Can a Shallow Ice Approximation - Mass Balance Model Be Used to Calculate the Water Output of Alpine Glaciers?

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Abstract

This study evaluates whether a simple one-dimensional Shallow Ice Approximation (SIA) coupled with a temperature–degree-day and precipitation mass balance model can accurately model runoff from small mountain glaciers. The South Cascade Glacier in Washington State will be used to test this theory. The model consists of a 500-year spinup run to create the modeled glacier in 1984, and a 1984–2024 data driven run forced by daily temperature and precipitation data (Diablo Dam weather station). Seven input paramter are used: ice and snow melt factors, upper and lower accumulation bounds, temperature lapse rates, a precipitation conversion factor and an avalanche percentage. The melt factors and accumulation factors were optimized using USGS mass balance data. Model performance was benchmarked against the Open Global Glacier Model (OGGM) over 1992–2007 runoff records. The SIA-Mass Balance mode achieved a relative RMSE of 30.35%, Nash–Sutcliffe efficiency of 0.80, and Kling–Gupta efficiency of 0.88 for daily average runnof per month, outperforming OGGM (RMSE 45.96%, NSE 0.53, KGE 0.72). Accuracy further improved on annual (RMSE 16.11%) and 16 year (5.06%) timescales. Disabling ice flux altered errors by less than 1%, indicating limited importance of complex ice dynamics modeling for runoff prediction in this context.

These results demonstrate that a computationally efficient SIA-Mass Balance model can reliably estimate long-term runoff from small, low-aspect-ratio glaciers. Future work should refine precipitation interpolation, extend application to other glacier basins, and incorporate projected climate forcing.

1 Introduction

1.1 Importance of Glacial Melting in Mountain Hydrology

Glacial melting plays a significant role in the hydrology of mountain basins as shown in (Fountain & Tangborn, 1985). In order to understand how the hydrology of these mountain basins will evolve as glaciers melt and retreat, scientists create computer models to predict this evolution. The runoff from these mountain basins is often used as a water source for downstream communities (Barnett et al., 2005). As shown in (Fountain & Tangborn, 1985), basins with greater glaciation produce more runoff. As the glaciers in these basins melt and retreat, change in meltwater patterns and amounts can affect communities that rely on it in a variety of ways.

1.2 Role of Numerical Modeling in Understanding Glacial Runoff

One of the best ways to understand how glaciers affect the runoff of the basins that they occupy is by using computer models. Scientists have been using computers to model glaciers for several decades, such as (Iken, 1981). As computational resources have grown, these models have grown in complexity and resolution, leading to highly computationally expensive models. These models have proven to be very accurate in modeling the past and present state of a variety of glacier types all around the world (Eis et al., 2021; Farinotti et al., 2020).

1.3 Challenges in Computational Modeling of Glaciers

Many of these more advanced models can be quite complex to run because they require a variety of input parameters and different configurations to run the model. The goal of this paper is to write a simple Shallow Ice Approximation (SIA)-Mass Balance model that is easy to run and understand while still being generalizable to a variety of small mountain glaciers. The advantage of using much more advanced models is that they often do a much better job at modeling the ice dynamics of glaciers. For some glaciers such as marine calving glaciers, accurate ice dynamics are crucial for accurate modeling, as shown in (Amaral et al., 2020). However, on smaller mountain glaciers, modeling in three dimensions is often unnecessary and simpler models such as the SIA can be used (Le Meur et al., 2004). This paper will compare the results of a simple SIA-Mass Balance model to the results of the Open Global Glacier Model (OGGM)(Maussion et al., 2019), a more advanced model using the ice-thickness continuity equation to model ice dynamics.

2 Literature Review

2.1 Prior Research on SIA vs. Stokes Models

Table 1 shows that there are significant differences in computational time between an SIA model and a Stokes model. (Le Meur et al., 2004) shows that there are some instances where SIA models do significantly worse than Stokes models, such as glaciers on steep slopes and in narrow valleys, because the SIA equations only approximate the Stokes equations. One of these approximations is to ignore horizontal stress gradients which can cause an SIA model to deviate from a Stokes model significantly in predicted glacier flow and expansion. In the 2D model, the bed characteristics and slope become the limiting factor of the SIA model. They found instances in which the SIA model performed well compared to Stokes model–particularly for large flat glaciers with relatively free edges. One thing to note about (Le Meur et al., 2004) is that the authors are looking at the shape, area, and velocity profile of the glacier, whereas this study will focus on the water runoff (surface mass loss) of the glacier.

Metric	**SIA model**	**Stokes model**
CPU time (2D free surface)	< 1 min	2 hrs
CPU time (3D free surface)	2 min	4 days
Upper free surface	1.15-1.2×	
Relevant surface velocities	2.0×	
Max velocity (3D model)	1.9×	
Max velocity (2D model)	1.3×	
Cross-sectional area	1.5×	

Table 1: Summary of key performance and output metric differences between the SIA and Stokes models (Le Meur et al., 2004). (×) indicates ratio of the metric between the SIA and Stokes models.

2.2 Applying SIA Models to Alpine Glaciers

There are several papers, (Le Meur & Vincent, 2003; Le Meur et al., 2004; Kessler et al., 2006), that use an SIA model for alpine glaciers. The consensus from these papers is that SIA models only work well on alpine glaciers with a low aspect ratio, defined as the thickness-to-extent ratio (Le Meur et al., 2004). The glacier used in this study will have a low aspect ratio and therefore the SIA equations should work well to model it.

2.3 Applying SIA Models to Glacial Runoff

Additionally, there is precedent for using an SIA-Mass Balance model for modeling water runoff from glaciers. In (Naz et al., 2014) an SIA model is used to approximate the ice dynamics and a mass balance model to approximate the accumulation and ablation patterns on the glacier. As shown in their paper, the SIA model was able to accurately predict the glacier, and the coupled hydrological model was able to predict the stream flow accurately—only overestimating the July flow by an average of 13% and underestimating the August and September flow by an average of 2%.

3 Thesis Question

Can a simple SIA - Mass Balance model be used to calculate the water runoff of small mountain glaciers?

If using a simple one-dimensional SIA-Mass Balance model on small mountain glaciers (with low aspect ratios), the mass balance profile will have a much larger effect on the output of the model and its overall accuracy than the modeled ice dynamics. The results of the SIA-Mass Balance model and the OGGM model will be compared to the actual stream flow data to answer this question.

3.1 Study Site: South Cascade Glacier, Washington State

This study will focus on modeling the South Cascade Glacier in the North Cascades region of Washington State. This glacier is roughly 1.68 km^2 with widths ranging from 400m-1200m. It has a mean elevation of roughly 1900m, (GLIMS Consortium, 2005) faces North, has an average thickness of 99m, and a maximum thickness of 195m (GlaThiDa Consortium, 2020). The glacier is not overly steep (average slope of 7.14 degrees along this study's centerline in 2021), and has a low aspect ratio (thickness to extent ratio). Therefore an SIA model should be able to accurately model its ice dynamics. On the other hand, the glacier is large enough to exhibit some movement and produce a measurable amount of runoff throughout the year.

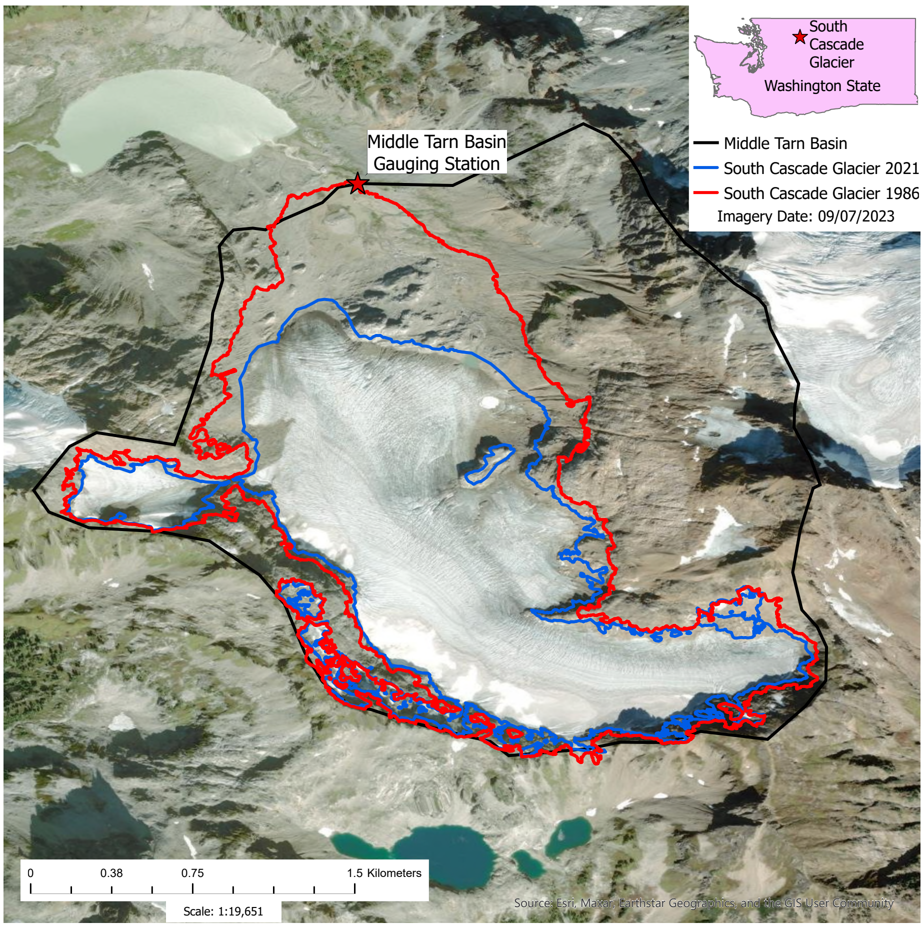

Figure 1: Map of the South Cascade Glacier in the North Cascades of Washington State.

Symbol	Description
Z_g	Glacier surface altitude (m)
ELA	Equilibrium line altitude (m)
γ	Spinup mass balance equation gradient (m/year)
Y	Current year
Q	Ice flux (m^2/day)
A	Flow rate factor ($5.87 * 10^{-19}\mathrm{Pa}^{-3}\mathrm{day}^{-1}$)
n	Flow law exponent (3)
ρ	Density of ice ($917\mathrm{kg/m}^3$)
g	Acceleration due to gravity (9.81 $\mathrm{m/s}^2$)
$\frac{\partial z_s}{\partial x}$	Slope of glacier
H	Ice thickness (m)
T	Temperature (°C)
m.w.e	meters water equivalent
M_s	Snow melt factor (m.w.e/°C/day)
M_i	Ice melt factor (m.w.e/°C/day)
P	Precipitation (m)
α	Precipitation conversion factor
b_s	Summer mass balance (m/day)
b_w	Winter mass balance (m/day)
$Accum_l$	Accumulation factor lower bound ($\mathrm{m.w.e_{ice}/m.w.e_{precipitation}}$)
$Accum_u$	Accumulation factor upper bound ($\mathrm{m.w.e_{ice}/m.w.e_{precipitation}}$)
S_d	Snow depth (m.w.e)
S_{mv}	Snow melt volume ($\mathrm{m.w.e}^3$)
$Area_b$	Basin area (m^2)
$Area_g$	Glacier area (m^2)
R_v	Rain volume (m^3)
G_{mv}	Glacial melt volume ($\mathrm{m.w.e}^3$)

Table 2: Symbols Table

4 Methods

4.1 Model Development

4.1.1 Model Overview

Model Structure:

1. SIA ice dynamics model
2. Temperature degree day and precipitation mass balance model
3. Temperature degree day and precipitation (snow fall/melt and rain) model

There are two sections to the complete model run, the spinup run and the data driven run. The spinup section of the model runs for 500 years and aims to replicate the state of the glacier in 1984 when weather data becomes readily available. The spinup run starts with the bed topography and no ice and uses a simple mass balance equation to accumulate ice to simulate the glacier state in 1984. The data driven run starts in 1984 with the simulated spinup glacier and applies a temperature and precipitation driven mass balance model to model how the glacier changes until 2024 when the weather data ends. The data driven run also contains the precipitation model to calculate the runoff from snow melt and rainfall.

4.1.2 Spinup Run

This section uses a simple mass balance equation 1 to model the accumulation and ablation of the glacier.

$$((Z_g - \text{ELA}) * \gamma)/365.25 \tag{1}$$

A γ is used to calculate the mass balance change in meters per day. When the spinup hits the year 1900 the ELA is shifted up from 1903m to 1930m to force the glacier to retreat.

4.1.3 Data Driven Run Setup

The data driven run of the model relies on a variety of data in order to run. It requires daily temperature and precipitation data, a bed topography, yearly glacier area, and total basin area. In order to tune the input parameters the model also needs winter and summer mass balance data. The model requires seven input parameters to run: ice melt factor, snow melt factor, temperature lapse rate, accumulation factor lower bound, accumulation factor upper bound, avalanche percentage and precipitation conversion factor. The details of how these input parameters are calibrated and used in the model are explained in the following sections.

4.1.4 Ice Dynamics

SIA Model The SIA model is a one-dimensional model that uses the shallow ice approximation equations to calculate the ice dynamics of the glacier. This model calculates the one-dimensional ice flux of the glacier using equation 2.

$$Q = \frac{2A}{n+2}(\rho g|\frac{\partial z_s}{\partial x}|)^n \frac{H^5}{5} \tag{2}$$

Assumptions The SIA ice flux equation makes several assumptions. First, the equation is one-dimensional, so it neglects longitudinal stress, and ice only flows downhill. Second, the equation assumes that there is no basal sliding of the glacier. Third, it only uses gravity as the driver of ice flow; it ignores other forces such as lateral and basal stress. Fourth, this equation assumes that the horizontal dimensions of the modeled glacier are much larger than the vertical dimensions.

4.1.5 Mass Balance Model

The mass balance of the glacier is calculated using temperature and precipitation data from the Diablo Dam weather station at 272m. The temperature at the glacier is calculated by using a month-specific lapse rate. The precipitation at the glacier is calculated by adjusting the precipitation at Diablo Dam with a precipitation conversion factor of 1.58 (Rasmussen, 2009). The ablation of the glacier is modeled using a combination of an ice melt factor and a snow melt factor. Above the ELA the ablation is calculated by equation 3 for $T \geq 0$. The ELA is computed by finding the elevation at which the previous year's mass balance changes from positive to negative.

$$b_s = T * M_s \tag{3}$$

Below the ELA the ablation is calculated by equation 4 for $T \geq 0$.

$$b_s = T * (M_s + \left(\frac{ELA - Z_g}{ELA - min(Z_g)}\right) * (M_i - M_s)) \tag{4}$$

The result of this equation is the snow melt factor being used at the ELA and a linear increase in the melt factor until it hits the ice melt factor at the base of the glacier. This set of equations assumes that in the accumulation zone (above the ELA) the glacier surface is always snow year round, and below the ELA the glacier surface transitions from snow to ice as you decrease in elevation.

The accumulation of the glacier is calculated using equation 5 for $T < 0$.

$$b_w = P * \alpha * (Accum_l + \left(\frac{Y - 1984}{2024 - 1984}\right) * (Accum_l - Accum_u)) \tag{5}$$

This equation starts the accumulation factor at the accumulation factor lower bound at the start of the data driven run in 1984 and linearly increases it to accumulation factor upper bound by the end of the run in 2024.

4.1.6 Precipitation Model

Snow Model The snow model uses precipitation and temperature data to accumulate and melt snow. To calculate the change in snow depth equation 6 is used.

$$S_d{+}= \begin{cases} P * \alpha & T \leq 0, \\ -\min(|M_s * T|, S_d) & T > 0 \end{cases} \tag{6}$$

The snow melt calculation is constrained so that there cannot be a negative snow depth. The total volume of snow melt is calculated by equation 7.

$$S_{mv} = (M_s * T) * (Area_b - Area_g) \tag{7}$$

This is the total volume of snow melt on the non-glacierized areas of the basin. Any snow that falls on the glacier is factored into the glacier mass balance equations and accumulated on the glacier.

Rain Model The rain is simply modeled by $P * \alpha$ for $T \geq 0$. The volume of rain is calculated by equation 8.

$$R_v = P * \alpha * Area_b \tag{8}$$

This calculates the volume of rain for the whole basin because rain is not factored into the glacier mass balance model. This assumes that that any rain that falls runs out of the basin on the day it falls.

Avalanche Model In order to stop snow accumulating to unrealistic depths at high elevations an avalanche model is used. This model simulates snow avalanches above 2120m once per year on a randomly chosen date between January and March, which changes every year. The amount of snow that avalanches is determined by the avalanche percentage input parameter. On the chosen day the snow depth above 2120m is decreased by the avalanche percentage and that snow is evenly distributed below 1900m. The result of this is the average monthly snow depth hitting approximately 2m.w.e at its lowest point in September and October before starting to accumulate again. Without the avalanche model it is approximately 5.5m.w.e in September and October. This decrease in snow depth yields a more realistic model and improves runoff model accuracy by about one percentage point. This is due to there being more snow available for melt during the summer at the lower elevations where the temperature is higher.

4.1.7 Glacial Melt Model

The glacial melt model uses the mass balance of the glacier to calculate how much volume the glacier is losing. The volume of runoff from the glacier is calculated by equation 9.

$$G_{mv} = b_s * Area_g \tag{9}$$

4.2 Model Calibration

4.2.1 Data Used for Model

The temperature and precipitation data used for the model is from the Diablo Dam weather station at 272m (U.S. Geological Survey Benchmark Glacier Program, 2020). This data is available from 1984-2024 and is missing 298 days of temperature data and 292 days of precipitation data. The missing temperature data was interpolated using the interp function from the NumPy python library (Harris et al., 2020); the missing precipitation data is assumed to be 0. Since the Diablo Dam weather station is at a significantly lower altitude than the South Cascade Glacier, a lapse rate is applied to the temperature data. This lapse rate was empirically calculated using data (Baker et al., 2018) from a weather station at 1830m next to the South Cascade Glacier. This data is available daily from 2010-2018. Using these two sets of data an monthly average lapse rate was calculated between Diablo Dam and the South Cascade Glacier (Table 3).

Jan	Feb	Mar	Apr	May	Jun
-3.35	-5.45	-5.77	-6.79	-6.61	-6.28
Jul	Aug	Sep	Oct	Nov	Dec
-5.30	-5.35	-4.95	-4.94	-4.73	-4.52

Table 3: Monthly temperature lapse rates (°C/km)

The glacier area data used in the model is from the USGS (U.S. Geological Survey Benchmark Glacier Program, 2020). The basin area was calculated from the basin outline shown in Figure 1.

The centerline bed topography was calculated using latitude, longitude and elevation bed topography data from Robert Jacobel (Fountain & Jacobel, 1997). The ArcGIS Kriging Interpolation function (Environmental Systems Research Institute, 2011) was used to interpolate this point elevation data into a complete bed topography across the whole glacier. Then a line was traced down the center of the glacier using the interpolated bed and elevations were extracted to create the bed topography of the centerline. Finally, these latitude, longitude, and elevation points were used to create the one-dimensional bed topography in the model.

4.2.2 Calibration

Spinup Calibration The spinup initial ELA, ELA in 1900 and γ were optimized using the minimize function with the Nelder-Mead method from the SciPy python library (Virtanen et al., 2020). The optimization was done by minimizing the Root Mean Squared Error (RMSE) of the modeled and measured glacier ice thicknesses in 1958 and 1986. The measured glacier ice thickness was calculated using digital elevation models (DEMs) of the glacier in 1958 and 1986

(McNeil et al., 2019). The result is an average RMSE of 20.24m with RMSE's of 21.36m and 19.11m in 1958 and 1986 respectively.

$$\mathrm{RMSE} = \left(\frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(O_i^{1958} - S_i^{1958}\right)^2} + \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(O_i^{1986} - S_i^{1986}\right)^2}}{2} \right) * 100 \tag{10}$$

where:

$$O = \text{Measured ice thicknesses (m)}, \quad S = \text{Modeled ice thicknesses (m)}$$

Summer Mass Balance Calibration The two input parameters that control the summer mass balance are the ice melt factor and the snow melt factor. They were calibrated using yearly summer mass balance data available from the USGS from 1984-2024 (U.S. Geological Survey Benchmark Glacier Program, 2020). The same minimize function used for the spinup calibration was used to minimize the RMSE between the modeled mass balance and the measured mass balance from the USGS. The result is a RMSE of 0.5m

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(O_i - S_i\right)^2} \tag{11}$$

where:

$$O = \text{Measured mass balance (m)}, \quad S = \text{Modeled mass balance (m)}$$

Winter Mass Balance Calibration There are also two factors that control the winter mass balance: the upper and lower bound accumulation factors. These were tuned using the same method as the summer mass balance input parameters using the USGS yearly winter mass balance data available from 1984-2024 (U.S. Geological Survey Benchmark Glacier Program, 2020). The result is an RMSE (equation 11) of 0.54m.

Avalance Percentage Calibration The avalanche percentage was optimized using the same minimize function as the mass balance parameters. Instead of minimizing the RMSE, the mean of the snow depth over the whole data driven run was minimized. The mean snow depth was minimized because almost all of the snow on the non-glacierized areas of the basin should melt during the summer. By minimizing the mean snow depth over the whole run, only the snow depth during the summer will be minimized since the avalanche percent cannot change the total amount of snow present in the whole basin, just the elevation it is at. If there is more snow at lower elevations then more snow will melt during the summer. This resulted in an avalanche percentage of 32%.

4.2.3 Model Comparison

Running OGGM for the South Cascade Glacier OGGM was run using the run_with_hydro task from the OGGM python library. This model run used the GSWP3_W5E5 historical temperature and precipitation data (Lange et al., 2021) to model the hydrology of the glacier from 1984-2019. The total runoff from the glacier was calculated using the melt_on_glacier_monthly, and liq_precip_on_glacier_monthly variables. In order to calculate the runoff from the non-glacierized areas, melt_off_glacier_monthly, and liq_precip_off_glacier_monthly variables were averaged over the SIA model glacier basin area.

5 Results

5.1 Accuracy of SIA Model

The calculated runoff of the SIA-Mass Balance model was validated using measured streamflow data from 1992-2007, consisting of 2418 daily measurements, which were aggregated into 91 months (Krimmel, 1993, 1994, 1995, 1996, 1997, 1998, 1999, 2000, 2001, 2002; Bidlake et al., 2004, 2005, 2007, 2010). This data was measured using a stream gauge (Middle Tarn Gauging Station) located just below the glacier (refer to Figure 1 for location). The data is in units of mm per day averaged over the basin area (4.46km^2). The runoff values were then converted to cubic meters per day and then summed over the month to calculate a monthly runoff. There were several months in the dataset that were incomplete, so in order to validate the model, the model runoff was saved for the same days as the measured data and then a daily average was calculated for each month. The error for this model will be in units of daily average runoff volume per month due to the incomplete months in the validation dataset. This paper will use three types of error to evaluate the model, relative Root Mean Square Error (RMSE), Nash–Sutcliffe Efficiency (NSE), and Kling–Gupta Efficiency (KGE), defined by the equations below.

$$\text{RMSE} = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(O_i - S_i\right)^2}}{\bar{O}} * 100 \tag{12}$$

$$\text{NSE} = 1 - \frac{\sum_{i=1}^{N}\left(O_i - S_i\right)^2}{\sum_{i=1}^{N}\left(O_i - \bar{O}\right)^2} \tag{13}$$

$$\text{KGE} = 1 - \sqrt{(r-1)^2 + (\alpha-1)^2 + (\beta-1)^2} \tag{14}$$

where:

$$O = \text{Observed runoff} \quad S = \text{Simulated runoff}$$
$$r = \text{Correlation coefficient}, \quad \alpha = \frac{\sigma_S}{\sigma_O}, \quad \beta = \frac{\mu_S}{\mu_O}$$

The results for these three methods are 30.35% (RMSE), 0.80 (NSE) and 0.88 (KGE). It is important to note that these errors are primarily the model error for

May-November due to the lack of runoff data outside of those months. There is one instance of January, February, March and April, and six instances of December.

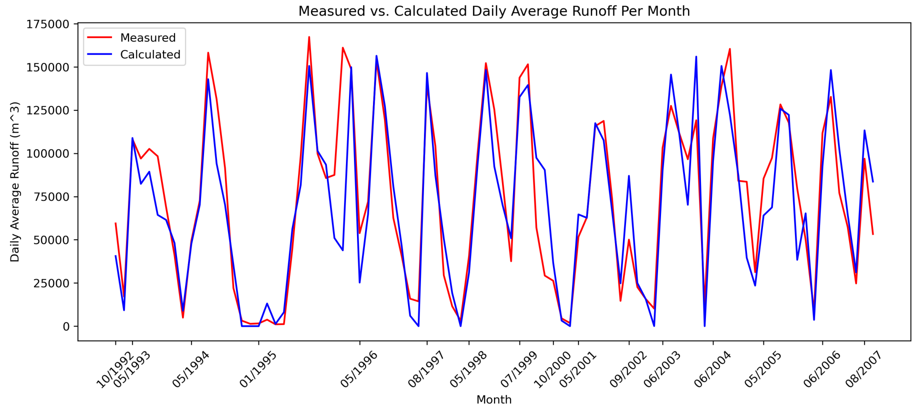

Figure 2: Measured vs. Calculated runoff data from 1992-2007

5.2 Accuracy of OGGM

The RMSE(12), NSE(13) and KGE(14) of OGGM are 45.96%, 0.53 and 0.72 respectively. OGGM proved to be slightly less accurate than the SIA-Mass Balance model when compared to the measured runoff data.

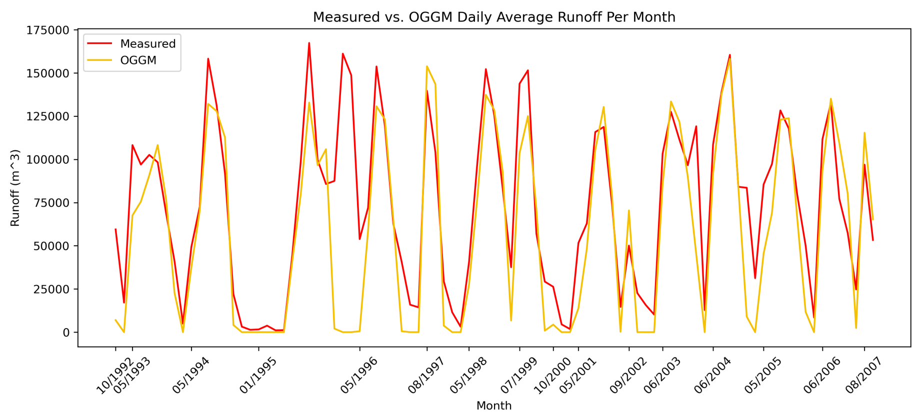

Figure 3: Measured vs. OGGM runoff data from 1992-2007

6 Discussion

6.1 Comparison of Model Accuracies

The SIA-Mass Balance model is roughly 2.1 times (averaged over the three error metrics) more accurate than OGGM. One reason for the higher accuracy of the SIA-Mass Balance model is likely due to it using real-world temperature and precipitation data and OGGM using modeled climate data which is less accurate. The SIA model input parameters are also tuned to the local climate of the glacier which may contribute to its higher accuracy.

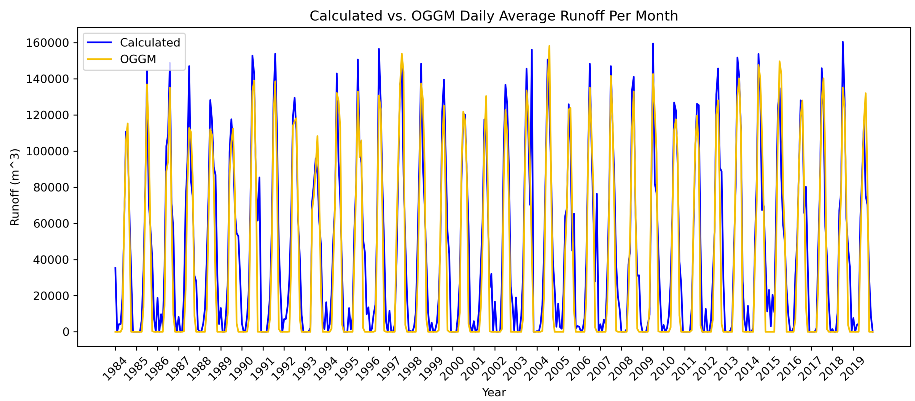

Figure 4: Calculated vs. OGGM runoff from 1984-2019

6.2 Spinup Model Error

Figure 5 shows the difference in the actual glacier vs the modeled glacier in 1958 and 1986. The difference between the modeled and measured glaciers at the top of the valley stands out. The modeled glacier is much thicker in this location compared to the actual glacier. This may be due to different accumulation patterns on the actual glacier compared to the model, or due to error in the bed topography data. The gaps in the measured ice are due to the DEM's not being complete over the entire glacier area.

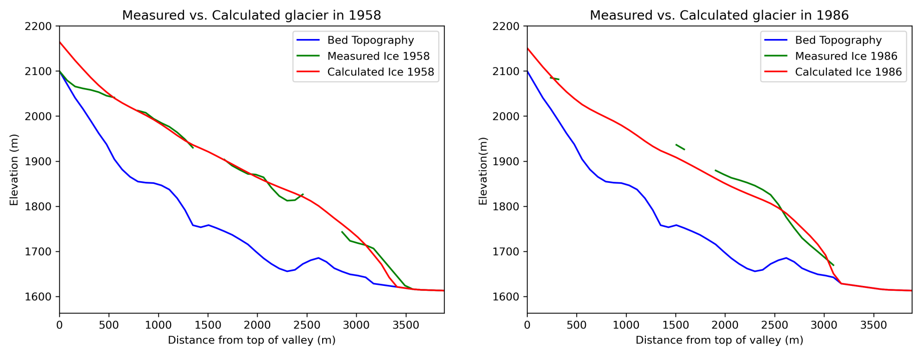

Figure 5: Glacier from DEM compared with calculated glacier in 1958 and 1986

6.3 Mass Balance Model Error

From the missing temperature and precipitation data one can note a few interesting errors in the mass balance model. There are five years with significant gaps (at least one month of missing data) in the temperature and precipitation data. These years are 1984 (missing December), 1985 (missing January), 1987 (missing July), 1997 (missing March and September) and 2009 (missing May-July). The year with the highest error in winter mass balance is 1997 probably due to it missing March data. The measured winter mass balance that year is 3.44m, and the model calculated a winter mass balance of 2.65m. One possibility for this is that there was a large amount of snowfall during March of that year, but since the precipitation data is missing for that month, the glacier accumulation is 0. According to PRISM (PRISM Group, 2014) the South Cascade Glacier received 0.57m of precipitation during March 1997. Using the mass balance equations above, the accumulation on the glacier during March 1997 with 0.57m of precipitation would be 0.97m. This would increase the winter mass balance for that year from 2.65m to 3.62m. With the updated value in the modeled winter mass balance, the RMSE (11) drops from 0.55m to 0.53m. This illustrates just how important consistent and reliable weather data is for models like these because a month of missing weather data can skew the results.

Figure 6 shows that the calculated summer mass balance does a good job of capturing the trends in the summer mass balance, but not the amplitude. The modeled summer mass balance consistently doesn't reach the same magnitude as the measured summer mass balance. One reason for this could be because the model does not take cloud cover into account. For years where the measured summer mass balance was particularly low, there could have been heavy cloud cover which would have reduced the amount of solar radiation reaching the glacier and thus reduced the amount of melt. There are many nuances in the weather that can change the amount of melt that occurs on the glacier. The

SIA-Mass Balance model used in this paper was designed to capture all of these nuances with two melt factors. Using two melt factors provides a good generalization of the glacier melt shown by the low RMSE for the summer mass balance data but perhaps does not accurately represent the glacial melt on shorter timescales.

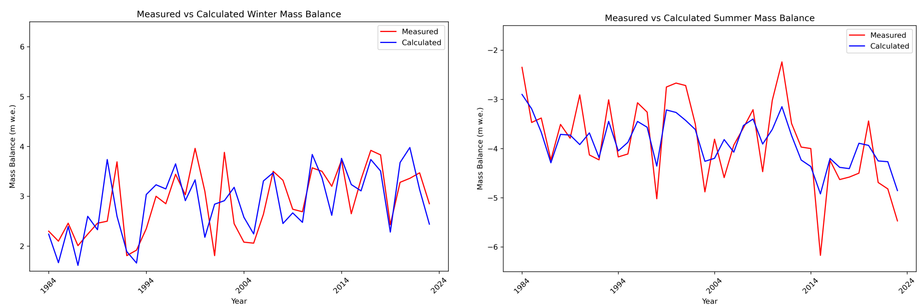

Figure 6: Comparison plots of the measured vs. calculated winter and summer mass balance.

6.4 Runoff Model Error

The largest problem with the runoff model lies with the precipitation model. This model calculates the type of precipitation that falls (rain or snow), based on the temperature. It assumes that if the temperature is greater than or equal to 0° Celsius then the precipitation is rain, and if the temperature is less then 0° Celsius then the precipitation is snow. While this is a good generalization, it is not always true as precipitation can fall as snow when the temperature is right above 0° Celsius and vice versa. Additionally, there is some amount of error in the calculated temperature lapse rate which can effect the calculated temperature at the glacier. When the actual temperature at the glacier is right around 0° Celsius, and precipitation is falling, the error in the lapse rate can cause the precipitation to fall as the wrong type.

Shown in Figure 2, there is a large difference between the modeled and measured runoff in November 1995. This is likely due to the model predicting the large amount of precipitation during that month falling as snow because the model temperatures are just below 0° Celsius, but in reality it most likely fell as rain. This also highlights the types of error that can be caused by using a proxy weather station that is not present at the glacier such as the weather data used by this model. With more accurate temperature data for the glacier, then the model might correctly predict the precipitation falling as rain.

It is also important to note that out of the 298 missing days of temperature and 292 missing days of precipitation, 63 days of temperature and 64 days of precipitation were during the 1992-2007 period where runoff data is available. Most notable of this missing data is the month of September 1997. This entire month is missing temperature and precipitation data, and this entire month is included in the measured runoff data. Looking at the error for this month it has a relative RMSE (12) of 27.24% which is very close to the overall relative RMSE (12) of 29.26%. This means that the error this month is quite typical for the model. The interpolated temperatures for this month range from 10° Celsius to 5° Celsius, so the glacier is melting and no glacier accumulation is missed. As shown in Figure 7, during September glacier melt is contributing the majority of the runoff to the basin, so having 0m of precipitation in the model for September 1997 would not cause as much error as if there was no precipitation for December or January when lack of snowfall could affect the runoff the following summer.

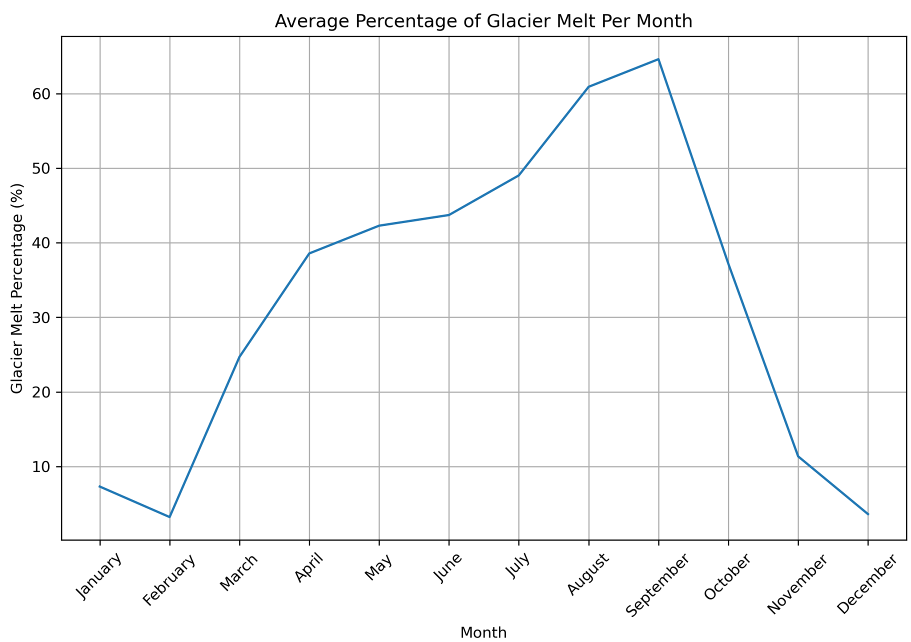

Figure 7: Average percentage from 1984-2024 that the modeled glacier (ice and snow melt) contributes to the monthly runoff

These two examples illustrate that missing precipitation data may be much more detrimental to model accuracy than missing temperature data due to temperature data being much easier to interpolate than precipitation data.

The model shows significant improvement in modeling the runoff on longer timescales. The relative RMSE (12) drops from 30.35% to 16.11% when the daily average runoff per month data is aggregated into daily average runoff per year. Part of this improvement may be due to all of the precipitation being accounted for over the course of a year. If the model calculates the precipitation falling as the wrong type for a certain month than the error will be off for that month but over the course of a year that precipitation will be accounted for. If it actually fell as rain but the model calculates it falling as snow, once the summer is over, that snow will have melted and will have been included in the runoff. This is further proved by the relative RMSE (12) of the summed runoff data over the whole period being 5.06% showing that on even longer timescales (16 years) the model performs even better.

6.5 Interpretations of Model Results

Even though the glacier is contributing slightly less than half of the total basin runoff on average (47.39%), it contributes much more during the summer months as shown in Figure 7. Due to the setup of the mass balance model there is no way to easily distinguish how much of the glacier melt is made up of glacial ice melting and snow melting off the surface of the glacier. So while Figure 7 shows the average monthly percent of runoff due to glacier melt, part of that is snow melting off the glacier surface.

As the glacier retreats it will contribute less and less water to the basin runoff resulting in a decrease in total runoff. With a smaller glacier surface area the peak snow melt will shift earlier in the summer as the snow melts off faster due the more snow accumulating on rocky surfaces, and not glacier ice which keeps the snow cool throughout the summer. As shown in Figure 7, the peak of the glacier runoff occurs in September, which coincides with some of the lowest average monthly precipitation amounts as shown in Figure 8.

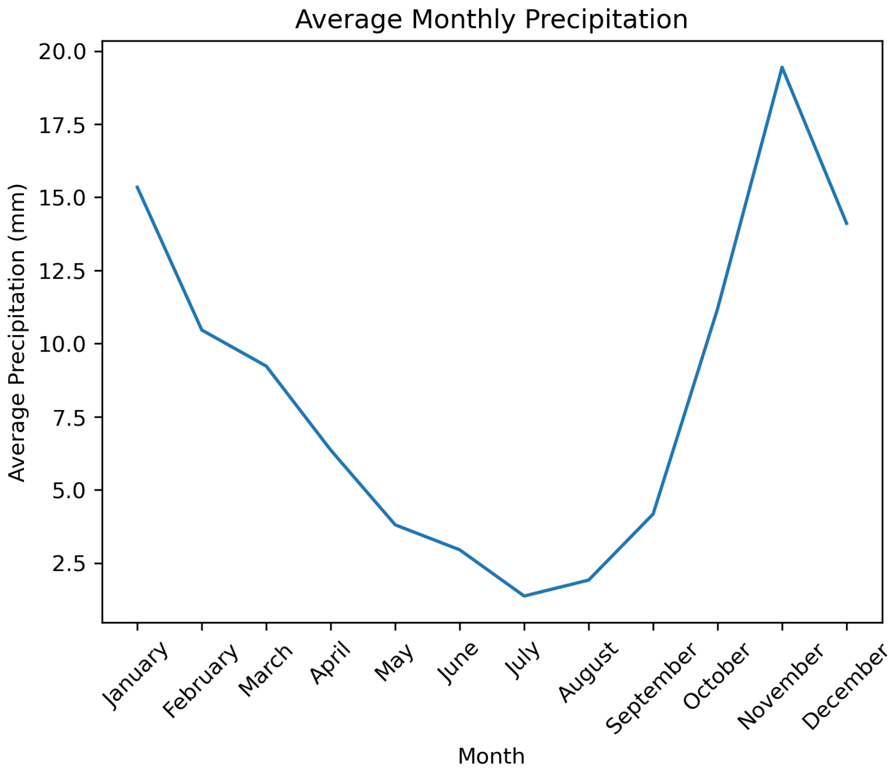

Figure 8: Average amount of precipitation per month from 1984-2024 at the Diablo Dam weather station. Most of the precipitation from October-April is snow

6.6 Overall Model Performance

On longer timescales (yearly to decadal) the SIA-Mass Balance model performs quite well. The error in the runoff data drops quite significantly as the timescale increases. The factors that contribute to the error on shorter (monthly) timescales such as the precipitation are better accounted for on longer timescales, and also are rendered less significant due to the aggregation of the data. Overall for the simplicity and generalizability of the SIA-Mass Balance model it performs quite well. With more accurate temperature and precipitation data at the glacier, the model accuracy could be improved even further.

6.7 Implications of Research

6.7.1 Importance of Simplified Ice Dynamics in Numerical Glacier Modeling

The results of the SIA-Mass Balance model show that complicated and computationally intensive mass balance and ice dynamics are not required to accurately model the runoff from small mountain glaciers. This means that much simpler and less computationally intensive models (such as the one used in this paper) can be used to to model the runoff from small mountain glaciers significantly faster than more complicated models. The modeling techniques used in this paper could easily be scaled to a much larger region if mass balance is available to tune the input parameters. Additionally, the relative simplicity of the SIA-Mass Balance model means that it is much easier to add new features and customize to a specific region or glacier.

The insignificance of ice dynamics in modeling the runoff from small mountain glaciers, is further proved by the runoff error of the SIA-Mass Balance model not changing significantly when the ice flux is set to 0. The relative RMSE increased by roughly 0.4% and the NSE and KGE decreased by roughly 0.1. This means that the ice dynamics of the glacier are not a significant factor contributing to the accuracy of the glacier runoff. A possible reason for this is that the South Cascade Glacier is in an active state of retreat which can be seen in the model and the glacier outline in Figure 1. Since the glacier is retreating, the model does not need the ice dynamics in order to move more glacier ice into the ablation zone where it can melt, since much of the glacier is already in this zone.

6.7.2 Applications

The work of this paper shows that complex ice dynamics and precipitation models are not always required to accurately model the runoff from small mountain glaciers. This can have applications in many areas, from regional glacier modeling to water resource management as glaciers are a significant source of water for many communities around the world. It could also be used to understand if a glacier may produce an outburst flood by measuring the water runoff from the glacial valley and comparing it to the modeled runoff. If the modeled runoff is significantly higher than the measured runoff, it could be due to a large amount of water being stored in the glacier.

7 Future Work

As addressed in earlier sections, this model has limited performance on shorter timescales (less than a year). A large portion of these limitations are probably due to the precipitation model. The easiest way to improve this would be to have more accurate temperature and precipitation data at the glacier, but there are some ways the model itself could be improved as well. For instance, having a dry and wet temperature lapse rate for each month could improve the accuracy of the modeled temperature at the glacier. Also, the missing precipitation data could be filled in using a supplementary data source or the average monthly precipitation. Additionally increasing the resolution of the model could help improve accuracy but would also increase the runtime.

The next step for this project would be to run the model for a glacier near the South Cascade Glacier with a similar climate to see how region specific the input parameters are. Due to the input parameters being tuned for the mass balance of the glacier which largely depends on the local climate, it is plausible that they would work well for a glacier with a similar climate.

Another idea to explore is to run the model with modeled climate data in order to run it into the future to see how this glacier will evolve and how the basin runoff will evolve. As shown in (Fountain & Tangborn, 1985), the peak runoff of a glacierized basin is a function of how much of the basin is glacierized. It would be interesting to see how the peak runoff of this basin shifts as the glacier retreats when the model is run into the future.

8 Conclusion

To answer the thesis question of this paper–yes–a simple SIA-Mass Balance model can be used to model the runoff from small mountain glaciers, but it works best on longer timescales. The SIA-Mass Balance model is able to model the runoff from the South Cascade Glacier with a RMSE, NSE and KGE of 30.35%, 0.80 and 0.88 respectively, for 91 months from 1992-2007, while OGGM has errors of 45.96%, 0.53 and 0.72 for the same time period. This shows that while complex models like OGGM might work well on much larger regions, for a single small glacier basin, a simpler model like the SIA-Mass Balance model used in this paper can work just as well, if not better.

From looking at the model error on various time scales, one can see that on longer time scales the SIA-Mass Balance model does quite well. More advanced models may be able to model the runoff on shorter timescales with more advanced methods and higher model resolution, but simple models like the one used in this study can work well on longer (yearly to decade) timescales.

The simple SIA-Mass Balance model used in this study has potential to work well for modeling the runoff from small glaciated mountain basins on longer timescales, and shorter timescales with less accuracy. There are many ways this model could be improved to be more accurate, while still remaining simple and computationally efficient. This model could also be used to model the runoff from other small mountain glaciers with similar climates, likely with limited tuning of the input parameters, but more work needs to be done to confirm this.

Appendix

ELA	ELA 1900	γ
1903	1930	0.0309

Table 4: Spinup Run Input Parameters

M_s	M_i	$Accum_l$	$Accum_u$	Avalanche %	α	Lapse rates
-0.003	-0.00347	0.91	1.41	32%	1.58	Table 3

Table 5: Data Driven Run Input Parameters

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Thanks to the foresight of the USGS in creating the Benchmark Glacier Program in the 1950s to study and document the South Cascade Glacier in Washington State, and eventually four more glaciers, there is an abundant amount of information on these glaciers. Due to this program this study was able to easily access and use temperature, precipitation, mass balance and DEM data from the USGS. Also, as part of this Benchmark Glacier program a stream gauge was installed just below the glacier to track the runoff from the basin, the data from which was essential to this project.

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