Can a Shallow Ice Approximation Model be Used to Model the Water Output of Alpine Glaciers?

Abstract

1 Introduction

1.1 Importance of glacial melting in mountain hydrology

Glacial melting plays a significant role in the hydrology of mountain catchment areas as shown in Fountain and Tangborn, 1985. In order to understand how the hydrology of these mountain catchment areas will evolve as glaciers melt an retreat, scientists create models to predict this evolution. The runoff from these mountain catchments are often used as water sources for communities down stream [1] and as shown in Figure 1, basins with greater glaciation produce more runoff. As the glaciers in these basins melt and retreat this change in mass takes the form of meltwater. This meltwater can effect communities that rely on the runoff from these glaciated basins in a variety of ways [2].

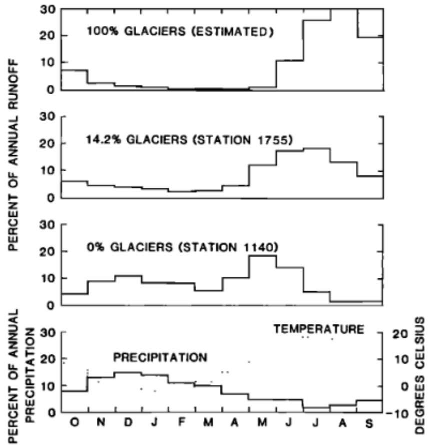

Fig. 3. Monthly fraction of the annual specific runoff for basins of various glacier covers. The monthly fraction of precipitation and mean monthly temperature (Snoqualmine Pass, Washington) are included for comparison.

(a) Monthly fraction of the annual runoff for basins of various glacier cover.

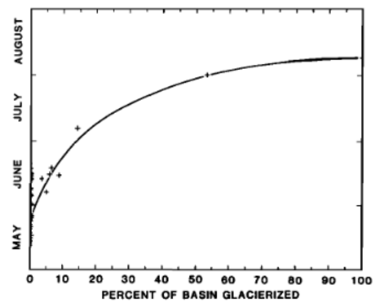

Fig. 4. Timing of peak specific runoff as a function of glacier cover for basins in the North Cascades, Washington.

(b) Timing of peak specific runoff as a function of glacier cover.

Figure 1: Figures 3 and 4 from Fountain and Tangborn (1985) showing how the percent of basin glacierized delays the peak runoff time of the basin.[Need to change these figs, maybe use fig 2 from that paper]

1.2 Role of numerical modeling in understanding glacial runoff

One of the most accurate ways to predict out how glaciers will affect the runoff of a glaciated mountain catchment is by using computer models to approximate the water discharge of the glaciers. Scientists have been using computers to model glaciers for several decades, such as this paper by Iken A, 1981. As computational resources have grown, these models have grown in complexity and resolution leading to very computationally expensive models. These models have proved to be very accurate in modeling the past and present state of a variety of glacier types all around the world.

1.3 Challenges in computational modeling of glaciers

Many of these more advanced models can be quite complex to run due to there being a variety of input parameters and different configurations to run the model. The goal of this paper is to write a simple Shallow Ice Approximation (SIA)-Mass Balance model that is easy to run and understand while still being reasonably generalizable to different small mountain glaciers. The advantage of using much more advanced models is that they often do a much better job at modeling the ice dynamics of glaciers. For some glaciers such as marine calving glaciers, accurate ice dynamics are crucial to accurately model them, as shown in Amaral et al., 2020. However on smaller mountain glaciers, modeling in 3-dimensions if often unnecessary and simpler models such as the SIA can be used. This paper will compare the results of a simple SIA-Mass Balance model to the results of OGGM, a much more advanced model using a simplified version of the Stokes Equations.

2 Literature Review

2.1 Prior research on SIA vs. Stokes models

As shown in Le Meur et al., 2004, there are significant differences in computational time between a SIA model and a Stokes model. When computing the free surface and associated velocity field, the SIA model took 1 minute of CPU time, whereas the Stokes model took 2 hours. This disparity grew even larger for 3D models. The authors show that there are some instances where SIA models do significantly worse than Stokes models, such as glaciers on steep slopes and glaciers in steep, narrow valleys because SIA models only approximate the Stokes equations. One of these approximations is to ignore horizontal stress gradients. This can cause a SIA model to deviate from a Stokes model significantly in predicted glacier flow and expansion. In one example, the resulting SIA model can have an upper free surface that is 15–20% greater than the Stokes model and velocities up to a factor of 2 greater (Le Meur et al., 2004). In the 2D model, the bed characteristics and slope become the limiting factor of the SIA model; Le Meur et al., 2004 note that the maximum velocity ratio of their SIA and Stokes models goes from 1.9 in a 3D model to 1.3 in a 2D model, which will differ depending on model configuration, but it tends to indicate that the horizontal stress gradients played a large part in this error. They found instances in which the SIA models performed well compared to Stokes models—particularly large flat glaciers with relatively free edges. One thing to note about this comparison study is that the authors are looking at the shape, area, and velocity profile of the glacier, whereas this study will focus on the water output (surface mass loss) of the glacier.

2.2 Using SIA models to model alpine glaciers

There are several papers, such as Le Meur et al., 2003 and Kessler et al., 2006, that use an SIA model for alpine glaciers. The consensus from those papers is that SIA models only work well on alpine glaciers with a low aspect ratio, defined as the thickness-to-extent ratio in Le Meur et al., 2004. The glacier used by this study will have a low aspect ratio and therefore a SIA model should work well to model it.

2.3 Using SIA models to model water runoff from glaciers

Additionally, there is precedent for using a SIA-Mass Balance model for modeling water runoff from glaciers (Naz et al., 2014). This paper used a SIA model to approximate the ice dynamics and a mass balance model to approximate the accumulation and ablation patterns on the glacier. As shown in their paper, the SIA model was able to accurately predict the glacier, and the coupled hydrological model was able to predict the stream flow accurately—only overestimating the July flow by an average of 13% and underestimating the August and September flow by an average of 2%.

3 Thesis Statement

How much do ice dynamics affect the model result when modeling small mountain glaciers for water runoff? I theorize that if using a simple 1-dimensional SIA-Mass Balance model on small mountain glaciers (with a low aspect ratio), the mass balance profile will have a much larger effect on the output of the model and its overall accuracy than the modeled ice dynamics. The results of the SIA-Mass Balance model and the OGGM model will be compared to the actual stream flow data to verify this hypothesis.

3.1 Study Site

South Cascade Glacier, Washington State

In this study, I will focus on modeling the South Cascade Glacier in the North Cascades region of Washington State. The South Cascade Glacier is roughly 1.68 square kilometers with widths ranging from 400m-1200m, has a mean elevation of roughly 1900 meters, (GLIMS) faces North, an average thickness of 99 meters, and a maximum thickness of 195 meters (GlaThiDa Consortium, 2020). The glacier is small, not overly steep (average slope of 7.14 degrees along my centerline in 2021), and has a low aspect ratio, therefore a SIA model should be able to accurately model its ice dynamics. On the other hand, the glacier is large enough to exhibit some movement and produce a measurable amount of runoff throughout the year.

Thanks to the foresight of the USGS in creating the Benchmark Glacier Program in the 1950s to study and document the South Cascade glacier, and eventually four more glaciers, there is an abundant amount of information on these glaciers. Due to this program I was able to easily access and use temperature, precipitation, mass balance and DEM data from the USGS. Also as part of this Benchmark Glacier program a stream gauge was installed just below the glacier to track the runoff from the basin, the data from which was essential to this project.

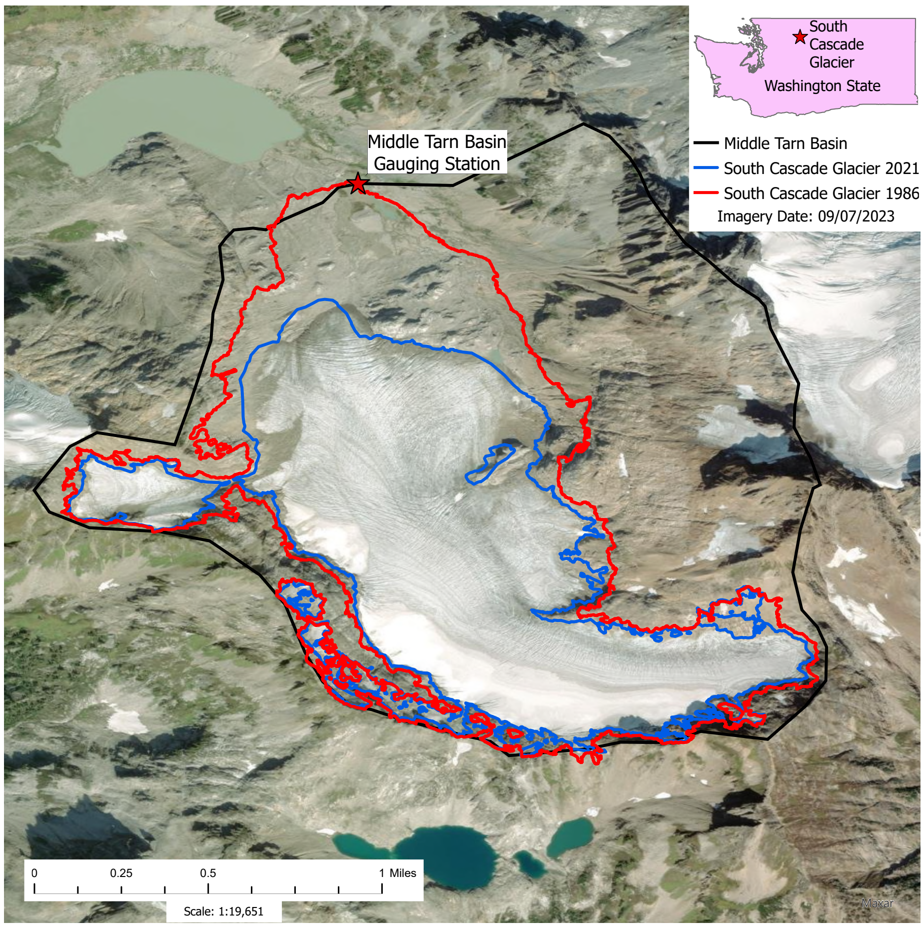

Figure 2: Map of the South Cascade Glacier in the North Cascades of Washington.

Symbol	Description
Z_g	Glacier surface elevation above sea level (m)
ELA	Equilibrium line altitude (m)
γ	Spinup mass balance equation gradient (m/year)
Q	Ice flux (m^2/day)
A	Flow rate factor ($5.87 * 10^{-19}Pa^{-3}day^{-1}$)
n	Flow law exponent (3)
p_{ice}	Density of ice ($917kg/m^3$)
g	Acceleration due to gravity ($9.81m/s^2$)
$\frac{\partial z_s}{\partial x}$	Slope of the glacier
H	Ice thickness (m)
b_s	Summer mass balance (m/day)
T	Temperature (°C)
M_{snow}	Snow melt factor (m/day/°C)
M_{ice}	Ice melt factor (m/day/°C)
b_w	Winter mass balance (m/day)
P	Precipitation (m/day)
α	Precipitation conversion factor
$Accum_{lower}$	Accumulation lower bound (m/day)
Y	Current year
$Accum_{upper}$	Accumulation upper bound (m/day)
S_{depth}	Snow depth (m water equivalent)
$S_{melt-vol}$	Snow melt volume (m^3 water equivalent)
$Area_{basin}$	Basin area (m^2)
$Area_{glacier}$	Glacier area (m^2)
R_{vol}	Rain volume (m^3 water equivalent)
$G_{melt-vol}$	Glacial melt volume (m^3 water equivalent)

Table 1: Symbols Table

4 Methods

4.1 Model Development

4.1.1 Model Overview

Model Structure:

1. Ice dynamics modeled using SIA
2. Temperature degree day and precipitation mass balance model
3. Temperature degree day and precipitation (snow fall/melt and rain) model

There are two sections to getting a complete model run, the spinup and the data driven run. The spinup section of the model runs for 500 years and aims to replicate the state of the glacier in 1984 when weather data becomes readily available. The spinup run starts with the bed topography and no ice and uses a simple mass balance equation to accumulate ice to simulate the glacier state in 1984. The data driven run starts in 1984 with the simulated spinup glacier and applies a temperature and precipitation driven mass balance model to model how the glacier changes until 2024. The data driven run also contains the precipitation model to calculate the runoff from snow melt and rainfall.

4.1.2 Spinup run

This section uses the simple mass balance equation below

$$(Z_g - \mathrm{ELA}) * \gamma/365.25 \tag{1}$$

This uses a γ of 0.0309 to calculate the mass balance in meters per day. When the spinup hits the year 1900 the ELA is shifted up from 1903m to 1930m to simulate the retreat state of the glacier. The figure below shows the model in 1958 (474 years of spinup), and 1986 (500 years of spinup and 2 years of data driven run) compared with the actual glacier in 1958 and in 1986 derived from a DEM [3]. The gaps in the actual glacier lines are due to the available DEMs being partially incomplete over the glacier area.

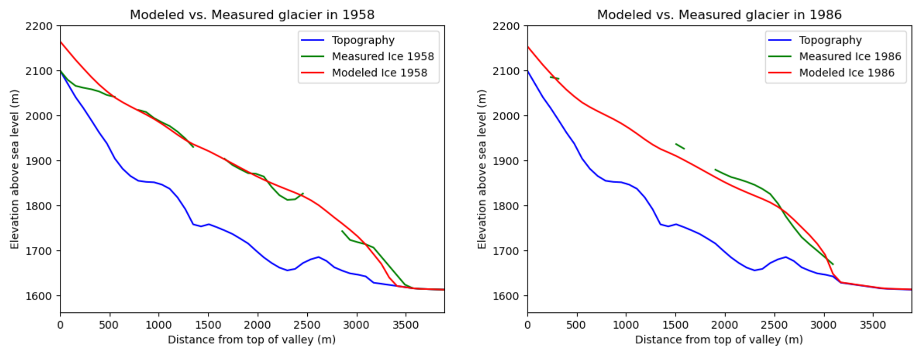

Figure 3: Comparisons of the modeled glacier and actual glacier in 1958 and 1986

4.1.3 Data driven run setup

The data driven section of the model run relies on a variety of data in order to run. It requires daily temperature and precipitation data, a bed topography, yearly glacier area, and total basin area to run. In order to tune the input parameters the model needs winter and summer mass balance data. The model needs seven input parameters to run: ice melt factor, snow melt factor, temperature lapse rate, start accumulation factor, end accumulation factor, avalanche percentage and precipitation conversion factor.

The centerline bed topography was calculated using latitude, longitude and elevation bed topography from Robert Jacobel [4]. I used the ArcGIS Kriging Interpolation function [5] to interpolate this point elevation data into a complete bed topography across the whole glacier. I then traced a line down the center of the glacier using the interpolated bed and got bed elevation points for the line from the interpolated data. I then used these new latitude, longitude, and elevation points to create the one-dimensional bed topography for the model. I explored using a centerline from RGI 7.0 [6], but ultimately chose not to use it because the RGI centerline did not go down the center of the interpolated bed.

4.1.4 Ice Dynamics

SIA Model The SIA model is a one-dimensional model that uses the shallow ice approximation to approximate the ice dynamics of the glacier. This model calculates the one dimensional ice flux of the glacier using equation 2.

$$Q = \frac{2A}{n+2}(p_{ice}g|\frac{\partial z_s}{\partial x}|)^n \frac{H^5}{5} \tag{2}$$

Assumptions The SIA ice flux equation make several assumptions. First, the equations are 1-dimensional, so they neglect longitudinal stress, and ice only flows downhill. Second, the equations also assume that there is no basal sliding of the glacier. Third, the equations only use gravity as the driver of ice flow; they ignore other forces such as lateral and basal stress. Fourth, this set of equations assumes that the horizontal dimensions of the modeled glacier are much larger than the vertical dimensions.

4.1.5 Mass Balance Model

The mass balance of the glacier is calculated using temperature and precipitation data from the Diablo Dam weather station at 272m. The temperature at the glacier is calculated by using a month-specific lapse rate. The precipitation at the glacier is calculated by multiplying the precipitation at the Diablo Dam weather station by the precipitation conversion factor of 1.58 obtained from (reference). The ablation of the glacier is calculated by using a combination of an ice melt factor and a snow melt factor. Above the ELA the ablation is calculated by the equation

$$b_s = T * M_{snow} \tag{3}$$

Below the ELA the ablation is calculated by the equation

$$b_s = T * (M_{snow} + ((ELA - Z_g)/(ELA - min(Z_g))) * (M_{ice} - M_{snow})) \tag{4}$$

The result of this equation is the snow melt factor being used at the ELA and a linear increase in the melt factor until it hits the ice melt factor at the base of the glacier. This set of equations assumes that in the accumulation zone (above the ELA) the surface is always snow year round, and below the ELA the surface transitions from snow to ice as you decrease in elevation. The accumulation of the glacier is calculated using a similar linear equation that increases with time.

$$b_w = P*\alpha*(Accum_{lower}+((Y-1984)/(2024-1984))*(Accum_{lower}-Accum_{upper})) \tag{5}$$

This results in the accumulation increasing with time until it hits the end accumulation at 2024.

4.1.6 Precipitation Model

Snow Model The snow melt model uses precipitation and temperature data to accumulate and melt snow. The equation below is used to calculate the change in snow depth per timestep

$$S_{depth} += \begin{cases} P * \alpha & T \leq 0, \\ -\min(|M_{snow} * T|, S_{depth}) & T > 0 \end{cases} \tag{6}$$

The snow melt is constrained so that there cannot a negative snow depth. The total volume of snow melt is calculated by equation 7.

$$S_{melt-vol} = (M_{snow} * T) * (Area_{basin} - Area_{glacier}) \tag{7}$$

This give us the total volume of snow melting off the non-glacierized areas of the basin. Any snow that falls on the glacier is factored into the glacier mass balance equations and accumulated on the glacier.

Rain Model The rain is simply modeled by $P * \alpha$ for positive temperature days. The volume of rain is calculated equation 8

$$R_{vol} = p * \alpha * Area_{basin} \tag{8}$$

This calculates the rain for the whole basin because rain is not factored into the glacier mass balance. This assumes that that any rain that falls runs out of the basin on the day it falls.

Avalanche Model In order to stop now accumulating to unrealistic depths at high elevations an avalanche model is used. This model avalanches the snow above 2123m once per year on a randomly chosen date (chosen each year) between January and March. The amount of snow that avalanches is controlled by the avalanche percentage. On the chosen that the snow depth above 2123m is decreased by the avalanche percentage and that snow is evenly distributed below 1900m. The result of this is the average snow depth being 2m water equivalent in September and October before starting to accumulate again. With an avalanche percentage of 0 it is 5.5m water equivalent in September and October.

4.1.7 Glacial Melt Model

The glacial melt model uses the mass balance of the glacier to calculate how much volume the glacier is losing. The volume of runoff from the glacier is calculated by the equation

$$G_{melt-vol} = b_s * Area_{glacier} \tag{9}$$

4.2 Model Calibration

4.2.1 Data used for model

The temperature and precipitation data used for the model is from the Diablo Dam weather station at 272m. The data is available from 1984-2024 and missing 298 days of temperature measurements and 292 days of precipitation measurements. The missing temperature data was interpolated using the interp function from the numpy python library [7], the missing precipitation data was assumed to be 0. The glacier area data used in the model is from the USGS [8]. The basin area data was calculated using a DEM from the USGS and the basin outline shown in figure 1.

4.2.2 Calibration

Spinup Calibration The spinup initial ELA, ELA in 1900 and γ were optimized using the minimize function with the Nelder-Mead method from the scipy library [9]. The optimization was done by minimizing the averaged RMSE's of the glacier areas's in 1958 and 1986. Since this is a 1-dimensional model this is the cross-sectional area of the modeled glacier. The result is a relative RMSE of 7.27% and 6.63% in 1958 and 1986 respectively.

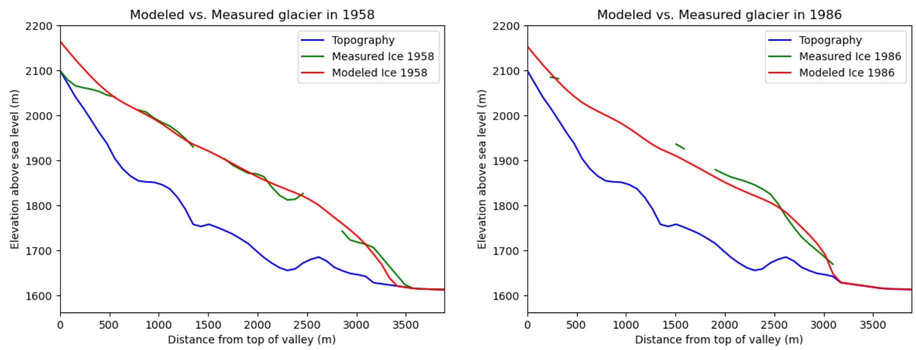

Figure 4: Glacier from DEM compared with modeled glacier in 1958 and 1986

Summer Mass Balance Calibration The two input parameters that control the summer mass balance are the ice melt factor and the snow melt factor. They were calibrated using yearly summer mass balance data available from the USGS from 1984-2024. I used the minimize function to minimize the mean squared error between the modeled mass balance and the USGS measured mass balance.

Winter Mass Balance Calibration There are also two factors that control the winter mass balance, the start and end accumulation factors. They were tuned using the same method as the summer mass balance factors using the USGS yearly winter mass balance data available from 1984-2024.

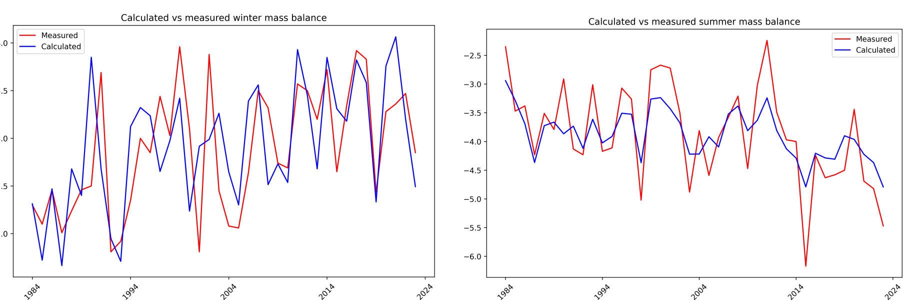

Figure 5: Comparison plots of the calculated vs measured winter and summer mass balance.

Avalance Percentage Calibration The avalanche percentage was optimized using the same minimize function as the mass balance variables. Instead of minimizing the MSE, the mean of the snow depth over the period 1984-2024 was minimized. The result of this is the snow depth during the summer being minimized due to there being more snow available at lower elevation with higher temperatures during the summmer.

4.2.3 Model Comparison

Running OGGM Model for the South Cascade Glacier The OGGM model was run using the run_with_hydro task from the oggm library. This model run used the GSWP3_W5E5 historical temperature and precipitation data [10] to model the hydrology of the glacier from 1984-2019. The total runoff from the glacier was calculated using runoff from the glacier using the melt_on_glacier_monthly, and liq_precip_on_glacier_monthly variables. In order to calculate the runoff from the non-glacierized areas melt_off_glacier_monthly, and liq_precip_off_glacier_monthly variables were averaged over the SIA model glacier basin area.

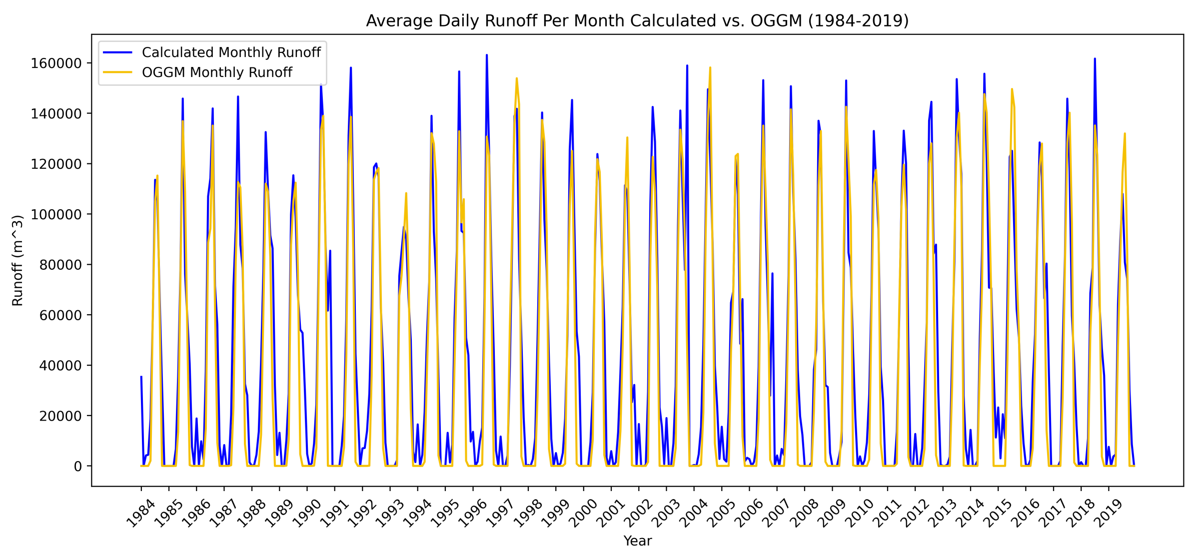

Figure 6: OGGM calculated daily average runoff per month compared to SIA-Mass Balance Model from 1984-2019

5 Results

5.1 Accuracy of SIA Model

The calculated runoff of the SIA-Mass Balance model was validated using measured streamflow data from 1992-2007, consisting of 2418 data points, which were aggregated into 91 months (reference list of papers). This data was measured using a stream gauge located just below the glacier (refer to Figure 1 for location). The data is in units of mm per day averaged over the basin area (4.46km^2). This was converted to cubic meters per day and then summed over the month to calculate a monthly runoff. Not every month was complete, so in order to validate the model, the model runoff was saved for the same days as the measured data and then a daily average was calculated for each month. This paper will look at the model error in terms of daily average runoff per month due to the several incomplete months of data. This paper will used three types of error to evaluate the model, relative Root Mean Square Error (RMSE), Nash–Sutcliffe Efficiency (NSE), and Kling–Gupta Efficiency (KGE), defined by the equations below.

$$\text{RMSE} = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(O_i - S_i\right)^2}}{\bar{O}} * 100, \tag{10}$$

$$\text{NSE} = 1 - \frac{\sum_{i=1}^{N}\left(O_i - S_i\right)^2}{\sum_{i=1}^{N}\left(O_i - \bar{O}\right)^2}, \tag{11}$$

$$\text{KGE} = 1 - \sqrt{(r-1)^2 + (\alpha - 1)^2 + (\beta - 1)^2}, \tag{12}$$

where:

$$O = \text{Observed data}, \quad S = \text{Simulated Data}$$
$$r = \text{Correlation coefficient}, \quad \alpha = \frac{\sigma_S}{\sigma_O}, \quad \beta = \frac{\mu_S}{\mu_O}$$

The error for these three methods are 29.26%, 0.81 and 0.88 respectively. It is important to note that this error is only the model error for May-November due to the lack of runoff data outside of those months. There is one instance of January, February, March and April, and six instances of December. What is interesting about this model is how good it does on longer timescales. The relative RMSE of the daily average runoff per year drops to 15.21%, and the average relative error for the whole time period is 4.51%, which shows that over longer time periods the over and underestimates of the model cancel each other out and the model becomes more accurate. [Compare to Naz paper, note that the precip model they use is more complicated]

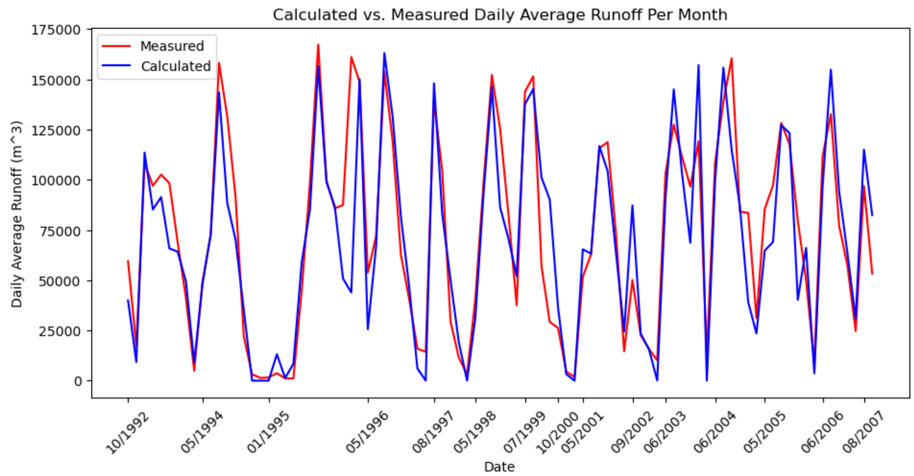

Figure 7: Measured vs. Calculated runoff data from 1992-2007

5.2 Accuracy of OGGM

The RMSE, NSE and KGE of the OGGM model are 45.96%, 0.53 and 0.72 respectively. The OGGM Model proved to be slightly less accurate when compared to the real-world measured runoff data. OGGM uses the GSWP3_W5E5 climate modeled data instead of measured temperature and precipitation data due to the purpose of the model being to model glaciers into the future.

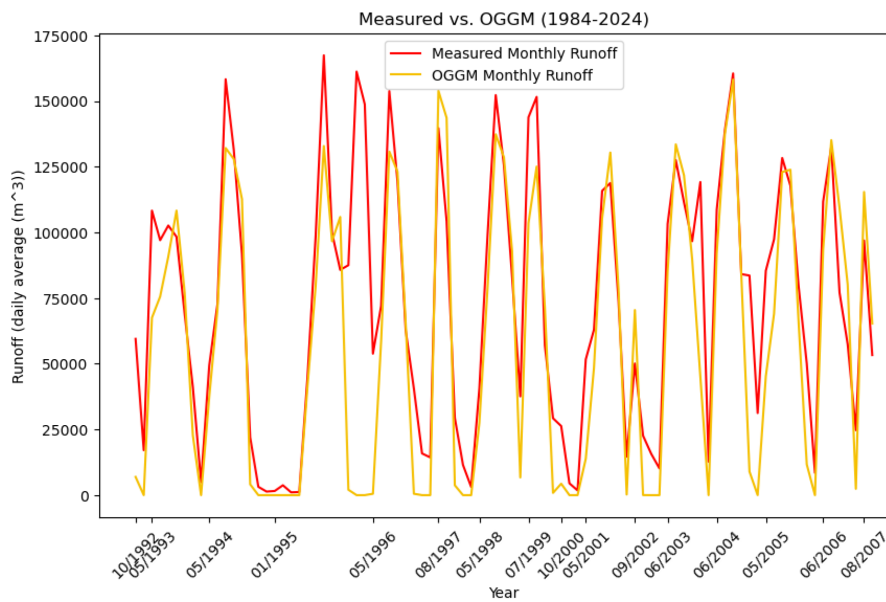

Figure 8: Measured vs. Calculated runoff data from 1992-2007

5.3 Comparison of Accuracy

The SIA-Mass Balance model is roughly 1.4 times (averaged over the three error metrics) more accurate than the OGGM model. One reason for the higher accuracy of the SIA-Mass Balance model could be due to it using real-world temperature and precipitation data and OGGM using modeled climate data which is less accurate. The SIA model input parameters are also tuned to the local climate of the glacier.

6 Implications of Research

6.1 Importance of Simplified Ice Dynamics in Numerical Glacier Modeling

These results show that complicated and computationally intensive mass balance and ice dynamics are not required to accurately model the runoff from small mountain glaciers. This means that we can use much simpler and less computationally intensive models such as the ones used in this paper to model the runoff from mountain glaciers much quicker than more complicated models. The modeling techniques used in this paper could easily be scaled to a much larger region if mass balance is available to tune the input parameters. Also the relative simplicity of the model used in this paper means that it is much easier to modify to add new features and customize to a specific region or glacier.

This point is further proved by the error of the SIA-Mass Balance model not changing significantly (less than 0.01 for the KGE and NSE, and less than 0.1% for the relative RMSE) when the ice flux is set to 0. This means that the ice dynamics of the glacier are not a significant factor contributing to the glacier runoff. One reason for this is the South Cascade Glacier is in an active state of retreat which can be seen in the model and the glacier outline in Figure 1. Since it is retreating, the model does not need the ice dynamics in order to move more glacier ice into the ablation zone where it can melt, much of the glacier is already in this ablation zone.

6.2 Applications

The work of this paper shows that complex ice dynamics and precipitation models are not always required to accurately model the runoff from small mountain glaciers. This can have applications in everything from regional glacier modeling to water resource management as glaciers are a significant source of water for many communities around the world.

7 Discussion

7.1 Improvements

The largest problem with the runoff model lies with the precipitation model. This model calculates the type of precipitation that falls (rain or snow), based on the temperature. It assumes that if the temperature is greater than 0° Celsius then the precipitation is rain, and if the temperature is less then 0° Celsius then then precipitation is snow. This is a good generalization, but it isn't always true. Looking at the error graph, there is a large spike in the error in November of 1995. This is likely due to the model predicting the large amount of precipitation during that month falling as snow because the temperatures are just below 0° Celsius, but in reality it most likely fell as rain. This also highlights the types of error that can be caused by using a proxy weather station that is not present at the glacier such as this model used. With more accurate temperature data for the glacier, then the model might correctly predict the precipitation falling as rain.

It is also important to note that out of the 298 missing days of temperature and 292 missing days of precipitation 63 days of temperature and 64 days of precipitation were during the 1992-2007 period where runoff data is available. Most notable of these missing days is the month of September 1997. This entire month is missing temperature and precipitation data, and this entire month is included in the measured runoff data. Looking at the error for the month it has a relative RMSE of 30.51% which is very close to the overall relative RMSE of 29.26%. This means that the error this month is quite typical for the model. The interpolated temperatures for this month range from 10° C to 5° during the month, so the glacier is melting and no glacier accumulation is missed. Also from looking at Figure 10, during September glacier melt is contributing the majority of the runoff to the basin, so having zero precipitation in the model for September 1997 would not cause as much error as if there was no precipitation for December or January when lack of snowfall could affect the runoff in the following summer.

These two examples illustrate that missing precipitation data may be much more detrimental to model accuracy than missing precipitation data, partly due to temperature data being much easier to interpolate than precipitation data.

7.2 What we can learn from the model results

Even though the glacier is contributing slightly less than half of the total basin runoff on average (47.56%), it contributes much more during the summer months as show in figure 11. As the glacier retreats it will start contributing less water to the basin runoff resulting in a decrease in total basin runoff. If the precipitation stays constant over time, but the glacier continues to retreat the basin runoff will see a steep decline particularly in early summer and fall when the glacier contributes most to the runoff. This is especially important considering that the Cascades see the least amount of precipitation during that portion of the year as shown in figure 12.

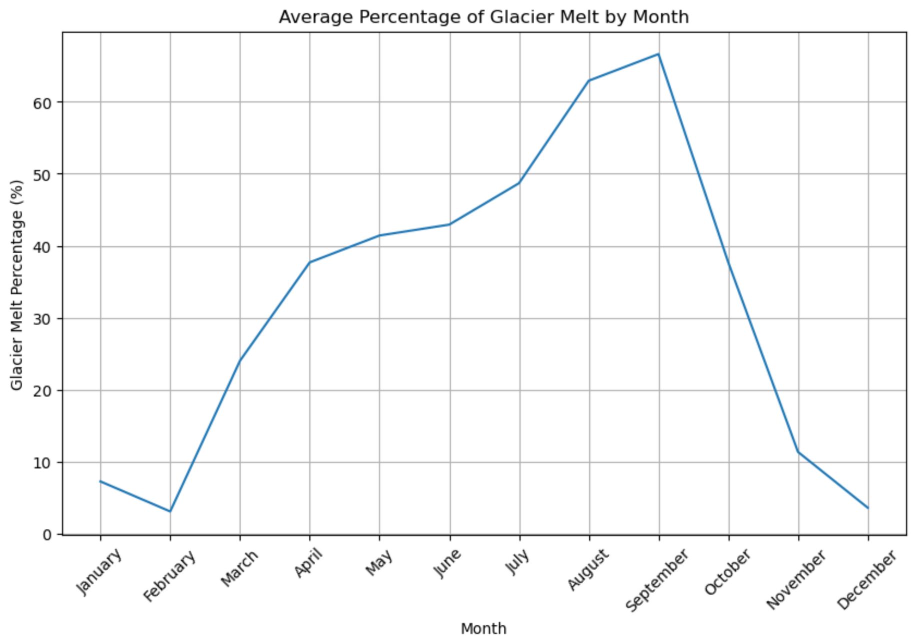

Figure 9: Average percentage from 1984-2024 that the glacier contributes to the monthly runoff

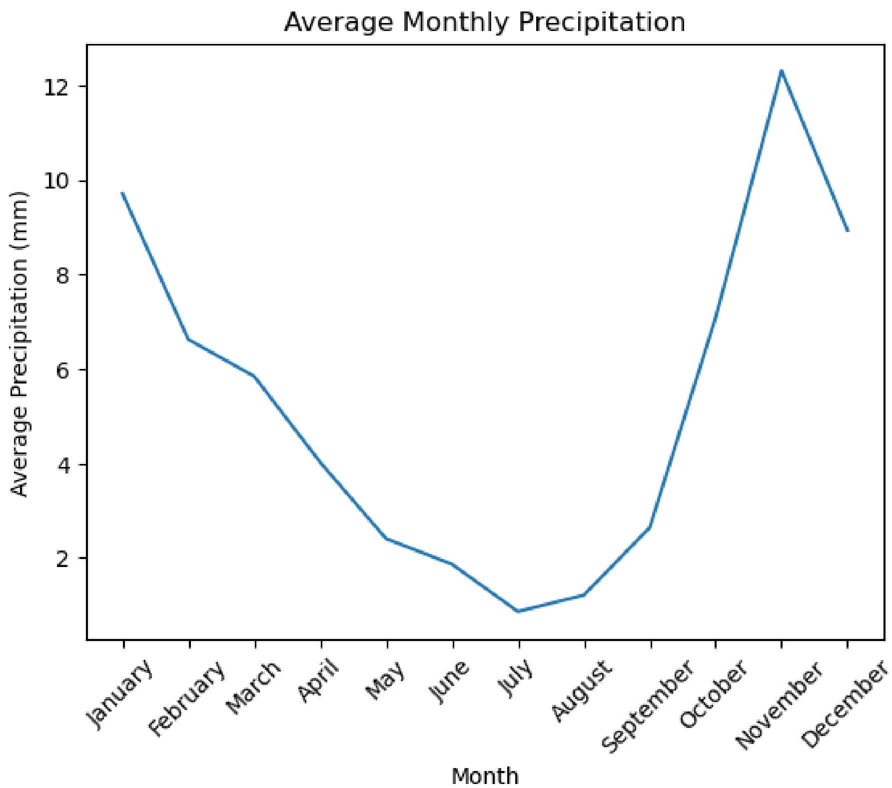

Figure 10: Average amount of precipitation per month from 1984-2024 at the Diablo Dam weather station

7.3 Mass Balance model error

The relative RMSE of the winter and summer mass balance are 19.01% and 13.33% respectively.

Looking at the missing temperature and precipitation one can note a few interesting things about the mass balance model error. There are five years with significant gaps (at least one month of missing data) in the temperature and precipitation data. These years are 1984 (missing December), 1985 (missing January), 1987 (missing July), 1997 (missing March and September) and 2009 (missing May-July). The year with the highest error in winter mass balance is 1997 probably due to it missing March data. The measured winter mass balance that year is 3.44m, and the model calculated a winter mass balance of 2.65m. One possibility for this is that there was a large amount of snowfall during March of that year, but since the precipitation data is missing for that month, the glacier accumulation is 0. According to PRISM (reference) the South

Cascade Glacier recieved 0.57m of precipitation during March 1997. Using the mass balance equations above, the accumulation on the glacier during March 1997 with 0.57m of precipitation would be 1m, which would increase the winter mass balance for that year form 2.65m to 3.65m, much closer to the measured value. This illustres just how important consistent and reliable weather data is for models like these because a month of missing weather data can skew the result.

7.4 Future Work

The next step for this project would be to run the model for a glacier near the South Cascade Glacier with a similar climate to see how region specific the input parameters are. Due to the input parameters being tuned for the mass balance of the glacier which largely depends on the local climate, it is plausible that they could work well for a glacier with a similar climate.

Another idea to explore is to run the model with modeled climate data in order to run the model into the future to see how this glacier will evolve and how the basin runoff will evolve.

8 Conclusion

The SIA-Mass Balance model is able to model the runoff from the South Cascade Glacier with a RMSE, NSE and KGE of 29.26%, 0.81 and 0.88 respectively, for 16 months from 1992-2007, while the OGGM model errors of 45.96%, 0.53 and 0.72 for the same time period. This show that complex models are not always the best way to model the runoff from small mountain glaciers. A simpler model with parameters tuned to the local climate of the glacier can perform just as well, if not better than a more complex model.

The work of this paper also highlights how important it is to have accurate and reliable weather data for modeling the runoff from glaciers. Inaccurate or missing weather data can cause large errors in the model, particularly in the case where the missng data was abnormal.

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