

GNF

Rule 1: $A \rightarrow B\alpha$ $B \rightarrow \beta_1 / \beta_2 / \beta_3 / \dots$

then

 $A \rightarrow \beta_1\alpha / \beta_2\alpha / \beta_3\alpha / \dots$

Rule 2:

 $A \rightarrow A\alpha_1 / A\alpha_2 / \dots$ $A \rightarrow \beta_1 / \beta_2 / \beta_3 / \dots$

then

 $B \rightarrow \alpha_i / \alpha_i B$ $A \rightarrow \beta_i / \beta_i B$

Q1) Convert the foll CFG to GNF

 $S \rightarrow AA / AB / BA$ $A \rightarrow aA / a$ $B \rightarrow bB / b$

Production

 $B \rightarrow b$ $B \rightarrow bB$ $A \rightarrow a$ $A \rightarrow aA$ $S \rightarrow BA$ $S \rightarrow AB$ $S \rightarrow AA$ solⁿ $B \rightarrow b$ $B \rightarrow bB$ $A \rightarrow a$ $A \rightarrow aA$ $S \rightarrow bB / bA$ $S \rightarrow aAB / aB$ $S \rightarrow aAA / aA$ Solⁿ is $S \rightarrow bA / bBA / aB / aAA / aA$ $A \rightarrow a / aA$ $B \rightarrow b / bB$

same CNF tuples

$$\begin{aligned} S &\rightarrow AA \mid a \\ A &\rightarrow SS \mid b \end{aligned}$$

- (1)

Production

Solⁿ \Rightarrow

$$S \rightarrow AA$$

$$S \rightarrow SSA \mid bA \mid a$$

$$S \rightarrow SSA \quad \alpha$$

$$S \rightarrow bA \mid a$$

 $\beta_1 \quad \beta_2$

then we will introduce

$$B \rightarrow \alpha_i / \alpha_i B$$

$$B \rightarrow SA / SA B$$

we have introduced it ourselves

- (2)

$$A \rightarrow \beta_i / \beta_i B$$

 \downarrow

$$S \rightarrow \underbrace{bA}_{\beta_1} / \underbrace{bAB}_{\beta_1 B} / \underbrace{a}_{\beta_1} / \underbrace{aB}_{\beta_1 B}$$

Put S in (2)

$$B \rightarrow bAA / bABA / aA / aBA / bAAB / bABAB / aAB / aBAB$$

Put S in (1)

$$A \rightarrow bAS / bABS / aS / aBS / b$$

$$\begin{aligned} Q3) \quad A_1 &\rightarrow A_2 A_2 / 0 \\ A_2 &\rightarrow A_1 A_1 / 1 \end{aligned} \quad - (1)$$

\Rightarrow

$$A_1 \rightarrow A_2 A_2 / 0$$

$$A_1 \rightarrow A_1 A_1 A_2 / 1 A_2 / 0$$

$$A_1 \rightarrow A_1 A_1 A_2$$

$$A_1 \rightarrow 1 A_2 / 0$$

then we will introduce

$$\hookrightarrow B \rightarrow \alpha_i / \alpha_i B$$

$$\hookrightarrow B \rightarrow A A_2 / A A_2 B$$

$$A \rightarrow \beta_i / \beta_i B$$

$$A_1 \rightarrow \underbrace{1 A_2}_{\beta_i} / \underbrace{1 A_2 B}_{\beta_i B} / \underbrace{0}_{\beta_i} / \underbrace{0 B}_{\beta_i B}$$

put A_1 in (1)

$$A_2 \rightarrow 1 A_2 A_1 / 1 A_2 B A_1 / 0 A_1 / 0 B A_1 / 1$$

put A_1 in (2)

$$\begin{aligned} B \rightarrow 1 A_2 A_2 / 1 A_2 B A_2 / 0 A_2 B B_2 / 1 A_2 A_2 B / \\ 1 A_2 A_2 B / 0 A_2 B / 0 B A_2 B \end{aligned}$$

q4) $S \rightarrow \overset{\text{terminal}}{\downarrow} \underline{a} S a / b S b / a b$

Note: If a terminal is followed by a variable, use selective replacement

\Rightarrow Grammar is already simplified

$$S \rightarrow a S C_1 / b S C_2 / a C_2$$

$$C_1 \rightarrow a$$

$$C_2 \rightarrow b$$

q5) $S \rightarrow S S / a S b / c$

\Rightarrow $G \rightarrow b$

$$S \rightarrow S S / a S C_1 / c$$

$$S \rightarrow S S \quad \alpha$$

$$S \rightarrow a \underline{S C_1} / c$$

$\beta_1 \quad \beta_2$

then we will introduce

$$B \rightarrow \alpha_i / \alpha_i B$$

$$B \rightarrow S / S B \quad - (1)$$

$$A \rightarrow \beta_i / \beta_i B$$

$$S \rightarrow a S C_1 / c / a S C_1 B / c B$$

solⁿ: put S in (1)

$$B \rightarrow a S C_1 / c / a S C_1 B / c B / a S C_1 B / c B / a S C_1 B B / c B B$$

$$C_1 \rightarrow b$$

⑥ $S \rightarrow aSA / aA$
 $A \rightarrow Aa / Sa / a$

$\Rightarrow [C_1 \rightarrow a]$

$A \rightarrow A a / SC_1 / a$

$A \rightarrow Aa / aSAC_1 / aAC_1 / a$

$A \rightarrow A \underline{a}$

$A \rightarrow \underline{aSAC_1} / \underline{aAC_1} / \underline{a}$

β_1

β_2

β_3

$B \rightarrow d_j / d_j B$

$[B \rightarrow a / aB]$

$A \rightarrow \beta_i / \beta_i B$

$[A \rightarrow aSAC_1 / aSAC_1 B / aAC_1 / aAC_1 B / a / aB]$

$[S \rightarrow aSA / aA]$