# FIT3155: Week 11 Tutorial - Answer Sheet

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**Question 1**

Use simplex method in the tableau form to solve the following linear program:

Steps:

1. Convert all 5 linear constraints (inequalities) into equations by introducing slack variables S1,S2,S3,S4,S5
2. Prepare the initial table

| $C_j$ | | 1 | 2 | 0 | 0 | 0 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **x** | **y** | **S1** | **S2** | **S3** | **S4** | **S5** | **RHS** | **Θ** |
| 0 | **S1** | 4 | 1 | 1 | 0 | 0 | 0 | 0 | 44 | 44 |
| 0 | **S2** | 3 | 2 | 0 | 1 | 0 | 0 | 0 | 39 | 19.5 |
| 0 | **S3** | 2 | 3 | 0 | 0 | 1 | 0 | 0 | 37 | 12.3 |
| 0 | **S4** | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 9 | 9 |
| 0 | **S5** | -1 | 1 | 0 | 0 | 0 | 0 | 1 | 6 | 6 |
| $C_j - Z_j$ | | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | | | | | | | | | | |

$C_j$ is the coefficient vector of the objective function Z = x + 2y

- Basic variables: S1,S2,S3,S4,S5
- Non-basic variables: x,y (set to 0)
- Initial exploration is the (0,0) point which gives Z = 0 (green highlighted cell)
- Next, decide which non basic variable we can increase as to improve Z.
  - For that, compute $C_j - Z_j$ for each column Col_j, where $Z_j$ is the dot product of $C_j$ and Col_j
  - Variable corresponding to max($C_j - Z_j$) enters the basis for the next iteration (yellow highlighted column)
- To decide which basic variable leaves,
  - compute Θ = RHS/entering_variable_column
  - Variable corresponding to min(Θ) leaves the basis (yellow highlighted row)

- This ensures we increase the entered variable value without violating the non-negativity constraint for slack variables.

- Intersection of the entering_variable_column and leaving_variable_row is the pivot element

3. Prepare the next table using previous information

- Divide the leaving_variable_row by pivot element and replace its variable name to the entering variable -- this updates the pivot element to be 1
- Next, perform row operations to make rest of the elements in the entering_variable_column 0

| $C_j$ | | 1 | 2 | 0 | 0 | 0 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **x** | **y** | **S1** | **S2** | **S3** | **S4** | **S5** | **RHS** | **Θ** |
| **0** | **S1** | 5 | 0 | 1 | 0 | 0 | 0 | -1 | **38** | 38/5 |
| **0** | **S2** | 5 | 0 | 0 | 1 | 0 | 0 | -2 | **27** | 27/5 |
| **0** | **S3** | 5 | 0 | 0 | 0 | 1 | 0 | -3 | **19** | 19/5 |
| **0** | **S4** | 1 | 0 | 0 | 0 | 0 | 1 | -1 | **3** | 3/1 |
| **2** | **y** | -1 | 1 | 0 | 0 | 0 | 0 | 1 | **6** | 6/-1 |
| $C_j$ - $Z_j$ | | **3** | **0** | **0** | **0** | **0** | **0** | **-2** | **12** | |
| | | | | | | | | | | |

- Now this iteration corresponds to exploring (x=0,y=6) point which gives Z = 12.
- Here the basic variables are S1,S2,S3,S4,y. Non-basic variables are x,S5
- Re_perform all above mentioned steps to decide the entering variable and leaving variable, and accordingly prepare the table for next iteration.

  At this stage, entering variable is x, and leaving variable is S4.

4. Next table

| $C_j$ | | 1 | 2 | 0 | 0 | 0 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **x** | **y** | **S1** | **S2** | **S3** | **S4** | **S5** | **RHS** | Θ |
| 0 | **S1** | 0 | 0 | 1 | 0 | 0 | -5 | 4 | 23 | 23/4 |
| 0 | **S2** | 0 | 0 | 0 | 1 | 0 | -5 | 3 | 12 | 12/3 |
| 0 | **S3** | 0 | 0 | 0 | 0 | 1 | -5 | 2 | 4 | 4/2 |
| 1 | **x** | 1 | 0 | 0 | 0 | 0 | 1 | -1 | 3 | 3/-1 |
| 2 | **y** | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 9 | Inf |
| $C_j$ -$Z_j$ | | 0 | 0 | 0 | 0 | 0 | -3 | 1 | 21 | |
| | | | | | | | | | | |

- At this stage we explore (x=3,y=9) point which gives Z = 21
- Re-perform the steps to decide entering and leaving variable, which in this case are: S5 and S3, respectively.

5. Next table

| $C_j$ | | 1 | 2 | 0 | 0 | 0 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **x** | **y** | **S1** | **S2** | **S3** | **S4** | **S5** | **RHS** | Θ |
| 0 | **S1** | 0 | 0 | 1 | 0 | -2 | 5 | 0 | 15 | |
| 0 | **S2** | 0 | 0 | 0 | 1 | -3/2 | 5/2 | 0 | 6 | |
| 0 | **S3** | 0 | 0 | 0 | 0 | 1/2 | -5/2 | 1 | 2 | |
| 1 | **x** | 1 | 0 | 0 | 0 | 1/2 | -3/2 | 0 | 5 | |
| 2 | **y** | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 9 | |
| $C_j$ -$Z_j$ | | 0 | 0 | 0 | 0 | -1/2 | -1/2 | 0 | 23 | |
| | | | | | | | | | | |

- At this stage we explore (x=5,y=9) point which gives Z = 23

Note that all $C_j$ -$Z_j$ values are either 0 or negative. This means no more improvement is possible, and we have reached the maximum for Z which is 23 with x=5,y=9. Therefore we can stop.