Lab Report – 5

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Answer 1:

(a)

$$p[n] = \cos(2\pi f_0 n/f_s)$$

$$P(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \cos(2\pi f_0 n/f_s)\, e^{-j\omega n}$$

$$\downarrow$$

$$\sum_{n=-\infty}^{\infty} \left[\frac{e^{j(2\pi f_0 n/f_s)} + e^{-j(2\pi f_0 n/f_s)}}{2}\right] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{e^{jn\left(\frac{2\pi f_0}{f_s}-\omega\right)} + e^{-jn\left(\frac{2\pi f_0}{f_s}+\omega\right)}}{2}\right]$$

Let $\frac{2\pi f_0}{f_s} - \omega = \omega_1$ and $\frac{2\pi f_0}{f_s} + \omega = \omega_2$

$$= \frac{1}{2}\left[\sum_{n=-\infty}^{\infty} e^{jn\omega_1}\cdot 1 + \sum_{n=-\infty}^{\infty} e^{-jn\omega_2}\cdot 1\right]$$

Using formula: $\displaystyle\sum_{n=-\infty}^{\infty} e^{j\omega n} = 2\pi\,\delta(\omega)$

$$\frac{1}{2}\Big(2\pi\,\delta(\omega_1) + 2\pi\,\delta(-\omega_2)\Big)$$

$$= \pi\Big(\delta(\omega_1) + \delta(-\omega_2)\Big)$$

$$= \pi\left(\delta\left(\frac{2\pi f_0}{f_s} - \omega_0\right) + \delta\left(\frac{-2\pi f_0}{f_s} - \omega\right)\right)$$

(b) First peak at $\omega = \dfrac{2\pi f_0}{f_s}$

Second peak at $\omega = \dfrac{-2\pi f_0}{f_s}$

(Impulses)

$\Rightarrow$ Peaks are symmetric about origin ie the centre of the signal.

(c) $x[n] = \begin{cases} \cancel{\sum_{n=0}^{L}} \cos(2\pi f_0 n/f_s) , & \forall\ n = 0, 1, 2 \dots, L-1 \\ 0 , & \text{otherwise} \end{cases}$

$$\text{DTFT}\big(x[n]\big) = \sum_{n=0}^{L-1} \cos(2\pi f_0 n/f_s)$$

$$= \frac{1}{2}\sum_{n=0}^{L-1}\left[e^{\left(\frac{2\pi f_0}{f_s} - \omega\right) jn} + e^{\left(\frac{-2\pi f_0}{f_s} - \omega\right) jn} \right]$$

$$\frac{1}{2}\left(\frac{\left(1-e^{\left(\frac{2\pi f_0}{f_s}-\omega\right)jL}\right)}{\left(1-e^{\left(\frac{2\pi f_0}{f_s}-\omega\right)}\right)}\right)+\frac{1}{2}\left(\frac{\left(1-e^{\left(\frac{-2\pi f_0}{f_s}-\omega\right)jL}\right)}{\left(1-e^{\frac{-2\pi f_0}{f_s}-\omega}\right.}\right)$$

(d)

Yes, the plots are consistent. They are symmetric about the center of the signal.

(e)

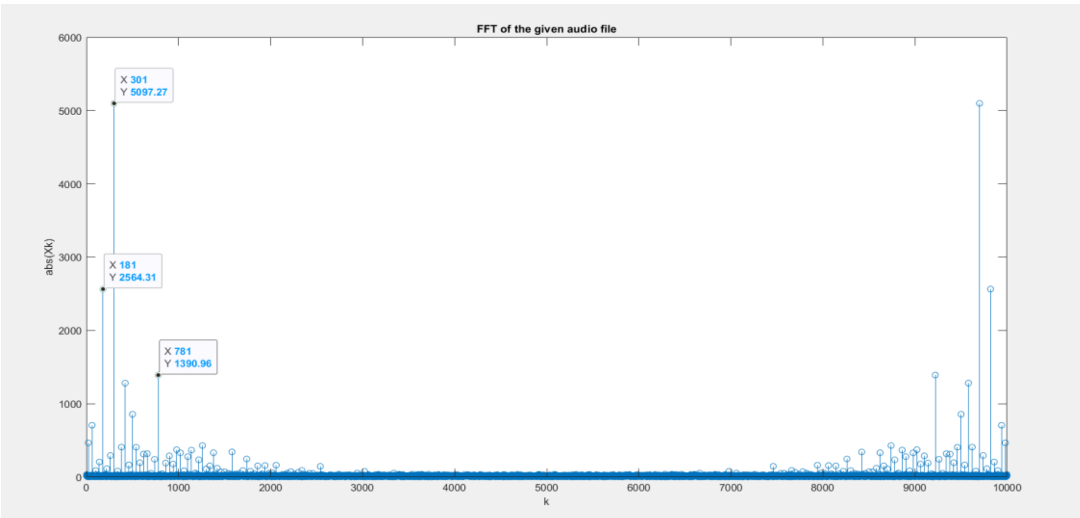
The signal is a sum of two impulses. When L value is smaller, the impulse is wider and less distinct. As L value increases the impulse becomes sharper and more distinct.

(g)

The values of signal increases in a gaussian like way. It means that the values in the middle get amplified.

(i)

3 Strongest frequencies are corresponding to the following k values:



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K = 181, 301 and 781.

Answer 2:

The method of using in-built function and convolution using DFT and IDFT, both yield the same result.