1. Obtain L { sin at } by the definition of Laplace Transform.

[KNR 07 APR 11] [KNR 07 Apr 13]

Ans:

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

$$\Rightarrow \quad L\{\sin at\} = \int_0^\infty e^{-st} \sin at\, dt$$

Using the standard result

$$\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2+b^2}\left[a \sin bx - b \cos bx\right] \qquad \text{where} \quad a = -s \quad \text{and} \quad b = a$$

we get

$$L\{\sin at\} = \left[\frac{e^{-st}}{s^2+a^2}\left[-s \sin at - a \cos at\right]\right]_0^\infty$$

$$= \left[0 - \left(\frac{1}{s^2+a^2}(0-a)\right)\right]$$

$$\Rightarrow \quad L\{\sin at\} = \frac{a}{s^2+a^2}$$

2. Obtain L { cosh at } by the definition of Laplace Transform.

[KNR 07 Jan 17]

Ans:

$$L\{\cosh at\} = L\left\{\frac{e^{at}+e^{-at}}{2}\right\}$$

$$= \frac{1}{2}\left[L\{e^{at}\} + L\{e^{-at}\}\right] \qquad \text{-----} \quad (1)$$

$$L\{e^{at}\} = \int_0^\infty e^{-st} e^{at}\, dt$$

$$= \int_0^\infty e^{-(s-a)t}\, dt$$

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty$$

$$= \left[0 - \frac{1}{-(s-a)}\right]$$

$$= \frac{1}{s-a}$$

$$L\{e^{-at}\} = \int_0^\infty e^{-st} e^{-at}\, dt$$

$$= \int_0^\infty e^{-(s+a)t}\, dt$$

$$= \left[\frac{e^{-(s+a)t}}{-(s+a)}\right]_0^\infty$$

$$= \left[0 - \frac{1}{-(s+a)}\right]$$

$$= \frac{1}{s+a}$$