Com S 573 HW 2 Kanak Choudhury

Problem 1:

We have, $P(\underset{\sim}{x}|\underset{\sim}{\theta}) = \prod_{i=1}^{d} \theta_i^{x_i}(1-\theta_i)^{1-x_i}$

$$L(\underset{\sim}{\theta}|\underset{\sim}{x}) = \prod_{j=1}^{n}\prod_{i=1}^{d} \theta_i^{x_i}(1-\theta_i)^{1-x_i}$$

$$\Rightarrow \ln L(\underset{\sim}{\theta}|\underset{\sim}{x}) = \sum_{i=1}^{d}\sum_{j=1}^{n}\left[x_{ij}\ln\theta_i + (1-x_{ij})\ln(1-\theta_i)\right]$$

$$l(\underset{\sim}{\theta}|\underset{\sim}{x}) = \sum_{i=1}^{d}\left[\sum_{j=1}^{n}x_{ij}\ln\theta_i + \left(n-\sum_{j=1}^{n}x_{ij}\right)\ln(1-\theta_i)\right]$$

Differentiating w.r.to θ_i

$$\Rightarrow \frac{\partial l(\underset{\sim}{\theta}|\underset{\sim}{x})}{\partial\theta_i} = \sum_{j=1}^{n}x_{ij}/\theta_i + \frac{n-\sum_{j=1}^{n}x_{ij}}{1-\theta_i}(-1) = 0$$

$$\Rightarrow \sum_{j=1}^{n}x_{ij} - \cancel{\theta_i\sum_{j=1}^{n}x_{ij}} = n\theta_i - \cancel{\theta_i\sum_{j=1}^{n}x_{ij}}$$

$$\Rightarrow \theta_i = \sum_{j=1}^{n}x_{ij}/n$$

So, MLE of $\theta_i = \sum_{j=1}^{n}x_{ij}/n$

i.e. $\underset{\sim}{\theta} = \frac{1}{n}\sum_{j=1}^{n}\underset{\sim}{x}_j$ where, $\underset{\sim}{x}_j = \{x_1, \dots, x_d\}_j$

Problem 2:

We have, $n = 1000$ and $\sum_{i=1}^{n} Y_i = 900$

So, the estimated accuracy is given by

$$\hat{\theta} = \frac{800}{1000} = 0.8$$

95% CI can be written as –

$$\hat{\theta} \pm Z_{1-\alpha/2}\, se(\hat{\theta})$$

Where, $\hat{\theta} = 0.8$, $se(\hat{\theta}) = \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}$

$= \sqrt{\frac{0.8 \times 0.2}{1000}}$

and $Z_{1-\alpha/2} = 1.96$

$= 0.012649$

So, $CI = [0.8 - 1.96 \times 0.012649,\ 0.8 + 1.96 \times 0.012649]$

$= [0.7752,\ 0.8247]$