120. Find $L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right]$

[KER 08 May 09] [KNR 07 June 09] [CUSAT 06 Nov 10] [MG 10 May 11] [CUSAT 06 Nov 11] [CUSAT 06 Nov 14] [CUSAT 06 Nov 15] [CUSAT 12 Nov 15] [CUSAT 12 Nov 16] [CLT 14 Apr 17] [MG 10 May 17] [CLT 09 Apr 18] [KER 13 Jan 19]

Ans:

Let $F(s) = \log\left(\frac{s+1}{s-1}\right)$

$= \log(s+1) - \log(s-1)$

$\therefore\quad F'(s) = \frac{1}{s+1} - \frac{1}{s-1}$

$L^{-1}[F'(s)] = L^{-1}\left[\frac{1}{s+1} - \frac{1}{s-1}\right]$

$= e^{-t} - e^{t}$

$L^{-1}[F'(s)] = -t\ L^{-1}[F(s)]$

$\therefore\quad e^{-t} - e^{t} = -t\ L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right]$

$\Rightarrow\quad L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right] = \frac{e^{t} - e^{-t}}{t}$

$= \frac{2}{t}\left(\frac{e^{t} - e^{-t}}{2}\right)$

$= \frac{2\sinh t}{t}$

121. Find $L^{-1}\left[\log\left(\frac{s^2-1}{s^2}\right)\right]$

[KNR 07 Apr 11] [KER 08 Jan 19] [CLT 14 Apr 19]

Ans:

$F(s) = \log\left(\frac{s^2-1}{s^2}\right)$

$= \log(s^2-1) - \log(s^2) = \log(s^2-1) - 2\log(s)$

$\therefore\quad F'(s) = \frac{2s}{s^2-1} - \frac{2}{s}$

$L^{-1}[F'(s)] = 2\ L^{-1}\left[\frac{s}{s^2-1} - \frac{1}{s}\right]$

$= 2(\cosh t - 1)$

$L^{-1}[F'(s)] = -t\ L^{-1}[F(s)]$

$\therefore\quad 2(\cosh t - 1) = -t\ L^{-1}\left[\log\left(\frac{s^2-1}{s^2}\right)\right]$

$\Rightarrow\quad L^{-1}\left[\log\left(\frac{s^2-1}{s^2}\right)\right] = 2\left(\frac{\cosh t - 1}{t}\right)$