ComS 573 HW4

Karak Choudhury

Problem 1:

1a) i) dsep(T, ϕ, B) = True

There are two path from T to B without any given information.

① $T \rightarrow P \leftarrow C \leftarrow S \rightarrow B$

(ii) $T \rightarrow P \rightarrow D \leftarrow B$

Now, if dsep(T, ϕ, B) is d-separated if all above two Paths are blocked.

Now, we can see. path $T \rightarrow P \leftarrow C$ is a convergent valve ⇒ it is closed because neither P nor any of decendants of P are given.

So Path ① is blocked

Now, Path $T \rightarrow P \rightarrow D$ is a sequencial valve and it is open.

But, $P \rightarrow D \leftarrow B$ is a convergent valve and it is closed ⇒ $T \rightarrow P \rightarrow D \leftarrow B$ is also blocked.

Since both paths are blocked ⇒ dsep(T, ϕ, B) is d-separated.

ii) $dsep(A, \{D, C\}, B) = \text{False}$

Paths, – ① $A \to T \to P \leftarrow C \leftarrow S \to B$

ⅱ $A \to T \to P \to D \leftarrow B$

For ①,

$A \to T \to P \Rightarrow$ open

$T \to P \leftarrow C \Rightarrow$ open since D is given

$P \leftarrow C \leftarrow S \Rightarrow$ closed since C is given

Thus, path ① is closed.

For ⅱ, $A \to T \to P \Rightarrow$ open since sequencial valve,

$T \to P \to D \Rightarrow$ open since sequencial valve

and $P \to D \leftarrow B \Rightarrow$ open since D is given.

$\Rightarrow$ path ⅱ is open.

So, $dsep\{A, \{D, C\}, B\}$ ~~is~~ not dseparated

iii) dsep(A, P, {X, D}) = False

We can move ~~to~~ from A to X as -

A → T → P → X

We can not ~~to~~ go to X in any way because P is given that is. T → P → X is closed

However, we can move from A to D, that is not all paths from A to D are closed. that is,

A → T → P ⇒ open sequencial

T → P ← C ⇒ open since P is given

~~C~~ C ← S → B ⇒ open divergent

B → D ⇒ open sequencial

⇒ We can move ~~from~~ from A → D and it is open

Thus, dsep(A, P, {X, D}) is not d-seperated.

iv) $\text{dsep}(\{A, X\}, \{P, S\}, \{C, D\}) = \text{False}$

If we can show that there is at least one open path $\Rightarrow (\{A, X\}, \{P, S\}, \{C, D\})$ is not d-separated.

We can see that we can move ~~fro~~ from A to C, that is

$$A \rightarrow T \rightarrow P \leftarrow C$$

$A \rightarrow T$ is open sequencial

$T \rightarrow P \leftarrow C$ is open since P is given

Thus, A to C is open $\Rightarrow \text{dsep}(\{A, X\}, \{P, S\}, \{C, D\})$ is not d-separated.

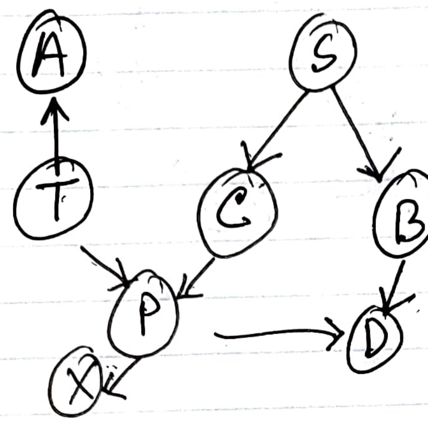
1. b) $P(a, s, t, c, p, b, x, d) = P(a) P(s) P(t|a) P(C|S) P(P|T, C) P(b|s) P(x|P) P(d|P, b)$

$= P(a) P(s) P(t|a) P(c|s) P(P|T, C) P(b|s) P(x|P) P(d|P, b)$

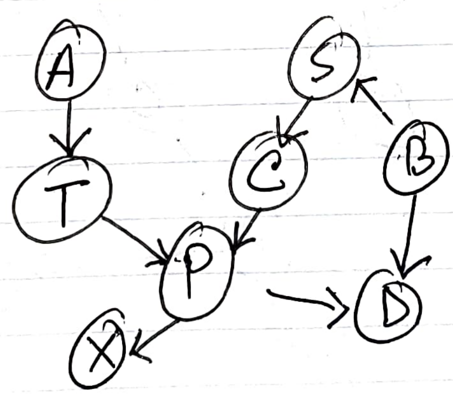
Problem 2:

(I) We can see that

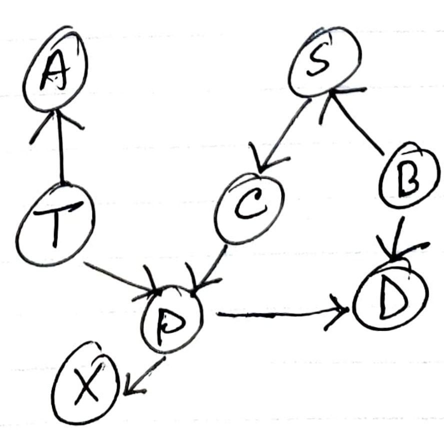
$$A \to T = A \leftarrow T$$



(II) Again, $S \to B = S \leftarrow B$

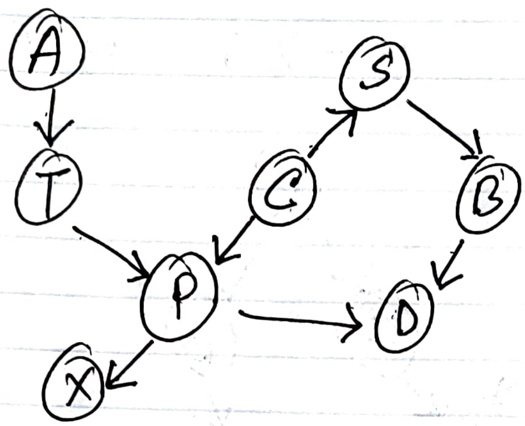


(III) Similarly,

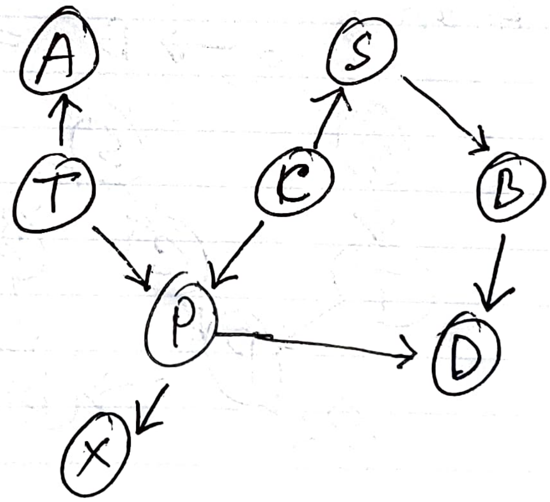


(iv) I

$S \rightarrow C = S \leftarrow C$



(v) Similarly,



Problem 3:

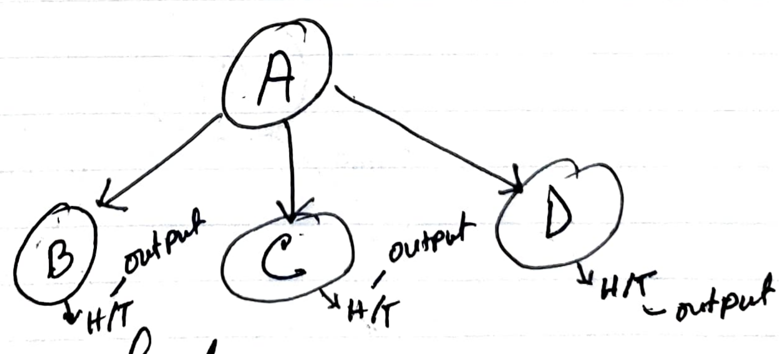
$$\text{Let } A = \begin{cases} 1 \ ; & \text{if 1st two flips in } C_1 \text{ are same} \\ 0 \ ; & \text{if " " " " " " different} \end{cases}$$

$$B = \begin{cases} 1 \ ; & H \text{ from 1st flip of } C_i / M \ ; i=2,3 \\ 0 \ ; & T \text{ " " " " } C_i/M \ ; i=2,3 \end{cases}$$

$$C = \begin{cases} 1 \ ; & H \text{ " 2nd " " } C_i/M \ ; i=2,3 \\ 0 \ ; & T \text{ " " " " " " " } \end{cases}$$

$$D = \begin{cases} 1 \ ; & H \text{ " 3rd " " " " " } \\ 0 \ ; & T \text{ " " " " " " " } \end{cases}$$

In Bayesian network, we can represent as.



we have to find,

$$P(A = \text{'same'} / B = T, \ C = H, \ D = T)$$

Here, $P(A = \text{'Same'}) = P_1^2 + (1-P_1)^2$

$$P(A = \text{'different'}) = 2P_1(1-P_1)$$

$$P(B=T,\ C=H,\ D=T \mid A=\text{'Same'}) = (1-P_2)^2 P_2$$

$$P(B=T,\ C=H,\ D=T \mid A=\text{'different'}) = (1-P_3)^2 P_3$$

So, $P(A=\text{'Same'} \mid B=T, C=H, D=T)$

$$= \frac{P(T,H,T \mid \text{Same})\ P(\text{Same})}{P(T,H,T \mid \text{Same})\ P(\text{Same}) + P(T,H,T \mid \text{Diff.})\ P(\text{Same})}$$

$$= \frac{P_2(1-P_2)^2\left(P_1^2+(1-P_1)^2\right)}{P_2(1-P_2)^2\left(P_1^2+(1-P_1^2)\right) + P_3(1-P_3)^2\left(2P_1(1-P_1)\right)}$$