Roll No. 511942 Experiment No. 1 Sheet No. 2

Procedure:

1) Open MATLAB Command window.
2) Click on file/new/M file to open the MATLAB editor window. In MATLAB editor window enter the program.
3) Save the program as .M file.
4) Execute the program by selecting run.
5) Obtain the specifications from the plot.

Theoretical Calculations:

$$TF = \frac{100}{s^2+10s+100}$$

$$\omega_n^2 = 100$$

$$\boxed{\omega_n = 10}$$

$$2\xi\omega_n = 10$$

$$\boxed{\xi = 0.5}$$

Resonance frequency $\omega_r = \omega_n\sqrt{(1-2\xi^2)} = 10\sqrt{(1-2\times0.5)^2} = 7.07106$

$$M_R = \frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{1}{2\times0.5\sqrt{1-0.5^2}} = 1.1547.$$

$$\omega_b = \omega_n\left[1-2\xi^2+\sqrt{2-4\xi^2+4\xi^4}\right]^{1/2}$$

$$= 10\left[1-2(0.5)^2+\sqrt{2-4(0.5)^2+4\times(0.5)^4}\right]^{1/2} = 12.7202.$$

$$OLTF = \frac{CLTF}{1-CLTF} = \frac{100/s^2+10s+100}{1-\frac{100}{s^2+10s+100}} = \frac{100}{s^2+10s} = \frac{100}{s(s+10)}$$

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$$TF = \frac{100}{s(s+10)} = \frac{100}{j\omega(j\omega+10)}$$

$$= \frac{10\angle 0^\circ}{\omega\angle 90^\circ (\sqrt{100+\omega^2})\angle\tan^{-1}(\frac{\omega}{10})}$$

$$= 0-(90^\circ + \tan^{-1}(\frac{\omega}{10})) = -180^\circ$$

$$\tan^{-1}(\frac{\omega}{10}) = 90^\circ$$

gain cross over frequency

$$\frac{100}{\omega(\sqrt{100+\omega^2})} = 1$$

$$\omega\sqrt{100+\omega^2} = 100$$

$$\omega^2(100+\omega^2) = 100\times100$$

$$\omega^4 + 100\omega^2 - 10^4 = 0$$

$$\omega^2 = t$$

$$t^2 + 100t - 10^4 = 0$$

$$t = -161.80 \times \quad , t = 61.80 \checkmark$$

$$\omega_{gc} = \sqrt{61.80} = 7.861$$

$$PM(\text{Phase Margin}) = 180 + (-90 - \tan^{-1}(\frac{7.861}{10}))$$

$$\tan^{-1}(\frac{7.861}{10}) = 38.170$$

$$PM = 51.829.$$

Phase cross over frequency

$$0 + (90 + \tan^{-1}(\frac{\omega_{pc}}{10}) = +180$$

$$\tan^{-1}(\frac{\omega_{pc}}{10}) = 90$$

$$\boxed{\omega_{pc} = \infty}$$

$$|G(j\omega_{pc})| = \frac{100}{\omega\sqrt{100+\omega^2}} = \frac{100}{\infty\sqrt{100+\infty^2}}$$

$$GM = \frac{1}{|G(j\omega_{pc})|} = \frac{1}{0} = \infty$$

for $\xi = 1$

$M_r = 1$, $\omega_r = 0$, $\omega_b = 6.4229$

for $\xi = 0$

$M_r = \infty$, $\omega_r = 10$, $\omega_b = 15.5323$.

$\xi = x$

$1 < \xi < \infty \rightarrow$ over damped system.

$\xi = 1 \rightarrow$ critically damped.

$0 < \xi < 1 \rightarrow$ under damped

$x = 0 \rightarrow$ undamped system.

Time domain Specifications

Delay time $T_d = (1 + 0.7\xi)/\omega_n$

Rise time $T_r = (\pi - \theta)/\omega_d$

peak overshoot time $= T_p = \pi/\omega_d$

$\% M_p = \exp(-\pi\xi/\mathrm{sqrt}(1-\xi^2))$

Settling time $T_s = 4/\xi\omega_n$ (2% tolerance).

Theoretical calculation:

$\omega_n = 30$; $\xi = 0.5 \Rightarrow \omega_d = \omega_n\sqrt{1-\xi^2} = 25.98$ rad/sec

$$t_d = \frac{1+0.7\xi}{\omega_n} = 0.045 \text{ sec}$$

$$t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \cos^{-1}(\xi)}{25.98} = \frac{\pi - \cos^{-1}(0.5)}{25.98} = 0.080615\text{ec}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{25.98} = 0.12 \text{ sec}$$

$$\% M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100 = e^{\frac{-\pi(0.5)}{\sqrt{1-(0.5)^2}}} \times 100 = 16.30\%$$

$$\text{Settling time } (t_s) = \frac{4}{\xi\omega_n} = \frac{4}{(0.5)30} = 0.266 \text{ sec}$$

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2) +ve feed back system:

Step info } Rise time : 5.8584 , settling time : 10.6547 , over shoot : 0
Peak : 0.9999 , Peak time : 25.9983 , steady state value $\simeq$ 1

→ The response took more than the previous system to settle down. So less stable. But steady state value = 1 (increased). So +ve feedback (unity) makes system less stable and increases the final value (or) DC gain.

3) Closed loop response $G = \frac{1}{(s+1)(s+2)}$, $H = \frac{1}{s+1}$ → pole at −1

−ve feed back:

$$CLTF = \frac{G}{1+GH} = \frac{1/(s+1)(s+2)}{1 + 1/(s+1)^2(s+2)} = \frac{s+1}{s^3+4s^2+5s+3}$$

so $s = -2.47, -0.767 \pm 0.793i$

→ Based on location of poles we can say that the system is stable and as poles have imaginary component the response is oscillatory too. (i.e $\xi = \cos\theta < 1$)
(has overshoot)

Step info:

Rise time : 1.2722 Settling time : 5.0078
Over shoot : 11.8450 Peak : 0.3728 Peak Time : 2.8764

From response

→ Due to addition of pole in feedback path the rise time got improved compared to unity feedback, But system became less stable (taking more settling time), Become oscillatory.

3) +ve feedback:

$$CLTF = \frac{G}{1-GH} = \frac{1/(s+1)(s+2)}{1 - \frac{1}{(s+1)(s+2)(s+1)}}$$

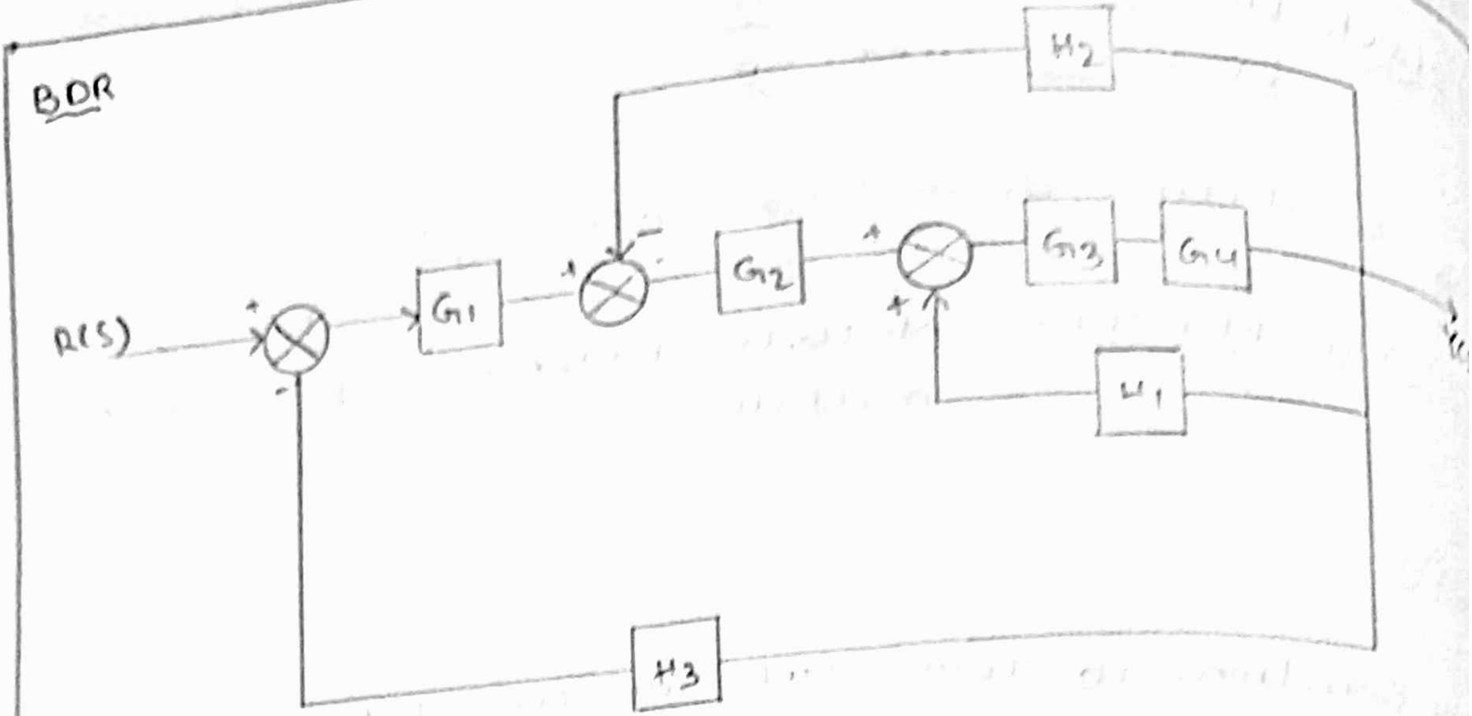
$$= \frac{1}{s^3+4s^2+5s+1}$$

Pole location:

$s = -1.88 \pm j0.745, -0.245$

→ Due to pole near to $j\omega$ axis the system stability is reduced (relatively).

BDR



$$G_1(S) = G_2(S) = G_3(S) = G_4(S) = 1/s$$

$$H_1(S) = H_2(S) = H_3(S) = 1$$

Theoretical calculations:

Step 1: $\frac{1}{s} \cdot \frac{1}{s} = \frac{1}{s^2}$

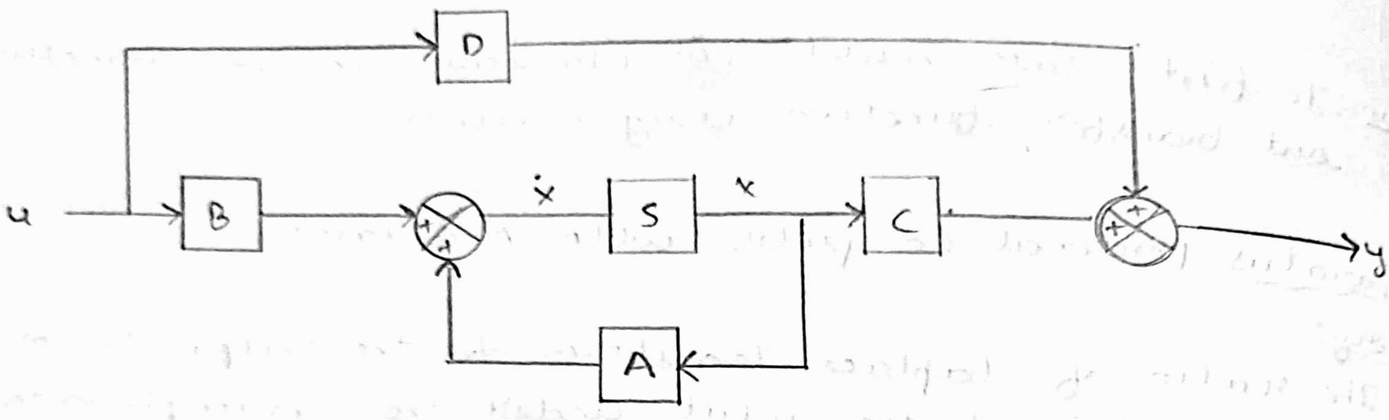
Step 2: $\frac{1/s^2}{1 - \frac{1}{s^2}} = \frac{1}{s^2 - 1}$

Step 3: $\left(\frac{1}{s^2-1}\right)\left(\frac{1}{s}\right) = \frac{1}{s^3 - s}$

Step 4: $\frac{\frac{1}{s(s^2-1)}}{1 + \frac{1}{s(s^2-1)}} = \frac{1}{s^3 - s + 1}$

Step 5: $\left(\frac{1}{s^3 - s + 1}\right)\left(\frac{1}{s}\right) = \frac{1}{s^4 - s^2 + s}$

Step 6: $\frac{\frac{1}{s^4 - s^2 + s}}{1 + \frac{1}{s^4 - s^2 + s}} = \frac{1}{s^4 - s^2 + s + 1}$



where x = State Vector

$\dot{x}$ = differential State Vector

U = input Vector

Y = Output Vector

$[A]_{n \times m}$ = System Matrix

$[B]_{n \times m}$ = Input Matrix

$[C]_{p \times n}$ = Output Matrix

$[D]_{p \times m}$ = feed forward Matrix

(n = order of the System

m = inputs to the System)

P = Outputs to the System

Program 1:-

```
a= [0 2; 1 -4];
b= [0 ; 1];
c= [1 0];
d= [0];
[num, den]= ss2tf[a, b, c, d, 1];
Transfer - function = tf(num,den);
```

Simulation Result;

$$T.F = \frac{2}{(s+2)(s-1)}$$

Theoretical Calculation:-

$$UTF = C[SI - A]^{-1}B + D$$

$$= C[SI - A]^{-1}B + [0]$$

$$= [1\ 0]\begin{bmatrix} s & -2 \\ -1 & s+1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{(s+2)(s+1)}[1\ 0]\begin{bmatrix} s+1 & 2 \\ 1 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{(s+2)(s-1)}[s+1\ 2]\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T.F = \frac{2}{(s+2)(s-1)}$$

Program 2:-

```
n= [0 2];
d= [1 1 -2];
H= tf (n, d1);
[a, b, c, d] = tf2ss (n, d1);
```

Simulation Result:

$$a = \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$c = [0\ 2]$$

$$d = [0]$$

$$T.F = \frac{2}{s^2+s-2}$$

2) $\frac{Y(s)}{U(s)} = \frac{2}{s^2+s-2}$

$$(s^2+s-2)Y(s) = 2U(s)$$

$$\frac{d^2y(t)}{dt} + \frac{dy(t)}{dt} - 2y(t) = 2u(t)$$

Let $y(t) = x_1$; $u(t) = u$

$$\frac{dy(t)}{dt} = \dot{x}_1 = x_2$$

$$\dot{x}_2 + x_2 - 2x_1 = 2u$$

$$\dot{x}_2 = 2x_1 - x_2 + 2u$$

Comparing with

$$\dot{x} = AX + BU$$
$$Y = CX + DU$$

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} ; B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$C = [1\ 0] ; D = [0]$$

Theoretical Calculations:

3) i) $Q_c = [B : AB : A^2B : \cdots : A^{n-1}B]$

$$Q_c = \left[\begin{bmatrix}1\\0\\1\end{bmatrix} : \begin{bmatrix}-3 & -1 & 0\\2 & 0 & 0\\0 & -1 & -1\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix} : \begin{bmatrix}-3 & -1 & 0\\2 & 0 & 0\\0 & -1 & -1\end{bmatrix}\begin{bmatrix}-3 & -1 & 0\\2 & 0 & 0\\0 & -1 & -1\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix}\right]$$

$$Q_c = \begin{bmatrix}1 & -3 & 7\\0 & 2 & -6\\1 & -1 & -1\end{bmatrix} \qquad |Q_c| = -4 \neq 0$$

$\therefore$ Given system is Controllable.

ii) $$Q_b = \begin{bmatrix}C\\CA\\CA^2\end{bmatrix} = \begin{bmatrix}[1\ 0\ 1]\\ [1\ 0\ 1]\,[A]_{3\times3}\\ [1\ 0\ 1]\,[A]_{3\times3}\,[A]_{3\times3}\end{bmatrix} = \begin{bmatrix}1 & 0 & 1\\-3 & -2 & -1\\5 & 4 & 1\end{bmatrix}$$

$|Q_b| = 0$ $\therefore$ Given system is not observable.

Observation Table:

	Theoretical Calculations	Matlab
Program 1	$T.F = \frac{2}{(S+2)(S-1)}$	$T.F = \frac{2}{(S+2)(S-1)}$
Program 2	$A = \begin{bmatrix}0 & 1\\2 & -2\end{bmatrix}$ $B = \begin{bmatrix}0\\2\end{bmatrix}$ $C = [1\ 0]$	$A = \begin{bmatrix}-1 & 2\\1 & 0\end{bmatrix}$ $B = \begin{bmatrix}1\\0\end{bmatrix}$ $C = [0\ 2]$ $D = [0]$
Program 3	$Q_c = \begin{bmatrix}1 & -3 & 7\\0 & 2 & -6\\1 & -1 & -1\end{bmatrix}$ $Q_b = \begin{bmatrix}1 & 0 & 1\\-3 & -2 & -1\\5 & 4 & 1\end{bmatrix}$	$Q_c = \begin{bmatrix}1 & -3 & 7\\0 & 2 & -6\\1 & -1 & -1\end{bmatrix}$ $Q_b = \begin{bmatrix}1 & 0 & 1\\-3 & -2 & -1\\5 & 4 & 2\end{bmatrix}$

Theoretical Calculation:

$$TF = \frac{36}{s^3+6s^2+11s+6} = \frac{36}{(s+1)(s+2)(s+3)}$$

no. of poles = 3 ; No. of zeros = 0 ; poles = $-1, -2, -3$

No. of branches = $P = 3$

No. of asymptotes = $|P-Z| = 3$

angle of asymptotes : $\phi = \frac{(2k+1)\pi}{|P-Z|}$; $k = 0, 1, 2 \ldots (P-Z-1)$

for $k=0$; $\phi_1 = -\frac{\pi}{3} = 60°$

for $k=1$; $\phi_2 = \frac{3}{3} \times \pi = 180°$

$k=2$; $\phi_3 = \frac{5\pi}{3} = 300°$

$$\text{Centroid} = \frac{\Sigma \text{ real part of pole} - \Sigma \text{ real parts of zeros}}{|P-Z|}$$

$$\sigma = \frac{(-1-2-3)-0}{3} = -2$$

root locus exist in b/w $(-2, -1)$ and $(-\infty, -3)$

Break away point : $\frac{dK}{ds} = 0$

$$1 + G(s)\,H(s) = 0$$

$$1 + \frac{K}{s^3+6s^2+11s+6} = 0$$

$$s^3+6s^2+11s+(6+K) = 0$$

$$K = -(s^3+6s^2+11s+6)$$

$$\frac{dK}{ds} = 0 \Rightarrow \frac{d}{ds}\left[-(s^3+6s^2+11s+6)\right] = 0$$

$$3s^2+12s+11 = 0$$

$$s = -1.42, -2.57$$

Break away points always exist in root locus path.

Valid break away point is $s = -1.42$.

Point of intersection with Imaginary axis:

$$1 + G(s)\,H(s) = 0$$

$$s^3 + 6s^2 + 11s + (6+K) = 0$$

s^3	1	11
s^2	6	$6+K$
s^1	$\frac{66-6-K}{6}$	0
s^0	$6+K$	0

From row s^1

$$\frac{60-K}{6} = 0$$

$$K = 60$$

From row s^2:

$$6s^2 + (6+K) = 0$$

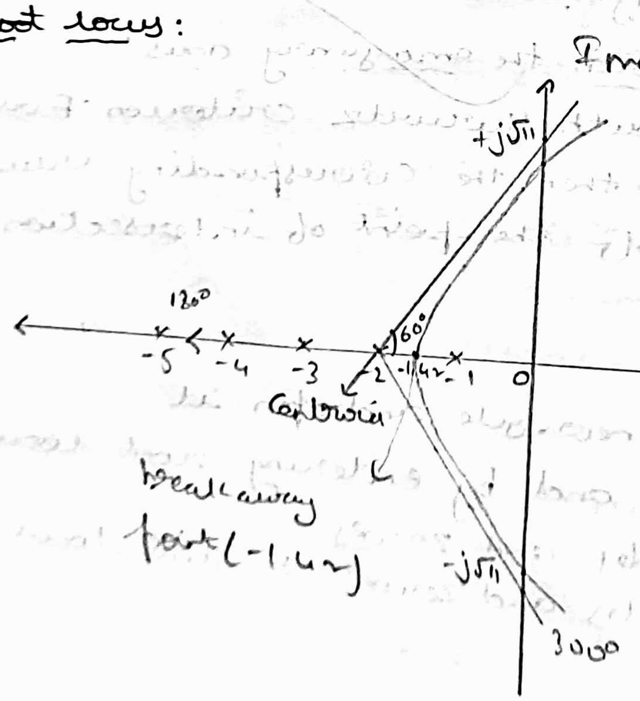
$$6s^2 + 66 = 0$$

$$s^2 = -11$$

$$s = \pm j\,3.316 = \pm j\sqrt{11}$$

Root locus intersects the Imaginary axis at $\pm j\sqrt{11}$

Root locus:



Theoretical Calculations:

Bode plot:

$$TF = \frac{36}{s^3+6s^2+11s+6} = \frac{36}{(s+1)(s+2)(s+3)} = \frac{6}{(1+s)(1+\frac{s}{2})(1+\frac{s}{3})}$$

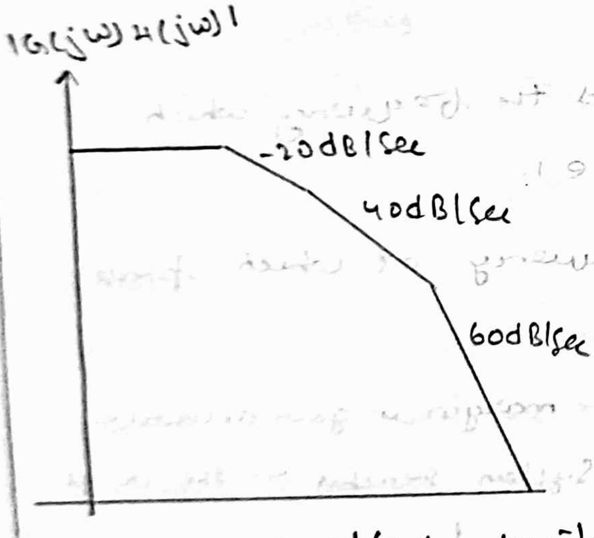
Corner frequency : $\omega_1 = 1$ rad/sec

$\omega_2 = 2$ rad/sec

$\omega_3 = 3$ rad/sec

Term	Corner frequency	Slope
$\frac{1}{1+s}$	1	-20 dB/sec upto 2.
$\frac{1}{1+s/2}$	2	-40 dB/sec
$\frac{1}{1+s/3}$	3	-60 dB/sec

slope = 0 upto 1.



$$\omega_{gco} = \left| \frac{36}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}\sqrt{9+\omega^2}} \right| = 1$$

$\omega_{gco} = \sqrt{6.7} = 2.58$ rad/sec

$PM = 180 - \tan^{-1}(\omega) - \tan^{-1}(\omega/2) - \tan^{-1}(\frac{\omega}{3})$

$PM = 19.09°$

$\omega_{pco} : -\tan^{-1}(\omega) - \tan^{-1}(\frac{\omega}{2}) - \tan^{-1}(\frac{\omega}{3}) = -180$; $\omega = \omega_{gco}$

$\omega_{pco} = 3.31$ rad/sec

$$\text{Gain Margin} = \left| \frac{36}{\sqrt{12}\sqrt{15}\sqrt{20}} \right|^{-1} = 4.436\,dB$$

Nyquist plot:

$$TF = \frac{36}{(s+1)(s+2)(s+3)}$$

$$T(j\omega) = \frac{36}{(1+j\omega)(2+j\omega)(3+j\omega)}$$

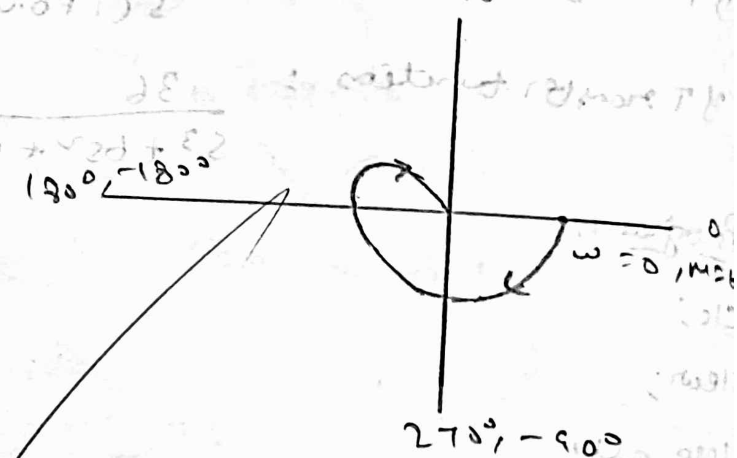
$$M = |G(j\omega)H(j\omega)| = \left| \frac{36}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}\sqrt{9+\omega^2}} \right|;$$

$$\phi = \angle G(j\omega)H(j\omega) = -\tan^{-1}(\omega) - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/3)$$

C_1: polar plot:

at $\omega = 0$; $M = 6$, $\phi = 0°$

at $\omega = \infty$; $M = 0$; $\phi = -270°$



pole dominate ∴ clockwise direction

starting

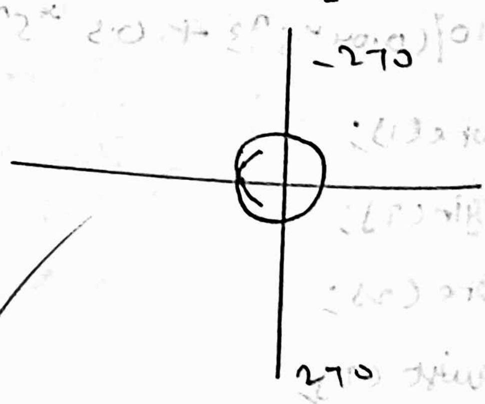
end direction $= -270 - 0 = -270 = -ve$

↳ c.w

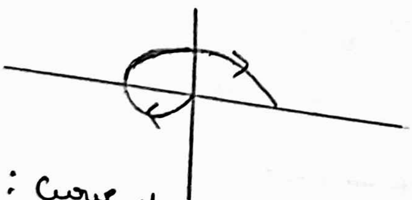
C_2: Curve of zero magnitude from $-(n-m)\frac{\pi}{2}$ to $(n-m)\frac{\pi}{2}$

$n - m = 3 - 0 = 3$

∴ $-\frac{3\pi}{2}$ to $\frac{3\pi}{2} = -270$ to $270°$

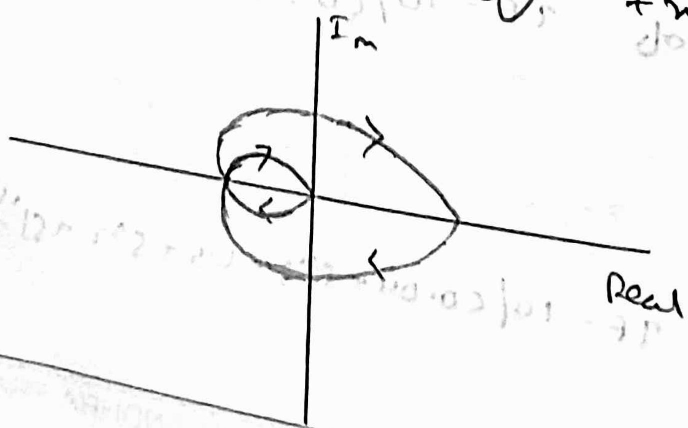


C_3: mirror polar plot



C_4: Curve of infinite magnitude from $+n\frac{\pi}{2}$ to $-n\frac{\pi}{2}$

Nyquist plot:



Theoretical calculations:-

1. Lead compensator:-

$G_f(s) = \frac{k}{s(s+1)}$, Given $k_v = 10s^{-1}$, $\phi_m = 35°$

Taking $k = 10$, $G_f(j\omega) = \frac{10}{j\omega(1+j\omega)}$

phase margin of uncompensated system $16°$

for lead compensator, phase lead required at new Gf is given by $\phi_l = 35 - 16 + 5 = 24$

magnitude contribution $= 10\log\left(\frac{1}{\alpha}\right) = -3.75\ dB$

i.e., frequency at which uncompensated system has magnitude of $-3.75\ dB$

ω_{g_2} becomes ω_m; $\omega_{g_2} = 4$ rad/sec

$\omega_1 = \frac{1}{T} = \omega_m\sqrt{\alpha} = 4\sqrt{0.4217} = 6.16$ rad/sec

$T.F = G(s) = \frac{1+sT}{1+\alpha sT} = \frac{1+\left(\frac{1}{2.6}\right)s}{1+\left(\frac{1}{6.16}\right)s} = \frac{1+0.385s}{1+0.162s}$

$A = \frac{1}{\alpha} = \frac{1}{0.4217} = 2.37$

$G(s) = G_f(s)\ G_c(s) = \frac{10(1+0.385s)}{s(1+s)(1+0.162s)}$

$\omega_{g_2} = 4$ rad/sec

$PM = 38°$

2. Lag compensator:

$G_f(s) = \frac{k}{s(1+s)(5+s)}$, $\xi = 0.5$, $t_s = 12sec$, $k_v \geq 8s^{-1}$

$t_s = \frac{3}{\xi\omega_n} \Rightarrow \omega_n = 0.83$ rad/sec

$\omega_b = \omega_n\sqrt{1-2\xi^2+\sqrt{2-4\xi^2+4\xi^4}} = 1.217$ rad/sec

$\psi_{PM} = \tan^{-1}\left(\frac{2\xi}{\sqrt{1+4\xi^2}-2\xi^2}\right)^{1/2} = 33°$

$G_f(s) = \frac{k/5}{s(1+s)(1+0.2s)}$

$k_v = \lim_{s\to 0} s\,G_f(s) = \frac{k}{5}(5) \Rightarrow k_v = 8\times5 = 40$

$$G_f(j\omega) = \frac{8}{(j\omega)(j\omega+1)(1+0.2j\omega)} \Rightarrow \omega_{gc} = 2.7 \text{ rad/sec}$$

$PM = 5^\circ$

$\psi_2 = 33^\circ + 10^\circ = 43^\circ$, $\omega_2 = \frac{1}{T} = \frac{\omega_{g2}}{2^2} = 0.2$ rad/sec

To bring lag magnitude curve to zero dB, at which ω_{g2} attenuation must be 18dB

$$20\log B = 18 \Rightarrow B = 8$$

$$\omega_1 = \frac{1}{BT} = \frac{1}{8\times 5} = 0.025 \text{ rad/sec}, \quad BT = 40$$

$$G(s) = \frac{1}{8}\left[\frac{(s+0.2)}{(s+0.025)}\right] = \frac{1+5s}{1+40s}$$

phase lag $= \tan^{-1}(5\omega_{g2}) - \tan^{-1}(40\omega_{g2}) = 75.96 - 88 = -12^\circ$

$$G(s) = G_f(s) \cdot G_c(s) = \frac{8(1+5s)}{s(1+s)(1+0.2s)(1+40s)}$$

3 Lag lead compensator:

$$G_f(s) = \frac{K}{s(1+0.5s)(1+0.1s)}; \quad K_v \geq 25s^{-1}, \ \psi_s \geq 60^\circ, \ \omega_b = 10 \text{ rad/sec}$$

$K_v = \lim_{s\to 0} sG(s) = K$, $K_v = 25 \Rightarrow G_f(j\omega) = \frac{25}{j\omega(1+0.5j\omega)(1+0.1j\omega)}$

Let G lag compensator provide $PM = 36^\circ$, so new crossover frequency is 2 rad/sec

For this to happen magnitude of bode plot must be 18dB

$$20\log\beta = 18 \Rightarrow \beta = 8$$

$$\omega_2 = \frac{1}{T} = \frac{G_{cf}}{2^2} = 0.5, \quad T = 2\text{sec}$$

$$\omega_1 = \frac{1}{\beta T} = 0.0625 \text{ rad/sec}, \quad \beta T = 16$$

$$G_c(s) = \frac{1+Ts}{1+\beta Ts} = \frac{25(1+2s)}{s(1+0.5s)(1+0.1s)(1+16s)}$$

$PM = 15^\circ$, $G_{cf} = 2.25$ rad/sec

$$\alpha = \frac{1}{\beta} = \frac{1}{8} = 0.125, \quad \psi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right) = 51.05^\circ$$

$$-10\log\left(\frac{1}{\alpha}\right) = -9\text{dB}$$

$$\omega_1 = \frac{1}{T_2} = \omega_m\sqrt{\alpha} = 3.8\sqrt{0.125} = 1.34 \text{ rad/sec}$$