R Outputs
related to PCA for n < p

Dimension of the Data

```
> View(X)
> dim(X)
[1]   64 6830
```

Dimension of the Variance-Covariance Matrix

```
> c = cov(X)
> dim(c)
[1] 6830 6830
```

Eigenvalues of Var-Cov Matrix

```
> e = eigen(c)
>
> round(e$values,4)
   [1] 633.2156 352.9278 279.9189 183.0830 163.5573 149.0968
   [7] 122.2882 119.7912 112.1777  91.7108  89.2895  85.1300
  [13]  77.8269  75.1452  71.0961  68.4686  67.0811  61.8224
  [19]  61.5536  60.0242  58.3950  54.9767  53.5923  50.3407
  [25]  50.1501  47.1681  45.1392  43.8107  43.2037  42.2685
  [31]  40.7568  39.9030  37.2435  36.8453  35.5733  35.2031
  [37]  34.4904  33.9733  32.2537  31.7271  31.0329  30.3893
  [43]  29.7626  28.9381  28.3175  26.7731  26.4679  25.1793
  [49]  23.2744  22.8368  22.0119  20.8277  20.1636  19.4607
  [55]  18.2677  17.7540  16.6965  15.3630  14.3497  12.4190
  [61]  10.4253   9.9400   8.9138   0.0000   0.0000   0.0000
  [67]   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  [73]   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  [79]   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  [85]   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  [91]   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
  [97]   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
 [103]   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
 [109]   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
 [115]   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
 [121]   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
 [127]   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
 [133]   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
 [139]   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
 [145]   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
```

Note that only first 63 eigenvalues are non-zero. The remaining (6830-63) eigenvalues are all equal to zero.

Rank of Var-Cov Matrix

```
> require(Matrix)
> r = rankMatrix(c)
> r[1]
[1] 63
```

Note that the rank of the variance – covariance matrix is 63, which is equal to the number of non-zero eigenvalues.