Can a Shallow Ice Approximation model be used to model the water output of alpine glaciers?

Introduction

Glacial melting plays a significant role in the hydrology of mountain catchment areas as shown in Fountain and Tangborn, 1985. Accurately capturing glacial dynamics with numerical models is important for understanding how water sources in high mountain catchments will evolve. The runoff produced by glaciers, especially later in the summer months, is a major contributor to stream flow. Without the runoff from the glaciers, the rivers would be even lower during the driest summer months as shown in Figure 1.

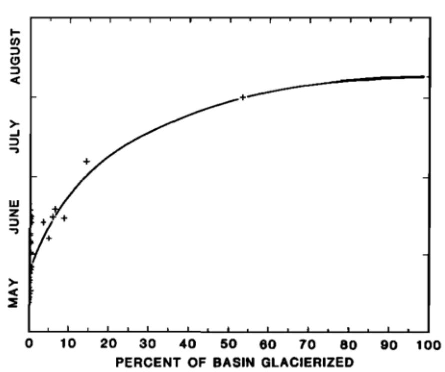

Fig. 4. Timing of peak specific runoff as a function of glacier cover for basins in the North Cascades, Washington.

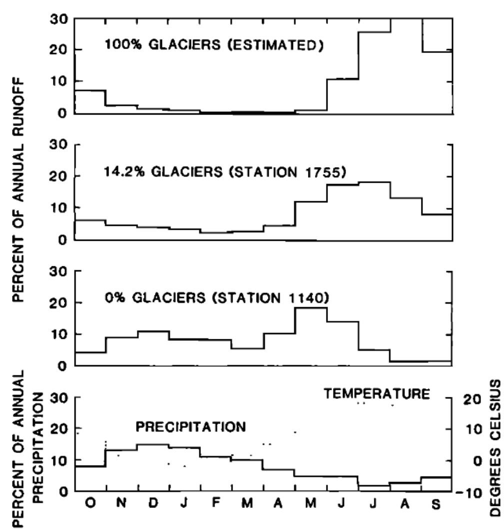

Fig. 3. Monthly fraction of the annual specific runoff for basins of various glacier covers. The monthly fraction of precipitation and mean monthly temperature (Snoqualmine Pass, Washington) are included for comparison.

Fig 1. Figures 3 and 4 from Fountain and Tangborn, 1985 showing how the percent of basin glacierized delays the peak runoff time of the basin

As glaciers melt and retreat, the change in mass takes the form of meltwater. This meltwater affects communities downriver in a variety of ways. For instance, many communities rely on these rivers as a water source for drinking and irrigation.

One of the best ways to find out how glaciers will affect the stream flow of rivers in the glacier basin is by using computer models to approximate the water discharge of the glaciers. Scientists have been using computers to model glaciers for several decades, such as this paper by Iken A, 1981. As computational resources have grown, these models have grown in complexity and resolution leading to very computationally expensive models, meaning that they take a lot of computing power to run. Many of these Advanced Ice Sheet Models (AISMs) use equations such as the 3-dimensional Stokes equations (Larour et al., 2012) to model the complicated ice dynamics of these glaciers.

AISMs are numerically very sophisticated; these models are also not very intuitive and often take a while to learn how to run. The advantage of AISMs is that they do a particularly good job of modeling ice dynamics. For some glaciers such as marine calving glaciers, accurate ice dynamics are crucial to accurately model them, as shown in Amaral et al., 2020. However, modeling glaciers in 3-dimensions is often unnecessary. If we can use simpler models, such as the Shallow Ice Approximation (SIA) model, to calculate the water runoff of small mountain glaciers in locations such as the Alps, then we can run these models over more glaciers and larger areas, and it is easier to incorporate them into larger hydrology models.

Literature review

As shown in Le Meur et al., 2004, there are significant differences in computational time between a SIA model and a Stokes model. When computing the free surface and associated velocity field, the SIA model took 1 minute of CPU time, whereas the Stokes model took 2 hours. This disparity grew even larger for 3D models. The authors show that there are some instances where SIA models do significantly worse than Stokes models, such as glaciers on steep slopes and glaciers in steep, narrow valleys because SIA models only approximate the Stokes equations. One of these approximations is to ignore horizontal stress gradients. This can cause a SIA model to deviate from a Stokes model significantly in predicted glacier flow and expansion. In one example, the resulting SIA model can have an upper free surface that is 15-20% greater than the Stokes model and velocities up to a factor of 2 greater (Le Meur et al., 2004). In the 2D model, the bed characteristics and slope become the limiting factor of the SIA model; Le Meur et al., 2004 note that the maximum velocity ratio of their SIA and Stokes models goes from 1.9 in a 3D model to 1.3 in a 2D model, which will differ depending on model configuration, but it tends to indicate that the horizontal stress gradients played a large part in this error. They found instances in which the SIA models performed well compared to Stokes models - particularly large flat glaciers with relatively free edges. One thing to note about this comparison study is that the authors are looking at the shape, area, and velocity profile of the glacier, whereas this study will focus on the water output (surface mass loss) of the glacier.

There are several papers, such as Le Muer et al., 2003 and Kessler et al., 2006 that use an SIA model for alpine glaciers. The consensus from those papers is that SIA models only work well on alpine glaciers with a low aspect ratio, defined as the thickness-to-extent ratio in Le Muer et al., 2004. The glacier used by this study will have a low aspect ratio and therefore a SIA model should work well to model it.

Additionally, there is precedent for using a SIA-Mass Balance model for modeling water runoff from glaciers (Naz et al. 2014). They used the SIA model to approximate the ice dynamics and a mass balance model to approximate the accumulation and ablation patterns on the glacier. As shown in their paper, the SIA model was able to accurately predict the glacier, and the coupled hydrological model was able to predict the stream flow accurately only overestimating the July flow by an average of 13% and underestimated the August and September flow by an average of 2%

Thesis Statement

How much do ice dynamics affect the model result when modeling small mountain glaciers for mass balance? I theorize that if using a simple 1-dimensional SIA-Mass Balance model on small mountain glaciers (with a low aspect ratio) in the Alps, the mass balance profile will have a much larger effect on the output of the model and its overall accuracy than the modeled ice dynamics. The results of the SIA-Mass Balance model and the Stokes model will be compared to the actual stream flow data to verify this hypothesis.

Methods

In this study, I will focus on modeling the Hochalmkees glacier in the Carinthia region of the Austrian Alps. Hochalmkees is roughly 2.415 square kilometers, has a mean elevation of 2936 meters, faces East, has an average slope of 18 degrees (Paul et al., 2020), an average thickness of 53 meters, and a max thickness of 155 meters (GlaThiDa Consortium, 2020). The glacier is small, not overly steep, and has a low aspect ratio such that a SIA model should be able to accurately model the ice dynamics. On the other hand, the glacier is large enough such that it has some movement and a measurable amount of water runoff throughout the year. It was also one of the few glaciers in the dataset (Brunner et al., 2024) where there is a stream gauge close to the glacier that measures a stream that contains only water from one glacier. This glacier is a small glacier compared to many of the other glaciers in the Alps and therefore does not have as much ice flow as many of the larger glaciers in the Alps which should be noted as a limitation of this study.

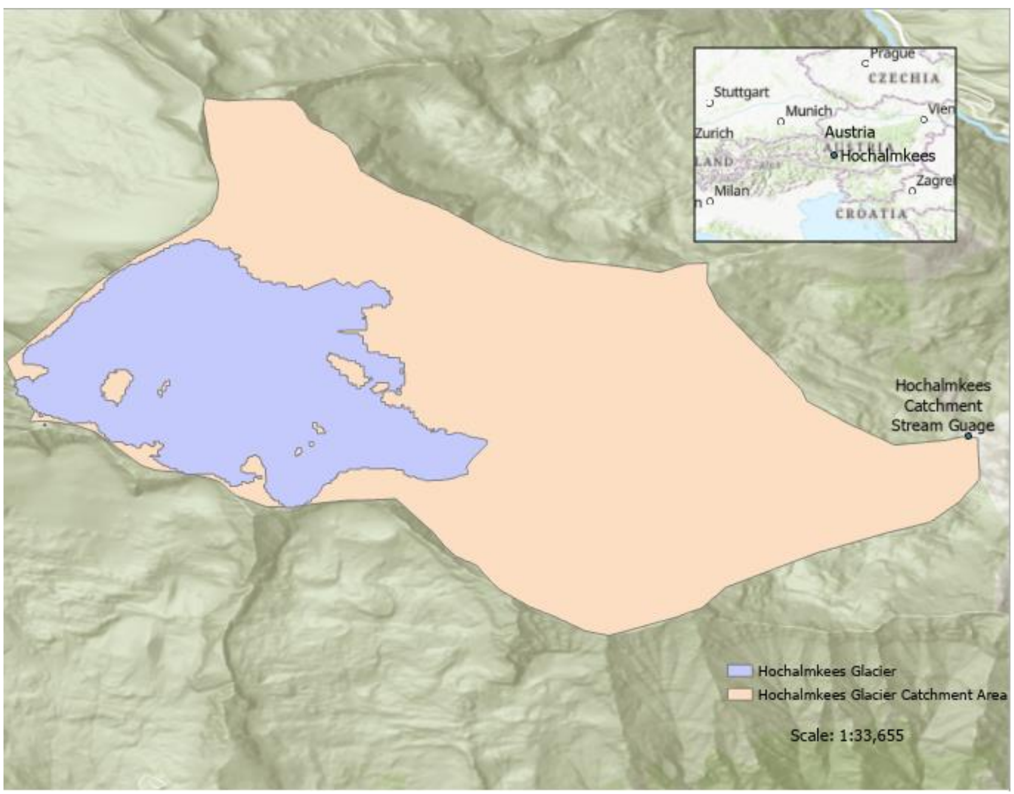

Fig 2. Hochalmkees Glacier area and catchment area

To test the hypothesis, a 1-dimensional SIA-Mass Balance model will be written in Python to model the mass balance of Hochalmkees. The model will account for ice dynamics by using the SIA approximation. To accurately calculate the water discharge of the glacier, the model needs to calculate the mass balance of the glacier. This model will use a mass balance profile created from historical precipitation and temperature data to model how much snow the glacier receives during the winter and how much the glacier melts during the summer. The SIA ice dynamics will account for the flow of the glacier down the valley. The model will calculate how much the glacier is melting and based on that, calculate the water discharge of the glacier. The SIA model will be compared to the Ice-sheet and Sea-level System Model (ISSM), which will be run with the Stokes equations. This will determine just how accurate the simplified ice dynamics of the SIA model are compared to the much more accurate ice dynamics of the Stokes model when modeling for the water discharge of a small mountain glacier. To validate both models, the results will be compared to stream flow data from Brunner et al., 2024.

To model a glacier, one must calculate the change in ice thickness at a given point in time. The SIA model utilized by this study will use the following set of equations to do this.

$$\frac{\partial H}{\partial t} = b_s + b_w - \frac{1}{W}\frac{\partial (QW)}{\partial x}$$

Where $\frac{\partial H}{\partial t}$ is the rate of change of ice thickness, b_s and b_w are the summer and winter mass balance respectively, W is the width of the glacier at ∂x , and $\frac{\partial Q}{\partial t}$is the along-flowline discharge defined as:

$$Q = \frac{2A}{n+2}\left(p_{ice} g \left|\frac{\partial z_s}{\partial x}\right|\right)^{n-1} \frac{H^5}{5}$$

Where A is the flow law parameter (this study will use the recommended value of $6.8 \cdot 10^{-15} s^{-1} (kPa)^{-3}$ (Paterson, 1994)), n is the flow law exponent (usually 3 for glaciers), p_{ice} is the density of ice (917 kg/m^3), g is the gravitational constant (9.81m/s^2), $\frac{\partial z_s}{\partial x}$ is the slope and H is the ice thickness. The width will be calculated from the glacier outlines available in LIMS Consortium, 2005. The mass balance equations will be calculated based on the temperature and precipitation data from Auer et al., 2007. The above equations make several assumptions. First, the equations neglect longitudinal stress,￼only flows downhill￼. Second, the equations also assume that there is no basal sliding of the glacier. Third, the equations only use gravity as the driver of ice flow; they ignore other forces such as lateral and basal stress. Fourth, this set of equations assumes that the horizontal dimensions of the modeled glacier are much larger than the vertical dimensions.

The Stokes equations used for the ISSM model are defined in Larour et al., 2012.

Expected Results

To calculate the accuracy of the SIA-Mass Balance model, the glacial water discharge (surface mass loss) data will be compared against the measured catchment water discharge (Brunner et al., 2024) minus the monthly average precipitation. This will allow me to compare the water discharge of the glacier directly with the measured catchment water discharge without snowmelt and rainfall skewing results. The accuracy will be calculated by comparing the difference in glacial water discharge between the two models and the measured catchment data using the equation $Error = \frac{|measured - modeled|}{modeled} \cdot 100$ to calculate a percentage accuracy. This will show how accurate the SIA model can be compared to the Stokes model. To easily visualize this, I will create plots of the water discharge per month. I will also compare the seasonal output of the glacier via the two models to understand how it is changing over time and how well each model can predict that. The goal of this research is to be able to say how accurate the SIA-Mass Balance model is in predicting water discharge.

Implications of Research

The result of this research project will tell us how important it is to accurately model the ice dynamics of small mountain glaciers when predicting mass balance changes. If the hypothesis is correct, then researchers will be able to run less computationally expensive models to achieve a similar output as a much more sophisticated and computationally expensive model such as the Stokes model. As referenced in Verbunt et al., 2003, glacier models are often just one component of hydrology or other types of models. If we can make certain assumptions, like using an SIA-Mass Balance model when modeling a small mountain glacier as part of a larger hydrology model, that glacier component will run much faster. As a result, the whole model will run faster which means researchers can run this model over a larger area or run it many more times. This could lead to more accurate water predictions for water management teams and subsequently better water management techniques.

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