89. Find the Laplace transform of the saw-toothed wave function of period T, defined by

$$f(t) = \frac{Kt}{T} \quad 0 < t < T$$

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Ans:

If f (t) is a continuous periodic function with period T, then

$$L\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T f(t)\, e^{-st}\, dt$$

Here

$$\begin{aligned}
L\{f(t)\} &= \frac{1}{1-e^{-sT}} \left[\int_0^T \frac{Kt}{T}\, e^{-st}\, dt\right] \\
&= \frac{K}{T(1-e^{-sT})} \left[\int_0^T t\, e^{-st}\, dt\right] \\
&= \frac{K}{T(1-e^{-sT})} \left[\left((t)\frac{e^{-st}}{-s} - \frac{e^{-st}}{s^2}\right)_0^T\right] \\
&= \frac{K}{T(1-e^{-sT})} \left[\left((T)\frac{e^{-sT}}{-s} - \frac{e^{-sT}}{s^2}\right) - \left((0) - \frac{1}{s^2}\right)\right] \\
&= \frac{K}{T(1-e^{-sT})} \left[\frac{-Te^{-sT}}{s} - \frac{e^{-sT}}{s^2} + \frac{1}{s^2}\right] \\
&= \frac{K}{T(1-e^{-sT})} \left[\frac{1}{s^2} - \frac{e^{-sT}}{s^2} - \frac{Te^{-sT}}{s}\right] \\
&= \frac{K}{T(1-e^{-sT})} \left[\frac{1}{s^2}(1-e^{-sT}) - \frac{Te^{-sT}}{s}\right] \\
&= \frac{K}{Ts^2} - \frac{Ke^{-sT}}{s(1-e^{-sT})} \\
&= K\left[\frac{1}{Ts^2} - \frac{e^{-sT}}{s(1-e^{-sT})}\right]
\end{aligned}$$

90. Find the Laplace transform of the rectified semi wave function

$$f(t) = \begin{cases} \sin at, & 0 < t < \pi/a \\ 0, & \pi/a < t < 2\pi/a \end{cases}$$

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Ans:

If f (t) is a continuous periodic function with period T, then

$$L\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T f(t)\, e^{-st}\, dt$$