assgn2

June 6, 2024

[ ]:
```
import numpy as np
from numpy.linalg import inv
```

1 Question 1

1.1 Rectangular model matrix

$$y = \begin{bmatrix} 5 \\ 0 \\ 6 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A_{32} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 5 & 1 \end{bmatrix}$$

[ ]:
```
y = np.array([[5],[0],[6]])
A32 = np.array([[1,2],[2,-1],[5,1]])
```

$$A_{32}^T = \begin{bmatrix} 1 & 2 & 5 \\ 2 & -1 & 1 \end{bmatrix}$$

[ ]:
```
A32T = np.transpose(A32)
A32T_A32 = np.matmul(A32T,A32)
inv_A32T_A32 = inv(A32T_A32)
```

$$(A_{32}^T A_{32})^{-1} = \begin{bmatrix} 0.039 & -0.032 \\ -0.032 & 0.194 \end{bmatrix}$$

[ ]:
```
A32T_y = np.matmul(A32T,y)
```

$$A_{32}^T y = \begin{bmatrix} 35 \\ 16 \end{bmatrix}$$

[ ]:
```
x_hat = np.matmul(inv_A32T_A32,A32T_y)
```

$$\hat{x} = \begin{bmatrix} 0.84 \\ 1.97 \end{bmatrix}$$

1.2 Square model matrix

$$y = \begin{bmatrix} 8 \\ 3 \\ -1 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A_{33} = \begin{bmatrix} 2 & 4 & -3 \\ 1 & 1 & -1 \\ 2 & -2 & 3 \end{bmatrix}$$

```
[ ]: y = np.array([[8],[3],[-1]])
     A33 = np.array([[2,4,-3],[1,1,-1],[2,-2,3]])
     A33T = np.transpose(A33)
     inv_A33T_A33 = inv(np.matmul(A33T,A33))
```

$$(A_{33}^T A_{33})^{-1} = \begin{bmatrix} 1.06 & -2.11 & -2.06 \\ -2.11 & 4.72 & 4.61 \\ -2.06 & 4.61 & 4.56 \end{bmatrix}$$

```
[ ]: A33T_y = np.matmul(A33T,y)
```

$$A_{33}^T y = \begin{bmatrix} 17 \\ 37 \\ -30 \end{bmatrix}$$

```
[ ]: x_hat = np.matmul(inv_A33T_A33,A33T_y)
```

$$\hat{x} = \begin{bmatrix} 1.5 \\ 0.5 \\ -1 \end{bmatrix}$$

2 Question 2

$$y = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A_{32} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 0 & 0 \end{bmatrix}$$

```
[ ]: y = np.array([[4],[5],[6]])
     A32 = np.array([[1,2],[1,3],[0,0]])
     A32T = np.transpose(A32)
```

```
A32T_A32 = np.matmul(A32T,A32)
inv_A32T_A32 = inv(A32T_A32)
```

$$(A_{32}^T A_{32})^{-1} = \begin{bmatrix} 13 & -5 \\ -5 & 2 \end{bmatrix}$$

[ ]:
```
A32T_y = np.matmul(A32T,y)
```

$$A_{32}^T y = \begin{bmatrix} 9 \\ 23 \end{bmatrix}$$

[ ]:
```
x_hat = np.matmul(inv_A32T_A32,A32T_y)
```

$$\hat{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

[ ]:
```
y_hat = np.matmul(A32,x_hat)
```

$$\hat{y} = \begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix}$$

3 Question 3

$$y = \begin{bmatrix} 4 \\ 5 \\ 9 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A_{32} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

[ ]:
```
y = np.array([[4],[5],[9]])
A32 = np.array([[1,-1],[1,0],[1,1]])
A32T = np.transpose(A32)
A32T_A32 = np.matmul(A32T,A32)
inv_A32T_A32 = inv(A32T_A32)
```

$$(A_{32}^T A_{32})^{-1} = \begin{bmatrix} 0.33 & 0 \\ 0 & 0.5 \end{bmatrix}$$

[ ]:
```
A32T_y = np.matmul(A32T,y)
```

$$A_{32}^T y = \begin{bmatrix} 18 \\ 5 \end{bmatrix}$$

[ ]:
```
x_hat = np.matmul(inv_A32T_A32,A32T_y)
```

$$\hat{x} = \begin{bmatrix} 6 \\ 2.5 \end{bmatrix}$$

[ ]:
```
y_hat = np.matmul(A32,x_hat)
```

$$\hat{y} = \begin{bmatrix} 3.5 \\ 6 \\ 8.5 \end{bmatrix}$$

[ ]:
```
e_hat = y - y_hat
```

$$\hat{e} = \begin{bmatrix} 0.5 \\ -1 \\ 0.5 \end{bmatrix}$$

4 Question 4

$$y = \begin{bmatrix} 2 \\ 0 \\ -3 \\ -6 \end{bmatrix}$$

$$x = \begin{bmatrix} x_0 \\ v \\ a \end{bmatrix}$$

$$A_{43} = \begin{bmatrix} 1 & -1 & 0.5 \\ 1 & 0 & 0 \\ 1 & 1 & 0.5 \\ 1 & 2 & 2 \end{bmatrix}$$

[ ]:
```
y = np.array([[2],[0],[-3],[-6]])
A43 = np.array([[1,-1,0.5],[1,0,0],[1,1,0.5],[1,2,2]])
A43T = np.transpose(A43)
inv_A43T_A43 = inv(np.matmul(A43T,A43))
```

$$(A_{43}^T A_{43})^{-1} = \begin{bmatrix} 0.55 & 0.15 & -0.5 \\ 0.15 & 0.45 & -0.5 \\ -0.5 & -0.5 & 1 \end{bmatrix}$$

[ ]:
```
A43T_y = np.matmul(A43T,y)
```

$$A_{43}^T y = \begin{bmatrix} -7 \\ -17 \\ -12.5 \end{bmatrix}$$

[ ]:
```
x_hat = np.matmul(inv_A43T_A43,A43T_y)
```

$$\hat{x} = \begin{bmatrix} -0.15 \\ -2.45 \\ -0.5 \end{bmatrix}$$

$v = -2.45$

$a = -0.5$

5 Question 5

System of equations

$x = y_1$

$x = y_2$

$\vdots$

$x = y_m$

5.1 5-a

$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} x$

$A_{m1} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$

$A_{m1}^T y = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$

$A_{m1}^T y = y_1 + y_2 + \cdots + y_m$

$A_{m1}^T A_{m1} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$

$A_{m1}^T A_{m1} = 1 + 1 + \cdots + 1 = m$

$\hat{x} = (A_{m1}^T A_{m1})^{-1} A_{m1}^T y$

$$\hat{x} = \frac{y_1 + y_2 + \cdots + y_m}{m}$$