Lab Report – 4

Krishna **2021112005**

Question – 1:

(b)

For (i) $\delta[n]$

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-jwn} = x[0]\, e^{0} = 1$$

For (ii) $\delta[n+3]$

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} \delta[n+3]\, e^{-jwn} = e^{3jw}$$

$|e^{3jw}| = 1$, Phase $= \tan 3w$, Realpart $= \cos 3w$

Imag part $= \sin 3w$

For (iii) Rect impulse from -3 to 3

$$X(e^{jw}) = \sum_{n=-3}^{3} 1 \cdot e^{-jwn} = e^{3jw} + e^{2jw} + e^{jw} + 1 + e^{-jw} + e^{-2jw} + e^{-3jw}$$

Realpart $\Rightarrow$ Overlapping cos waves (superimposed/added)

Imag part $\Rightarrow$ Overlapping sin waves (superimposed/added)

(iv) $\sin(\pi n/4)$ for $n = -200$ to 200

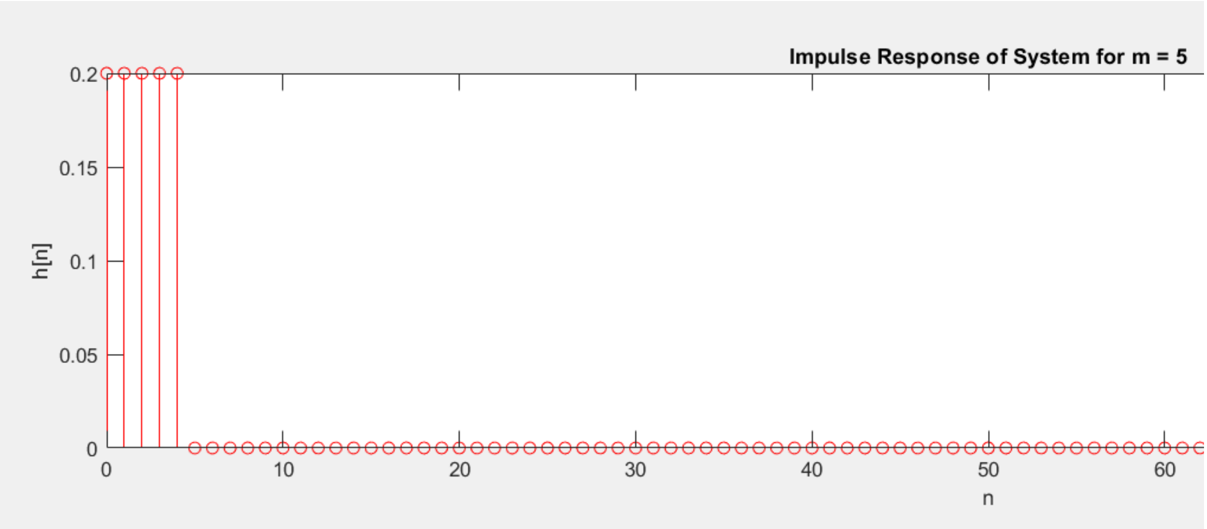
$$X(e^{jw}) = \sum_{n=-200}^{200} \sin(n\pi/4) \cdot e^{-jwn}$$

(c) For smaller value of b (0.01), the graph of magnitude spectrum is sinusoidal in shape. As the value of b increases (0.5, 0.99), the sinusoid starts to get distorted and tends to get sharper peaks.

Question – 2:

(a) The impulse response of the given moving average system has value of 1/m for n = [0, m-1] and 0 for all other n.

For example, the below figure shows impulse response of the system for m = 5. It has value (1/5) for n = [0, 4].



(e) On increasing the M value, noise gets filtered out better.

(f) The DTFT of both noisy and filtered signals have quite similar peaks, but the filtered signal has smoother DTFT than the noisy signal.

(g) The simple digital differentiator does not filter out noise much as it takes just two adjacent values. Whereas in the moving average, the number of points can be increased by increasing the M value.

(h) The moving average filter is a low-pass filter as it filters out the high frequency noise from the filter

Question – 3:

(a)

Let $w_c = \frac{\pi}{k}$, $K = 1$ or 2 or 4 or 8 or 16

Thus, $$x[n] = \frac{1}{2\pi}\int_{-w_c}^{w_c} e^{jwn}\,dw = \left.\frac{e^{jwn}}{2\pi jn}\right|_{-w_c}^{w_c}$$

$$= \frac{1}{2\pi jn}\left(e^{jnw_c} - e^{-jnw_c}\right)$$

$$= \frac{1}{\cancel{2}\pi \cancel{j}n} \times \left(\cancel{2}\cancel{j}\sin(nw_c)\right)$$

$$= \frac{\sin(nw_c)}{\pi n}$$ $\Rightarrow$ Real Valued

Thus, it can also be seen in our matlab plots for Imaginary part of $x[n]$ that it is either 0 or negligibly small value (10^{-13} order for $\pi/16$)

(b)

We found out that $x[n] = \dfrac{\sin(n w_c)}{\pi n}$

For $w_c = \pi \Rightarrow x[n] = \dfrac{\sin(n \pi)}{\pi n} = 0$ $\left(\text{as } \sin(n\pi) = 0 \ \forall n \in I\right)$

but for $n = 0$, $x[0] = \lim_{n \to 0} \dfrac{\sin(n\pi)}{\pi n} = 1$

Thus, $x[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$

(c)

For band-pass signal, the imaginary part of x[n] comes out to be non-zero.