exercise

June 6, 2024

[ ]:
```
import numpy as np
from numpy.linalg import inv
import matplotlib.pyplot as plt
```

1 Excercise 11

$$y = \begin{bmatrix} 3.5 \\ 0.5 \\ 3.7 \\ 2.8 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 1 & 3 \\ 1 & 1 \end{bmatrix}$$

$$W = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

[ ]:
```
y = np.array([[3.5],[0.5],[3.7],[2.8]])
A = np.array([[1,2],[1,0],[1,3],[1,1]])
W = 3 * np.identity(4)

AT = np.transpose(A)
AT_W_A = np.matmul(AT,np.matmul(W,A))
inv_AT_W_A = inv(AT_W_A)
AT_W_y = np.matmul(AT,np.matmul(W,y))

x_hat = np.matmul(inv_AT_W_A,AT_W_y)
```

$$\hat{x} = \begin{bmatrix} 1.08 \\ 1.03 \end{bmatrix}$$

[ ]:
```
y_hat = np.matmul(A,x_hat)
e_hat = y - y_hat
```

$$\hat{e} = \begin{bmatrix} 0.36 \\ -0.58 \\ -0.47 \\ 0.69 \end{bmatrix}$$

$$W = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

[ ]:
```
W = 5 * np.identity(4)

AT = np.transpose(A)
AT_W_A = np.matmul(AT,np.matmul(W,A))
inv_AT_W_A = inv(AT_W_A)
AT_W_y = np.matmul(AT,np.matmul(W,y))

x_hat = np.matmul(inv_AT_W_A,AT_W_y)
y_hat = np.matmul(A,x_hat)
e_hat = y - y_hat
```

$$\hat{e} = \begin{bmatrix} 0.36 \\ -0.58 \\ -0.47 \\ 0.69 \end{bmatrix}$$

$$W = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

2 Excercise 12

$h(t) = a\cos\omega t + b\sin\omega t$

$x = \begin{bmatrix} a & b \end{bmatrix}^T$

$h = \begin{bmatrix} -3 & -16 & 6 & 9 & -8 \end{bmatrix}^T$

$t = \begin{bmatrix} -0.4 & -0.2 & 0.0 & 0.2 & 0.4 \end{bmatrix}^T$

[ ]:
```
h = np.array([[-3],[-16],[6],[9],[-8]])
t = np.array([[-0.4],[-0.2],[0.0],[0.2],[0.4]])
omega = 10

Acol1 = np.cos(omega * t)
Acol2 = np.sin(omega * t)

A = np.column_stack((Acol1,Acol2))
```

2.1 12-a

$$A = \begin{bmatrix} -0.65 & 0.76 \\ -0.42 & -0.91 \\ 1 & 0 \\ -0.42 & 0.91 \\ -0.65 & -0.76 \end{bmatrix}$$

[ ]:
```
AT = np.transpose(A)
AT_A = np.matmul(AT,A)
inv_AT_A = inv(AT_A)
AT_h = np.matmul(AT,h)

x_hat = np.matmul(inv_AT_A,AT_h)
```

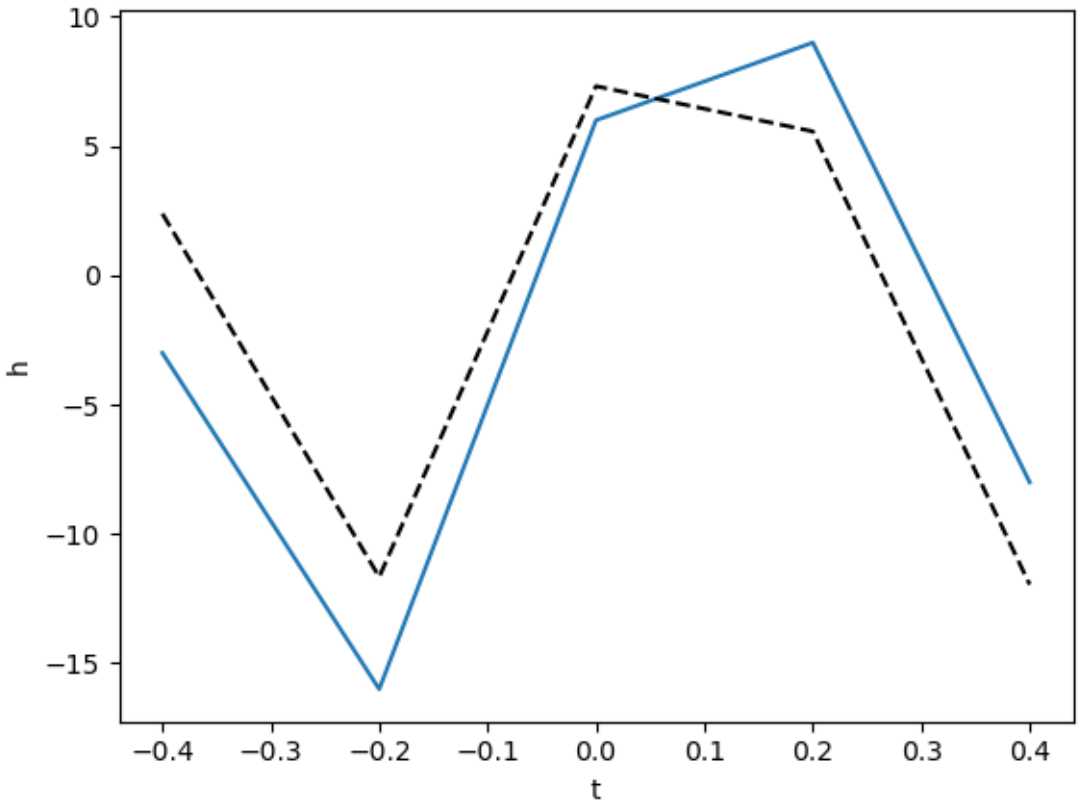
$\hat{x} = \begin{bmatrix} 7.32 & 9.47 \end{bmatrix}^T$

[ ]:
```
h_hat = np.matmul(A,x_hat)
```

$\hat{h} = \begin{bmatrix} 2.39 & -11.66 & 7.32 & 5.57 & -11.95 \end{bmatrix}^T$

[ ]:
```
fig, ax = plt.subplots()
ax.plot(t,h, label="raw")
ax.plot(t,h_hat, "k--", label="fitted")
ax.set_xlabel("t")
ax.set_ylabel("h")
```

[ ]:
```
Text(0, 0.5, 'h')
```


10
5
0
−5
−10
−15
h
−0.4
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
0.4
t