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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION(2019 SCHEME), DECEMBER 2019

Course Code: MAT101

Course Name: LINEAR ALGEBRA AND CALCULUS

(2019-Scheme)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

1	Determine the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$	(3)
2	If 2 is an eigen value of $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, without using its characteristic equation, find the other eigen values.	(3)
3	If $f(x,y) = xe^{-y} + 5y$ find the slope of f (x,y) in the x-direction at (4,0).	(3)
4	Show that $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$, where $z = e^x \sin y + e^y \cos x$	(3)
5	Find the mass of the square lamina with vertices (0,0) (1,0) (1,1) and (0,1) and density function $x^2 y$	(3)
6	Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} dxdy$ by changing to polar coordinates.	(3)
7	Test the convergence of the series $\sum_{k=1}^{\infty} \dfrac{k}{2k+1}$	(3)
8	Check the convergence of $\sum_{k=1}^{\infty} \dfrac{1}{k^{k/2}}$	(3)
9	Find the Taylors series for $f(x) = \cos x$ about $x = \dfrac{\pi}{2}$ up to third degree terms.	(3)
10	Find the Fourier half range sine series of $f(x) = e^x$ in $0 < x < 1$	(3)

PART B

Answer one full question from each module, each question carries 14 marks

Module-I

11 a) Solve the system of equations by Gauss elimination method. (7)

$$x + 2y + 3z = 1$$
$$2x + 3y + 2z = 2$$
$$3x + 3y + 4z = 1$$

b) Find the eigenvalues and eigenvectors of (7)

$$\begin{bmatrix} 4 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix}$$

12 a) Find the values of λ and μ for which the system of equations (7)

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

has (i) no solution (ii) a unique solution and (iii) infinite solution

b) Find the matrix of transformation that diagonalize the matrix (7)

$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$. Also write the diagonal matrix.

Module-II

13 a) Let f be a differentiable function of three variables and suppose that (7)

$w = f(x-y, y-z, z-x)$,show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$

b) Locate all relative extrema of $f(x,y) = 4xy - y^4 - x^4$ (7)

14 a) Find the local linear approximation L to the function $f(x,y) = \sqrt{x^2 + y^2}$ at the point P(3,4).Compare the error in approximating f by L at the point Q(3.04,3.98) with the distance PQ. (7)

b) The radius and height of a right circular cone are measured with errors of at most 1% and 4%, respectively. Use differentials to approximate the maximum percentage error in the calculated volume. (7)

Module-III

15 a) Evaluate $\iint_R y\,dx\,dy$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. (7)

b) Use double integral to find the area of the region enclosed between the parabola $y = \frac{x^2}{2}$ and the line $y = 2x$. (7)

16 a) Evaluate $\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2}\,dx\,dy$ by reversing the order of integration (7)

b) Use triple integrals to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$. (7)

Module-IV

17 a) Find the general term of the series $1 + \frac{1\cdot 2}{1\cdot 3} + \frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5} + \frac{1\cdot 2\cdot 3\cdot 4}{1\cdot 3\cdot 5\cdot 7} + \ldots\ldots$ and use the ratio test to show that the series converges. (7)

b) Test whether the following series is absolutely convergent or conditionally convergent $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k(k+1)}}$ (7)

18 a) Test the convergence of $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \ldots\ldots\ldots \frac{x^k}{k(k+1)} + \text{--------}$ (7)

b) Test the convergence of the series $\sum_{k=1}^{\infty} \frac{(k+1)!}{4!\,k!\,4^k}$ (7)

Module-V

19 a) Find the Fourier series of periodic function with period 2 which is given below $f(x) = \begin{cases} -x\,; & -1 \le x \le 0 \\ x\,; & 0 \le x \le 1 \end{cases}$. Hence prove that $1 + \frac{1}{3^2} + \frac{1}{5^2} + ---- = \frac{\pi^2}{8}$ (7)

b) Find the half range cosine series for $f(x) = \begin{cases} kx & 0 \le x \le L/2 \\ k(L-x) & L/2 \le x \le L \end{cases}$ (7)

20

a) Find the Fourier series of $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x^2 & 0 < x < \pi \end{cases}$ (7)

b) Obtain the Fourier series expansion for $f(x) = x^2, \quad -\pi < x < \pi$. (7)

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Reg No.:_______________ 00000MA101121804 Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech examinations (S) September 2020 S1/S2 (2015 Scheme)

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks. Marks

1 a) Determine whether the series $\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^{k+2}$ converges. If so, find the sum (2)

b) Find the Maclaurin series expansion of $f(x) = \boldsymbol{ln}(\mathbf{1}-\boldsymbol{x})$up to 3 terms (3)

2 a) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $ye^{x} - 5cos2z = 3z$ (2)

b) Use chain rule to find $\frac{dw}{dx}$ at (0,1,2) for $w = xy + yz$, $y = \sin x, z = e^{x}$. (3)

3 a) Find the velocity of a particle moving along the curve $\vec{r}(t) == e^{t}\sin t\,\vec{i} + e^{t}\cos t\,\vec{j} + t\,\vec{k}$ at t = π (2)

b) Find the unit normal to the surface $yz + zx + xy = c$ at (-1,2,3) (3)

4 a) Evaluate $\int_{1}^{2}\int_{y}^{3-y} dx\, dy$ (2)

b) Evaluate $\int_{1}^{2}\int_{0}^{x}\frac{dy\,dx}{x^2+y^2}$. (3)

5 a) Find the value of constant a so that if $\bar{F} = (3x-2y+z)\,\boldsymbol{i} + (4x-ay+z)\boldsymbol{j} + (x-y+2z)\boldsymbol{k}$ is solenoidal. (2)

b) Find the work done by a force field $F(x,y) = -y\boldsymbol{i} + x\boldsymbol{j}$ acting on a particle moving along the circle $x^2 + y^2 = 3$ from $(\sqrt{3},0)$ to $(0,\sqrt{3})$ (3)

6 a) Determine the source and sink of the vector field $F(x,y,z) = 2(x^3-2x)\boldsymbol{i} + 2(y^3-2y)\boldsymbol{j} + 2(z^3-2z)\boldsymbol{k}$ (2)

b) Using Stoke's theorem prove that $\int_C \bar{r}\,.\,d\bar{r} = 0$ where $\bar{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$ and C is any closed curve. (3)

PART B
Module 1
Answer any two questions, each carries 5 marks.

7	Test the convergence of the infinite series $\sum_{k=1}^{\infty}\left(\frac{k}{k+1}\right)^{k^2}$.	(5)
8	Examine the convergence of $\sum_{k=0}^{\infty}\frac{(k+4)!}{4!k!4^k}$	(5)
9	Find the radius of convergence of the power series $\sum_{k=1}^{\infty}\frac{(2x-3)^k}{4^{2k}}$	(5)

Module 1I
Answer any two questions, each carries 5 marks.

10	The height and radius of a circular cone is measured with errors of atmost 3% and 5% respectively. Use differentials to approximate the maximum percentage error in calculated volume.	(5)
11	If $u = x^2 tan^{-1}\left(\frac{y}{x}\right) - y^2 tan^{-1}\left(\frac{x}{y}\right)$, find $\frac{\partial^2 u}{\partial x \partial y}$	(5)
12	Find relative extrema and saddle points, if any, of the function $f(x,y) = x^3 + y^3 - 15xy$.	(5)

Module 1II
Answer any two questions, each carries 5 marks.

13	Find where the tangent line to the curve $\boldsymbol{r}(t) = e^{-2t} i + \cos t\, j + 3 \sin t\, k$ at the point (1,1,0) intersects the YZ plane.	(5)
14	Find the position and velocity vectors of the particle given $\mathbf{a}(t)=(t+1)^{-2}\mathbf{j} - e^{-2t}k$, $\mathbf{v}(0) = 3\mathbf{i}-\mathbf{j}$, $r(0)= k$	(5)
15	A particle moves along a curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$ where t is the time. Find the component of acceleration at time t = 1 in the direction of $\vec{i} - 3\vec{j} + 2\vec{k}$	(5)

Module 1V
Answer any two questions, each carries 5 marks.

16	Evaluate $\iiint_R xysin\, z\, dV$ where R is the rectangular box defined by $0 \le x \le 1, 0 \le y \le 1, \quad 0 \le z \le \frac{\pi}{6}$	(5)
17	Sketch the region of integration and evaluate $\int_1^2 \int_y^{y^2} dx\, dy$ by changing the order of integration.	(5)
18	Use double integral to find the area bounded by the x – axis $y = 2x \;\; and \;\; x + y = 1$	(5)

Module V

Answer any three questions, each carries 5 marks.

19 Prove that $\int_C (x^2 - yz)\bar{\imath} + (y^2 - zx)\bar{\jmath} + (z^2 - xy)\bar{k}\ .\ d\bar{r}$ is independent of the path and evaluate the integral along any curve from (0,0,0) to (1,2,3). (5)

20 Evaluate $\int_C xy^2\,dx + xy\,dy$ where C is a triangle with vertices at $(0,0), (0,1)$ and (2,1) (5)

21 Evaluate $\int_C 2xy\,dx + (x^2 + y^2)dy$ along the curve $C: x = \cos t\,, y = \sin t$, $0 \le t \le \frac{\pi}{2}$ (5)

22 Determine whether $F(x, y) = 6y^2\, i + 12xy\, j$ is a conservative vector field. If so find the potential function for it. (5)

23 If $\bar{F} = (sinz + y\cos x)\boldsymbol{i}$ + $(\sin x + 2\cos y)\boldsymbol{j}$ $+(\sin y + x\cos z)\boldsymbol{k}$,find $Div\ \bar{F}$ and $Curl\ \bar{F}$. (5)

Module VI

Answer any three questions, each carries 5 marks.

24 Using Stoke's theorem, evaluate $\int_C \bar{F}\ .\ d\bar{r}$ where C is the boundary of the projection of the sphere $x^2 + y^2 + z^2 = 1$ on the XY plane with (5)

$$\bar{F} = (2x - y)\bar{\imath} - yz^2\bar{\jmath} - y^2 z\,\bar{k}$$

25 Using Green's theorem evaluate $\int_C (y^2 - 7y)dx + (2xy + 2x)\,dy$ where C is the circle $x^2 + y^2 = 1$ (5)

26 Evaluate using divergence theorem for $\vec{F} = x^2 i + zj + yzk$ taken over the cube bounded by $x = 0,\ x = 1, y = 0,\ y = 1,\ \ z = 0\ and\ z = 1$ (5)

27 Evaluate the surface integral $\iint_\sigma z^2\,ds$, where σ is the portion of the curve $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ (5)

28 Use Green' theorem to find the area enclosed by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ (5)

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Reg No.:_______________ Name:____________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION(S), DECEMBER 2019

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks. Marks

1 a) Find the sum of the series $\sum_{k=1}^{\infty} \frac{2}{3^{(k+1)}}$ (2)

b) Determine whether the alternating series $\sum_{k=2}^{\infty} (-1)^k \frac{k}{k-1}$ converges. (3)

2 a) Find the slope of the function $f(x,y) = x\cos(xy) + y\sin(xy)$ at $(\pi,1)$ along the x- direction. (2)

b) If $z= f(x^2-y^2)$, show that

$$y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$$ (3)

3 a) Find $\lim_{t\to 0} \boldsymbol{r}(t)$, where $\boldsymbol{r}(t) = \langle 1+t^3, te^{-t}, \frac{\sin t}{t} \rangle$ (2)

b) Find the directional derivative of $f(x,y) = e^x \cos y$ at $P(0,\pi/4)$ in the direction of negative Y-axis (3)

4 a) Evaluate $\int_0^1\int_0^1\int_0^1 e^{x+y+z}\, dx\, dy\, dz$ (2)

b) Evaluate $\iint_R \left(x^2+y^2\right)dx\,dy$ where R is the region taken over the first quadrant for which $x+y\le 1$. (3)

5 a) Find the divergence of the vector field $F(x,y,z) = x^2y\, i + 2y^3z\, j + 3z\, k$ (2)

b) Evaluate $\int_c x^2dy + y^2dx$ where C is the path $y = x$ from (0,0) to(1,1) (3)

6 a) Determine the source and sink of the vector field $F(x,y,z) = 2(x^3-2x)i + 2(y^3-2y)j + 2(z^3-2z)k$ (2)

b) If S is any closed surface enclosing a volume V and if $A = axi + byj + czk$ prove that $\iint A.nds = (a+b+c)\,V$ (3)

PART B

Module 1

Answer any two questions, each carries 5 marks.

7 Test for convergence of the series $\sum_{k=0}^{\infty}\frac{(k!)^2}{(2k)!}$. (5)

8 Find the radius of convergence of $\sum_{k=0}^{\infty}\frac{(2x-1)^k}{3^{2k}}$. (5)

9 Expand $f(x) = sin\pi x$ into a Taylors series about $x = \frac{1}{2}$,up to third derivative. (5)

Module 1I

Answer any two questions, each carries 5 marks.

10 If $u = f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$ find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. (5)

11 Find the local linear approximation L(x ,y) of f(x ,y)=$\frac{1}{\sqrt{x^2+y^2}}$ at the point P(4,3).Compare the error in the approximation to f by L at the point Q(3.92,3.01) with the distance between P and Q. (5)

12 Locate all relative extrema and saddle point for the function $f(x,y) = x^3 + y^3 - 6xy + 20$. (5)

Module 1II

Answer any two questions, each carries 5 marks.

13 Find the equation of the unit tangent and unit normal to the curve $x = e^t\cos t, y = e^t\sin t, z = e^t$ $; at\ t = 0$. (5)

14 A particle moves along the curve $r(t) = \left(\frac{1}{t}\right)i + t^2 j + t^3 k$, where t denotes time. Find

1) The scalar tangential and normal components of acceleration at time$t = 1$. (5)

2) The vector tangential and normal component of acceleration at time $t = 1$

15 Find the equation of the tangent plane and the parametric equations of the normal line to the surface $z = 4x^3y^2 + 2y - 2$ at (1,-2,10). (5)

Module 1V

Answer any two questions, each carries 5 marks.

16 Use double integral to find the area of the plane enclosed by $y^2 = 4x \text{ and } x^2 = 4y$ (5)

17 Change the order of integration to evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dy\,dx$ (5)

18	Use triple integral to find the volume of the solid with in the cylinder $x^2 + y^2 = 4$ and between the planes z = 0 and $y + z = 3$.	(5)

Module V

Answer any three questions, each carries 5 marks.

19	If $\bar{r} = x\,\bar{\imath} + y\,\bar{\jmath} + z\,\bar{k}$ and $r = \lvert\bar{r}\rvert$, prove that $\nabla^2 r^n = n(n+1)\,r^{n-2}$	(5)
20	Evaluate $\int_c (3x^2 + y^2)\,dx + 2xydy$ along the curve $C: x = \cos t, y = \sin t,\ 0 \le t \le \frac{\pi}{2}$	(5)
21	Find the scalar potential of $\vec{F} = (2xy + z^3)\vec{\imath} + x^2\vec{\jmath} + 3xz^2\overline{k}$	(5)
22	Find the work done by $F(x,y) = (x+y)i + xy\,j - z^2\,k$ along the line segments from $(0,0,0)$ to $(1,3,1)$ to $(2,-1,5)$	(5)
23	Show that $\int_{(0,0)}^{(1,\frac{\pi}{2})} e^x \sin y\ dx + e^x \cos y\ dy$ is independent of path. Hence evaluate $\int_{(0,0)}^{(1,\frac{\pi}{2})} e^x \sin y\ dx + e^x \cos y\ dy$	(5)

Module VI

Answer any three questions, each carries 5 marks.

24	Evaluate using Green's theorem in the plane $\int_c (x^2dx - xydy)$ where C is the boundary of the square formed by $x = 0, y = 0, x = a, y = a$	(5)
25	Evaluate the surface integral $\iint_\sigma f(x,y,z)\ ds$ where $f(x,y,z) = x + y,\ \sigma$ is the portion of the surface $z = 6 - 2x - 4y$ in the first octant.	(5)
26	Using divergence theorem find the flux across the surface σ which is the surface of the tetrahedron in the first octant bounded by $x + y + z = 1$ and the coordinate planes, $\bar{F} = (x^2 + y)\bar{\imath} + xy\bar{\jmath} - (2xz + y)\bar{k}$	(5)
27	Evaluate $\int_c (e^x dx + 2ydy - dz)$ *where C* is the curve $x^2+y^2 = 4, z = 2$ using Stoke's theorem	(5)
28	Evaluate the surface integral $\iint_\sigma f(x,y,z)\ ds$ where $f(x,y,z) = x^2 + y^2,\ \sigma$ is the surface of the sphere $x^2 + y^2 + z^2 = a^2$	(5)

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Reg No.:________________ Name:____________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION (S), MAY 2019

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks. Marks

1 a) Check the convergence of the series $\sum_{k=1}^{\infty}\left(\frac{3k-4}{4k-5}\right)^k$ (2)

b) Find the Maclaurin series of $f(x)=\frac{1}{1+x}$, up to 3 terms (3)

2 a) If $z=(3x-2y)^4$, find $\frac{\partial^4 z}{\partial x\,\partial y^3}$ (2)

b) If $w=\log(\tan x+\tan y+\tan z)$ then prove that (3)

$\sin 2x\frac{\partial w}{\partial x}+\sin 2y\,\frac{\partial w}{\partial y}+\sin 2z\frac{\partial w}{\partial y}=2$

3 a) Find the speed of a particle moving along the path $x=2\cos t, y=2\sin t, z=t$ (2)

at $t=\pi/2$

b) If $y'(t)=\cos t\ i+\sin t\ j$; $y(0)=i-j$. Find $y(t)$. (3)

4 a) Evaluate $\int_0^1\int_0^{x^2}\int_0^2 dy\,dz\,dx$ (2)

b) Evaluate $\iint xy$ dx dy over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and lying in the first quadrant. (3)

5 a) Show that $F(x,y)=2xy^3 i+3x^2y^2 j$ is conservative. (2)

b) If $\bar{r}=x\,\bar{i}+y\,\bar{j}+z\,\bar{k}$ and $r=\|\bar{r}\|$, prove that $\nabla.\frac{\bar{r}}{r^3}=0$ (3)

6 a) Evaluate by Stoke's theorem $\oint_C (e^x\,dx+2y\,dy-dz)$, where C is the curve (2)

$x^2+y^2=4,\ z=2$

b) Using Green's theorem evaluate $\int_C x\,dy-y\,dx$ where C is the circle $x^2+y^2=4$ (3)

PART B

Module 1

Answer any two questions, each carries 5 marks.

7 Test for convergence of the series $\sum_{k=1}^{\infty}\frac{1}{(8k^2-3k)^{1/3}}$ (5)

8 Find the radius of convergence and interval of convergence of the power series $\sum_{k=0}^{\infty}\frac{(2x-1)^k}{3^{2k}}$. (5)

9 Show that the series $\sum_{k=1}^{\infty}(-1)^k\left(\frac{k}{k+1}\right)^{k^2}$ is convergent. (5)

Module 1I

Answer any two questions, each carries 5 marks.

10 Let $w = \sqrt{x^2 + y^2 + z^2}$, where $x - \cos\theta, y - \sin\theta,\ z - \tan\theta$. Find $\frac{dw}{d\theta}$ at $\theta = \frac{\pi}{4}$, using chain rule. (5)

11 Find the local linear approximation $L(x,y)$ to $f(x,y) = \ln\ (xy)$ at the point $P(1,2)$. Compare the error in approximating f by L at the point $Q(1.01,2.01)$ with the distance between P and Q . (5)

12 Find relative extrema and saddle points, if any, of the function $f(x,y) = xy + \frac{8}{x} + \frac{8}{y}$. (5)

Module 1II

Answer any two questions, each carries 5 marks.

13 Find the unit tangent T(t) and unit normal N(t) to the curve $x = a\cos t,\ y = a\sin t,\ z = ct$ a >0 (5)

14 Find the velocity and position vectors of the particle ,if the acceleration vector $a(t) = \sin t\, i + \cos t\, j + e^t\, k$; $v(0) = k$; $r(0) = -i + k$. (5)

15 Find the equation of the tangent line to the curve of intersection of surfaces $z = x^2 + y^2$ and $3x^2 + 2y^2 + z^2 = 9$ and the point (1,1,2). (5)

Module 1V

Answer any two questions, each carries 5 marks.

16 Evaluate by reversing the order of integration $\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} x\,dx\,dy$ (5)

17 Evaluate $\iint_R xy\,dA$, where R is the sector in the first quadrant bounded by $y = \sqrt{x},\ y = 6 - x,\ y = 0$. (5)

18 Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x}$ x dz dx dy (5)

Module V

Answer any three questions, each carries 5 marks.

19 Find the work done by $F(x, y) = (x^2 + y^2)i - xj$ along the curve $C: x^2 + y^2 = 1$ counter clockwise from $(1,0)$ to $(0,1)$ (5)

20 Determine whether $F(x, y) = 6y^2\, i + 12xy\, j$ is a conservative vector field. If so find the potential function for it. (5)

21 Find the divergence and curl of the vector field $F(x, y, z) = xyz^2 i + yzx^2\, j + zxy^2\, k$ (5)

22 Prove that $\int_C (x^2 - yz)\bar{\imath} + (y^2 - zx)\bar{\jmath} + (z^2 - xy)\bar{k} \,.\, d\bar{r}$ is independent of the path and evaluate the integral along any curve from (0,0,0) to (1,2,3). (5)

23 If $\bar{r} = x\,\bar{\imath} + y\,\bar{\jmath} + z\,\bar{k}$ and $r = \|\bar{r}\|$, prove that $\nabla^2 f(r) = \frac{2}{r} f'(r) + f''(r)$. (5)

Module VI

Answer any three questions, each carries 5 marks.

24 Using Green's theorem evaluate $\int_C (xy + y^2)dx + x^2\, dy$ where C is the boundary of the region bounded by $y = x^2$ and $x = y^2$ (5)

25 Evaluate the surface integral $\iint_\sigma z^2\, ds$, where σ is the portion of the curve $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ (5)

26 Determine whether the vector field $F(x, y, z)$ is free of sources and sinks. If not, locate them. (5)

$i) F(x, y, z) = (y + z)\bar{\imath} - xz\bar{\jmath} + x^2 \sin y\, \bar{k}$

$ii) F(x, y, z) = x^3\bar{\imath} + y^3\bar{\jmath} + 2z^3\bar{k}$

27 Use divergence theorem to find the outward flux of the vector field $F(x, y, z) = (2x + y^2)i + xy\, j + (xy - 2z)k$ across the surface σ of the tetrahedron bounded by $x + y + z = 2$ and the coordinate planes. (5)

28 Using Stoke's theorem evaluate $\int_C \bar{F} \,.\, d\bar{r}$; where $\bar{F} = xy\,\bar{\imath} + yz\,\bar{\jmath} + xz\,\bar{k}$; C triangular path in the plane $x + y + z = 1$ with vertices at $(1,0,0), (0,1,0)$ and (0,0,1) in the first octant (5)

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Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION,DECEMBER 2018

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks. Marks

1 a) Test the convergence of $\sum_{k=1}^{\infty} \frac{\cos k}{k^2}$ (2)

b) Discuss the convergence of $\sum_{k=1}^{\infty} \frac{(2k)!}{4^k}$ (3)

2 a) Find the slope of the surface $z = \sin(y^2 - 4x)$ in the $x -$ direction at the point $(3,1)$. (2)

b) Find the differential dz of the function $z = \tan^{-1}(x^2 y)$. (3)

3 a) Find the direction in which the function $f(x,y) = xe^y$ decreases fastest at the point $(2,0)$. (2)

b) Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at $(1,1,3)$ (3)

4 a) Evaluate $\iint_R y \sin xy \; dA$, where $\boldsymbol{R} = [1,2] \times [0,\pi]$. (2)

b) Evaluate $\int_0^2 \int_0^1 \frac{x}{(1+xy)^2}$ dy dx (3)

5 a) if $\vec{A} = (3x^2 + 6y)\boldsymbol{i} - 14yz\boldsymbol{j} + 20xz^2\boldsymbol{k}$,evaluate $\int \vec{A}.d\vec{r}$ from (0,0,0) to(1,1,1) along the path,$x = t, y = t^2, z = t^3$ (2)

b) Prove that $\vec{F}=(x^2 - yz)\boldsymbol{i} + (y^2 - xz)\boldsymbol{j} + (z^2 - xy)\boldsymbol{k}$ is irrotational. (3)

6 a) Determine the source and sink of the vector field (2)

$\boldsymbol{F}(x,y,z) = 2(x^3 - 2x)\boldsymbol{i} + 2(y^3 - 2y)\boldsymbol{j} + 2(z^3 - 2z)\boldsymbol{k}$

b) Evaluate $\iint_S \bar{F}.\bar{n}ds$ where S is the surface of the cylinder $x^2 + y^2 = 4$, $z = 0$, $z = 3$ where $\bar{F} = (2x - y)\bar{\imath} + (2y - z)\bar{\jmath} + z^2\bar{k}$ (3)

PART B

Module 1

Answer any two questions, each carries 5 marks.

7 Check the convergence of the series $\frac{3}{4} + \frac{3.4}{4.6} + \frac{3.4.5}{4.6.8} + \frac{3.4.5.6}{4.6.8.10} + \cdots$ (5)

8 Find the radius of convergence of the power series $\sum_{k=1}^{\infty} \frac{(-1)^k (x-4)^k}{3^k}$ (5)

9 Determine whether the alternating series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{3^{2k-1}}{k^2+1}$ is absolutely (5)

convergent.

Module 1I

Answer any two questions, each carries 5 marks.

10 If $u = x^2 tan^{-1}\left(\frac{y}{x}\right) - y^2 tan^{-1}\left(\frac{x}{y}\right)$, find $\frac{\partial^2 u}{\partial x \partial y}$ (5)

11 Let $z = xye^{\frac{x}{y}}, x = r\,cos\theta,\; y = r\,sin\theta$, use chain rule to evaluate $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ at $r = 2$ and $\theta = \frac{\pi}{6}$ (5)

12 A rectangular box open at the top is to have volume $32m^3$. Find the dimensions of the box requiring least material for its construction. (5)

Module III

Answer any two questions, each carries 5 marks.

13 Suppose that a particle moves along a circular helix in 3-space so that its position vector at time **t** is **r**(t) = $4\cos \pi t\, \mathbf{i} + 4\sin \pi t\, \mathbf{j} + t\, \boldsymbol{k}$. Find the distance travelled and the displacement of the particle during the time interval $1 \le t \le 5$. (5)

14 Suppose that the position vector of a particle moving in a plane $\bar{r} = 12\sqrt{t}\, \boldsymbol{i} + t^{\frac{3}{2}} \boldsymbol{j}, t > 0$. Find the minimum speed of the particle and locate when it has minimum speed? (5)

15 Find the parametric equation of the tangent line to the curve $x = \cos t, y = \sin t, z = t$ where $t = t_0$ and use this result to find the parametric equation of the tangent line to the point where $t = \pi$. (5)

Module 1V

Answer any two questions, each carries 5 marks.

16 Evaluate $\iint_R y\, dA$ where R is the region in the first quadrant enclosed between the circle $x^2 + y^2 = 25$ and the line $x + y = 5$. (5)

17 $Evaluate \int_1^2 \int_0^x \frac{dy\, dx}{x^2 + y^2}$ (5)

18 Evaluate $\iiint_V x\, dxdydz$ where V is the volume of the tetrahedron bounded by the plane $x = 0, y = 0, z = 0\;\; x + y + z = a$. (5)

Module V

Answer any three questions, each carries 5 marks.

19 Find the scalar potential of $\vec{F} = (2xy + z^{3)}\boldsymbol{i} + x^2\boldsymbol{j} + 3xz^2\, \boldsymbol{k}$ (5)

20 Find the work done by $F(x, y) = (x^2 + y^2)\boldsymbol{i} - x\boldsymbol{j}$ along the curve $C: x^2 + y^2 = 1$ counter clockwise from $(1,0)$ to $(0,1)$. (5)

21 Evaluate $\int_C \bar{F} . d\bar{r}$ where $\bar{F} = y^2\boldsymbol{i} + xy\,\boldsymbol{j}$ and $\bar{r}(t) = t\boldsymbol{i} + 2t\boldsymbol{j}$, $1 \le t \le 3$. (5)

22 Evaluate $\int y\,dx + z\,dy + x\,dz$ along the path $x = \cos \pi t, y = \sin \pi t\,,\ z = t$ from $(1,0,0)$ to $(-1,0,1)$ (5)

23 If $\bar{r} = x\,\bar{\imath} + y\,\bar{\jmath} + z\,\bar{k}$ and $= \|\bar{r}\|$, prove that $\nabla^2 f(r) = \frac{2}{r} f'(r) + f''(r)$. (5)

Module VI

Answer any three questions, each carries 5 marks.

24 Using Stoke's theorem evaluate $\int_C \overline{F} . d\bar{r}$; where $\bar{F} = xy\,\bar{\imath} + yz\,\bar{\jmath} + xz\,\bar{k}$; C triangular path in the plane $x + y + z = 1$ with vertices at $(1,0,0), (0,1,0)$ and $(0,0,1)$ in the first octant (5)

25 Using Green's theorem evaluate $\int_C (y^2 - 7y)dx + (2xy + 2x)\,dy$ where C is the circle $x^2 + y^2 = 1$ (5)

26 Find the mass of the lamina that is the portion of the cone $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ if the density is $\emptyset(x, y, z) = x^2 z$. (5)

27 Use divergence theorem to find the outward flux of the vector field $F(x, y, z) = x^3 i + y^3 j + z^3 k$ across the surface σ bounded by $x^2 + y^2 = 4,\ z = 0$ and $z = 4$. (5)

28 If S is the surface of the sphere $x^2 + y^2 + z^2 = 1$,Evaluate

$$\iint_s (xi + 2yj + 3zk) . dS$$ (5)

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Reg No.:_______________ Name:___________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, JULY 2018

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks

			Marks
1	a)	Express the repeating decimal 1.454545….as a fraction.	(2)
	b)	Expand $f(x) = sin\pi x$ using Taylor series about $x = \frac{1}{2}$.	(3)
2	a)	Find the slope of the surface $z=\sqrt{3x+2y}$ in the x-direction at the point (1,3).	(2)
	b)	Find a linear approximation of $f(x,y,z)=x^2-xy+3\sin z$ at the point (2,1,0).	(3)
3	a)	Determine whether $\vec{r}(t)=3\sin t\hat{i}+3t\hat{j}$ is continuous at t = 0.	(2)
	b)	Find the gradient of $f(x,y)=x^2e^y$ at (1,0).	(3)
4	a)	Evaluate $\int_0^1\int_0^{\log y}\frac{dxdy}{\log y}$.	(2)
	b)	Using double integral find the area bounded by $x=y^2, x+y=2$ in the positive quadrant.	(3)
5	a)	Find the divergence of vector field $\bar{V}=x^2y^2\hat{i}+2xy\hat{j}$.	(2)
	b)	Show that the vector field $\bar{F}=2x(y^2+z^3)\hat{i}+2x^2y\hat{j}+3x^2z^2\hat{k}$ is conservative.	(3)
6	a)	Locate sources or sinks of $\bar{F}=xy\hat{i}-2xy\hat{j}+y^2\hat{k}$.	(2)
	b)	By Green's theorem evaluate $\oint_C (3x+y)dx+(2x+y)dy$, C is the circle $x^2+y^2=a^2$.	(3)

PART B

Module I

Answer any two questions, each carries 5 marks

7	Determine whether $\sum_{k=1}^{\infty}\frac{1}{(3k-1)(3k+2)}$ is convergent.	(5)
8	Test the convergence of $1+\frac{1.3}{3!}+\frac{1.3.5}{5!}+\frac{1.3.5.7}{7!}+........$	(5)
9	Expand $f(x)=\frac{1}{x}$ in powers of (x-2). Find the interval of convergence and radius of convergence of the power series obtained.	(5)

Module II

Answer any two questions, each carries 5 marks

10 If $W = f(x-y, y-z, z-x)$, Show that $\frac{\partial W}{\partial x}+\frac{\partial W}{\partial y}+\frac{\partial W}{\partial z}=0$. (5)

11 a) If $f(x,y) = y\sin x + e^{2x}\cos y$, find f_{yxx}. (3)

b) State the second partial test for local(relative)extreme values. (2)

12 An aquarium with rectangular sides and bottom (no top) is to hold 32 liters of water. Find its dimension so that least quantity of material is required for its construction. (5)

Module III

Answer any two questions, each carries 5 marks

13 Find the unit tangent $\hat{T}(t)$ and unit normal $\hat{N}(t)$ to the curve $\overline{r}(t) = e^t \cos t\hat{i} + e^t \sin t\hat{j} + e^t\hat{k}$ at t=0. (5)

14 Suppose the position vector of a moving particle is $\overline{r} = t^2\hat{i} + \frac{1}{3}t^3\hat{j}$, find the displacement and distance traveled over the time interval $2 \le t \le 4$. (5)

15 Find the equation of tangent plane to the surface $x^2 + y^2 + z^2 = 25$ at P(3,0,4).Also find the parametric equation for the normal line to the surface at P. (5)

Module IV

Answer any two questions, each carries 5 marks

16 Evaluate $\iint\limits_R e^{x^2}dxdy$, where the region R is given by $2y \le x \le 2$ and $0 \le y \le 1$. (5)

17 Evaluate $\iint r\sin\theta drd\theta$ over the cardioid $r = a(1-\cos\theta)$ above the initial line. (5)

18 Evaluate the triple integral $\iiint xy\sin(yz)dV$ over the rectangular box $0 \le x \le \frac{\pi}{2}, 0 \le y \le 1, 0 \le z \le \frac{\pi}{6}$. (5)

Module V

Answer any three questions, each carries 5 marks

19 Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$ where $r = \sqrt{x^2+y^2+z^2}$. (5)

20 Evaluate $\int\limits_C ydx + zdy + xdz$ along the twisted cubic $x = t, y = t^2, z = t^3$ from (0,0,0) to (1,1,1). (5)

21 Find the potential function for the vector field $\overline{F} = (\sin z + y\cos x)\hat{i} + (\sin x + z\cos y)\hat{j} + (\sin y + x\cos z)\hat{k}$. (5)

22 Find the work done in moving a particle in the force field $\overline{F} = (x+y)\hat{i} - x^2\hat{j} + (y+z)\hat{k}$ along the curve defined by $x^2 = 4y, z = x, 0 \le x \le 2$. (5)

23 Show that $\int\limits_C (yz-1)dx + (z+xz+z^2)dy + (y+xy+2yz)dz$ is independent of the path of integration. Find the scalar potential and the value of the integral from (1,2,2) to (2,3,4). (5)

Module VI

Answer any three questions, each carries 5 marks

24 Use Green's theorem to evaluate $\int_C x^2 y dx + x dy$ where C is the boundary of triangular region enclosed by $y = 0, y = 2x, x = 1$. (5)

25 Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ using Green's theorem. (5)

26 Evaluate the surface integral $\iint_\sigma xy dS$ where σ is the portion of the plane $x + y + z = 2$ lying in the first octant. (5)

27 Using divergence theorem evaluate $\iint_S \vec{F}.\hat{n} dS$ where $\vec{F} = 2x\hat{i} + 4y\hat{j} - 3z\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. (5)

28 Use Stokes' theorem to calculate the circulation around the bounding circle $x^2 + y^2 = 9$ and the field $\bar{A} = y\, i - x\, j$ where S is the disk of radius 3 centered at origin in the XY-plane. (5)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100　　Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks.

Marks

1 a) Determine whether the series $\sum_{k=0}^{\infty}\frac{5}{4^k}$ converges. If so, find the sum (2)

b) Examine the convergence of $\sum(\frac{k}{k+1})^{k^2}$ (3)

2 a) Find the slope of the surface $z = x\,e^{-y} + 5y$ in the y direction at the point (4, 0) (2)

b) Show the function $f(x,y) = e^x \sin y + e^y \cos x$ satisfies the Laplace's equation $f_{xx} + f_{yy} = 0$ (3)

3 a) Find the directional derivative of $f(x,y,z) = x^3z - yx^2 + z^2$at P (2, -1, 1) in the direction of 3 $\vec{\imath}$- $\boldsymbol{j}$ +2**k** (2)

b) Find the unit tangent vector and unit normal vector to the curve (3)

$$\boldsymbol{r}(t) = 4\cos t\,\boldsymbol{i} + 4\sin t\,\boldsymbol{j} + t\,\boldsymbol{k}\ at\ t = \frac{\pi}{2}$$

4 a) Using double integration, evaluate the area enclosed by the lines (2)

$$x = 0,\quad y = 0, \frac{x}{a}+\frac{y}{b} = 1$$

b) Evaluate $\int_{-1}^{2}\int_{0}^{2}\int_{0}^{1}(x^2+y^2+z^2)dx\,dy\,dz$ (3)

5 a) If $\boldsymbol{F}(x,y,z) = x^2\boldsymbol{i} - 3\boldsymbol{j} + yz^2\boldsymbol{k}$ find div $\boldsymbol{F}$ (2)

b) Find the work done by the force field $\boldsymbol{F} = xy\,\boldsymbol{i} + yz\,\boldsymbol{j} + zx\,\boldsymbol{k}$ on a particle that moves along the curve C: $x = t, y = t^2, z = t^3, 0 \le t \le 1$ (3)

6 a) Use Green's theorem to evaluate $\int_c(xdy - ydx)$, where c is the circle $x^2 + y^2 = a^2$ (2)

b) If S is any closed surface enclosing a volume V and $\boldsymbol{F} = x\boldsymbol{i} + 2y\boldsymbol{j} + 3\,z\boldsymbol{k}$ show that $\iint_S \mathrm{F.n}\,ds = 6V$ (3)

PART B

Module I

Answer any two questions, each carries 5 marks.

7 Determine whether the alternating series $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k+7}{k(k+4)}$ is absolutely convergent (5)

8 Find the Taylor series expansion of $f(x) = \frac{1}{x+2}$ about $x = 1$ (5)

9 Find the interval of convergence and radius of convergence of $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{(x+1)^k}{k}$ (5)

Module II

Answer any two questions, each carries 5 marks.

10 Find the local linear approximation L to the function$f(x,y,z) = xyz$ at the pointP (1,2,3). Also compare the error in approximating f by L at the point Q (1.001, 2.002, 3.003) with the distance PQ. (5)

11 Locate all relative extrema and saddle points of $f(x,y) = 2xy - x^3 - y^2$ (5)

12 If $u = f(\frac{x}{y},\frac{y}{z},\frac{z}{x})$prove that$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ (5)

Module III

Answer any two questions, each carries 5 marks.

13 Write the parametric equations of the tangent line to the graph of $\boldsymbol{r}(t) = ln\, t\, i + e^{-t}\boldsymbol{j} + t^4\boldsymbol{k}$ at $t = 2$ (5)

14 A particle moves along the curve $\boldsymbol{r} = (t^3 - 4t)\,\boldsymbol{i} + (t^2 + 4t)\,j + (8t^2 - 3t^3)\boldsymbol{k}$ where t denotes time. Find (5)

(i) the scalar tangential and normal components of acceleration at time t = 2

(ii) the vector tangential and normal components of acceleration at time t = 2

15 Find the equation to the tangent plane and parametric equations of the normal line to the ellipsoid $x^2 + y^2 + 4z^2 = 12$ a t the point (2, 2, 1) (5)

Module IV

Answer any two questions, each carries 5 marks.

16 Reverse the order of integration and evaluate $\int_0^1\int_x^1 \frac{x}{x^2+y^2}dydx$ (5)

17 If R is the region bounded by the parabolas $y = x^2 and\, y^2 = x$ in the first quadrant, evaluate $\iint_R (x+y)dA$ (5)

18 Use triple integral to find the volume of the solid bounded by the surface $y = x^2$ and the planes $y + z = 4, z = 0$. (5)

Module V

Answer any three questions, each carries 5 marks.

19 If $\boldsymbol{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$ and $r = ||\boldsymbol{r}||$, show that $\nabla log\, r = \frac{\boldsymbol{r}}{r^2}$ (5)

20 Examine whether $\boldsymbol{F} = (x^2 - yz)\boldsymbol{i} + (y^2 - zx)\boldsymbol{j} + (z^2 - xy)\boldsymbol{k}$ is a conservative field. If so, find the potential function (5)

21 Show that $\nabla^2 f(r) = 2\frac{f'(r)}{r} + f''(r)$, where $\boldsymbol{r} = x\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$, $r = ||\boldsymbol{r}||$ (5)

22 Compute the line integral $\int_c (y^2dx - x^2dy)$ along the triangle whose vertices are (1, 0), (0,1)and (-1, 0) (5)

23 Show that the line integral $\int_c (y\sin x dx - \cos x dy)$ is independent of the path and hence evaluate it from (0, 1) and (π, - 1) (5)

Module VI

Answer any three questions, each carries 5 marks.

24 Using Green's theorem, find the work done by the force field $\vec{f}(x,y) = (e^x - y^3)\vec{i} + (\cos y + x^3)\vec{j}$ on a particle that travels once around the unit circle $x^2 + y^2 = 1$ in the counter clockwise direction. (5)

25 Using Green's theorem evaluate $\int_c (xy + y^2)dx + x^2dy$, where c is the boundary of the area common to the curve $y = x^2 and\, y = x$ (5)

26 Evaluate the surface integral $\iint_S xzds$ **,**where S is the part of the plane $x + y + z = 1$ that lies in the first octant (5)

27 Using divergence theorem, evaluate $\iint_S \mathsf{F.n}\,ds$ where $\boldsymbol{F} = (x^2 + y)\,\boldsymbol{i} + z^2\,\boldsymbol{j} + (e^y - z)\,\boldsymbol{k}$ and S is the surface of the rectangular solid bounded by the co ordinate planes and the planes x = 3, y = 1, z = 3 (5)

28 Apply Stokes's theorem to evaluate $\int_c F.dr$, where $\boldsymbol{F} = (x^2 - y^2)\boldsymbol{i} + 2xy\boldsymbol{j}$ and c is the rectangle in the xy plane bounded by the lines $x = 0, y = 0, x = a\; and\; y = b$ (5)

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries5 marks. Marks

1	a)	Test the convergence of the series $\sum_{k=1}^{\infty}\frac{1}{\sqrt[3]{2k-1}}$.	(2)
	b)	Find the radius of convergence of $\sum_{n=1}^{\infty}\frac{x^n}{2n+3}$.	(3)
2	a)	Find the Slope of the surface $z = xe^{-y} + 5y$ in the y-direction at the point (4,0).	(2)
	b)	Find the derivative of $z = \sqrt{1+x-2xy^4}$ with respect to t along the path $x = \log t,\ y = 2t$.	(3)
3	a)	Find the directional derivative of $f = x^2y - yz^3 + z$ at $(-1,2,0)$ in the direction of $a = 2i + j + 2k$.	(2)
	b)	Find the unit tangent vector and unit normal vector to $r(t) = 4\cos t i + 4\sin t j + tk$ at $t = \frac{\pi}{2}$.	(3)
4	a)	Evaluate $\int_0^{\log 3}\int_0^{\log 2} e^{x+2y}\,dydx$.	(2)
	b)	Evaluate $\iint_R xy\,dA$,where R is the region bounded by the curves $y = x^2$ and $x = y^2$.	(3)
5	(a)	Find the divergence and curl of the vector $F(x,y,z) = yz\,i + xy^2\,j + yz^2k$.	(2)
	(b)	Evaluate $\int_C (3x^2 + y^2)dx + 2xy\,dy$ along the circular arc C given by $x = \cos t, y = \sin t$ for $0 \le t \le \frac{\pi}{2}$.	(3)
6	(a)	Use line integral to evaluate the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.	(2)
	(b)	Evaluate $\int_C (x^2 - 3y)dx + 3xdy$,where C is the circle $x^2 + y^2 = 4$.	(3)

PART B

Module 1

Answer any two questions, each carries 5 marks.

7	Test the convergence or divergence of the series $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}$.	(5)

8 Test the absolute convergence of $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{(2k)!}{(3k-2)!}$. (5)

9 Find the Taylor series for $\frac{1}{1+x}$ at $x=2$. (5)

Module 1I

Answer any two questions, each carries 5 marks.

10 Find the local linear approximation L to $f(x,y)=\log(xy)$ at P(1,2) and compare the error in approximating f by L at Q(1.01, 2.01) with the distance between P and Q. (5)

11 Let $w=4x^2+4y^2+z^2, x=\rho\sin\phi\cos\theta, y=\rho\sin\phi\sin\theta, z=\rho\cos\phi$. Find $\frac{\partial w}{\partial\rho},\frac{\partial w}{\partial\phi}$ and $\frac{\partial w}{\partial\theta}$. (5)

12 Locate all relative extrema and saddle points of $f(x,y)=4xy-x^4-y^4$. (5)

Module 1II

Answer any two questions, each carries 5 marks.

13 Find the equation of the tangent plane and parametric equation for the normal line to the surface $x^2+y^2+z^2=25$ at the point $(3,0,4)$. (5)

14 A particle is moving along the curve $r(t)=(t^3-2t)i+(t^2-4)j$ where t denotes the time. Find the scalar tangential and normal components of acceleration at $t=1$. Also find the vector tangential and normal components of acceleration at $t=1$. (5)

15 The graphs of $r_1(t)=t^2i+tj+3t^3k$ and $r_2(t)=(t-1)i+\frac{1}{4}t^2j+(5-t)k$ are intersect at the point $P(1,1,3)$ .Find, to the nearest degree, the acute angle between the tangent lines to the graphs of $r_1(t)\,\&\,r_2(t)$ at the point $P(1,1,3)$. (5)

Module 1V

Answer any two questions, each carries5 marks.

16 Change the order of integration and evaluate $\int_0^1\int_{4x}^4 e^{-y^2}dydx$. (5)

17 Use triple integral to find the volume bounded by the cylinder $x^2+y^2=9$ and between the planes $z=1$ and $x+z=5$. (5)

18 Find the area of the region enclosed between the parabola $y=\frac{x^2}{2}$ and the line $y=2x$. (5)

Module V

Answer any three questions, each carries5 marks.

19 Determine whether $F(x,y)=(\cos y+y\cos x)i+(\sin x-x\sin y)j$ is a conservative vector field. If so find the potential function for it. (5)

20 Show that the integral $\int_{(1,1)}^{(3,3)}(e^x\log y-\frac{e^y}{x})dx+(\frac{e^x}{y}-e^y\log x)dy$,where x and y are positive is independent of the path and find its value. (5)

21 Find the work done by the force field $F(x,y,z)=xy\,i+yz\,j+xz\,k$ on a particle that moves along the curve $C: r(t)=ti+t^2j+t^3k\,(0\le t\le 1)$. (5)

22 Let $\bar{r} = xi + yj + zk$ and $r = \|\bar{r}\|$, let f be a differentiable function of one variable, then show that $\nabla f(r) = \frac{f'(r)}{r}\bar{r}.$ (5)

23 Find $\nabla.(\nabla \times F)$ and $\nabla \times (\nabla \times F)$ where $F(x,y,z) = e^{xz}i + 4xe^{y}j - e^{yz}k.$ (5)

Module VI

Answer any three questions, each carries5 marks.

24 Use Green's Theorem to evaluate $\int_C \log(1+y)dx - \frac{xy}{(1+y)}dy$, where C is the triangle with vertices (0,0), (2,0) and (0,4). (5)

25 Evaluate the surface integral $\iint_\sigma xzds$, where σ is the part of the plane $x+y+z=1$ that lies in the first octant. (5)

26 Using Stoke's Theoremevaluate $\int_C F.dr$ where $F(x,y,z) = xzi + 4x^2y^2j + yxk$, C is the rectangle $0 \le x \le 1, 0 \le y \le 3$ in the plane $z = y$. (5)

27 Using Divergence Theorem evaluate $\iint_\sigma \overline{F}.n\,ds$ where $F(x,y,z) = x^3i + y^3j + z^3k$, σ is the surface of the cylindrical solid bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 4$. (5)

28 Determine whether the vector fields are free of sources and sinks. If it is not, locate them (5)

(i) $(y+z)i - xz^3j + x^2 \sin y\,k$ (ii) $xy\,i - 2xy\,j + y^2k$

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Reg. No.______________ Name:______________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, JUNE/JULY 2017

Course Code: **MA 101**

Course Name: **CALCULUS**

(For 2015 Admission and 2016 Admission)

Max. Marks :100 Duration: 3 hours

PART A

Answer all questions. Each question carries 5 marks.

1. (a) Find the interval of convergence and radius of convergence of the infinite series $\sum_{n=0}^{\infty} n!\,x^n$ **(2)**

   (b) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{5n-1}$ converges or not **(3)**

2. (a) Find the slope of the surface $z = \sqrt{3x+2y}$ in the y-direction at the point (4,2) **(2)**

   (b) Find the derivative of $w = x^2 + y^2$ with respect to t along the path $x = at^2, y = 2at$ **(3)**

3. (a) Find the directional derivative of $f(x,y) = xe^y$ at (1,1) in the direction of the vector $i - j$ **(2)**

   (b) If $\vec{F}(t)$has a constant direction, then prove that $\vec{F} \times \frac{d\vec{F}}{dt} = 0$ **(3)**

4. (a) Evaluate $\int_0^1 \int_0^1 \frac{1}{\sqrt{(1-x^2)(1-y^2)}}\,dxdy$ **(2)**

   (b) Evaluate $\iint_R \frac{\sin x}{x}\,dxdy$ where R is the triangaular region bounded by the x-axis, $y = x$ and $x = 1$. **(3)**

5. (a) Show that $\int_A^B (2xy + z^3)\,dx + x^2dy + 3xz^2dz$ is independent of the path joining the points A and B. **(2)**

   (b) If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, then prove that $\nabla^2 r^n = n(n+1)r^{n-2}$ **(3)**

6. (a) Using line integral evaluate the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **(2)**

   (b)Evaluate $\int_C (e^x\,dx + 2y\,dy - dz)$ where C is the curve $x^2 + y^2 = 4, z = 2$. **(3)**

PART B

Answer any two questions each Module I to IV

Module I

7. Determine whether the series converge or diverge $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$ **(5)**

8. Check the absolute convergence or divergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{(2n-1)!}{3^n}$ **(5)**

9. Find the Taylor series expansion of $\log \cos x$ about the point $\frac{\pi}{3}$ **(5)**

Module II

10. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, Show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$ **(5)**

11. The length, width and height of a rectangular box are measured with an error of atmost 5%. Use a total differential to estimate the maximum percentage error that results if these quantities are used to calculate the diagonal of the box. **(5)**

12. Locate all relative extrema and saddle points of $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ **(5)**

Module III

13. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$ **(5)**

14. Let $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, then prove that $\nabla f(r) = \frac{f'(r)}{r}\vec{r}$. **(5)**

15. Find an equation of the tangent plane to the ellipsoid $2x^2 + 3y^2 + z^2 = 9$ at the point $(2,1,1)$ and determine the acute angle that this plane makes with the XY plane. **(5)**

Module IV

16. Change the order of integration and hence evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$ **(5)**

17. Evaluate $\int_0^2 \int_0^{\sqrt{(4-x^2)}} y(x^2 + y^2)\, dx\, dy$ using polar co-ordinates **(5)**

18. Find the volume of the paraboloid of revolution $x^2 + y^2 = 4z$ cut off by the plane $z = 4$ **(5)**

Module V

Answer any 3 questions.

19. Evaluate the line integral $\int_C (xy + z^3)\, ds$ from $(1,0,0)$ to $(-1,0,\pi)$ along the helix C that is represented by the parametric equations $x = \cos t,\ y = \sin t,\ z = t$ **(5)**

20. Evaluate the line integral $\int_C (y - x)\, dx + x^2 y\, dy$ along the curve C, $y^2 = x^3$ from (1, -1) to (1,1) **(5)**

21. Find the work done by the force field $\vec{F} = (x + y)i + xyj - z^2k$ along the line segment from $(0,0,0)$ to $(1,3,1)$ and then to $(2, -1, 5)$. **(5)**

22. Show that $\vec{F} = (2xy + z^3)i + x^2j + 3xz^2k$ is a conservative vector field. Also find its scalar potential. **(5)**

23. Find the values of constants a, b, c so that $\vec{F} = (axy + bz^3)i + (3x^2 - cz)j + (3xz^2 - y)k$ may be irrotational. For these values of a, b, c find the scalar potential of $\vec{F}$ **(5)**

Module VI

Answer any 3 questions.

24. Verify Green's theorem for $\int_C (xy + y^2)dx + x^2\,dy$ where C is bounded by $y = x$ and $y = x^2$ **(5)**

25. Apply Green's theorem to evaluate $\int_C (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$ **(5)**

26. Apply Stokes theorem to evaluate $\int_C (x + y)dx + (2x - y)dy + (y + z)dz$ where C is the boundary of the triangle with vertices (0,0,0), (2,0,0) and (0,3,0) **(5)**

27. Use Divergence theorem to evaluate $\iint_S \vec{F}\cdot\vec{n}dS$ where $\vec{F} = xi + zj + yzk$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$.Also verify this result by computing the surface integral over S **(5)**

28. State Divergence theorem. Also evaluate $\iint_S \vec{F}\cdot\vec{n}dS$ where $\vec{F} = axi + byj + czk$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ **(5)**

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A B1A003 Pages:3

(2016 ADMISSIONS)

Reg. No:...................... Name:...............................

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, FEBRUARY 2017

MA101: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer All Questions and each carries 5 marks)

1. a) Test the convergence of $\sum_{k=1}^{\infty} \frac{99^k}{k!}$ (2)

   b) Test the convergence of $\sum_{k=1}^{\infty} \frac{1}{3^k+1}$. (3)

2. a) Find the slope of the sphere $x^2+y^2+z^2=1$ in the y- direction at $(\frac{2}{3},\frac{1}{3},\frac{-2}{3})$. (2)

   b) Find the critical points of the function $f(x,y) = 2xy-x^3-y^3$. (3)

3. a) Find the velocity at time $t=\pi$ of a particle moving along the curve

   $\vec{r}(t) = e^t \sin t\, i + e^t \cos t\, j + t\, k$. (2)

   b) Find the directional derivative of $f(x,y) = xe^y-ye^x$ at the point P(0,0) in the direction of $5i - 2j$. (3)

4. a) Change the order of integration in $\int_0^1 \int_{y^2}^{\sqrt{2-y^2}} f(x,y)dxdy$. (3)

   b) Find the area of the region enclosed by $y= x^2$ and $y = x$. (2)

5. a) Find the divergence of the vector field $f(x,y,z) = x^2y\, i + 2y^3z\, j+ 3z\, k$. (2)

   b) Find the work done by $\overrightarrow{F} = xy\, i + x^3 j$ on a particle that moves along the

   curve $y^2=x$ from (0,0) to (0,1). (3)

6. a) Using Green's theorem to evaluate $\int_C 2xy\ dx + (x^2+x)\, dy$ where C is the triangle with vertices (0,0), (1,0) and (1,1). (2)

   b) Use Stoke's theorem to evaluate $\int_C \vec{F}.\, dr$ where $\vec{F} = (x-2y)i + (y-z)\, j + (z-x)\, k$ and C is the circle $x^2+y^2 = a^2$ in the xy plane with counter clockwise orientation looking down the positive z- axis. (3)

PART B

MODULE I *(Answer Any Two Questions)*

7. a) Test the convergence of the following series (5)

i) $\sum_{k=1}^{\infty} \frac{(k+4)!}{4!k!\,4^k}$ ii) $\sum_{k=2}^{\infty} \left(\frac{4k-5}{2k+1}\right)^k$

8. Use the alternating series test to show that the series $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{(k+3)}{k(k+1)}$ converge. (5)

9. Find the Taylor's series of f(x) = x sinx about the point $x=\frac{\pi}{2}$. (5)

MODULE II *(Answer Any Two Questions)*

10. Find the local linear approximation L to f(x,y) = ln (xy) at P(1,2) and compare the error in approximating f by L at Q(1.01, 2.01) with the distance between P and Q. (5)

11. Show that the function $f(x,y) = 2\tan^{-1}(y/x)$ satisfies the Laplace's equation

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0.$$ (5)

12. Find the relative minima of $f(x,y) = 3x^2-2xy+y^2-8y$. (5)

MODULE III *(Answer Any Two Questions)*

13. Find the unit tangent vector and unit normal vector to $\vec{r} = 4\cos t\,i + 4\sin t\,j + t\,k$ at $t=\frac{\pi}{2}$. (5)

14. Suppose a particle moves through 3- space so that its position vector at time t is $\vec{r} = t\,i + t^2 j + t^3 k$. Find the scalar tangential component of acceleration at the time t=1. (5)

15. Given that the directional derivative of f(x,y,z) at (3,-2, 1) in the direction of $2i - j - 2k$ is -5 and that $\|\nabla f(3,-2,1)\| = 5$. Find $\nabla f(3,-2,1)$. (5)

MODULE IV *(Answer Any Two Questions)*

16. Evaluate the integral $\int_0^4 \int_{\sqrt{y}}^2 e^{x^3}\,dx\,dy$ by reversing the order of integration. (5)

17. Evaluate $\int_0^1 \int_0^{y^2} \int_{-1}^{z} z\,dx\,dy.$ (5)

18. Find the volume of the solid in the first octant bounded by the co-ordinate planes and the plane x+2y+z = 6. (5)

MODULE V *(Answer Any Three Questions)*

19. Let $\vec{r} = xi + yj + zk$ and let $r = \|\vec{r}\|$ and f be a differentiable function of one variable show that $\nabla f(r) = \frac{f'(r)}{r}\vec{r}$. (5)

20. Evaluate the line integral $\int_C [-y\,dx + x\,dy]$ along $y^2 = 3x$ from (3,3) to (0,0). (5)

21. Show that $\overrightarrow{F}(x,y) = (\cos y + y\cos x)\,i + (\sin x - x\sin y)\,j$ is a conservative vector field. Hence find a potential function for it. (5)

22. Show that the integral $\int_C (3x^2e^y\,dx + x^3e^y\,dy)$ is independent of the path and hence evaluate the integral from (0,0) to (3,2). (5)

23. Find the work done by the force field $\overrightarrow{F} = xy\,i + yz\,j + xz\,k$ on a particle that moves along the curve C: $\vec{r}(t) = t\,i + t^2\,j + t^3\,k$ where $0 \le t \le 1$. (5)

MODULE VI *(Answer Any Three Questions)*

24. Use Green's theorem to evaluate the integral $\int_C (x\cos y\,dx - y\sin x\,dy)$ where C is the square with vertices (0,0), $(\pi,0)$, (π,π) and $(0,\pi)$. (5)

25. Evaluate the surface integral $\int_\sigma \int z^2\,ds$ where σ is the portion of the cone $z = \sqrt{x^2 + y^2}$ between the planes z = 1 and z = 3. (5)

26. Use divergence theorem to find the outward flux of the vector field $\overrightarrow{F} = 2x\,i + 3y\,j + z^2\,k$ across the unit cube x = 0, x = 1, y=0, y = 1, z = 0 and z = 1. (5)

27. Use Stoke's theorem to evaluate the integral $\int_C \overrightarrow{F}.d\vec{r}$ where $\overrightarrow{F} = (x-y)\,i + (y-z)\,j + (z-x)k$ and C is the boundary of the portion of the plane x+y+z = 1 in the first octant. (5)

28. Use Stoke's theorem to evaluate the integral $\int_C \overrightarrow{F}.d\vec{r}$ where $\overrightarrow{F} = 2z\,i + 3x\,j + 5y\,k$ and C is the boundary of the paraboloid $x^2+y^2+z = 4$ for which $z \ge 0$ and C is positively oriented. (5)

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A **B1A216S (2015 Admission)** **Pages:2**

Reg.No...................... Name............................

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE (SUPPLEMENTARY) EXAMINATION, FEBRUARY 2017 **(2015 ADMISSION)**

Course Code: **MA 101**

Course Name: **CALCULUS**

Max.Marks : 100 Duration : 3 Hours

PART A

(Answer all questions. Each question carries 3 marks)

1) Show that the series $\sum_{n=1}^{\alpha}\left(\frac{1}{2}\right)^{n}$ converges.

2) Classify the surface $z = (x-1)^2 + (y+2)^2 + 3$

3) Find the Maclaurin series for $\cos x$

4) Evaluate $\underset{(x,y)\to(-1,2)}{Lt}\frac{xy}{x^2+y^2}$

5) Convert the cylindrical co-ordinate into rectangular co ordinate of $(4, \pi/3 - 3)$.

6) Find the slope of the surface $z = xy^2$ in the x direction at the point (2,3).

7) Find the directional derivative of $f = x^2y - yz^3 + z$ at (1,-2,0) in the direction of

$$\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$$

8) Find the unit normal to the surface $xy + xz + yz = c$ at (-1,2,3)

9) Evaluate $\int_1^a\int_1^b x^2 y\, dxdy$

10) Find the area of the region R enclosed by $y=1, y=2, x=0, x=y$.

PART B

(Answer any 2 questions. Each question carries 7 marks)

11) Test the absolute convergence of $\sum_{n=1}^{\alpha}\frac{(-1)^n n^4}{4^n}$

12) Determine the Taylor's series expansion of f(x) = $\sin x$ at x = $\pi/2$.

13) Test the convergence of $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}$........

(Answer any 2 questions. Each question carries 7 marks)

14) Find the equation of the paraboloid $z = x^2 + y^2$ in the cylindrical and spherical coordinates.

15) Find $F(f(x),g(y),h(z))$ if $F(x,y,z)=y\,e^{xyz}$, $f(x)=x^2, g(y)=y+1, h(z)=2z^2$

16) By converting into polar coordinate evaluate $\underset{(x,y)\to(0,0)}{Lt} \sqrt{x^2+y^2}\ln\left(\left(x^2+y^2\right)^2\right)$

(Answer any 2 questions. Each question carries 7 marks)

17) Find the local linear approximation L of $f(x,y,z) = xyz$ at the point P(1,2,3). Compare the error in approximating f by L at the point Q(1.001, 2.002, 3.003) with the distance PQ.

18) Find the relative extrema of $f(x,y) = 3x^2 - 2xy + y^2 - 8y$

19) If f is a differentiable function of three variables and suppose that

$w = f(x-y, y-z, z-x)$ Show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$

(Answer any 2 questions. Each question carries 7 marks)

20) Suppose that a particle moves along a curve in 3-space so that its position vector at time t is $r(t) = 4\cos \pi t\, i + 4\sin \pi t\, j + t\, k$. Find the distance travelled and the displacement of the particle during the time interval $1 \le t \le 5$

21) A particle is moving along the curve, $\vec{r} = (t^3 - 2t)\vec{i} + (t^2 - 4)\vec{j}$ where t denotes the time. Find the scalar tangential and normal components of acceleration at t = 1. Also find the vector tangential and normal components of acceleration at t = 0.

22) Find the arc length of the parametric curve $x = 5\cos t,\ y = 5\sin t,\ z = 2t\ ;\ 0 \le t \le \pi$

(Answer any 2 questions. Each question carries 7 marks)

23) Evaluate the integral by converting into polar co ordinates $\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2+y^2)\, dy\, dx$

24) Using triple integral to find the volume bounded by the cylinder

$x^2 + y^2 = 4\ and\ \ the\ \ planes\ \ z=0\ \ and\ \ y+z=3$

25) Change the order of integration and evaluate $\int_0^1 \int_x^1 \frac{x}{x^2+y^2}\, dx\, dy$

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A **B1A215S (2015 Admission)**

Reg No.................................. Name:..

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2016

(2015 ADMISSION)

Course Code: MA 101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer all questions. Each question carries 3 marks)

1 Find the derivative of $y = (1 + x\cosh^{-1} x)^2$

2 Test the convergence of $\sum_{n=1}^{\alpha} \left(\frac{n}{n+1}\right)^{n^2}$

3 Classify the surface $4x^2 + 4y^2 + z^2 + 8y - 4z = 4$

4 Convert the rectangular co-ordinate into spherical co-ordinate of $(2, 2\sqrt{3}, -4)$

5 Prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ where $f = x^2 y$.

6 Find the velocity, acceleration and speed of a particle moving along the curve

$x = 1 + 3t,\ y = 3 - 4t,\ z = 1 + 3t\ \ at = 2$

7 Given $z = e^{xy}$, $x = 2u + v$, $y = \frac{u}{v}$ Find $\frac{\partial z}{\partial u}$ *and* $\frac{\partial z}{\partial v}$

8 Find the unit tangent vector and unit normal vector to the curve

$x = e^t \cos t,\ y = e^t \sin t,\ z = e^t$ at $t = 0$.

9 Evaluate $\int_0^3 \int_0^{\sqrt{9-y^2}} 2y\,dx\,dy$

10 Find the area of the region R enclosed between the parabola $y = \frac{x^2}{2}$ and the line $y = 2x$

(10*3=30 Marks)

PART B

(Answer any 2 questions each question carries 7 marks)

11 Find the radius of curvature and interval of curvature of $\sum_{n=1}^{\alpha} \frac{x^n}{2n+3}$

12 Test the convergence of $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + - - - - - -$

13 Determine the Taylor's series expansion of f(x) = sinx at x = π/4.

(Answer any 2 questions each question carries 7 marks)

14 Find the nature of domain of the following function

1. $f(x,y) = \sqrt{x^2 - y^2}$
2. $f(x,y) = \ln(x^2 - y)$

15 Show that the function $f(x,y) = \frac{x^3 y}{2x^6 + y^2}$ approaches zero as $(x,y) \to (0,0)$ along the line y = mx.

16 Find the trace of the surface $x^2 + y^2 - z^2 = 0$ in the plane x = 2 and y = 1.

$x^2 + y^2 - z^2 = 0$

(Answer any 2 questions each question carries 7 marks)

17 Find the local linear approximation of $f(x,y) = \sqrt{(x^2 + y^2)}$ at (3,4) and compare the error in approximation by L(3.04,3.98) with the distance between the points.

18 Find the relative extrema of $f(x,y) = 3x^2 - 2xy + y^2 - 8y$

19 If $z = e^{xy}, x = 2u + v, \; y = \frac{u}{v}$ Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$

(Answer any 2 questions each question carries 7 marks)

20 If $r(t) = e^t i + e^{-2t} j + tk$

1) Find the scalar tangential and normal component of acceleration at t = 0

2) Find the vector tangential and normal component of acceleration at t = 0.

21 Find the equation of the tangent plane and parametric equations of the normal

line to the surface $z = 4x^3y^2 + 2y - 2$ at the point P (1, -2, 10).

22 Find the directional derivative of $f = x^2y - yz^3 + z$ at (1,-2,0) in the direction of $\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$

(Answer any 2 questions each question carries 7 marks)

23 Evaluate $\iint_R y\,dA$ where R is the region in the first quadrant enclosed between the circle $x^2 + y^2 = 25$ and the line x+y=5

24 Change the order of integration and evaluate $\int_1^2 \int_y^{y^2} y^2\,dx\,dy$

25 Find the volume bounded by the cylinder $x^2 + y^2 = 4$ the planes $y + z = 3$ and $z = 0$.

A **B1A004** **(2016)**

Reg. No.:.................... Name...............................

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, DEC 2016
(2016 ADMISSION)

Course Code: MA 101
Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer ALL questions

1 (a) Determine whether the series $\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^{k+2}$ converges and if so, find its sum. (2)

(b) Find the Maclaurin series for the function xe^x (3)

2 (a) If $= x^y$, then find $\frac{\partial^2 z}{\partial x \partial y}$ (2)

(b) Compute the differential dz of the function $z = tan^{-1}(xy)$. (3)

3 (a) Find the domain of $r(t) = \langle\sqrt{5t+1}\,, t^2\rangle, t_0 = 1$ and $r(t_0)$ (2)

(b) Find the directional derivative of $f(x,y) = e^{2xy}\ at\ P\,(5,0\,)$, in the direction of $u = -\frac{3}{5}i + \frac{4}{5}j$ (3)

4 (a) Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{1-x^2}\sqrt{1-y^2}}$ (2)

(b) Use double integration to find the area of the plane region enclosed by the given curves $y = \sin x\ \ and\ \ y = cosx$ for $0 \le x \le \frac{\pi}{4}$ (3)

5 (a) Confirm that $\varphi(x,y,z) = x^2 - 3y^2 + 4z^3$ is a potential function for $F(x,y,z) = 2xi - 6yj + 12\ z^2\,k$. (2)

(b) Evaluate $\int F.dr$ where $F(x,y) = sinx\ i + \cos x\ j$ where C is the curve $r(t) = \pi i + tj,\ 0 \le t \le 2$ (3)

6 (a) Using Green's theorem evaluate $\oint ydx + xdy$, where C is the unit circle oriented counter clockwise. (2)

(b) If σ is any closed surface enclosing a volume V and $= 2xi + 2yj + 3zk$, Using Divergence theorem show that $\int_\sigma \int F.n\, dS = 7V$ (3)

PART B

(Each question carries 5 Marks)

Answer any TWO questions

7 Test the nature of the series $\sum_{k=1}^{\infty} \frac{4k^3-6k+5}{8k^7+k-8}$

8 Check whether the series $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^k}{k!}$ is absolutely convergent or not.

9 Find the radius of convergence and interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x-5)^k}{5^k}$

Answer any TWO questions

10 If $u = f(y-z, z-x, x-y)$, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = 0$

11 A function $f(x,y) = x^2+y^2$; is given with a local linear approximation $L(x,y) = 2x+4y-5$ to $f(x,y)$at a point P. Determine the point P.

12 Find the absolute extrema of the function $f(x,y) = xy-4x$ on R where R is the triangular region with vertices (0,0) (0,4) and (4,0).

Answer any TWO questions

13 Evaluate the definite integral $\int_0^1 (e^{2t}i + e^{-t}j + 2\sqrt{t}\,k)dt$.

14 Find the velocity, acceleration, speed, scalar tangential and normal components of acceleration at the given t of
$r(t) = 3\,sint\;i + 2cost\;j - sin2t\;k\;; t = \frac{\pi}{2}$

15 Find the equation of the tangent plane and parametric equation for the normal line to the surface $z = 4x^3y^2+2y-2$ at the point (1,-2,10)

Answer any TWO questions

16 Evaluate the integral $\int_0^4\int_y^4 \frac{x}{x^2+y^2}dxdy$ by first reversing the order of integration.

17 Evaluate $\int_0^1\int_{y^2}^1\int_0^{1-x} x\,dzdxdy$

18 Find the volume of the solid in the first octant bounded by the co-ordinate

planes and the plane $x + y + z = 1$

PART C

(Each question carries 5 Marks)

Answer any THREE questions

19 Find div F and curl F of $F(x, y, z) = x^2yi + 2y^3zj + 3zk$

20 Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ where $r = ||xi + yj + zk||$

21 Find the work done by the force field $F(x, y, z) = (x^2 + xy)\ i + (y - x^2y)j$ on a particle that moves along the curve $C: x = t, y = \frac{1}{t}$ $,1 \leq t \leq 3$

22 Evaluate $\int F.dr$ where $F(x, y) = y\ i - x\ j$ along the triangle joining the vertices (0,0), (1,0), and (0,1).

23 Determine whether $F(x, y) = 4y\ i + 4xj$ is a conservative vector field. If so, find the potential function and the potential energy.

Answer any THREE questions

24 Using Green's theorem evaluate $\oint_C (e^x + y^2)\ dx + (e^y + x^2)\ dy)$ where C is the boundary of the region between $y = x^2$ and $y = 2x$.

25 Evaluate the surface integral $\int\int_\sigma \frac{x^2+y^2}{y}\ dS$ over the surface σrepresented by the vector valued function $r(u, v) = 2cosvi + uj + 2sinvk$ $, 1 \leq u \leq 3, \quad 0 \leq v \leq \pi$

26 Using Divergence Theorem evaluate $\iint_\sigma F.n\ ds$ where $F(x, y, z) = (x - z)i + (y - x)j + (2z - y)k$. σ is the surface of the cylindrical solid bounded by $x^2 + y^2 = a^2, z = 0\ , z = 1$.

27 Determine whether the vector field $F\ (x, y, z) = 4(x^3 - x)i + 4(y^3 - y)j + 4(z^3 - z)k$ is free of sources and sinks. If it is not, locate them.

28 Using Stokes theorem evaluate $\int_C F.dr$ where $F(x, y, z) = x^2i + 4xy^3j + y^2xk$,
C is the rectangle: $0 \leq x \leq 1, 0 \leq y \leq 3$ in the plane $= y$

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10105

Reg. No.:__________________ Name: ____________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE SPECIAL EXAMINATION, AUGUST 2016

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer ALL questions. Each question carries 3 marks

1. Find derivative of y = sinh (4x-8)
2. Test whether the series converges or diverges, $\sum_{k=1}^{\infty} \frac{k}{2^k}$
3. Identify the surface $z = y^2 - x^2$
4. Convert from rectangular to spherical co-ordinates, $(2\sqrt{3}, 2, -4)$
5. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $Z = \cos \ (xy^3)$
6. Show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ if $z = x^2y + 5y^3$.
7. Evaluate $\int_0^2 (2t\hat{\imath} + 3t^2\hat{\jmath})dt$
8. Find the arc length of the parametric curve $x=e^t$, $y=e^{-t}$,$z = \sqrt{2}\,t$,$0\le t\le 1$.
9. Evaluate $\int_1^3 \int_2^4 (40 - 20 \ xy \ ) \ dy \ dx$
10. Evaluate $\int_0^3 \int_0^2 \int_0^1 (xyz \ ) \, dx \ dy \ dz$

PART B

Answer any 2 complete questions each having 7 marks

11. Test the convergence of the series $\sum_{k=1}^{\infty} \frac{k(k+3)}{(k+1)(k+2)(k+5)}$
12. Show that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$
13. Find the Taylor series of $\frac{1}{x+2}$ about x =1.

Answer any 2 complete questions each having 7 marks

14. Express the equation $x^2 - y^2 - z^2 = 0$ in cylindrical And Spherical coordinates

A

15. Evaluate $\lim_{(x,y)\to(0,0)}$ [sin $(\sqrt{x^2+y^2})]/(x^2+y^2)$ by converting to polar coordinates.

16. Show that the functions $f(x, y) = 3x^2y^5$ and $f(x, y) = \sin(3x^2y^5)$ are continuous everywhere.

Answer any 2 complete questions each having 7 marks

17. Let L(x, y) denote the local linear approximation to $f(x, y) = \sqrt{x^2+y^2}$ at the point (3, 4). Compare the error in approximating $f(3.04, 3.98) = \sqrt{(3.04)^2+(3.98)^2}$ by L (3.04, 3.98) with the distance between the points (3,4) and (3.04, 3.98).

18. Suppose that $w = x^2+y^2-z^2$ and $x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$, $z = \rho \cos\phi$. Use appropriate form of the chain rule to find $\frac{\partial w}{\partial \rho}$ and $\frac{\partial w}{\partial \theta}$

19. Locate the relative extrema and saddle points of $f(x,y) = 3x^2-2xy+y^2-8y$

Answer any 2 complete questions each having 7 marks

20. Let $f(x,y) = x^2e^y$. Find the maximum value of a directional derivative at (-2,0) and find the unit vector in the direction in which the maximum value occur.

21. Find the angle between the tangent lines to the graphs of $r_1(t) = \tan^{-1} t\, i + \sin t\, j + t^2 k$ $r_2(t) = (t^2-t)i + (2t-2)j + \log t\, k$

22. Suppose that a particle moves through 3-space so that its position vector at time t is $r(t) = ti + t^2 j + t^3 k$.

    Find the scalar tangential and normal components of acceleration at time t = 1.

Answer any 2 complete questions each having 7 marks

23. Use a polar double integral to find the area enclosed by the circle $r = \sin\theta$

24. Use a triple integral to find the volume of the solid within the cylinder $x^2+y^2 = 9$ and between the planes $z = 1$ and $z = 5$

25. Evaluate $\iint_R \frac{x-y}{x+y}\, dA$ where R is the region enclosed by $x-y=0, x-y=1, x+y=1,$ $x+y=3$

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10103

Reg. No.:________________ Name: ________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, JUNE 2016

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer ALL questions. Each question carries 3 marks

1. Evaluate $\int_0^1 sinh^2(2x)\,dx$
2. Check whether the series $\sum_{k=1}^{\infty} \frac{1}{2k-1}$ converges or not.
3. Identify the quadric surface $6x^2 + 3y^2 + 4z^2 = 24$
4. Convert $(2\sqrt{3}, \pi/3, 6)$ from cylindrical to spherical co-ordinates.
5. Find the rate of change of $f(x,y)=xe^{-y} + 5y$ with respect to x at the point (4,0) with **y** held fixed.
6. If $f(x,y) = x^2y^3 + x^4y$. Find f_{xy}
7. Evaluate $\int_1^9 ((\frac{t}{2})\,i + (t - \frac{1}{2})\hat{j})\,dt$
8. Find $\frac{d\bar{u}}{dt}$ if $\bar{u}(t) = (3t\,i + 5\,t^2 j + 6\,k) \cdot (t^2 i + 2t\,j + t\,k)$
9. Sketch the region of integration in $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)\,dydx$
10. Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{(x+y+z)}\,dx\,dy\,dz$

PART B

Answer any 2 complete questions each having 7 marks

11. A ball is dropped from a height of **h** feet and on each bounce rises 75% of the distance it has fallen previously. If it travels a distance of 21 feet what is h?

12. Use Ratio Test for absolute convergence to find whether the series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} 2^k}{k!}$ converges.

13. Find the Maclaurin's Series for $\frac{1}{1-x}$

Answer any 2 complete questions each having 7 marks

14. For the surface $4x^2 + 9y^2 + 18z^2 = 72$

   a. Find the equation of the elliptical trace in the plane

   b. $z = \sqrt{2}$

   c. Find the length of the major and minor axes of the ellipse.

15. Find $\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2)$

16. Let $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$. Determine the limit of f (x, y) as (x, y) approaches (0, 0) along the curve C. where C is

   (a) $x = 0$ (b) $y = 0$ (c) $y = x$

   (d) $y = x^2$ (e) $x = y^2$

Answer any 2 complete questions each having 7 marks

17. Use chain rule to find $\frac{dw}{ds}$ $at\ s = 1/4$ if

$$w = r^2 - r\tan\theta;\ \ r = \sqrt{s};\ \ \theta = \pi s$$

18. Locate all relative extrema and saddle points of $f(x,y)=x^2+xy-2y-3x+1$

19. The volume V of a right circular cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. Suppose that the height decreases from 20 to 19.95 units and the radius

increases from 4 to 4.05 units. Compare the change in volume of the cone with an approximation of this change using a total differential.

Answer any 2 complete questions each having 7 marks

20.The temperature in degree Celsius at a point in the (x, y) plane is

$$T(x, y) = \frac{xy}{1 + x^2 + y^2}$$

Find the rate of change of temperature at (1,1) in the direction of $(2\hat{\imath} - \hat{\jmath})$.

21.Find the scalar tangential and normal components of acceleration at time t of a particle with position vector at time t is $r(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$

22. Find the equation of the tangent plane and parametric equation for the normal line to the surface $x^2 + y^2 + z^2 = 25$ at P(3,0,4)

Answer any 2 complete questions each having 7 marks

23.Evaluate $\iint_R \sin\theta \, dA$ where R is the region in the first quadrant that is outside the circle $r = 2$ and inside the cardioid $r = 2(1 + \cos\theta)$.

24.Find the Jacobian $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ where x = 4u + v, y = u – 2w, z = v + w.

25.By changing the order of integration evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy \, dx$

10101 **A**

Reg. No.:________________ Name: ________________

FIRST SEMESTER B.TECH DEGREE EXAMINATION, JANUARY 2016

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each question carries 3 marks

1. Show that the series $\sum_{k=1}^{\infty} \frac{\cos k}{k^2}$ is convergent.
2. Find $\frac{d}{dx}\left(e^x \operatorname{sech}^{-1}\sqrt{x}\right)$
3. Identify the surfaces $5x^2 - 4y^2 + 20z^2 = 0$
4. Equation of a surface in spherical coordinates is $\rho = \sin\theta \sin\varphi$
   Find the equation of this surface in rectangular coordinates.
5. Given $f = e^x \sin y$; show that the function satisfies the Laplace equation $f_{xx} + f_{yy} = 0$
6. Let $w = 4x^2 + 4y^2 + z^2$,where $x = \rho \sin\varphi \cos\theta$, $y = \rho \sin\varphi \sin\theta$, $z = \rho \cos\varphi$ Find $\frac{\partial w}{\partial \rho}$ using chain rule.
7. A particle moves along a circular helix in 3-space so that its position vector at time t is $r(t) = (4\cos\pi t)i + (4\sin\pi t)j + tk$ Find the displacement of the particle during theinterval $1 \le t \le 5$.
8. Find the tangent to the curve $r(t) = (t^2 - 1)\vec{i} + t\vec{j}$ at $t = 1$
9. Evaluate $\int_1^a \int_1^b \frac{dydx}{xy}$
10. The line y = 2- x and the parabola $y = x^2$ intersect at the points (-2, 4) and (1, 1).If R is the region enclosed by y=2-x and $y=x^2$,then find $\iint_R (y)\, dA$

(10 x 3 = 30 Marks)

PART B

Answer any 2 complete questions each having 7 marks

11. Find the radius of convergence and interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x-5)^k}{k^2}$.
12. Test the convergence of $\frac{x}{12} + \frac{x^2}{23} + \frac{x^3}{34} + \cdots$ ....
13. Find the Taylors series of $\frac{1}{x}$ about x = 1.

Answer any 2 complete questions each having 7 marks

14. Find the domains of (i) $f(x,y)=\sqrt{25-x^2-y^2-z^2}$ (ii) $f(x,y)=\ln(x-y^2)$ and describe them in words.
15. Find the limit of $f(x,y)=\frac{-xy}{x^2+y^2}$ as (x,y)→ (0,0) along (i) the X-axis , (ii) the Y- axis (iii) the line y = x.
16. Find the spherical and cylindrical coordinates of the point that has rectangular coordinates(x,y,z)=(4,-4,$4\sqrt{6}$)

Answer any 2 complete questions each having 7 marks

17. Locate all relative maxima, relative minima and saddle point if any,of $f(x,y) = y^2 + xy + 4y + 2x + 3$
18. Let f be a differentiable fuction of 3 varaiables and suppose that $W = f(x-y, y-z, z-x)$. Prove that $\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}=0$.
19. Find the local linear approximation L(x,y) to f(x,y) $=\frac{1}{\sqrt{x^2+y^2}}$ at the point P(4,3). Compare the error in approximating 'f' by L at the specified point Q (3.92, 3.01) with the distance between P and Q.

Answer any 2 complete questions each having 7 marks

20. Find y(t) where $y''(t) = 12t^2 i - 2tj$, $y(0) = 2i - 4j$, $y'(0) = 0$.
21. Find the arc length parametrization of the line x = 1 + t,y = 3 − 2t, z = 4 + 2t that has the same direction as the given line and has reference point (1, 3, 4).
22. Find the directional derivative of f(x, y) = e^xsec y at P (0, π/4) in the direction of PQ where Q is the origin.

Answer any 2 complete questions each having 7 marks

23. Find the area bounded by the x-axis, $y = 2x$ and x+y=1 using double integration.
24. Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes z = 1and x + z = 5.
25. Sketch the region of integration and evaluate the integral $\int_1^2 \int_y^{y^2} dxdy$ by changing the order of integration.

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2015

MA101 CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer ALL Questions

1. Find the derivative of $\tanh\sqrt{1+x^2}$ 2
2. Examine the convergence of the series $\sum_{k=1}^{\infty}\frac{k^k}{k!}$ 3
3. Convert the rectangular coordinates $(0,4,\sqrt{3})$ to cylindrical and spherical coordinates 2
4. Find equations of the paraboloid $z^2 = x^2 + y^2$ in cylindrical and spherical coordinates 3
5. If $U=\frac{x^3+y^3}{x-y}$, . Find $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}$ 2
6. The length, width, and height of a rectangular box are measured with an error of at most 5%. Use a total differential to estimate the maximum percentage error that results if these quantities are used to calculate the diagonal of the box. 3
7. Find $\nabla z, if, z = 4x - 10y.$ 2
8. A particle moves on the curves x=$2t^2$, y=t^2-4t,z=3t-5 where t is the time .Find the component of acceleration at the time t=1 in the direction$\hat{\imath}$ -3$\hat{\jmath}$ +2$\hat{k}$. 3
9. Evaluate$\int_0^1\int_{x^2}^{2-x} xydydx$ 2
10. Find the Jacobian of the transformations x = uv and y = $\frac{u}{v}$ 3
11. Find curl $\vec{F}$ at the point (1,-1,1) where
   $\vec{F}$ =$xz^3\hat{\imath} - 2x^2yz\hat{\jmath} + 2yz^4\hat{k}$ 2
12. The function ϕ(x,y,z) = xy+yz+xz is a potential for the vector field $\vec{F}$, find the vector field $\vec{F}$. 3

PART B

MODULE 1

Answer ANY TWO Questions

13. Find the Maclaurin series for cosx and also find cos 1,calculate the absolute error 5

14. Prove that $\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$,-1< x< 1 5

15. Show the series $\sum_{k=1}^{\infty}(\frac{1}{2})^k$ converges and $\sum_{k=1}^{\infty}(-1)^k$ diverges 5

MODULE 2

Answer ANY TWO Questions

16. Find the natural domain of the following functions.

    i. $f(x,y) = 3x^2\sqrt{y} - 1$

    ii. $f(x,y) = \log(x^2 - y)$ 5

17. Evaluate $\underset{(x,y)\to(-1,2)}{Lt} \frac{x^2+y}{x^2+y^2}$. State the properties used in the evaluation. 5

18. Find the traces of the surface $x^2 + y^2 - z^2 = 0$ in the planes x=2 and y=1 and identify the same. 5

MODULE 3

Answer ANY TWO Questions

19. Find maximum and minimum values of

    $f(x, y)=x^3+3xy^2-15x^2-15y^2+72x$ 5

20. Let L(x, y,z) denote the local linear approximation to $f(x,y,z) = \frac{x+y}{y+z}$ at the point P(-1,1,1). Compare the error in approximating f by L at Q(-0.99,0.99, 0.01)with the distance between P and Q . 5

21. $z = 3xy^2z^3$; $y = 3x^2 + 2$; $z = \sqrt{x-1}$ Find $\frac{dw}{dx}$ *and* $\frac{dw}{dy}$ 5

MODOULE 4

Answer ANY TWO Questions

22. Given a circular helix r(t)=acost$\hat{\imath} + asint\hat{\jmath} + bt\hat{k}$,a, b>0 , 0 ≤ t ≤ ∞ ,find its arc length and unit tangent vector. 5

23. The position vector at any time t of a particle moving along a curve is $\vec{r}$(t)=t$\hat{\imath}$ + $t^2\hat{\jmath}$+$t^3\hat{k}$.

    Find the scalar and vector tangential and normal component of the acceleration at time t=1 5

24. Find the parametric equation of the tangent line to the curve of intersection of the paraboloid $z = x^2$+$y^2$and the ellipsoid $3x^2 + 2y^2 + z^2 = 9\ at\ (1,1,2)$ 5

MODULE 5

Answer ANY THREE Questions

25. Evaluate$\iint (x^2 + y^2)dxdy$ over the region in the positive quadrant for which x+ y ≤ 1 5

26. Change the order of integration in$\int_0^1 \int_x^1 \frac{x}{x^2+y^2} dxdy$ and hence evaluate the same. 5

27. Find the area bounded by the Parabolas y^2=4xand x^2=-(y/2). 5

28. Find the volume bounded by the cylinder x^2+y^2=4 the planes y+ z=3 and z=0 5

29. Evaluate$\int_0^\infty \int_0^\infty e^{-(x+y)} \sin\left(\frac{\pi y}{x+y}\right) dxdy$

    by means of the transformation u = x+y, v=y 5

MODULE 6

Answer ANY THREE Questions

30. Use Green's theorem to evaluate $\oint_C\ (xcosydx - ysinxdy)$ where C is the square with vertices (0, 0), (π, 0), (π, π) and (0, π) 5

31. Use Stoke's theorem to evaluate the integral $\oint_C \vec{F}.dr$, where $\vec{F} = xy\ \hat{\imath} + yz\ \hat{\jmath} + zx\ \hat{k}$; C is the triangle in the plane x+y+z = 1 with vertices (1,0,0) ,(0,1,0) and (0,0,1)with a counter clockwise orientation looking from the first octant towards the origin. 5

32. Use Gauss Divergence Theorem to find the outward flux of vector field $\vec{F}(x,y,z) = x^3 i + y^3 j + z^3 k$ across the surface of the region enclosed by circular cylinder $x^2 + y^2 = 9$ and the plane $z = 0$ and $z = 2$ 5

33. Use Gauss Divergence Theorem to find the outward flux of vector field $\vec{F}(x,y,z) = x^3 i + y^3 j + z^3 k$ across the surface of the region enclosed by circular cylinder $x^2 + y^2 = 9$ and the plane $z = 0$ and $z = 2$ 5

34. Find the work done by the force field $\vec{F}(x,y) = (e^x - y^3)\hat{\imath} + (\cos y + x^3)\hat{\jmath}$ on a particle that travels once around the unit circle $x^2 + y^2 = 1$ in the counter clockwise direction. 5