Can a Shallow Ice Approximation Model be Used to Model the Water Output of Alpine Glaciers?

Abstract

1 Introduction

1.1 Importance of glacial melting in mountain hydrology

Glacial melting plays a significant role in the hydrology of mountain catchment areas as shown in Fountain and Tangborn, 1985. Accurately capturing glacial dynamics with numerical models is important for understanding how water sources in high mountain catchments will evolve. The runoff produced by glaciers, especially later in the summer months, is a major contributor to stream flow. Without the runoff from the glaciers, the rivers would be even lower during the driest summer months as shown in Figure 1.

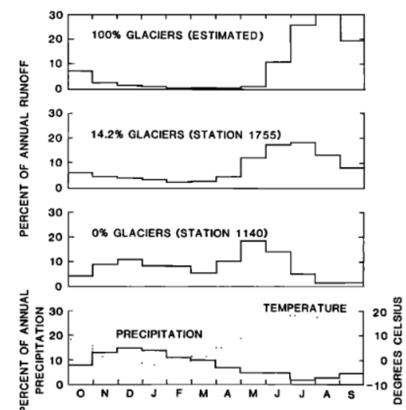

Fig. 3. Monthly fraction of the annual specific runoff for basins of various glacier covers. The monthly fraction of precipitation and mean monthly temperature (Snoqualmine Pass, Washington) are included for comparison.

(a) Monthly fraction of the annual runoff for basins of various glacier cover.

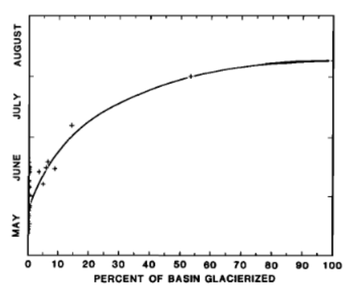

Fig. 4. Timing of peak specific runoff as a function of glacier cover for basins in the North Cascades, Washington.

(b) Timing of peak specific runoff as a function of glacier cover.

Figure 1: Figures 3 and 4 from Fountain and Tangborn (1985) showing how the percent of basin glacierized delays the peak runoff time of the basin.

As glaciers melt and retreat, the change in mass takes the form of meltwater. This meltwater affects communities downriver in a variety of ways. For instance, many communities rely on these rivers as a water source for drinking and irrigation.

1.2 Role of numerical modeling in understanding glacial runoff

One of the best ways to find out how glaciers will affect the stream flow of rivers in the glacier basin is by using computer models to approximate the water discharge of the glaciers. Scientists have been using computers to model glaciers for several decades, such as this paper by Iken A, 1981. As computational resources have grown, these models have grown in complexity and resolution leading to very computationally expensive models, meaning that they take a lot of computing power to run. Many of these Advanced Ice Sheet Models (AISMs) use equations such as the 3-dimensional Stokes equations (Larour et al., 2012) to model the complicated ice dynamics of these glaciers.

1.3 Challenges in computational modeling of glaciers

AISMs are numerically very sophisticated; these models are also not very intuitive and often take a while to learn how to run. The advantage of AISMs is that they do a particularly good job of modeling ice dynamics. For some glaciers such as marine calving glaciers, accurate ice dynamics are crucial to accurately model them, as shown in Amaral et al., 2020. However, modeling glaciers in 3-dimensions is often unnecessary. If we can use simpler models, such as the Shallow Ice Approximation (SIA) model, to calculate the water runoff of small mountain glaciers in locations such as the Alps, then we can run these models over more glaciers and larger areas, and it is easier to incorporate them into larger hydrology models.

2 Literature Review

As shown in Le Meur et al., 2004, there are significant differences in computational time between a SIA model and a Stokes model. When computing the free surface and associated velocity field, the SIA model took 1 minute of CPU time, whereas the Stokes model took 2 hours. This disparity grew even larger for 3D models. The authors show that there are some instances where SIA models do significantly worse than Stokes models, such as glaciers on steep slopes and glaciers in steep, narrow valleys because SIA models only approximate the Stokes equations. One of these approximations is to ignore horizontal stress gradients. This can cause a SIA model to deviate from a Stokes model significantly in predicted glacier flow and expansion. In one example, the resulting SIA model can have an upper free surface that is 15–20% greater than the Stokes model and velocities up to a factor of 2 greater (Le Meur et al., 2004).

In the 2D model, the bed characteristics and slope become the limiting factor of the SIA model; Le Meur et al., 2004 note that the maximum velocity ratio of their SIA and Stokes models goes from 1.9 in a 3D model to 1.3 in a 2D model, which will differ depending on model configuration, but it tends to indicate that the horizontal stress gradients played a large part in this error. They found instances in which the SIA models performed well compared to Stokes models—particularly large flat glaciers with relatively free edges. One thing to note about this comparison study is that the authors are looking at the shape, area, and velocity profile of the glacier, whereas this study will focus on the water output (surface mass loss) of the glacier.

There are several papers, such as Le Muer et al., 2003 and Kessler et al., 2006, that use an SIA model for alpine glaciers. The consensus from those papers is that SIA models only work well on alpine glaciers with a low aspect ratio, defined as the thickness-to-extent ratio in Le Muer et al., 2004. The glacier used by this study will have a low aspect ratio and therefore a SIA model should work well to model it.

Additionally, there is precedent for using a SIA-Mass Balance model for modeling water runoff from glaciers (Naz et al., 2014). They used the SIA model to approximate the ice dynamics and a mass balance model to approximate the accumulation and ablation patterns on the glacier. As shown in their paper, the SIA model was able to accurately predict the glacier, and the coupled hydrological model was able to predict the stream flow accurately—only overestimating the July flow by an average of 13% and underestimating the August and September flow by an average of 2%.

3 Thesis Statement

4 Methods

4.1 Study Site

South Cascade Glacier in the North Cascades of Washington

4.2 Physical Characteristics and Relevance

4.3 Availability of Hydrological and Glacial Data

4.4 Model Development

4.4.1 Model Overview

4.4.2 Ice Dynamics

SIA Model

Assumptions

4.4.3 Snow and Rain Model

The snow melt model uses precipitation and temperature data to melt and accumulate snow. This model uses 14 elevation bins and keeps track of the snow depth in each bin. The equation below is used to calculate the change in snow depth per timestep

$$\text{snow depth}+ = \begin{cases} p * \alpha & \text{if } T \leq 0, \\ -\text{minimum}((s * T), \text{snow depth}) & \text{if } T > 0 \end{cases}$$

Where p is the precipitation, α is the precipitation conversion factor, s is the snow melt factor, and T is the temperature. The snow melt is constrained so that there cannot be more melt than there is snow. The rain is simply modeled by $p * \alpha$ for positive temperatures

The total volume of snow is calculated by the equation

$$\text{snow melt volume} = (s * T) * (\text{area of bin} - \text{area of glacier})$$

This give us the total volume of snow melting off the glacier. The glacial melt is calculated elsewhere. The volume of rain is calculated by

$$\text{rain volume} = p * \alpha * (\text{area of bin})$$

This calculates the rain for the whole basin, assuming that any rain that falls off the glacier runs off immediately.

4.4.4 Mass Balance Model

The mass balance of the glacier is calculated using temperature and precipitation data from the Diablo Dam weather station at 272m. The temperature at the glacier is calculated by using a month-specific lapse rate. This month-specific lapse rate was empiraclly calculated using data from the Diablo Dam weather station and the South Cascade Glacier weather station at 1830m from 2010-2018. The precipitation at the glacier is calculated by multiplying the precipitation at the Diablo Dam weather station by the precipitation conversion factor of 1.58 (reference). The ablation of the glacier is calculated by using a combination of an ice melt factor and a snow melt factor. Above the ELA the ablation is calculated by the equation

$$\text{ablation} = T * \text{snow melt factor}$$

Below the ELA the ablation is calculated by the equation

$$\text{ablation} = T * (\text{snow melt factor} + ((\text{ELA} - \text{elevation})/(\text{ELA} - \text{minimum}(\text{elevation}))) * (\text{ice melt factor} - \text{snow melt factor}))$$

The result of this equation is the snow melt factor being used at the ELA and a linear increase in the melt factor until it hits the ice melt factor at the base of the glacier. The accumulation of the glacier is calculated using a similar linear equation that increases with time.

$$\text{accumulation} = p * \alpha * (\text{start accumulation} + ((\text{year} - 1984)/(2024 - 1984)) *$$

$$(\text{start accumulation} - \text{end accumulation}))$$

This resuls in the accumulation increasing with time until it hits the end accumulation at 2024.

4.4.5 Glacial Melt Model

The glacial melt model uses the mass balance of the glacier to calculate how much volume the glacier is losing. The volume of runoff from the glacier per timestep is calculated by the equation

$$\text{glacial melt volume} = (\text{mass balance} < 0) * \text{area of glacier}$$

Data Used for Model

4.4.6 Model Calibration

4.4.7 Model Comparison

Running OGGM Model for the Same Glacier

Comparing Outputs of Both Models

Validation Using Real-World Streamflow Data (Brunner et al., 2024)

5 Expected Results

5.1 Accuracy of SIA

5.2 Accuracy of OGGM

5.3 Comparison of Accuracy

6 Implications of Research

6.1 Importance of Simplified Ice Dynamics in Numerical Glacier Modeling

6.2 Applications

7 Discussion

7.1 What Worked

7.2 What Didn't Work and Why

7.3 What Can Be Improved

8 Conclusion

8.1 Summary of Results

8.2 Conclusion of Model Accuracy

9 References