

Assignment 2 - ICSE 12 2018 - Q4

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Question:

Use properties of determinant to solve for x :

$$\begin{vmatrix} x+a & b & c \\ c & x+b & a \\ a & b & x+c \end{vmatrix} = 0 \text{ and } x \neq 0$$

Solution:

$$\begin{vmatrix} x+a & b & c \\ c & x+b & a \\ a & b & x+c \end{vmatrix} = 0 \quad (1)$$

Transforming $C_1 \rightarrow C_1 + C_2 + C_3$,

$$\begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & a \\ x+a+b+c & b & x+c \end{vmatrix} = 0 \quad (2)$$

$$\Rightarrow (x+a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & x+b & a \\ 1 & b & x+c \end{vmatrix} = 0 \quad (3)$$

Transforming $R_3 \rightarrow R_3 - R_1$,

$$(x+a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & x+b & a \\ 0 & 0 & x \end{vmatrix} = 0 \quad (4)$$

Transforming $R_2 \rightarrow R_2 - R_1$,

$$(x+a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & x & a-c \\ 0 & 0 & x \end{vmatrix} = 0 \quad (5)$$

Expanding the determinant, we get

$$(x+a+b+c)x^2 = 0 \quad (6)$$

Since $x \neq 0$, we have

$$x+a+b+c = 0 \quad (7)$$

And so we have our answer,

$$x = -(a+b+c) \quad (8)$$