ComS 573 Machine Learning

Problem Set 1

Note: Please do not hesitate to contact the instructor or the TA if you have difficulty understanding or getting started with solving any of the problems.

1. (20 pts.) Consider a two category classification problem with one dimensional feature x. Assume that the priors are $P(\omega_1) = 1/3$ and $P(\omega_2) = 2/3$, and that the class-conditional distributions have normal densities $p(x|\omega_1) \sim N(1, 1)$ and $p(x|\omega_2) \sim N(3, 1)$, where

$$N(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

   (a) Derive the Bayes decision rule for minimum-error-rate classification.

   (b) Let $\lambda_{ij} = \lambda(\alpha_i|\omega_j)$ be the loss incurred for deciding ω_i when the true category is ω_j. Assume $\lambda_{11} = 0$, $\lambda_{12} = 2$, $\lambda_{21} = 1$, $\lambda_{22} = 0$. Derive the Bayes decision rule for the minimum risk classification.

2. (20 pts.) Consider the Binary Independence Model for text document. Given a vocabulary $V : (w_1, \ldots, w_d)$ of all English words (and tokens), assume that a text document is represented as a vector of binary features $\vec{x} = (x_1, \ldots, x_d)^t$ such that x_i is 1 if the word w_i appears in the document, and x_i is 0 otherwise. We want to classify text documents into c categories. Let $P(\omega_j)$ be the prior probability for the class ω_j for $j = 1, \ldots, c$. Assume that the components of $\vec{x}$ are statistically independent given the category (Naive Bayes model), i.e.,

$$P(\vec{x}|\omega_j) = \prod_{i=1}^{d} P(x_i|\omega_j).$$

   Assume we could estimate the following probability from the training data

$$p_{ij} = P(x_i = 1|\omega_j) \quad i = 1, \ldots, d, \quad j = 1, \ldots, c.$$

   Show that the minimum probability of error is achieved by the following decision rule: Decide ω_k if $g_k(\vec{x}) \geq g_j(\vec{x})$ for all $j \neq k$, where the discriminant function is given in the form of

$$g_j(\vec{x}) = \sum_{i=1}^{d} c_{ij} x_i + b_j.$$

   Give the expressions of c_{ij} and b_j.