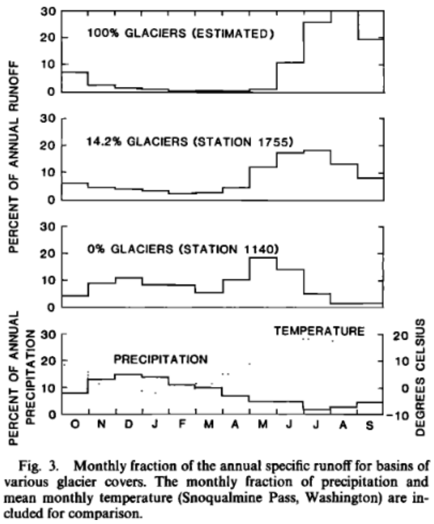
Can a Shallow Ice Approximation Model be Used to Model the Water Output of Alpine Glaciers?

Abstract

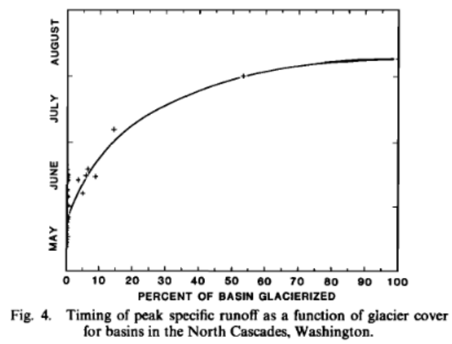
1 Introduction

1.1 Importance of glacial melting in mountain hydrology

Glacial melting plays a significant role in the hydrology of mountain catchment areas as shown in Fountain and Tangborn, 1985. In order to understand how the hydrology of these mountain catchment areas will evolve as glacier melt an retreat, scientists create models to predict this evolution. The runoff from these mountain catchments are often used as water sources for communities down stream (reference) and as shown in Figure 1, basins with greater glaciation produce more runoff. As the glaciers in these basins melt and retreat this change in mass takes the form of meltwater. This meltwater can effect communities that rely on the runoff from these glaciated basins in a variety of ways (reference).



(a) Monthly fraction of the annual runoff for basins of various glacier cover.



(b) Timing of peak specific runoff as a function of glacier cover.

Figure 1: Figures 3 and 4 from Fountain and Tangborn (1985) showing how the percent of basin glacierized delays the peak runoff time of the basin.

1.2 Role of numerical modeling in understanding glacial runoff

One of the most accurate ways to predict out how glaciers will affect the runoff of a glaciated mountain catchment is by using computer models to approximate the water discharge of the glaciers. Scientists have been using computers to model glaciers for several decades, such as this paper by Iken A, 1981. As computational resources have grown, these models have grown in complexity and resolution leading to very computationally expensive models. These models have proved to be very accurate in modeling the past and present state of a variety of glacier types all around the world.

1.3 Challenges in computational modeling of glaciers

Many of these more advanced models can be quite complex to run due to there being a lot of different configurations and ways to run it. The goal of this paper is to write a simple Shallow Ice Approximation (SIA) - Mass Balance model that is easy to run and understand while still being reasonably generalizable to different small mountain glaciers. The advantage of using much more advanced models is that they often do a much better job at modeling the ice dynamics of glaciers. For some glaciers such as marine calving glaciers, accurate ice dynamics are crucial to accurately model them, as shown in Amaral et al.,

2020. However on smaller mountain glaciers, modeling in 3-dimensions if often unnecessary and simpler models such as the SIA can be used. This paper will compare the results of a simple SIA - Mass Balance model to the results of OGGM, a much more advanced model using a simplified version of the Stokes Equations.

2 Literature Review

2.1 Prior research on SIA vs. Stokes models

As shown in Le Meur et al., 2004, there are significant differences in computational time between a SIA model and a Stokes model. When computing the free surface and associated velocity field, the SIA model took 1 minute of CPU time, whereas the Stokes model took 2 hours. This disparity grew even larger for 3D models. The authors show that there are some instances where SIA models do significantly worse than Stokes models, such as glaciers on steep slopes and glaciers in steep, narrow valleys because SIA models only approximate the Stokes equations. One of these approximations is to ignore horizontal stress gradients. This can cause a SIA model to deviate from a Stokes model significantly in predicted glacier flow and expansion. In one example, the resulting SIA model can have an upper free surface that is 15–20% greater than the Stokes model and velocities up to a factor of 2 greater (Le Meur et al., 2004). In the 2D model, the bed characteristics and slope become the limiting factor of the SIA model; Le Meur et al., 2004 note that the maximum velocity ratio of their SIA and Stokes models goes from 1.9 in a 3D model to 1.3 in a 2D model, which will differ depending on model configuration, but it tends to indicate that the horizontal stress gradients played a large part in this error. They found instances in which the SIA models performed well compared to Stokes models—particularly large flat glaciers with relatively free edges. One thing to note about this comparison study is that the authors are looking at the shape, area, and velocity profile of the glacier, whereas this study will focus on the water output (surface mass loss) of the glacier.

2.2 Using SIA models to model alpine glaciers

There are several papers, such as Le Meur et al., 2003 and Kessler et al., 2006, that use an SIA model for alpine glaciers. The consensus from those papers is that SIA models only work well on alpine glaciers with a low aspect ratio, defined as the thickness-to-extent ratio in Le Meur et al., 2004. The glacier used by this study will have a low aspect ratio and therefore a SIA model should work well to model it.

2.3 Using SIA models to model water runoff from glaciers

Additionally, there is precedent for using a SIA-Mass Balance model for modeling water runoff from glaciers (Naz et al., 2014). They used the SIA model to approximate the ice dynamics and a mass balance model to approximate the accumulation and ablation patterns on the glacier. As shown in their paper, the SIA model was able to accurately predict the glacier, and the coupled hydrological model was able to predict the stream flow accurately—only overestimating the July flow by an average of 13% and underestimating the August and September flow by an average of 2%.

3 Thesis Statement

How much do ice dynamics affect the model result when modeling small mountain glaciers for water runoff? I theorize that if using a simple 1-dimensional SIA-Mass Balance model on small mountain glaciers (with a low aspect ratio), the mass balance profile will have a much larger effect on the output of the model and its overall accuracy than the modeled ice dynamics. The results of the SIA-Mass Balance model and the OGGM model will be compared to the actual stream flow data to verify this hypothesis.

3.1 Study Site

South Cascade Glacier, Washington State

In this study, I will focus on modeling the South Cascade Glacier in the North Cascades region of Washington State. The South Cascade Glacier is roughly 1.68 square kilometers, has a mean elevation of roughly 1900 meters, (GLIMS) faces North, an average thickness of 99 meters, and a maximum thickness of 195 meters (GlaThiDa Consortium, 2020). The glacier is small, not overly steep, and has a low aspect ratio, therefore a SIA model should be able to accurately model its ice dynamics. On the other hand, the glacier is large enough to exhibit some movement and produce a measurable amount of runoff throughout the year.

Thanks to the foresight of the USGS in creating the Benchmark Glacier Program in the 1950s to study and document the South Cascade glacier, and eventually four more glaciers, there is an abundant amount of information on these glaciers. Due to this program I was able to easily access and use temperature, precipitation, mass balance and DEM data from the USGS. Also as part of this Benchmark Glacier program a stream gauge was installed just below the glacier to track the runoff from the basin, the data from which was essential to this project.

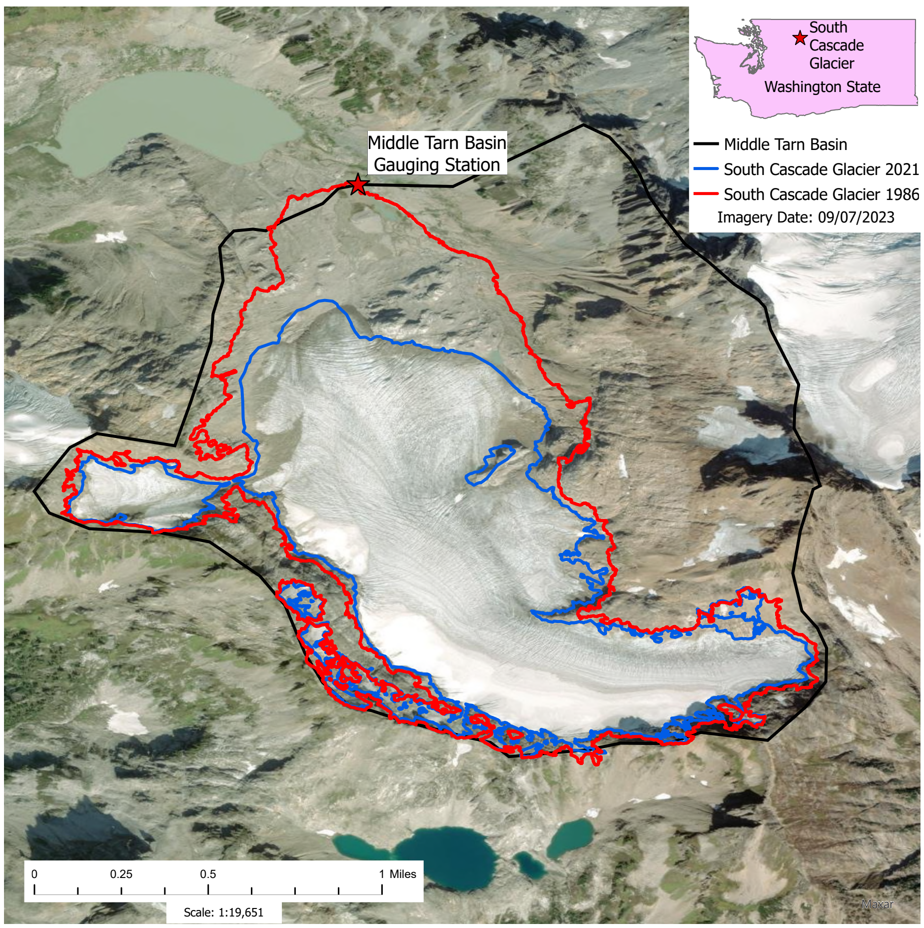


Figure 2: Map of the South Cascade Glacier in the North Cascades of Washington.

Symbol	Description
Z_g	Glacier surface elevation above sea level (m)
ELA	Equilibrium line altitude (m)
γ	Spinup mass balance equation gradient (m/year)
Q	Ice flux (m^2/day)
A	Flow rate factor ($5.87 * 10^{-19}\mathrm{Pa}^{-3}\mathrm{day}^{-1}$)
n	Flow law exponent (3)
p_{ice}	Density of ice ($917\mathrm{kg/m}^3$)
g	Acceleration due to gravity ($9.81\mathrm{m/s}^2$)
$\frac{\partial z_s}{\partial x}$	Slope of the glacier
H	Ice thickness (m)
b_s	Summer mass balance (m/day)
T	Temperature (°C)
M_{snow}	Snow melt factor (m/day/K)
M_{ice}	Ice melt factor (m/day/K)
b_w	Winter mass balance (m/day)
p	Precipitation (m/day)
α	Precipitation conversion factor
$Accum_{lower}$	Accumulation lower bound (m/day)
y	Year
$Accum_{upper}$	Accumulation upper bound (m/day)
s	Snow depth (m water equivalent)
$s_{melt-vol}$	Snow melt volume (m^3 water equivalent)
$Area_{basin}$	Basin area (m^2)
$Area_{glacier}$	Glacier area (m^2)
r_{vol}	Rain volume (m^3 water equivalent)
$g_{melt-vol}$	Glacial melt volume (m^3 water equivalent)

Table 1: Symbols Table

4 Methods

4.1 Model Development

4.1.1 Model Overview

Model Structure:

1. Ice dynamics modeled using SIA
2. Temperature degree day and precipitation mass balance model
3. Temperature degree day and precipitation (snow fall/melt and rain) model

There are two sections to getting a complete model run, the spinup and the data driven run. The spinup section of the model runs for 500 years and aims to replicate the state of the glacier in 1984 when weather data becomes readily available. This section uses the simple mass balance equation below

$$(Z_g - \text{ELA}) * \gamma / 365.25 \tag{1}$$

This uses a γ of 0.031 to calculate the mass balance in meters per day. When the spinup hits the year 1900 the ELA is shifted up from 1900m to 1930m to simulate the retreat state of the glacier. The figure below shows the model in 1958 (474 years of spinup), and 1986 (500 years of spinup and 2 years of data driven run) compared with the actual glacier in 1986 and in 1958 derived from a DEM (reference). The gaps in the actual glacier lines are due to the available DEMs being incomplete over the glacier area.

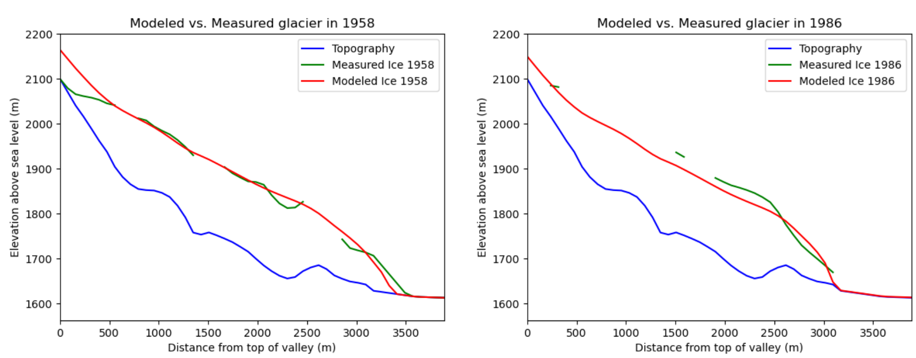


Figure 3: Comparisons of the modeled glacier and actual glacier in 1958 and 1986

Three factors were tuned to create the spinup glacier, the initial ELA, the shifted ELA in 1900 and γ. These variables were tuned to minimize the mean squared error (MSE) between the modeled glacier and the actual glacier in 1958 and 1986 when DEM's are available. The result is a MSE of 1.26% in 1958 and a MSE of 0.99% in 1986.

The data driven run uses the mass balance model described below to calculate the mass balance for the glacier instead of the simple mass balance equation used for spinup. This mass balance model is driven by temperature and precipitation data. The data driven run also contains a precipitation model in order to calculate the snow melt and rainfall in the basin.

4.1.2 Data driven run setup

The data driven section of the model run relies on a variety of data in order to run. It requires daily temperature and precipitation data, a bed topography, yearly glacier area, and total basin area to run. In order to tune the input parameters the model needs winter and summer mass balance data, and ice thickness data along the bed centerline to tune the glacier spinup. The model needs seven input parameters to run: ice melt factor, snow melt factor, lapse rate, start accumulation factor, end accumulation factor, avalanche percentage and precipitation conversion factor. The methods for tuning these parameters are described later in the paper

The centerline bed topography was calculated using latitude, longitude and elevation bed topography from Robert Jacobel (reference). This was calculated using ground penetrating radar. I used the ArcGIS (reference) Kriging Interpolation function to interpolate this point elevation data into a complete bed topography across the whole glacier. I then traced a line down the center of the glacier using the interpolated bed and got bed elevation points for the line from the interpolated data. I then used these new latitude, longitude, and elevation points to create the one-dimensional bed topography for the model. I explored using a centerline from RGI 7.0 (reference), but ultimately chose not to use it because the RGI centerline did not go down the center of the interpolated bed.

4.1.3 Ice Dynamics

SIA Model The SIA model is a one-dimensional model that uses the shallow ice approximation to approximate the ice dynamics of the glacier. This model calculates the one dimensional ice flux of the glacier using equation 2.

$$Q = \frac{2A}{n+2}(p_{ice}g|\frac{\partial z_s}{\partial x}|)^n \frac{H^5}{5} \tag{2}$$

Assumptions The SIA ice flux equation make several assumptions. First, the equations are 1-dimensional, so they neglect longitudinal stress, and ice only flows downhill. Second, the equations also assume that there is no basal sliding of the glacier. Third, the equations only use gravity as the driver of ice flow; they ignore other forces such as lateral and basal stress. Fourth, this set of equations assumes that the horizontal dimensions of the modeled glacier are much larger than the vertical dimensions.

Should I move the paragraph below???

The South Cascade Glacier was chosen because of some of these assumptions, it's horizontal dimensions are significant larger than its vertical dimensions (widths ranging from 400m-1200m and maximum ice thickness of 195m), and it is not a steep (average slope of 7.14 degrees along my centerline in 2021) or fast flowing glacier, allowing the SIA assumptions to hold.

4.1.4 Mass Balance Model

The mass balance of the glacier is calculated using temperature and precipitation data from the Diablo Dam weather station at 272m. The temperature at the glacier is calculated by using a month-specific lapse rate. The precipitation at the glacier is calculated by multiplying the precipitation at the Diablo Dam weather station by the precipitation conversion factor of 1.58 obtained from (reference). The ablation of the glacier is calculated by using a combination of an ice melt factor and a snow melt factor. Above the ELA the ablation is calculated by the equation

$$b_s = T * M_{snow} \tag{3}$$

Below the ELA the ablation is calculated by the equation

$$b_s = T * (M_{snow} + ((ELA - Z - g)/(ELA - min(Z_g))) * (M_{ice} - M_{snow})) \tag{4}$$

The result of this equation is the snow melt factor being used at the ELA and a linear increase in the melt factor until it hits the ice melt factor at the base of the glacier. This set of equations assumes that in the accumulation zone (above the ELA) the surface is always snow year round, and below the ELA the surface transitions from snow to ice as you decrease in elevation. The accumulation of the glacier is calculated using a similar linear equation that increases with time.

$$b_w = p*\alpha*(Accum_{lower}+((y-1984)/(2024-1984))*(Accum_{lower}-Accum_{upper})) \tag{5}$$

This results in the accumulation increasing with time until it hits the end accumulation at 2024.

4.1.5 Precipitation Model

The snow melt model uses precipitation and temperature data to melt and accumulate snow. The equation below is used to calculate the change in snow depth per timestep

$$s+= \begin{cases} p * \alpha & \text{if } T \leq 0, \\ -min((s * T), s) & \text{if } T > 0 \end{cases} \tag{6}$$

The snow melt is constrained so that there cannot be more melt than there is snow. The rain is simply modeled by $p * \alpha$ for positive temperatures.

The total volume of snow melt is calculated by the equation

$$s_{melt-vol} = (s * T) * (Area_{basin} - Area_{glacier}) \tag{7}$$

This give us the total volume of snow melting off the glacier. Any snow that falls on the glacier is factored into the mass balance equations and is either accumulated on the glacier or factored into the glacier melt. The volume of rain is calculated by

$$r_{vol} = p * \alpha * Area_{basin} \tag{8}$$

This calculates the rain for the whole basin, assuming that any rain that falls runs off immediately.

4.1.6 Glacial Melt Model

The glacial melt model uses the mass balance of the glacier to calculate how much volume the glacier is losing. The volume of runoff from the glacier is calculated by the equation

$$g_{melt-vol} = b_s * Area_{glacier} \tag{9}$$

4.2 Data Used for Model

The temperature and precipitation data used for the model is from the Diablo Dam weather station at 272m. The data is available from 1984-2024 and missing 298 days of temperature measurements and 292 days of precipitation measurements. The missing temperature data was interpolated using the interp function from the numpy python library (reference), the missing precipitation data was assumed to be 0. The glacier area data used in the model is from the USGS (reference). The basin area data was calculated using a DEM from the USGS and the basin outline shown in figure 1.

4.2.1 Model Calibration

The spinup initial ELA, ELA in 1900 and γ were optimized using the minimize function with the Nelder-Mead method from the scipy library (reference). The average of the MSE of the glacier in 1958 and the modeled glacier in 1958 and the MSE of the glacier in 1986 and the modeled glacier in 1986 was minimized.

The ice and snowmelt factors were calibrated using summer mass balance data available from the USGS from 1984-2024. I used the minimize function to minimize the mean squared error between the model mass balance and the USGS measured mass balance. The accumulation factors were calculated using the same methodology for the winter mass balance data available from the USGS from 1984-2024.

The avalanche percentage was optimized using the same methods as the mass balance variables, but instead of a mean squared error being minimized, the mean of the snow depth over the period 1984-2024 was minimized. The figures below show the modeled vs. measured winter and summer mass balance data.

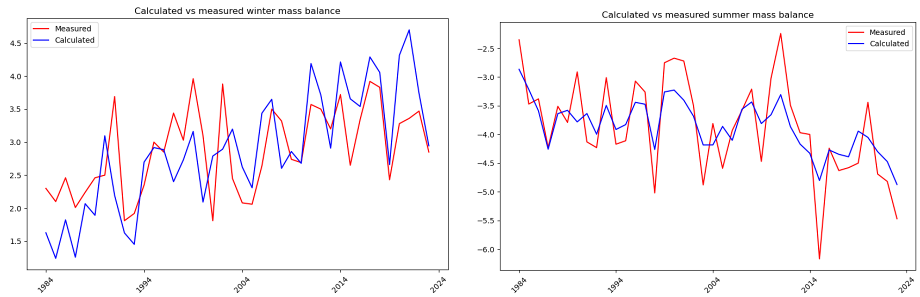


Figure 4: Comparison plots of the calculated vs measured winter and summer mass balance.

The figure below shows the resulting ice thicknesses computed by the model in 2021 compared to the 2021 DEM.

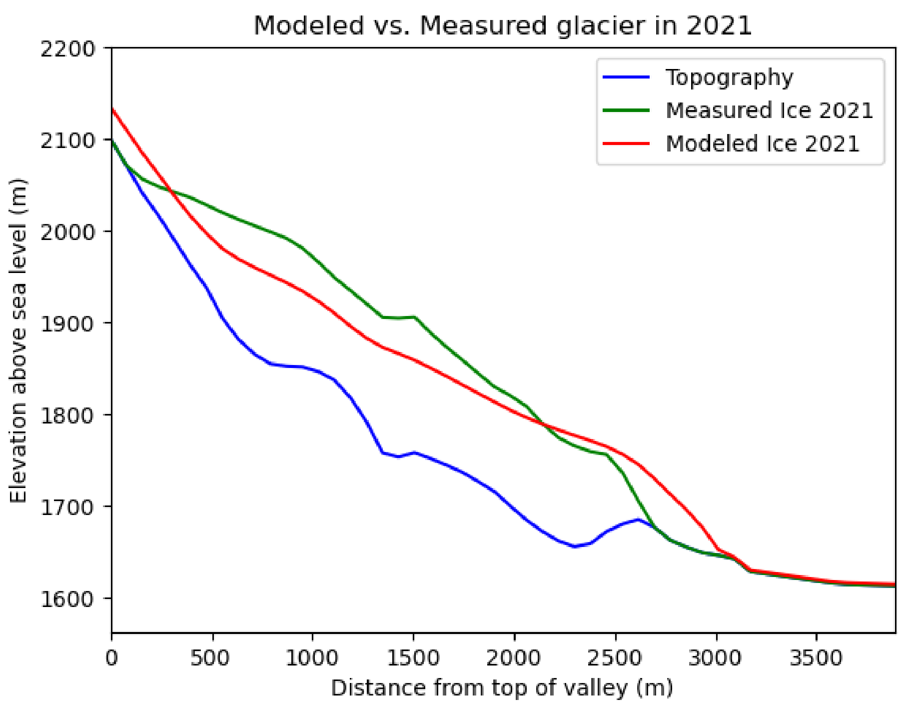


Figure 5: Model comparison of the ice thickness in 2021 compared to the actual glacier in 2021.

This figure shows the evolution of the glacier from 1984-2024

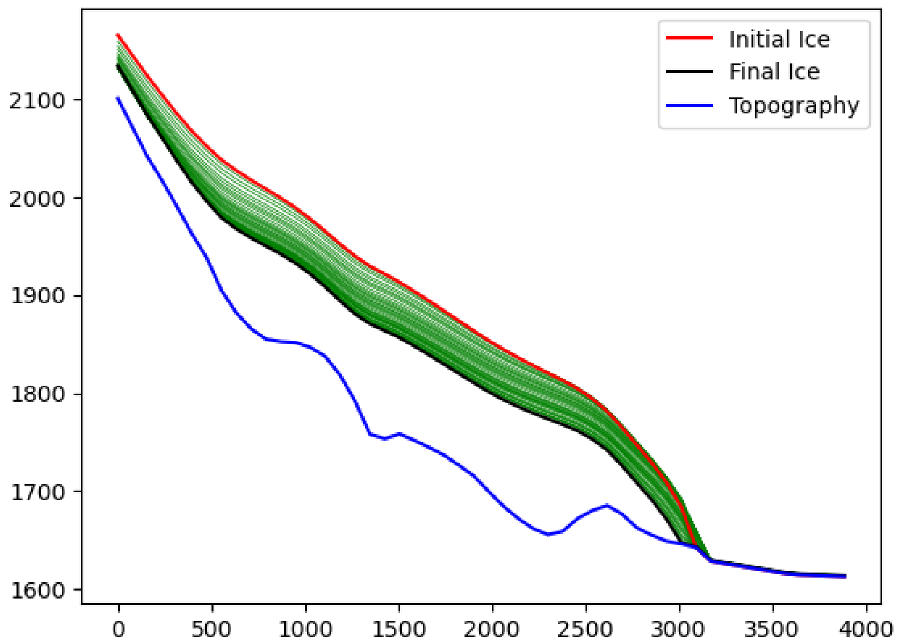


Figure 6: Output of the model run from 1984-2024.

4.2.2 Model Comparison

Running OGGM Model for the South Cascade Glacier The OGGM model was run using the run_with_hydro task from the oggm library. This model run used the GSWP3_W5E5 historical temperature and precipitation data (reference) to model the hydrology of the glacier from 1984-2019. Using the output of this model I was able to calculate the total runoff from the glacier using the melt_on_glacier_monthly, and liq_precip_on_glacier_monthly variables. In order to calculate the runoff off the glacier the melt_off_glacier_monthly, and liq_precip_off_glacier_monthly variables were averaged over the SIA model glacier basin area. The MSE of the OGGM model and my model is 1.55% and the MSE of the OGGM model and the measured runoff data is 3.22%.

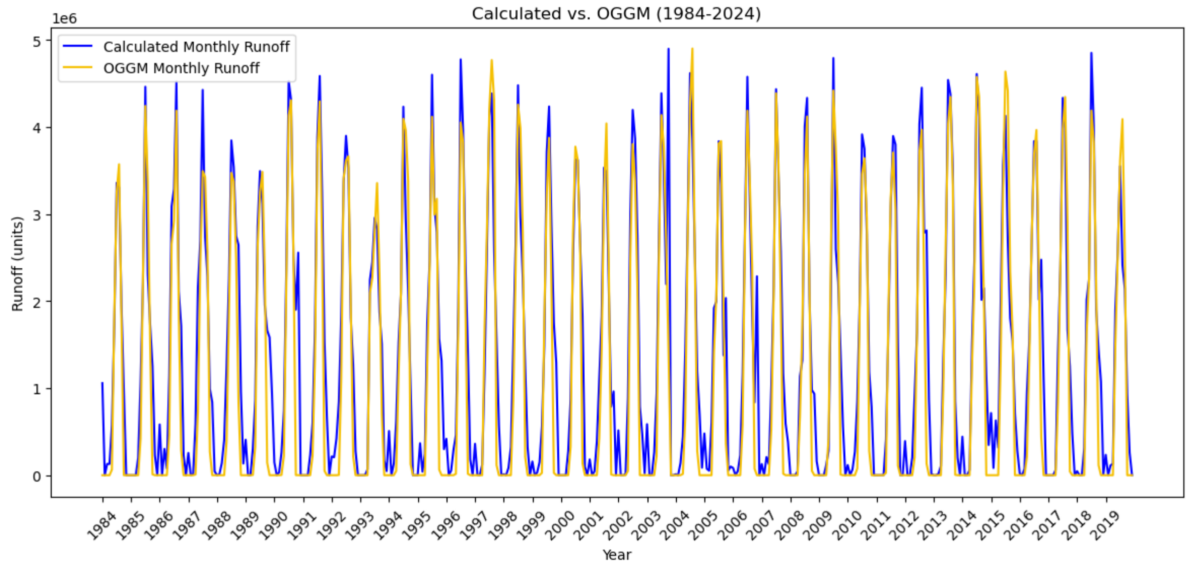


Figure 7: OGGM calculated runoff compared to SIA Modeled runoff form 1984-2019

5 Expected Results

5.1 Accuracy of SIA Model

The calculated runoff of the SIA-Mass Balance model was validated using measured streamflow data from 1992-2007, consisting of 2418 data points, which were condensed into 91 months. This data was measured using a stream gauge located just below the glacier (refer to Figure 1 for location). The data is in units of mm per day averaged over the basin area (4.46km^2). This was converted to cubic meters per day by dividing each value by 1000 and multiplying by the basin area in square meters. This daily data was then summed over the month to calculate a monthly runoff. Not every month was complete, so in order to validate the model, the model runoff was saved for the same days as the measured data and then summed over the month. The SIA model was able to calculate the runoff from the whole basin (snow melt, glacier melt and rainfall) with a mean squared error of 1.41%. Below is a graph of the real-world measured runoff data and my model calculated runoff

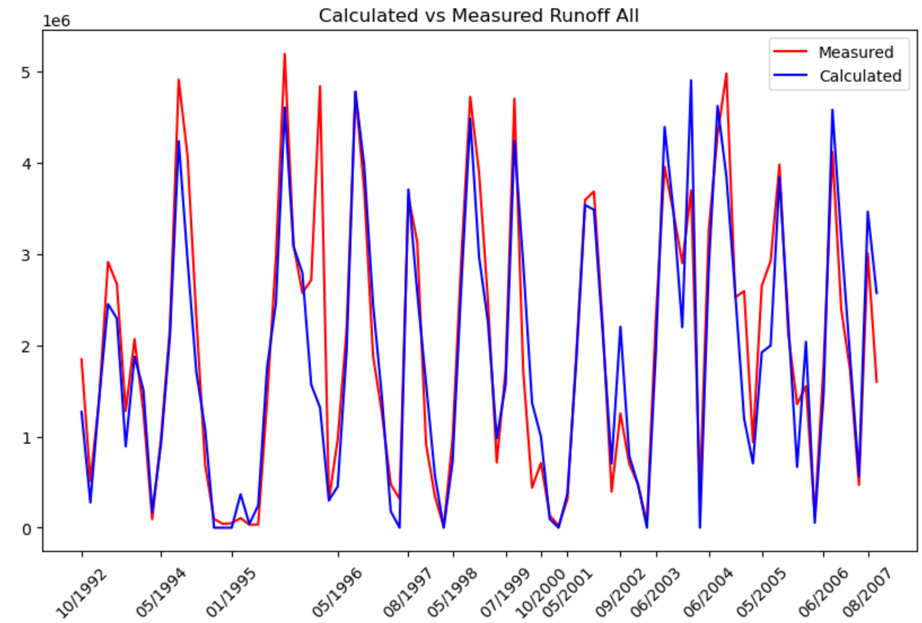


Figure 8: Measured vs. Calculated runoff data from 1992-2007

5.2 Accuracy of OGGM

add in info about comparing OGGM for complete months, and same for model runoff

The OGGM Model proved to be slightly less accurate when compared to the real-world measured runoff data. It was also run on slightly different temperature and precipitation data compared to my SIA-Mass Balance model. It used the GSWP3_W5E5 climate modeled data which goes back to 1970 (check this). It had a mean squared error of 3.22% for all of the runoff data from 1992-2007.

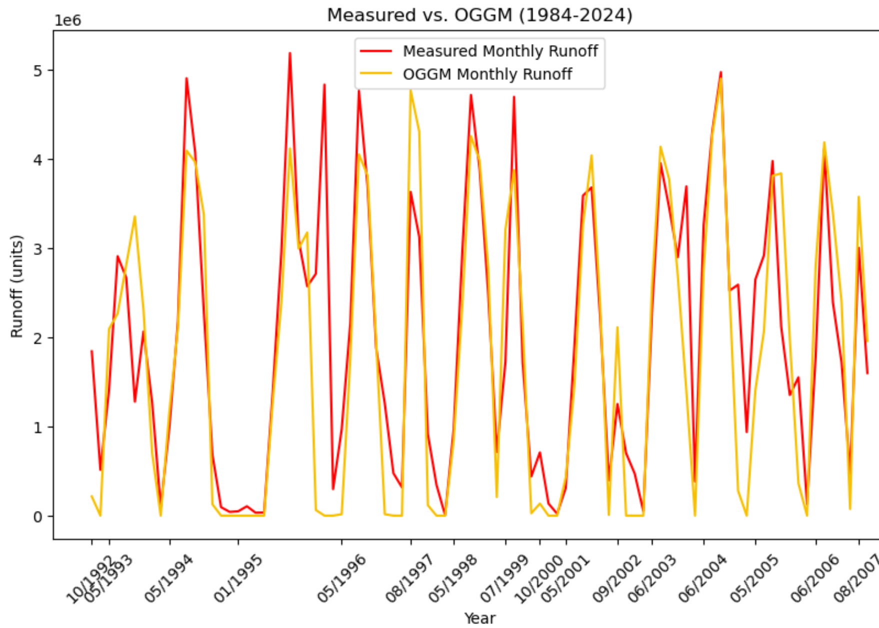


Figure 9: Measured vs. Calculated runoff data from 1992-2007

5.3 Comparison of Accuracy

The accuracy of the SIA-Mass Balance model is 2.3 times better than that of the more advanced OGGM model. This is partly due to the SIA model using real-world temperature and precipitation data and the OGGM model using modeled climate data which is less accurate. The SIA model input parameters are also tuned to the local climate of the glacier, whereas the OGGM model is not.

6 Implications of Research

6.1 Importance of Simplified Ice Dynamics in Numerical Glacier Modeling

These results show that complicated and computationally intensive mass balance and ice dynamics are not required to accurately model the runoff from small mountain glaciers. This means that we can use much simpler and less computationally intensive models such as the ones used in this paper to model the runoff from mountain glaciers much quicker than more complicated models. The modeling techniques used in this paper could easily be scaled to a much larger region if mass balance is available to tune the input parameters. Also the relative simplicity of the model used in this paper means that it is much easier to modify to add new features and customize to a specific region or glacier.

6.2 Applications

The work of this paper shows that complex models are not always required to accurately model the runoff from small mountain glaciers. This can have applications in everything from regional glacier modeling to water resource management as glaciers are a significant source of water for many communities around the world.

7 Discussion

7.1 What Worked

The runoff was modeled very accurately using only temperature and precipitation data. The model was calibrated using mass balance data, but none of the runoff data was used to tune any of the input parameters. The MSE of the modeled winter mass balance is 3.62% and the MSE of the modeled summer mass balance is 1.70%, but the MSE of the annual mass balance is 0%. In the parameter tuning the annual mass balance data was not used, only the individual summer and winter yearly data which means that through minimizing the error of those two parts of the mass balance, the yearly mas balance was modeled perfectly. While this study this study did not validate the precipitation model its accuracy can be assumed to be equal to that of the runoff. This is derived from the fact that the runoff consists of the glacier melt plus the snow melt and rainfall. If the yearly mass balance of the glacier has a 0% error then the error of the runoff must be solely due to the precipitation model. So therefore the MSE of the precipitation model must be 1.41%. Even though the glacier is contributing less than half of the total basin runoff on average, it contributes much more during the summer during the summer months as show in figure 11. As the glacier retreats it will start contributing less water to the basin runoff resulting in a decrease in total basin runoff. If the precipitation stays constant over time, but the glacier continues to retreat the basin runoff will see a steep decline particularly in early summer and fall when the glacier contributes most to the runoff. This is especially important considering that the Cascades see the least amount of precipitation during that portion of the year (reference). One might expect the glacier runoff to be reducing with time, but instead is stays constant, which I speculate is due to a rapidly warming climate is causing the glacier's melt to accelerate, meaning it is loosing a greater percentage of its total mass each year. This has a critical point where eventually the glacial runoff will start to decrease even if the glacier is still melting at an increasing rate, there simply isn't enough mass left to provide the same volume of runoff as the year before.

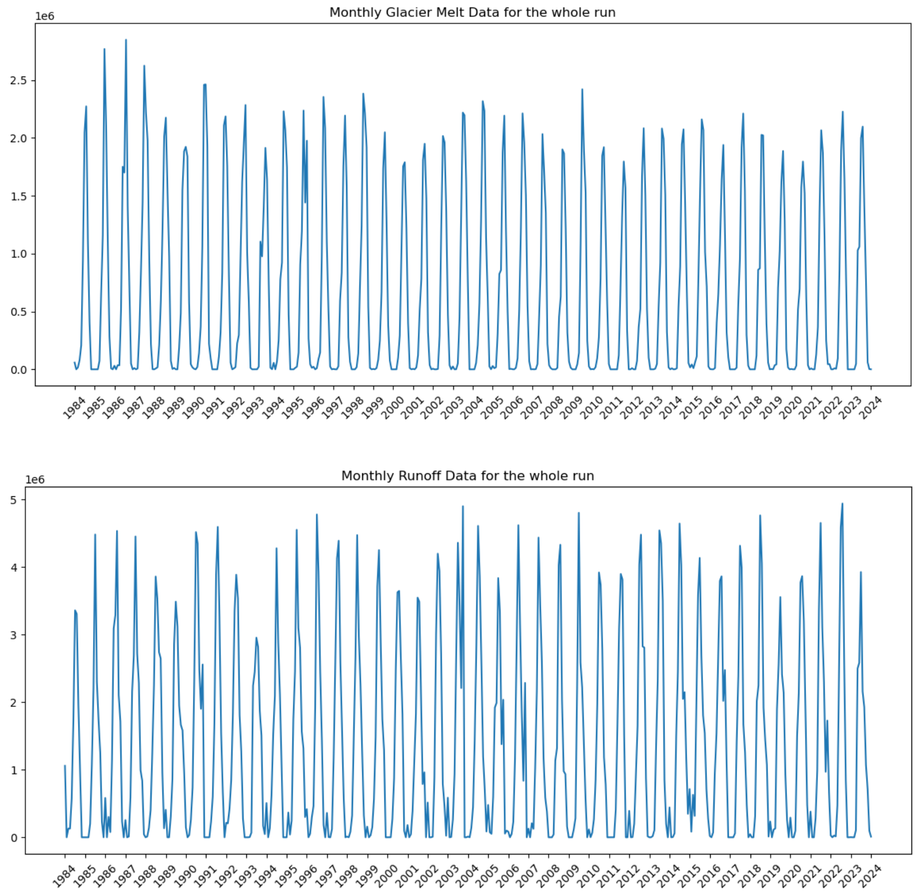


Figure 10: Comparison plots of the glacier runoff and total basin runoff.

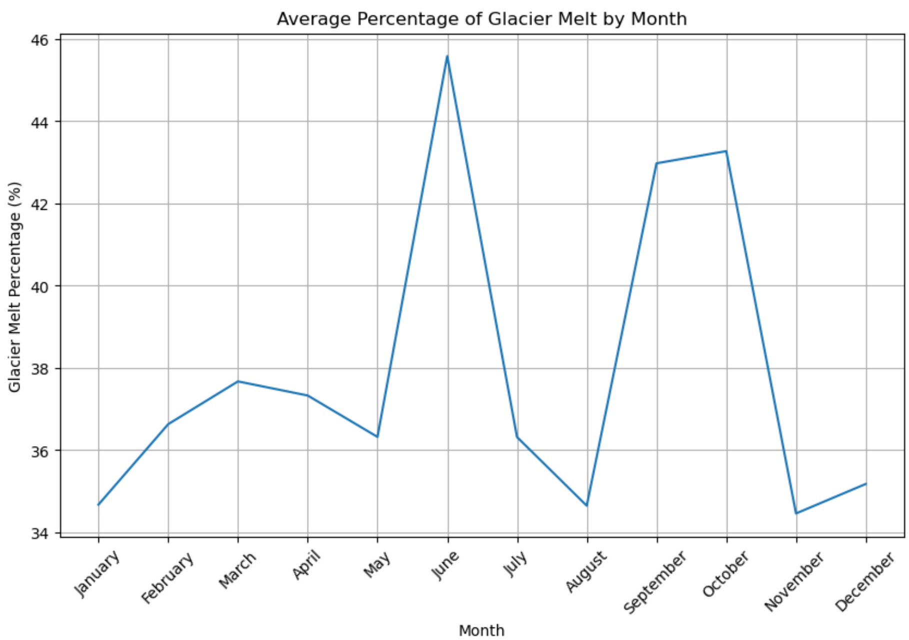


Figure 11: Average percentage from 1984-2024 that the glacier contributes to the monthly runoff

7.2 What Can Be Improved

The largest improvement to this model would be its runtime. It currently takes about 30 seconds to run for 1984-2024, which is most likely due to the use of the Python datetime library which is not very efficient. Eliminating the use of this library would make the model run much faster, but would make it more difficult to run. The datetime library is very useful when using date driven data such as the temperature and precipitation data in this model to keep track of the current date, but it is not very efficient.

7.3 Future Work

The next step for this project would be to run the model for a glacier near the South Cascade Glacier with a similar climate to see how region specific the input parameters are. Due to the input parameters being tuned for the mass balance of the glacier which largely depends on the local climate, it seems plausible that they would work reasonably well for a glacier with a similar climate. Another idea to explore would be to run the model with modeled climate data in order to run the model into the future to see how this glacier will evolve and how the basin runoff will evolve.

8 Conclusion

The SIA-Mass Balance model was able to accurately model the runoff from the South Cascade Glacier with a mean squared error of 1.41% for 16 months from 1992-2007, while the OGGM model has an accuracy of 3.22% for the same time period. This show that complex models are not always the best way to model the runoff from small mountain glaciers. A simpler model with parameters tuned to the local climate of the glacier can perform just as well, if not better than a more complex model.

9 References