142. Apply Convolution theorem to evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$

[KER 08 May 09] [CUSAT 06 Nov 11] [MG 10 May 12] [CUSAT 06 June 13] [KER 13 Apr 14]
[CUSAT 06 Nov 15] [KER 13 Dec 15] [CUSAT 06 Nov 16] [KER 13 Dec 16]
[CLT 09 Apr 17] [CUSAT 12 Apr 18] [KTU May 19]

Ans:

By Convolution theorem,

$$L^{-1}[F(s).G(s)] = L^{-1}[F(s)] * L^{-1}[G(s)]$$

$$= \int_0^t f(u)\, g(t-u)\, du$$

$$= \int_0^t g(u)\, f(t-u)\, du$$

$$\therefore \quad L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = L^{-1}\left[\frac{s}{s^2+a^2}\right] * L^{-1}\left[\frac{1}{s^2+a^2}\right]$$

$$= \cos at * \frac{1}{a}\sin at$$

$$= \frac{1}{a}\int_0^t \cos au\ \sin a(t-u)\ du$$

$$= \frac{1}{a}\int_0^t \cos au\ \sin(at-au)\ du$$

$$\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$$

$$\therefore \quad L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = \frac{1}{a}\int_0^t \frac{1}{2}[\sin at - \sin(2au-at)]\ du$$

$$= \frac{1}{2a}\int_0^t [\sin at - \sin(2au-at)]\, du$$

$$= \frac{1}{2a}\left[\sin at\,(u) + \frac{\cos(2au-at)}{2a}\right]_0^t$$

$$= \frac{1}{2a}\left[\sin at\,(t) + \frac{\cos at}{2a} - \frac{\cos at}{2a}\right]$$

$$= \frac{1}{2a}(t \sin at)$$

143. Apply Convolution theorem to evaluate $L^{-1}\left[\frac{1}{(s^2+a^2)^2}\right]$

[CUSAT 06 Nov 08] [KNR 07 Apr 13] [CLT 14 Apr 15]

Ans:

By Convolution theorem,

$$L^{-1}[F(s).G(s)] = L^{-1}[F(s)] * L^{-1}[G(s)]$$

$$= \int_0^t f(u)\, g(t-u)\, du$$

$$= \int_0^t g(u)\, f(t-u)\, du$$