Review of "A New Status Index Derived From Sociometric Analysis by Leo Katz"

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I. Introduction

The ordinary indices of *status* for investigating inter-personal and inter-group relations are based on the number of direct *votes* received by each individual and most of the researchers were not satisfied with these indices. In this article, author proposed a new method of computing status based on the number of direct *votes* received by each individual and the steps of choosing the status. In other words, the new index considered who chooses and how many choose.

II. The New Status Index

To illustrate the new status index, a matrix representation for sociometric data (Page 40) taken from [1] was used. The data matrix should be a square matrix with principal diagonal elements are zeroes. The data matrix was represented as C, with element c_{ij} is the response of individual i to individual j and the response can be either 0 or 1. From this data matrix, if we want to find the higher order chooses, i.e. i chooses k and k chooses j that is chain of length two from i to j, can be obtained by $C^2 = (c_{ij}^{(2)})$, where $c_{ij}^{(2)} = \sum_k c_{ik}c_{kj}$; $c_{ik}c_{kj}$ is equal to one if and only if i chooses k and k chooses j, otherwise zero. This representation is exactly equal to Markov chain. We can find this higher order Markov chain by simply multiplying C matrix.

The column sums of C give the numbers of direct choices made by members of the group to the individual corresponding to each column. Also, the column sums of higher order matrix i.e. C^2, give the numbers of two-step choices from the group to individuals and so on.

To find the index, it was proposed by weighted sum of all higher order matrices. The weighted sum was used to reduce the effect of longer chains and finding appropriate weight method called *attenuation* were described.

The new index calculation was based on two assumptions. (i) the information is accurate that is certain links between individuals exist and if the information indicates no link that means there is no communication, influence, or whatever else it is measured. (ii) Each link independently has the same probability of being effective [2].

Based on the second assumption, a constant a indicated as the force of a probability of effectiveness of a single link was considered which measures the non-attenuation in a link, i.e. $a = 0$ corresponding to complete attenuation and $a = 1$ to absence of any attenuation. This constant a (for $C^2, C^3,$ higher order $a^2, a^3,$ respectively) was considered as the weight for the index calculation.

Let s_i be the sum of jth column of the matrix C and s be a column vector with elements s_i. Then, the column sums of the matrix

$$T = aC + a^2C^2 + \cdots + a^kC^k + \cdots = (I - aC)^{-1} - I$$
$$\Big(\frac{1}{a}I - C^{'}\Big)t = s \tag{1}$$

where, t be a column vector with $t^{'} = u^{'}[(I - aC)^{-1} - I]$ and u be a column vector with unit elements. So, to obtain t, it is needed to solve the system of linear equations with given $a, C,$ and s. It is mentioned that the general-purpose values of $1/a$ should be between the largest root and twice that root.

Dividing the column sum s_i by $n-1$ $\big((n-1)^{(k)} = (n-1)(n-2)\cdots(n-k)\big)$ that is the number of possible choices for C $(C^{(k)})$, index of status can be obtained. So

$$\begin{aligned} m &= a(n-1) + a^2(n-1)^{(2)} + a^3(n-1)^{(3)} + \cdots \\ &\approx (n-1)! \cdot a^{n-1}e^{1/a} \end{aligned}$$

Finally, the new status index vector is given by t/m, where t is the vector solution to the system of equations (1).

III. A Numerical Example

Based on the above index calculation, a numerical example was illustrated and found that the new status indices were able to capture the relative position based on the objectives of this article than the ordinary indices.

References

[1] E. Forsyth and L. Katz, "A matrix approach to the analysis of sociometric data: preliminary report," *Sociometry*, vol. 9, no. 4, pp. 340–347, 1946.
[2] L. Katz, "A new status index derived from sociometric analysis," *Psychometrika*, vol. 18, no. 1, pp. 39–43, 1953.