

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} [F(s)]$$

$$\begin{aligned} \therefore L\{t e^{-t} \cos t\} &= (-1) \frac{d}{ds} \left[\frac{s+1}{s^2+2s+2} \right] \\ &= - \left[\frac{(s^2+2s+2)(1) - (s+1)(2s+2)}{(s^2+2s+2)^2} \right] \\ &= - \left[\frac{-s^2-2s}{(s^2+2s+2)^2} \right] \\ &= \frac{s^2+2s}{(s^2+2s+2)^2} \\ &= \frac{s(s+2)}{(s^2+2s+2)^2} \end{aligned}$$

41. Find $L\{t e^{-t} \sin t\}$

[CUSAT 06 Nov 10] CLT 14 Apr 16] [CUSAT 06 Apr 18] [CUAT 12 Apr 18]
[KER 08 Jan 19] [KTU May 19]

Ans:

$$\begin{aligned} L\{\sin t\} &= \frac{1}{s^2+1} \\ L\{e^{-t} \sin t\} &= \frac{1}{(s+1)^2+1} = \frac{1}{s^2+2s+2} \\ L\{t^n f(t)\} &= (-1)^n \frac{d^n}{ds^n} [F(s)] \\ \therefore L\{t e^{-t} \sin t\} &= (-1) \frac{d}{ds} \left[\frac{1}{s^2+2s+2} \right] \\ &= - \left[\frac{(s^2+2s+2)(0) - (2s+2)}{(s^2+2s+2)^2} \right] \\ &= \frac{2s+2}{(s^2+2s+2)^2} \\ &= \frac{2(s+1)}{(s^2+2s+2)^2} \end{aligned}$$

42. Find $L\{t e^{-2t} \cos 3t\}$

[KER 08 Apr 15]

Ans:

$$\begin{aligned} L\{\cos 3t\} &= \frac{s}{s^2+9} \\ L\{e^{-2t} \cos t\} &= \frac{s+2}{(s+2)^2+9} = \frac{s+2}{s^2+4s+13} \\ L\{t^n f(t)\} &= (-1)^n \frac{d^n}{ds^n} [F(s)] \end{aligned}$$