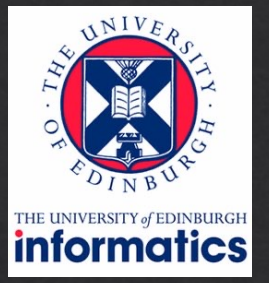
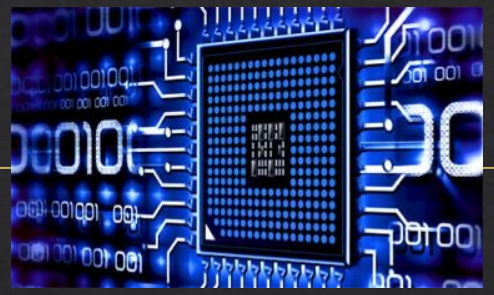



# Computer Graphics

Lecture 2: Cameras

Kartic Subr

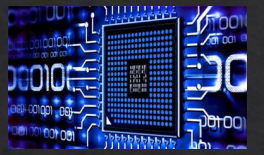
photography

rendering

Virtual

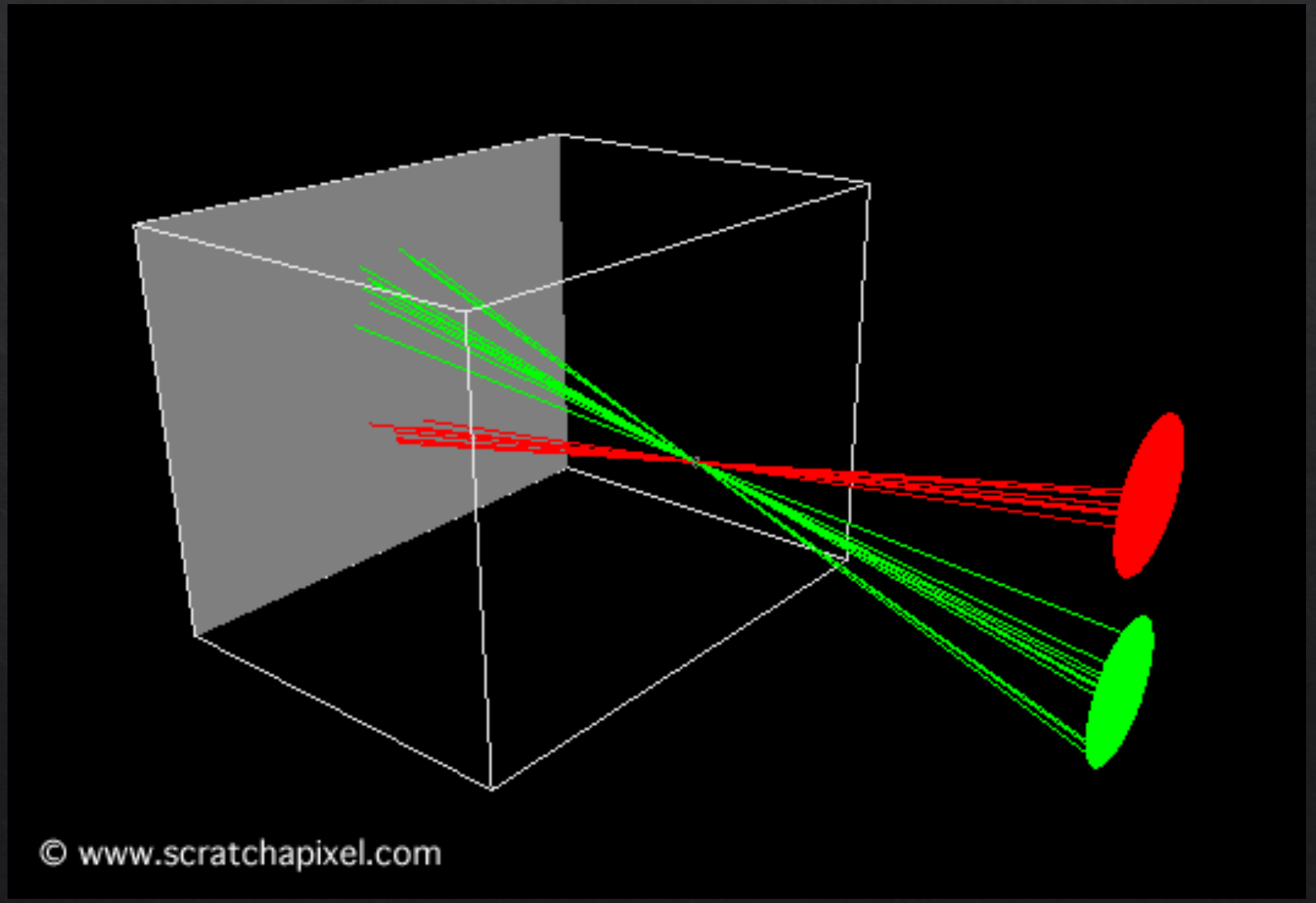
# Cameras

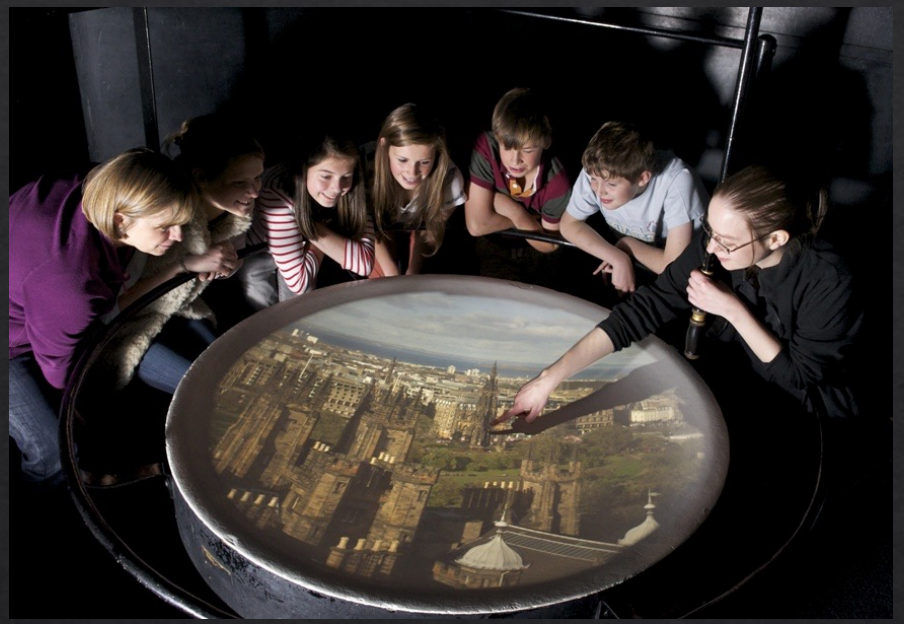
# The pinhole camera

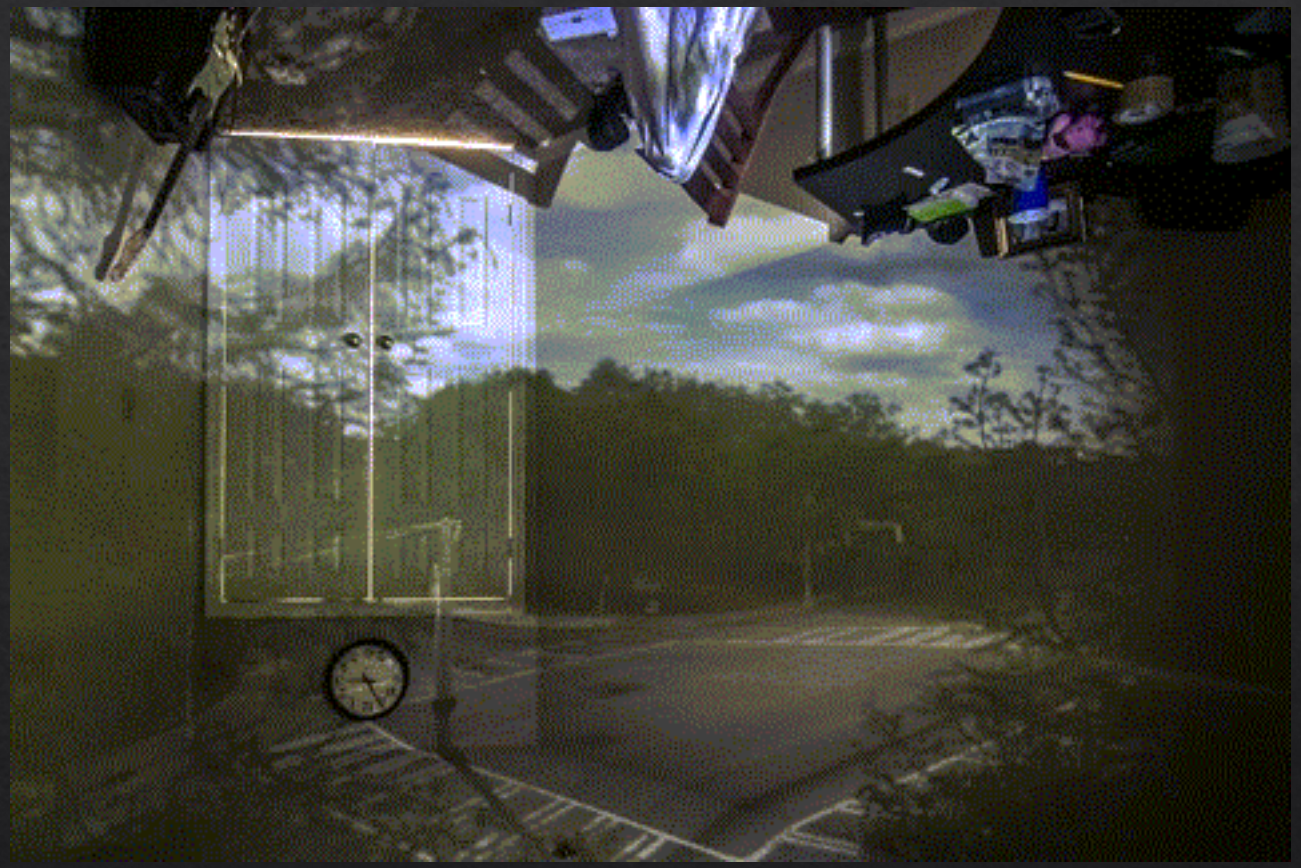
# Pinhole camera

Ibn al-Haytham (965-1040 AD)



# Camera Obscura



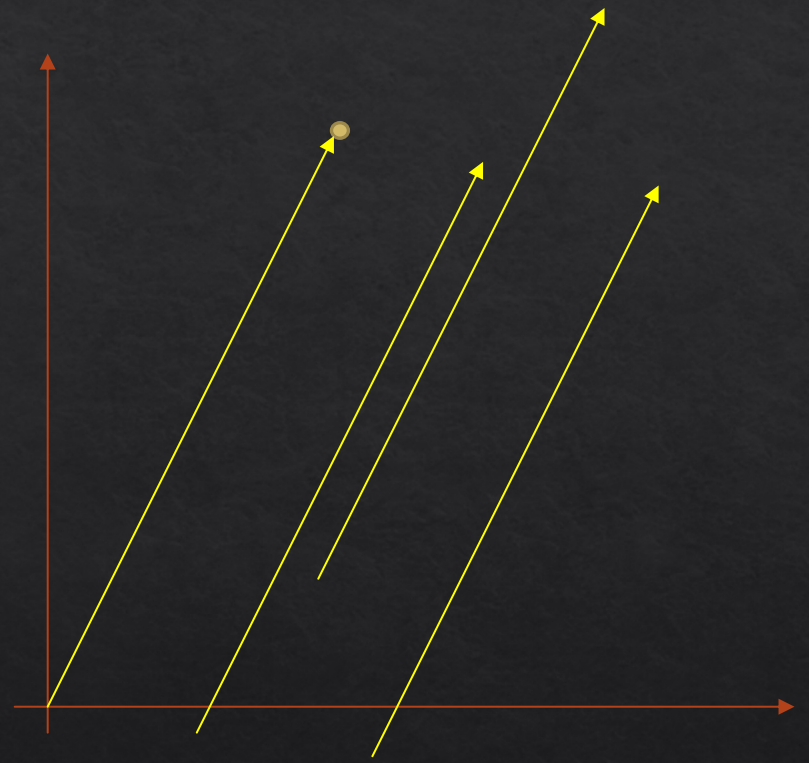


The making of …

# Projection

# What is a vector? e.g. 2D

$$\begin{bmatrix} u \\ v \end{bmatrix}$$



# What is a matrix? e.g. 2x2

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

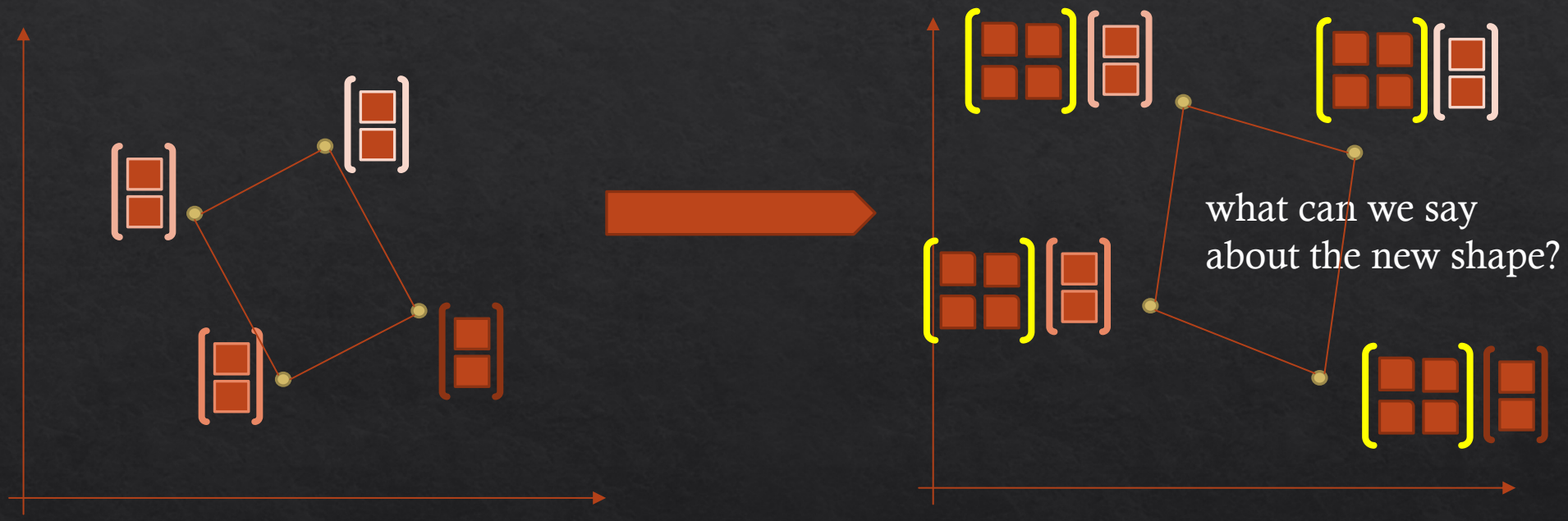
# Can we ‘operate on’ a vector?

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} au + bv \\ cu + dv \end{pmatrix}$$

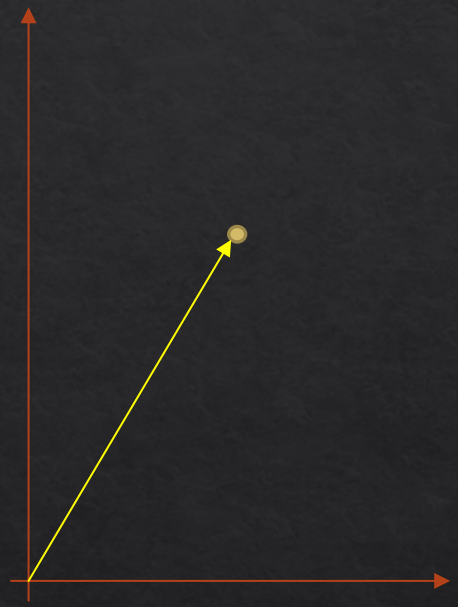
# Can we ‘operate on’ a vector?

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} au + bv \\ cu + dv \end{pmatrix}$$

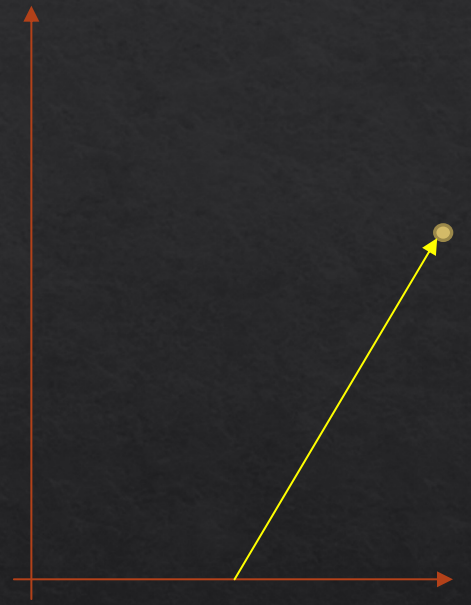
# What operations can it achieve?



# What operation achieves translation?

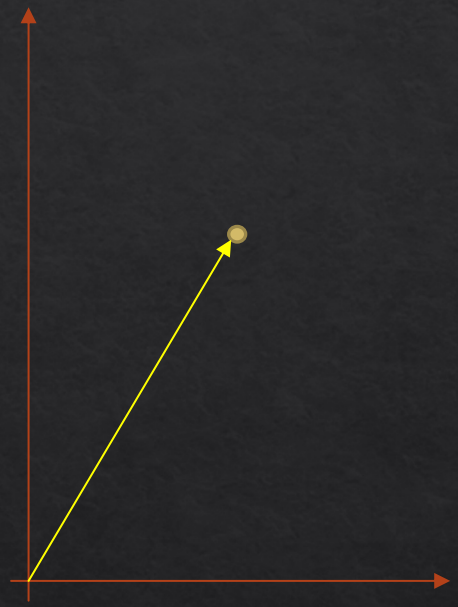


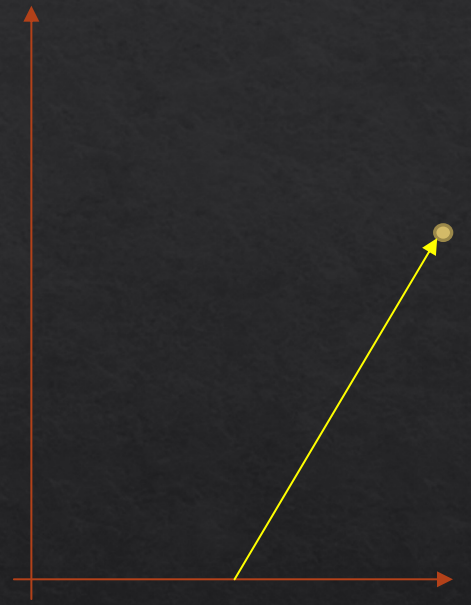
?



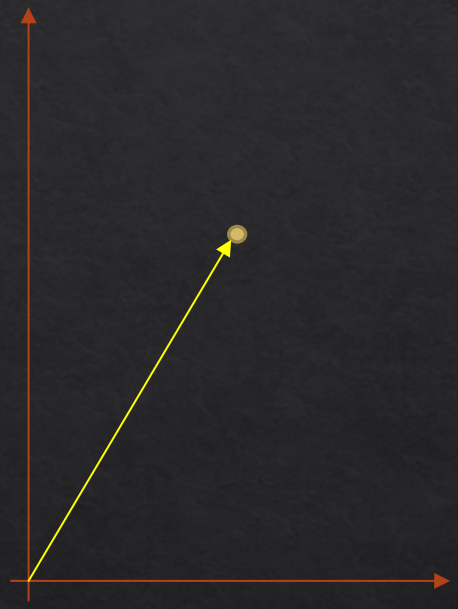
$$\begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} c_x \\ 0 \end{bmatrix}$$

# Can we achieve this with a matrix?

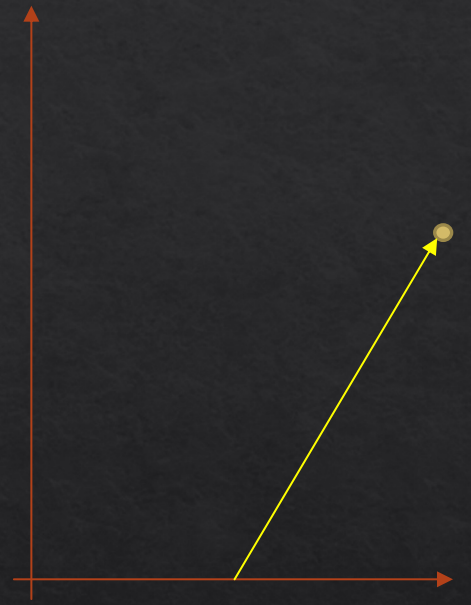


# Can we achieve this with a matrix?




Ans: Not with a 2x2 matrix



# What if we add a dimension?

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

matrix is 3x3

still 2D vectors

# Now, translation is possible as an operation

$$\begin{pmatrix} 0 & 0 & c_x \\ 0 & 0 & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} u + c_x \\ v + c_y \\ 1 \end{pmatrix}$$

# Homogeneous coordinates are useful!

$$\begin{pmatrix} u' \\ v' \\ s \end{pmatrix}$$

equivalent to

$$\begin{pmatrix} u'/s \\ v'/s \\ 1 \end{pmatrix}$$

point in 3D homogenous space

point in 2D space

# Homogeneous coordinates are useful!

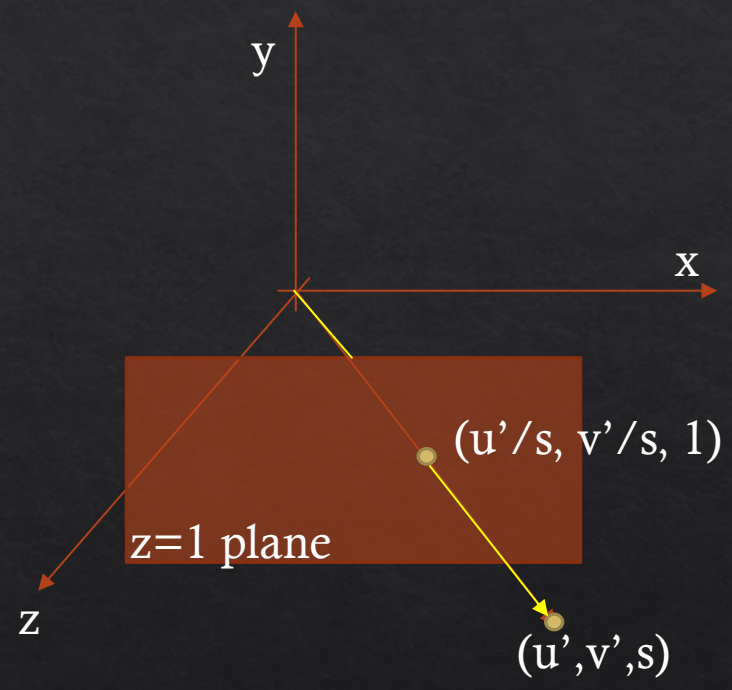
$$\begin{bmatrix} u' \\ v' \\ s \end{bmatrix}$$

point in 3D homogenous space

equivalent to

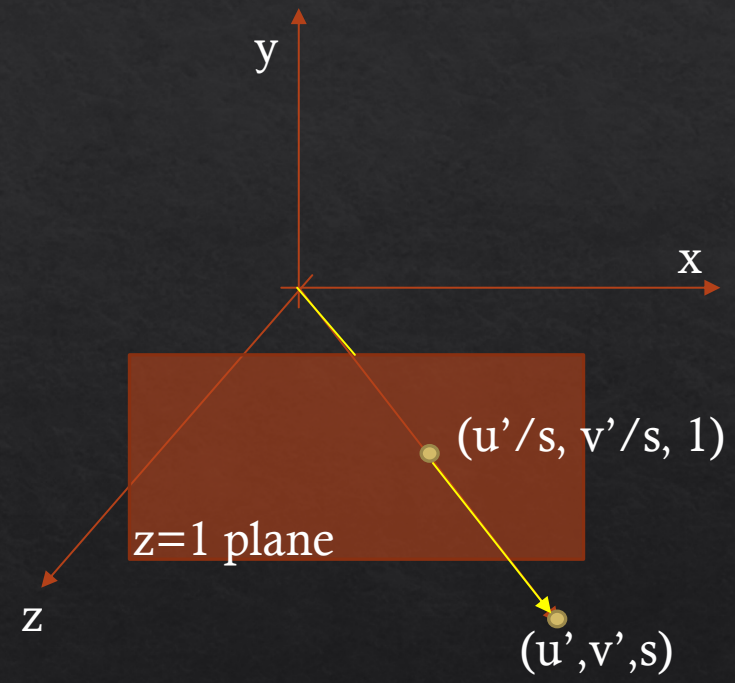
$$\begin{bmatrix} u'/s \\ v'/s \\ 1 \end{bmatrix}$$

point in 2D space



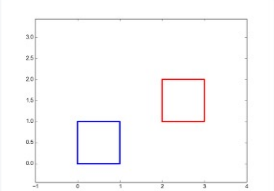
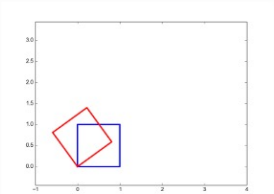
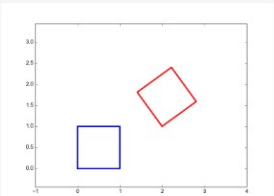
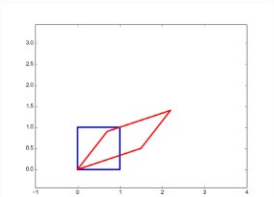
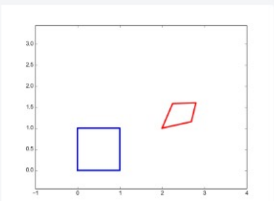
# Homogeneous coordinates are useful!

$$\begin{bmatrix} u' \\ v' \\ s \end{bmatrix} \quad \text{equivalent to} \quad \begin{bmatrix} u'/s \\ v'/s \\ 1 \end{bmatrix}$$




$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \\ 2 \end{bmatrix} = \begin{bmatrix} 3x \\ 3y \\ 3 \end{bmatrix} = \begin{bmatrix} 4x \\ 4y \\ 4 \end{bmatrix} \cdots$$

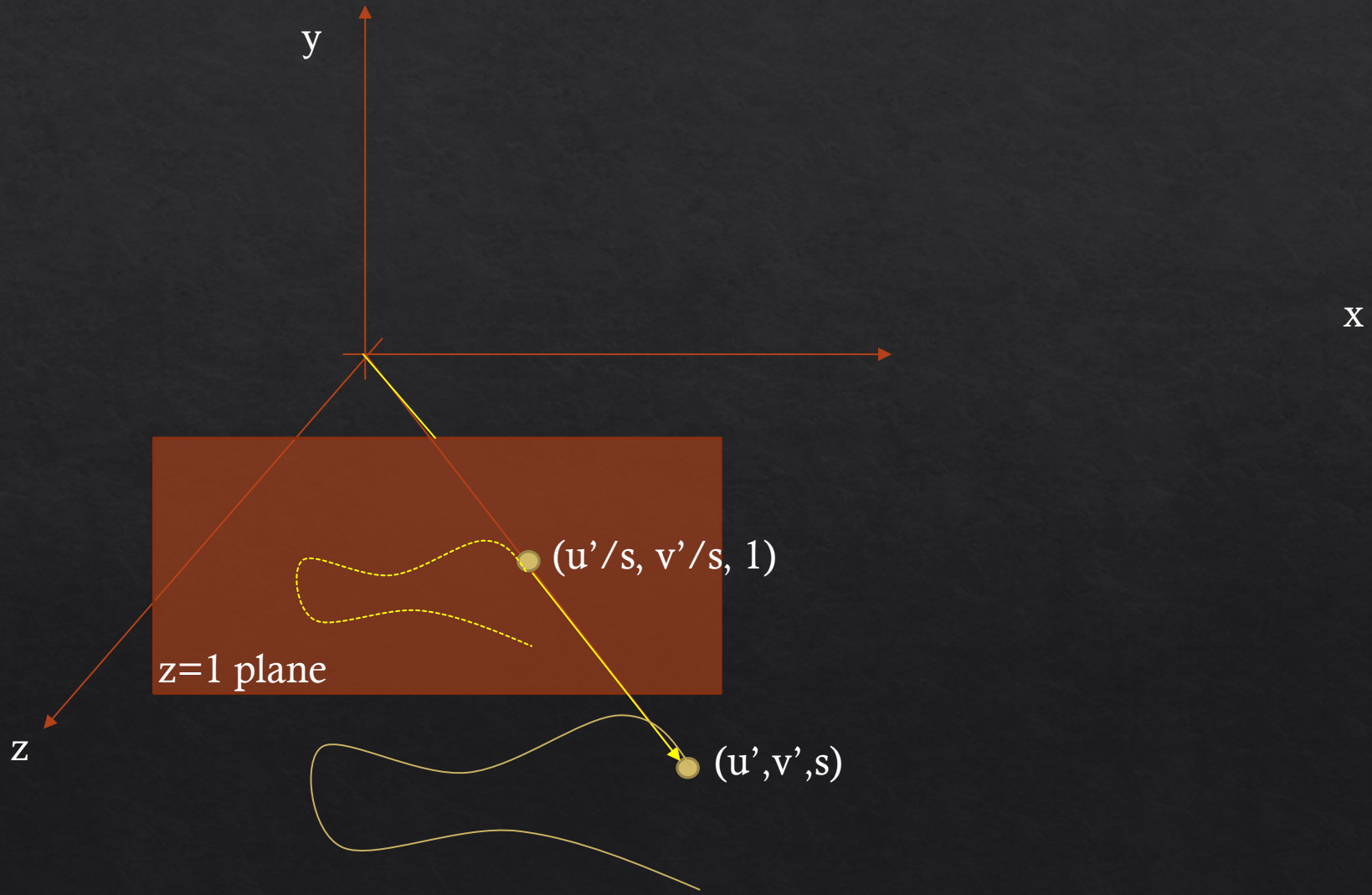
# What operations are possible now?

| | | |
|---|---|---|
|  | Translation | $\begin{pmatrix} 1 & 0 & t_1 \\ 0 & 1 & t_2 \\ 0 & 0 & 1 \end{pmatrix}$ |
|  | Rotation | $\begin{pmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$ |
|  | Rigid Body | $\begin{pmatrix} \cos(\phi) & -\sin(\phi) & t_x \\ \sin(\phi) & \cos(\phi) & t_y \\ 0 & 0 & 1 \end{pmatrix}$ |
|  | Affine | $\begin{pmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{pmatrix}$ |
|  | Projective Transform | $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{pmatrix}$ |

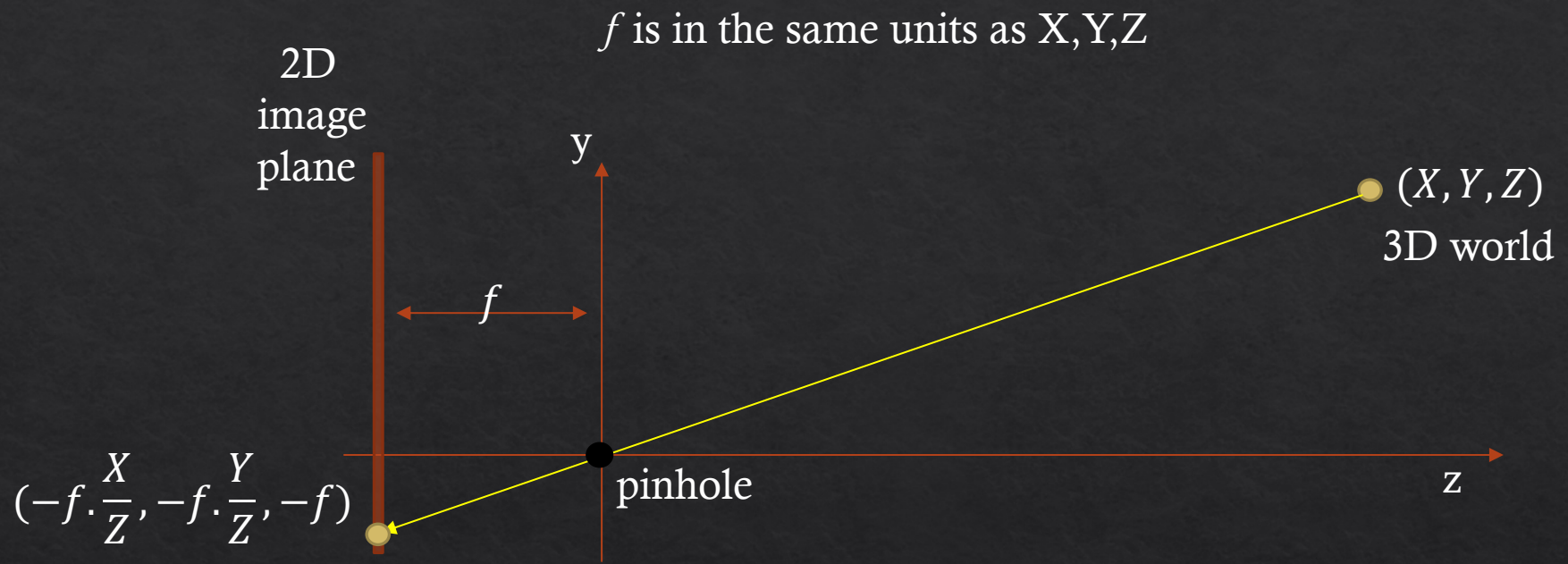
https://staff.fnwi.uva.nl/r.vandenboomgaard/IPCV20162017/LectureNotes/MATH/homogenous.html#sec-homogeneous

# Remind you of a camera sensor plane?

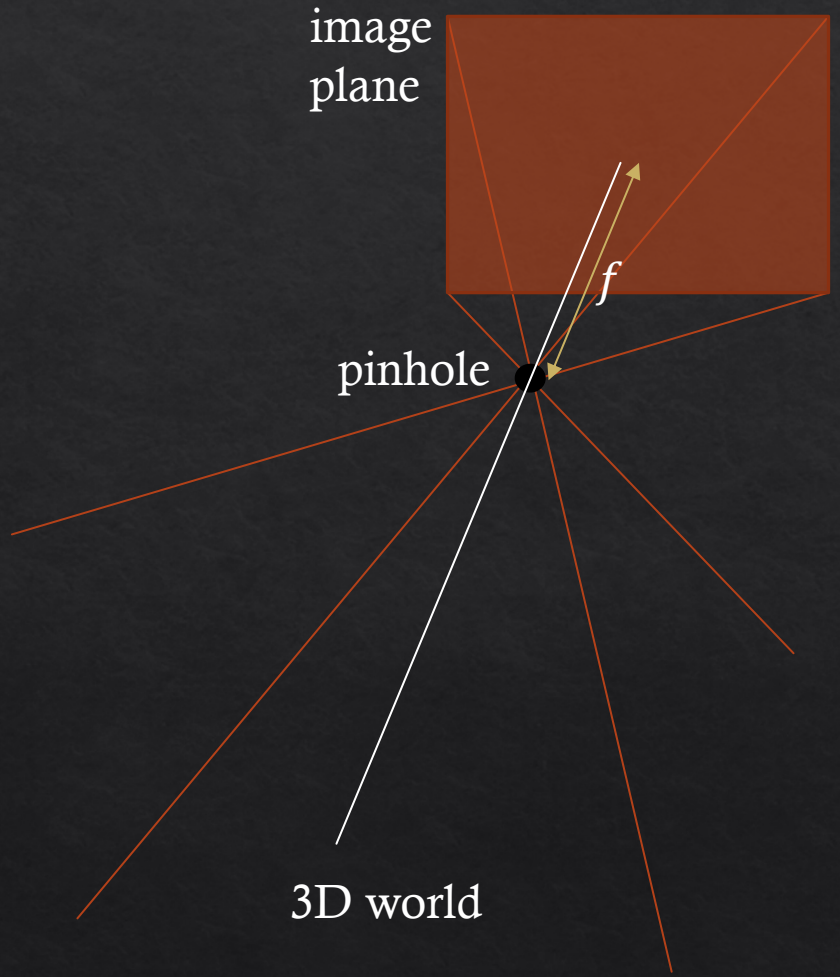
Yes, if the camera is at the origin looking down the Z-axis

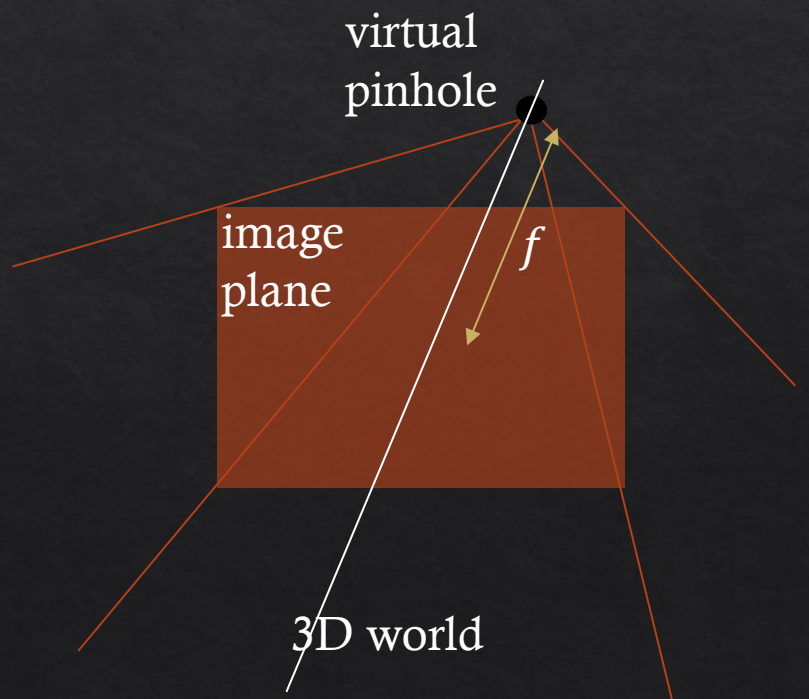


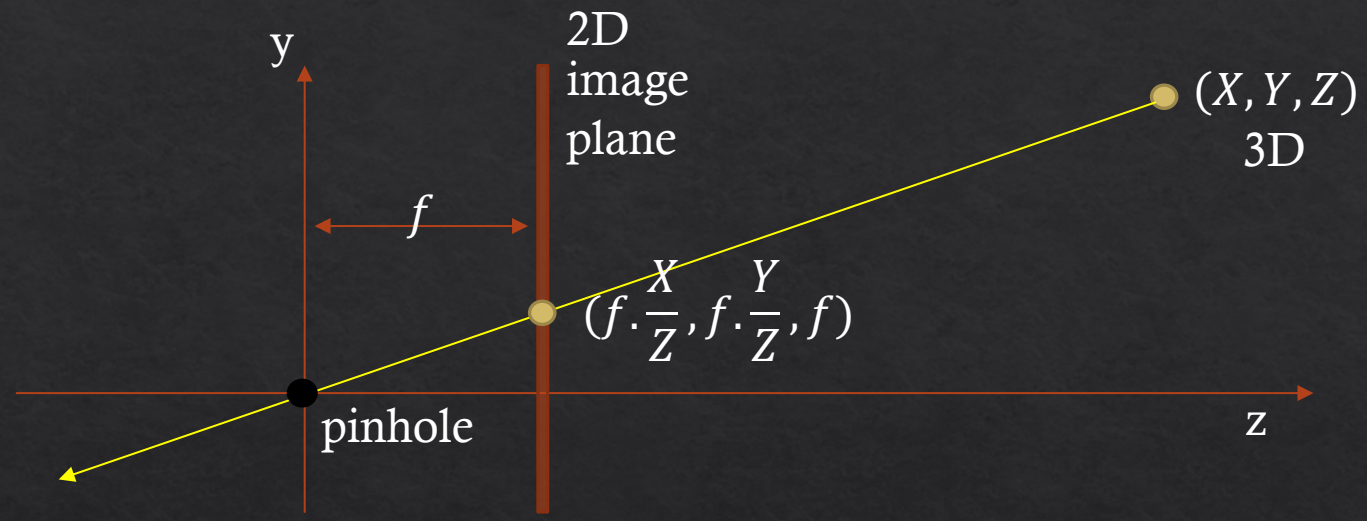
# Ideal pinhole camera 3D



# Ideal vs virtual pinhole model






$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \begin{bmatrix} fX \\ fY \\ Z \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Pixel coordinates
(u,v)
on z=1 plane

# Pixel coordinates from 3D point

1. Projection from 3D to 2D

$$\begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

2. Scaling pixels by pixel resoln.

$$\begin{pmatrix} s_x f & 0 & 0 & 0 \\ 0 & s_y f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3. Translation to positive quadrant

$$\begin{pmatrix} f_x & 0 & u & 0 \\ 0 & f_y & v & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

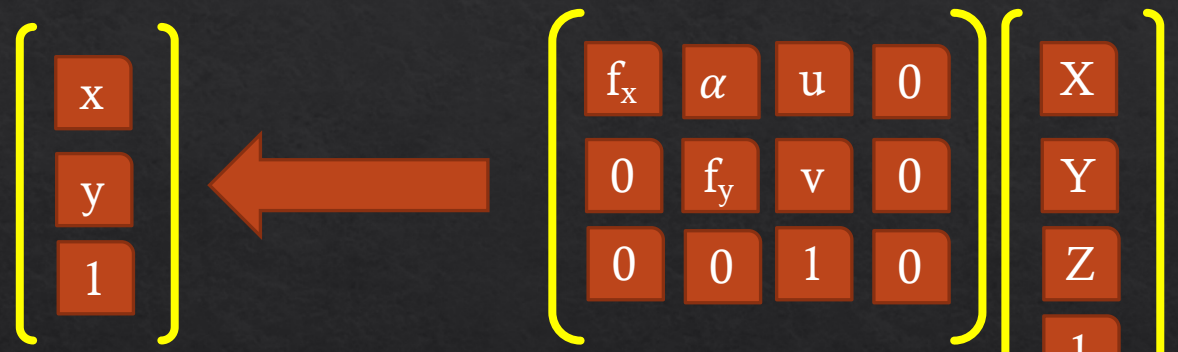
4. Skew, if sensor not perpendicular to optic axis

$$\begin{pmatrix} f_x & \alpha & u & 0 \\ 0 & f_y & v & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

More details here

# Pixel coordinates from 3D point

When the camera is at the origin looking towards Z

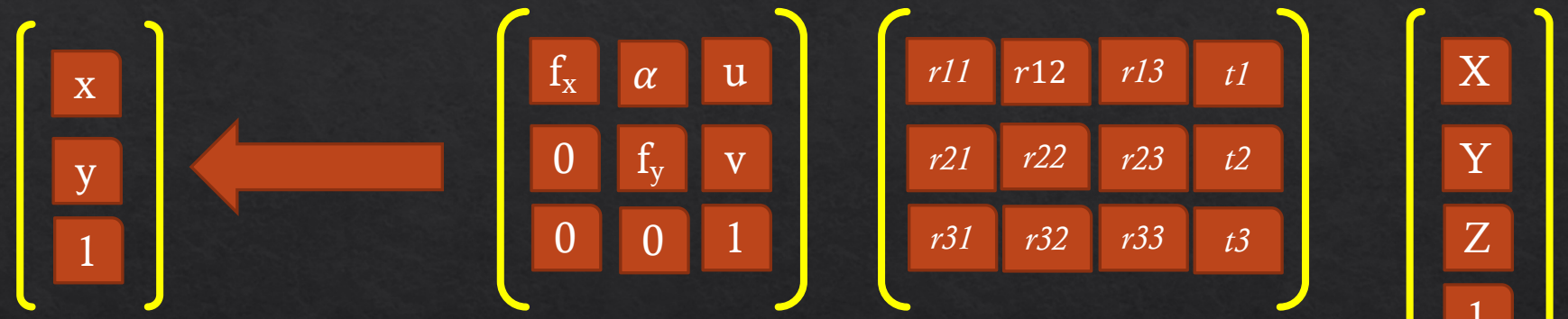



2D pixel coordinates

3D world coordinates

# Pinhole camera matrix

When the camera is at an arbitrary location




2D pixel coordinates

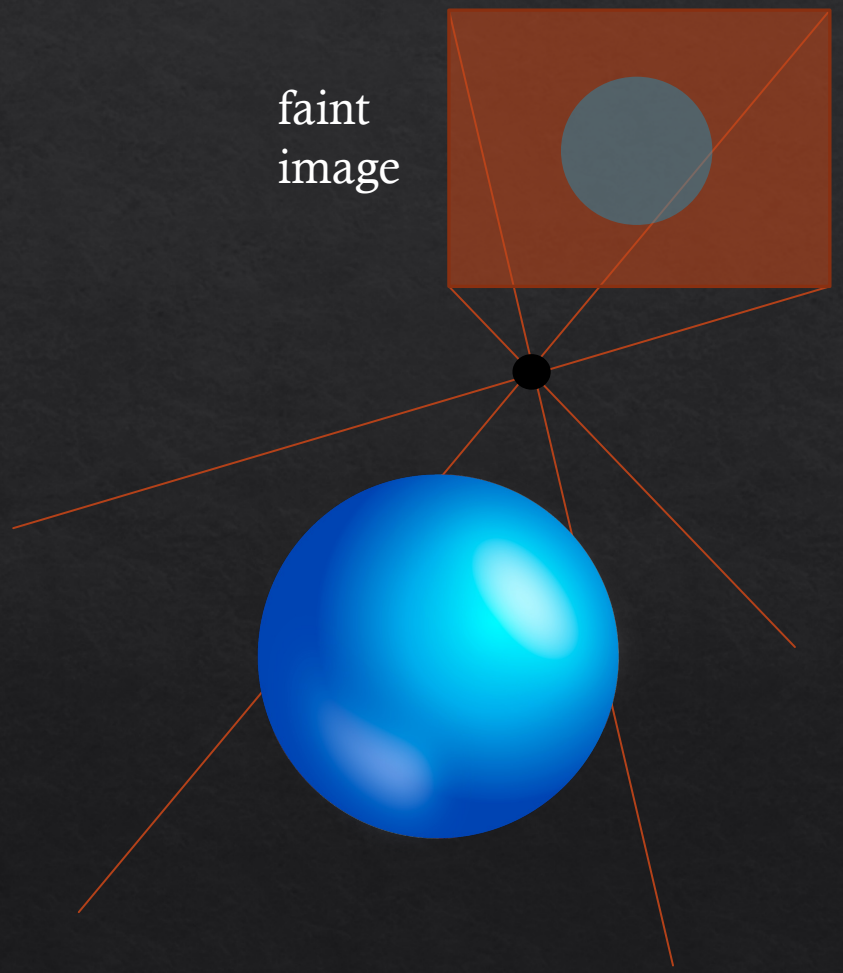
intrinsic

extrinsic

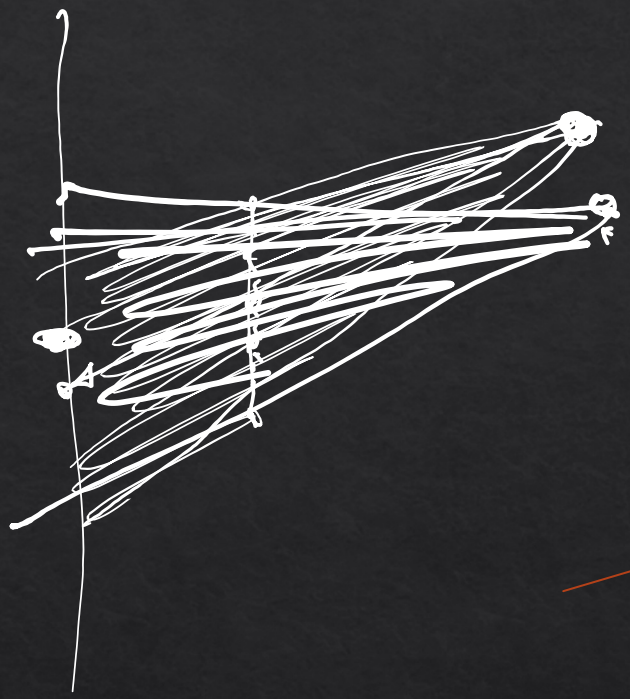
3D world coordinates

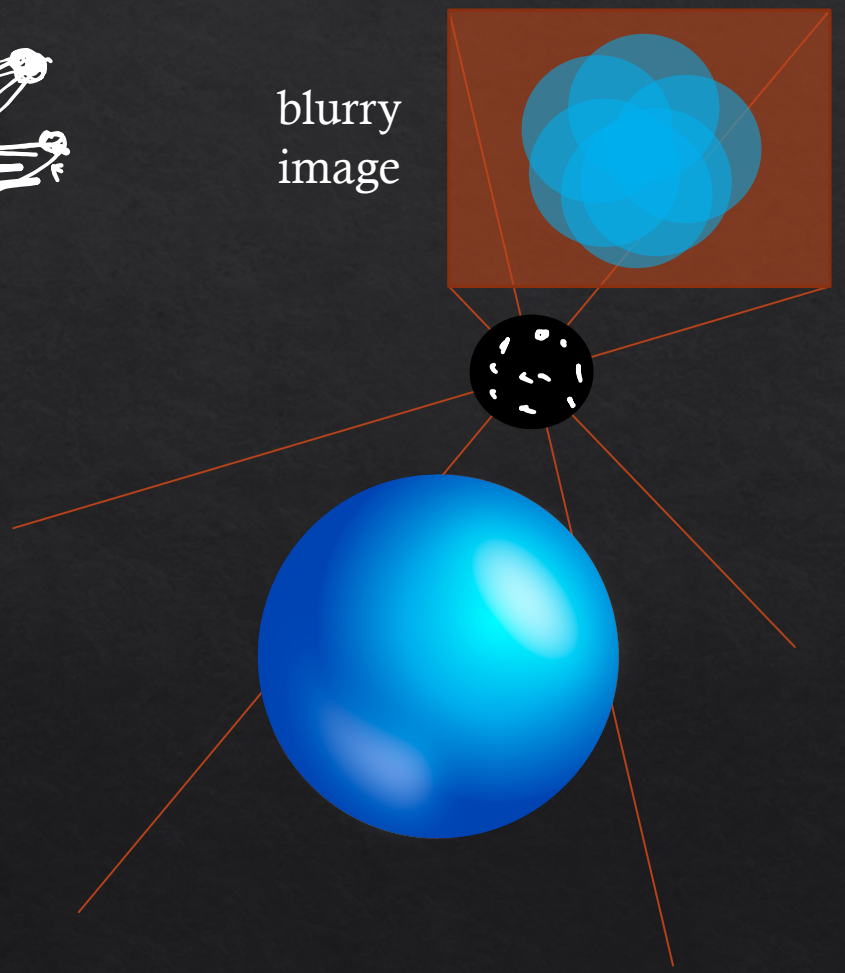
# Problems with pinhole camera?

# Pinhole only allows little light through

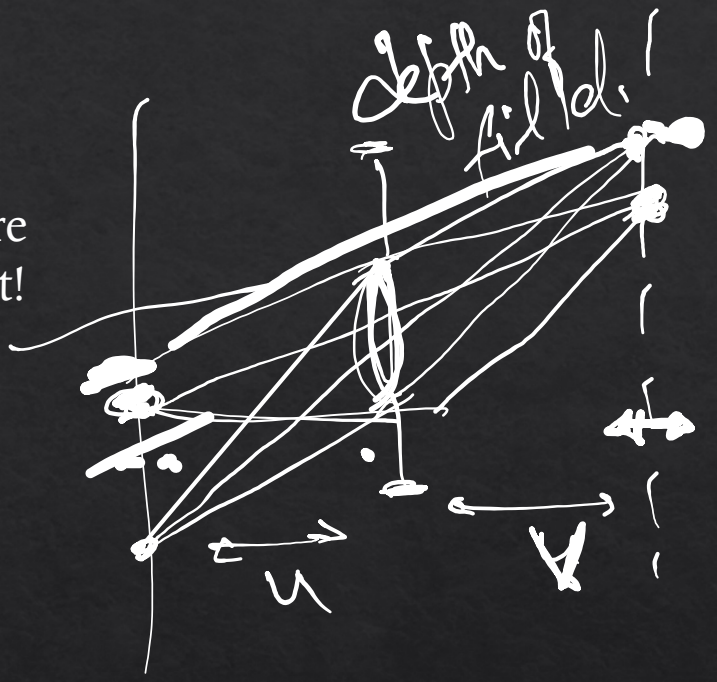


# Large hole: many superposed images



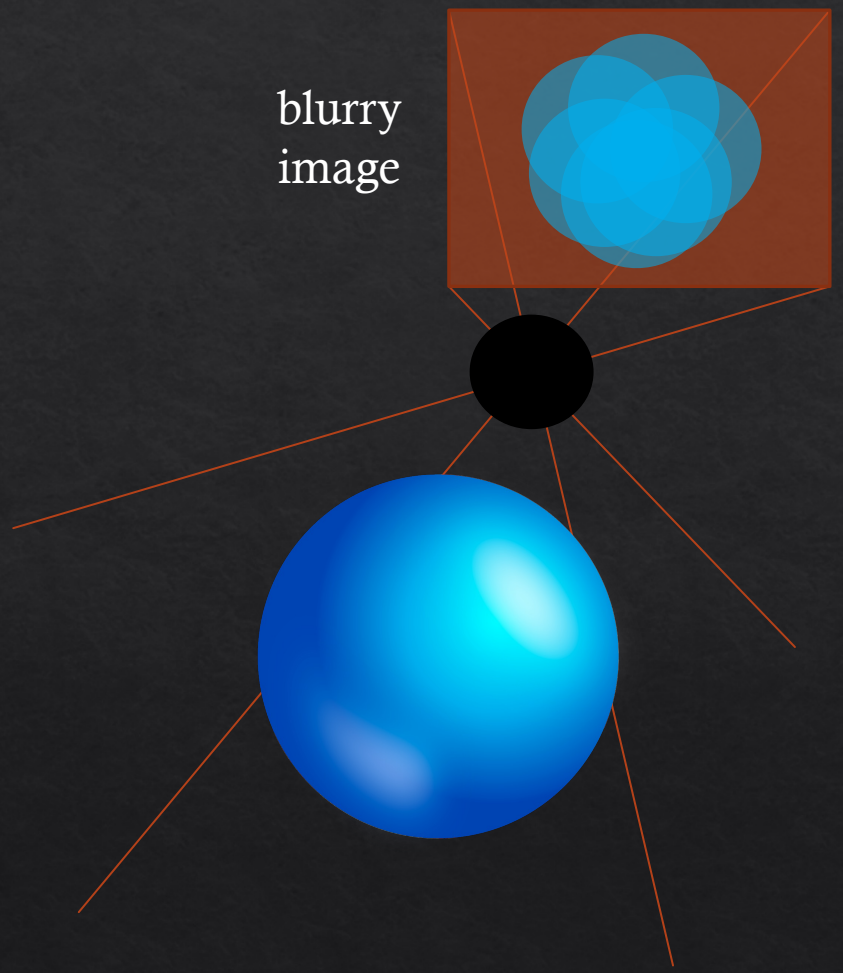



but
more
light!

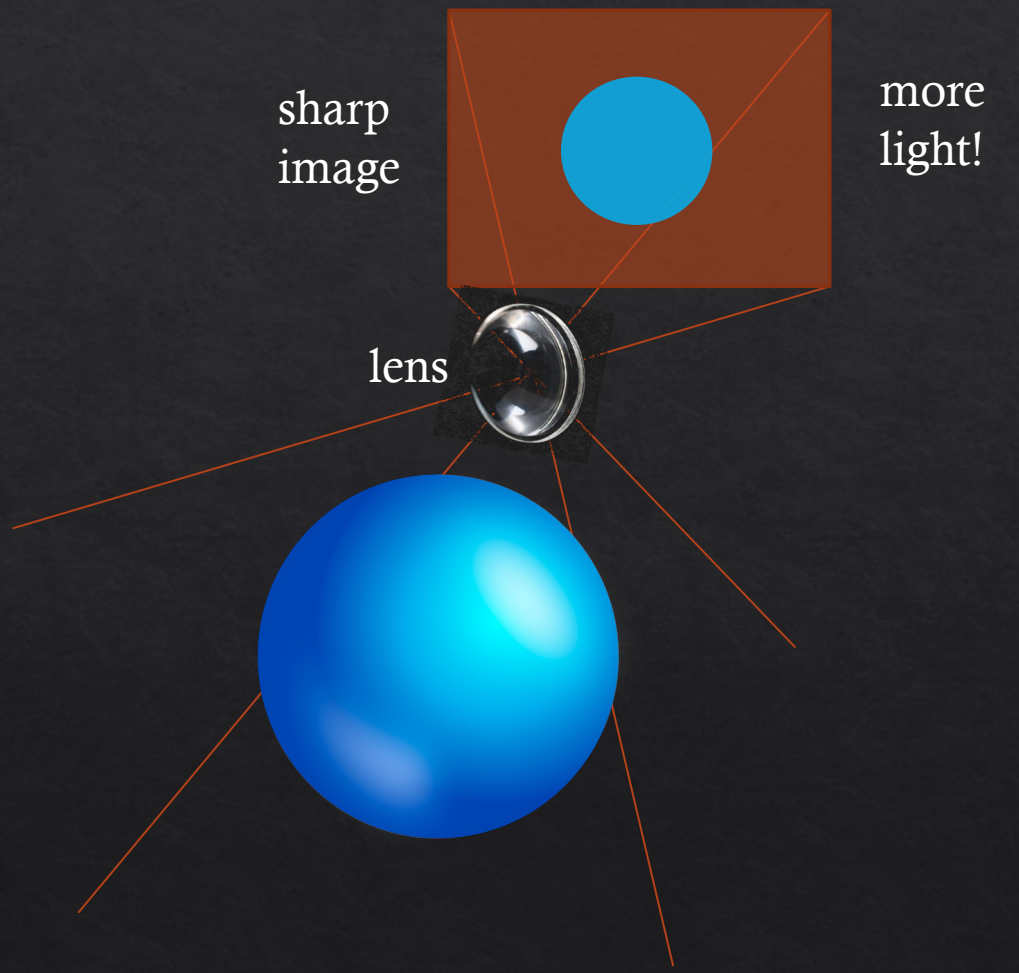



$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

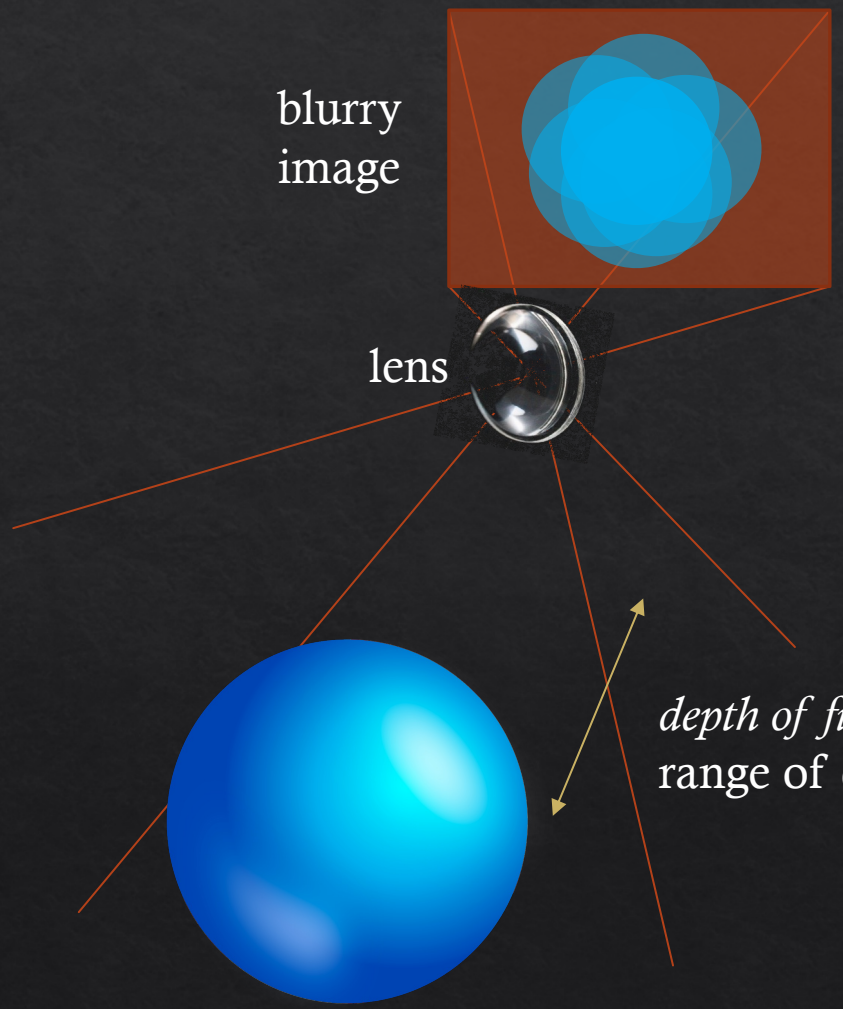
# Can we improve light and avoid blur?



# Lens improves light efficiency, but …

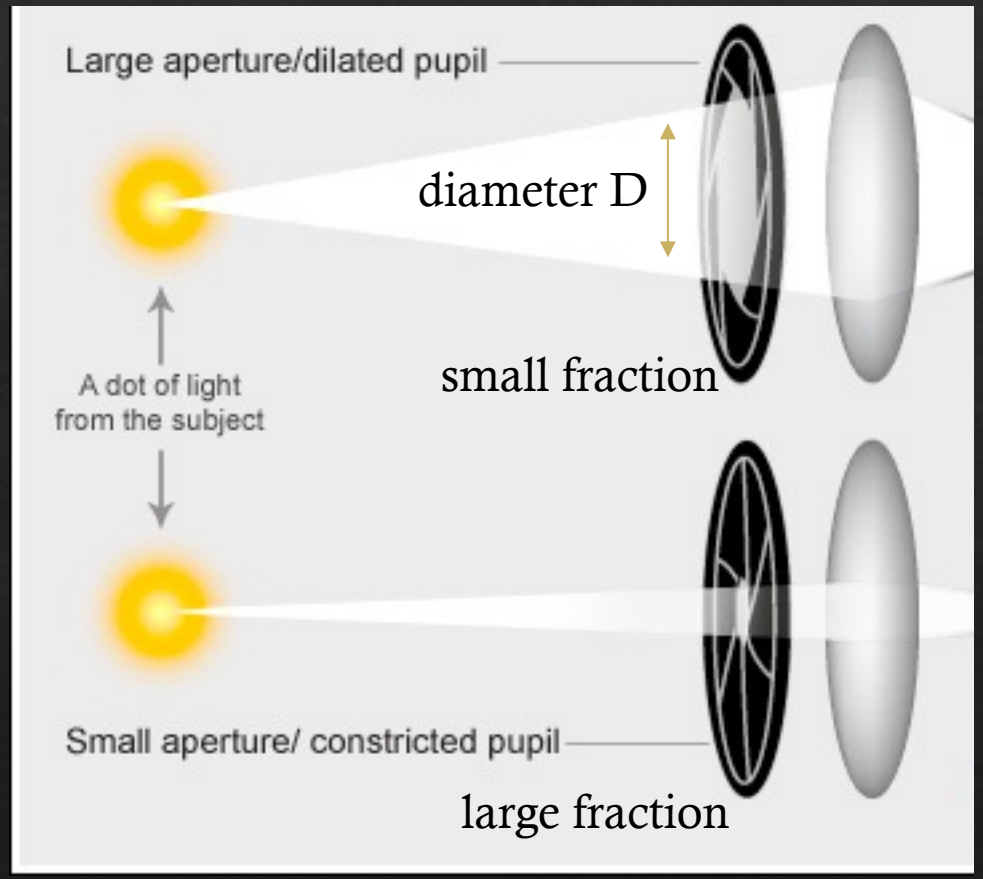


# … only focusses part of the world




*depth of field*
range of depths for which image is sharp

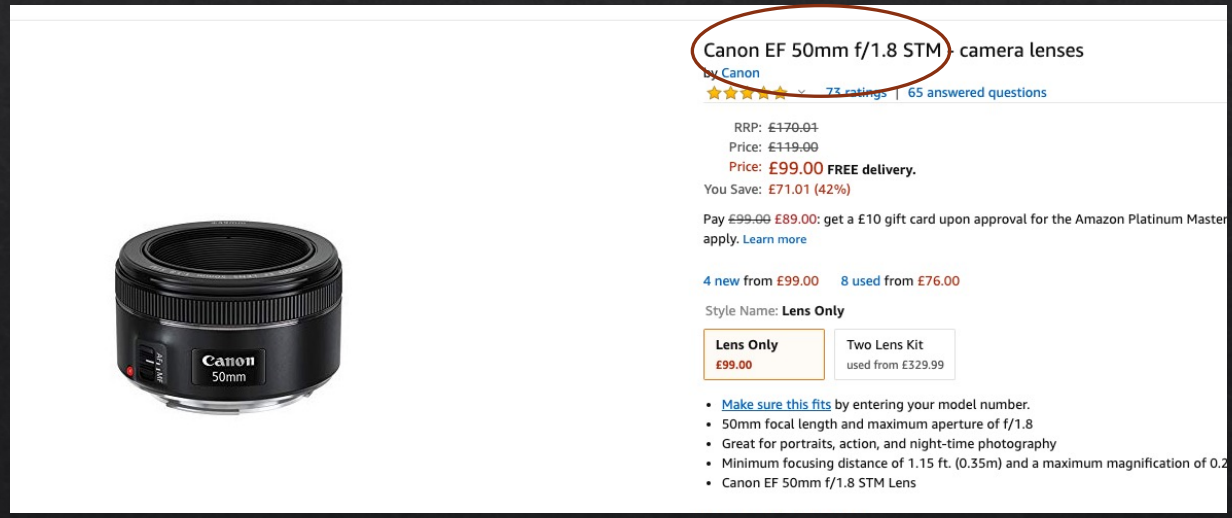
# Finite-sized pinhole = aperture




https://www.dpreview.com/forums/post/59717839

aperture specification is a fraction: $\frac{f}{D}$

called f-number of a lens

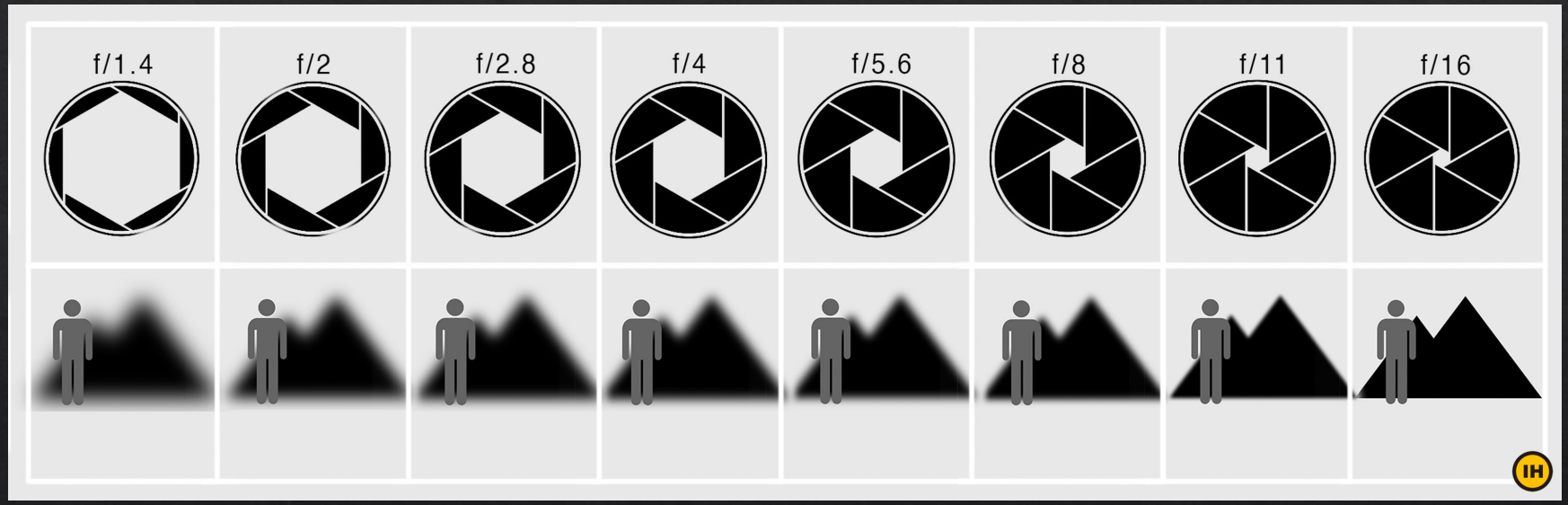


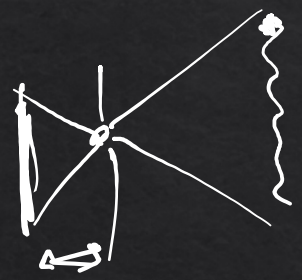

amazon purchase

# Depth of field depends on aperture size

more light allows
fast shutter speed –
good for dark scenes

less light but large
depth of field –
good for landscape



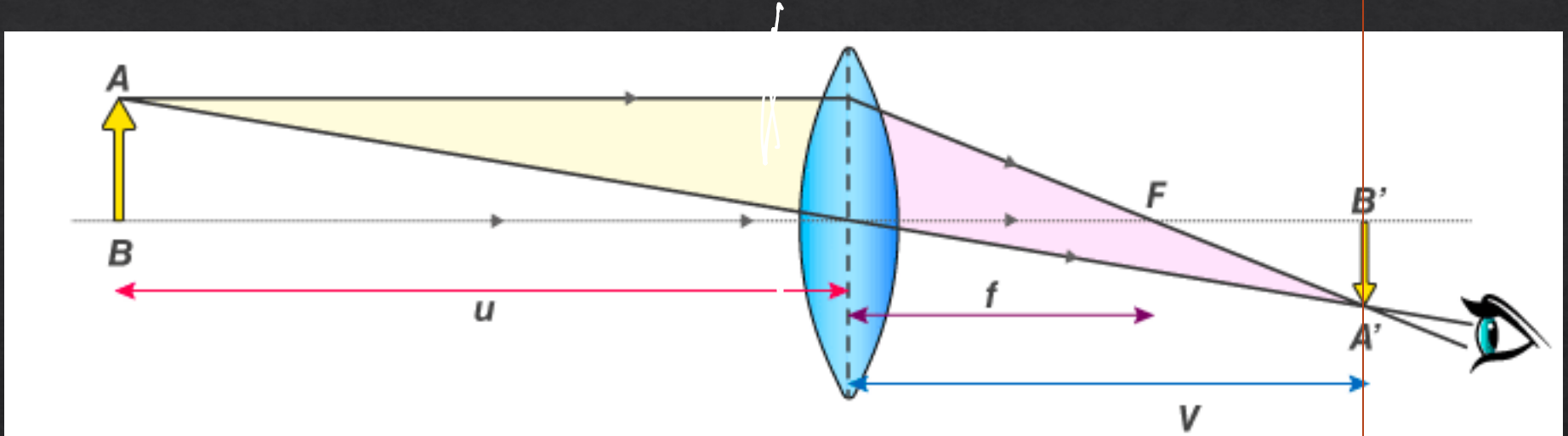



'fast lens'

‘slow lens’

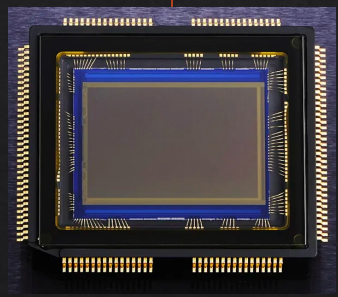

https://www.zippi.co.uk/thestudio/landscape-photography/aperture-diagram-indiahikes/

# Thin lens formula, independent of aperture

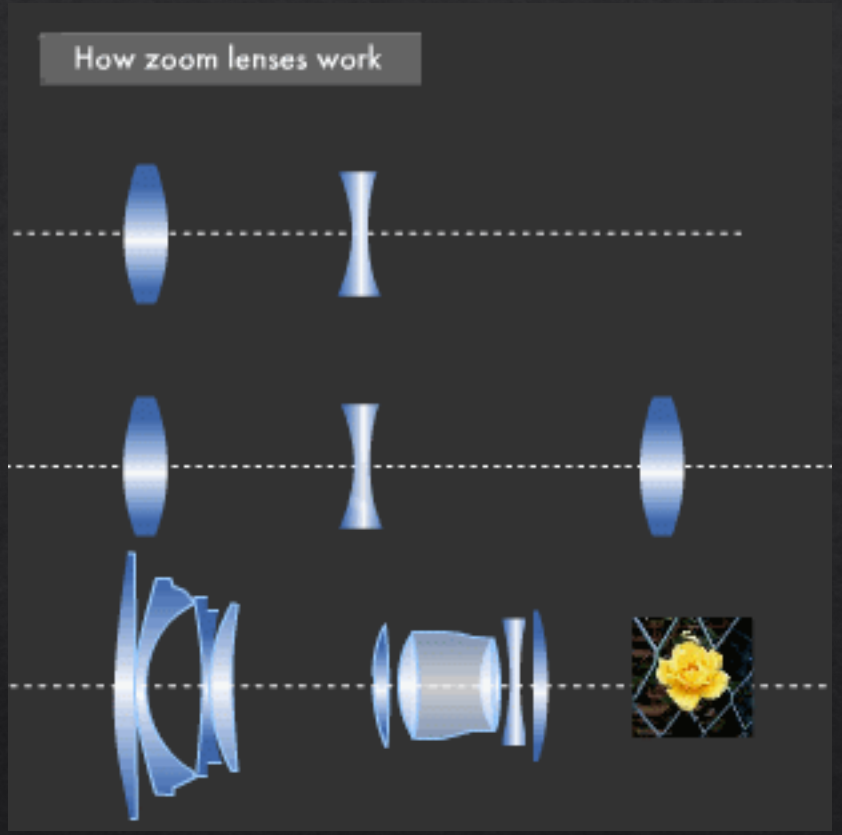



https://byjus.com/physics/derivation-of-lens-formula/

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$



# Zooming-- changing $f$



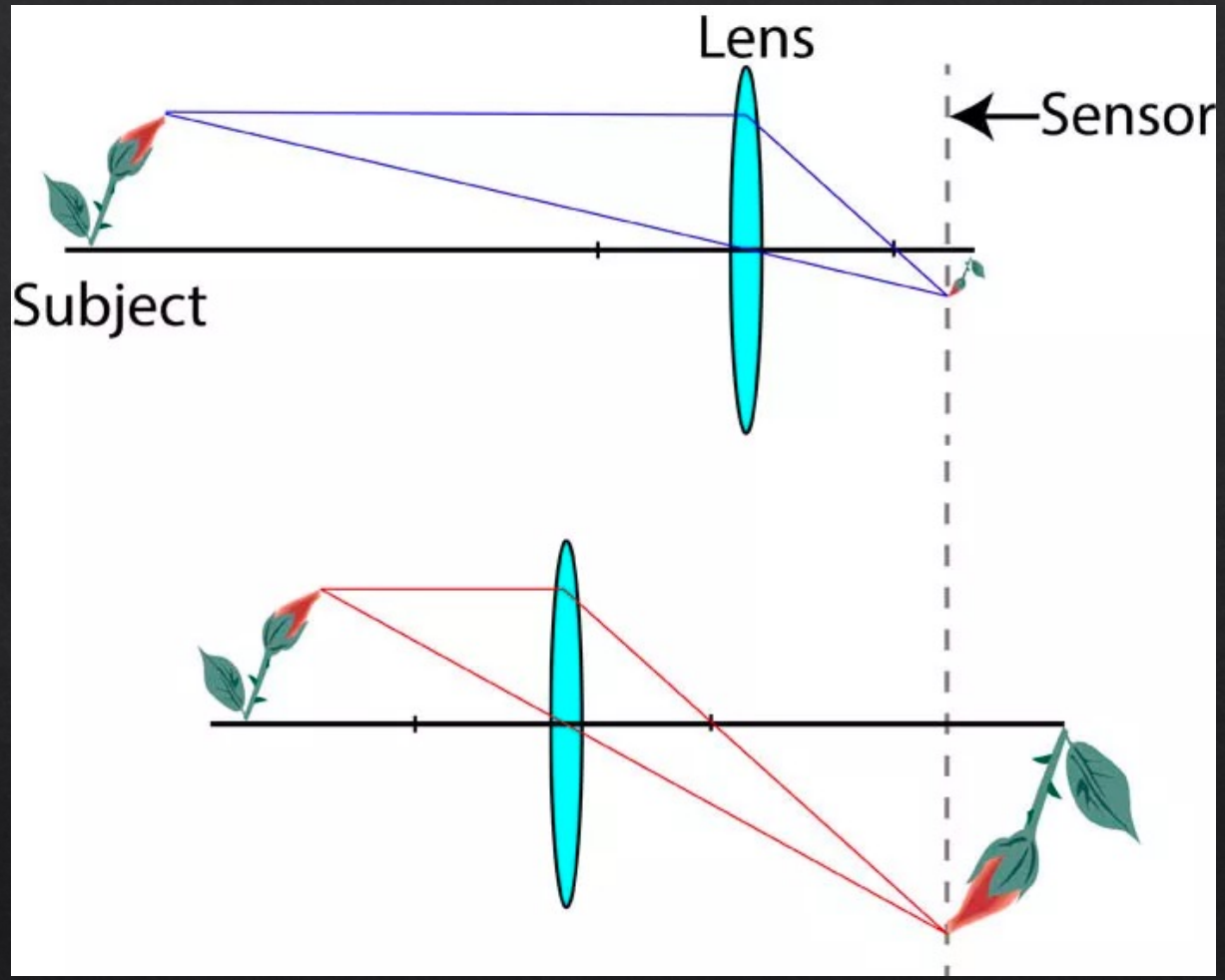

https://global.canon/en/technology/s_labo/light/003/02.html

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

effective
focal
length

# Same lens (fixed *f*), increase *v*

extension tube



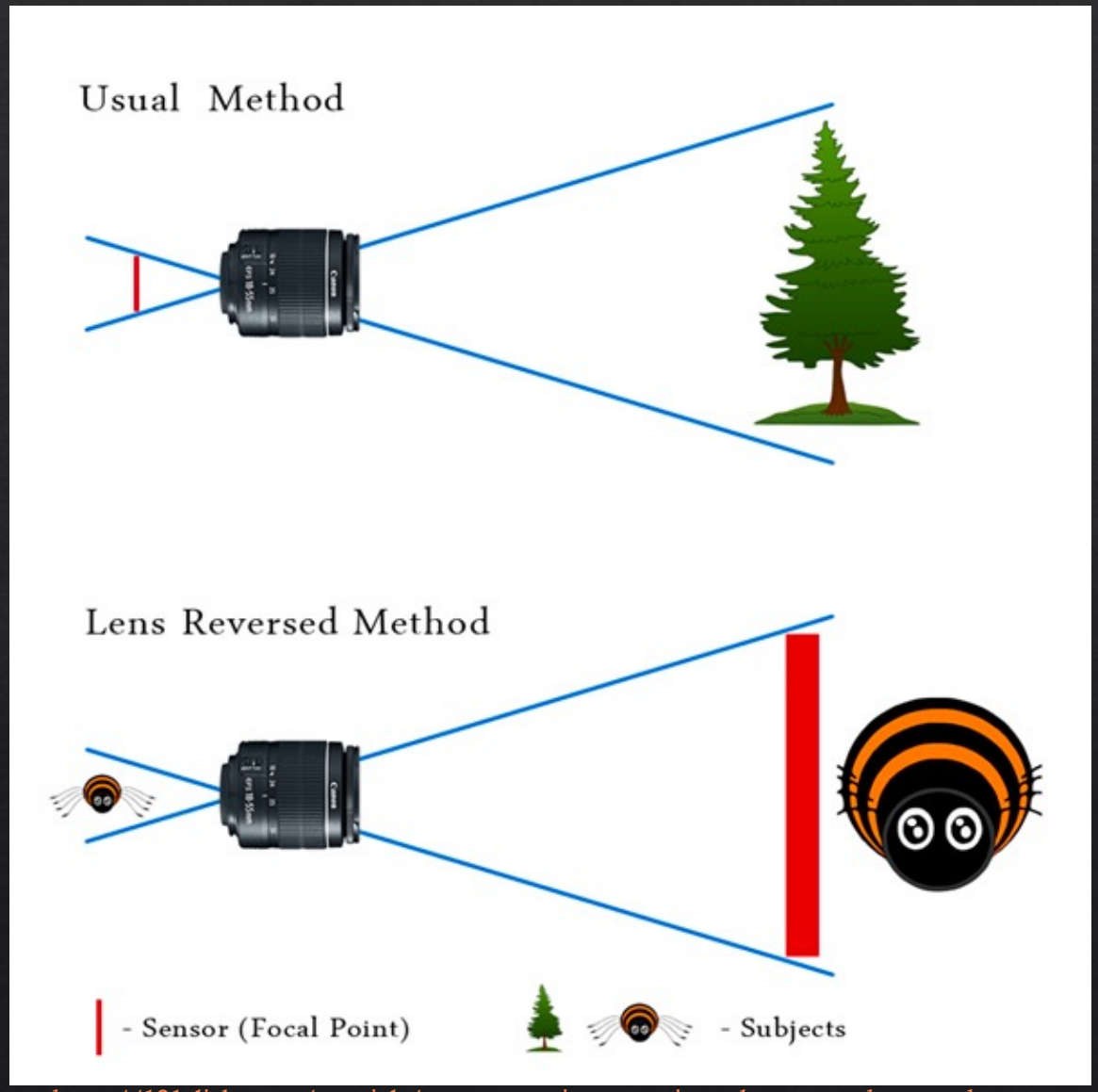

https://expertphotography.com/difference-between-macro-micro-and-close-up-photography/

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

larger image!

# Also achieved by swapping subject and sensor!

reverse ring extension tube

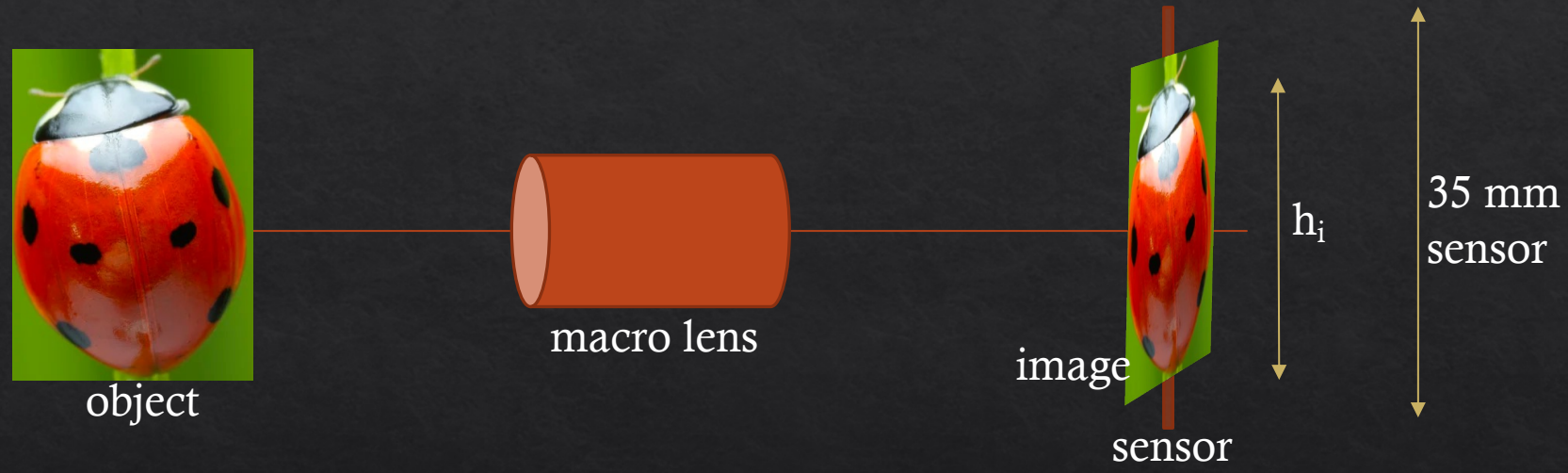



$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

https://121clicks.com/tutorials/use-reverse-ring-extension-tube-macro-photography

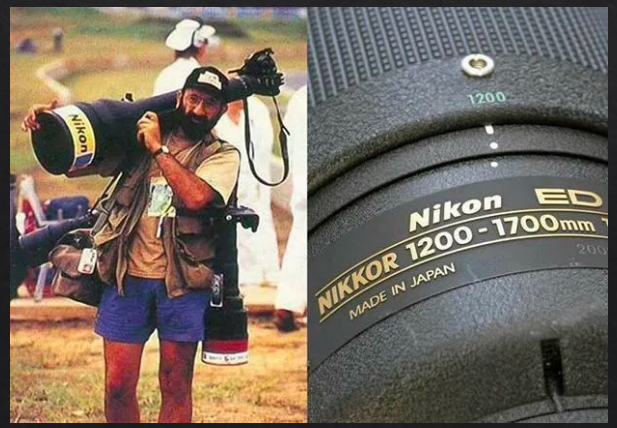
# Macro photography

high magnification ratio $\frac{h_i}{h_o}$




# Types of lenses

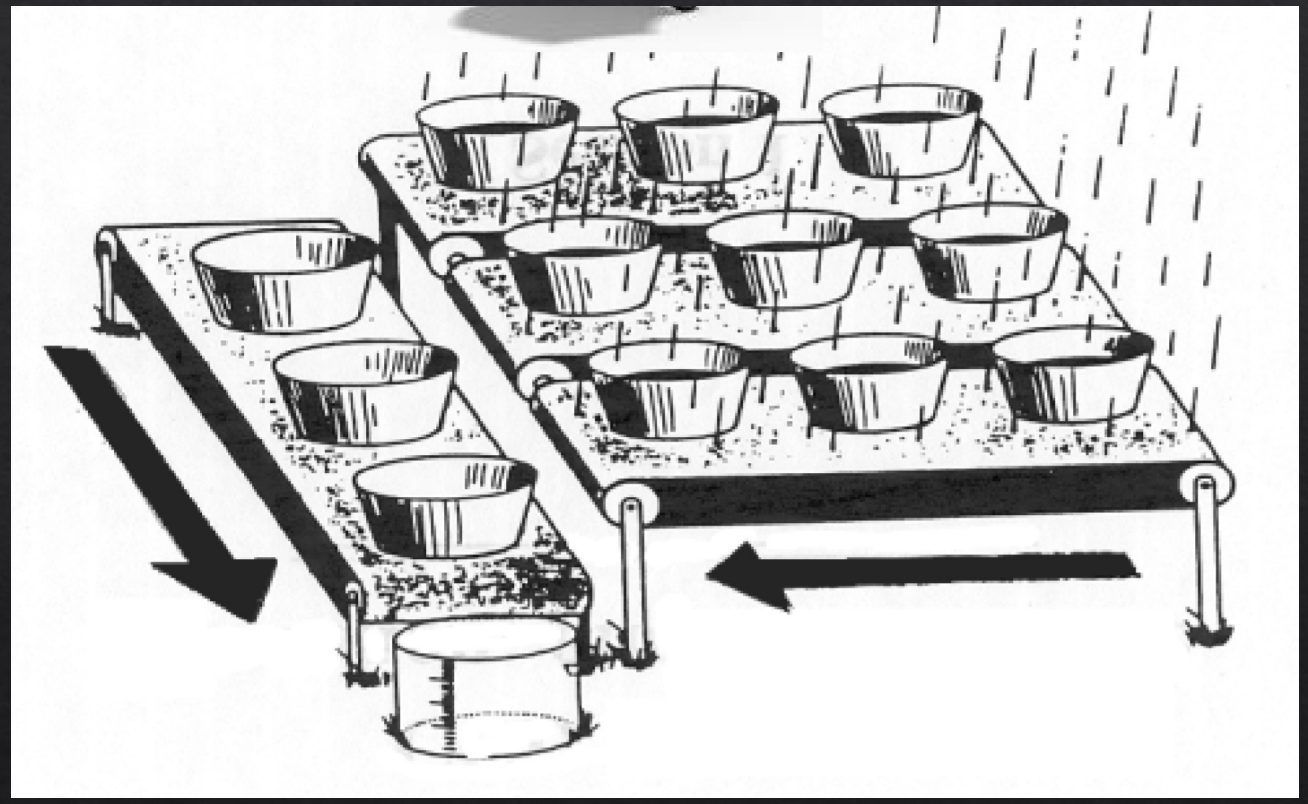
telephoto
- f larger than length of lens construction
- useful to zoom
- compresses range of depths
- usually variable focal lengths
- and variable f-number (depending on f)




standard/prime
- f fixed
- no zoom capability
- usually high quality build = better image quality


wide angle
- f shorter than lens construction
- good for landscape
- could introduce more distortion

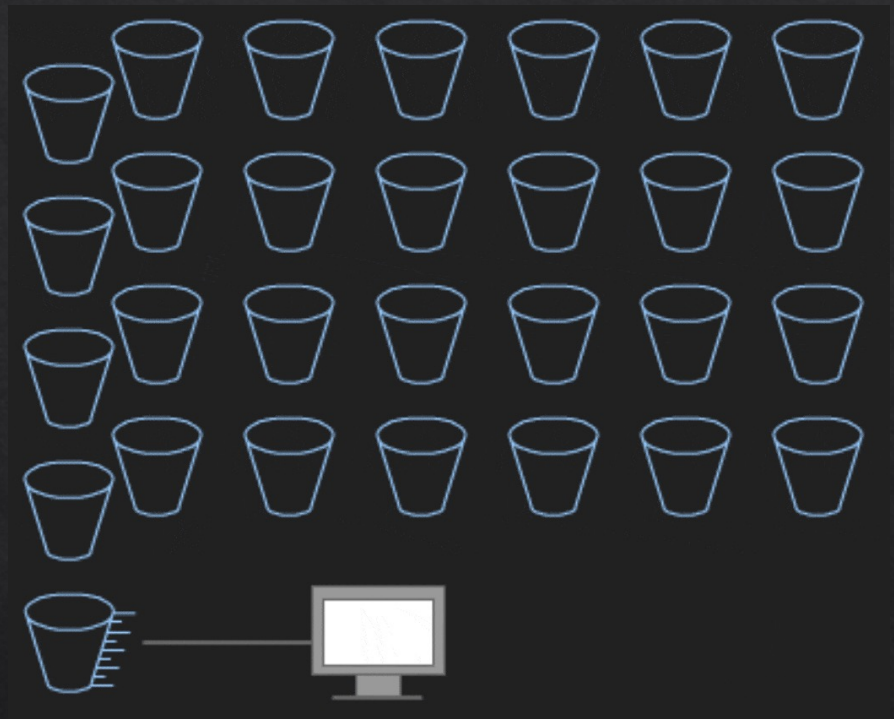
# Cameras – sensors



https://www.visiononline.org/userassets/aiauploads/file/cvp_the-fundamentals-of-camera-and-image-sensor-technology_jon-chouinard.pdf

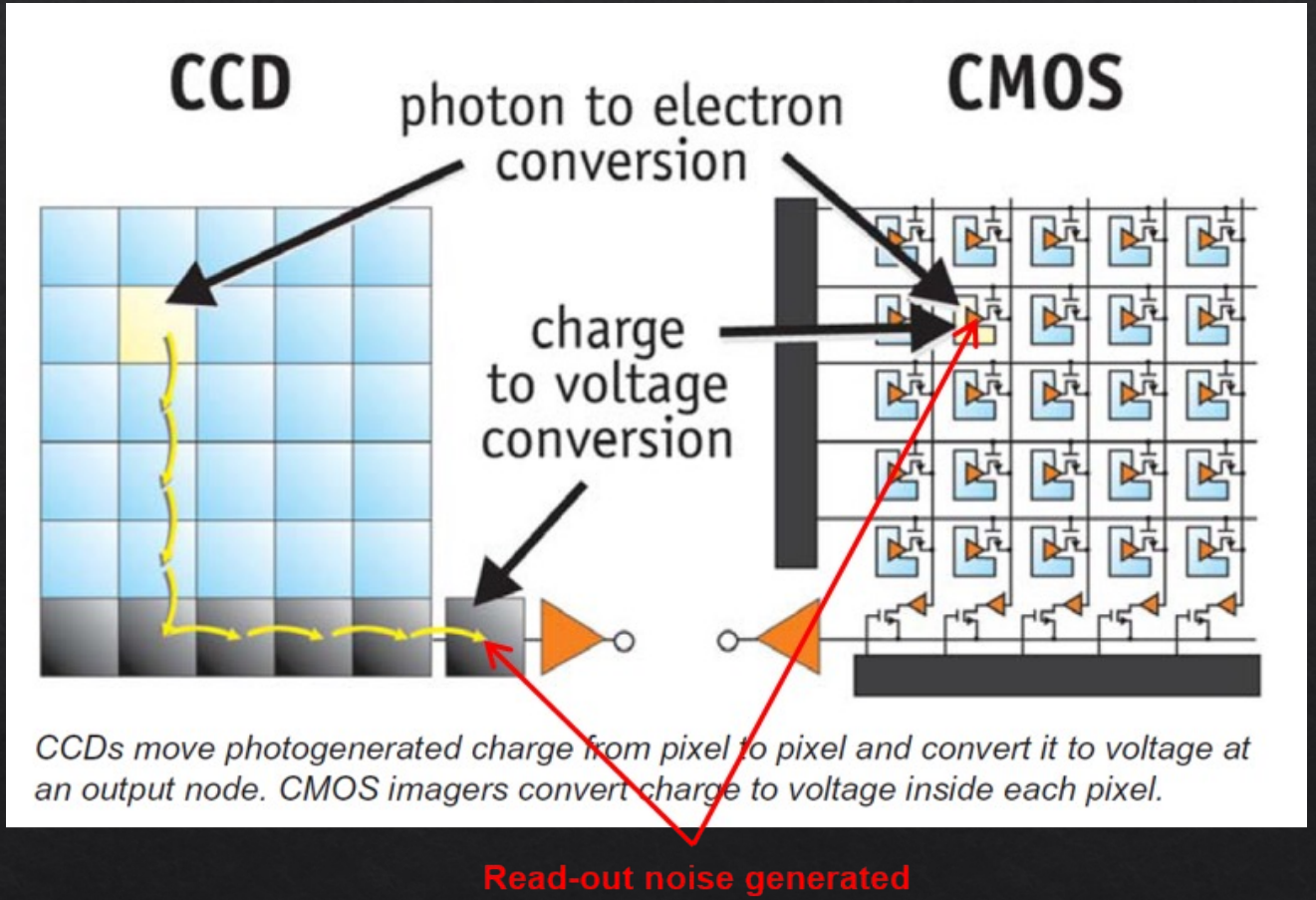
# Sensor sensitivity and response

# Types of sensors



Link to gif

# Types of sensors

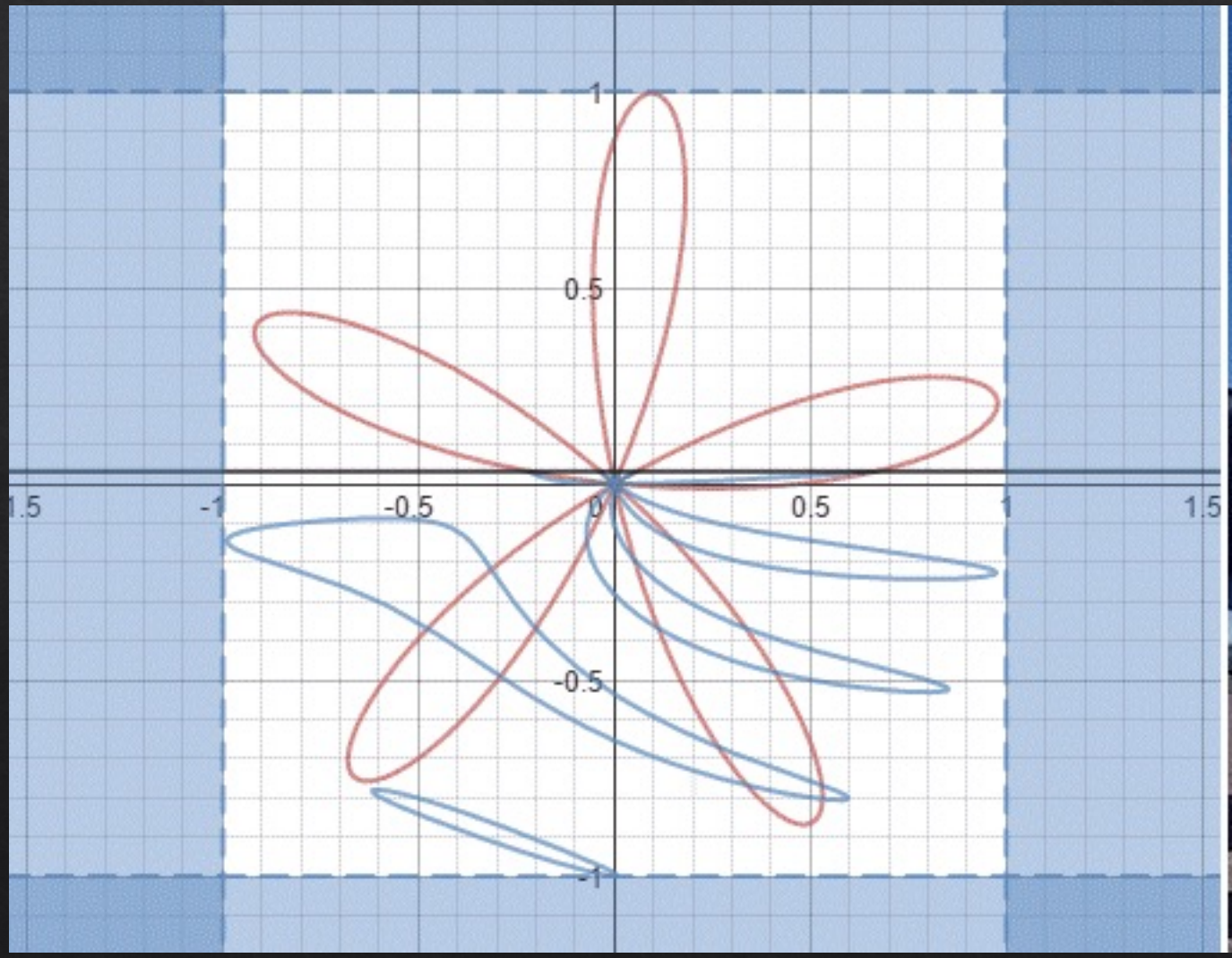



Read more here, and on this canon website (marketing-speak alert)

Link to gif

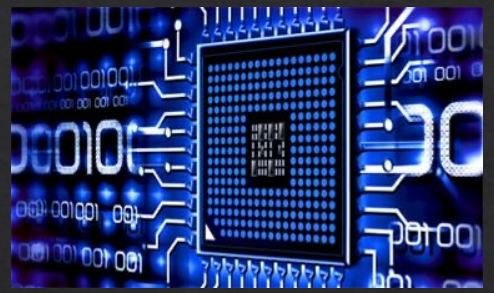
Link to gif

# Rolling shutter




Link to gif

# The big picture!

# CG – account for all factors!