

Assignment 6

Papoulis Example 4.23

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Question

Over a period of 12 hours, 180 calls are made at random. What is the probability that in a 4 hour interval, the number of calls is between 50 and 70?

Variables

Let the Bernoulli variable $X \in \{0, 1\}$ denote whether a call lies within the 4 hour interval with $p_X(1) = p = 1 - q$.

Let $Y \in \{0, 1, \dots, 180\}$ denote the number of calls in the four hour interval. Y represents 180 trials of X .

Calculations

The probability that a single call in the 12 hour interval lies in the 4 hour interval is

$$p_X(1) = p = \frac{4hr}{12hr} = \frac{1}{3} \quad (1)$$

Since Y is a binomial variable, we have that,

$$p_Y(k) = \binom{180}{k} p^k q^{180-k} \quad (2)$$

$$= \binom{180}{k} \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{180-k} \quad (3)$$

We need to find

$$\Pr(50 \leq Y \leq 70) = \sum_{k=50}^{70} p_Y(k) \quad (4)$$

$$= \sum_{k=50}^{70} \binom{180}{k} \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{180-k} \quad (5)$$

$$\approx 0.9025098... \quad (6)$$

So we have that the required probability, that the number of calls in the interval is between 50 and 70, is approximately 0.9025.

Graph

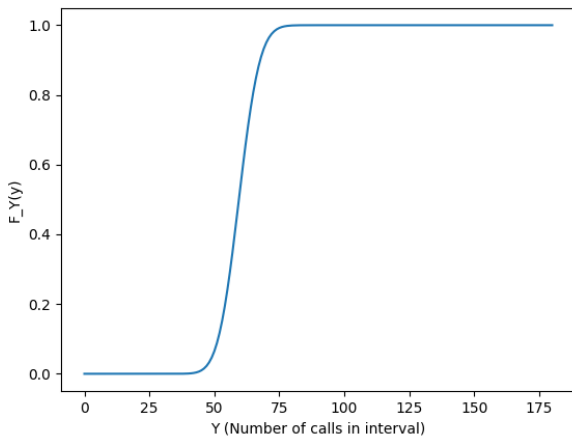


Figure: Cumulative Probability Function