Prepared by Julian García

# Lecture 26
# More Dynamic Programming

## FIT 1008
## Introduction to Computer Science

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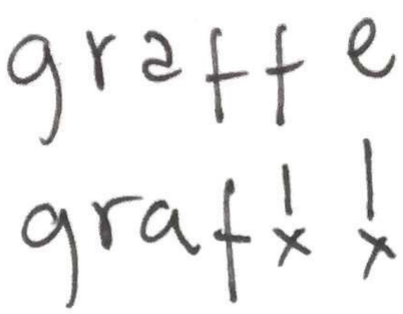
# Edit Distance Problem

Find the smallest number of **operations** to convert the following first string into the second string

atggctaagtctatgctttctggaattgttttgctggtctt
gttgctgctgcagcggccagttcggccaacaacag
cgccgccaacgtctccgttttggagagtgggcccg
ctgtgcaggaagtgccagcga

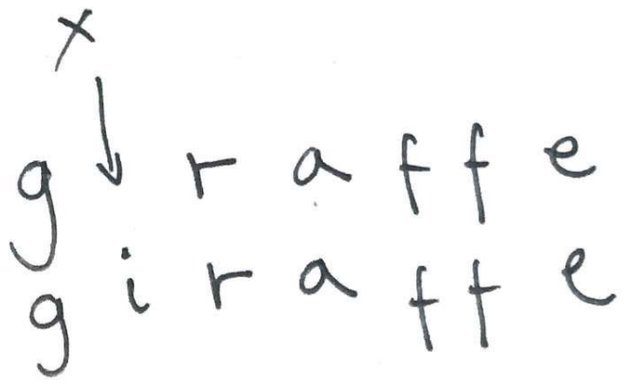
acggtcacagctcgcctggcgaagcctttgctgct
tcttttctgctcttgctgcgactttggcagcagctttcc
tcgttttgcaatgcttcaacatcatctccagcaacaa
ccagcaaaccagcgtcagg




**2 deletions**

**1 deletion + 1 substitution**

**1 deletion + 2 substitutions**



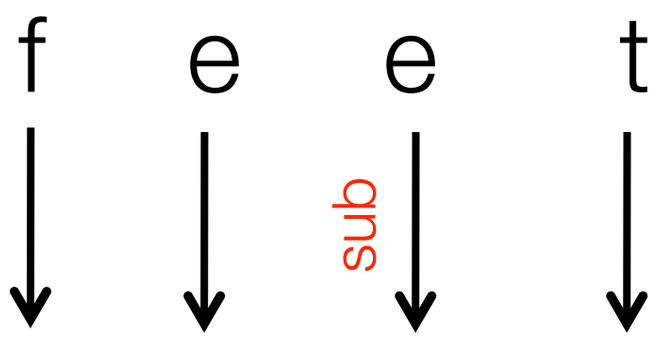

**1 insertion**

# Edit Distance Problem

**insertions**, **deletions** and **substitutions**

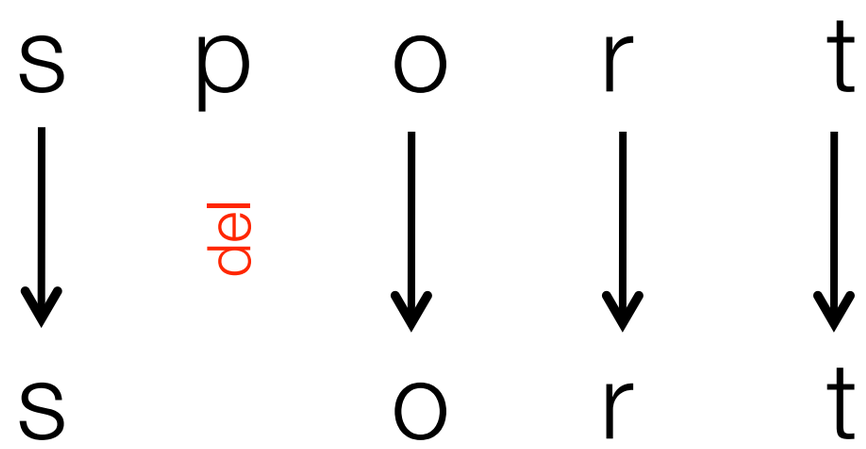
- We can transform the string “feet” into “felt” by **substituting** one letter for another letter.
- We can transform “sport” into “sort” by the **deletion** of one letter.
- We can also transform “mere” into “merge” by the **insertion** of one letter.
- The **smallest total number** of substitutions, deletions, and insertions to **transform one string**, s, into another string, t, is the ***edit distance* between s and t**.

# Transform feet into felt

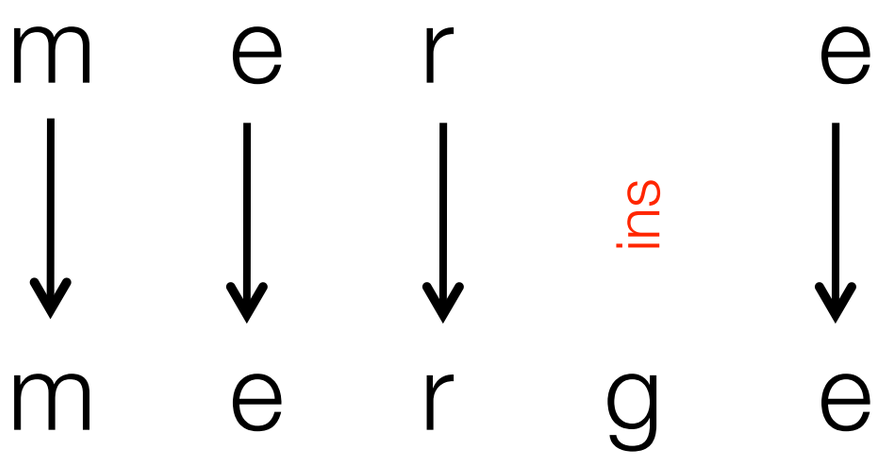



f e l t

# Transform sport into sort



# Transform mere to merge



# Subproblems

- Computing the edit distance between:

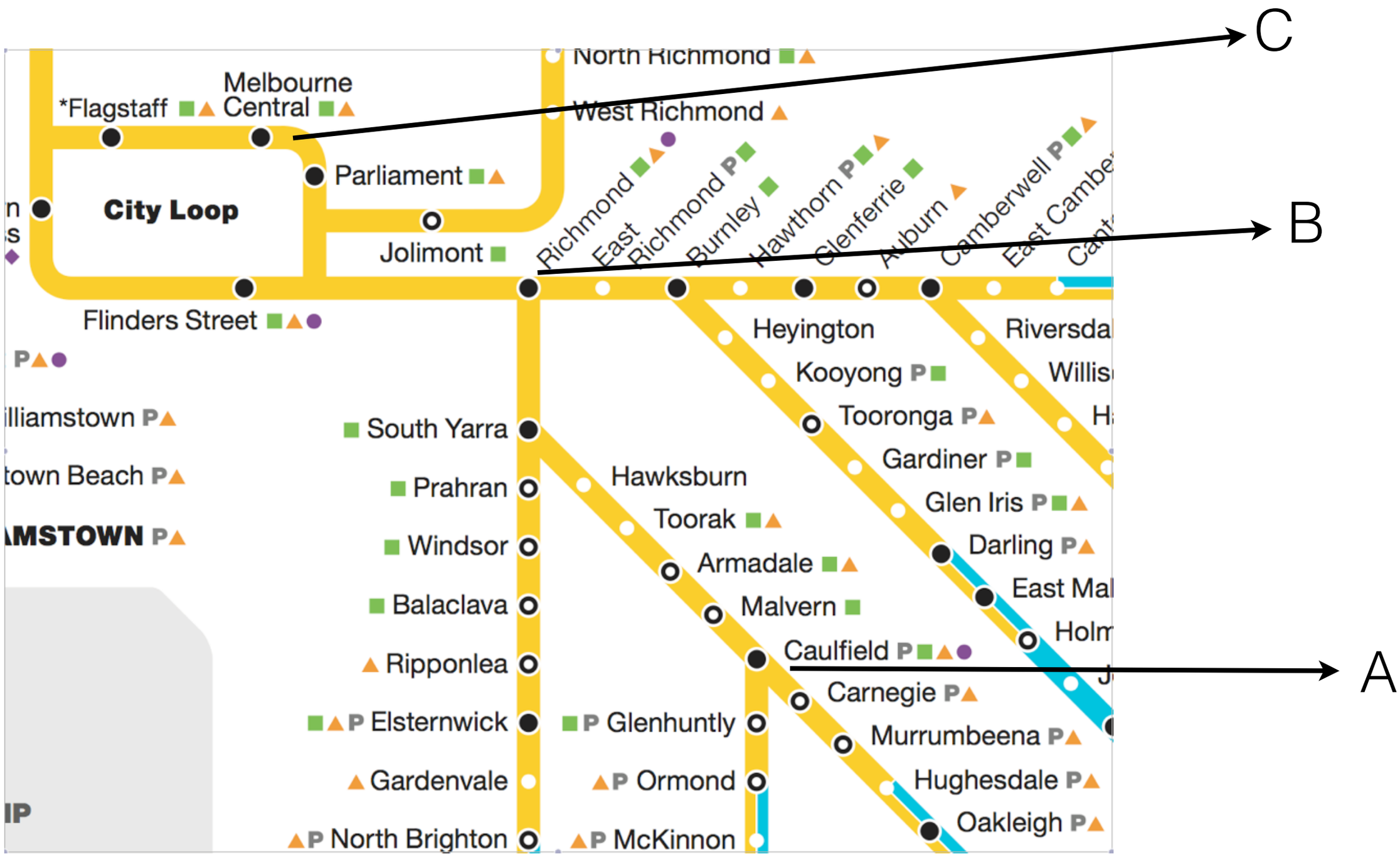
s[0..n]  t[0..k]

- Simpler problems: s and t have fewer characters

s[0..n-1]  t[0..k-1]

# Distance

Time needed to go from A to C




http://ptv.vic.gov.au

Problem I want to solve

d( feel ,feet )

Smaller subproblems

All possibilities with one element less

d( fee , feet )

d( feel , fee )

d( fee , fee )

| | fee | feet |
|---|---|---|
| fee | d( fee , fee ) | d( fee , feet ) |
| feel | d( feel, fee ) | d( feel , feet ) → ? |

# Edit distance

s[0..n] ⟶ t[0..k]

# Edit distance

d( s[0..n] ,“ “ ) = n + 1  deletions

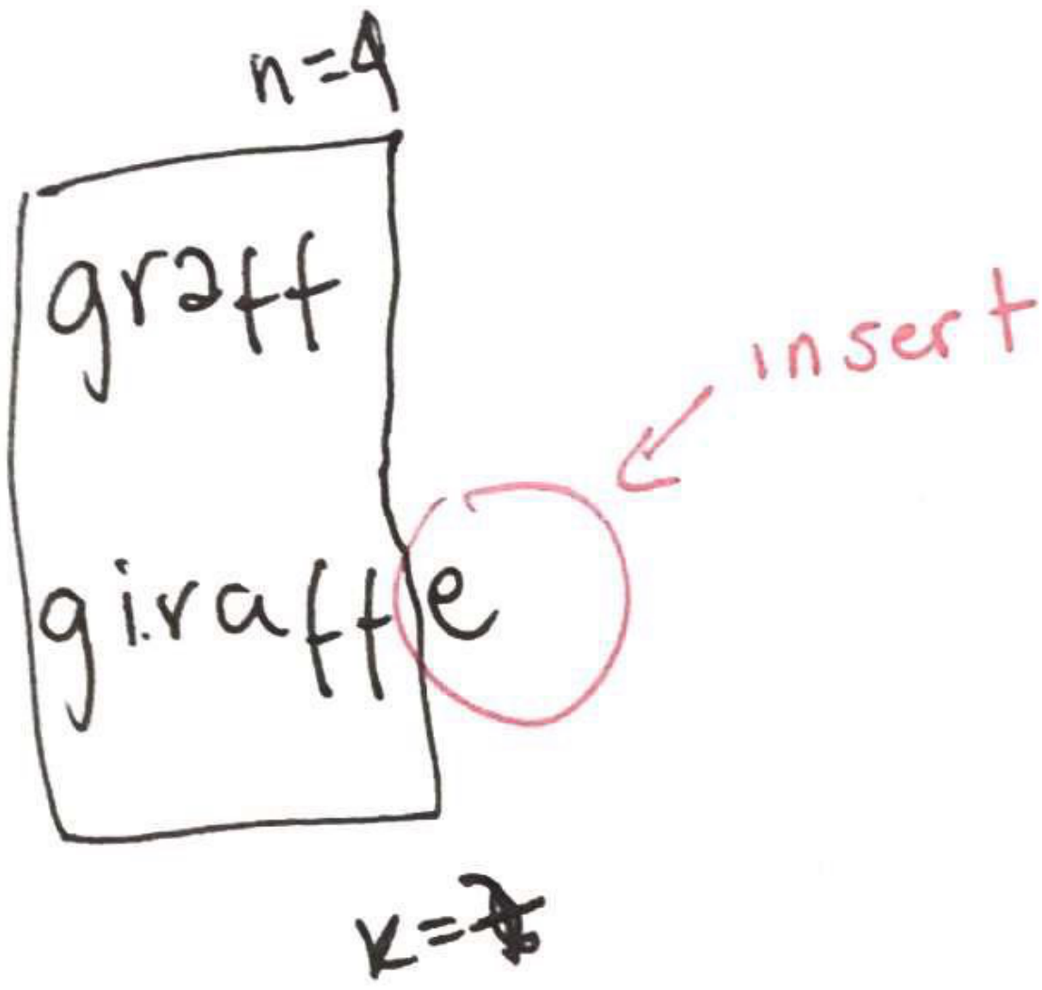
d( “ ” , t[0..k] ) = k + 1  insertions

# Edit distance

d( s[0..n] ,“ “ ) = n + 1  deletions

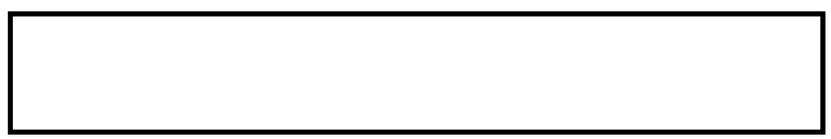
d( “ ” , t[0..k] ) = k + 1  insertions

d(s[0..n], t[0..k] )


n=9
graff
giraffe
insert
K=7

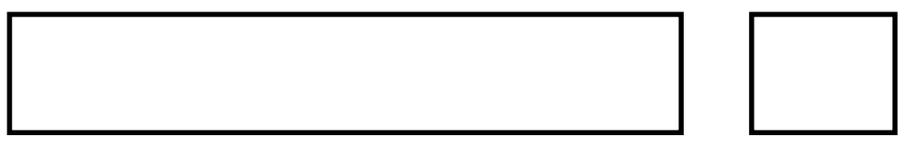
s[0..n] → t[0..k]

s[0..n]



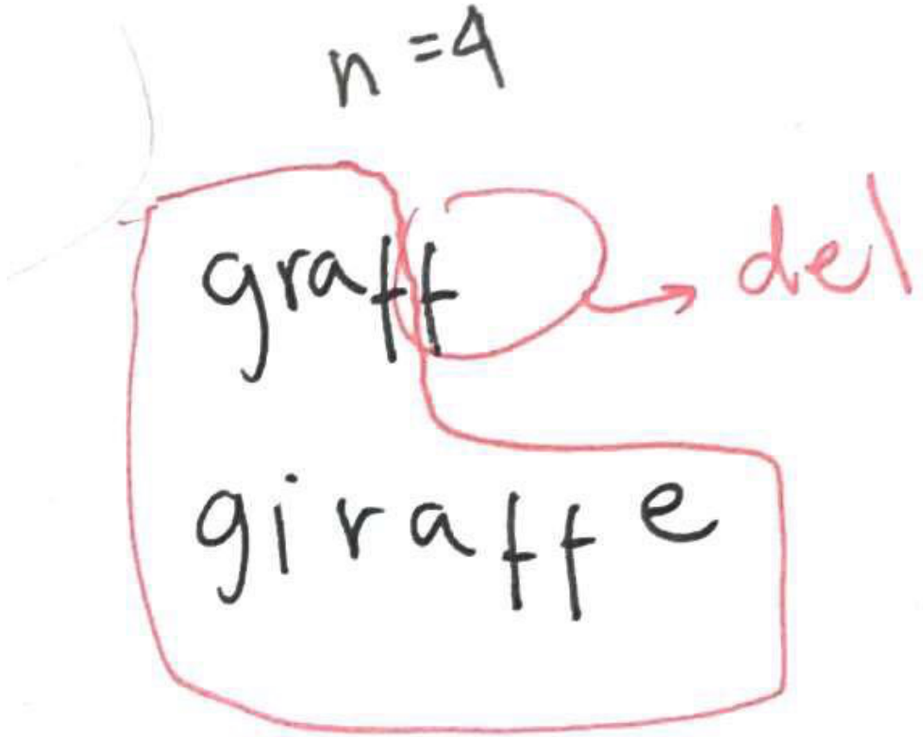
t[0..k-1]

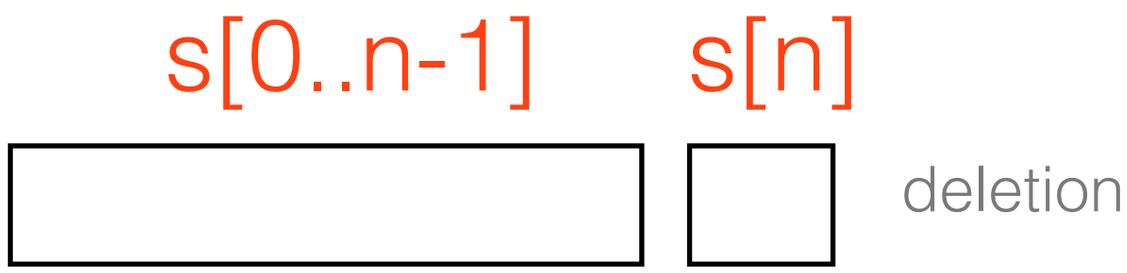
t[k]

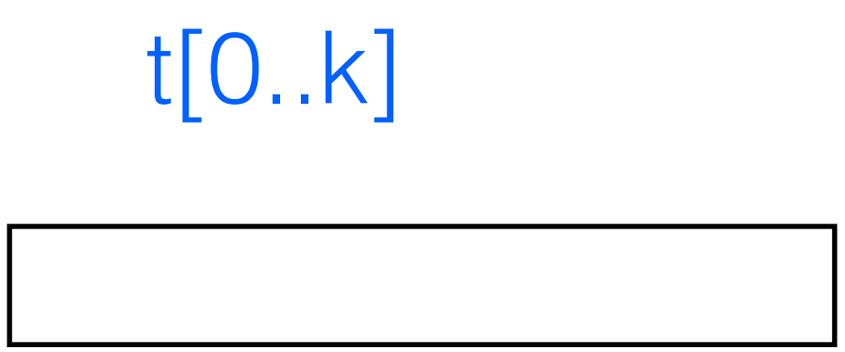


insertion

d(s[0..n], t[0..k-1]) + 1


n=4
graff
del
giraffe
k=7




d(s[0..n-1], t[0..k]) + 1

n=4

graft

giraffe

subs

k=7

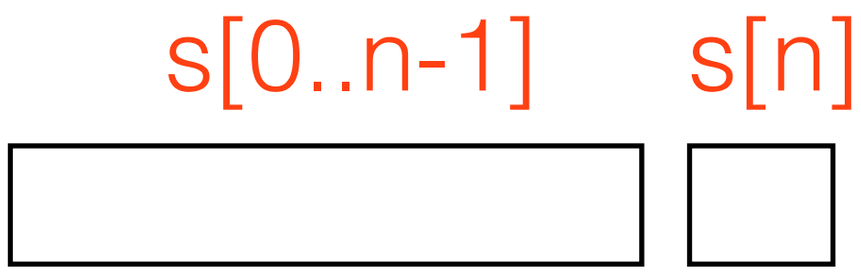
f ≠ e

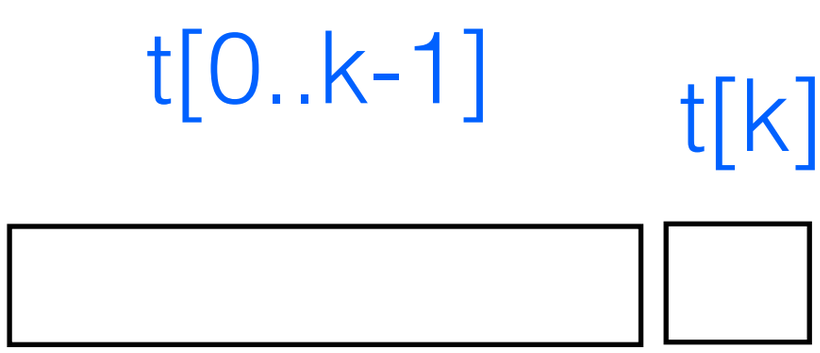
grafte

giraffe

nothing to do

e = e




$d(s[0..n-1], t[0..k-1]) + 0$ $s[n] = t[k]$ nothing

$d(s[0..n-1], t[0..k-1]) + 1$ $s[n] \neq t[k]$ substitution

# Edit distance

d( s[0..n] ," " ) = n + 1 deletions

d( " " , t[0..k] ) = k + 1 insertions

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

# Distance between feet and felt

| | “ ” | f | fe | fel | felt |
|---|---|---|---|---|---|
| “ ” | 0 | | | | |
| f | | | | | |
| fe | | | | | |
| fee | | | | | |
| feet | | | | | |

$$d(\, s[0..n] \,, \text{“ ”} \,) = n + 1$$

$$d(\, \text{“ ”} \,, t[0..k] \,) = k + 1$$

# Distance between feet and felt

| | “ ” | f | fe | fel | felt |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | | | | |
| fe | 2 | | | | |
| fee | 3 | | | | |
| feet | 4 | | | | |

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

# Distance between feet and f

$d(s[0..n], t[0..k])$

| | “ ” | f | fe | fel | felt |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | | | | |
| fe | 2 | | | | |
| fee | 3 | | | | |
| feet | 4 | | | | |

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

# Distance between feet and felt

| | “ ” | f | fe | fel | felt |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | 0 | | | |
| fe | 2 | | | | |
| fee | 3 | | | | |
| feet | 4 | | | | |

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

# Distance between feet and felt

| | “ ” | f | fe | fel | felt |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | 0 | | | |
| fe | 2 | | | | |
| fee | 3 | | | | |
| feet | 4 | | | | |

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

# Distance between feet and felt

| | “ ” | f | fe | fel | felt |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | 0 | 1 | | |
| fe | 2 | | | | |
| fee | 3 | | | | |
| feet | 4 | | | | |

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

# Distance between feet and felt

| | “ ” | f | fe | fel | felt |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | 0 | 1 | | |
| fe | 2 | | | | |
| fee | 3 | | | | |
| feet | 4 | | | | |

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

# Distance between feet and felt

| | “ ” | f | fe | fel | felt |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | 0 | 1 | 2 | |
| fe | 2 | | | | |
| fee | 3 | | | | |
| feet | 4 | | | | |

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

# Distance between feet and felt

| | “ ” | f | fe | fel | felt |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | 0 | 1 | 2 | |
| fe | 2 | | | | |
| fee | 3 | | | | |
| feet | 4 | | | | |

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

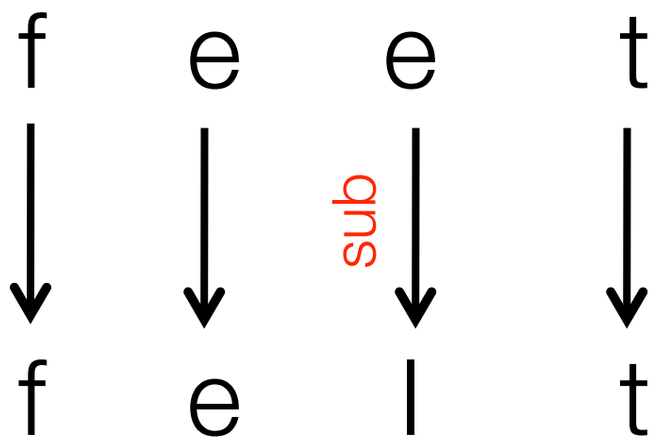
# Distance between feet and felt

| | “ ” | f | fe | fel | felt |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | 0 | 1 | 2 | 3 |
| fe | 2 | | | | |
| fee | 3 | | | | |
| feet | 4 | | | | |

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

# Distance between feet and felt

| | “ ” | f | fe | fel | felt |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | 0 | 1 | 2 | 3 |
| fe | 2 | 1 | 0 | 1 | 2 |
| fee | 3 | 2 | 1 | 1 | 2 |
| feet | 4 | 3 | 2 | 2 | 1 |



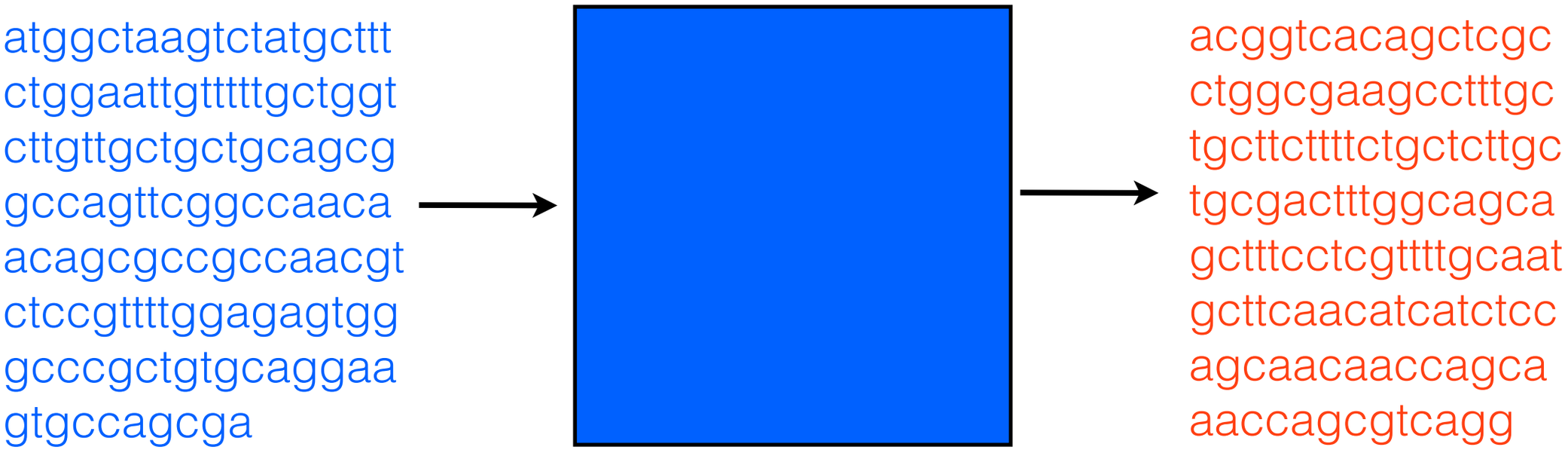

# Distance between feet and felt

| | “ ” | f | fe | fel | felt |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | 0 | 1 | 2 | 3 |
| fe | 2 | 1 | 0 | 1 | 2 |
| fee | 3 | 2 | 1 | 1 | 2 |
| feet | 4 | 3 | 2 | 2 | **1** |

# Distance between sport and sort

| | “ ” | s | so | sor | sort |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| s | 1 | 0 | 1 | 2 | 3 |
| sp | 2 | 1 | 1 | 2 | 3 |
| spo | 3 | 2 | 1 | 2 | 3 |
| spor | 4 | 3 | 2 | 1 | 2 |
| sport | 5 | 4 | 3 | 2 | **1** |

**insertions**, **deletions** and **substitutions**



# Edit Distance Problem

**insertions**, **deletions** and **substitutions**

- We can transform the string “feet” into “felt” by **substituting** one letter for another letter.
- We can transform “sport” into “sort” by the **deletion** of one letter.
- We can also transform “mere” into “merge” by the **insertion** of one letter.
- The **smallest total number** of substitutions, deletions, and insertions to **transform one string**, s, into another string, t, is the ***edit distance* between s and t**.

# Subproblems

- Computing the edit distance between:

s[0..n]  t[0..k]

- Simpler problems: s and t have fewer characters

s[0..n-1]  t[0..k-1]

Problem I want to solve

d( feel ,feet )

Smaller subproblems

All possibilities with one element less

d( fee , feet )

d( feel , fee )

d( fee , fee )

| | fee | feet |
|---|---|---|
| fee | d( fee , fee ) | d( fee , feet ) |
| feel | d( feel, fee ) | d( feel , feet ) → ? |

# Edit distance

s[0..n] ⟶ t[0..k]

# Edit distance

d( s[0..n] ,“ “ ) = n + 1  deletions

d( “ ” , t[0..k] ) = k + 1  insertions

# Edit distance

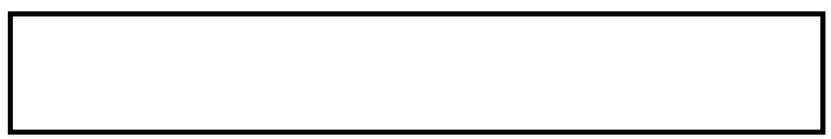
d( s[0..n] ,“ “ ) = n + 1  deletions

d( “ ” , t[0..k] ) = k + 1  insertions

d(s[0..n], t[0..k] )

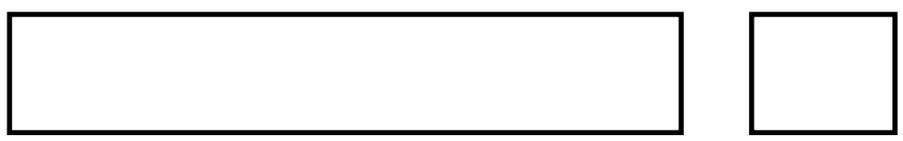
s[0..n] → t[0..k]

s[0..n]



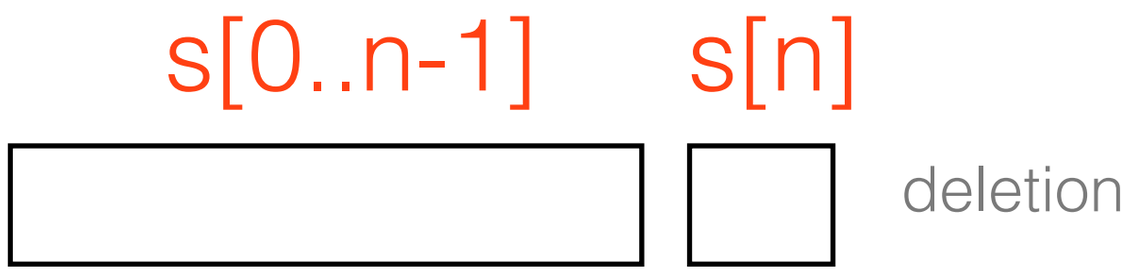
t[0..k-1]

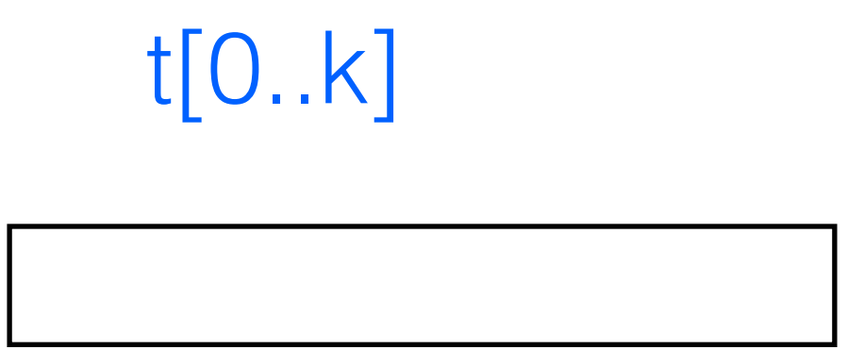
t[k]



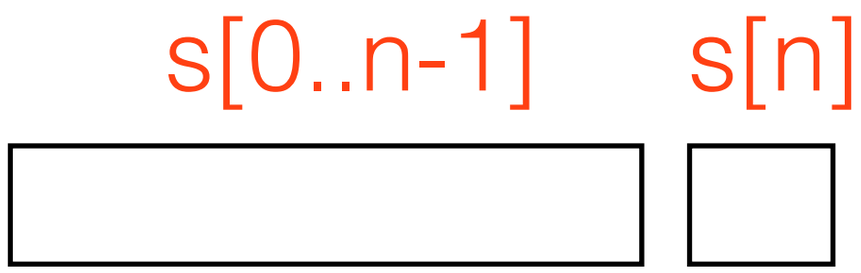
insertion

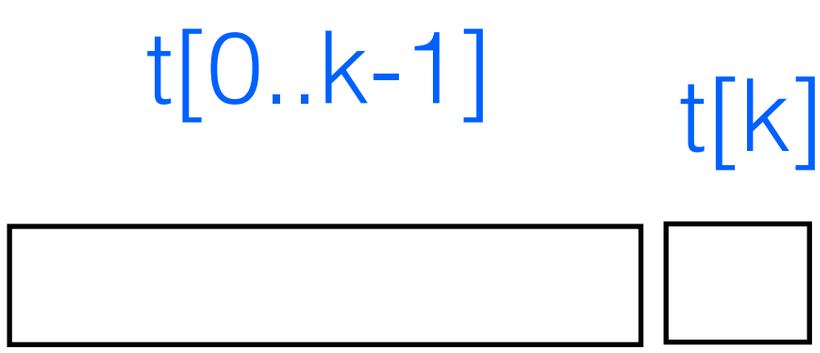
d(s[0..n], t[0..k-1]) + 1




d(s[0..n-1], t[0..k]) + 1




$d(s[0..n-1], t[0..k-1]) + 0$ $s[n] = t[k]$ nothing

$d(s[0..n-1], t[0..k-1]) + 1$ $s[n] \neq t[k]$ substitution

# Edit distance

d( s[0..n] ," " ) = n + 1 deletions

d( " " , t[0..k] ) = k + 1 insertions

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

# Distance between feet and felt

| | “ ” | f | fe | fel | felt |
|---|---|---|---|---|---|
| “ ” | | | | | |
| f | | | | | |
| fe | | | | | |
| fee | | | | | |
| feet | | | | | |

# Distance between feet and felt

| | “ ” | f | e | l | t |
|---|---|---|---|---|---|
| “ ” | | | | | |
| f | | | | | |
| e | | | | | |
| e | | | | | |
| t | | | | | |

d( s[0..n] ,“ ” ) = n + 1  deletions

d( “ ” , t[0..k] ) = k + 1  insertions

# Distance between feet and felt

| | “ ” | f | e | l | t |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | | | | |
| e | 2 | | | | |
| e | 3 | | | | |
| t | 4 | | | | |

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

# Distance between feet and felt

| | “ ” | f | e | l | t |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | 0 | 1 | 2 | 3 |
| e | 2 | 1 | 0 | 1 | 2 |
| e | 3 | 2 | 1 | 1 | 2 |
| t | 4 | 3 | 2 | | |

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

# Distance between feet and felt

| | “ ” | f | e | l | t |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | 0 | 1 | 2 | 3 |
| e | 2 | 1 | 0 | 1 | 2 |
| e | 3 | 2 | 1 | 1 | 2 |
| t | 4 | 3 | 2 | 2 | |

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

# Distance between feet and felt

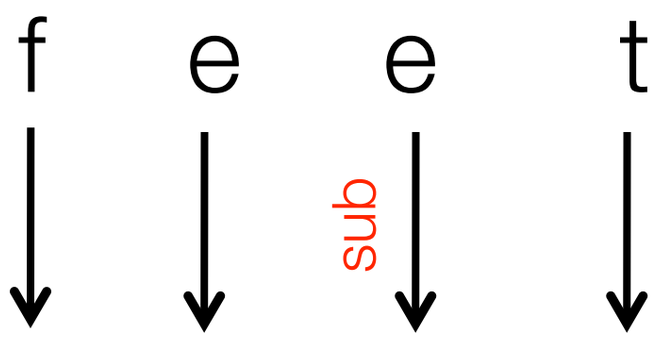
| | “ ” | f | e | l | t |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | 0 | 1 | 2 | 3 |
| e | 2 | 1 | 0 | 1 | 2 |
| e | 3 | 2 | 1 | 1 | 2 |
| t | 4 | 3 | 2 | 2 | |

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

# Distance between feet and felt

| | “ ” | f | e | l | t |
|---|---|---|---|---|---|
| “ ” | 0 | 1 | 2 | 3 | 4 |
| f | 1 | 0 | 1 | 2 | 3 |
| e | 2 | 1 | 0 | 1 | 2 |
| e | 3 | 2 | 1 | 1 | 2 |
| t | 4 | 3 | 2 | 2 | 1 |

# Transform feet into felt




f e l t

# Edit distance

cgggtatccaa

ccctaggtccca

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | | | | | | | | | | | | | |
| c | | | | | | | | | | | | | |
| g | | | | | | | | | | | | | |
| g | | | | | | | | | | | | | |
| t | | | | | | | | | | | | | |
| a | | | | | | | | | | | | | |
| t | | | | | | | | | | | | | |
| c | | | | | | | | | | | | | |
| c | | | | | | | | | | | | | |
| a | | | | | | | | | | | | | |
| a | | | | | | | | | | | | | |

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | | | | | | | | | | | | |
| g | 2 | | | | | | | | | | | | |
| g | 3 | | | | | | | | | | | | |
| g | 4 | | | | | | | | | | | | |
| t | 5 | | | | | | | | | | | | |
| a | 6 | | | | | | | | | | | | |
| t | 7 | | | | | | | | | | | | |
| c | 8 | | | | | | | | | | | | |
| c | 9 | | | | | | | | | | | | |
| a | 10 | | | | | | | | | | | | |
| a | 11 | | | | | | | | | | | | |

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| g | 2 | | | | | | | | | | | | |
| g | 3 | | | | | | | | | | | | |
| g | 4 | | | | | | | | | | | | |
| t | 5 | | | | | | | | | | | | |
| a | 6 | | | | | | | | | | | | |
| t | 7 | | | | | | | | | | | | |
| c | 8 | | | | | | | | | | | | |
| c | 9 | | | | | | | | | | | | |
| a | 10 | | | | | | | | | | | | |
| a | 11 | | | | | | | | | | | | |

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| g | 2 | | | | | | | | | | | | |
| g | 3 | | | | | | | | | | | | |
| g | 4 | | | | | | | | | | | | |
| t | 5 | | | | | | | | | | | | |
| a | 6 | | | | | | | | | | | | |
| t | 7 | | | | | | | | | | | | |
| c | 8 | | | | | | | | | | | | |
| c | 9 | | | | | | | | | | | | |
| a | 10 | | | | | | | | | | | | |
| a | 11 | | | | | | | | | | | | |

$$d(s[0..n], t[0..k]) = \min \begin{cases} d(s[0..n-1], t[0..k]) + 1 & \text{deletion} \\ d(s[0..n], t[0..k-1]) + 1 & \text{insertion} \\ d(s[0..n-1], t[0..k-1]) & s[n] = t[k] \text{ nothing} \\ d(s[0..n-1], t[0..k-1]) + 1 & s[n] \neq t[k] \text{ substitution} \end{cases}$$

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| g | 2 | | | | | | | | | | | | |
| g | 3 | | | | | | | | | | | | |
| g | 4 | | | | | | | | | | | | |
| t | 5 | | | | | | | | | | | | |
| a | 6 | | | | | | | | | | | | |
| t | 7 | | | | | | | | | | | | |
| c | 8 | | | | | | | | | | | | |
| c | 9 | | | | | | | | | | | | |
| a | 10 | | | | | | | | | | | | |
| a | 11 | | | | | | | | | | | | |

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| g | 2 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| g | 3 | | | | | | | | | | | | |
| g | 4 | | | | | | | | | | | | |
| t | 5 | | | | | | | | | | | | |
| a | 6 | | | | | | | | | | | | |
| t | 7 | | | | | | | | | | | | |
| c | 8 | | | | | | | | | | | | |
| c | 9 | | | | | | | | | | | | |
| a | 10 | | | | | | | | | | | | |
| a | 11 | | | | | | | | | | | | |

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| g | 2 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| g | 3 | 2 | 2 | 2 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| g | 4 | | | | | | | | | | | | |
| t | 5 | | | | | | | | | | | | |
| a | 6 | | | | | | | | | | | | |
| t | 7 | | | | | | | | | | | | |
| c | 8 | | | | | | | | | | | | |
| c | 9 | | | | | | | | | | | | |
| a | 10 | | | | | | | | | | | | |
| a | 11 | | | | | | | | | | | | |

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| g | 2 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| g | 3 | 2 | 2 | 2 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| g | 4 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| t | 5 | | | | | | | | | | | | |
| a | 6 | | | | | | | | | | | | |
| t | 7 | | | | | | | | | | | | |
| c | 8 | | | | | | | | | | | | |
| c | 9 | | | | | | | | | | | | |
| a | 10 | | | | | | | | | | | | |
| a | 11 | | | | | | | | | | | | |

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| g | 2 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| g | 3 | 2 | 2 | 2 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| g | 4 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| t | 5 | 4 | 4 | 4 | 3 | 4 | 5 | 5 | 4 | 5 | 6 | 7 | 8 |
| a | 6 | | | | | | | | | | | | |
| t | 7 | | | | | | | | | | | | |
| c | 8 | | | | | | | | | | | | |
| c | 9 | | | | | | | | | | | | |
| a | 10 | | | | | | | | | | | | |
| a | 11 | | | | | | | | | | | | |

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| g | 2 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| g | 3 | 2 | 2 | 2 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| g | 4 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| t | 5 | 4 | 4 | 4 | 3 | 4 | 5 | 5 | 4 | 5 | 6 | 7 | 8 |
| a | 6 | 5 | 5 | 5 | 4 | 3 | 4 | 5 | 5 | 5 | 6 | 7 | 7 |
| t | 7 | | | | | | | | | | | | |
| c | 8 | | | | | | | | | | | | |
| c | 9 | | | | | | | | | | | | |
| a | 10 | | | | | | | | | | | | |
| a | 11 | | | | | | | | | | | | |

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| g | 2 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| g | 3 | 2 | 2 | 2 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| g | 4 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| t | 5 | 4 | 4 | 4 | 3 | 4 | 5 | 5 | 4 | 5 | 6 | 7 | 8 |
| a | 6 | 5 | 5 | 5 | 4 | 3 | 4 | 5 | 5 | 5 | 6 | 7 | 7 |
| t | 7 | 6 | 6 | 6 | 5 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 8 |
| c | 8 | | | | | | | | | | | | |
| c | 9 | | | | | | | | | | | | |
| a | 10 | | | | | | | | | | | | |
| a | 11 | | | | | | | | | | | | |

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| g | 2 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| g | 3 | 2 | 2 | 2 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| g | 4 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| t | 5 | 4 | 4 | 4 | 3 | 4 | 5 | 5 | 4 | 5 | 6 | 7 | 8 |
| a | 6 | 5 | 5 | 5 | 4 | 3 | 4 | 5 | 5 | 5 | 6 | 7 | 7 |
| t | 7 | 6 | 6 | 6 | 5 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 8 |
| c | 8 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 6 | 5 | 6 | 6 | 7 |
| c | 9 | | | | | | | | | | | | |
| a | 10 | | | | | | | | | | | | |
| a | 11 | | | | | | | | | | | | |

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| g | 2 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| g | 3 | 2 | 2 | 2 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| g | 4 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| t | 5 | 4 | 4 | 4 | 3 | 4 | 5 | 5 | 4 | 5 | 6 | 7 | 8 |
| a | 6 | 5 | 5 | 5 | 4 | 3 | 4 | 5 | 5 | 5 | 6 | 7 | 7 |
| t | 7 | 6 | 6 | 6 | 5 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 8 |
| c | 8 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 6 | 5 | 6 | 6 | 7 |
| c | 9 | 8 | 7 | 6 | 7 | 6 | 6 | 6 | 6 | 6 | 5 | 6 | 7 |
| a | 10 | | | | | | | | | | | | |
| a | 11 | | | | | | | | | | | | |

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| g | 2 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| g | 3 | 2 | 2 | 2 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| g | 4 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| t | 5 | 4 | 4 | 4 | 3 | 4 | 5 | 5 | 4 | 5 | 6 | 7 | 8 |
| a | 6 | 5 | 5 | 5 | 4 | 3 | 4 | 5 | 5 | 5 | 6 | 7 | 7 |
| t | 7 | 6 | 6 | 6 | 5 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 8 |
| c | 8 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 6 | 5 | 6 | 6 | 7 |
| c | 9 | 8 | 7 | 6 | 7 | 6 | 6 | 6 | 6 | 6 | 5 | 6 | 7 |
| a | 10 | 9 | 8 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 6 | 6 | 6 |
| a | 11 | | | | | | | | | | | | |

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| g | 2 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| g | 3 | 2 | 2 | 2 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| g | 4 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| t | 5 | 4 | 4 | 4 | 3 | 4 | 5 | 5 | 4 | 5 | 6 | 7 | 8 |
| a | 6 | 5 | 5 | 5 | 4 | 3 | 4 | 5 | 5 | 5 | 6 | 7 | 7 |
| t | 7 | 6 | 6 | 6 | 5 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 8 |
| c | 8 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 6 | 5 | 6 | 6 | 7 |
| c | 9 | 8 | 7 | 6 | 7 | 6 | 6 | 6 | 6 | 6 | 5 | 6 | 7 |
| a | 10 | 9 | 8 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 6 | 6 | 6 |
| a | 11 | 10 | 9 | 8 | 8 | 7 | 8 | 8 | 8 | 8 | 7 | 7 | 6 |

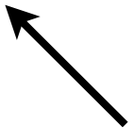
```
import numpy as np


def edit_distance(string_1, string_2):
    distance_table = np.zeros((len(string_1) + 1, len(string_2) + 1))
    distance_table[:, 0] = range(len(string_1) + 1)
    distance_table[0, :] = range(len(string_2) + 1)
    for i in range(1, len(string_1) + 1):
        for j in range(1, len(string_2) + 1):
            diff = 1 - (string_1[i - 1] == string_2[j - 1])
            distance_table[i, j] = min(distance_table[i - 1, j] + 1,
                                       distance_table[i, j - 1] + 1,
                                       distance_table[i - 1, j - 1] + diff)
    return distance_table[-1, -1]
```

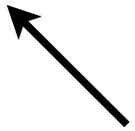
# Determining the transformation

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| g | 2 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| g | 3 | 2 | 2 | 2 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| g | 4 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| t | 5 | 4 | 4 | 4 | 3 | 4 | 5 | 5 | 4 | 5 | 6 | 7 | 8 |
| a | 6 | 5 | 5 | 5 | 4 | 3 | 4 | 5 | 5 | 5 | 6 | 7 | 7 |
| t | 7 | 6 | 6 | 6 | 5 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 8 |
| c | 8 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 6 | 5 | 6 | 6 | 7 |
| c | 9 | 8 | 7 | 6 | 7 | 6 | 6 | 6 | 6 | 6 | 5 | 6 | 7 |
| a | 10 | 9 | 8 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 6 | 6 | 6 |
| a | 11 | 10 | 9 | 8 | 8 | 7 | 8 | 8 | 8 | 8 | 7 | 7 | 6 |

# Where did the value come from....



Same number:
Nothing



Different number:
Substitution



Insertion

Deletion

| | - | c | c | c | t | a | g | g | t | c | c | c | a |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| - | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| c | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| g | 2 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| g | 3 | 2 | 2 | 2 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| g | 4 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
| t | 5 | 4 | 4 | 4 | 3 | 4 | 5 | 5 | 4 | 5 | 6 | 7 | 8 |
| a | 6 | 5 | 5 | 5 | 4 | 3 | 4 | 5 | 5 | 5 | 6 | 7 | 7 |
| t | 7 | 6 | 6 | 6 | 5 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 8 |
| c | 8 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 6 | 5 | 6 | 6 | 7 |
| c | 9 | 8 | 7 | 6 | 7 | 6 | 6 | 6 | 6 | 6 | 5 | 6 | 7 |
| a | 10 | 9 | 8 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 6 | 6 | 6 |
| a | 11 | 10 | 9 | 8 | 8 | 7 | 8 | 8 | 8 | 8 | 7 | 7 | 6 |