# Discussion 04

List mutation, Orders of growth, Nonlocal

# How to figure out orders of growth

1. Walk through the function.
   - What does it do?
   - How does it change its input?
   - When does it stop iterating/recursion?

2. After you get an expression for the order of growth, simplify it!
   - Remove any constants
   - Remove smaller terms

# Techniques

**Draw a tree:**

1. Start with the first function call.
2. Then draw a branch for each recursive call from that function call. (We used this technique to figure out recursive_fib)

**Draw a graph:**

1. Figure out what should be on the x axis.
2. Start with the largest input and determine how the input changes for each recursive call.
3. Find the area.
4. Simplify

# 1.2 #3

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    prev, curr, i = 0, 1, 0
    while i < n:
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So we have $\Theta(n)$ total iterations, and $\Theta(1)$ work at each iteration.
This gives us $\Theta(1 * n)$ total amount of work we must do.

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$$\begin{aligned} n &= 2^k \\ \log(n) &= \log(2^k) \\ \log(n) &= k*\log(2) \\ \log(n) &= k \end{aligned}$$

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$\Theta(1 * \log(n)) = \Theta(\log(n))$

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Plug in some values for n and walk through the function. The function keeps decrementing the input until it reaches a number that is divisible by 7. At this point it returns 0. Then it goes back up the recursive calls and adds 1 for each recursive call. So this function just returns a number between 0 and 6 that is equivalent to n in mod 7

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$\Theta(1)$

# 2.1 #1, #2

What is wrong with the code?

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a = 5
def another_add_one():
    nonlocal a
    a += 1
another_add_one()
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def adder(x):
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y isn’t defined in the parent! It’s passed into add, so it’s a local variable.

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s = stepper(3)
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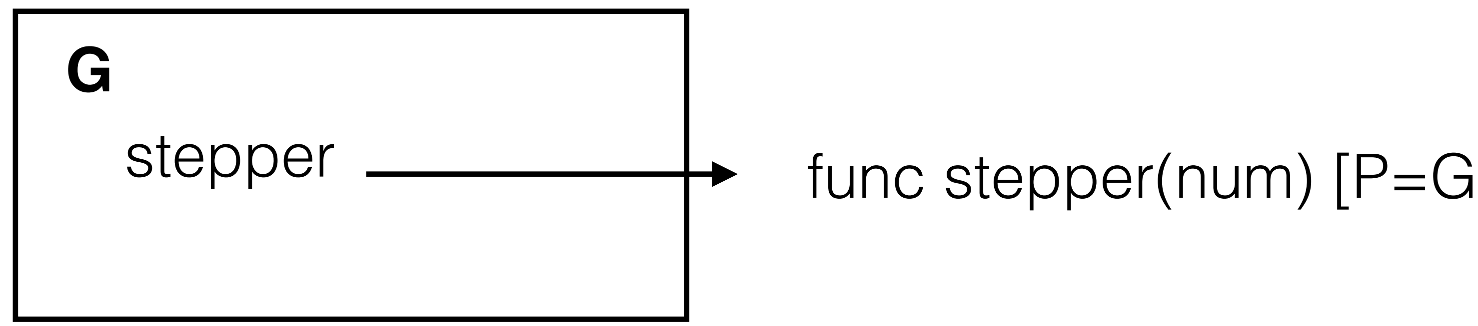

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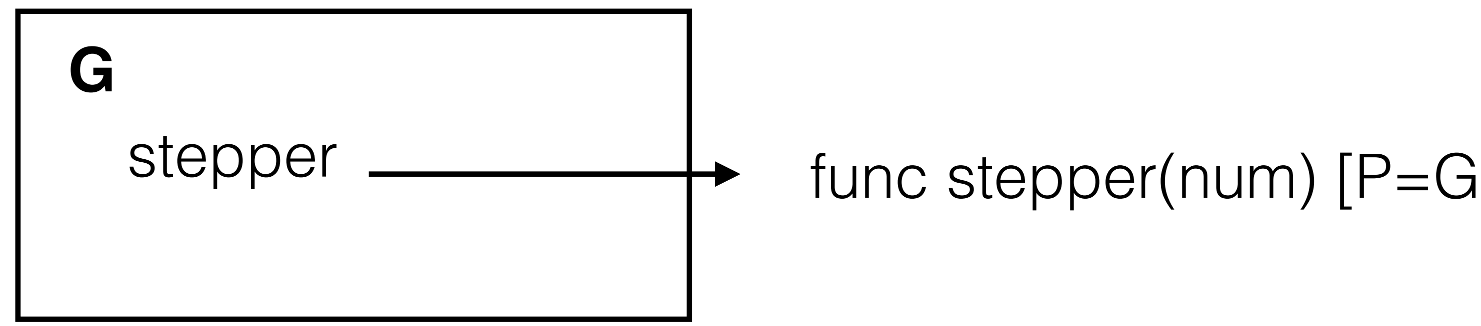

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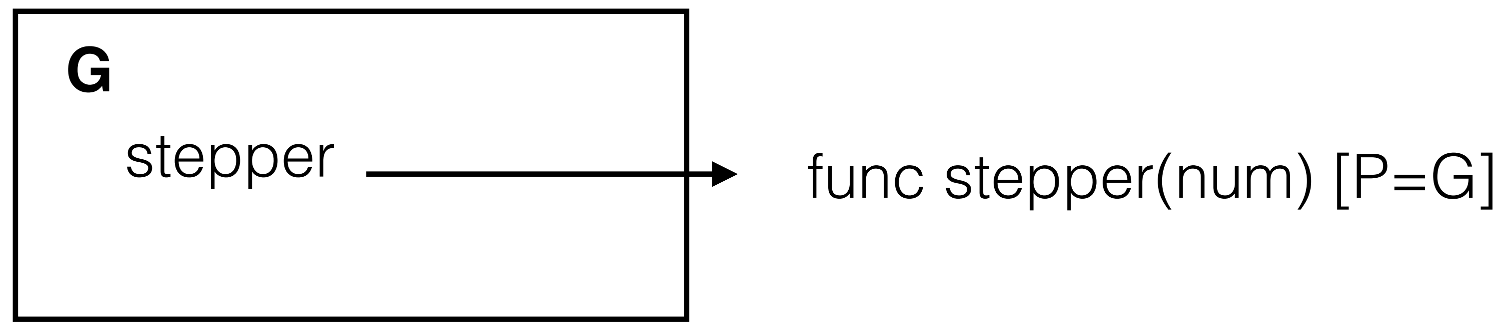

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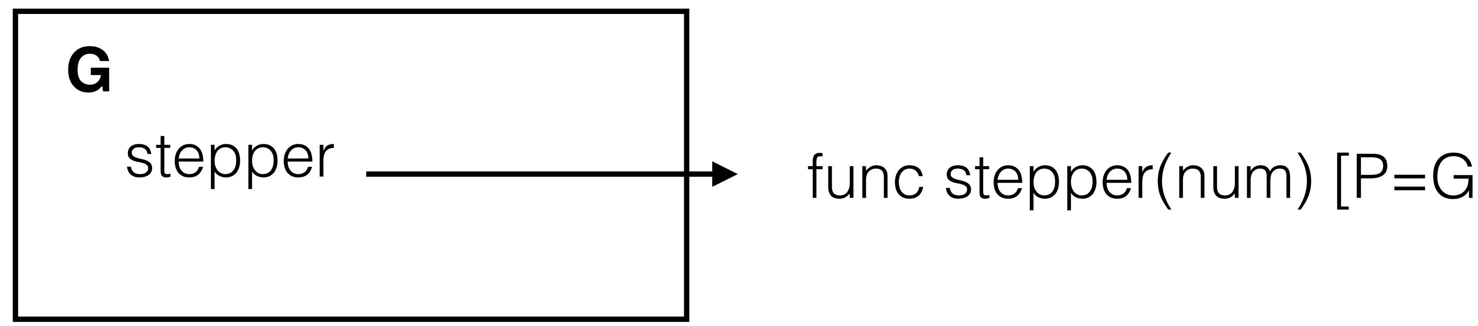

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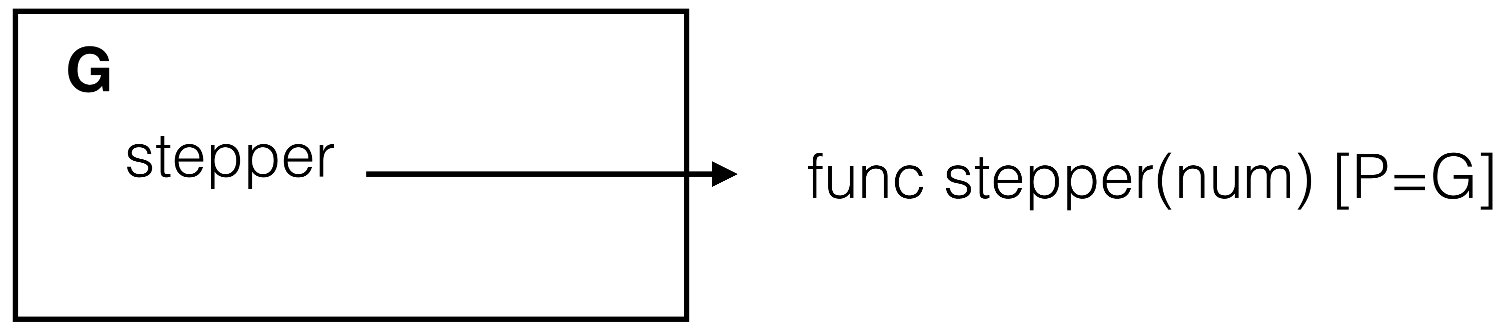

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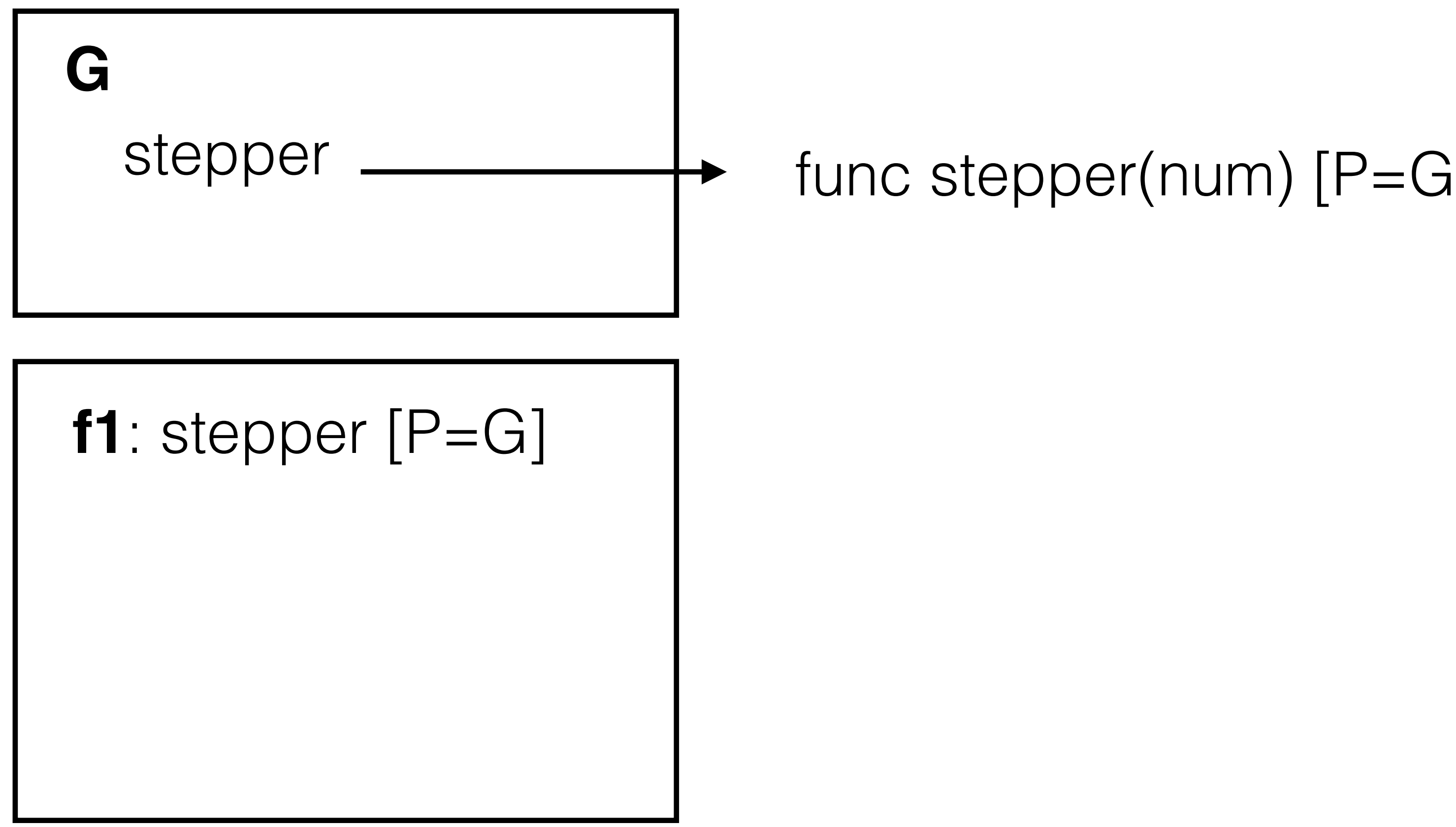

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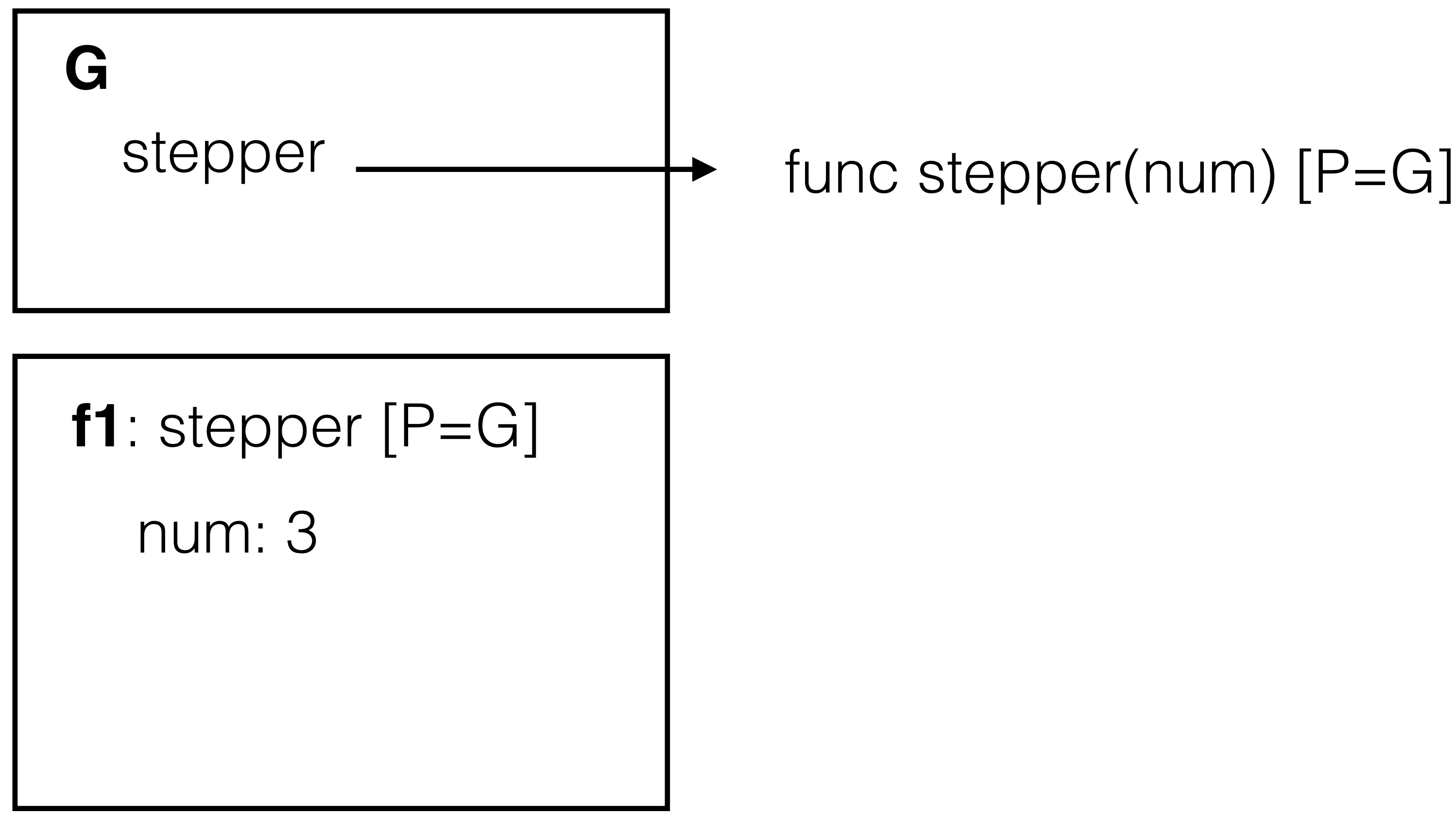

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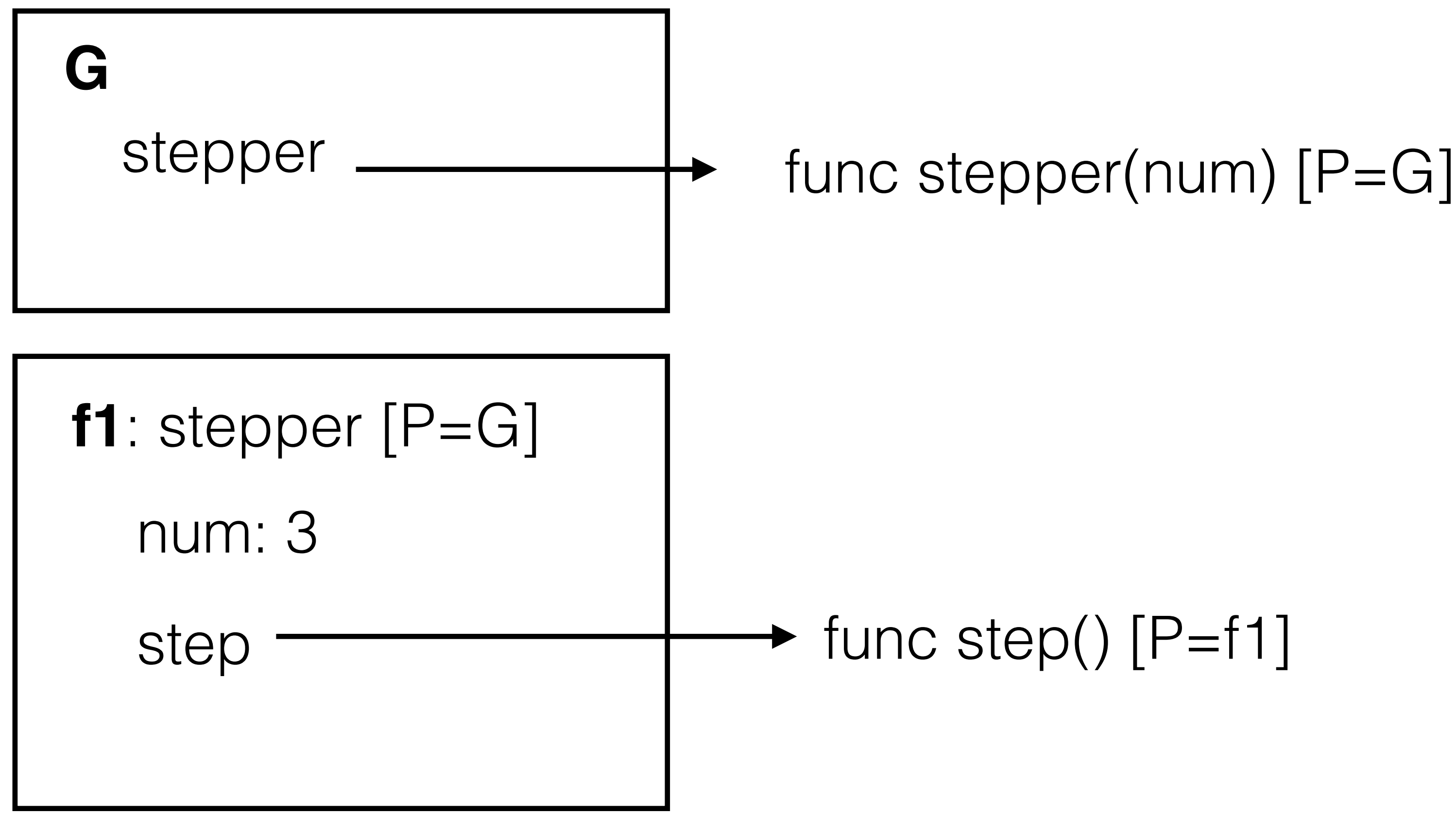

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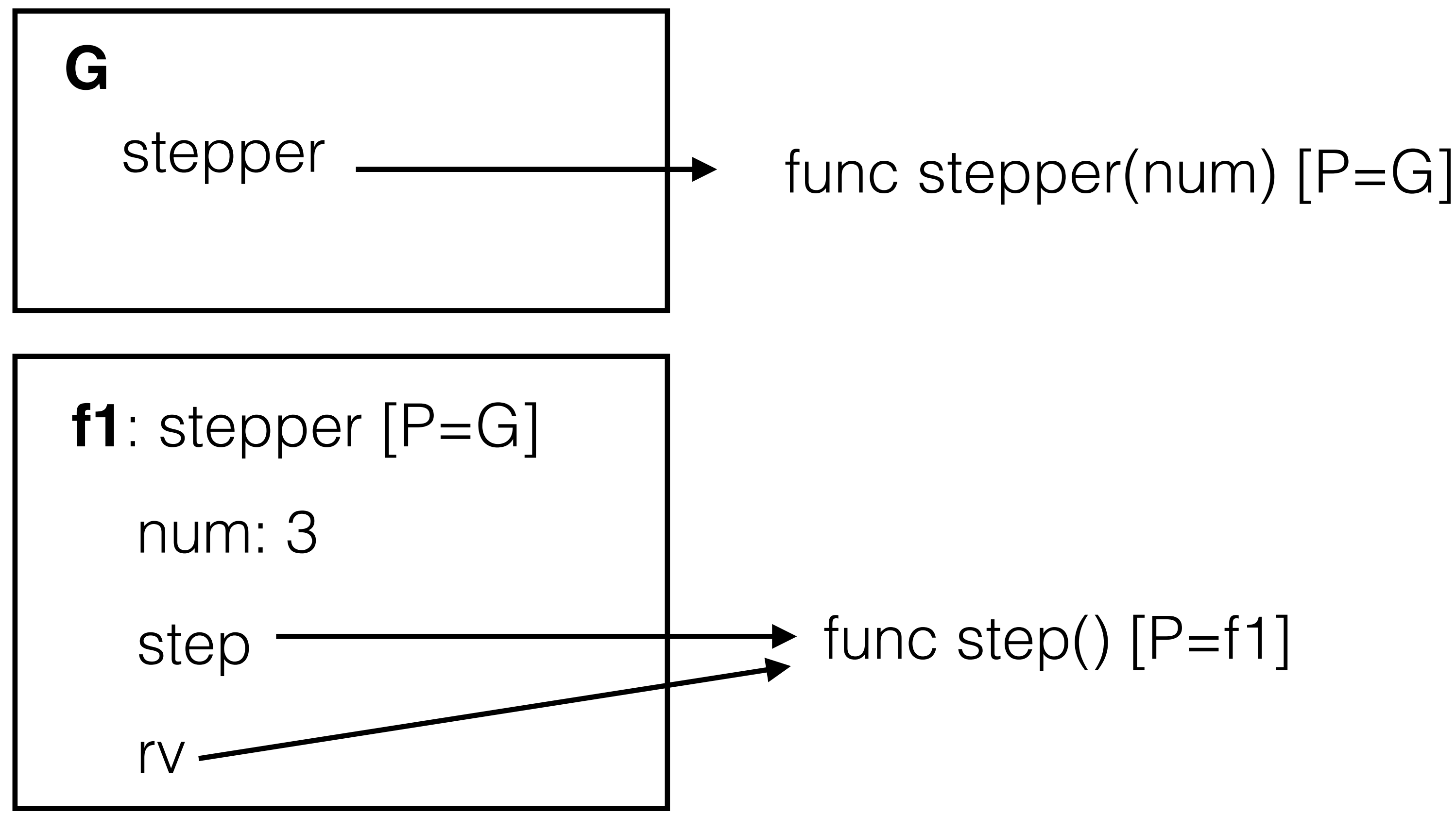

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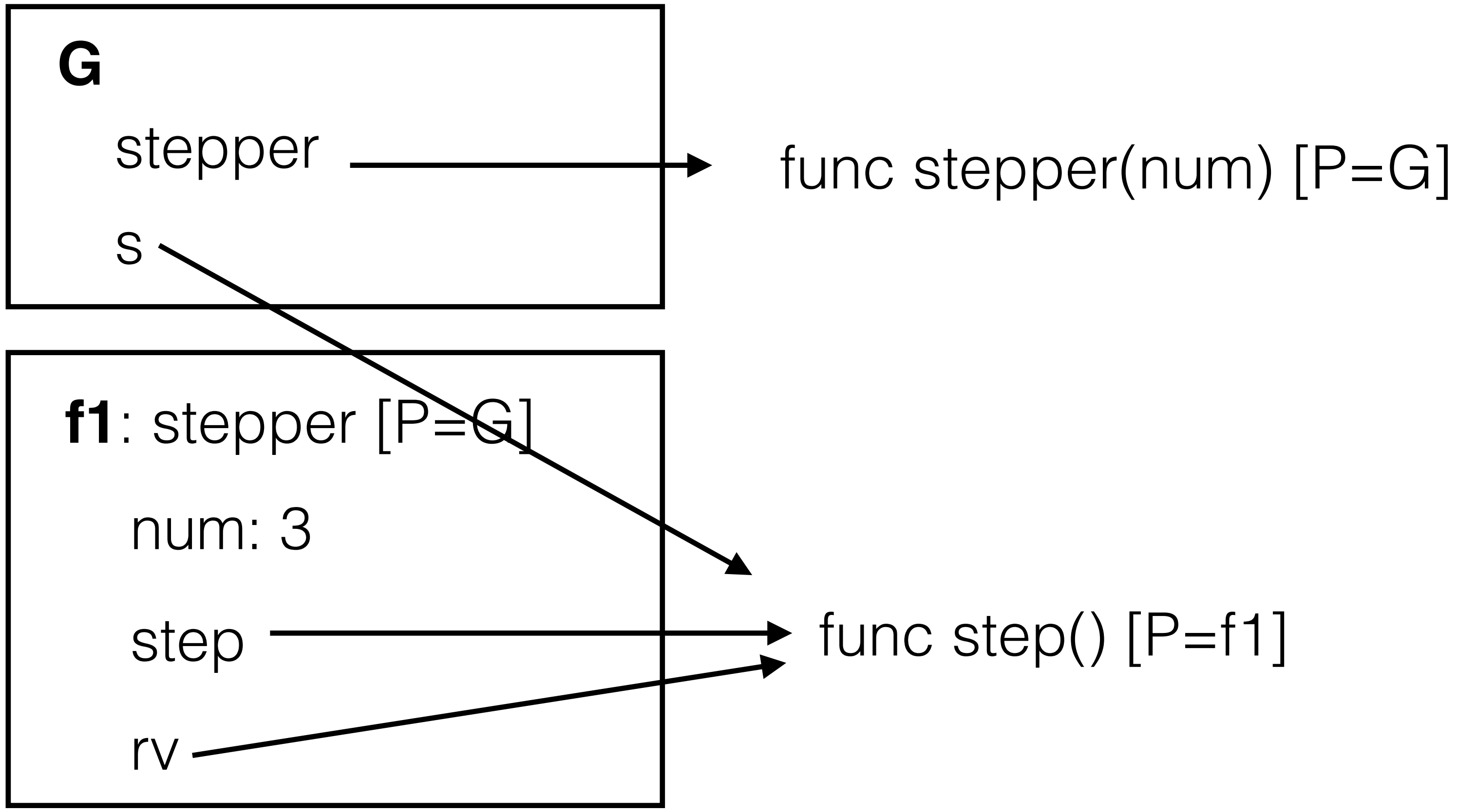

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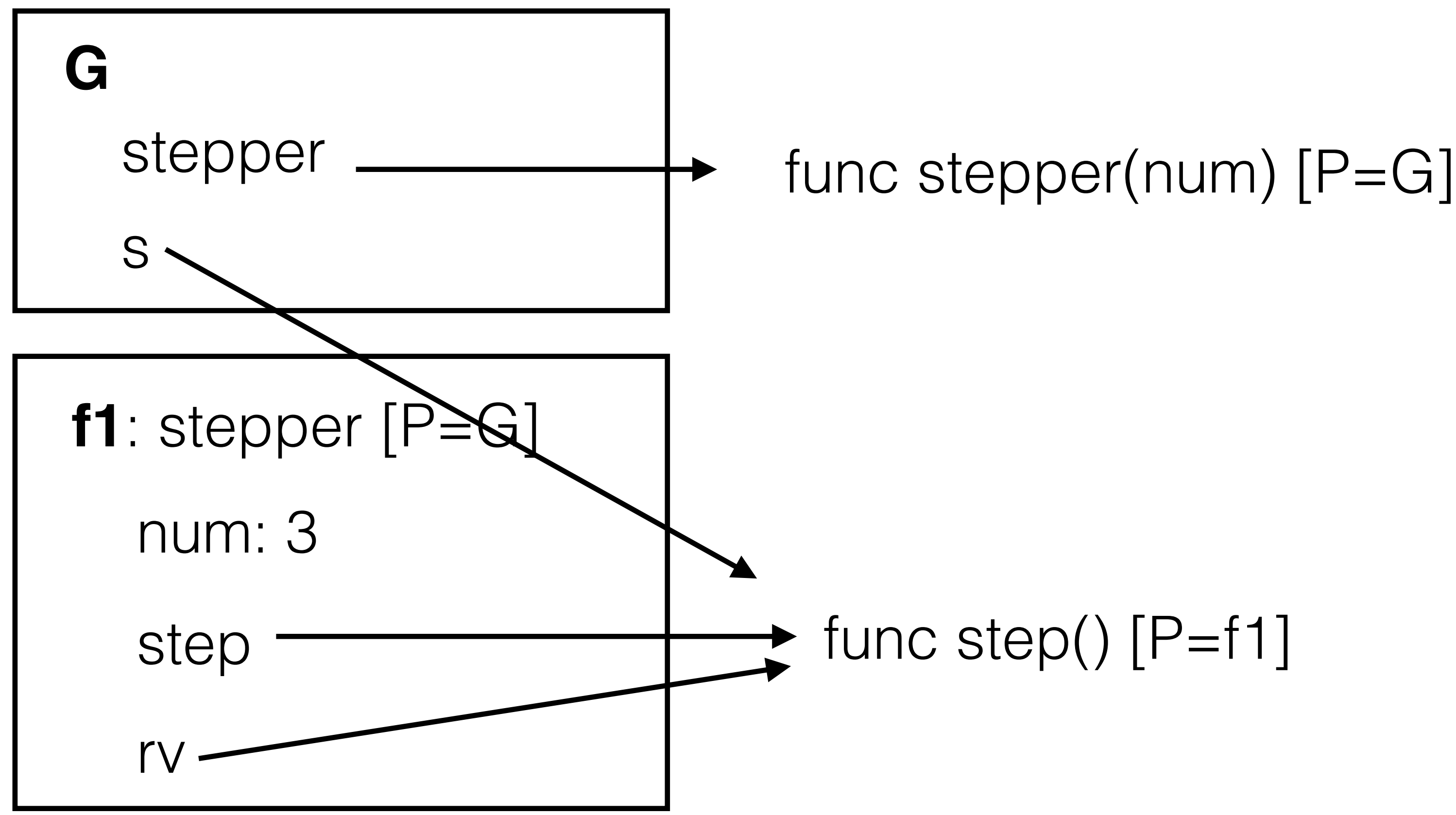

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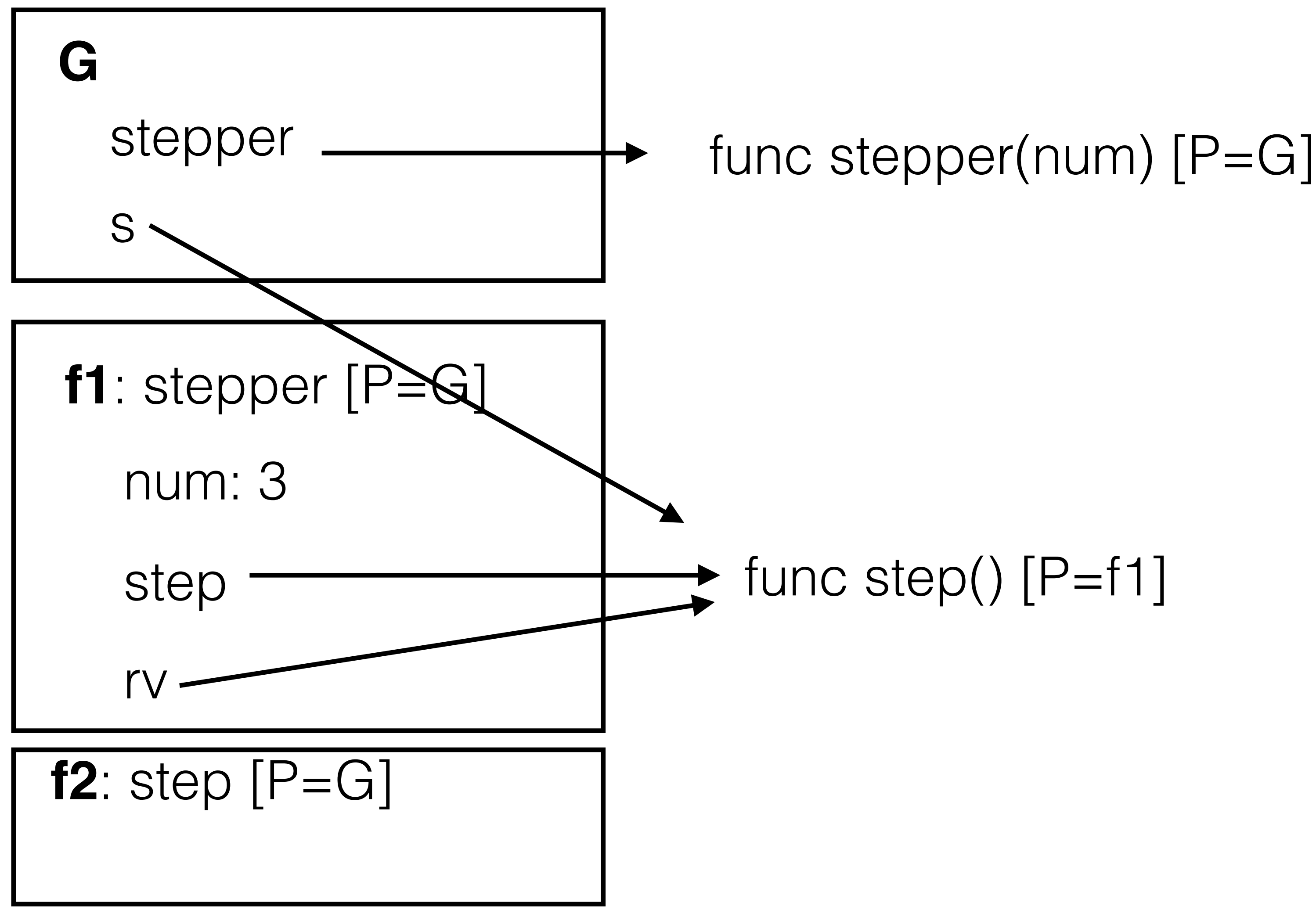


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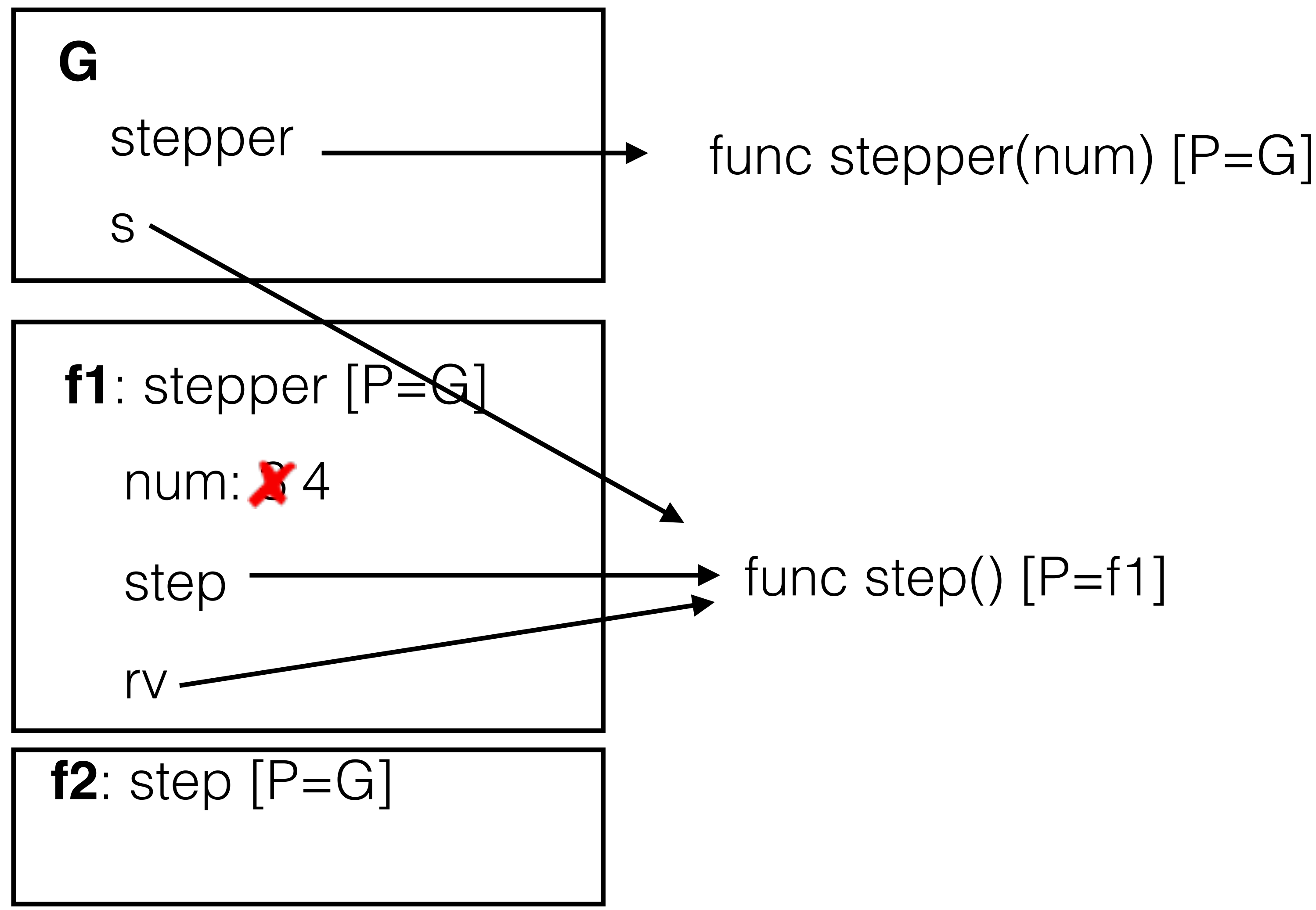


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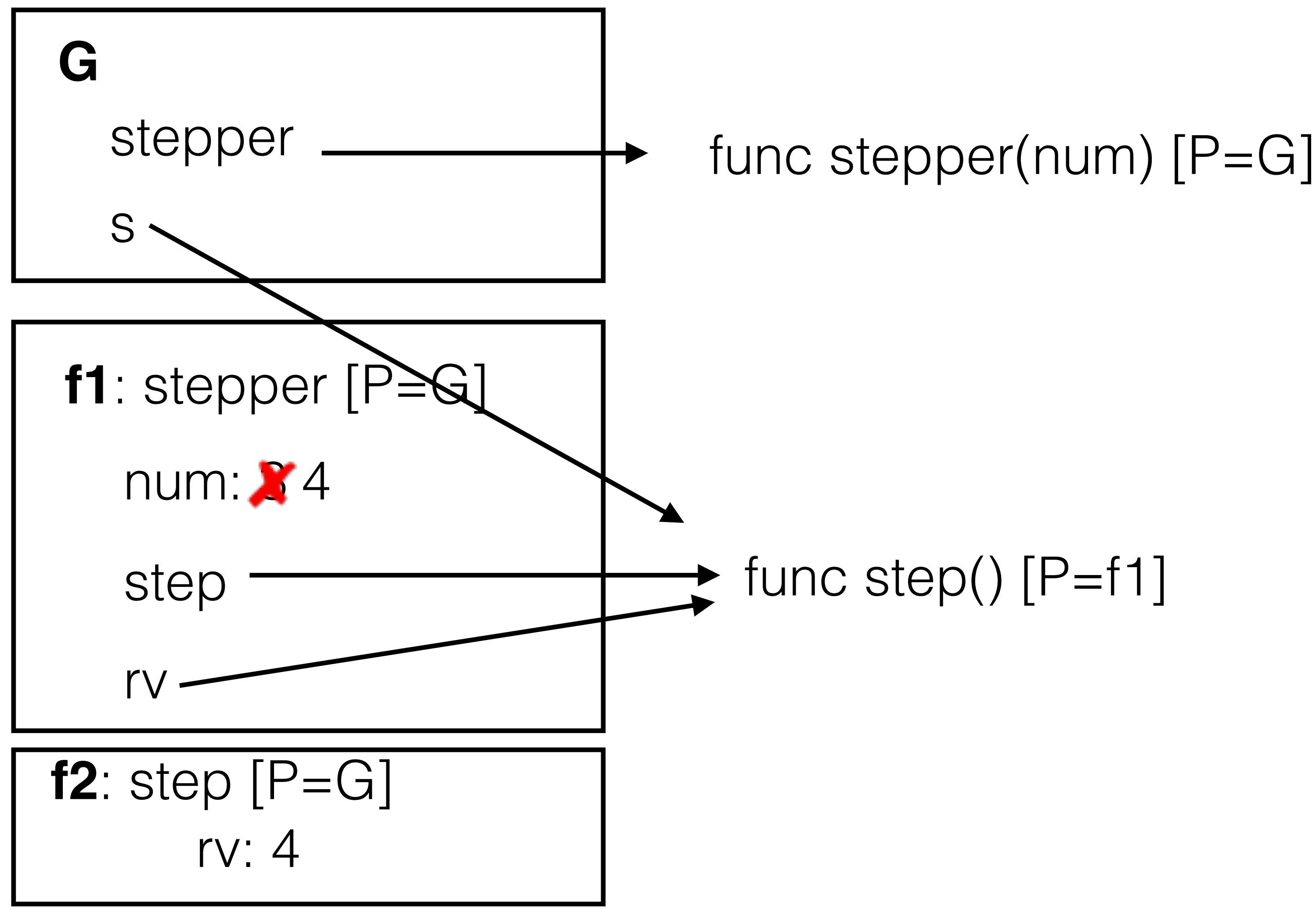

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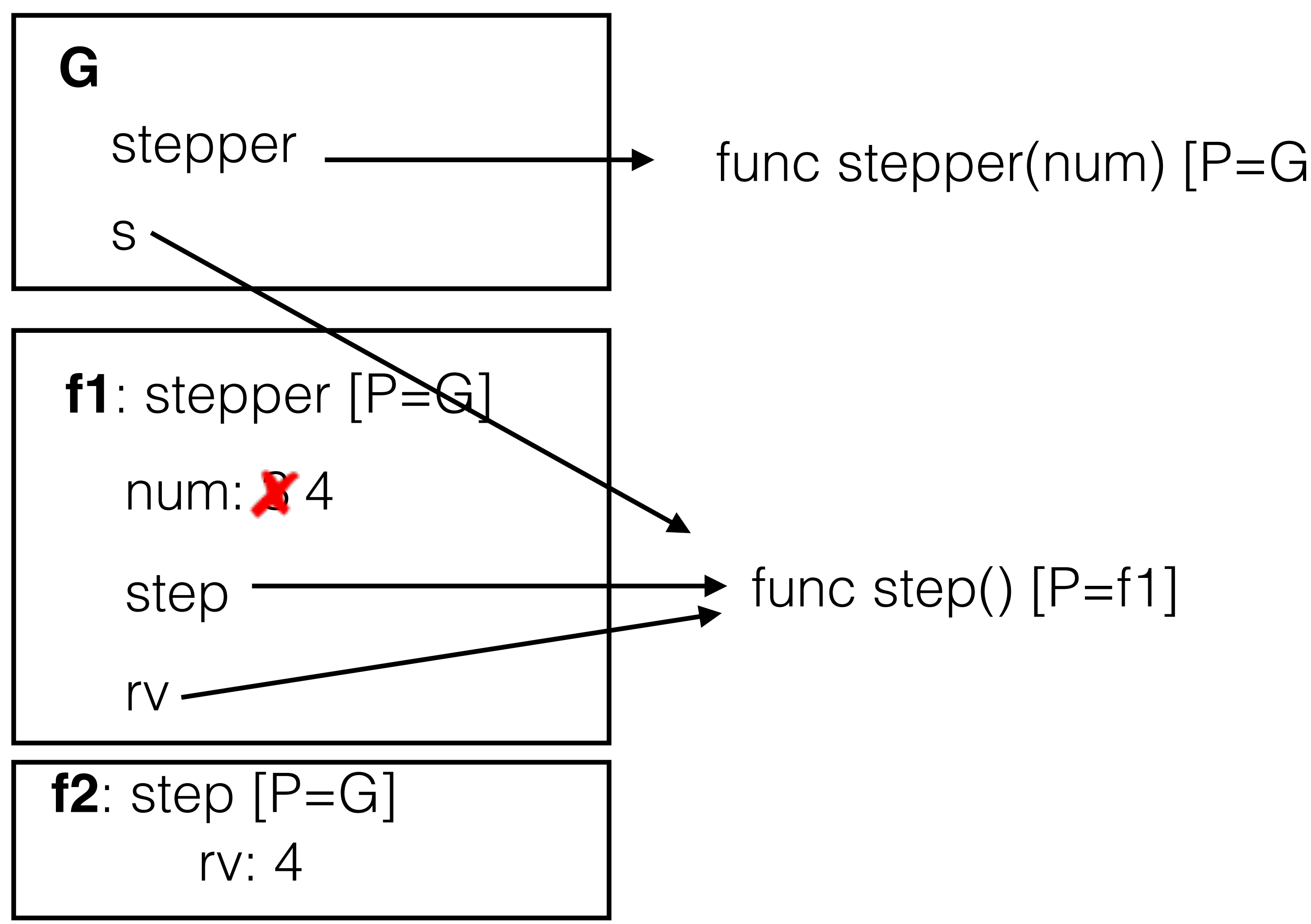
        nonlocal num
        num = num + 1
        return num
    return step

s = stepper(3) f1
s() f2
s() f3
```

nonlocal num! (so the name `num` cannot appear anywhere on the left hand side in this frame!)



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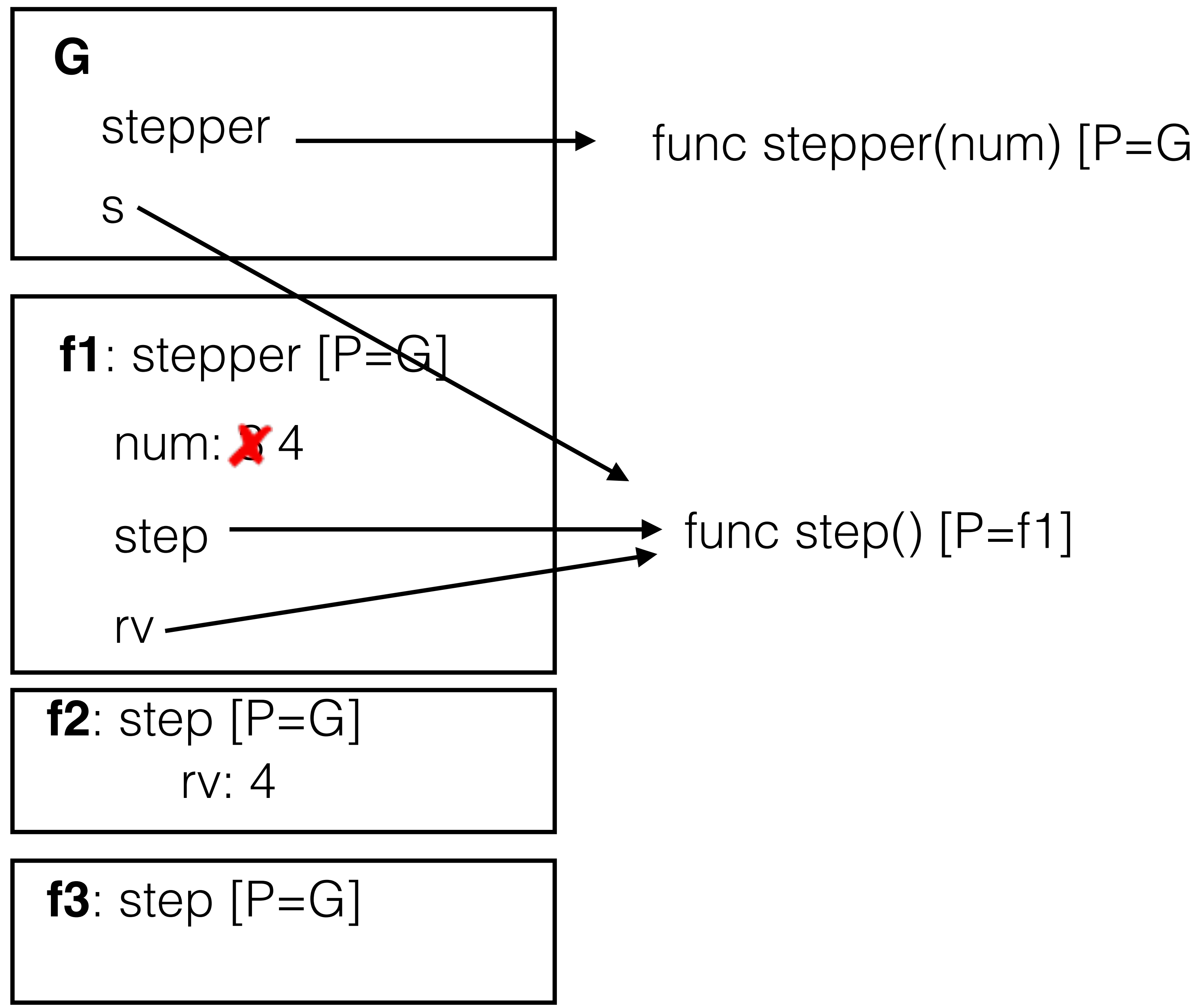

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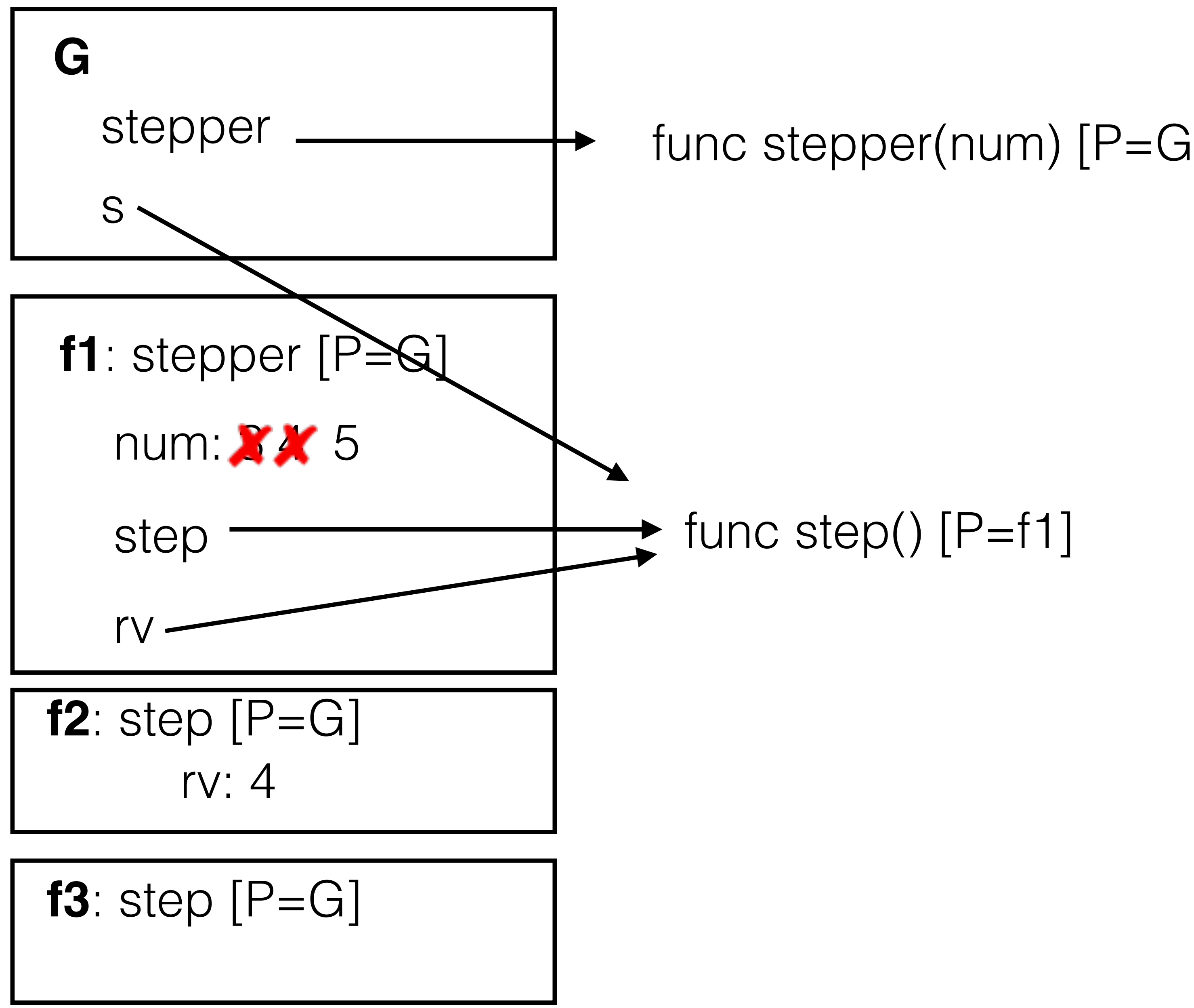

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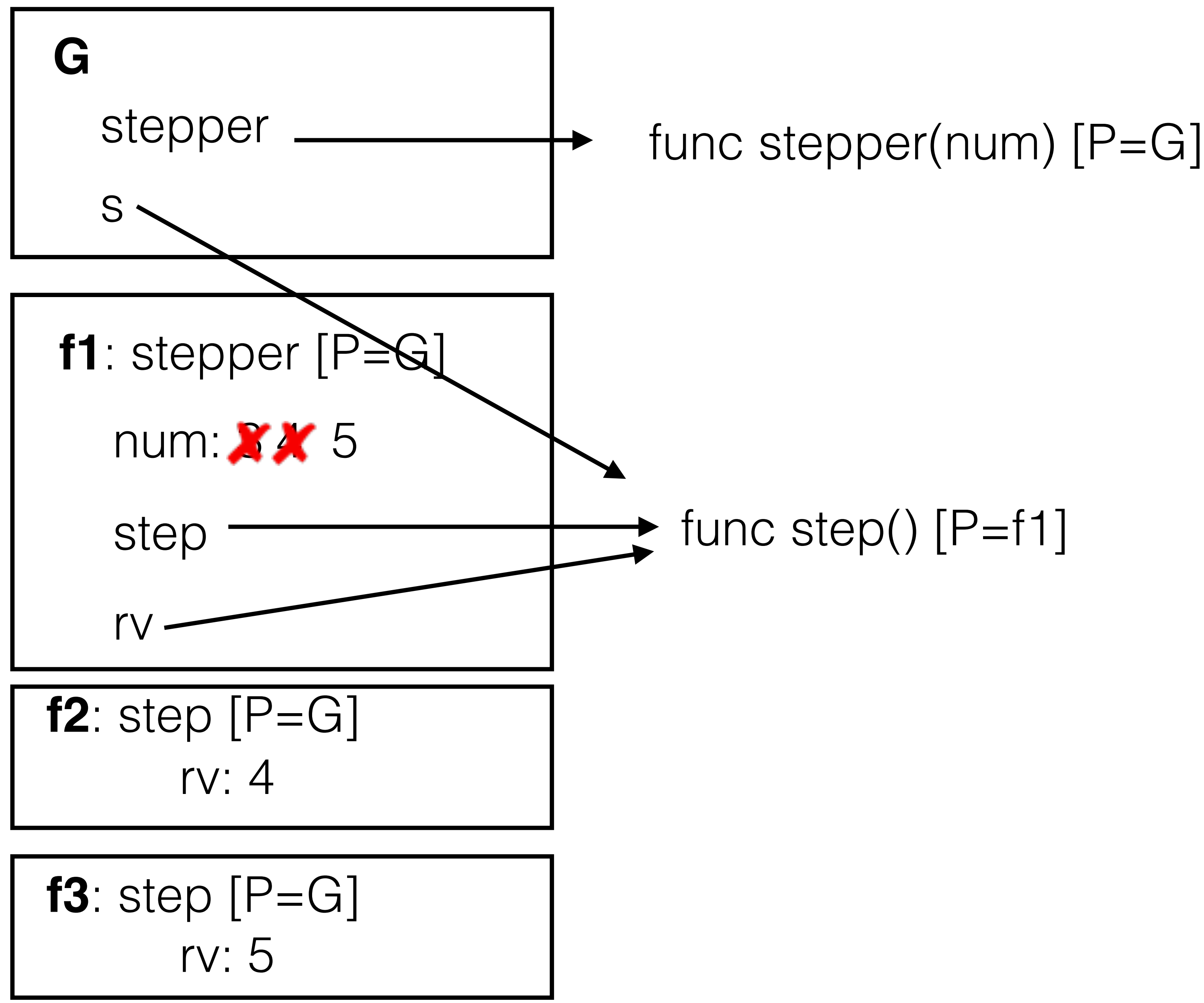


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