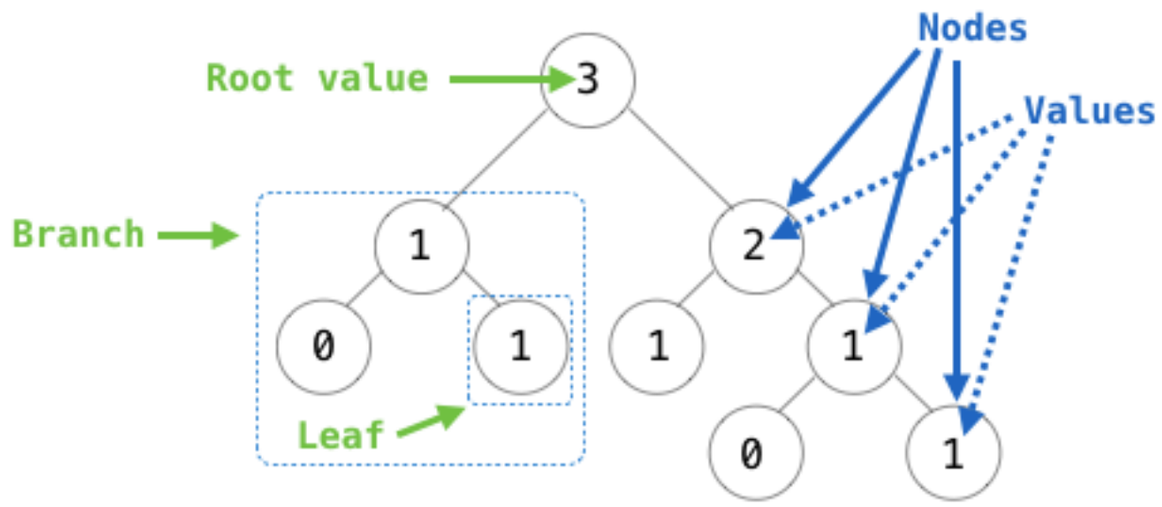
Discussion 03

Trees for Days

What's a tree?



Recursive description (wooden trees):

A **tree** has a **root** value and a list of **branches**

Each branch is a **tree**

A tree with zero branches is called a **leaf**

Relative description (family trees):

Each location in a tree is called a **node**

Each **node** has a **value**

One node can be the **parent**/**child** of another

People often refer to values by their locations: "each parent is the sum of its children"

This slide from lecture covers all of the terminology we use to discuss trees
Memorize this slide.

Python and Trees

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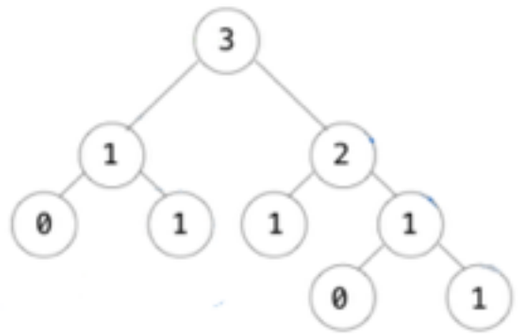
How do we **represent** trees? We don't know a lot of data types yet. So let's use lists!



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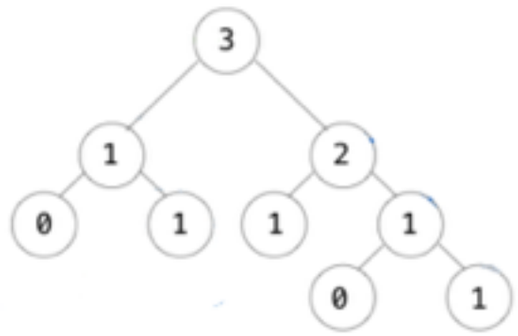

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def make_city(name, lat, lon):
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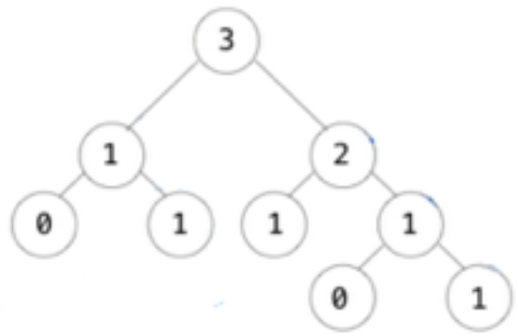

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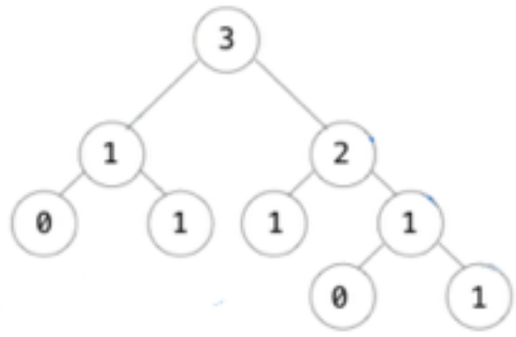

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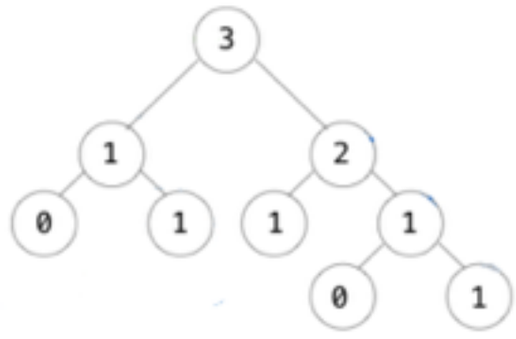

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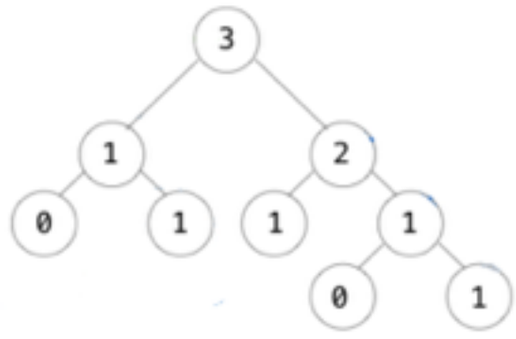

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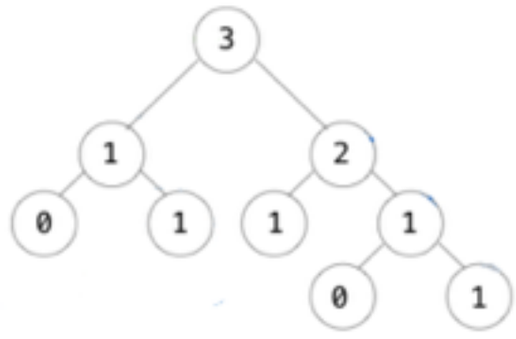

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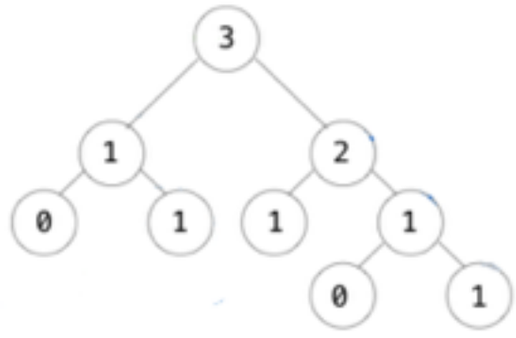


```
def is_leaf(t):
    return not branches(t)
```

if a tree has no branches, it is a leaf!
not [] —> not False —> True

Tree Function Example

Write a function that returns a new tree and increments every value by 1. This is the example from lecture.

Plan:

— “return a new tree” —> call tree()

— “increment by 1” —> root(t)+1

— input: tree!

— output: tree!

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def increment(t):
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def increment(t):
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accumulate the result of the recursive call in a variable. Check in Q: Why can we do this? Doesn’t this variable get “erased” with every recursive call?

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create a new tree!
what will be the root?
what are the branches?

the value of the new root is the value of the current root, incremented by 1

where did we accumulate the incremented branches?

How to write functions

There is a structure to how most functions involving trees are written.
You'll notice this as you do more problems!

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def tree_function(tree):
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depending on what your function is supposed to do, new_variable can accumulate branches, numbers, lists…

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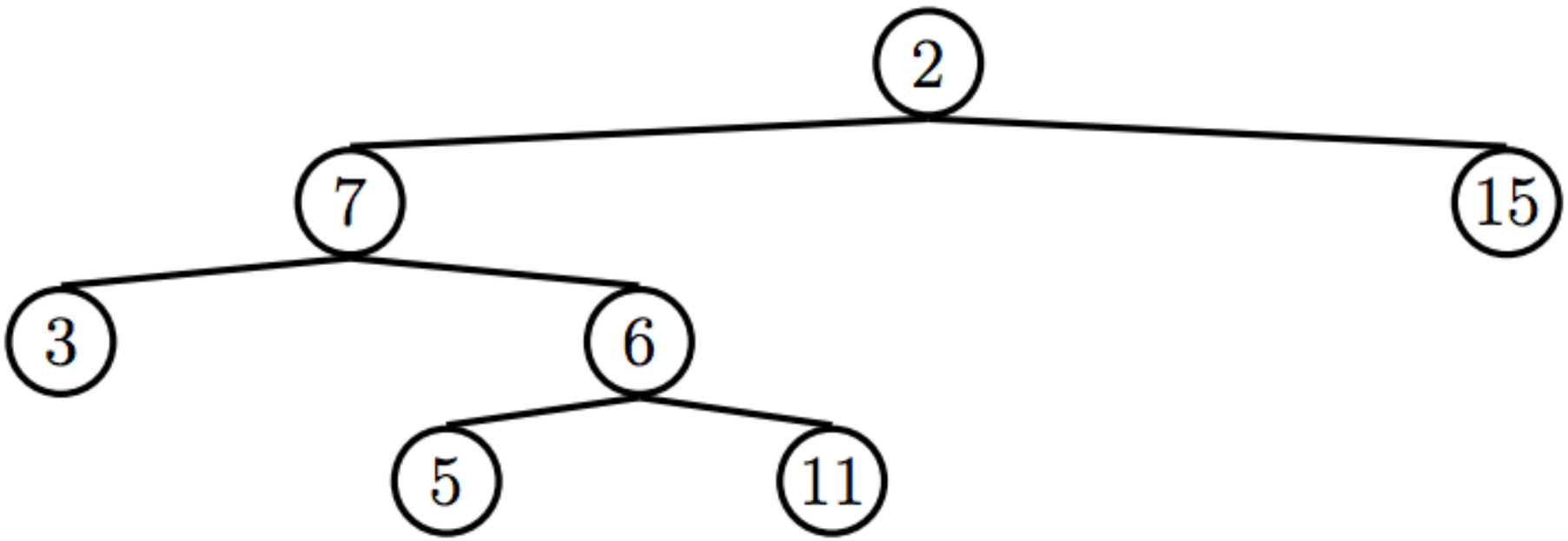
figure out what the return type is!!

Find path

Write a function that takes in a tree t and a value x, and returns the path to x as a list of values.




```
>>> find_path(t, 5)
[2, 7, 6, 5]
>>> find_path(t, 10)   # returns None
```

Look at the tree and doctests. Make sure you understand what this function is supposed to do!

Find path

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def find_path(t, x):
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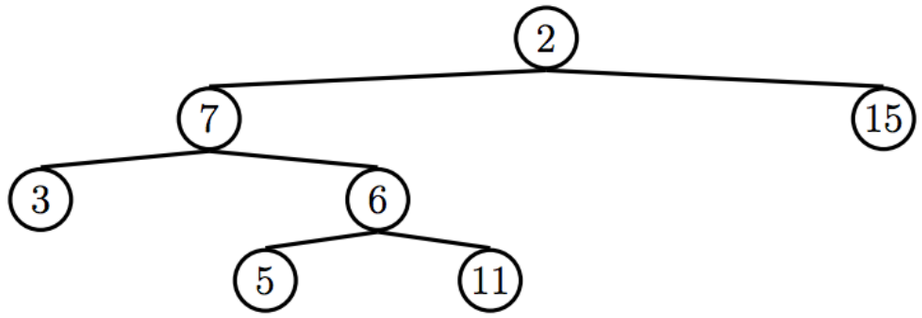

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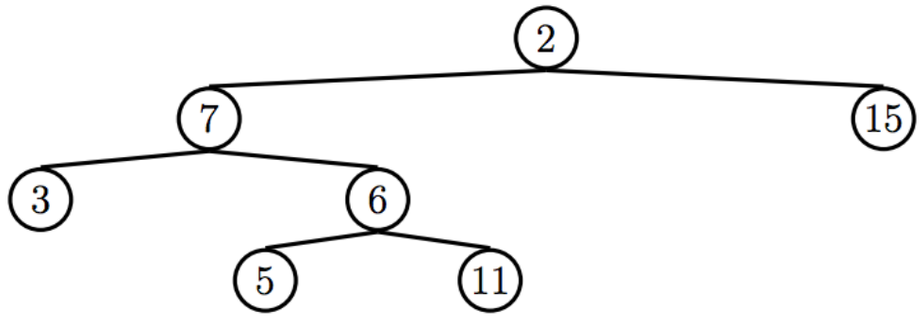

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we must check all the branches for a path
so iterate through them using a for loop (horizontal)

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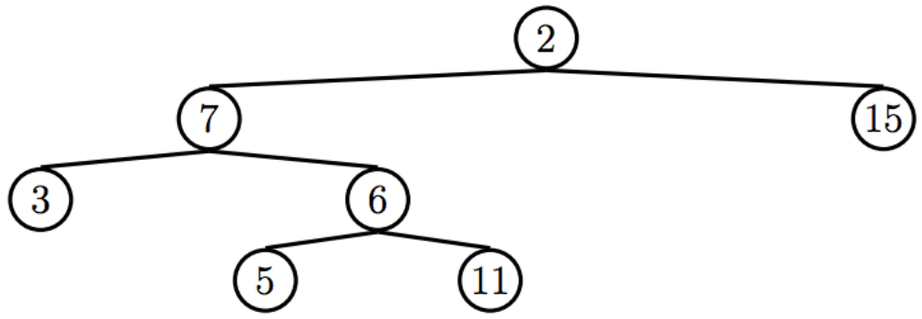

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try to find a path in each branch

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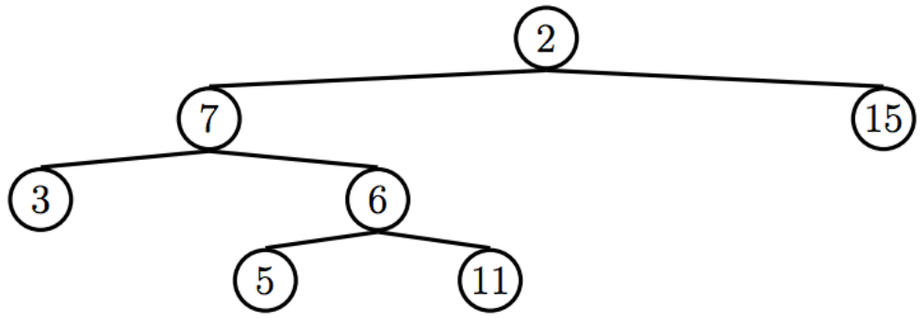

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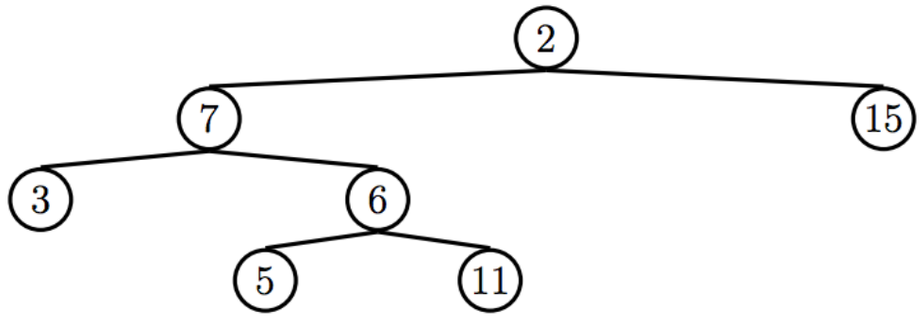

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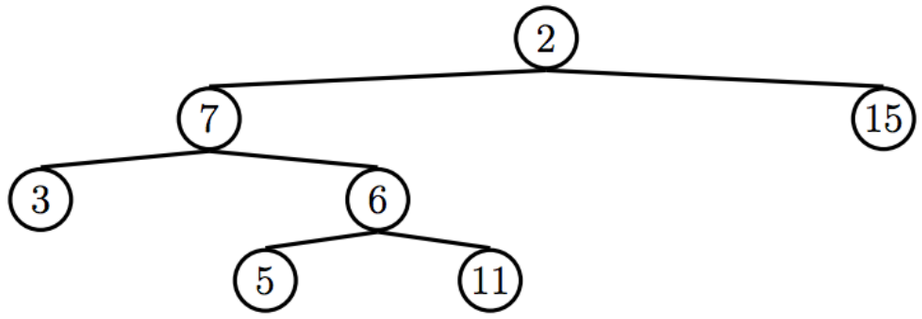

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            return [root(t)] + new_path
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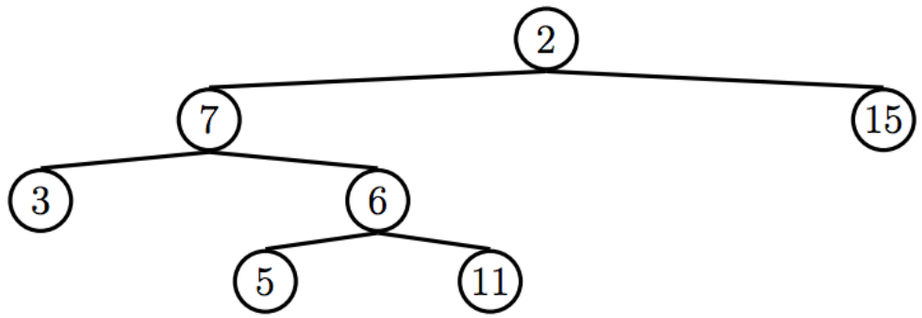

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we're missing out base case! how do we know for sure, that a node is included in the path?

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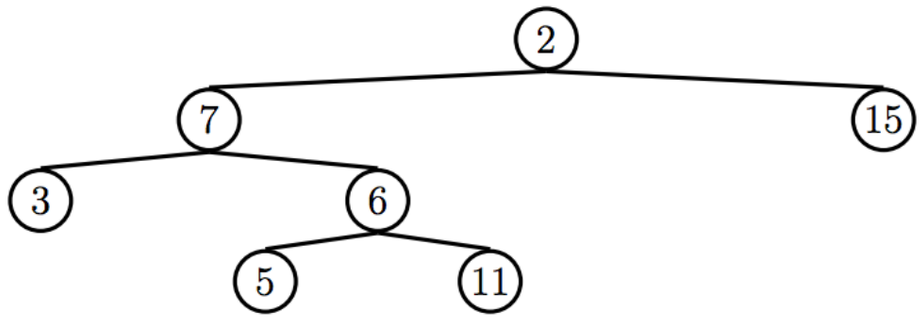

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