Module 4

Q . Multiclass Classification and Techniques

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1. Introduction

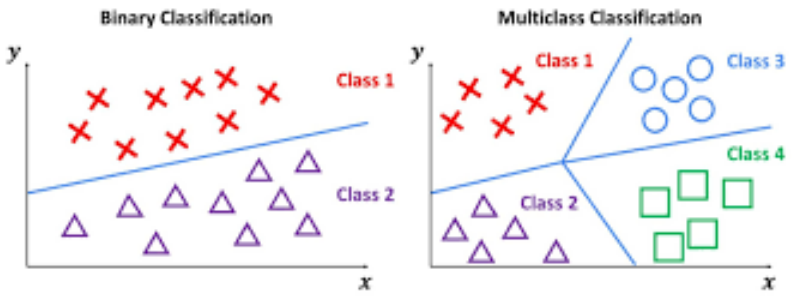
1. Multiclass Classification is a type of supervised learning problem where the model predicts one label from three or more possible classes.
2. It is also called Multinomial Classification.
3. In this type, every data sample belongs to exactly one class out of several classes.
4. Example – Identifying the type of fruit (Apple, Banana, or Orange) from images.
5. Unlike binary classification, which predicts between two outcomes (Yes/No), multiclass classification handles multiple categories.
6. It is used in applications like digit recognition, text categorization, image labeling, and disease diagnosis.

2. Techniques for Multiclass Classification

Multiclass classification can be handled using several approaches:

A. One-vs-Rest (OvR) or One-vs-All (OvA)

1. In this method, one classifier is trained for each class, treating that class as positive and all others as negative.
2. For example, if there are 4 classes (A, B, C, D), 4 classifiers are trained:
    a. Classifier 1: A vs (B, C, D)
    b. Classifier 2: B vs (A, C, D)
    c. and so on.
3. During prediction, the classifier with the highest output score decides the final class.
4. This method is simple and widely used in models like Logistic Regression and SVM.
5. Example – Classifying animals into Dog, Cat, and Horse using three binary classifiers.



B. One-vs-One (OvO)

1. In this approach, a separate classifier is trained for every pair of classes.
2. For *n* classes, the number of classifiers required is *n(n-1)/2*.
3. Example – For 3 classes (A, B, C), 3 classifiers are trained: A vs B, B vs C, and A vs C.
4. When predicting, each classifier votes for a class, and the class with the **maximum votes** is chosen.
5. OvO works well for models like **SVM**, where separating pairs of classes gives better accuracy.
6. However, it becomes computationally expensive when the number of classes is large.

C. Softmax Regression (Multinomial Logistic Regression)

1. Softmax is an **extension of Logistic Regression** for multiple classes.
2. It calculates the **probability of each class** using the Softmax function.
3. The class with the **highest probability** is selected as the output.
4. Example – Predicting the type of flower (Setosa, Versicolor, Virginica) in the **Iris dataset** using Softmax regression.
5. Softmax ensures that all class probabilities add up to 1.

D. Decision Tree and Random Forest

1. **Decision Trees** can handle multiclass problems **directly**, without converting them to binary.
2. They split the dataset based on feature values to reach leaf nodes representing different classes.
3. **Random Forest**, an ensemble of decision trees, improves performance by combining multiple trees.
4. Example – Classifying plants into different species based on leaf size and color.

E. K-Nearest Neighbors (KNN)

1. KNN can also handle multiple classes naturally.

2. It classifies a new point based on the **majority class among its k-nearest neighbors**.
3. Example – A new fruit image is classified as “Mango” if most nearby samples are labeled as Mango.
4. KNN is simple but can be slow for large datasets.

Q. Explain support vector machine as a constrained optimization problem. also explain Hyper plane, Support Vectors, Hard Margin, Soft Margin, Kernel.

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1. Introduction

1. **Support Vector Machine (SVM)** is a **supervised machine learning algorithm** mainly used for **classification** and sometimes for regression.
2. It works by finding the **best boundary (hyperplane)** that separates data points of different classes.
3. SVM aims to **maximize the margin** between different classes, ensuring the best generalization on unseen data.
4. It is based on the concept of **constrained optimization**, where we find an optimal solution under certain conditions.
5. Example – Classifying emails into "spam" or "not spam" using a boundary line that separates the two categories.

2. SVM as a Constrained Optimization Problem

1. The objective of SVM is to **find the hyperplane that maximizes the margin** while correctly classifying the training data.
2. The optimization problem is defined as follows:
    a. **Minimize:** $\frac{1}{2}||w||^2$
    b. **Subject to constraints:**
    $y_i (w \cdot x_i + b) \geq 1$ for all i
3. Here,
    c. **w** = weight vector (defines orientation of the hyperplane)
    d. **b** = bias term (defines the position of the hyperplane)
    e. **xi** = input feature vector
    f. **yi** = class label (+1 or –1)
4. This is called a **constrained optimization problem** because we want to minimize $||w||^2$ while satisfying certain class constraints.
5. By minimizing $||w||^2$, we are effectively **maximizing the margin** between two classes.
6. The solution to this optimization gives the **optimal hyperplane** that best separates the data.

3. Concept of Hyperplane

1. A **hyperplane** is a decision boundary that separates data into classes.

2. In **2D**, the hyperplane is a line; in **3D**, it is a plane; and in **higher dimensions**, it is called a hyperplane.
3. The equation of a hyperplane is given by:
   $w \cdot x + b = 0$
4. Points for which $w \cdot x + b > 0$ belong to one class, and points for which $w \cdot x + b < 0$ belong to another.
5. The position and slope of the hyperplane depend on **w** (weights) and **b** (bias).
6. Example – In classifying cats vs dogs using image features, the hyperplane acts as a line dividing the two categories.

4. Support Vectors

1. **Support Vectors** are the **data points closest to the hyperplane** from each class.
2. They play a **key role** in defining the optimal boundary.
3. If any of these points are moved or removed, the hyperplane position changes.
4. Only these vectors are used to define the model; other data points have no effect once the boundary is set.
5. Example – In a graph separating circles and squares, the few data points nearest to the boundary are the support vectors.

5. Hard Margin SVM

1. **Hard Margin SVM** is used when data is **perfectly linearly separable**.
2. It finds a hyperplane that separates all data points without any errors.
3. The constraint ensures every point lies on the correct side of the margin:
   $y_i (w \cdot x_i + b) \geq 1$
4. Hard margin aims to **maximize margin** and **minimize errors to zero**.
5. However, it is very **sensitive to noise or outliers**.
6. Example – A dataset of linearly separable shapes where no data point is misclassified.

6. Soft Margin SVM

1. **Soft Margin SVM** is used when the data is **not perfectly separable**.
2. It allows **some misclassifications** but keeps the margin as large as possible.
3. A **slack variable (ξ_i)** is introduced to allow certain violations of constraints.
4. The optimization becomes:
   Minimize: $\frac{1}{2} ||w||^2 + C \sum \xi_i$
   Subject to: $y_i (w \cdot x_i + b) \geq 1 - \xi_i$, and $\xi_i \geq 0$

5. Here, **C** is a penalty parameter that controls the trade-off between **maximizing margin** and **minimizing error**.
6. Large C → less tolerance to errors; Small C → more tolerance to errors.
7. Example – Separating customer groups where a few wrong classifications are acceptable.

7. Kernel Trick

1. Many datasets are **not linearly separable** in their original feature space.
2. The **Kernel Trick** helps to transform data into a **higher-dimensional space** where separation becomes possible.
3. It uses a **mathematical function** called a **kernel function** to compute relationships between points in this new space.
4. Example – Transforming circular data (non-linear) into a linear form in higher dimension.
5. Common kernel functions are:
    a. **Linear Kernel:** $K(xi, xj) = xi^T xj$
    b. **Polynomial Kernel:** $K(xi, xj) = (xi^T xj + 1)^d$
    c. **Radial Basis Function (RBF):** $K(xi, xj) = \exp(-\gamma||xi - xj||^2)$
    d. **Sigmoid Kernel:** $K(xi, xj) = \tanh(\alpha xi^T xj + c)$
6. Kernel functions allow SVM to handle **complex non-linear problems** like image or text classification.

Q. Kernel Trick in Support Vector Machine (SVM)

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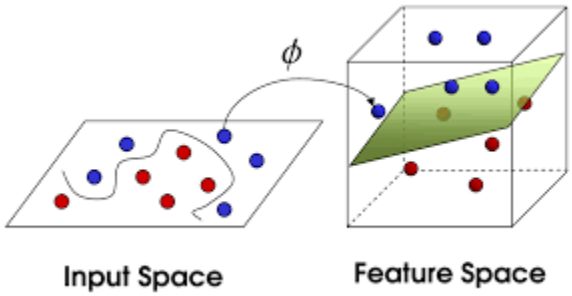
1. Introduction



1. The **Kernel Trick** is a mathematical technique used in **Support Vector Machines (SVMs)** to handle **non-linear data**.
2. In real-world problems, data is often not linearly separable — meaning it cannot be divided by a straight line or a simple plane.
3. The Kernel Trick allows SVM to classify such data by **mapping it into a higher-dimensional space** where it becomes linearly separable.
4. This mapping is done **without explicitly calculating the new coordinates**, making the process faster and more efficient.
5. In simple words, the Kernel Trick helps SVMs create curved or complex boundaries instead of straight lines.
6. Example – If two circles of different radii represent two classes, they cannot be separated by a straight line in 2D, but after mapping to 3D using a kernel, they can be separated by a plane.

2. Concept of the Kernel Trick

1. The main idea of the Kernel Trick is to **replace the dot product** of two data points with a **kernel function**.
2. In the higher-dimensional feature space, this kernel function acts as if the data points were transformed there.
3. This avoids direct computation of transformation, saving both **time and memory**.

Mathematically, if
$\phi(x)$ = transformation function,
then the kernel function satisfies
$K(x_i, x_j) = \phi(x_i) \cdot \phi(x_j)$.
Here, $K(x_i, x_j)$ represents the **similarity measure** between two data points.
This similarity helps in determining the boundary that best separates the classes.

3. Need for the Kernel Trick

1. Many real-world datasets (like images, text, or biological data) are **non-linear** in nature.
2. A **linear SVM** fails to classify such data properly because it assumes a straight boundary.
3. The Kernel Trick allows SVM to **build complex decision boundaries** that can separate non-linear data.
4. This method increases the **flexibility and power** of SVM without increasing computation cost significantly.
5. Example – In an email spam classifier, the relationship between words and spam probability is not linear, so kernel-based SVM performs better.

4. Common Types of Kernel Functions

(i) Linear Kernel:
   a. Used when data is linearly separable.
   b. Example – For simple numeric datasets where a straight line can divide classes.

(ii) Polynomial Kernel:
   a. Introduces polynomial relations between features.
   b. Example – Useful in cases like curve fitting or pattern recognition.

(iii) Radial Basis Function (RBF) or Gaussian Kernel:
   a. Measures similarity based on distance between data points.
   b. Very effective for **non-linear data**.
   c. Example – Used in image recognition or medical data classification.

(iv) Sigmoid Kernel:
   a. Works similar to a neural network's activation function.
   b. Example – Useful when data behaves like neural patterns.

7. Example to Understand Kernel Trick

1. Suppose we have two classes forming **two concentric circles** in a 2D plane.
2. No straight line can separate them in that plane.

3. Using a **non-linear transformation** (for example, adding a third dimension (z = x^2 + y^2)), the circles become **two separate planes** in 3D space.
4. In this new space, a **simple plane (linear separator)** can divide them perfectly.
5. The Kernel Trick performs this process automatically using a kernel function like the **RBF kernel**, without manually transforming the data.

8. Applications of Kernel Trick

1. **Image classification:** Recognizing handwritten digits or faces.
2. **Text categorization:** Classifying emails as spam or non-spam.
3. **Medical diagnosis:** Identifying patterns in biological or genetic data.
4. **Speech recognition:** Mapping complex sound patterns.