ComS 573 Machine Learning – HW3 solution

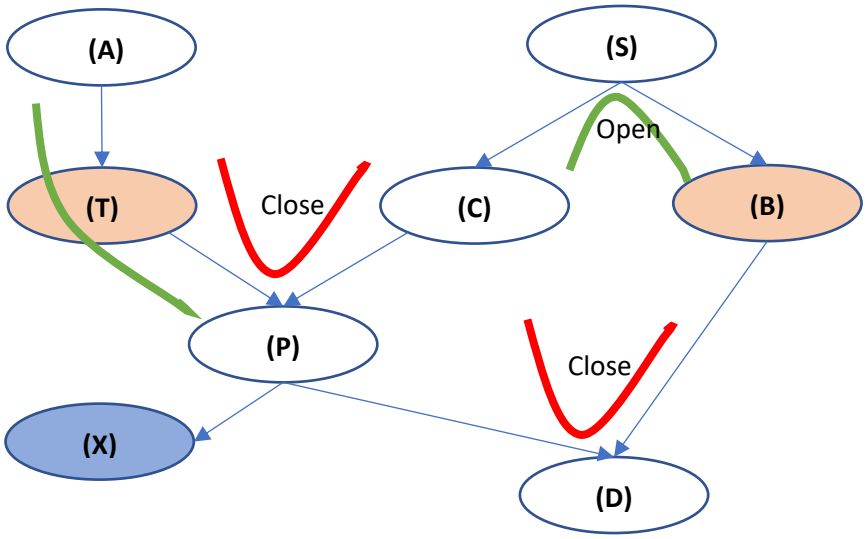
Problem 1:

(a) Rules: Using three rules in the slides to determine if some path is closed or open.

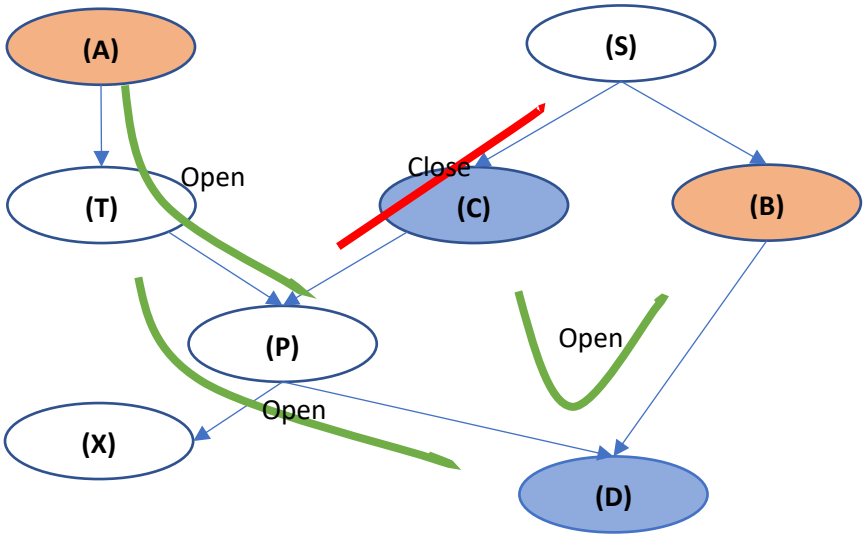
True: All paths should be closed.

False: At least one path should be open

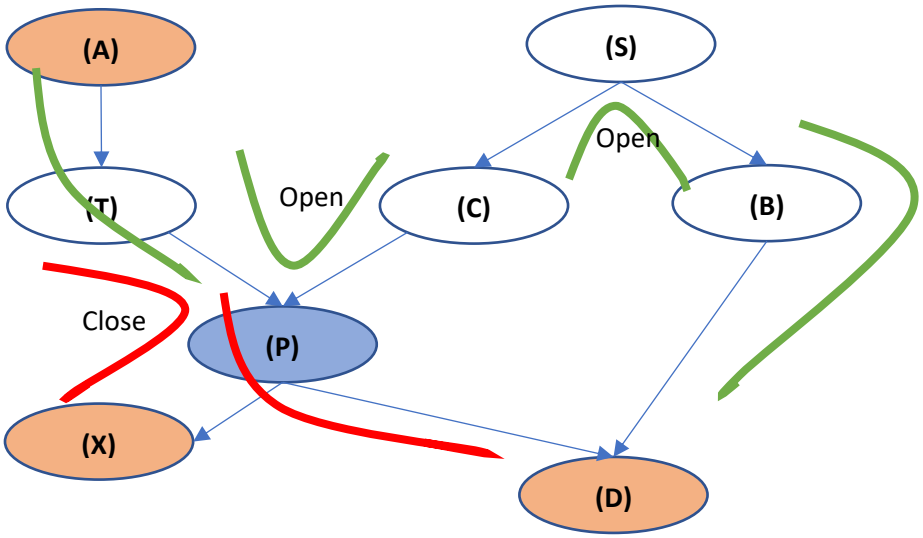
I. dsep(T, ∅, B) : True, $(T \rightarrow P \rightarrow C \rightarrow S \rightarrow B)$ is close.



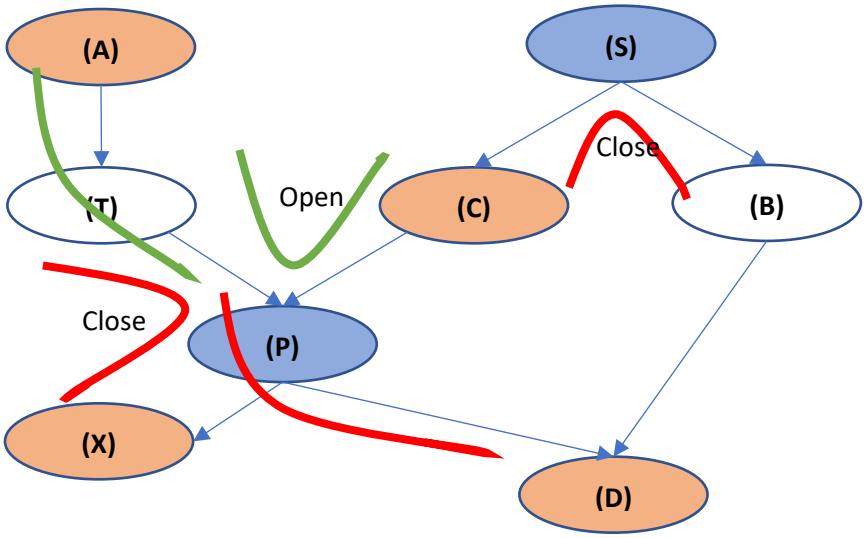
II. dsep(A, {D, C}, B): False,. $A \rightarrow T \rightarrow P \rightarrow D \rightarrow B$ is open.



III. dsep(A, P, {X, D}): False, $(A \rightarrow T \rightarrow P \rightarrow C \rightarrow S \rightarrow B \rightarrow D)$ is open.



IV. dsep({A, X}, {P, S}, {C, D}): False, $(A \rightarrow T \rightarrow P \rightarrow C)$ is open.



(b)(3 pts.) Express P(a, s, t, c, p, b, x, d) in factored form (the chain rule for BNs).

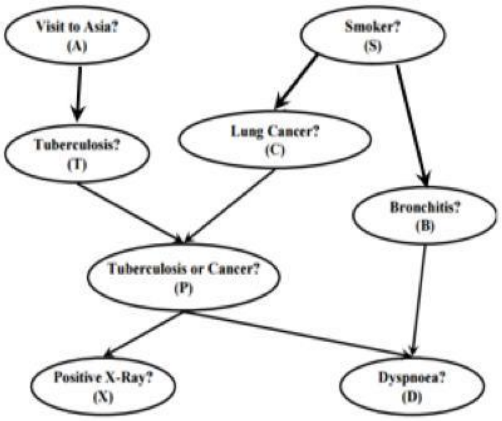
$P(X1,X2, \ldots ,Xn) = \prod_i^n P(X_i \mid Pa(X_i))$

Therefore,

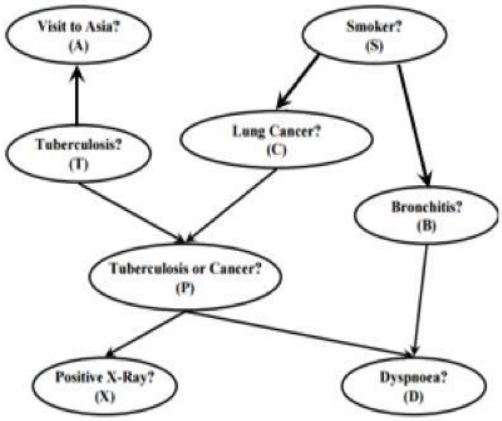
$P(a, s, t ,c,p,b,x,d) = P(a)\, P(s)\, P(t \mid a)\, P(c \mid s)\, P(p \mid t ,c)\, P(b \mid s)\, P(x \mid p)\, P(d \mid p,b)$

Problem: 2

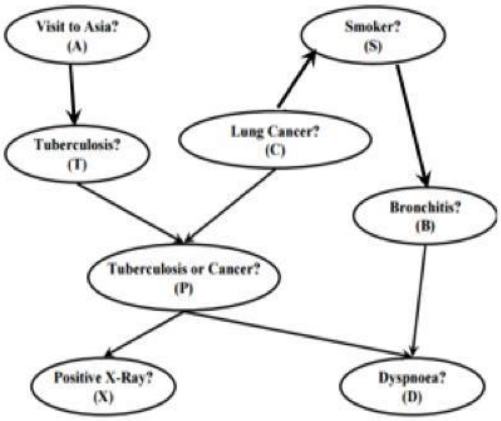
Using the theorem to get the equivalent Graphs (include the original one)



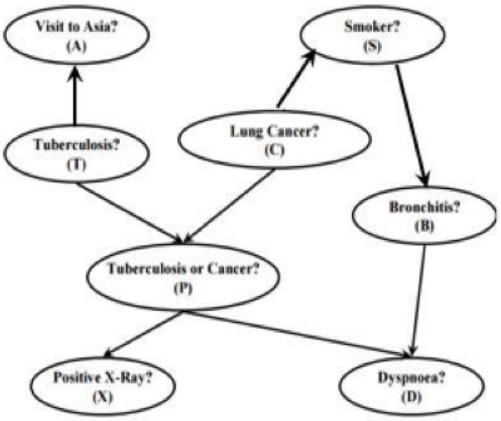
(a)



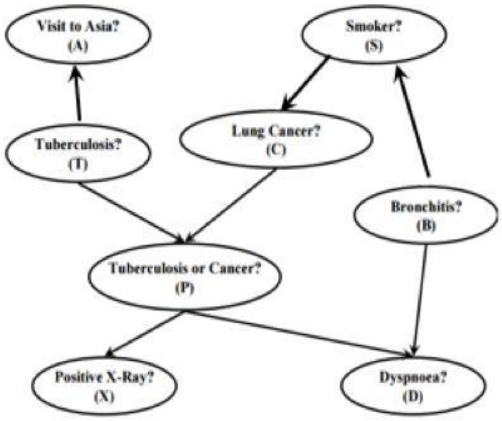
(b)



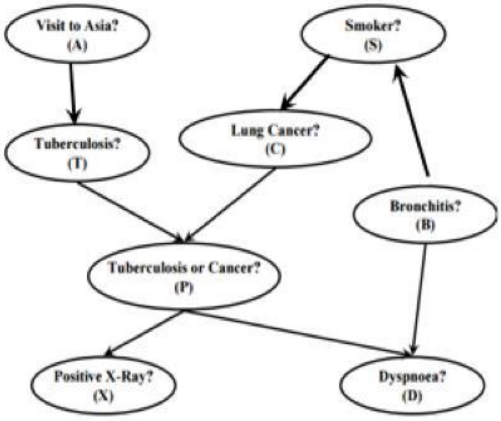
(c)



(d)



(e)



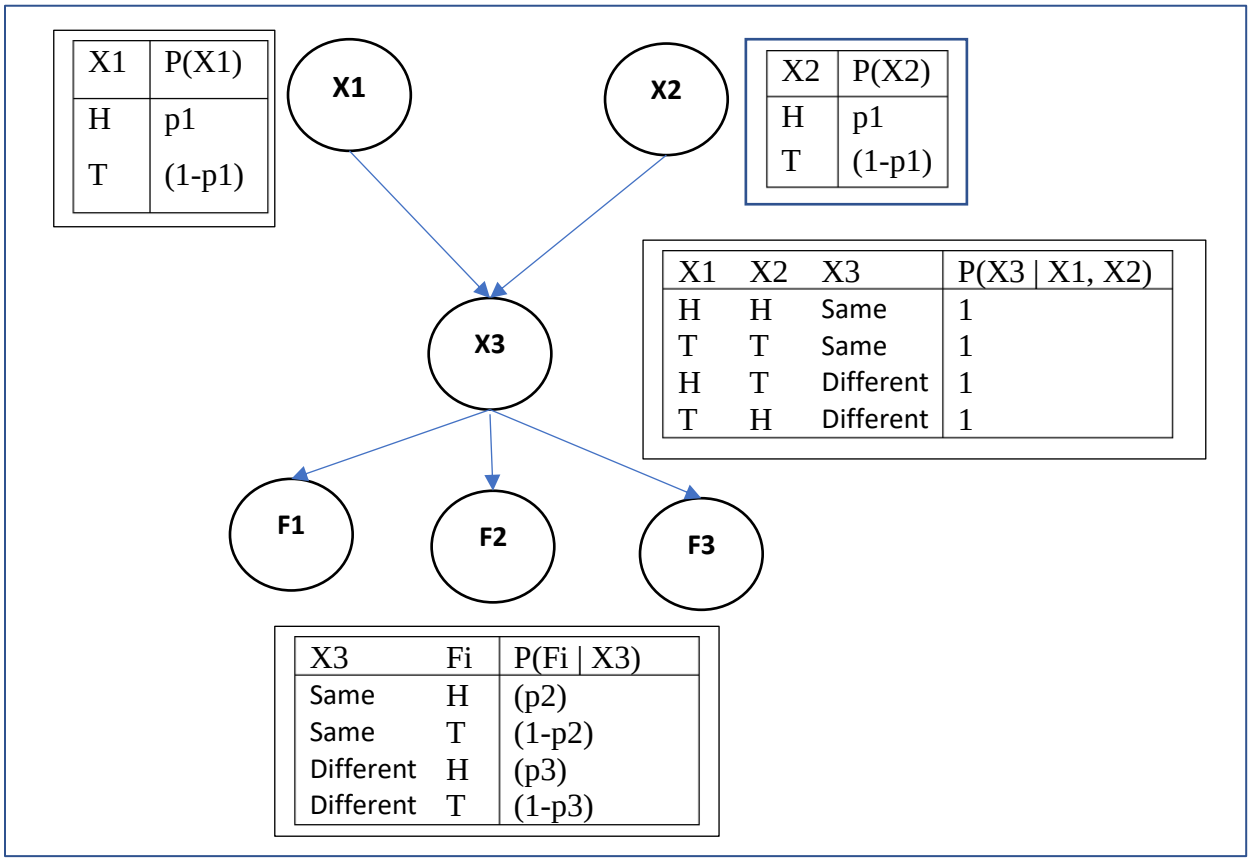
(f)

Problem 3:

(20 pts.) Jack has three coins C_1, C_2, and C_3 with p_1, p_2, and p_3 as their corresponding probabilities of landing heads. Jack flips coin C_1 twice and then decides, based on the outcomes, whether to flip coin C_2 or C_3 next: if the two C_1 flips come out the same, Jack flips coin C_2 three times; if the two C_1 flips come out different, Jack flips coin C_3 three times. Given the outcome of Jack's last three flips as: tails, heads, tails, we want to know whether his first two flips came out the same. Describe a Bayesian network and a corresponding query that solves this problem.

Solution:

Bayesian network and CPTs.



X1	P(X1)
H	p1
T	(1-p1)

X2	P(X2)
H	p1
T	(1-p1)

X1	X2	X3	P(X3 | X1, X2)
H	H	Same	1
T	T	Same	1
H	T	Different	1
T	H	Different	1

X3	Fi	P(Fi | X3)
Same	H	(p2)
Same	T	(1-p2)
Different	H	(p3)
Different	T	(1-p3)

Random variables:

X1 ∈ $\{H, T\}$ // First flip of C1

X2 ∈ $\{H, T\}$ // Second flip of C1

X3 ∈ $\{Same, Different\}$ // X3 = Same if X1 = X2 else X3 = Different

Fi ∈ $\{H, T\}$ // I^{th} flip of C2 if X3 = Same or I^{th} flip of C3 if X3 = Different, i ∈ {1,2,3}

Query: P(X3 = Same| F1 = T; F2 = H; F3 = T) ?

P(X3 = Different| F1 = T; F2 = H; F3 = T) ?