Monash University
Faculty of Information Technology

Lecture 3
(A) Predicate logic
(B) Linux

Slides by David Albrecht (2011) and Graham Farr (2013-2014, 2017).

FIT2014 Theory of Computation

Overview

Predicate logic

- Sentences
- Quantifiers
- Knowledge Representation

Linux

Predicate logic: Example

All men are mortal

Socrates is a man.

Therefore Socrates is mortal.

- Objects: **socrates, set of people**.
- Properties: **man, mortal**

Example

There is an app which is loved by every student.

Therefore every student loves some app.

- Objects: **set of people.**
- Properties: **app, student.**
- Relation: **loves**

Objects

- **Constant symbols**
  - Names which refer to exactly one object.
  - socrates, wumpus, 1, 2, …
- **Function symbols**
  - relates some objects to exactly one object.
  - motherOf, kingOf, plus, times, …
  - Complex name.
- **Individual variables**
  - a variable which can refer to any object.
  - X, Y, …

Term

A **term** is a logical expression which refers to an object.

E.g.

- **Constant symbols.**
- **Individual variables.**
- **Functions of constant symbols.**
- **Functions of other terms.**

Equality symbol (=)

Used to state that two objects are the same.

rebecca = rebecca

fatherOf(john) = henry

X = kingOf(sweden)

Predicates

- E.g.: man, mortal, app, student, loves, …
- Properties (1 place)
- Relations (2 or more places)

- Can only be **True** or **False** (like *propositions*)
- they take **arguments** (unlike *propositions*)

Sentences

- **Atomic sentences**
  - A predicate symbol followed by a list of terms in brackets.
  - E.g.

    taller(motherOf(claire), mary)
- **Complex sentences**
  - Atomic sentences joined together by logical connectives
  - E.g.

    man(socrates) ⇒ mortal(socrates)

Universal Quantification

- Used to make a statement about **every** object.
- ∀ **"for all"**

All dogs are happy

$$\forall X\ (dog(X) \Rightarrow happy(X))$$

No dog is happy

All dogs are unhappy

$$\forall X\ (dog(X) \Rightarrow \neg\ happy(X))$$

Existential Quantification

- Used to make a statement about **some** object.
- ∃ **"there exists"**

Some dogs are happy

$$\exists X\ (dog(X) \wedge happy(X))$$

Some dogs are not happy

$$\exists X\ (dog(X) \wedge \neg\ happy(X))$$

Universe of Discourse

- The set of objects that are being referred to.
- Often it is unstated or assumed.
- Can affect the truth of a statement.

- Consider the predicate **greaterThanZero**.

$$\forall \mathbf{X}\ \mathbf{greaterThanZero(X)}$$

Doing logic with quantifiers

If we know that

$\forall$ **X blah(X)**

and **obj** is any specific object

(in the universe of discourse),

then we can deduce that

blah(obj)

We have:

($\forall$ X blah(X)) $\Rightarrow$ blah(obj)

Also:

blah(obj) $\Rightarrow$ ($\exists$ X blah(X))

Doing logic with quantifiers

$$\forall \mathbf{X}\ (\ \mathbf{p(X)} \wedge \mathbf{q(X)}\ )$$

is logically equivalent to

$$(\ \forall \mathbf{X}\ \mathbf{p(X)}\ ) \wedge (\ \forall \mathbf{X}\ \mathbf{q(X)}\ )$$

$$\exists \mathbf{X}\ (\ \mathbf{p(X)} \vee \mathbf{q(X)}\ )$$

is logically equivalent to

$$(\exists \mathbf{X}\ \mathbf{p(X)}\ ) \vee (\exists \mathbf{X}\ \mathbf{q(X)}\ )$$

What about the logical relationship between …

$$\forall \mathbf{X}\ (\ \mathbf{p(X)} \vee \mathbf{q(X)}\ )$$

and $(\ \forall \mathbf{X}\ \mathbf{p(X)}\ ) \vee (\ \forall \mathbf{X}\ \mathbf{q(X)}\ )$ …?

… etc

Relationship between quantifiers

$\neg \forall y$ means the same as $\exists y \neg$

Not all dogs are happy.

is the same as … There exists an unhappy dog.

$\neg \forall X\ (dog(X) \Rightarrow happy(X))$ Not all dogs are happy

$= \exists X \neg\ (dog(X) \Rightarrow happy(X))$

$= \exists X \neg\ (\neg dog(X) \vee happy(X))$ (see last lecture)

$= \exists X\ (\neg\ \neg dog(X) \wedge \neg\ happy(X))$ (by De Morgan)

$= \exists X\ (\quad dog(X) \wedge \neg\ happy(X))$ There exists an unhappy dog

Relationship between quantifiers

Similarly,

$\neg \exists y$ means the same as $\forall y \neg$

$\neg \forall y \neg$ means the same as ……

$\neg \exists y \neg$ means the same as ……

Socrates Example

- All men are mortal

  $\forall$ **X (man(X)** $\Rightarrow$ **mortal(X))**

- Socrates is a man.

  man(socrates)

- Socrates is mortal.

  mortal(socrates)

Love Example

- There is an app which is loved by every student.
- There is an app X and if Y is a student then Y loves it.

  $\exists$ **X (app(X)** $\wedge$ $\forall$ **Y(student(Y)** $\Rightarrow$ **loves(Y,X)))**

- Every student loves some app.
- For every student Y there exists an app X that she loves.

  $\forall$ **Y(student(Y)** $\Rightarrow$ $\exists$ **X (app(X)** $\wedge$ **loves(Y,X)))**

Unix

Origin:

- Bell Laboratories, 1969; first published in 1974
- Dennis M. Ritchie and Ken Thompson
- cost-effective, simple, elegant, easy to use
- widely used, e.g. for servers
- security

https://www.acm.org/turing-award-50/spotlight-on-turing-laureates

Unix

- Dennis M. Ritchie and Ken Thompson, The Unix Time-Sharing System, *Communications of the ACM* **17** (no. 7) (July 1974) 365-375.
- https://people.eecs.berkeley.edu/~brewer/cs262/unix.pdf
- From the Abstract:
  - "It offers a number of features seldom found even in larger operating systems, including: (1) a hierarchical file system incorporating demountable volumes; (2) compatible file, device, and inter-process I/O; (3) the ability to initiate asynchronous processes; (4) system command language selectable on a per-user basis; and (5) over 100 subsystems including a dozen languages."
- written in high-level language, C (mostly)
- can combine programs to make more complex ones

GNU

http://www.cahilig.net/2008/06/02/what-linux

- GNU project
- Richard Stallman, from 1983

- Aim: free software, developed by large-scale collaboration
- including operating system based on Unix
- GNU = **G**NU's **N**ot **U**nix (recursive acronym!)
- Much software, but lacked kernel …

- GNU General Public Licence
  - https://www.gnu.org/licenses/gpl.html

Linux

- Linux kernel:
- Linus Torvalds, 1991
  - (independently of GNU)
- Then released under GNU Public Licence, 1992
- The OS is sometimes called GNU/Linux

Drawing by Pekka Vuori, 2000

https://finland.fi/business-innovation/good-taste-with-passion-21-years-rewarded/

MSc thesis, University of Helsinki, 1997: *Linux: A Portable Operating System.* https://www.cs.helsinki.fi/u/kutvonen/index_files/linus.pdf

"As of November 2017, all of the world's fastest 500 supercomputers run Linux-based operating systems."

Wikipedia, based on https://www.top500.org/statistics/details/osfam/1