Can a Shallow Ice Approximation Model be Used to Model the Water Output of Alpine Glaciers?

Abstract

1 Introduction

2 Literature Review

As shown in Le Meur et al., 2004, there are significant differences in computational time between a SIA model and a Stokes model. When computing the free surface and associated velocity field, the SIA model took 1 minute of CPU time, whereas the Stokes model took 2 hours. This disparity grew even larger for 3D models. The authors show that there are some instances where SIA models do significantly worse than Stokes models, such as glaciers on steep slopes and glaciers in steep, narrow valleys because SIA models only approximate the Stokes equations. One of these approximations is to ignore horizontal stress gradients. This can cause a SIA model to deviate from a Stokes model significantly in predicted glacier flow and expansion. In one example, the resulting SIA model can have an upper free surface that is 15–20% greater than the Stokes model and velocities up to a factor of 2 greater (Le Meur et al., 2004). In the 2D model, the bed characteristics and slope become the limiting factor of the SIA model; Le Meur et al., 2004 note that the maximum velocity ratio of their SIA and Stokes models goes from 1.9 in a 3D model to 1.3 in a 2D model, which will differ depending on model configuration, but it tends to indicate that the horizontal stress gradients played a large part in this error. They found instances in which the SIA models performed well compared to Stokes models—particularly large flat glaciers with relatively free edges. One thing to note about this comparison study is that the authors are looking at the shape, area, and velocity profile of the glacier, whereas this study will focus on the water output (surface mass loss) of the glacier.

There are several papers, such as Le Muer et al., 2003 and Kessler et al., 2006, that use an SIA model for alpine glaciers. The consensus from those papers is that SIA models only work well on alpine glaciers with a low aspect ratio, defined as the thickness-to-extent ratio in Le Muer et al., 2004. The glacier used by this study will have a low aspect ratio and therefore a SIA model should work well to model it.

Additionally, there is precedent for using a SIA-Mass Balance model for modeling water runoff from glaciers (Naz et al., 2014). They used the SIA model to approximate the ice dynamics and a mass balance model to approximate the accumulation and ablation patterns on the glacier. As shown in their paper, the SIA model was able to accurately predict the glacier, and the coupled hydrological model was able to predict the stream flow accurately—only overestimating the July flow by an average of 13% and underestimating the August and September flow by an average of 2%.

3 Thesis Statement

4 Methods

4.1 Study Site

South Cascade Glacier in the North Cascades of Washington

4.2 Physical Characteristics and Relevance

4.3 Availability of Hydrological and Glacial Data

4.4 Model Development

4.4.1 Model Overview

4.4.2 Ice Dynamics

SIA Model

Assumptions

4.4.3 Snow and Rain Model

The snow melt model uses precipitation and temperature data to melt and accumulate snow. This model uses 14 elevation bins and keeps track of the snow depth in each bin. The equation below is used to calculate the change in snow depth per timestep

$$\text{snow depth}+ = \begin{cases} p * \alpha & \text{if } T \leq 0, \\ -\text{minimum}((s * T), \text{snow depth}) & \text{if } T > 0 \end{cases}$$

Where p is the precipitation, α is the precipitation conversion factor, s is the snow melt factor, and T is the temperature. The snow melt is constrained so that there cannot be more melt than there is snow. The rain is simply modeled by $p * \alpha$ for positive temperatures

The total volume of snow is calculated by the equation

$$\text{snow melt volume} = (s * T) * (\text{area of bin} - \text{area of glacier})$$

This give us the total volume of snow melting off the glacier. The glacial melt is calculated elsewhere. The volume of rain is calculated by

$$\text{rain volume} = p * \alpha * (\text{area of bin})$$

This calculates the rain for the whole basin, assuming that any rain that falls off the glacier runs off immediately.

4.4.4 Glacial Melt Model

Data Used for Model

4.4.5 Model Comparison

Running OGGM Model for the Same Glacier

Comparing Outputs of Both Models

Validation Using Real-World Streamflow Data (Brunner et al., 2024)

5 Expected Results

5.1 Accuracy of SIA

5.2 Accuracy of ISSM

5.3 Comparison of Accuracy

6 Implications of Research

6.1 Importance of Simplified Ice Dynamics in Numerical Glacier Modeling

6.2 Applications

7 Discussion

7.1 What Worked

7.2 What Didn't Work and Why

7.3 What Can Be Improved

8 Conclusion

8.1 Summary of Results

8.2 Conclusion of Model Accuracy

9 References