# Review of "A Model for Dynamic Communicators by Alexander V. Mantzaris and Desmond J. Higham"

Kanak Choudhury
Department of Statistics
University of Central Florida
Orlando, FL, USA
Email: kanakc@knights.ucf.edu

## I. Introduction

FOR a single static network model, random graph models and centrality measures are very useful tools. However, for dynamic network model where links between nodes may appear and disappear in a time-dependent manner, a new models and algorithms are needed. Examples of dynamic network include email activity, voice calls, online social interaction, geographical proximity of mobile device users, dynamic transportation infrastructure, voting and trading patterns and neural activity etc. For a dynamic network model, final state of an iterative process is not the main concern rather continual change in topological structure is the main consideration in this article.

## II. Background And Motivation

In this section, authors tried to present the problems of static network model for dynamic scenario of communication with some hypothetical examples. Figure 1 [1] showed undirected, unweighted network communication between a set of 21 nodes over three days. From three different subfigures of Figure 1, we might consider that node 21 was not unusually important. However, node 21 was a special node based on *timing*. Because, at day one node 21 reached to nodes 19 and 20 and then the network spread on subsequent day, that is, nodes 16 to 18 on day two and nodes 12 to 15 on day three. It was explained node 21 as an *influential* player because whenever other individuals receive a message that can be traced back to node 21 they burst into action and pass the message on. For example, when a terrorist group get a message from the leader, they tried to spread it into other networks. So, node 21 can be responsible for future network structure. For example, Donald Trump, precedent of USA, might post a tweet and after that it can spread over other networks. For all the network based on that tweet, he is the most influential on the whole network although his direct communication is just a tweet. Because of this reason authors introduced a new, general dynamic network model based on simple but intuitively reasonable principles that can captures the effect.

For a xed set of $N$ nodes and time points $t_0 \leq t_1 \leq \; \leq t_M$, it was considered an ordered sequence of unweighted graph adjacency matrices $A^{[k]} \in \mathbb{R}^{N\times N}$, so that $(A^{[k]})_{ij} = 1$ if there was a link from node $i$ to node $j$ at time $t_k$ and $(A^{[k]})_{ij} = 0$ otherwise. It was found that even in the case of undirected networks, where each $A^{[k]}$ is symmetric, dynamic walks lack symmetry with dynamic walk of length $w$. The computation of the matrix $Q \in \mathbb{R}^{N\times N}$, for which $(Q)_{ij}$ is a weighted count of the number of dynamic walks of length $w$ from node $i$ to node $j$, was shown in [2] where walks of length $w$ are scaled by a factor $a^w$. To find effectiveness of broadcast and receive dynamic messages, the following row and column sums were used for centrality measures.

$$C_n^{\text{broadcast}} := \sum_{k=1}^{N} Q_{nk} \text{ and } C_n^{\text{receive}} := \sum_{k=1}^{N} Q_{kn}$$

A normalized matrix $Q/\|Q\|$ were used in order to avoid numerical under or overflow [1].

Using new method, authors found largest dynamic broadcast centrality for the node 21 in Figure 1 although it was ranking much lower based on static measures. They used the term *dynamic communicator* to describe a node that has excellent centrality in the dynamic sense while it was not represented by snapshot or aggregate views of the network sequence.

## III. Practical Observations

In this section, authors presented a real example covers dynamic broadcasters. They used two weeks of Enron email data [2] and presented the solution in Figure 2. For one-day time interval, it was found in Figure 2 (a) that nodes 1 and 2, an executive and the vice president, had the highest broadcast centrality but modest total out degree which are considered as dynamic communicators. On the other hand, nodes 3, 4 (both correspond to traders) and 5 (unknown role) had high bandwidth but relatively poor broadcast centrality. It was mentioned that for the time-dependent setting, nodes having very high out degree was neither necessary nor sufficient to guarantee influence amongst other nodes. Using two-day period (Figure 2 (b)), the same nodes (1 and 2) were observed as dynamic communicators while an extra node, labeled number 6 (an employee), was found with other 3, 4, and 5 nodes with high bandwidth but relatively poor broadcast centrality.

Department of Statistics, University of Central Florida, FL, USA

Similar results were found (Figure 3) using 30 days of voice call data between academics [3].

## IV. New Model

The new model that the authors proposed was considered as the generalizing the concept of network hierarchy from the static case [4]. It was assumed for the new model that there was an underlying hierarchy which can be explained that some nodes have high importance to expand the future network. This type of hierarchy might arise through an imposed chain of command, i.e. business or military organizations, criminal networks, completion of tasks as in on-line gaming.

This new model was based on two objectives, "(i) identifying a key feature in dynamic human inter action data sets and (ii) oering a simple, intuitively reasonable, explanatory mechanism" [1]. It was considered that where was the particular pairs of nodes involved in communication not the particular time of the events for a single node.

The proposed model was started by assigning a fixed level of importance,$l_n$, to each node n so that $0 < l_1 \leq l_2 \leq \ \leq l_n$. The concept behind this was that low ranked nodes might generate high communication when received messages from highly ranked nodes. Mathematically, given the time $t_k$ network, $A^{[k]}$, $A^{[k+1]}$ was generated based on basal and responsive.

*Basel*: with probability $b$ node $n$ generates a xed number $c_b$ of links, with the new neighbors chosen uniformly and independently at random. Otherwise no basal links are generated from node $n$ [1].

*Responsive*: with probability

$$r_n^{[k]} := \frac{\sum_{i=1}^{N} l_i (A^{[k]})_{in}}{1 + l_N \sum_{i=1}^{N} l_i (A^{[k]})_{in}}$$

node $n$ generates a xed number $c_r$ of links, with the new neighbors chosen uniformly and independently at random. Otherwise no responsive links are generated from node $n$ [1].

Results of this model were shown in Figure 4 and found consistent result for different scaling parameters of $a = 0.75, 0.5, 0.25$. For every case, model found the same node as a dynamic broadcaster while shoed ranking 26th in terms of aggregate degree.

The authors added that the dynamic broadcasters bear higher importance because links from the broadcasters and links from the links might become active and create the future network. So, dynamic receivers might have added global, historical knowledge that they know which nodes are currently most informative and deliberately form links with them.

## V. Conclusion

In this article, authors presented a new model that can describes the dynamic presence or absence of connections in an evolving network.

## References


[1] A. V. Mantzaris and D. J. Higham, "A model for dynamic communicators," *European Journal of Applied Mathematics*, vol. 23, no. 06, pp. 659–668, 2012.

[2] P. Grindrod, M. C. Parsons, D. J. Higham, and E. Estrada, "Communicability across evolving networks," *Physical Review E*, vol. 83, no. 4, p. 046120, 2011.

[3] N. Eagle, A. S. Pentland, and D. Lazer, "Inferring friendship network structure by using mobile phone data," *Proceedings of the national academy of sciences*, vol. 106, no. 36, pp. 15274–15278, 2009.

[4] L. Muchnik, R. Itzhack, S. Solomon, and Y. Louzoun, "Self-emergence of knowledge trees: Extraction of the wikipedia hierarchies," *Physical Review E*, vol. 76, no. 1, p. 016106, 2007.