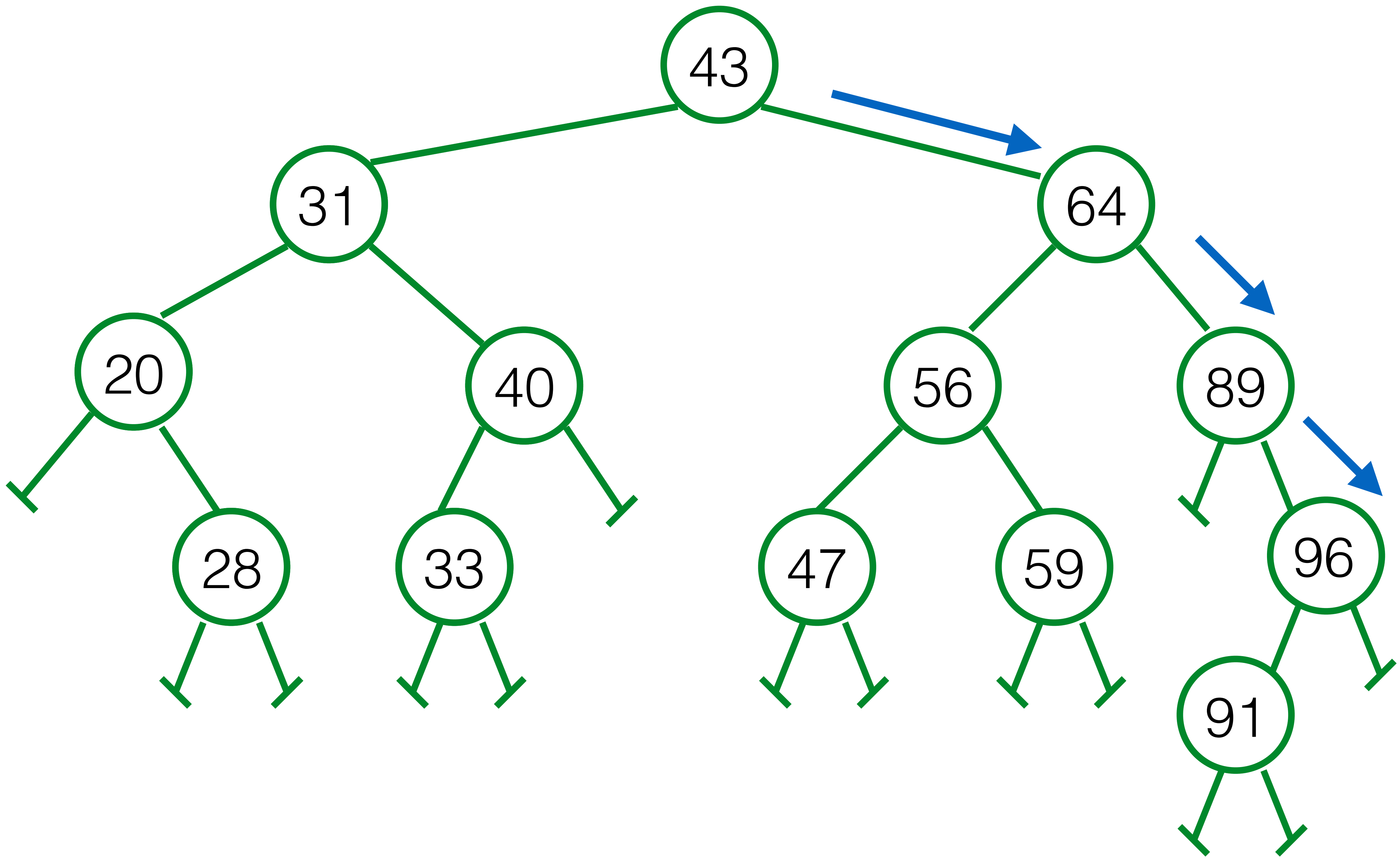
`get_max()`?



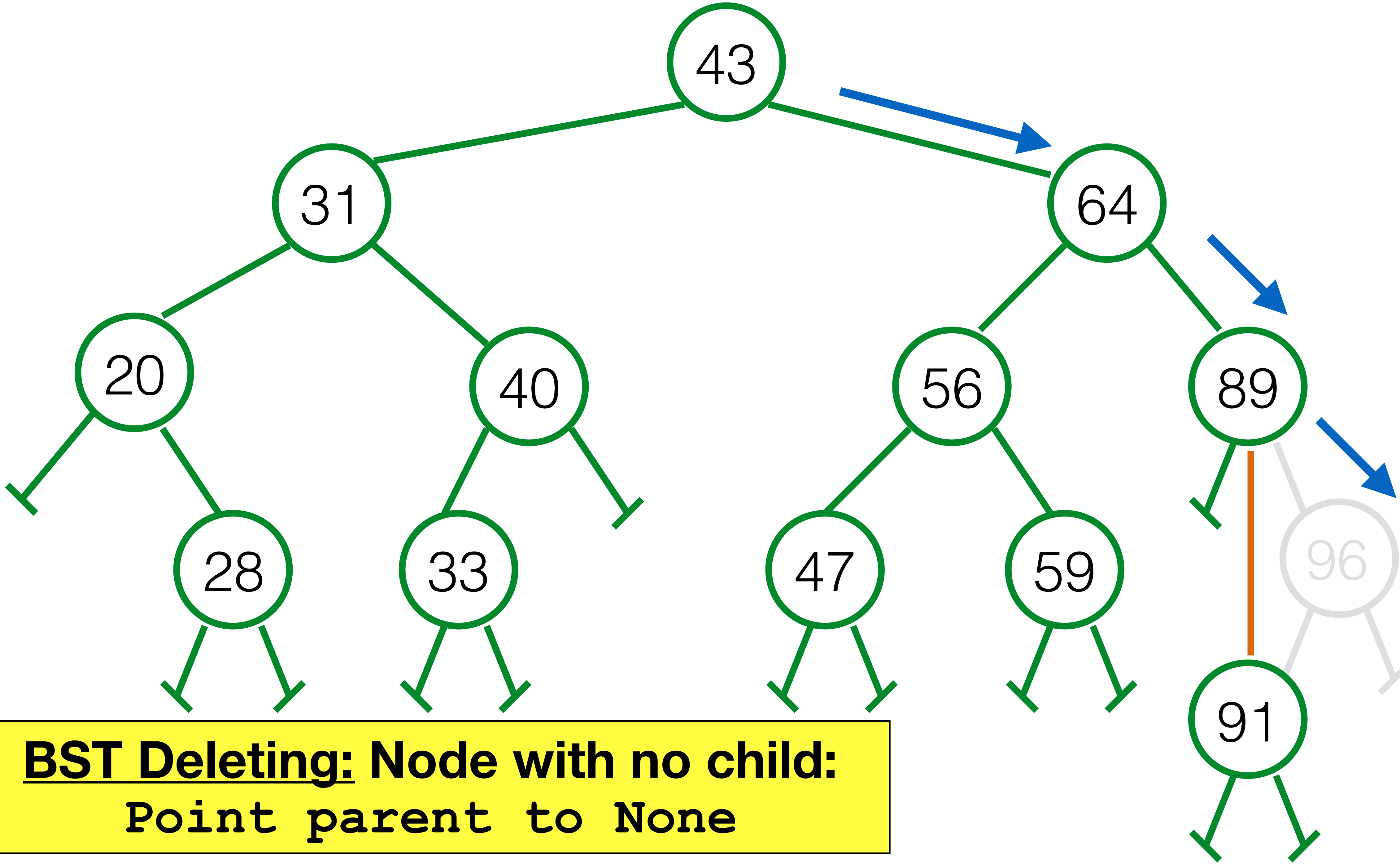
Right-most node: Go right until you can

Complexity depends on the height!

`get_max()`



`get_max()`



BST Deleting: Node with no child:
`Point parent to None`

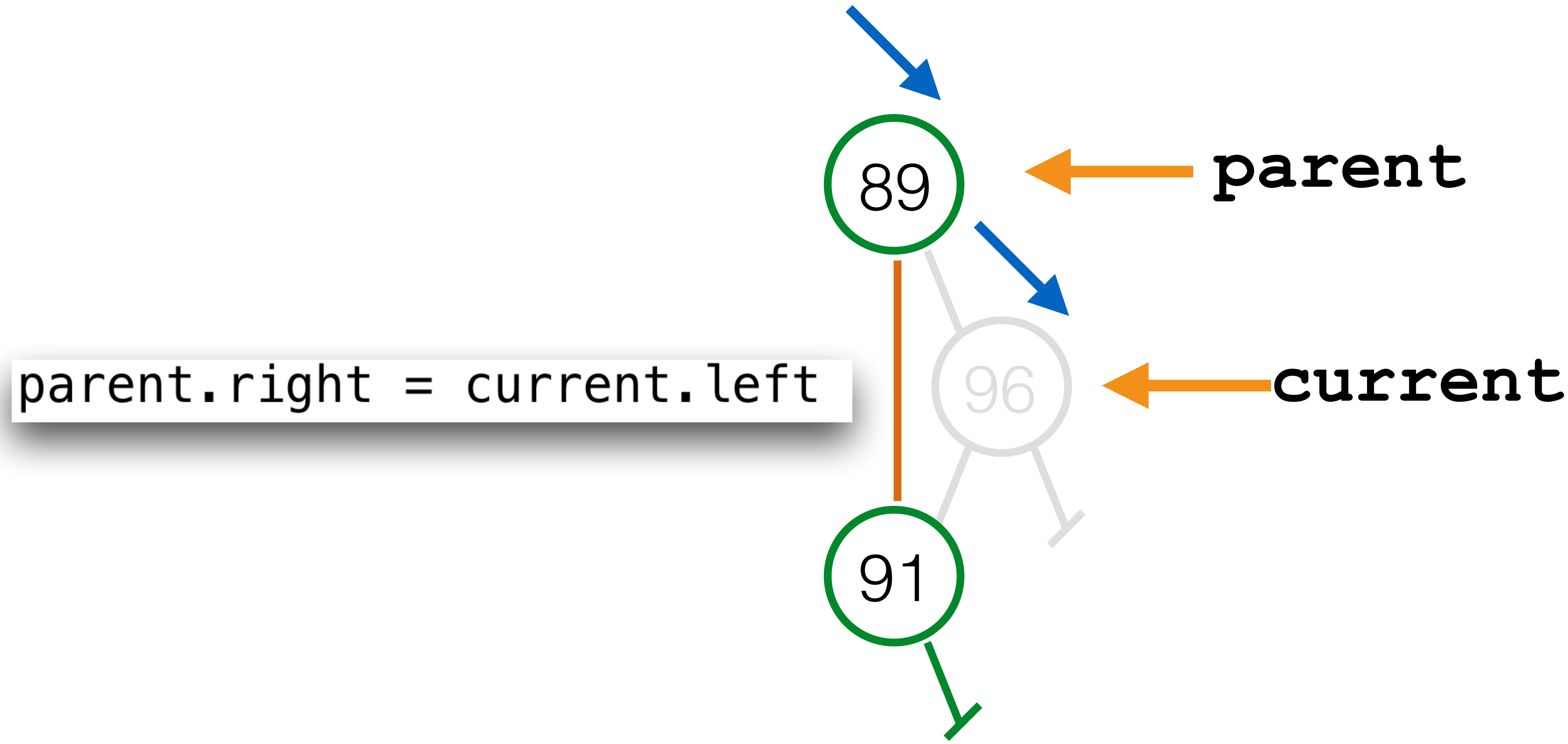
BST Deleting: Node with one child:
`Point parent to child of deleted node`

Simple version (without deleting)

```python
def get_max(self):
    if self.root is None:
        raise ValueError("Empty Priority Queue")
    else:
        return self.get_max_aux(self.root)

def get_max_aux(self, current):
    if current.right is None:  # base case: at max
        return current.item
    else:
        return self.get_max_aux(current.right)
```

With delete?

[Remember the parent]



<u>Node with no child:</u>
`Point parent to None`

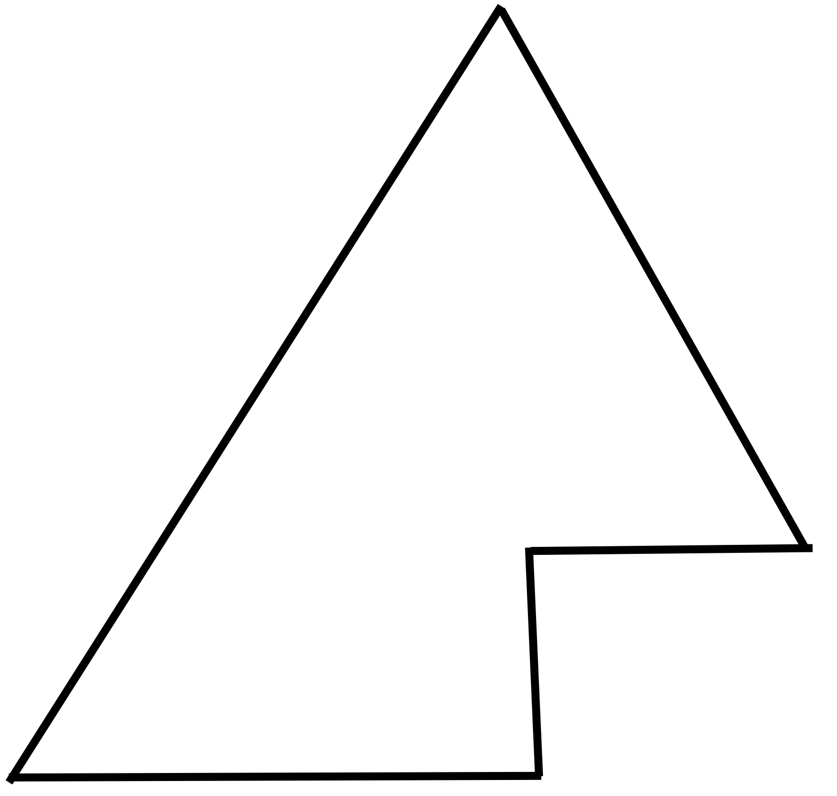
<u>Node with one child:</u>
`Point parent to child of deleted node`

```python
def get_max(self):
    if self.root is None:
        raise ValueError("Priority Queue is empty")
    elif self.root.right is None: # root has the max
        temp = self.root.item
        self.root = self.root.left # delete root
        return temp
    else:
        return self.get_max_aux(self.root.right, self.root)


def get_max_aux(self, current, parent):
    if current.right is None: # base case: at max
        parent.right = current.left
        return current.item
    else:
        return self.get_max_aux(current.right,current)
```

Alternative implementation

```
def get_max(self):
    if self.root is None:
        raise ValueError("Priority Queue is empty")
    elif self.root.right is None: # root has the max
        temp = self.root.item
        self.root = self.root.left # delete root
        return temp
    else:
        return self.get_max_aux(self.root)


def get_max_aux(self, parent):
    if parent.right.right is None: # base case: at max
        temp = parent.right.item
        parent.right = parent.right.left
        return temp
    else:
        return self.get_max_aux(parent.right)
```

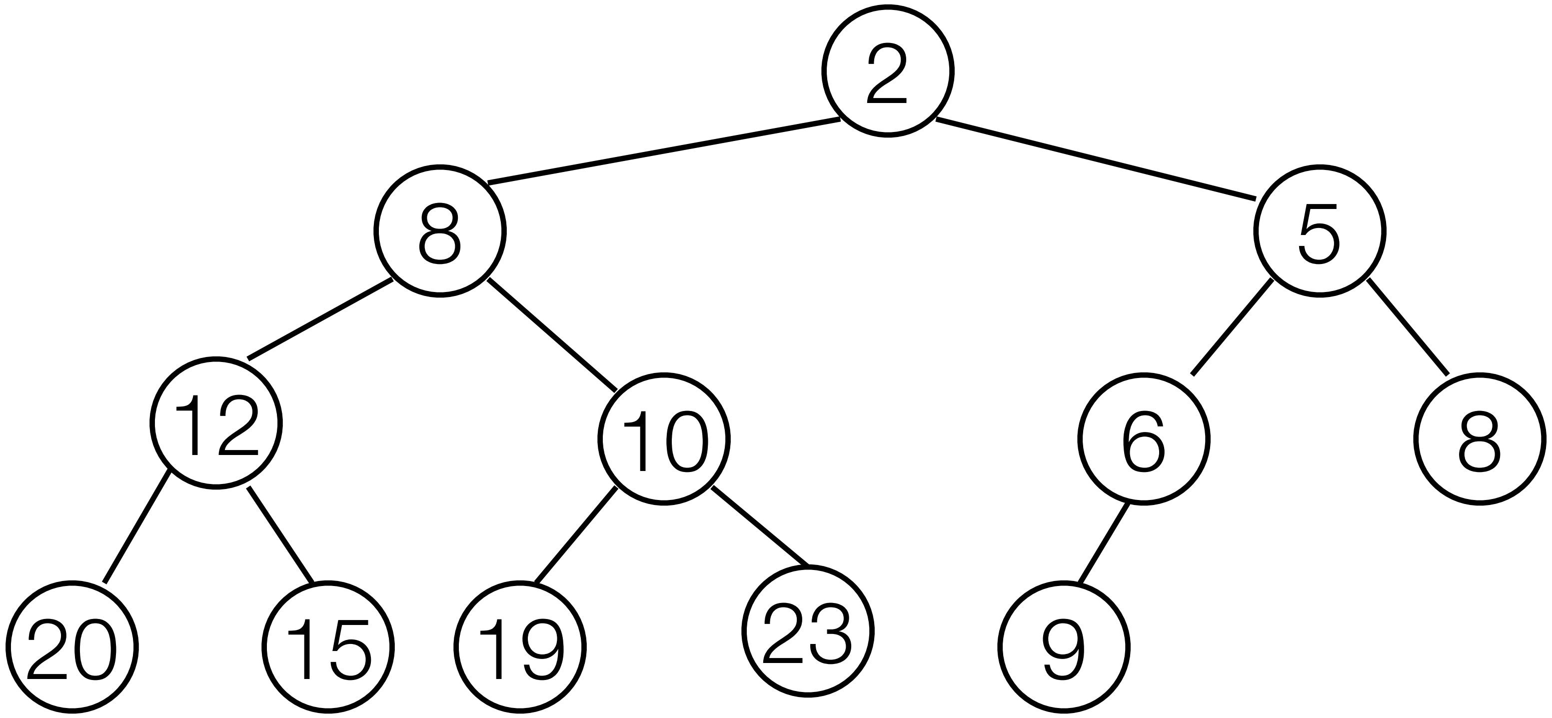
only passing the parent, but “looking down” two levels



A better implementation

- Each of the previous choices has one **O(N)** operation.
- O(Depth) for the binary tree with depth being N-1 if unbalanced.
- Can we do better? Use a (max) **heap.**
  - ➡ One could also use a min-heap
  - ➡ In FIT1008 we use max-heaps but the ideas are the same for a min-heap.

Heap (Min-Heap)

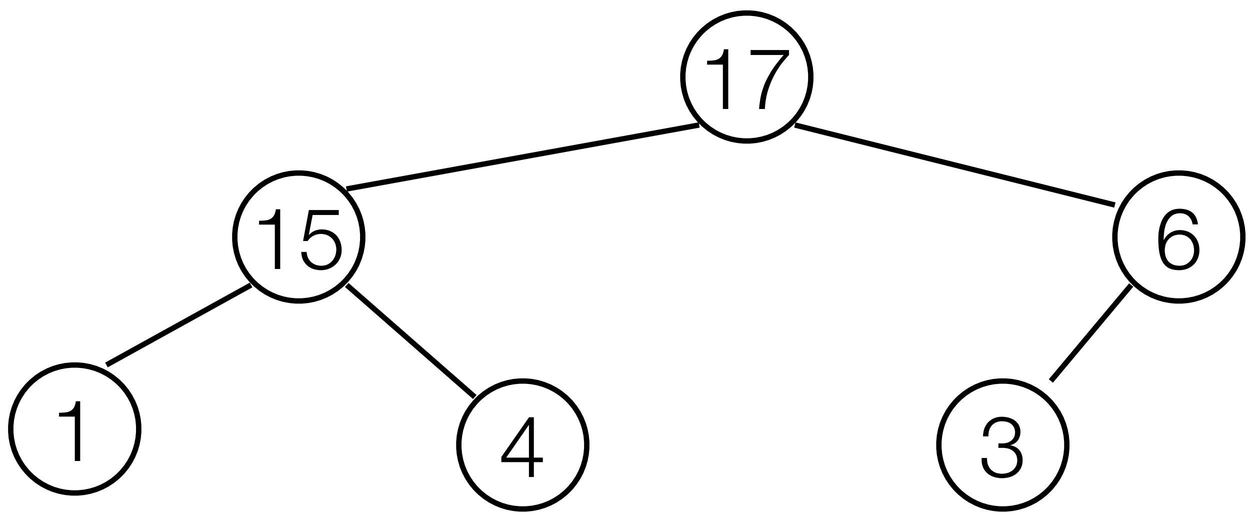


For **every** node:

- The values of the children are **greater or equal** to its value.
- **All the levels are filled**, except possibly the last one, which is filled left to right.

Note: The minimum is always at the root of the tree.

Heap (Max-Heap)

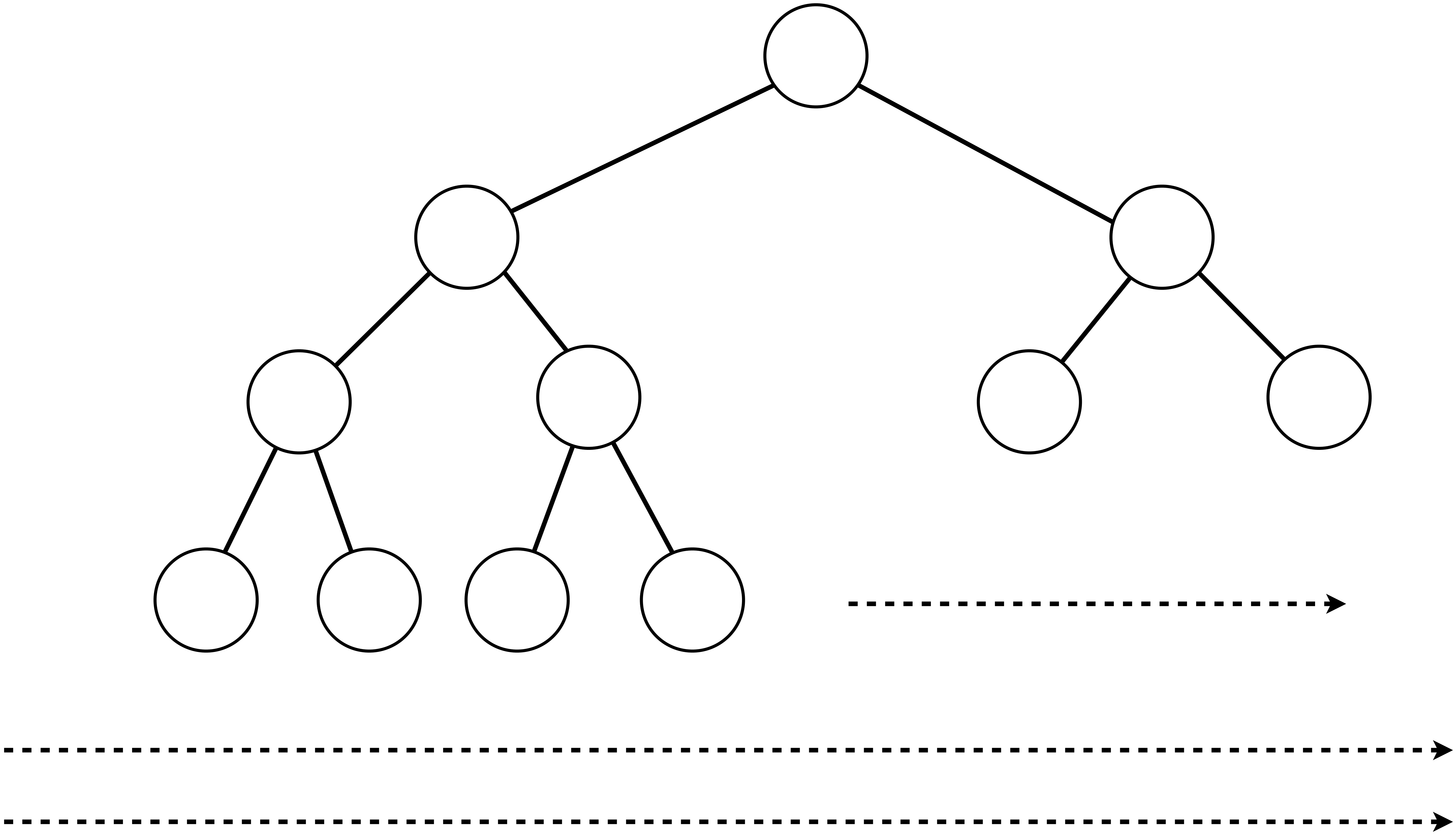


For **every** node:

- The values of the children are **smaller or equal** to its value.
- **All the levels are filled**, except possibly the last one, which is filled left to right.

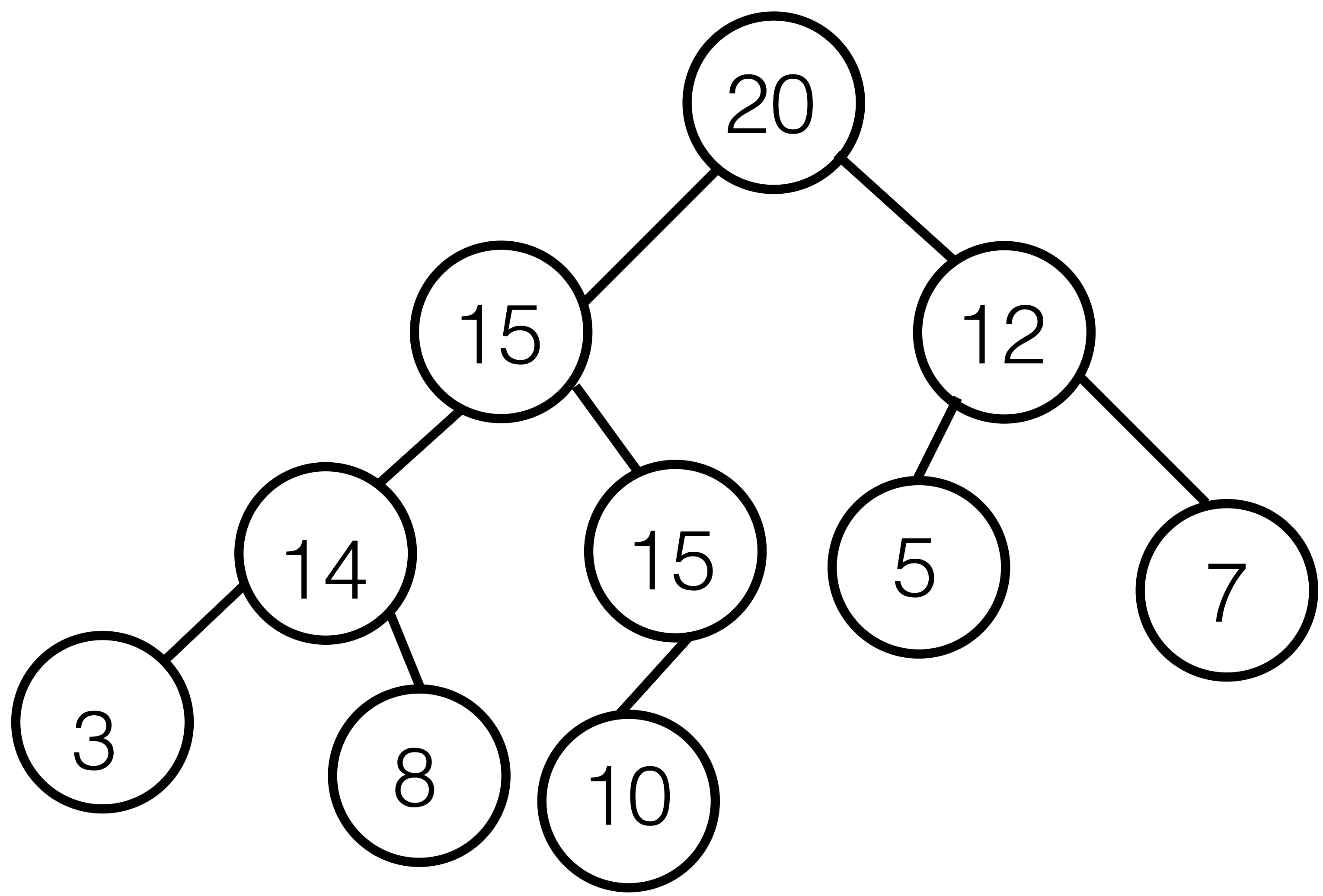
Note: The **maximum** is always at the root of the tree.

Building a *binary* heap



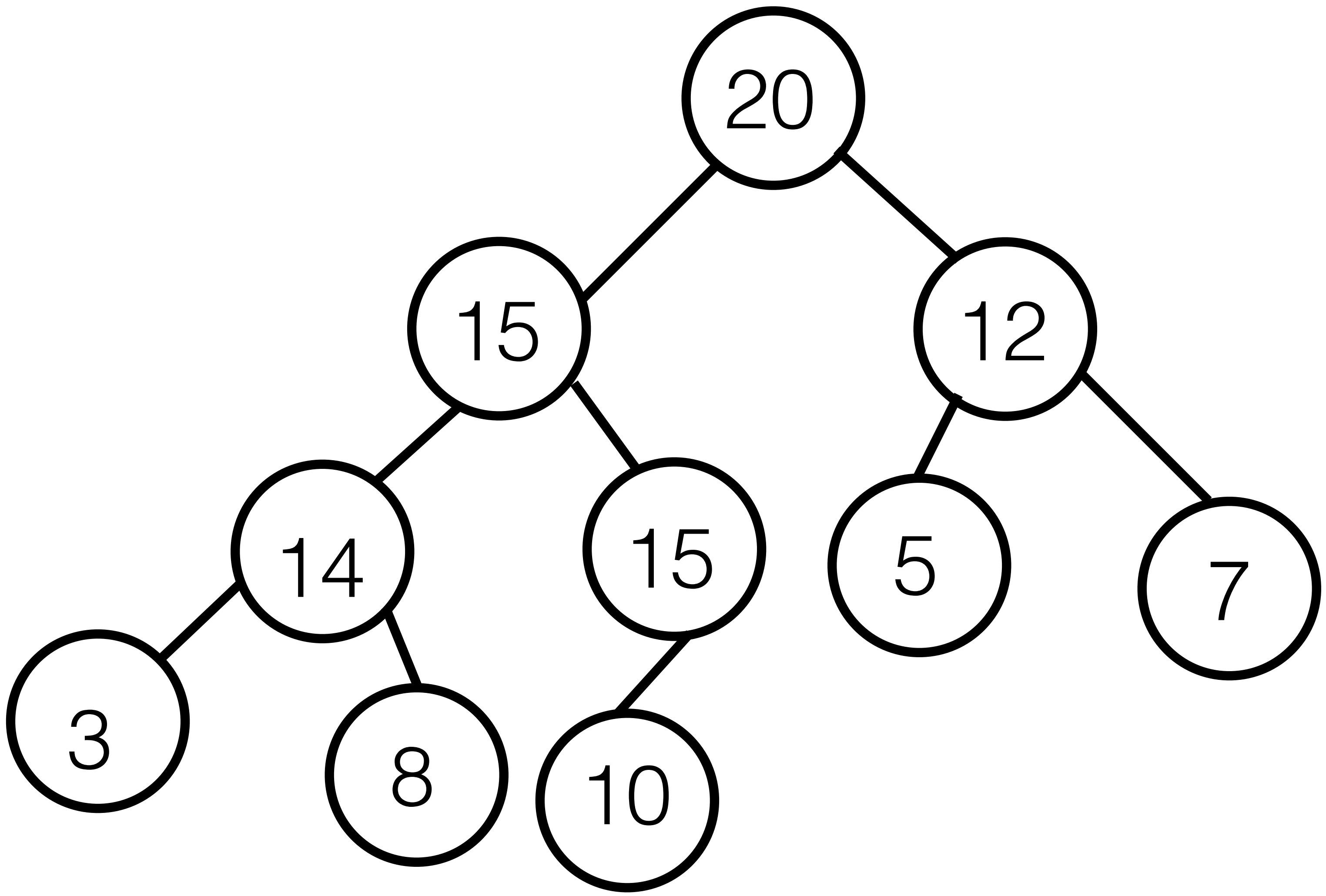
Force the tree to be balanced…

Example

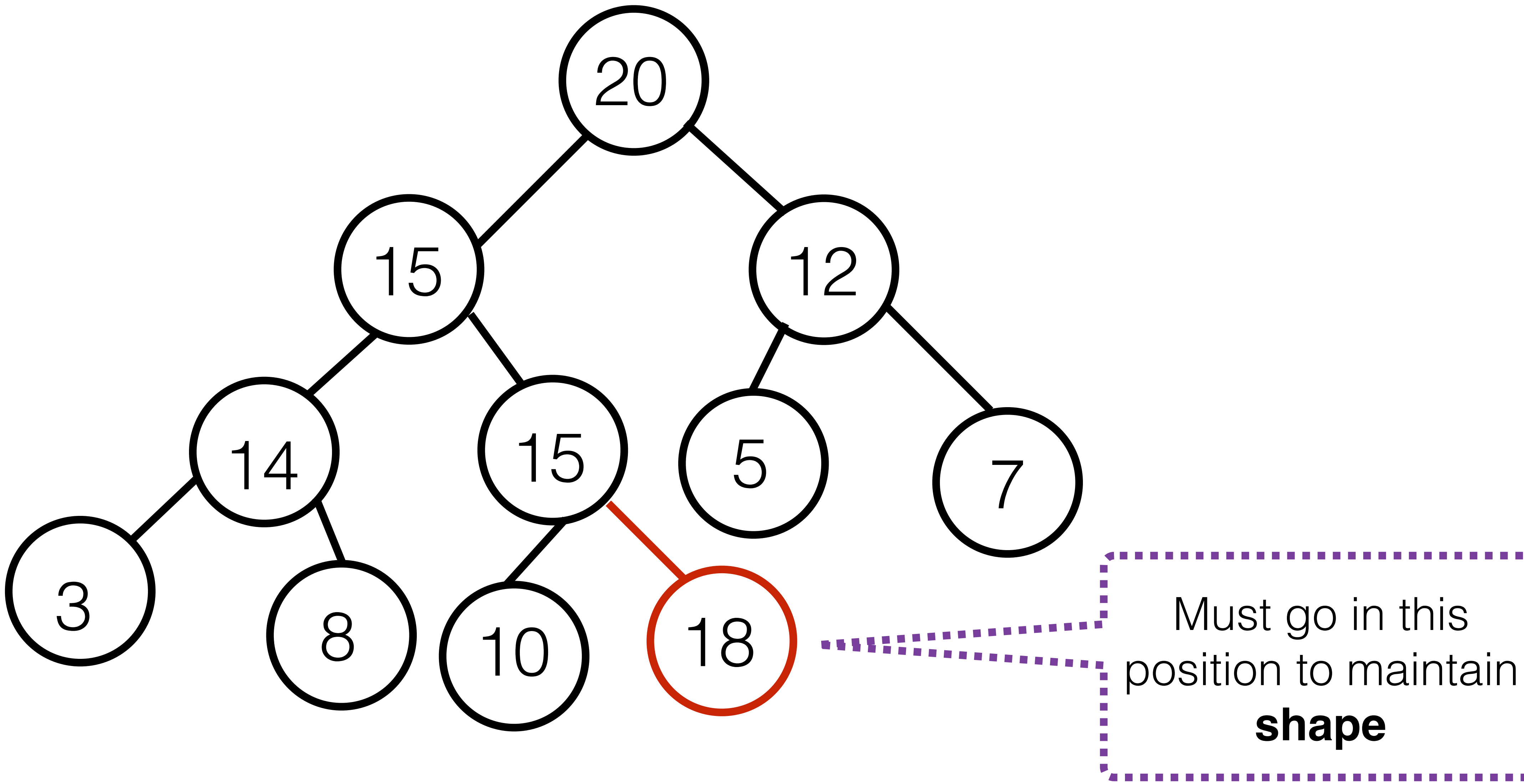


"Not a binary search tree"

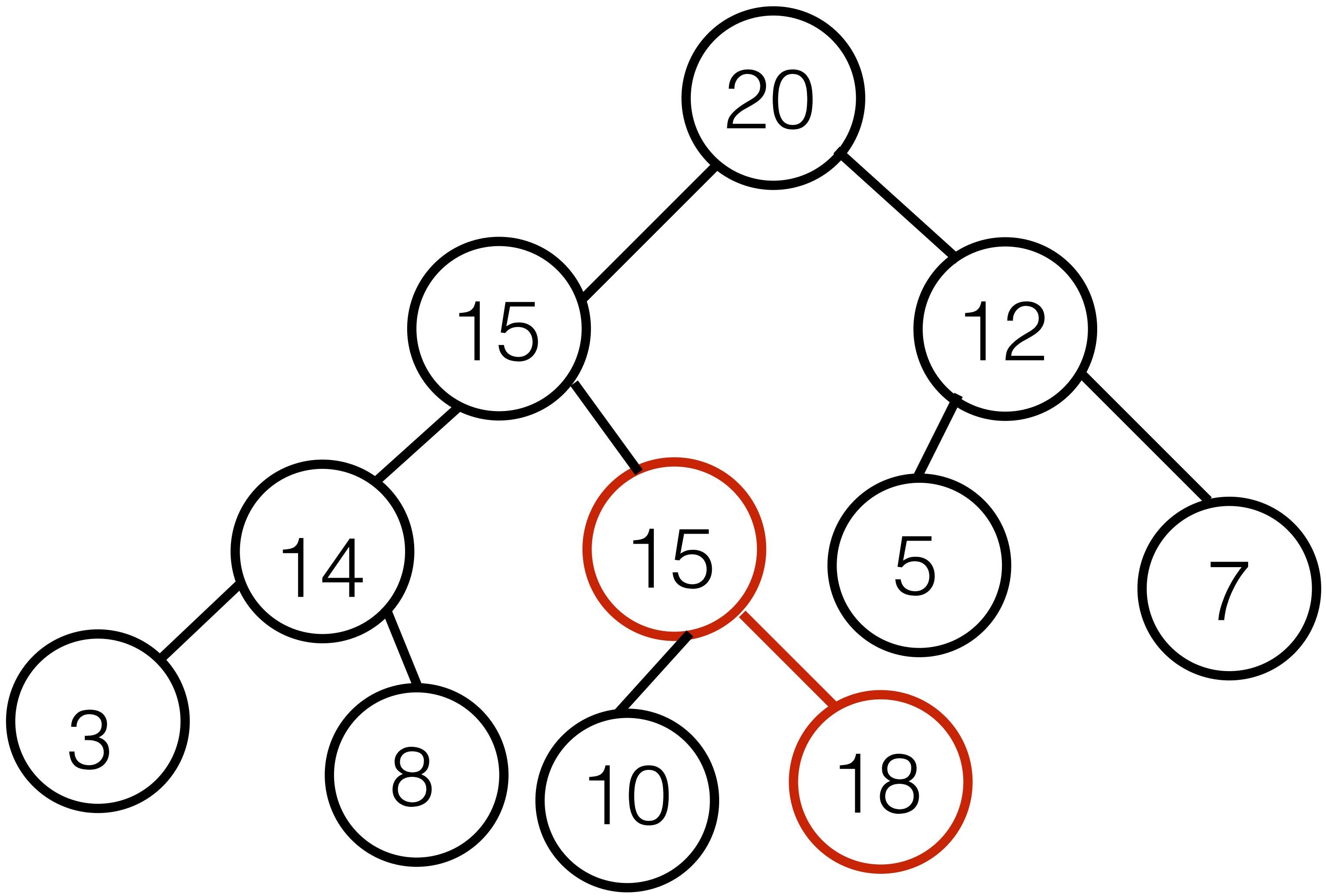
Add 18



Add 18

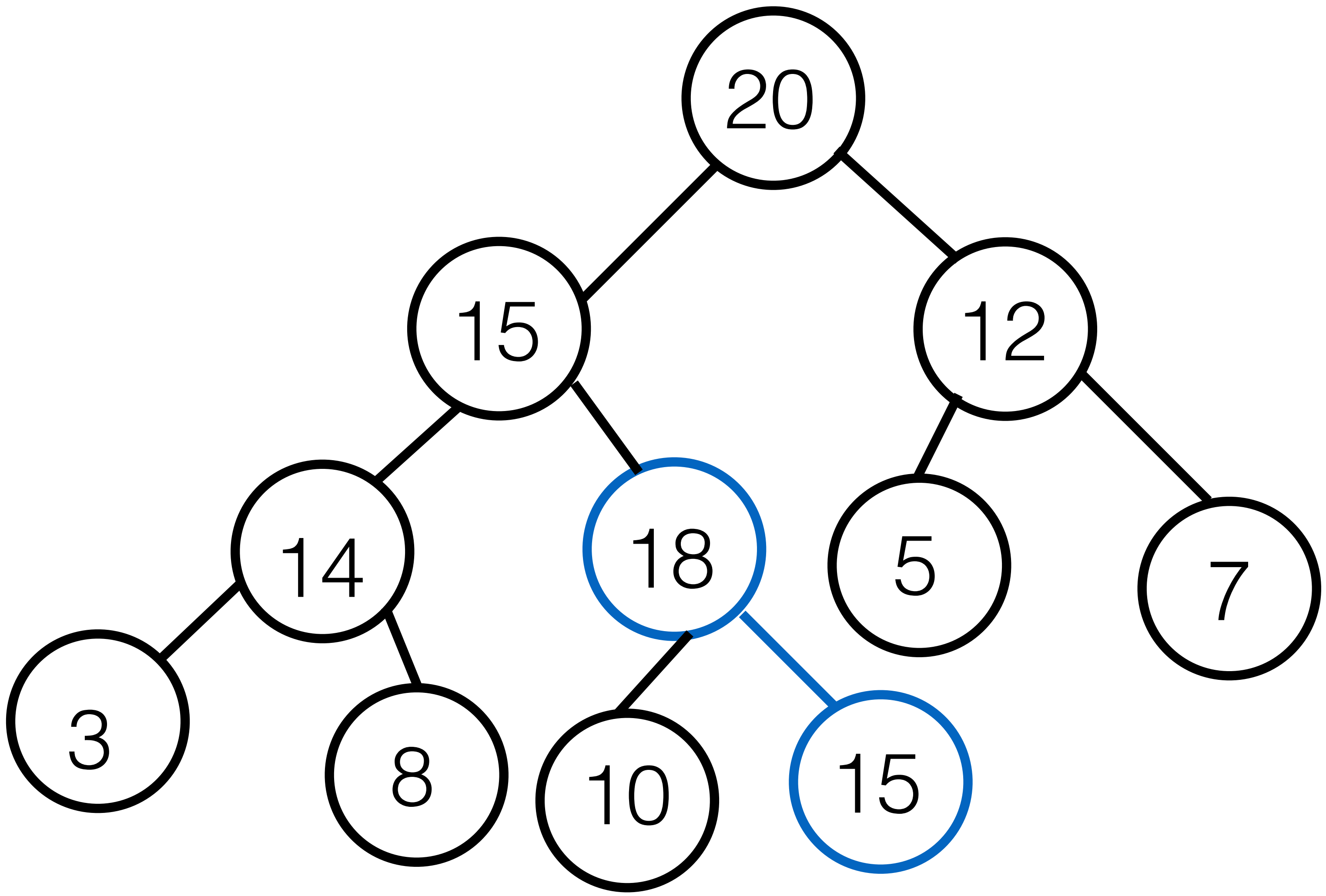

20
15
12
14
15
5
7
3
8
10
18
Must go in this position to maintain **shape**

Add 18



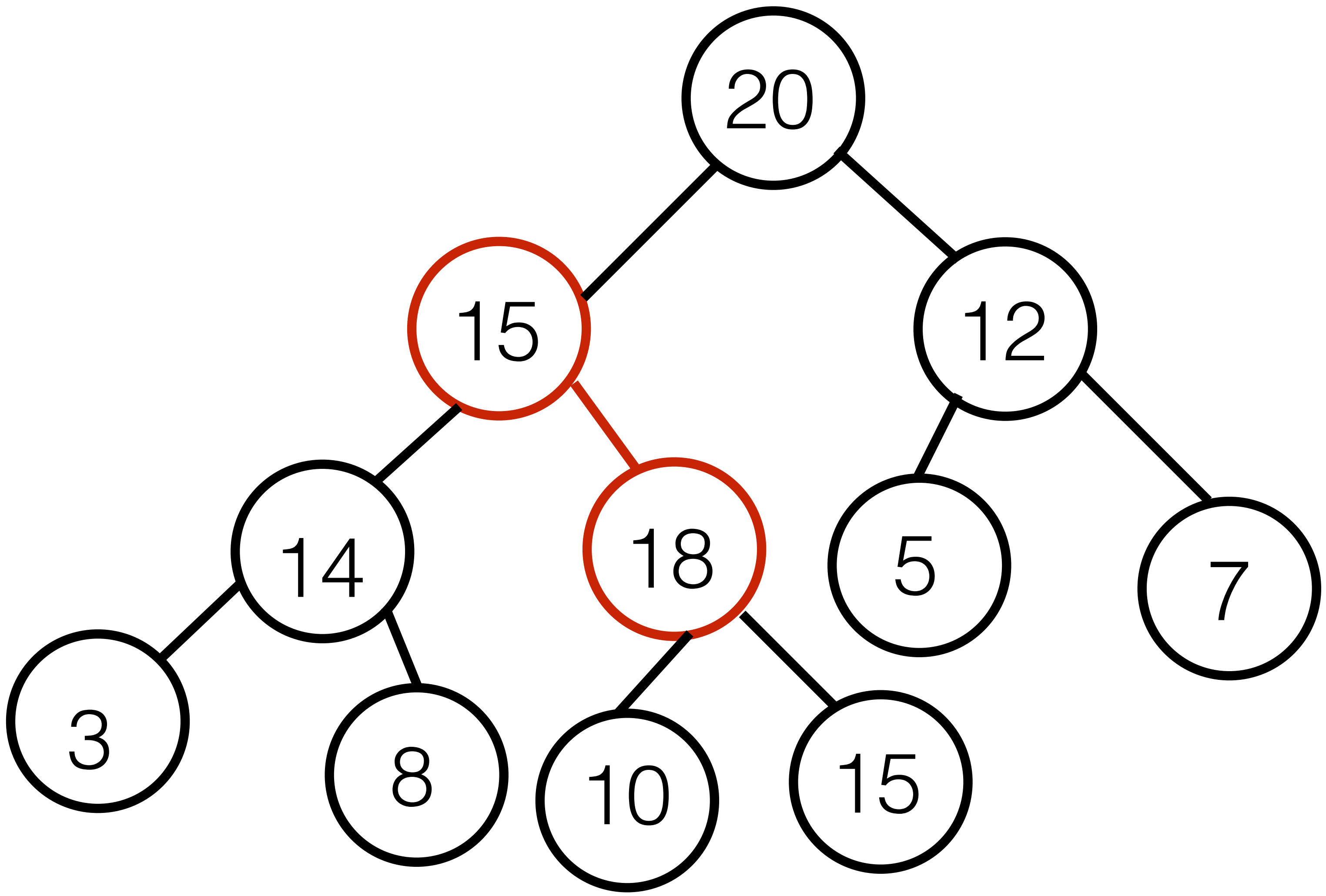
order is broken.

Add 18



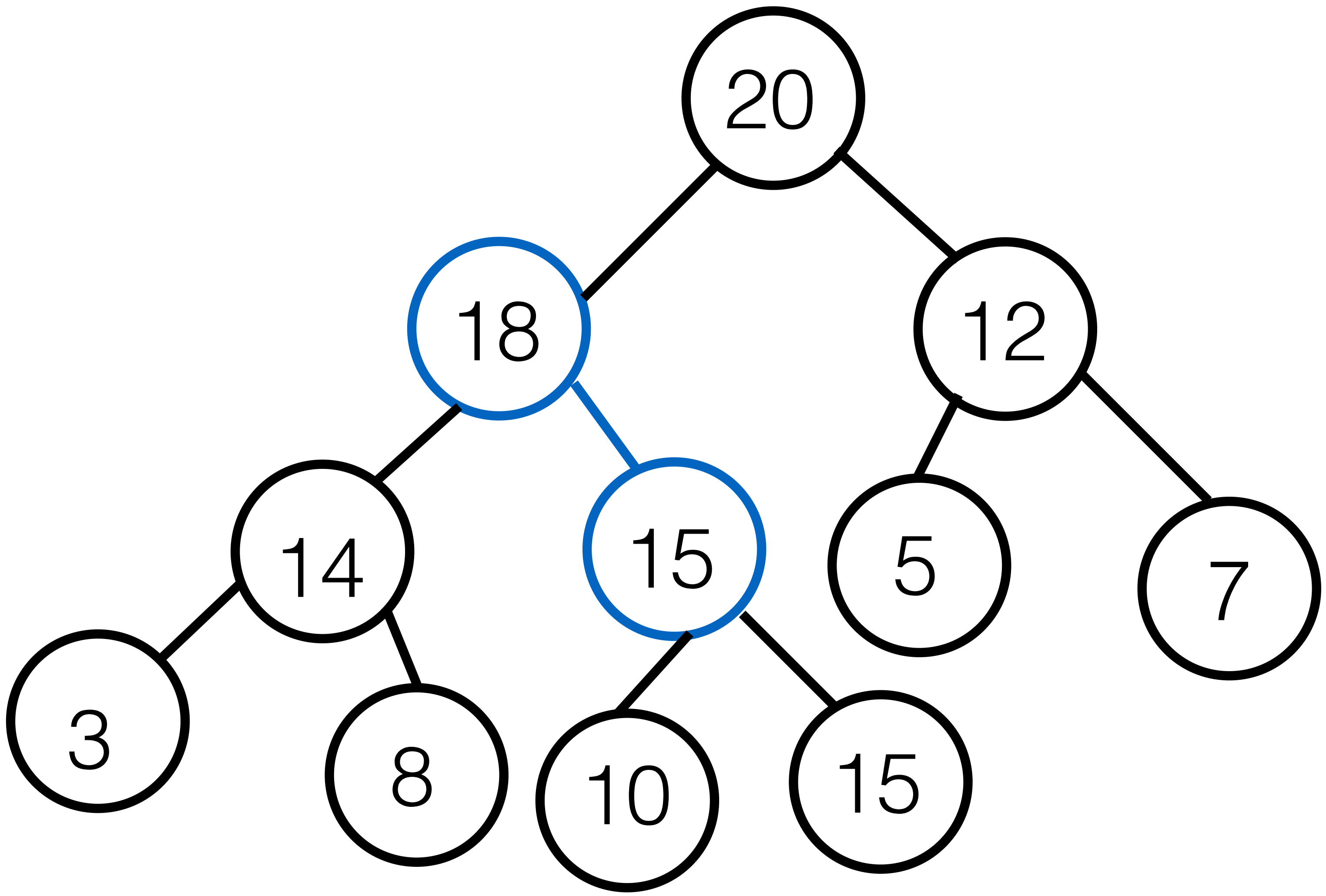
swap

Add 18



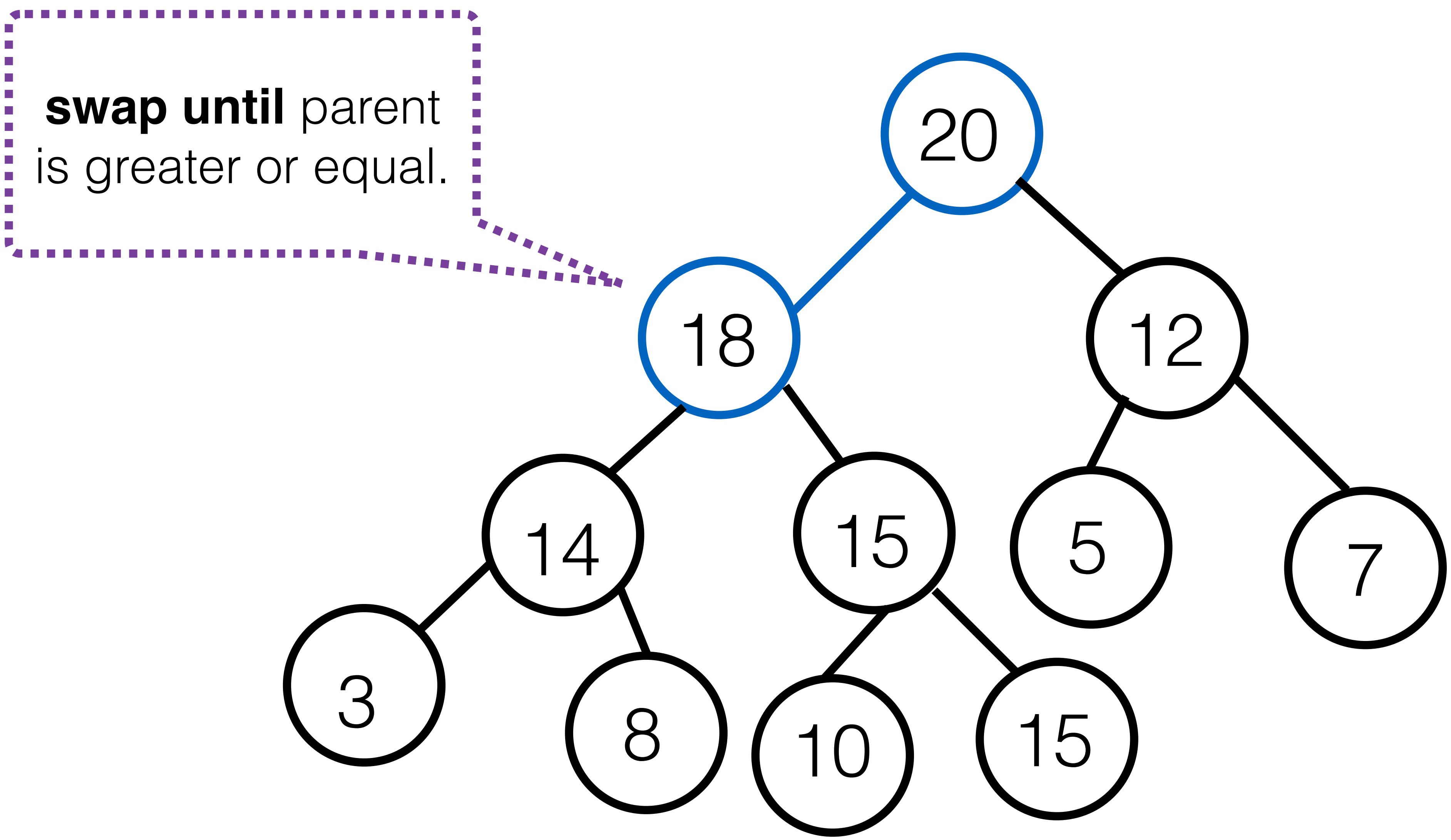
order is broken.

Add 18

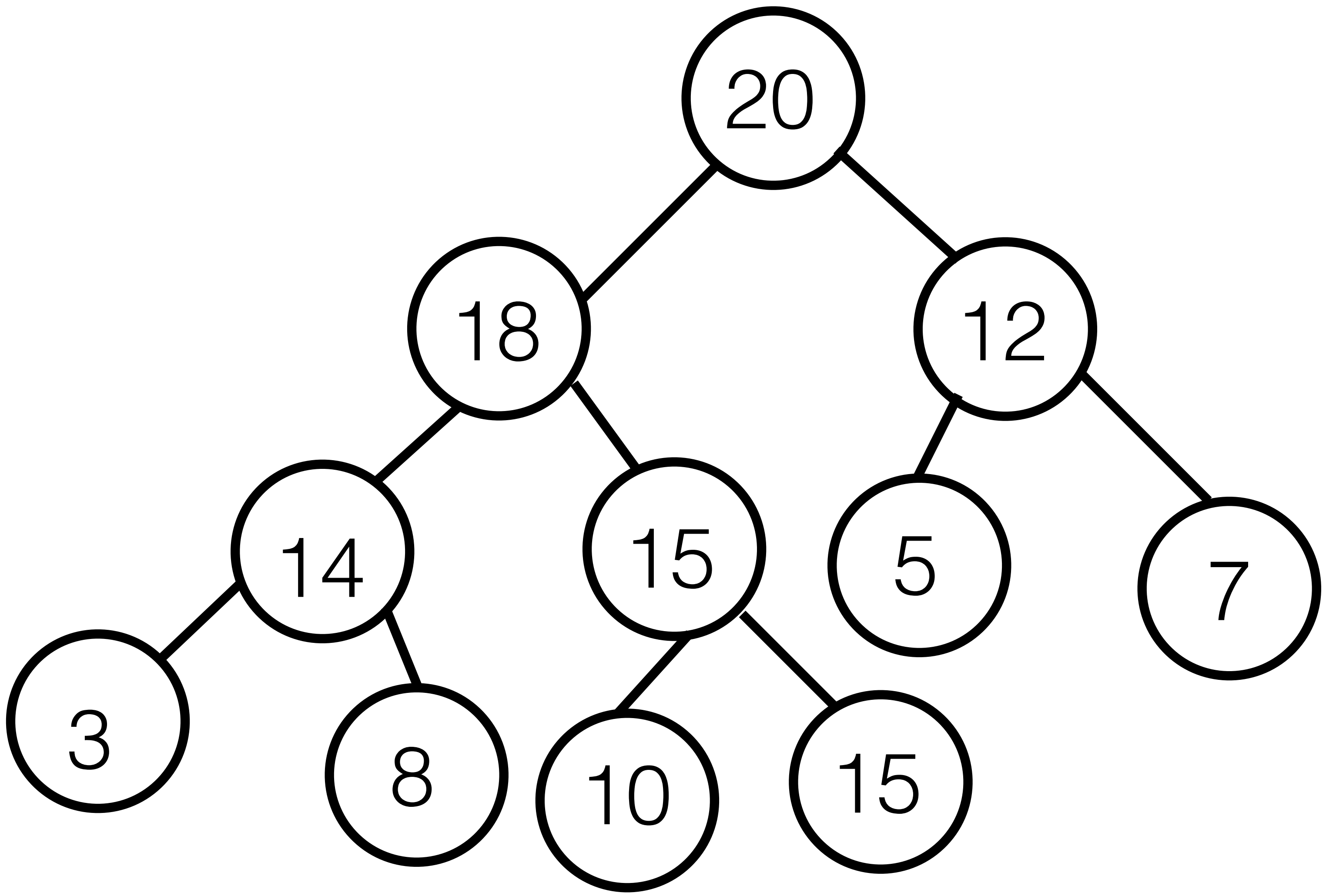


swap

Add 18



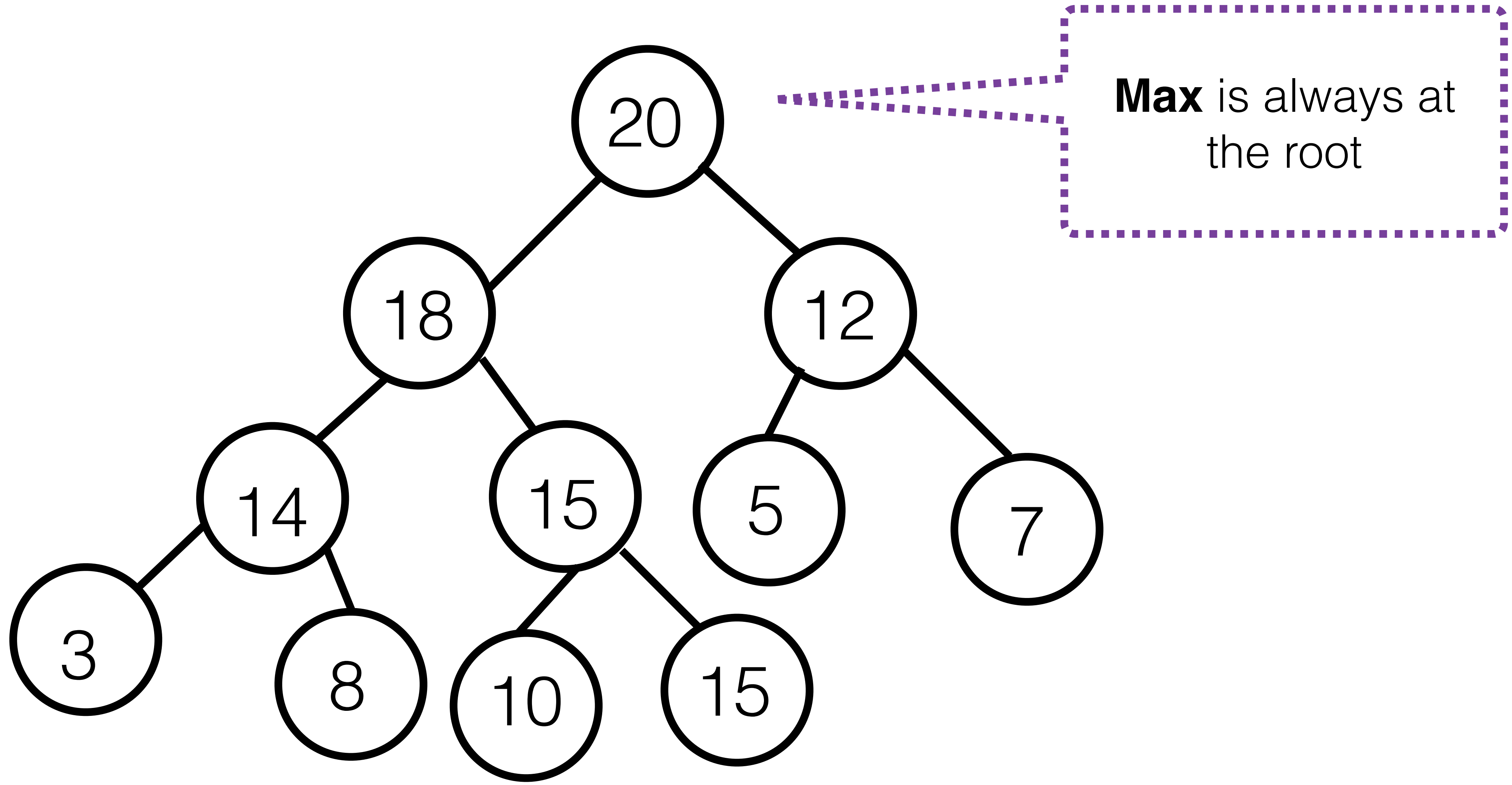
Add 18

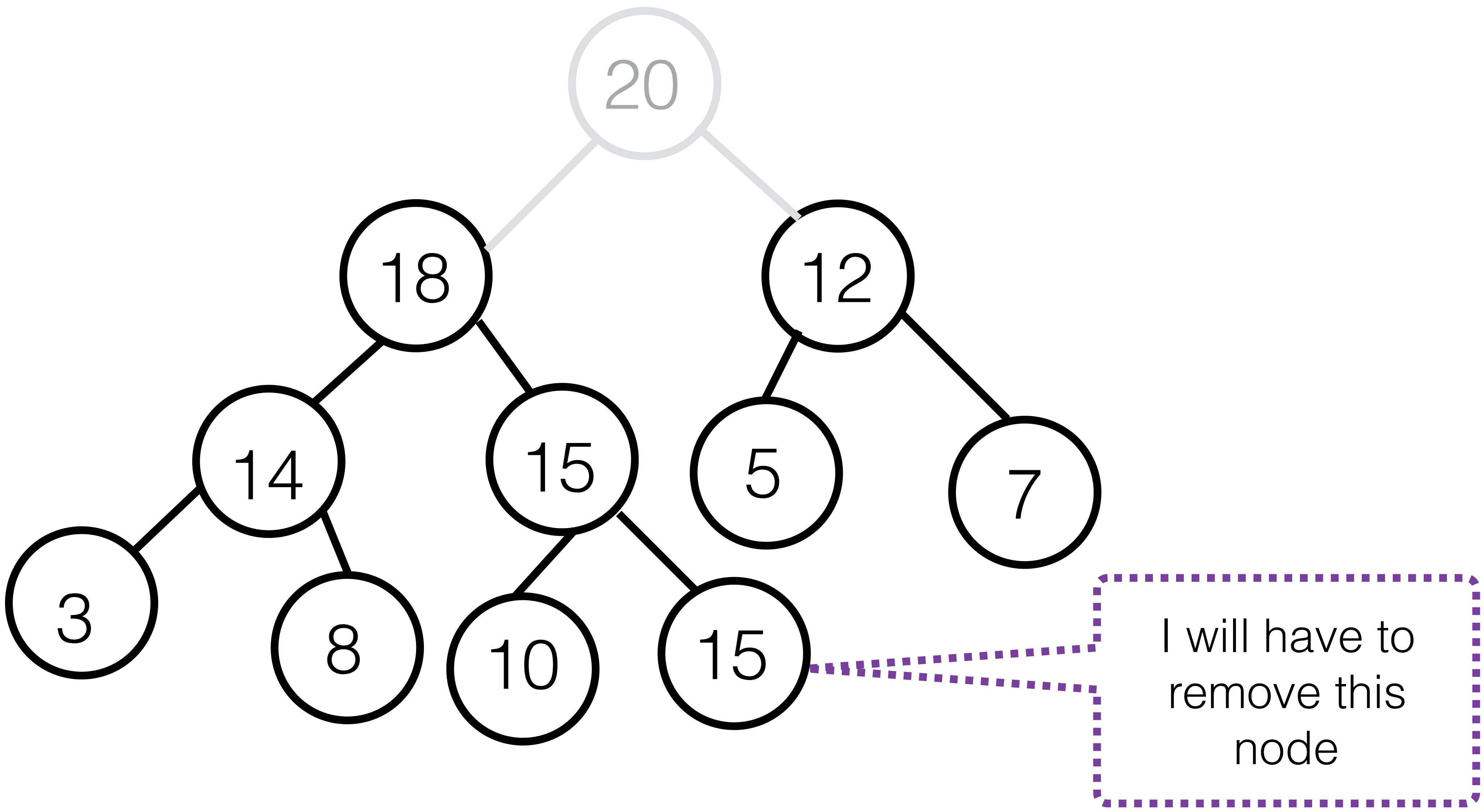


all good now!

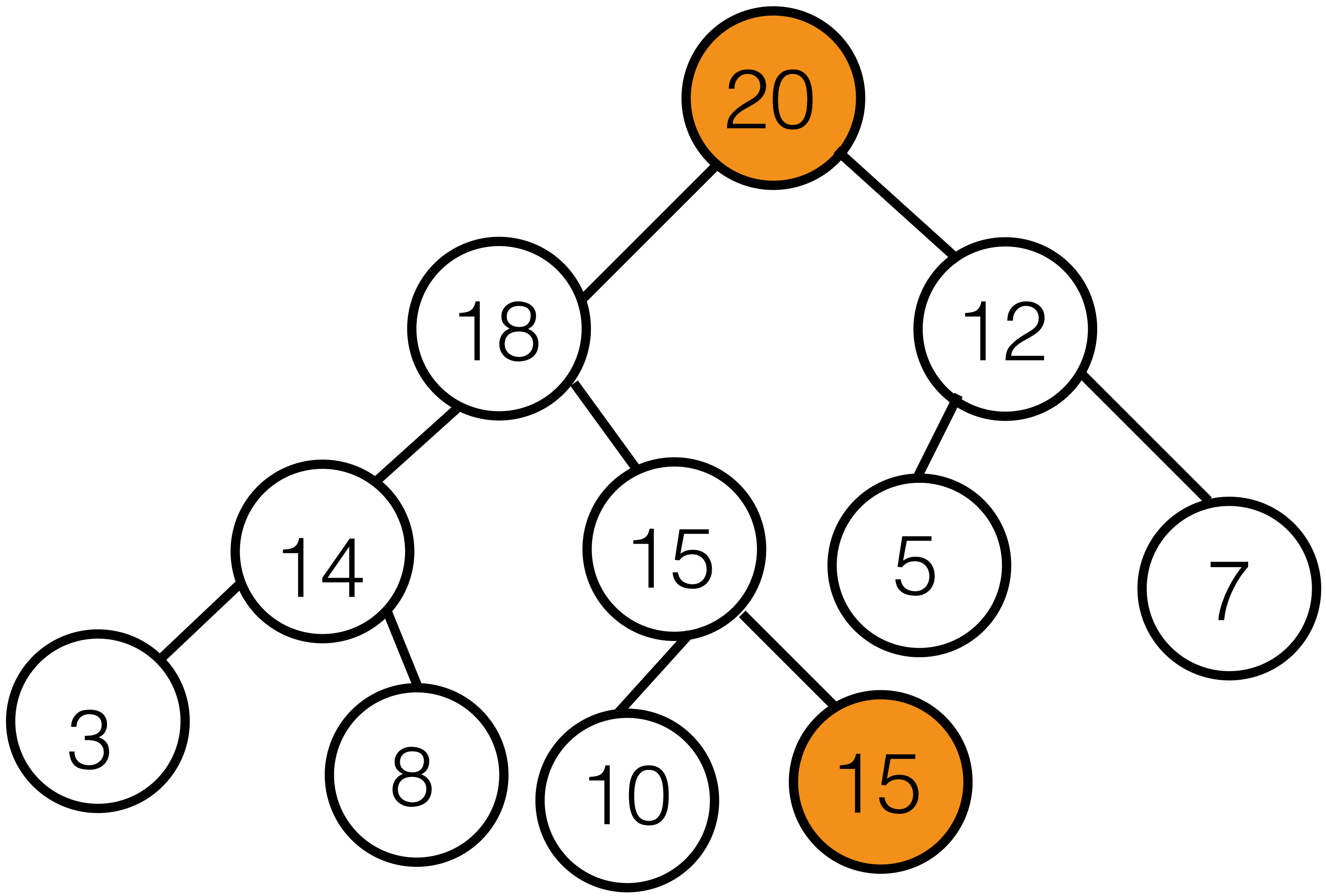
issue: swaps need a reference to parent

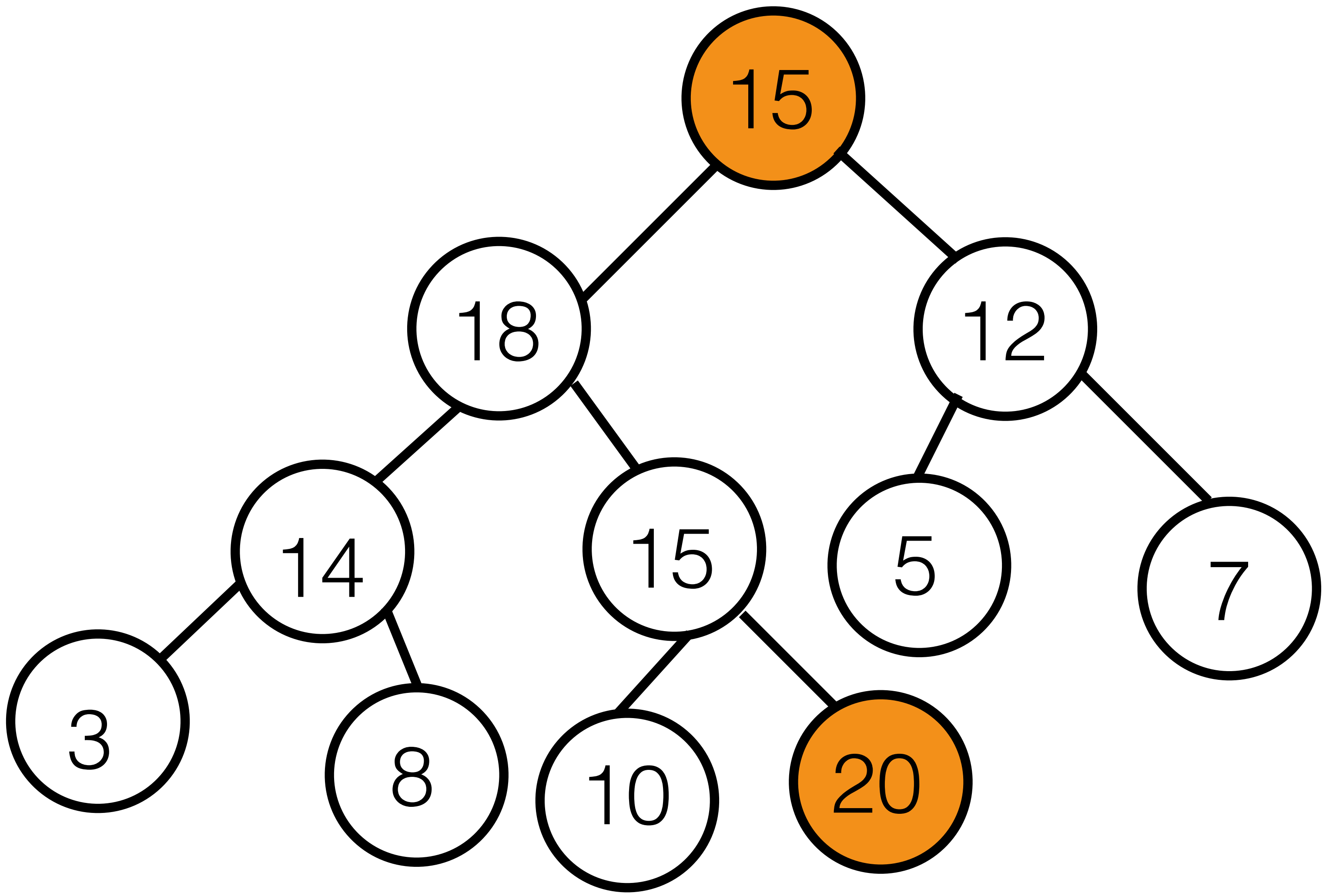
```
get_max()
```

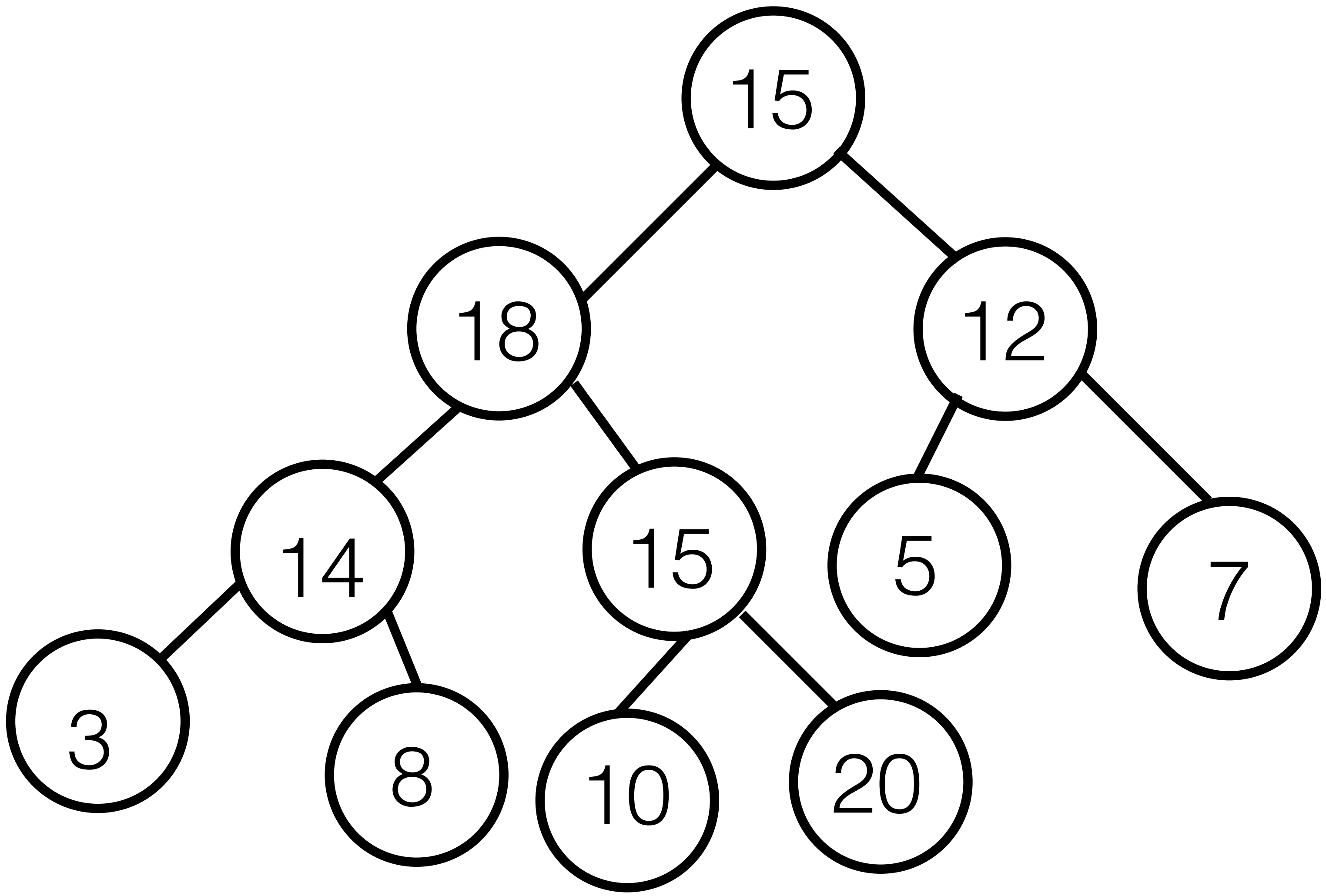

20
18
12
14
15
5
7
3
8
10
15
Max is always at the root

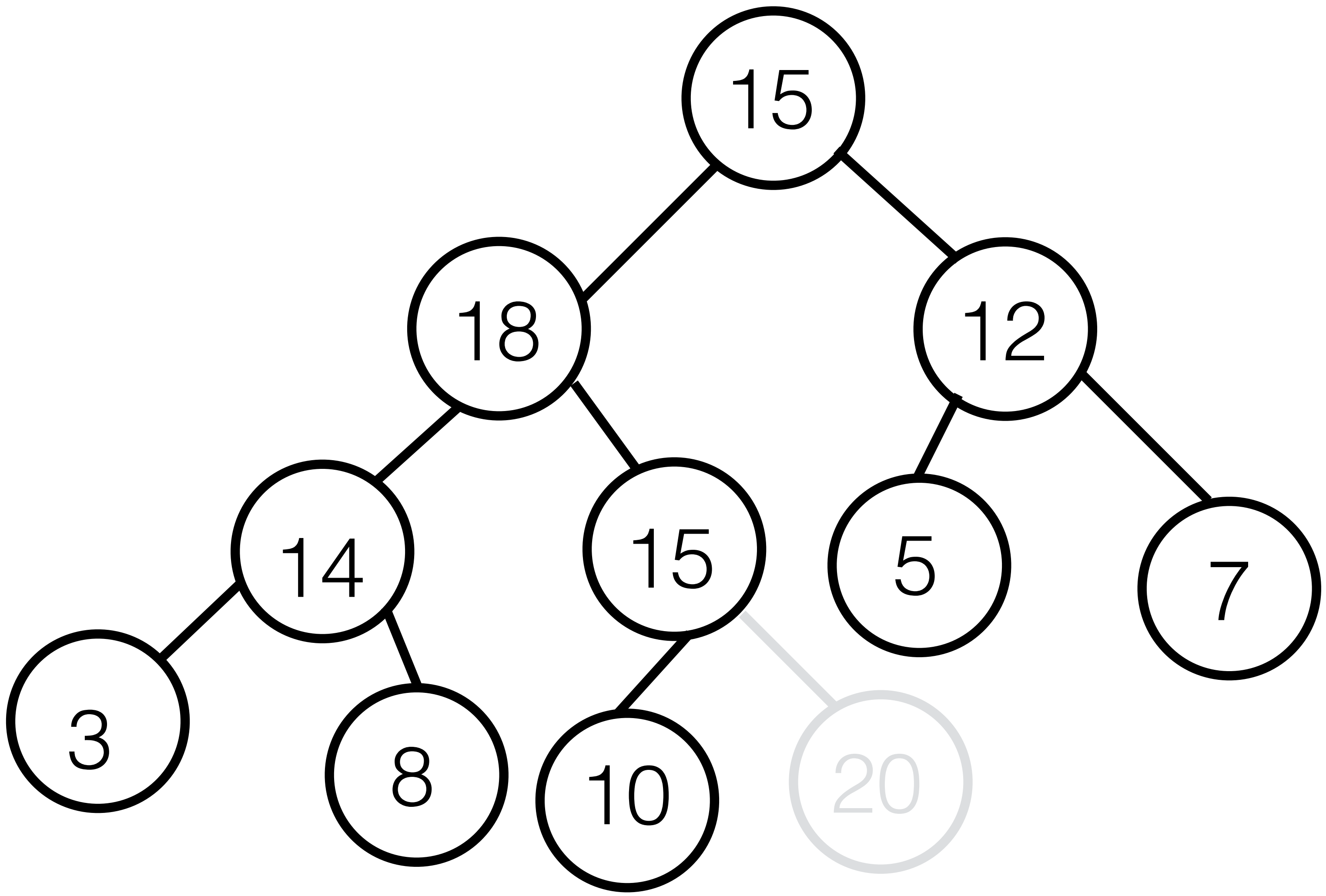


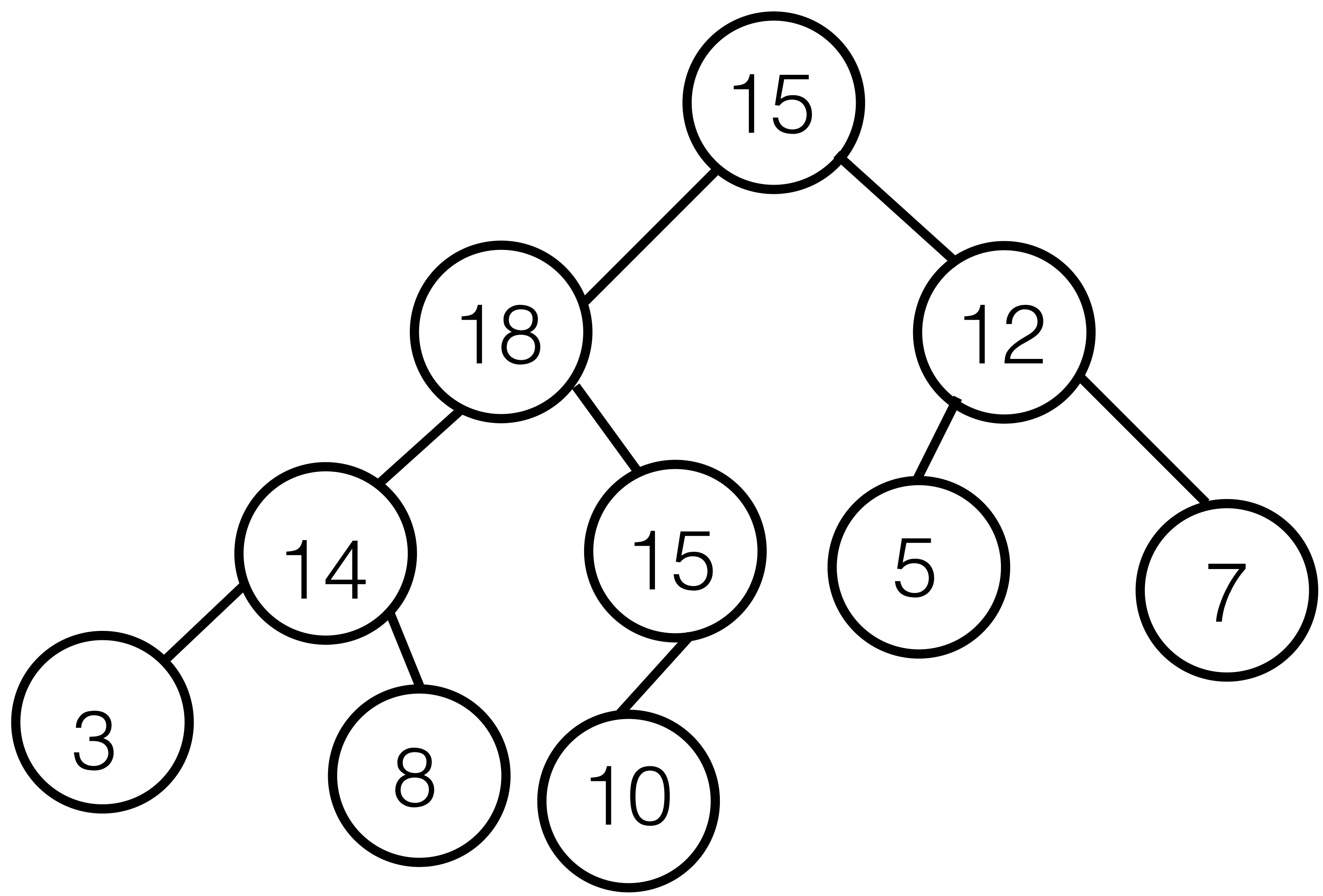
20
18
12
14
15
5
7
3
8
10
15
I will have to remove this node


20
18
12
14
15
5
7
3
8
10
15

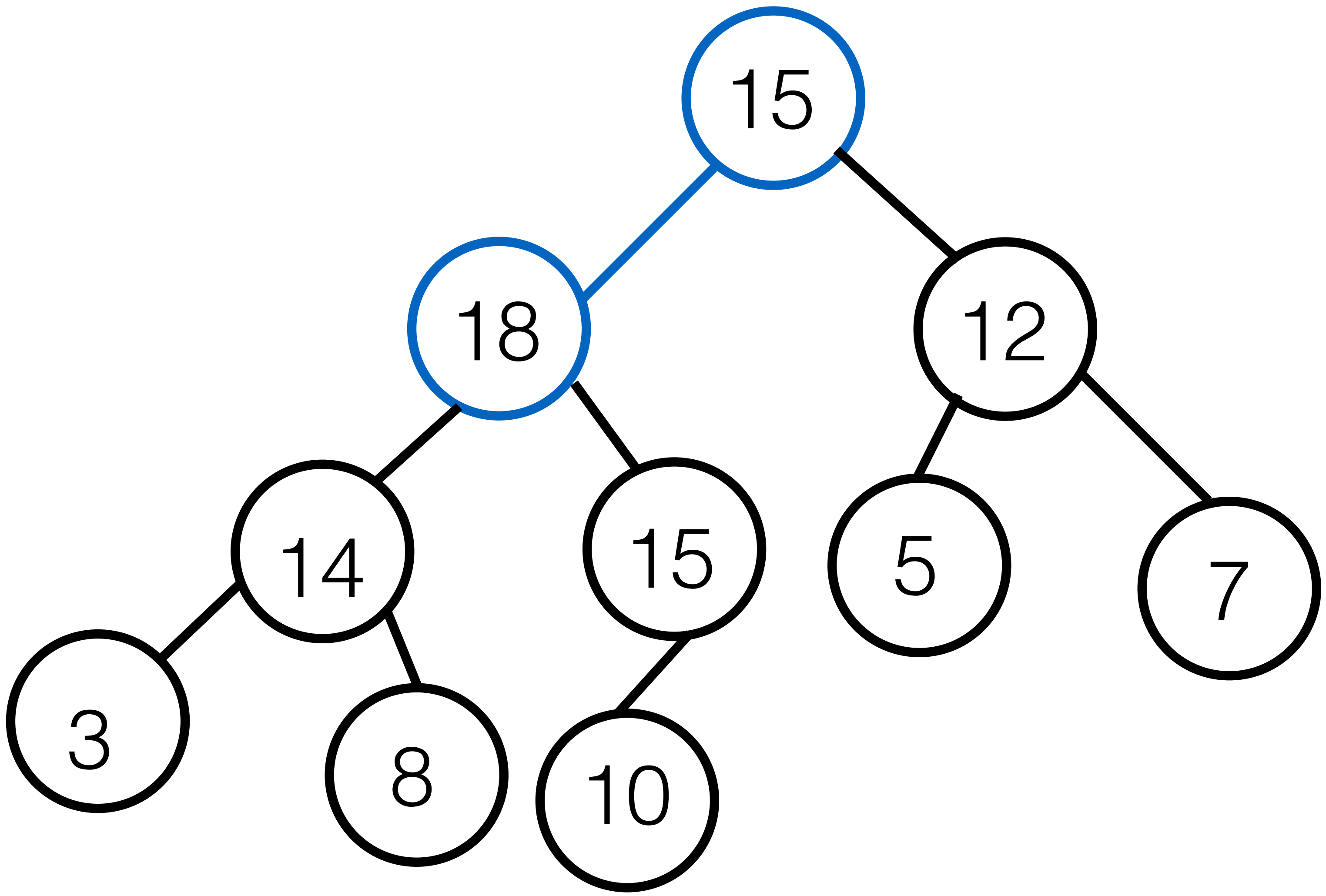

15
18
12
14
15
5
7
3
8
10
20


15
18
12
14
15
5
7
3
8
10
20

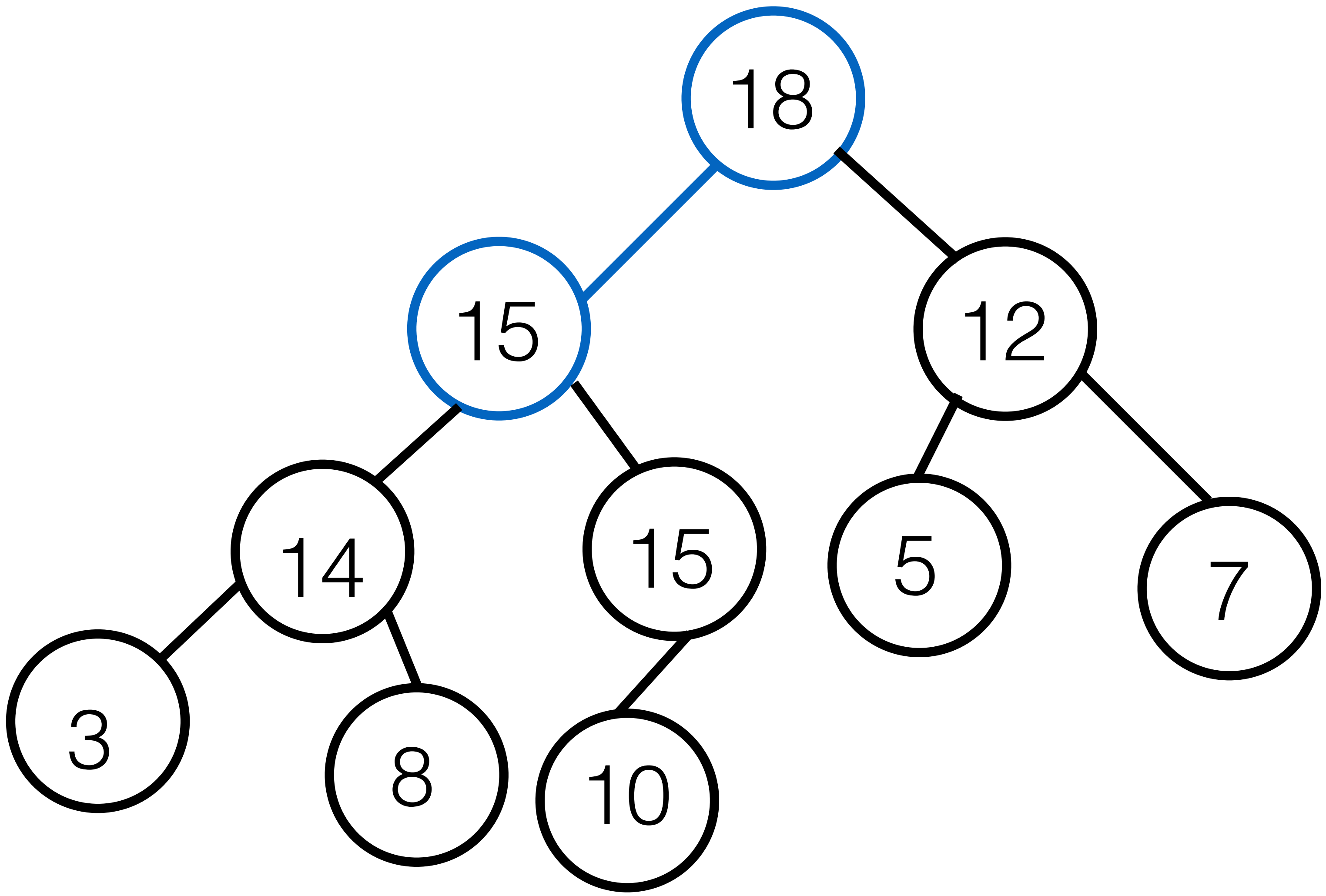

15
18
12
14
15
5
7
3
8
10
20



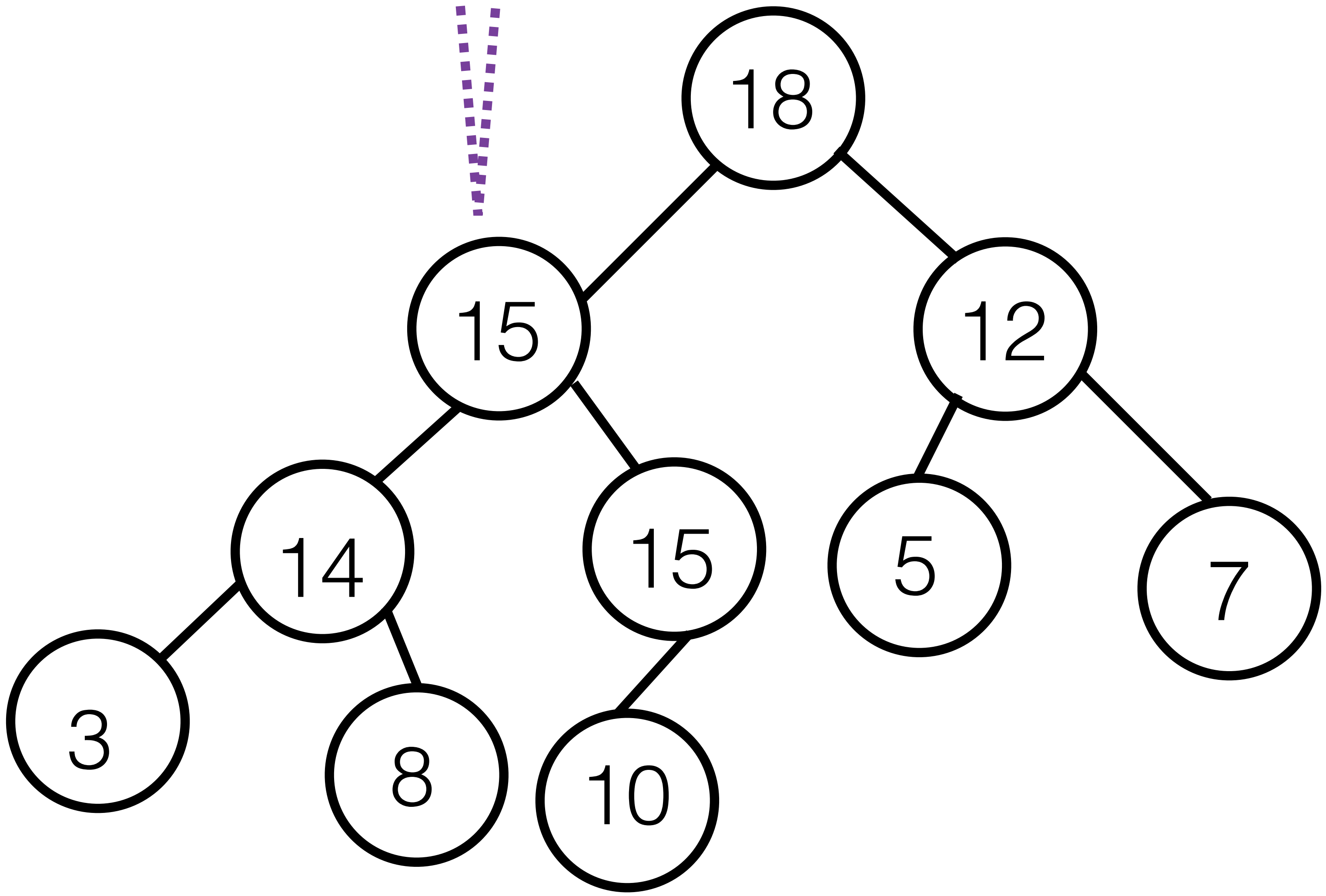
order is broken, we need max at the root



swap


18
15
12
14
15
5
7
3
8
10

“Sink” until the element is larger than or equal to children



Summary

- <u>Priority Queues:</u>
  - **`add`**
  - **`get_max`**

- Possible implementations: Lists, BST.

- **Heaps**: binary tress that are
  - Complete
  - Heap ordered

- Heap **operations** for Priority Queues: Complexity and correctness