Filtered historical simulation for initial margin of interest rate swap under Korean market

Abstract

We discuss how to determine the margin of interest rate portfolio under Korean interest rate market when the trades are cleared through a clearing house. The analysis is based on the filtered historical simulation using the EWMA and GARCH model for the interest rate process. Due to the irregular feature in the short tenor rates, we observe the instabilities of the filtered processes by the EWMA model and we propose how to mitigate the instability. We also explain the properties of the inferred volatility processes depending on the volatility model, the observation interval of the interest rate series and the parameter choice.

1 Introduction

Central clearing for OTC derivatives such as interest rate swap (IRS) and credit default swap (CDS) is now a standard financial system. For stable central clearing, margin or collateral is required for members who trade derivatives via central couterparty (CCP). It is essential to establish a efficient margin model which is reasonable and satisfiable both trading members and CCP. Nowadays, the methodology to determine the margin is similar worldwide (except details) and based on filtered historical simulation and Value-at-Risk (VaR) type risk measure.

The filtered historical simulation is widely used in financial instruments for risk managements and the early studies of the simulation methods include Hendricks (1996), Barone-Adesi et al. (1998), Hull and White (1998) and Barone-Adesi et al. (2002). The basic idea is that the financial risk is not well described by constant volatility and hence risk measure should reflect the time varying conditional volatility such as in GARCH(Bollerslev, 1986) or EWMA model. In these days, combined with Value-at-Risk or Expected Shortfall(Acerbi and Tasche, 2002), the filtered historical simulation becomes a standard method to measure risks associated with positions of members in CCPs (Murphy, 2013; Gregory, 2014).

Under the suggestion by Principles for Financial Market Infrastructure(PFMI), the margin models for OTC derivatives should satisfy some conditions, for example, more than 99% confidence level, sufficient samples to reflect market history, liquidation period for settlements considering extreme market condition ans CCP's hedging ability, procyclicality, daily basis test for margin and test for margin model by independent institution. Similar regulations are also found by European market infrastructure regulation(EMIR) and U.S. Commodity Futures Trading Commission(CFTC). These guidelines are followed by CCPs worldwide.

The margin model of CME is called HVaR, under which the volatilities are estimated by EWMA historical simulation with 1,250 scenarios based on 5 years samples. The historical simulation of the margin model of Eurex, called Prisma, uses filtered historically simulated VaR with 3 years data combined with special one year sample for stress test. JSCC uses similar filtered historical simulation method but using specially generated interest rate curve scenario for stress test.

There have been growing number of literature about clearing model ans collateral in CCP in academics and practice, consult Sidanius and Zikes (2012), Cont et al. (2011), Heller and Vause (2012) and Duffie et al. (2015). This paper focus on the filtered historical simulation model for the margin of interest rate swap traded via CCP under Korean interest rate market. We studies the dynamics of interest rate volatility and filtered innovation in Korean interest rate market and numerical features dealing with sudden changes in the interest rates. Our empirical analysis is based on the portfolio positions traded via CCP.

The remainder of the paper is organized as follows. Section 2, the basic properties of the EWMA and GARCH models for the filtered historical simulations are explained. In Section 3, we analyze the interest rate time series in the Korean interest rate market especially for the choices of time interval and the persistence parameter. In Section 4, we explain the roles of the persistence parameter and volatility floor in the filtered historical simulation based on the EWMA volatility model. Section 5 shows the behaviors of the inferred volatility depending on how to choose the time interval and parameter. Our empirical studies based on Korea interest rate market and the clients' portfolios in CCP are demonstrated in Section 6. Section 7 shows the differences between the filtered and non-filtered simulations. Section 8 concludes the paper.

2 EWMA and GARCH

For the filtered historical simulation of the interest rate series, we consider two basic volatility models, EWMA and GARCH. The EWMA and the GARCH models are both weighted average models for volatility estimation with recursive relationships between the volatilities and past realized returns but the fundamental assumptions are different from each other. The EWMA model can be considered as a volatility estimation method but rigorously speaking, it is not a volatility generating procedure since the underlying returns are assumed to be i.i.d. and hence the variance of return is constant over time. More detailed explanation, consult Alexander (2009). Meanwhile, the GARCH model has a time varying conditional volatility with mean reverting property by the following relationship for the conditional variance:

$$\sigma_{t+1}^2 = \omega + \alpha u_t^2 + \beta \sigma_t^2$$

where u is the underlying return series, σ_t is the conditional volatility of the return upon the information up to time t, and ω, α, β are the GARCH parameters. In our analysis, u is the difference or log-return of the interest rates.

Although the EWMA is basically not a volatility generating model, the EWMA recursive formula is used like a conditional volatility formula as in the GARCH model so that we can mimic the volatility generating procedure:

$$\sigma_{t+1}^2 = (1-\lambda)u_t^2 + \lambda\sigma_t^2$$

where σ is a hypothetical volatility process, u is the underlying time series and λ is the EWMA parameter. By doing so, we can perform historical simulation, likelihood estimation and generate future volatilities in a relatively short time horizon (since in the long run, the EWMA generated volatility process dies out in the absence of constant term such as ω in GARCH). Although these methods based on EWMA formula may not be mathematically rigorous, in general, we have quiet reasonable results with estimations and simulations. Thus, we consider the EWMA volatility model is one of the candidate for the filtered historical simulation for the interest rates.

Similarly in the GARCH model, the log-likelihood function in the EWMA model can be defined by

$$L(\lambda) = \sum_{i=1}^{N} \left(-\frac{1}{2}\log(2\pi) - \log\sigma_i - \frac{u_i^2}{2\sigma_i^2} \right)$$

with initial σ_1 being the sample standard deviation of the returns. One can find the proper value of λ that maximize the function as in the case of the GARCH model. However, note that there is no mathematical justification for EWMA volatility procedure and corresponding likelihood estimation method, λ is chosen subjectively in many practical cases.

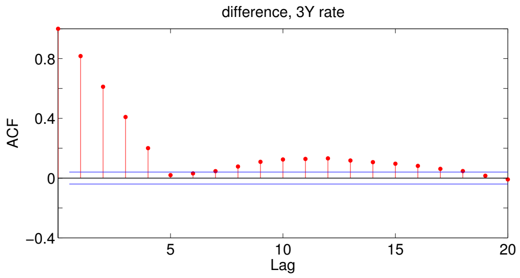
Since there is no rigorous method of choosing λ, we use the value maximizing $L(\lambda)$ as the estimator of λ in the empirical study later and discuss about the practical and subjective choice of λ. In addition, by defining $\varepsilon_i = u_i/\sigma_i$, we can perform a filtered historical simulation under the EWMA or GARCH framework, for example, i-th scenario of the tomorrow return is generated by

$$u_{N+1}^{(i)} = \sigma_{N+1}\varepsilon_i$$

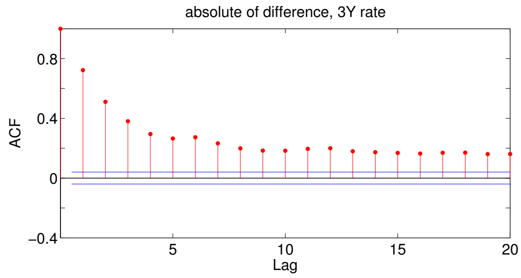
for $1 \leq i \leq N$, where $N+1$ denotes the tomorrow's time index,σ_{N+1} is the tomorrow's volatility computed by the GARCH or EWMA model and ε_i is a past realized standardized innovation.

3 Time series analysis

The liquidation day or the risk horizon for the clearing of interest rate swaps is generally set to five days and hence it is important to generate the future scenarios of the given portfolio and associated risk over five days interval. We discuss the three methods of generating scenarios over five days horizon and the corresponding interest rate time series. The data set used for



(a) autocorrelation of difference data



(b) autocorrelation of absolute of difference

Figure 1: Correlograms of the interest rate difference series, five days interval, overlapped, 3Y

test in this section is the interest rate from 2005 to 2014 with eighteen tenors from 1D overnight call rate and 3 month CD91 rate to 20Y IRS rate in the Korean interest rate market. Due to the lack of the space, in this section we generally discuss the properties of the interest rate with tenor 3Y. The IRS rates in Korea have similar time series properties over the tenors. For the short rate with 1D and 3M tenor, we discuss the details in the next section.

First, one can perform the filtered historical simulation with the five days interval time series with one day lag, i.e., two consecutive difference or log-return data overlap over four days. This is similar to the method used by CME for the filtered historical simulation. We examine the properties of the overlapped difference time series of the interest rate with five days interval. (The properties of the difference and log-return series are similar so we consider only difference series.) Due to the overlap, the difference series has autocorrelation up to four lags.

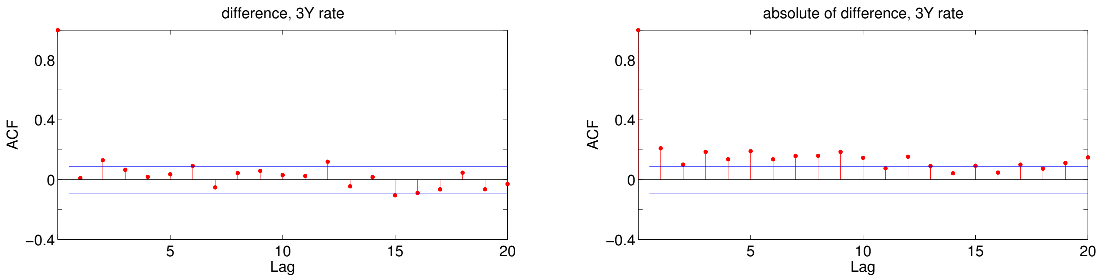
We plot the autocorrelation of the difference series of the interest rate with three years tenor in Figure 1a where significant autocorrelations are observed for lags less than five. These autocorrelations affect the autocorrelation of the absolute series of the difference which are used to diagnose whether the underlying return has the GARCH property. Recall that strong autocorrelation in the absolute series implies the volatility clustering, a typical property of the GARCH process.

Figure 1b shows the more strong but rather quickly diminishing autocorrelation for lags less than five due to the overlaps than the lags larger than or equal to five. Due to this effect, the autocorrelation of the absolute series have the exponentially decreasing shape. This is also represented by the GARCH parameters $\alpha = 0.5223$, $\beta = 0.4777$ and EWMA parameter $\lambda = 0.5733$ estimated by the likelihood method. The estimate of α is larger than the ones estimated from typical financial return series such as index returns and β is smaller, which representing the quickly diminishing shape of autocorrelation of the absolute series for lags< 5. Note that in typical financial return, β is much larger and close to 1 representing the long persistence of the volatility.

When we estimate the GARCH model based on the difference series, we scale the underlying series for the numerical procedure since the difference of the interest rate is quiet small. The estimates are presented in Table 1.

Table 1: Estimates of the difference series with tenor 3Y

interval	overlap	λ	ω	α	β
5	Y	0.5733	5.39×10^{-8}	0.5837	0.4163
5	N	0.7526	8.49×10^{-7}	0.2383	0.0469
1	N	0.8303	3.87×10^{-9}	0.1153	0.8826



(a) autocorrelation of difference data

(b) autocorrelation of absolute of difference

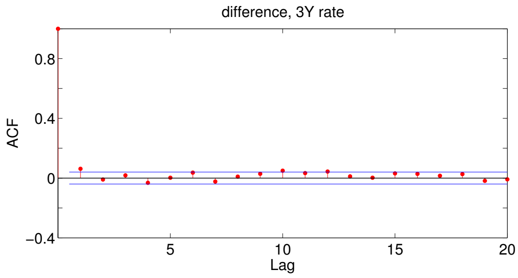
Figure 2: Correlograms of the interest rate difference series, five days interval, nonoverlapped, 3Y

Second, the filtered historical simulation can be performed with the non-overlapped five days interval time series. We examine the difference series of the interest rate with five days interval without overlapping. There is no significant autocorrelation in the difference series as in Figure 2a contrast with the previous case of the overlapped series. However, the sparse interval leads to weak autocorrelation in the absolute series. Note that ten lags implies fifty business days. We observe positive autocorrelation in the absolute series as in Figure 2b but the persistence of the volatility clustering is relatively weak as the lags increase. The weak persistence is clear when compared with the next case with daily data. The persistence parameter β of the interest rate difference series with 3Y tenor is estimated as 0.0469 which is quiet small. (In addition, the reduced numbers of the sample size can be a drawback in practice.)

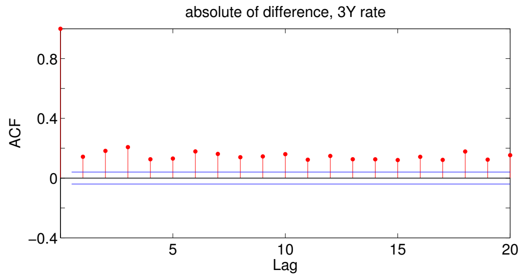
Third, we can perform the filtered historical simulation on a daily basis. To generate the five days scenarios of the liquidation, we can employ five steps simulations based on daily ε in the sense of the statistical bootstrapping. The autocorrelations of the one day time series are almost zeros and the absolute series has significant autocorrelations as in Figure 3. Theses correlograms are similar to the ones of typical financial returns. The estimates of $\beta = 0.8387$ and $\lambda = 0.8857$ are relatively close to one compared with the previous cases, implying strong persistence of the volatility clustering compared with the previous cases.

To sum up, for the filtered historical simulation, the daily basis filtering is natural way in the aspects of the parameter estimation and generating volatility. However, for a practical purpose, to use five days interval data with or without overlap is fine, when we are careful about some technical issues. We discuss it in the next section.

Before proceeding, we explain how we choose the parameter values for the filtered historical



(a) autocorrelation of difference data



(b) autocorrelation of absolute of difference

Figure 3: Correlograms of the interest rate difference series, one day interval, 3Y

simulation. There are eighteen tenors for the interest rates from 1D to 20Y and the properties of the series are not identical among tenors. Especially 1D and 3M data are different from the others in Korean interest rate market. However, for the model parsimony, we assume that the parameter values of the volatility model are the same among tenors so that we only consider one λ or a set of ω, α, β for all tenors. For simplicity, when we estimate the parameters, we find the values that maximize the sum of log-likelihood functions over all tenors k:

$$\sum_k L_k(\theta)$$

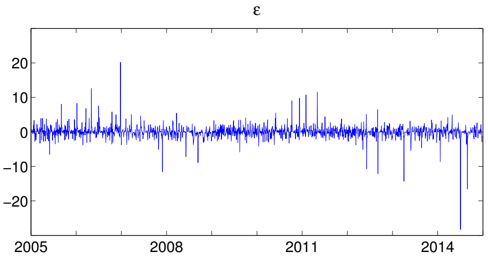
where θ is λ for the EWMA model and ω, α, β for the GARCH model.

4 EWMA λ and volatility floor to the past

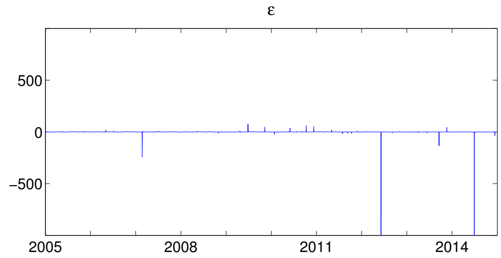
In this section, we explain the roles of λ and volatility floor in the filtered historical simulation based on the EWMA volatility method. The volatility floor in this section is applied to the past volatility series and different from the floor applied to the today's or future volatilities to make sure that the future volatilities are higher than a certain level and to prevent large breaks in the initial margin computed by the simulation. About the floor to the future volatility, we will discuss later. Our analysis focus on the strange behavior of ε generated from the EWMA volatility formula based on the difference series with tenors 1D and 3M.

For the time being, we examine the difference series of the interest rates with overlapped five days interval, the first case explained in the previous section. The estimates of λ by the maximum likelihood method of the overlapped series are smaller than the typical subjective choice of the EWMA parameter in financial risk managements. We have $\lambda = 0.7297$ and 0.8303 for tenor 1D and 3M, respectively, under the likelihood estimations. By the relatively small λs compared with the subjective choice of the EWMA parameter in financial analysis and along with the inherent natures of 1D and 3M interest rates in Korean interest rate market, we observe unstable dynamics of inferred ε.

The computed ε for tenor 1D and 3M shows extreme behavior, especially for 3M. The outliers of ε in Figure 6a quiet large in the aspect that ε follows a standardized and normalized

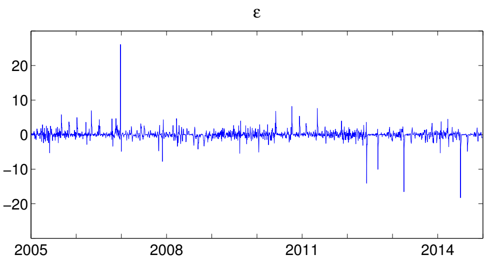


(a) 1D, $\lambda = 0.7297$

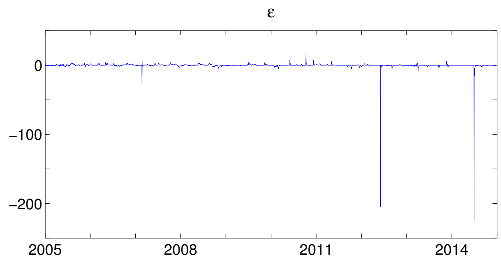


(b) 3M, $\lambda = 0.8303$

Figure 4: The behaviors of ε generated by the EWMA volatility formula with λ calibrated from market data



(a) 1D, $\lambda = 0.94$

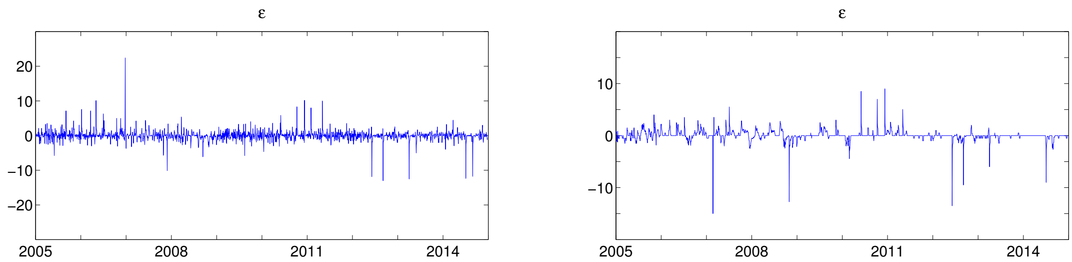


(b) 3M, $\lambda = 0.94$

Figure 5: The behaviors of ε generated by the EWMA volatility formula with one of typical choice of $\lambda = 0.94$

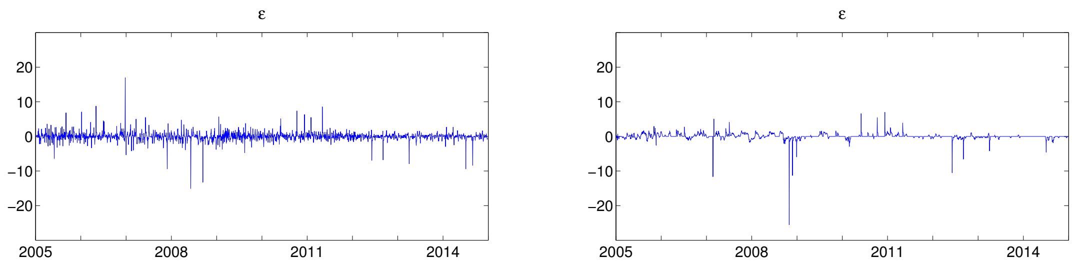
empirical distribution. The outliers of ε in Figure 6b are even more extreme. The reason is that fundamentally, the EWMA is not the best model to describe the behaviors of the interest rate series with 1D and 3M tenors. More precisely, the computed volatility based on EWMA formula can be very small especially when λ is not large enough.so that the absolute values of $\varepsilon_t = u_t/\sigma_t$ can be very large. For example, at 2014-08-14, there is an announcement by the Bank of Korea about lowering the standard interest rate in the middle of low volatility regime. Due to the low volatility regime, the computed σ_t is close to zero but by the sudden change in the standard interest rate, the realized u_t for 3M data is large enough to generate the extreme outliers in ε_t. The extreme distributions of ε affect on the IRS portfolio's simulated distribution especially for short maturity.

When λ is close to 1, for example, 0.94, one of the typical choice of λ in financial risk managements, we expect a mitigation effect on the extreme distribution of ε since the strong volatility clustering effect caused by large λ makes the volatility level tend to be relatively high even during a low volatility period. With sufficiently large λ, we observe that the extreme values of ε in 1D and especially in 3M data are diminished in magnitudes, see Figure 5b. There is also an mitigation effect on the tail of the P&L distribution of the IRS swap generated by the filtered historical simulation based on ε. About the mitigation effect in P&L distribution, we discuss later.



(a) 1D (b) 3M

Figure 6: The behaviors of ε generated by the EWMA volatility formula with volatility floor = 0.002



(a) 1D (b) 3M

Figure 7: The behaviors of ε generated by the GARCH model

The volatility floor to the past volatility also plays an important and more powerful role to mitigate ε and to prevent the initial margin from being extreme. As mentioned before, the reason for extreme ε is too small inferred volatility in the volatility model. The volatility floor guarantees the minimum of volatility all over time and make there is no too extreme value of ε. In Figure 6, we plot ε for 1D and 3M data where the mitigation effects are more pronounced than the previous case using large λ to mitigate the effect. The volatility floor is chosen as 0.002.

In the GARCH model, ω plays a similar role of the volatility floor since the volatility is always greater than $\sqrt{\omega}$. (This does not imply that ω and the volatility floor to the past are equivalent.) We set $\omega = 3.29 \times 10^{-8}, \alpha = 0.4953, \beta = 0.5037$ and plot the inferred ε in Figure 7 The results in ε are similar to the EWMA with the volatility floor.

Based on the above analysis, we discuss how to deal with extreme values of ε when computing the expected shortfall (ES) for the initial margin. First, we can assume that the extreme distribution of ε is the true nature of the interest rate series dynamics and apply directly to the computation of ES without any adjustment. Consider an illustrative interest rate swap with fixed rate 2.035% and six month time-to-maturity on 2015-2-9 and notional ten billions won. Table 2 presents the worst twelve outcomes of a fixed receiver of the swap computed based on the different methods of the filtered historical simulation with 1250 samples ranged from 2011 to 2015-2-9. The column A of the table is based on direct application of ε computed by the EWMA formula for five days difference overlapped data with $\lambda = 0.83$ without further adjustments. The value of λ is calibrated as maximizing $L(\lambda)$ for overall tenors. The worst outcome is larger than one billion one, more than ten percent of the notional value of the swap and it is hard to see this is a reasonable value. The initial margin based on the 99% expected shortfalls computed by the average of the worst twelve outcomes are presented in the last row. The margin is almost 10% of the notional value. This phenomenon is more extreme when today's volatility is larger than the volatility of the past day where the extreme ϵ is observed.

Second, if we use sufficiently large λ, then we mitigate the extremeness in ε and ES to some degree. The column B is the results of the filtered historical simulation where we subjectively choose $\lambda = 0.94$. Whether the worst scenario is too extreme or not might be controversial but the worst outcome is still about ten times of third worst outcome. In addition, to use this method, it is always a important concern how to choose proper value of λ. One possible suggestion is to use one day interval data where the persistence is stronger than the five days interval data and hence the persistence parameter λ is naturally large enough to avoid too extremeness.

Third, we can set the volatility floor $= 1.67 \times 10^{-4}$ for the EWMA historical simulation. The mitigation effect of the floor to the P&L distribution is presented in the column C where, for example, the increment between the worst and second worst outcomes is diminished compared with the previous case. In addition, the margin based on ES is reduced.

Fourth, we use the GARCH model where the volatility floor is implicitly given by $\sqrt{\omega}$. The simulation is performed with $\omega = 2.79 \times 10^{-8}, \alpha = 0.5873, \beta = 0.4117$ which are estimated by the unified maximum likelihood estimation. The shape of the distribution of the P&L is similar to the EWMA historical simulation with the volatility floor.

To sum up, if we want to stabilize the distribution of ε in 1D and 3M data and corresponding simulation outcomes, use large enough persistence parameter λ or set a volatility floor. A simple way is to use daily series where the persistence of the volatility is naturally large and use the GARCH model where the volatility floor is implicit and hence there is no need to subjectively choose persistence parameter or volatility floor. However, if we choose λ and the volatility floor properly, the EWMA volatility model still works fine for the filtered historical simulation under Korean interest rate market.

5 Inferred volatility

We examine the behaviors of the inferred volatilities computed by the EWMA or GARCH model when we choose underlying series differently in the length of intervals and whether the intervals are overlapped or not. As in the previous section, we can examine three types of the underlying series: the difference series of the interest rates under daily basis, the series based on five days interval with or without overlap. When the difference series is overlapped, two consecutive five days differences share four days. The underlying can be the difference or log-return series

Table 2: Worst scenarios of an IRS swap by filtered historical simulation with various methods

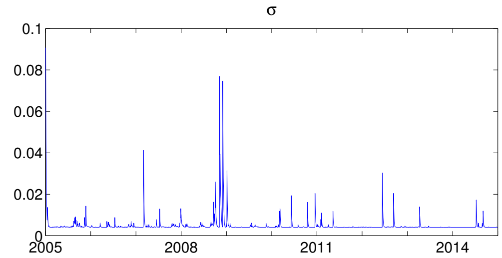
A	B	C	D
-2,268,703	-2,092,855	-1,824,430	-3,098,794
-2,273,937	-2,118,326	-1,832,176	-3,102,530
-2,302,042	-2,332,237	-1,847,114	-3,133,531
-2,395,042	-2,339,449	-1,972,055	-3,149,069
-2,425,370	-2,340,576	-1,978,937	-3,293,471
-2,527,856	-2,536,548	-2,012,346	-3,317,941
-2,598,009	-2,769,046	-2,095,297	-3,464,800
-2,886,836	-2,836,921	-2,109,377	-3,597,027
-3,142,333	-3,601,720	-2,186,658	-3,662,049
-3,990,394	-4,996,013	-2,527,857	-3,915,927
-15,651,230	-29,989,667	-2,794,476	-3,959,191
-1,133,053,493	-48,138,666	-3,986,751	-4,993,393
97,959,604	8,841,002	2,263,956	3,557,311

but two series are not much different in their volatility properties so that we only examine the difference series in this section.

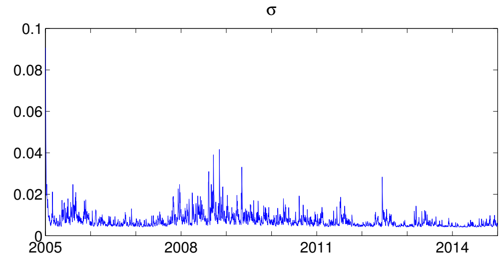
In Figures 8a and 8b, we plot the annualized volatility series computed by the GARCH(1,1) on a daily basis from 2005 to 2014 for tenors 3M and 3Y, respectively. The parameters are set to $\omega = 8.981 \times 10^{-10}, \alpha = 0.0513, \beta = 0.9423$ which are estimated by the unified likelihood estimation explained in Section 3. As mentioned before, we observe volatility floors induced by ω in both figures as the volatilities are larger than a certain level.

We plot the inferred volatility series based on the overlapped five days interval difference series for tenor 3M and 3Y, in Figures 8c and 8d, respectively. The volatility series based on non-overlapped five days interval for tenor 3M and 3Y also plotted in Figures 8e and 8f, respectively. For the volatility calculations for both cases of five days interval, we employ the EWMA model with subjectively chosen $\lambda = 0.97$. These specific methods are chosen with the purpose of replicating the descriptions of the risk methodology used by the international clearing houses such as CME and EUREX.

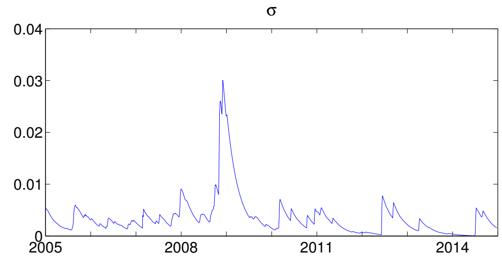
The volatilities based on five days interval data show the longer persistences compared with the volatilities on a daily basis since we subjectively impose the large persistence parameter $\lambda = 0.97$. Recall that this is larger than the value computed by the likelihood method in the previous section. In addition, the volatilities based on non-overlapped interval shows the most persistent behaviors. The peak of volatilities right after the financial crisis in the five days non-overlapped data (Figures 8e and 8f) are smaller than peaks in the daily and five days overlapped data but the persistences are longer enough to reach around 2011. This is contrast with the observations that the large volatilities during the financial crisis based on daily or



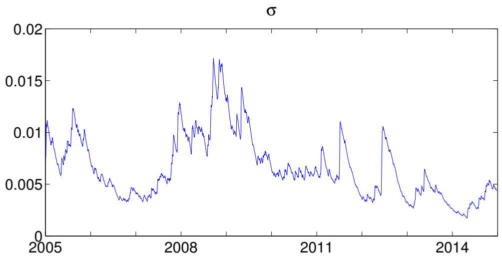
(a) 3M, daily data



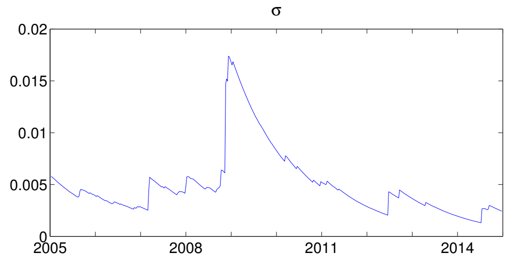
(b) 3Y, daily data



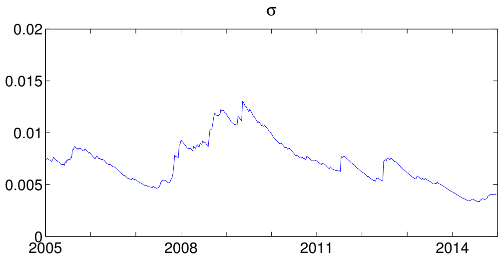
(c) 3M, 5 days, overlapped



(d) 3Y, 5 days, overlapped



(e) 3M, 5 days, non-overlapped



(f) 3Y, 5 days, non-overlapped

Figure 8: Comparison of inferred volatilities

five days overlapped data are rather quickly vanishes. The longer persistence tend to make the volatility to be smoothed over time. There are lots of volatility peaks in the data on a daily basis (Figures 8a and 8b) but theses hikes are smoothed in five days non-overlapped data (Figures 8e and 8f). Of course, theses properties affect the time variations of the initial margin computed by the filtered historical simulations as we will show in the next section.

Since the sudden changes in the volatility cause the sudden changes in the initial margin, one can intentionally smooth the volatility to prevent the abrupt changes. For example, the following algorithm is the one used by CME. Once all volatility process σ is computed by the EWMA of GARCH formula, then the smoothed volatility $\tilde{\sigma}_t$ is defined by

$$\tilde{\sigma}_t = \sigma_{t-1} + \eta(\sigma_t - \tilde{\sigma}_{t-1})$$

with a smoothing parameter η. Assuming that the same portfolio of IRS exists over time, we compare the margin determined as the 99% expected shortfall (ES) computed by FHS with the margin adjusted by the volatility smoothing in Figure 9. For the FHS, we use daily data and the GARCH model with $\omega = 9 \times 10^{-10}, \alpha = 0.0513, \beta = 0.9423$ and the smoothing parameter

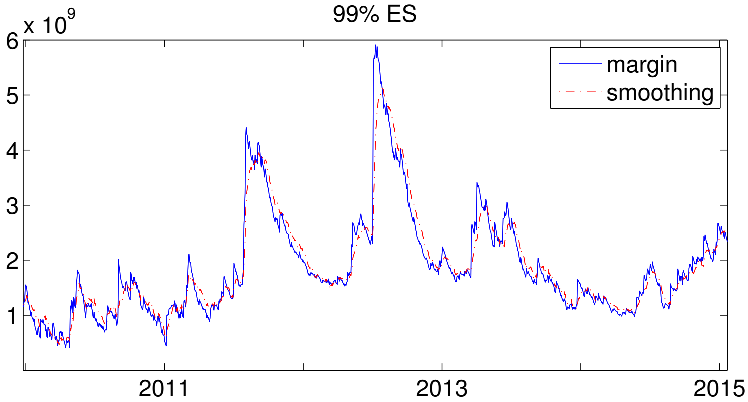


Figure 9: The variations of margins with and without volatility smoothing, $\eta = 0.1$

$\eta = 0.1$. If we choose smaller η, then smoother the initial margin variation. Meanwhile, EUREX use five days non-overlapped data with λ close to 1 as in Figures 8e and 8f where the volatility processes are smoothed enough and hence the behavior of the initial margin is also smoothed correspondingly.

6 Back test of initial margin

6.1 Stress period

Some CCPs such as EUREX introduce an additional stress period to determine the initial margin to the filtered historical simulation. The historical simulation for the stress period may not be filtered by volatility and the realization of the differences are directly applied to the future simulation. In our analysis, the stress period is set to be 250 days from 2008-07-01. We use the same method proposed by EUREX for the risk calculation based on the stress period and the initial margin are determined by the maximum of the margin by the GARCH filtered historical simulation and the margin by the stress period simulation multiplied by 0.7. Briefly speaking about the EUREX methodology of stress test, the stress period based margin is determined by the average of five 99.7% VaR generated by five subgroups of the stress period where each group consists of five days non-overlapped historical series. More detailed information, see.

By including the stress period for the historical simulation, there is a floor effect for the initial margin over time, see Figure 10. The solid line is for the initial margin computed only by the filtered historical simulation and the dash-dotted line is for the margin computed by the maximum of the FHS based margin and stress period based margin. In the left of the figure, we plot the time variations of the initial margin of an illustrative basic interest swap and in the right, the dynamics of a specific client's initial margin are plotted. For the filtered historical simulation, we use the GARCH(1,1) model with the daily difference series of the interest rates and the margin is set to be 99.5% expected shortfall.

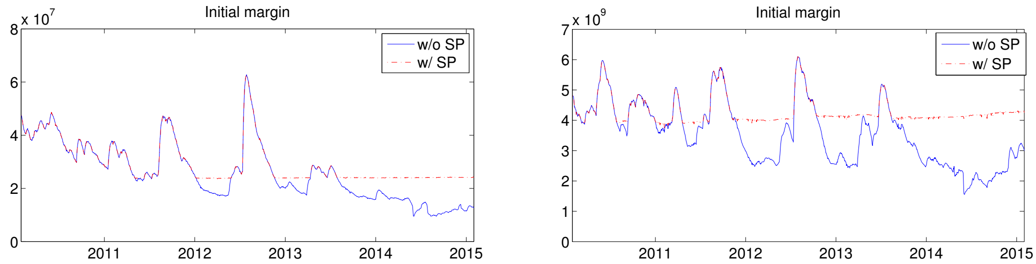


Figure 10: Comparison of the margin variations with and without stress period

6.2 Example study

Example 1. In this example, we examine the dynamics of the initial margin for an illustrative IRS with 6 month maturity. This example is along the line of the discussion about the instability of ε in 3M data in Section 4. Two initial margins are compared where the one is computed by the EWMA model with five days non-overlapped interval mimicking the method of CME and the other is computed by the GARCH(1,1) with daily data. The parameter values are $\lambda = 0.97$, $\omega = 8.981 \times 10^{-10}, \alpha = 0.0513, \beta = 0.9423$ where λ is subjectively chosen and the GARCH parameters are estimated from data.

Figure 11 presents the the initial margin dynamics of the IRS short position computed based on daily data (solid) and five days overlapped data (dash-dotted). In general, the overall levels of the margin between two methods are similar over time except the second half of 2014. The initial margin computed using five days overlapped data and the EWMA model is larger than the four times of the initial margin computed by daily data and the GARCH model after 2014-08-14. At 2014-08-14, there was an announcement of the standard rate cut by Bank of Korea and large drops in rates especially for short maturities as in Table 3. The margin hike is largely due to the outliers of ε generated by the EWMA model without a volatility floor to the past, recall Figure 5b and its explanation in Section 3. Note that the given example is a rather extreme case with an exemplary IRS of short maturity and in general, the initial margins of typical clients computed based on five days overlapped series under the EWMA model is similar to the initial margin computed by daily data under the GARCH model due to the diversification of clients' IRS portfolio.

Example 2. The risk measured based on non-overlapped five days interval series shows a different result from the risk computed by daily data or five days overlapped data. In this example, we compare the initial margin computed based on daily data under the GARCH model and the margin computed based on non-overlapped five interval data under the EWMA model. The later method replicates the method of EUREX and applies the stress period. We do not focus on the differences in the EWMA and GARCH in this example, but we demonstrate the effect caused by the long persistence under the non-overlapped five days interval series and subjectively chosen large λ. In addition, we apply the non-filtered stress period simulation to both cases.

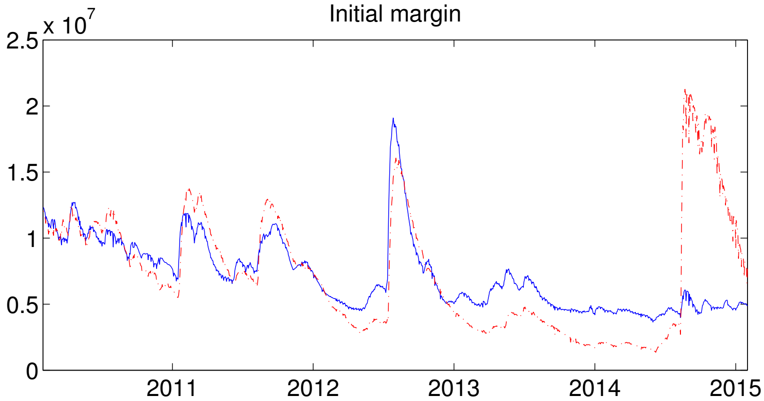


Figure 11: Comparison of the margin variations by daily time series and 5 days overlapped series

Table 3: Korean interest rates around 2014-08-14

date	1D	3M	6M	9M	1Y
2014-08-07	0.02503	0.02650	0.02532	0.02490	0.02465
2014-08-08	0.02508	0.02650	0.02527	0.02480	0.02454
2014-08-11	0.02504	0.02630	0.02530	0.02485	0.02458
2014-08-12	0.02500	0.02620	0.02532	0.02490	0.02463
2014-08-13	0.02493	0.02610	0.02512	0.02484	0.02460
2014-08-14	0.02256	0.02460	0.02438	0.02431	0.02429
2014-08-18	0.02247	0.02430	0.02425	0.02424	0.02424
2014-08-19	0.02251	0.02420	0.02400	0.02401	0.02403
2014-08-20	0.02244	0.02420	0.02400	0.02400	0.02405
2014-08-21	0.02246	0.02420	0.02395	0.02400	0.02405

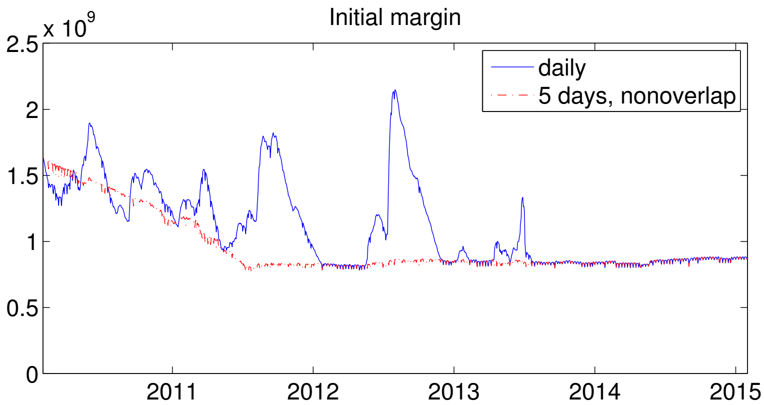


Figure 12: Comparison of the margin variations by daily time series and 5 days nonoverlapped series with stress period

The different initial margins between two methods are due to the different inferred volatilities as mentioned in the previous section. For example, at 2012-10-11, there is a standard rate cut by Bank of Korea and there is a hike in the initial margin (solid) computed by the daily data as in Figure 12. The abrupt rate changes represent dramatically increasing volatilities. Whereas there is no sudden hike in the initial margin (dash-dotted) computed by five days non-overlapped series around 2012-10-11 in the figure.

We observe the increasing volatilities in Figures 8e and 8f which represent the inferred volatilities based on five days non-overlapped series right after 2012-10-11, but the volatilities are not large enough to make the initial margin exceed the margin level induced by the stress period simulation. Thus, there is no change of the initial margin by the five days non-overlapped data. In contrast, the volatilities based on daily data is quiet large so that the initial margin computed by the filtered historical simulation peaks around 2012-10-11.

Now consider the dashed-dotted line of the five days non-overlapped data around 2010. The relatively large initial margin by the FHS exceeding the stress period margin level is due to the persistent volatilities started from the financial crisis, see again Figures 8e and 8f, where the financial crisis volatilities are persistent around 2011. The shapes of the initial margin and volatility dynamics are similar around 2010 and the first half of 2011.

Example 3. In this example, we compare four methods of computing the initial margin with the portfolios of various clients in CCP. First, the initial margin is computed by 99.5% expected shortfall by the GARCH filtered 1250 historical simulation on a daily basis. Second, the margin is determined by the maximum of the margin computed in the first method and the margin based on the 250 days stress period simulation multiplied by 0.7.

Third method is replicating EUREX method where we use five days non-overlapped data over 750 days with the EWMA filtered simulation. Briefly speaking, the samples are divided into five subgroups and the initial margin is determined by the average of 99.7% VaR of each group. In addition, 250 days stress period is applied as in the second method. Fourth method

Table 4: Korean interest rates around 2012-10-11

date	1D	3M	6M	9M	1Y	1.5Y
2012-10-04	0.03000	0.03090	0.02953	0.02865	0.02801	0.02746
2012-10-05	0.02990	0.03090	0.02960	0.02878	0.02808	0.02755
2012-10-08	0.02980	0.03080	0.02973	0.02890	0.02812	0.02760
2012-10-09	0.03000	0.03080	0.02973	0.02883	0.02803	0.02754
2012-10-10	0.03030	0.03080	0.02938	0.02844	0.02760	0.02711
2012-10-11	0.02740	0.02900	0.02880	0.02830	0.02771	0.02729
2012-10-12	0.02740	0.02870	0.02840	0.02810	0.02778	0.02741
2012-10-15	0.02740	0.02870	0.02833	0.02803	0.02770	0.02732
2012-10-16	0.02740	0.02870	0.02860	0.02835	0.02810	0.02775
2012-10-17	0.02750	0.02870	0.02860	0.02840	0.02820	0.02794
2012-10-18	0.02740	0.02870	0.02855	0.02845	0.02833	0.02813

is replicating CME method where five days overlapped EWMA filtered log-return series with 1260 sample are used without additional stress period. The risk horizon is fixed as five days for all cases.

In Figure 13, the solid, dash-dotted, dotted and dashed lines are for margins of various IRS portfolios computed by the first, second, third and fourth methods, respectively. In general, the behaviors of the initial margins of CME and the FHS based on daily data without stress period are similar to each other for all portfolios. If we add the stress period test, then there is a margin floor effect as mentioned before. By the third method mimicking EUREX, all portfolios have larger initial margins up to around 2011 due to the high volatility induced by the financial crisis, and after that in a low volatility regime, the margins are floored by the values determined by the stress period simulation.

7 Comparison with historical VaR

In this section, we compare the initial margin determined by the non-filtered historical simulation. The filtered historical simulation is based on the GARCH model with parameter settings $\omega = 8.981 \times 10^{-8}, \alpha = 0.0513, \beta = 0.9423$ and 99.5% ES. We examine the initial margins of all clients who trades IRS via the CCP at 2015-02-09. The initial margins by the current method on historical simulation (left bars, blue) is generally larger than the margins by the filtered historical simulation (right bars, red) as in Figure 14 in a low volatility period, in this example, the date is 2015-02-09. Figure 14a is for the comparison of the initial margins of all clients between FHS without the stress period the and current method. Figure 14a is for the comparison of the initial margins between the FHS including the stress period and the current method.

We compare the initial margins computed at 2008-11-06 in Figure 15 by assuming the port-

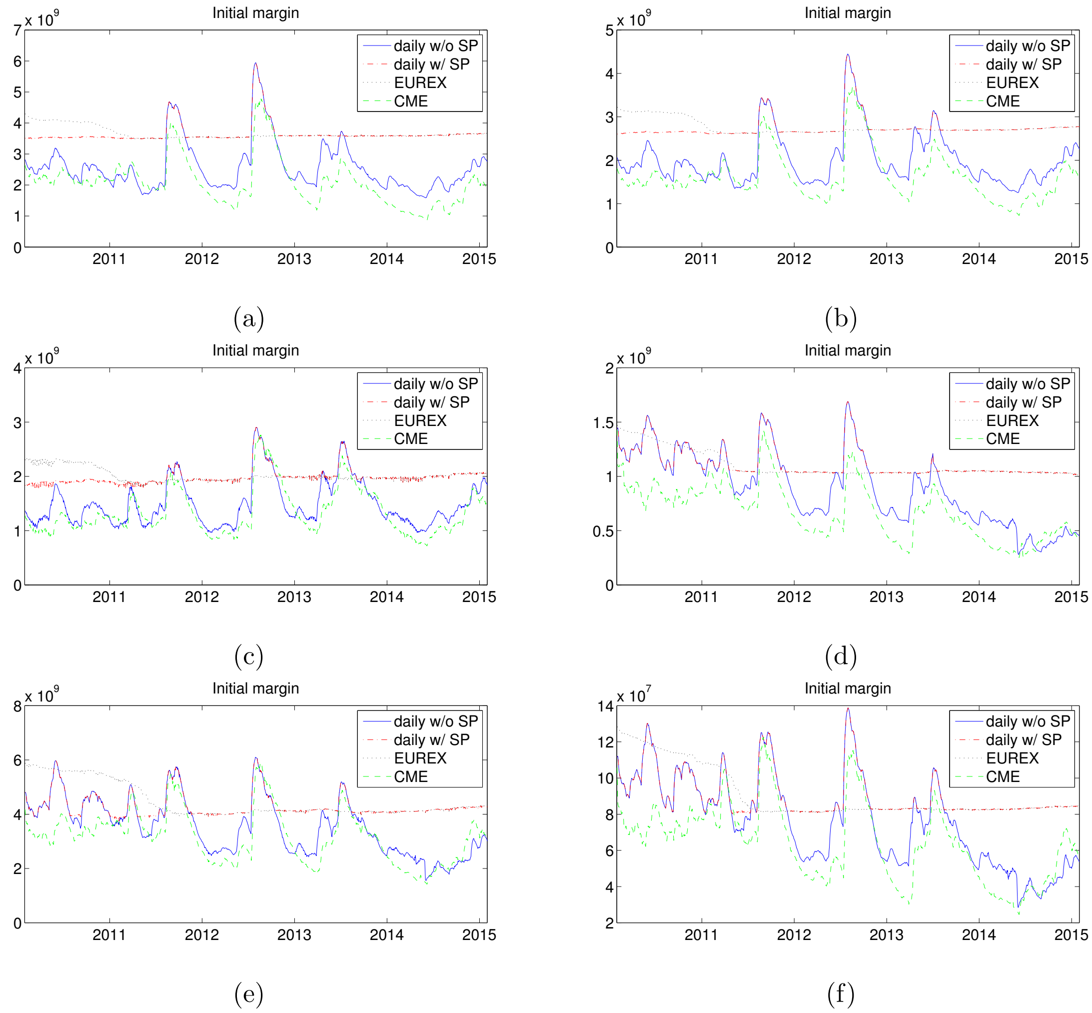


Figure 13: Comparison of the dynamics of the margins computed by various methods

folio statuses of the clients at 2015-02-09 are the same at 2008-11-06. The initial margin by the FHS are increased compared with the previous cases for both simulations without (Figure 15a) and with (Figure 15b) the stress period. The gaps between the margins by the FHS and the margins by the non-filtered historical simulation are diminished and some of the margins by the FHS are larger than the margins by the current methods.

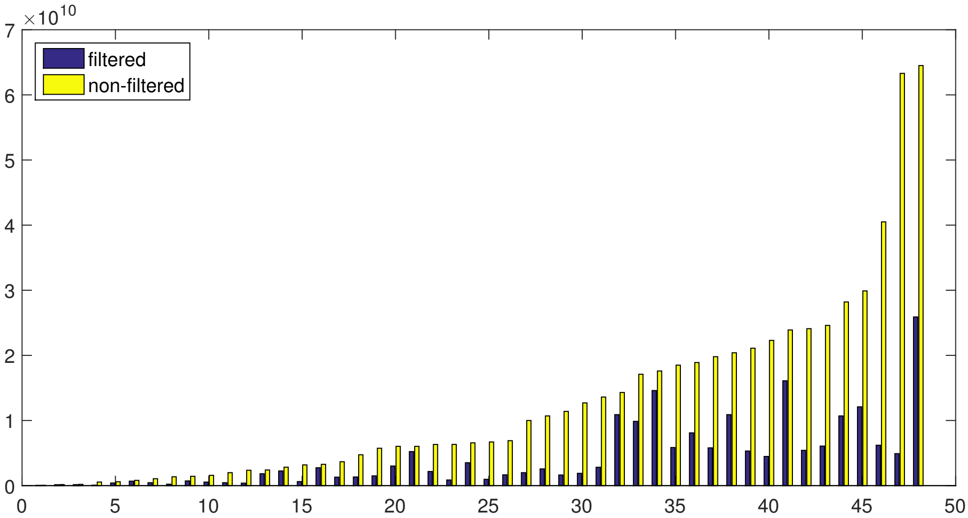
This tendency is prominent for basic interest rate swaps. The x-axis in Figure 16 represents the maturities of the basic swaps. In the middle of the global financial crisis (2008-11-06), the margins by the FHS (left bars, blue) are larger than the margin by the non-filtered historical simulation (right bars, red) as in Figure 16a. Meanwhile, during a low volatility regime (2015-02-09), the margins by the FHS are generally smaller than the margin by the non-filtered historical simulation as in Figure 16b.

Figure 17 plots the dynamics of the initial margins of clients from 2008 to 2014 by the non-filtered historical simulation (dashed) and FHS (solid). The behaviors of the margins depend on the clients' portfolios. The client in the left has larger FHS based margin (solid) than the margin by the non-filtered simulation (dashed) in the financial crisis and smaller FHS based margin in a low volatility period. However, for the client in the right figure, the initial margin by the non-filtered simulation (dashed) always larger than the initial margin by the FHS (solid). The time variation of the initial margins by the FHS fluctuates over time by the change of the volatility. The time variation of the initial margins by the current method have rather flat and step shape as the includes and excludes of large shocks are crucial to the determination of the margin level.

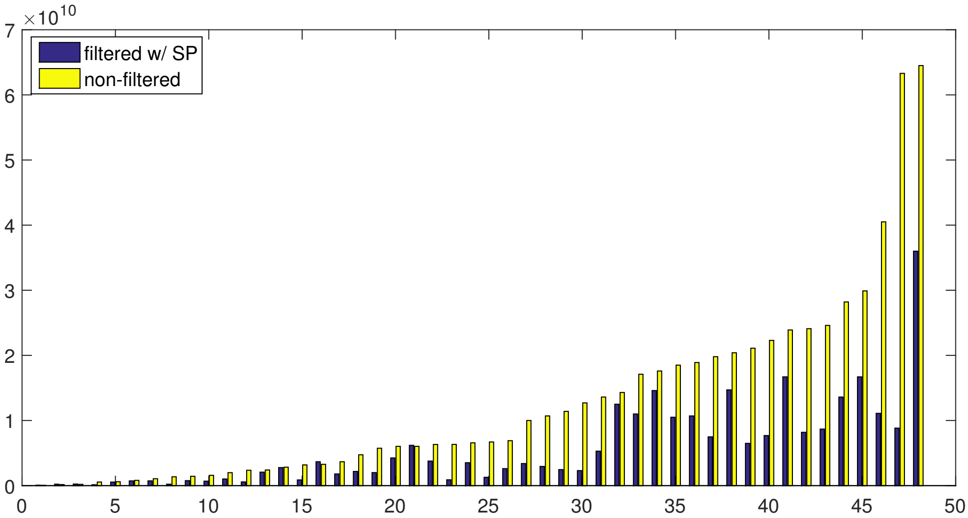
For the basic IRS, generally the margins determined by the FHS is larger than the margins by the non-filtered historical simulation in a high volatility period, and the margins determined by the FHS is smaller in a low volatility regime as in Figure 18. In the figure, we plot the dynamics of margins of IRS with 6M, 3Y, 6Y, 10Y maturities.

8 Conclusion

We explained the detailed method of the filtered historical simulation to determined the margin of the interest rate swap portfolios traded in a central clearing party under Korean interest rate market. Two widely used volatility models, the EWMA and the GARCH models for the filtered historical simulation are examined. The filtered process denoted by ε in our paper shows the instability especially for short tenors under the EWMA model and the instability can lead to the unintended results in the margin calculation. To mitigate the effect, one can use large enough persistent parameter λ, apply a floor to the past volatility series in the EWMA model, or use the GARCH model where ω plays a role similar to the volatility floor. The combination of choices of λ and the observation interval in the EWMA model determined the shapes of the volatility and hence the behavior of the computed margin. Our empirical studies based on the clients' portfolios traded in CCP supported our explanations.

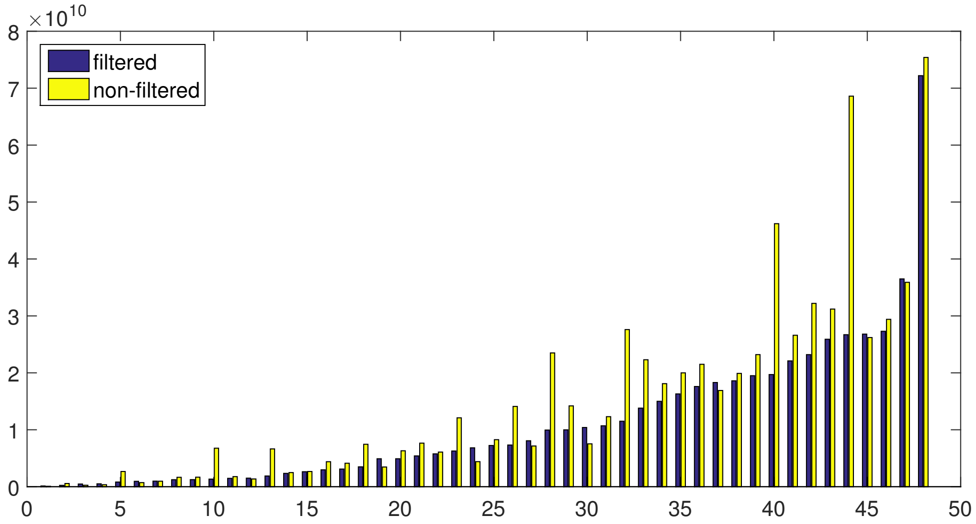


(a) without stress period for the filtered historical simulation

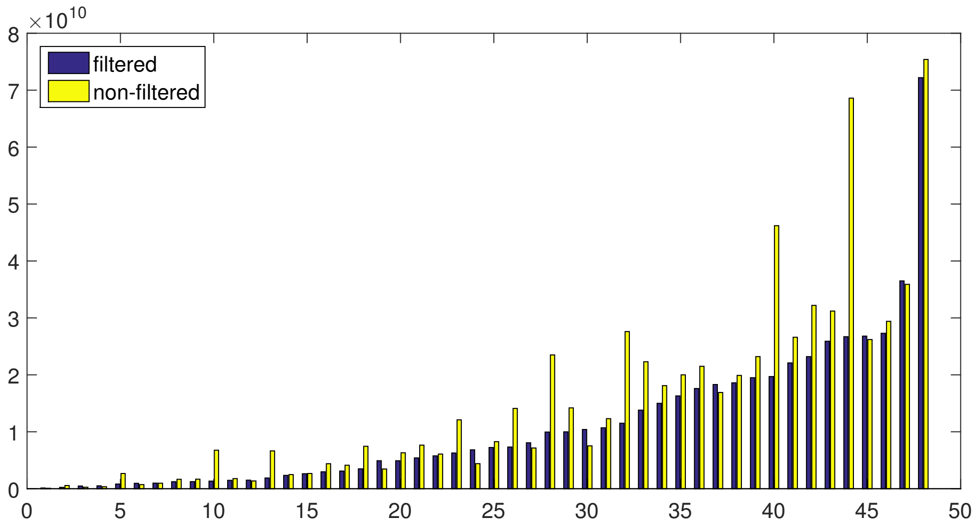


(b) with stress period for the filtered historical simulation

Figure 14: Comparison of the initial margin by filtered and non-filtered historical simulation, 2015-02-09

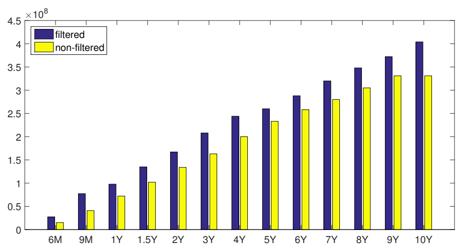


(a) without stress period for the filtered historical simulation

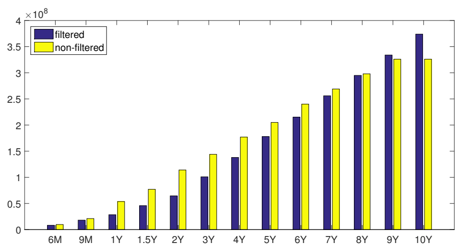


(b) with stress period for the filtered historical simulation

Figure 15: Comparison of the initial margin by filtered and non-filtered historical simulation, 2008-11-06

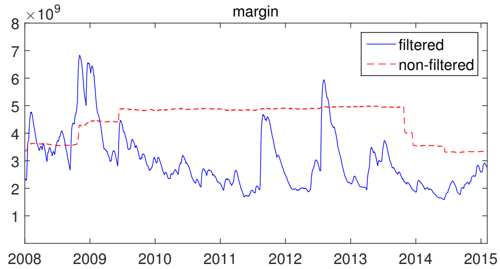


(a) 2008-11-06

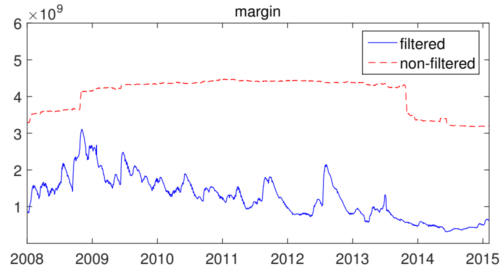


(b) 2015-02-09

Figure 16: Comparison of the initial margin by filtered and non-filtered historical simulation of the basic IRS

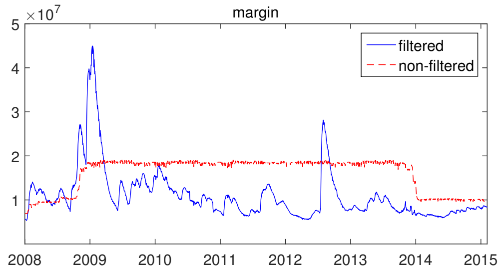


(a)

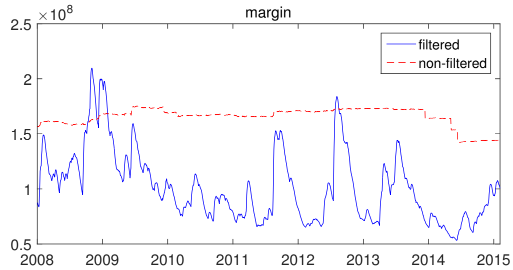


(b)

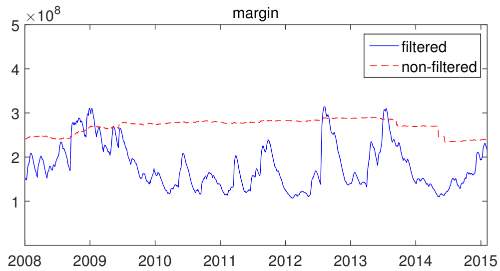
Figure 17: Time variations of margins by filtered and non-filtered historical simulation



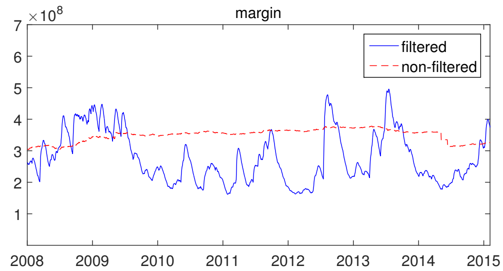
(a) 6M



(b) 3Y



(c) 6Y



(d) 10Y

Figure 18: Time variations of margins of the basic IRS by filtered and non-filtered historical simulation

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