



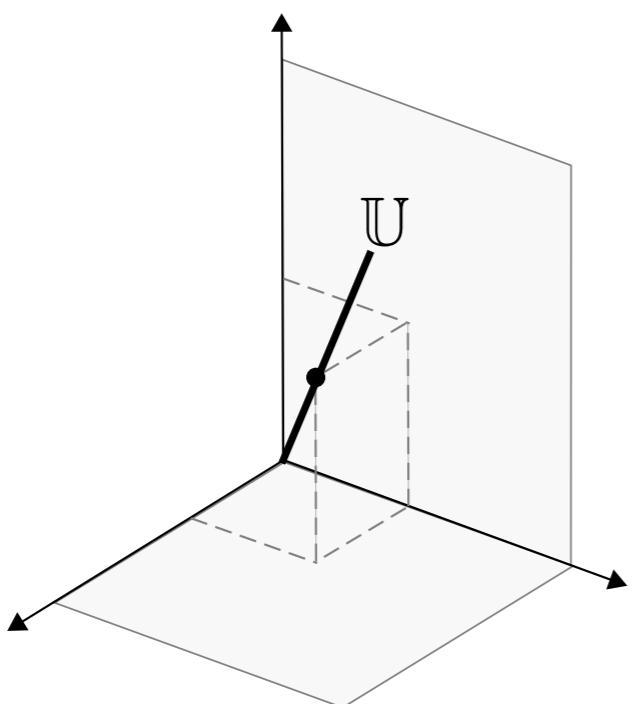
*Karan Srivastava*  
*The person you're looking at*



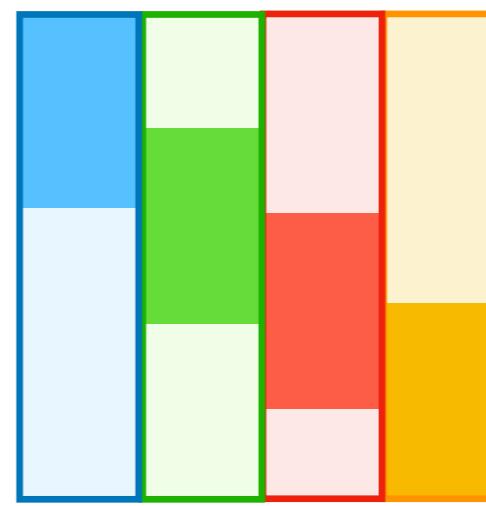
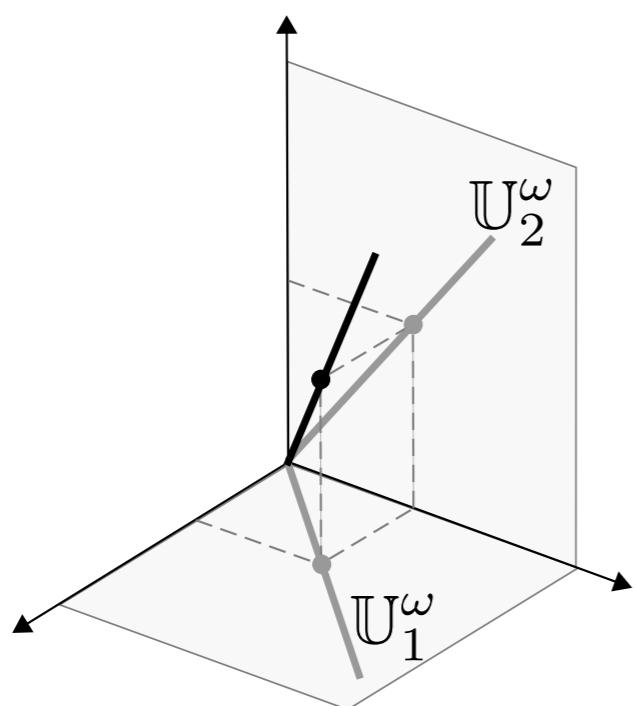
*Daniel Pimentel-Alarcón*  
*Not the person you're looking at*

Subspace Reconstruction

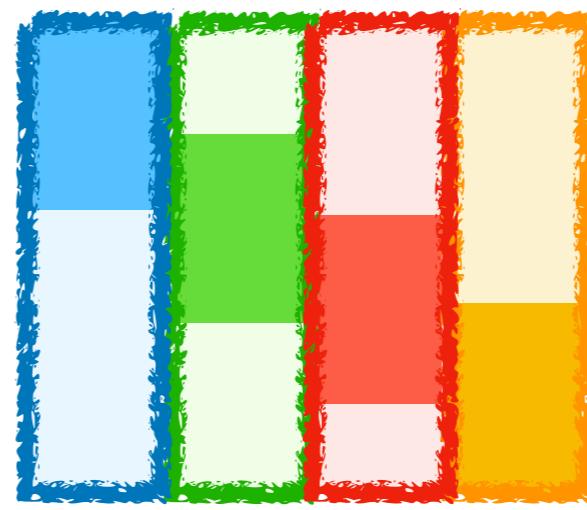
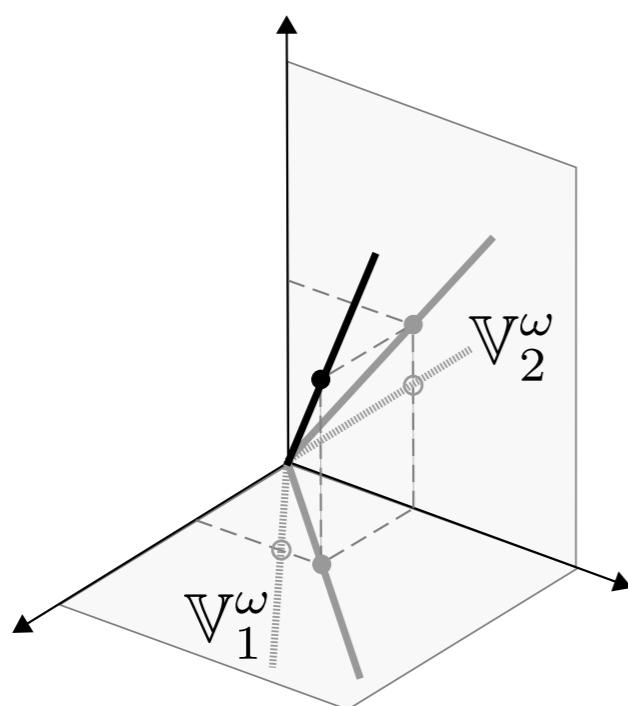
# Problem Description



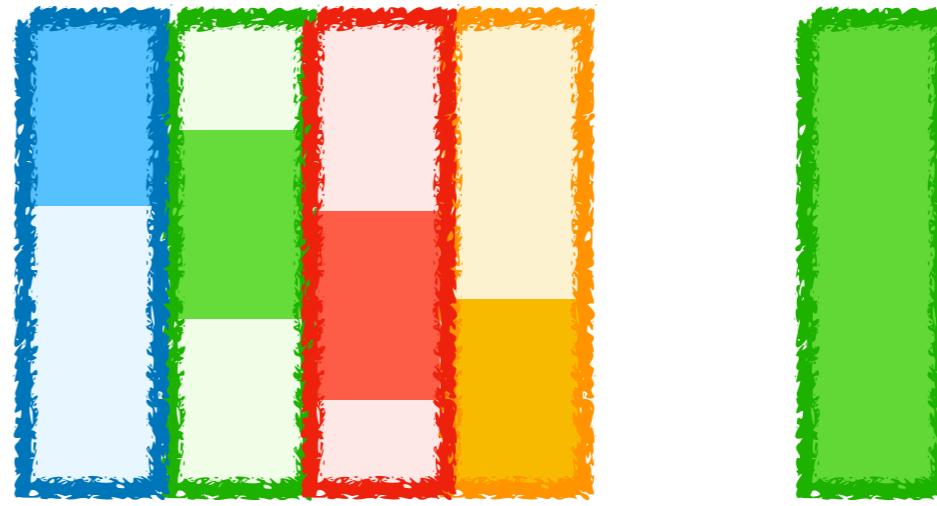
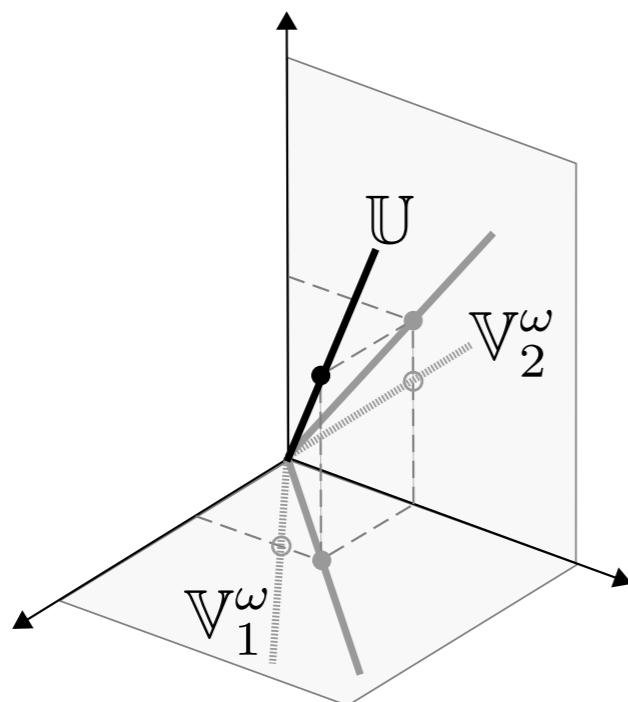
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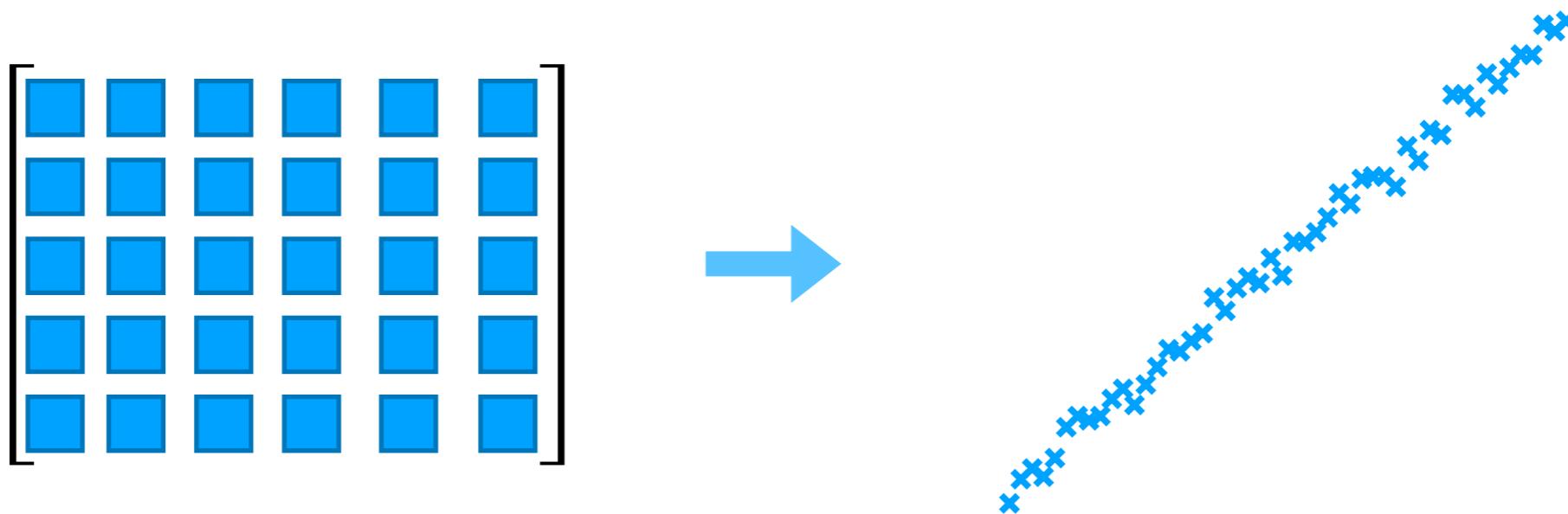
# Problem Description



Goal: To estimate the line (or linear shape)  
from the noisy pieces and bound the error?

# Motivation

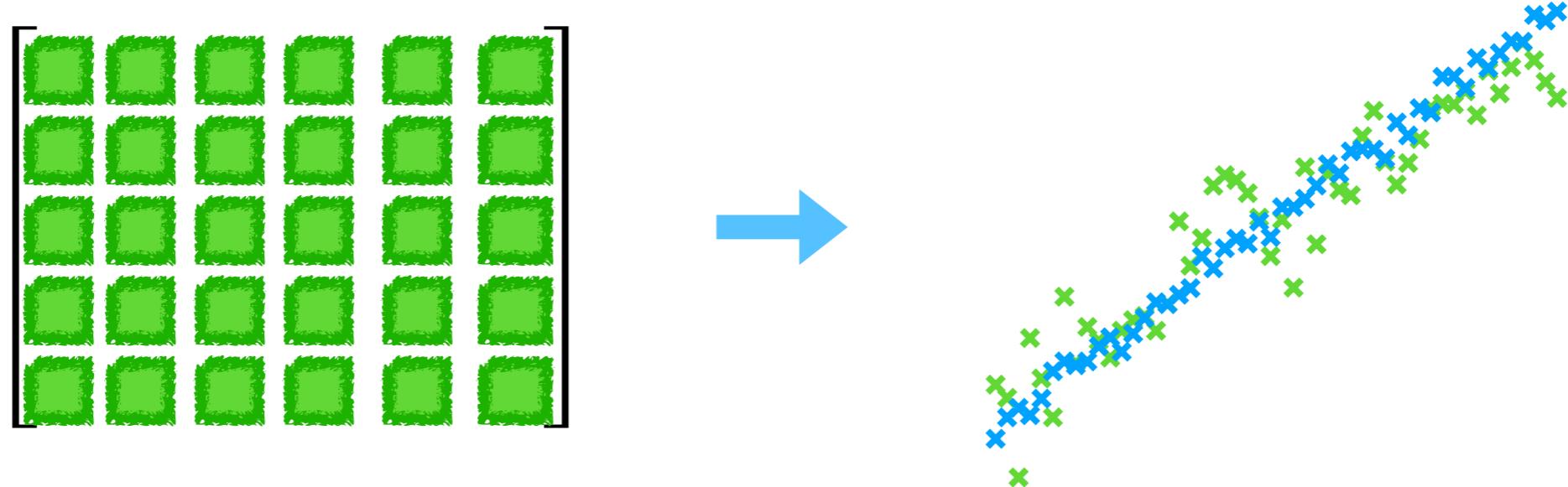
Our main tool for modelling data is linear algebra



Since data is often best modelled by **subspaces**

# Motivation

Our main tool for modelling data is linear algebra



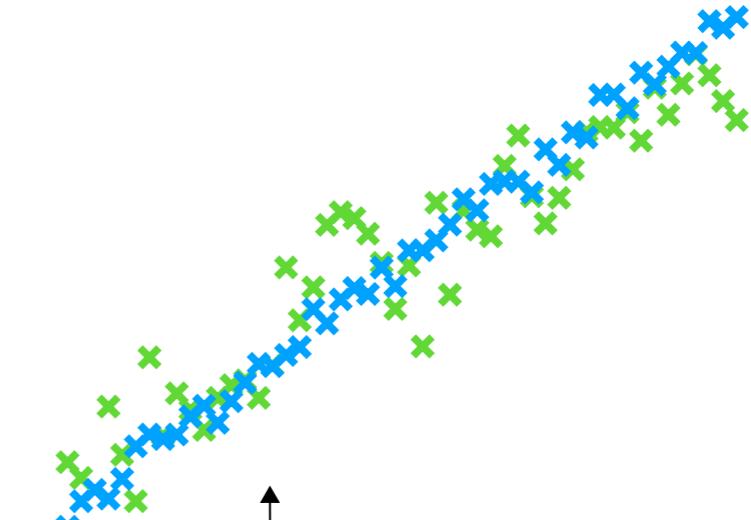
Since data is often best modelled by **subspaces**

But data is often **noisy**

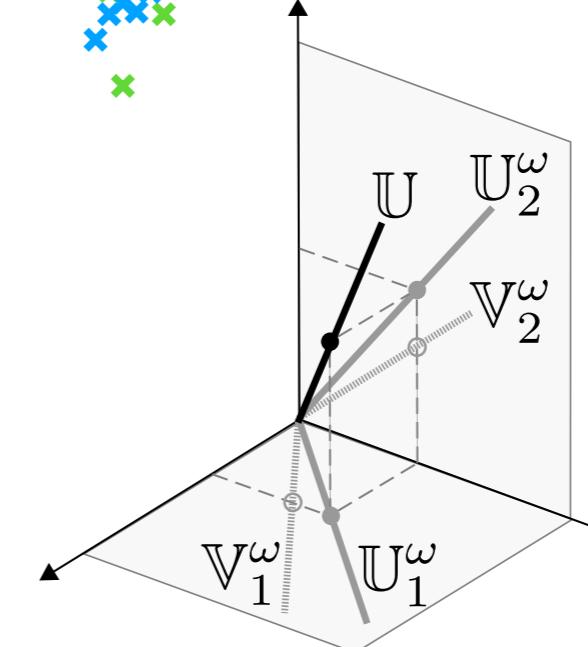
# Motivation

Our main tool for modelling data is linear algebra

$$\begin{bmatrix} \text{green squares} \\ \text{green squares} \\ \text{green squares} \\ \text{green squares} \\ \text{green squares} \end{bmatrix}$$

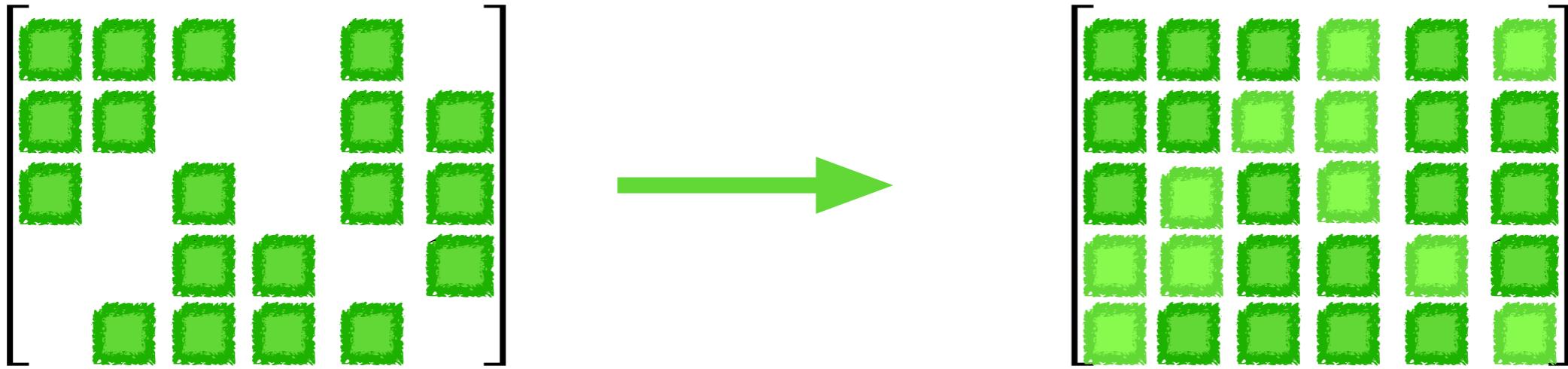


$$\begin{bmatrix} \text{green squares} & \text{green squares} & \text{green squares} \\ \text{green squares} & \text{green squares} & \text{green squares} \\ \text{green squares} & \text{green squares} & \text{green squares} \\ \text{green squares} & \text{green squares} & \text{green squares} \end{bmatrix}$$



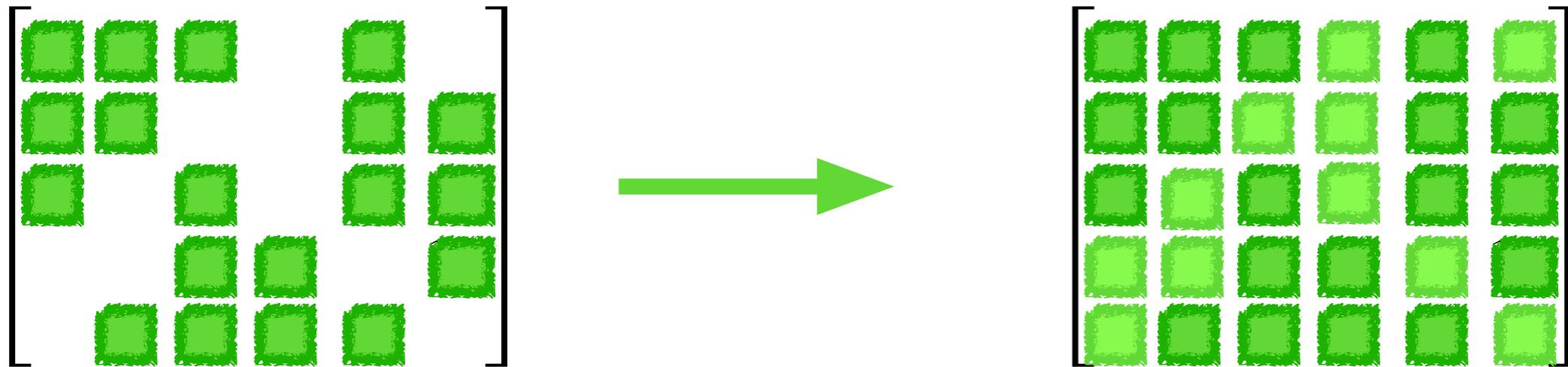
Since data is often best modelled by **subspaces**  
But data is often **noisy** and **missing**

# Motivation

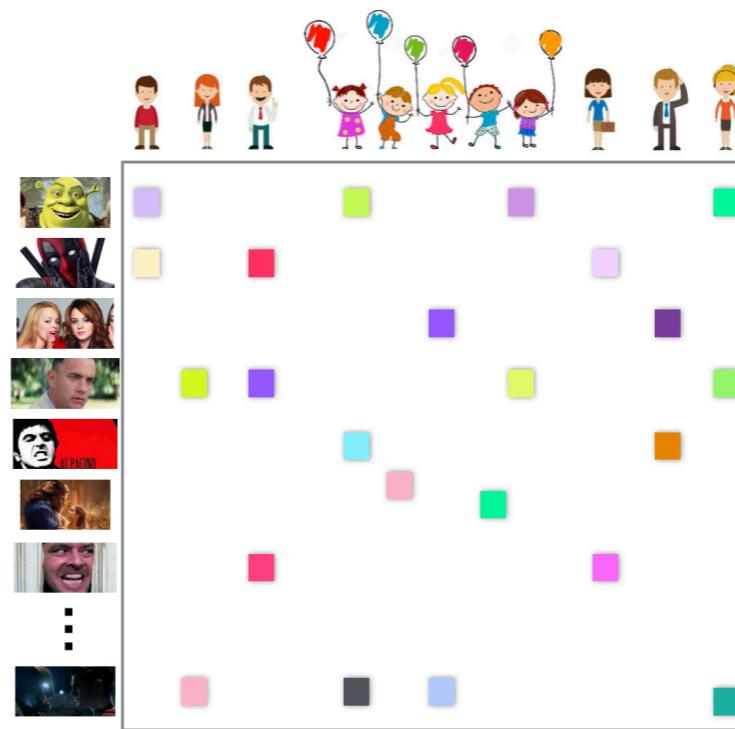
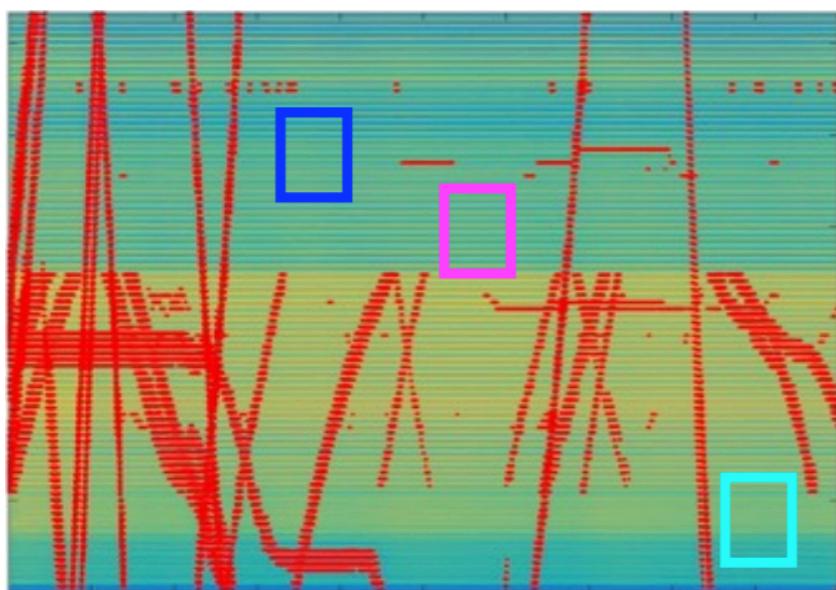


The problem of completing matrices is  
Matrix Completion

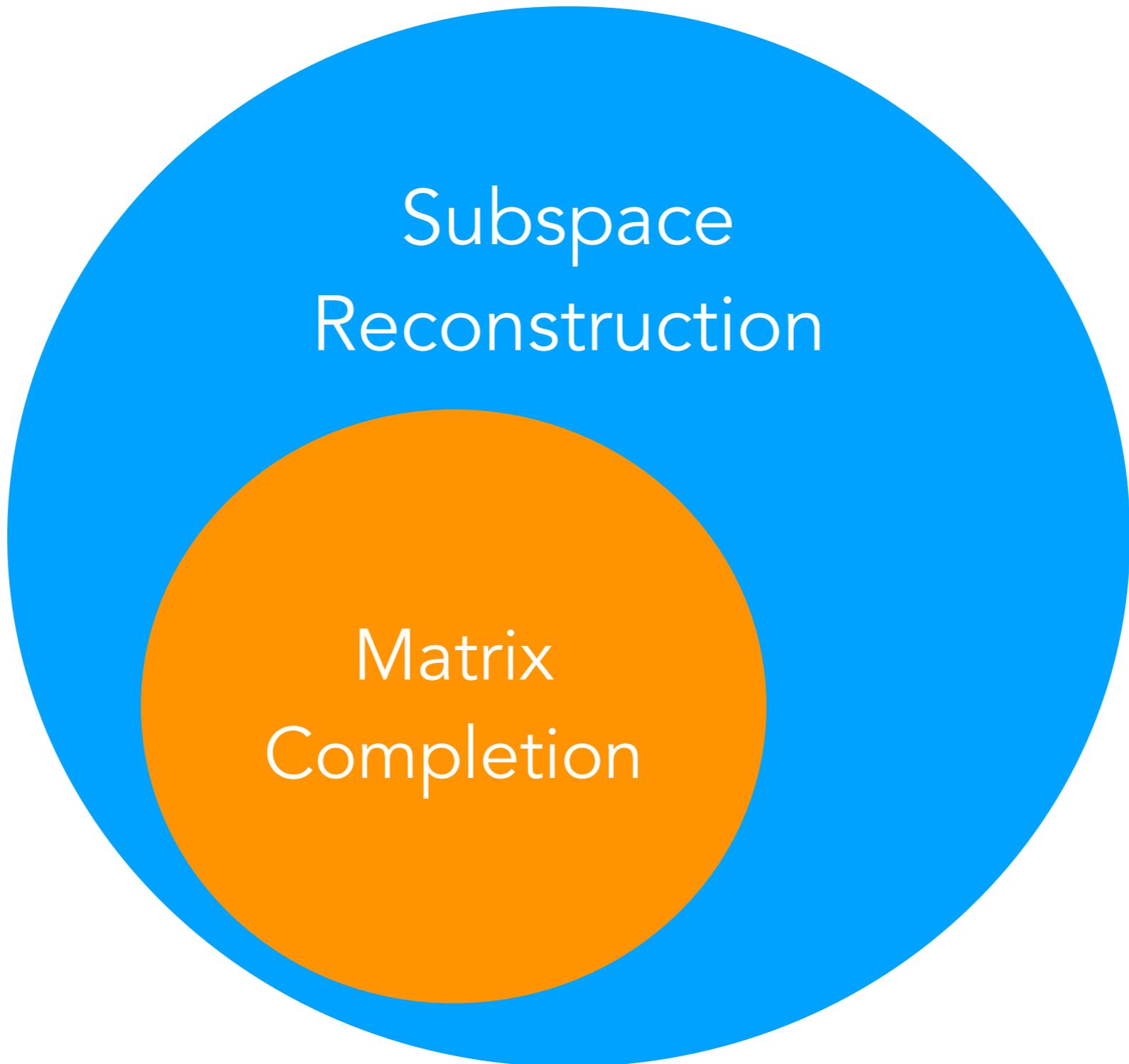
# Motivation



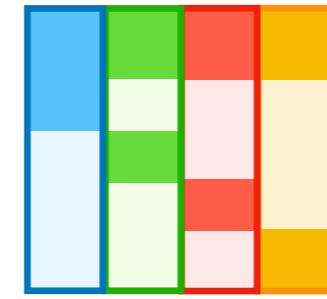
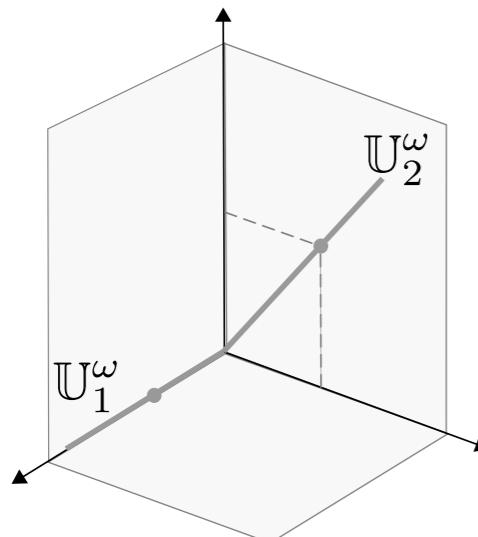
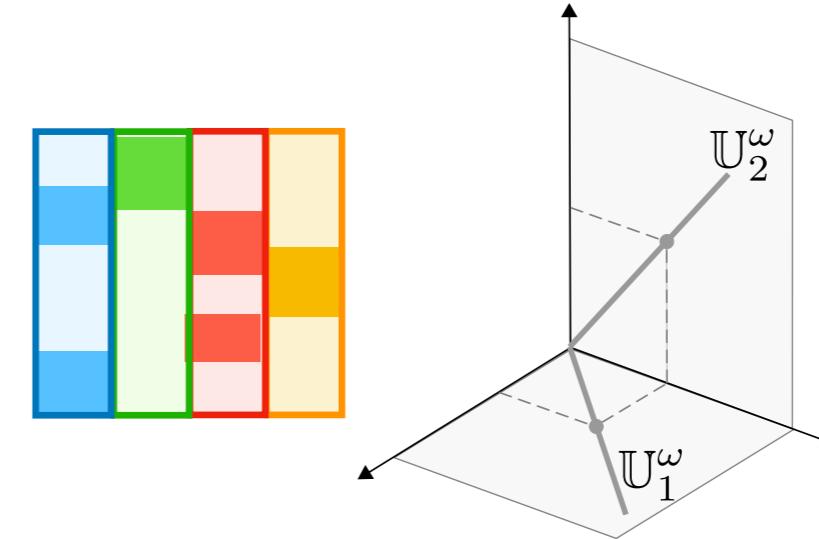
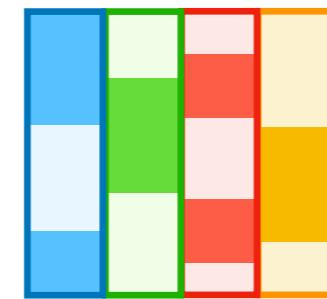
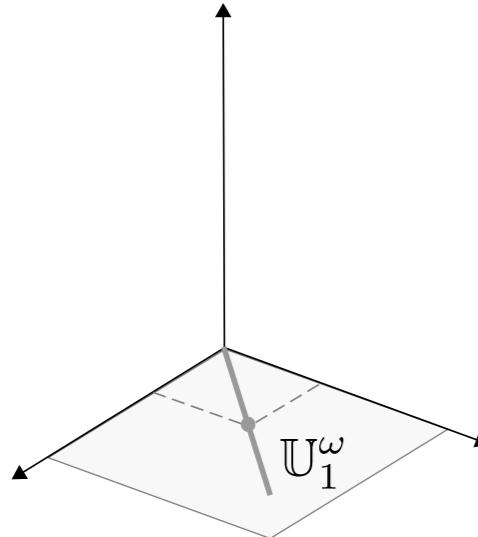
The problem of completing matrices is  
Matrix Completion



# Motivation



## Previous Work - Noiseless Case



Question: Which projections do we need to observe for a **unique** reconstruction?

D. L. Pimentel-Alarcón, N. Boston, and R. D. Nowak, “Deterministic conditions for subspace identifiability from incomplete sampling,” in *Information Theory (ISIT), 2015 IEEE International Symposium on*. IEEE, 2015, pp. 2191–2195.

## Our Work - Noisy Case

Theorem (S., P.-A.)

For almost every  $\mathbb{U}$ ,

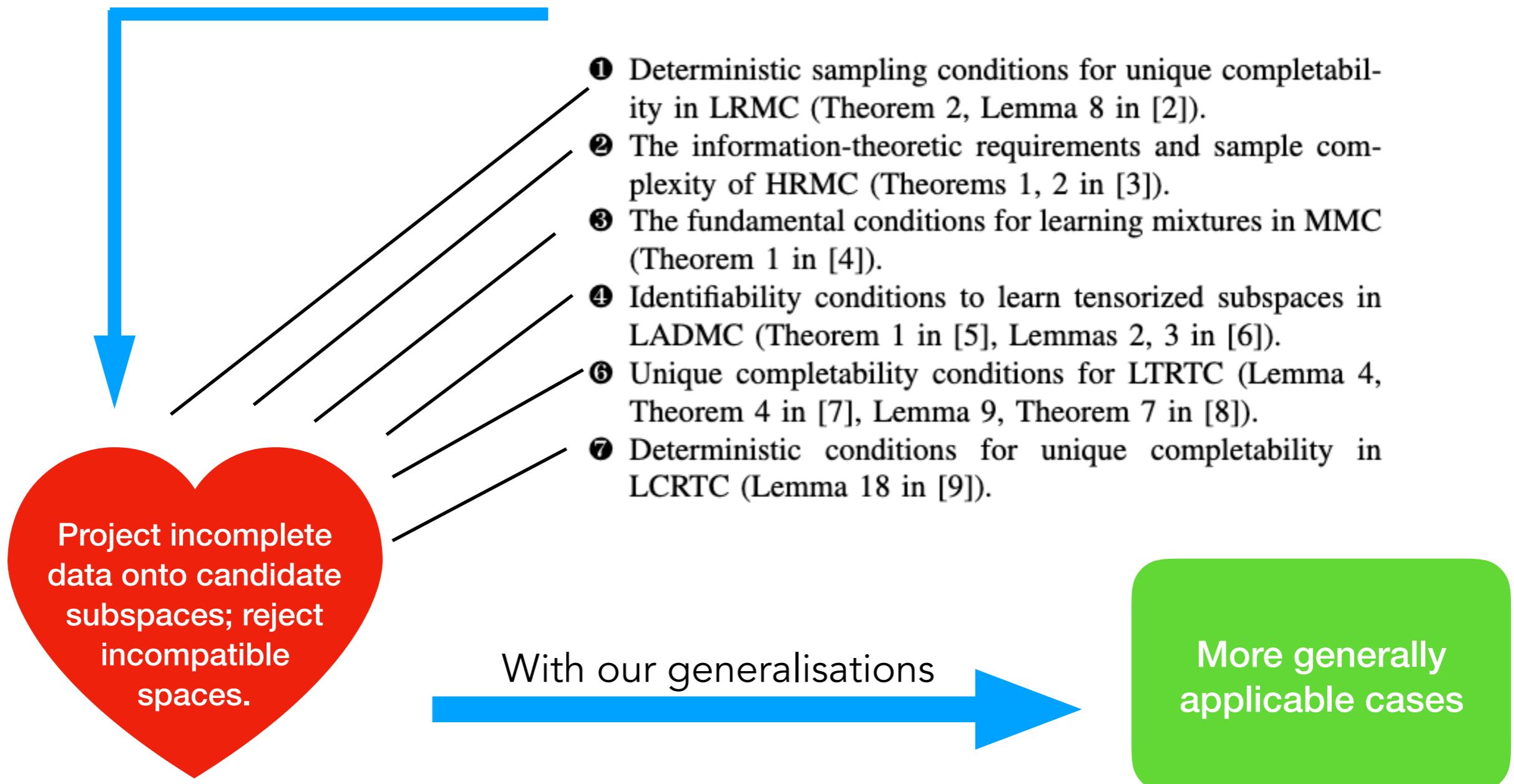
$$d_G(\mathbb{U}, \hat{\mathbb{U}}) \leq \frac{\epsilon \sqrt{2r(d-r)}}{\delta \cdot \sigma(B)}$$

Measure of how good  
the guess is

The cool bound we found

# Applications - LRMC Theory

## Noiseless Theory



# Applications - RPCA

**Original Frame**



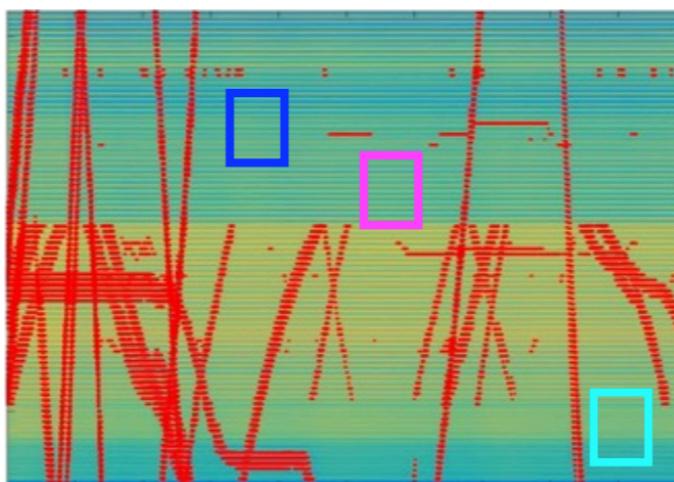
**Subspace Reconstruction  
Based Background Segmentation**



**RPCA-ALM**  
(Lin et.al 2011-206)



D. Pimentel-Alarcón and R. Nowak, “Random consensus robust pca,”  
*Electronic Journal of Statistics*, vol. 11, no. 2, pp. 5232–5253, 2017.



## Future Directions

- Line -> Curved Shapes - Can we do this in a computationally feasible way?
- How to find these projections - Can we find an algorithm to find projections given sparse data?
- Can we generalize these bounds to cases where we have multiple subspaces?

THANKS A BUNCH!



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