71. Find $L\left\{\int_0^t \frac{e^{-4t}\sin 3t}{t}\,dt\right\}$

[KNR 07 Apr 12] [MG 10 May 14]

Ans:

$$L\left\{\int_0^t f(t)\,dt\right\} = \frac{1}{s}\,L\{f(t)\}$$

$$L\left\{\int_0^t \frac{e^{-4t}\sin 3t}{t}\,dt\right\} = \frac{1}{s}\,L\left\{\frac{e^{-4t}\sin 3t}{t}\right\} \qquad ----- \quad (1)$$

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\,ds \quad \text{where} \quad F(s) = L\{f(t)\}$$

$$\therefore \quad L\left\{\frac{\sin 3t}{t}\right\} = \int_s^\infty L\{\sin 3t\}\,ds$$

$$= \int_s^\infty \frac{3}{s^2+9}\,ds$$

$$\left[\because \int \frac{a}{s^2+a^2}\,ds = \tan^{-1}\left(\frac{s}{a}\right)\right]$$

$$= \left[\tan^{-1}\left(\frac{s}{3}\right)\right]_s^\infty$$

$$= \tan^{-1}(\infty) - \tan^{-1}\left(\frac{s}{3}\right)$$

$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{3}\right)$$

$$= \cot^{-1}\left(\frac{s}{3}\right)$$

$$\Rightarrow \quad L\left\{\frac{e^{-4t}\sin 3t}{t}\right\} = \cot^{-1}\left(\frac{s+4}{3}\right)$$

$$\therefore (1) \Rightarrow \quad L\left\{\int_0^t \frac{e^{-4t}\sin 3t}{t}\,dt\right\} = \frac{1}{s}\cot^{-1}\left(\frac{s+4}{3}\right)$$

72. Find $L\left\{\int_0^\infty t\,e^{-2t}\sin t\,dt\right\}$

[CUSAT 06 Nov 07] [KNR 07 May 08] [CUSAT 06 Nov 09] [CUSAT 06 June 13] [CUSAT 12 Apr 18]

Ans:

$$\int_0^\infty t\,e^{-2t}\sin t\,dt = \int_0^\infty t\,e^{-st}\sin t\,dt \quad \text{where} \quad s = 2$$

$$= L\{t\sin t\} \quad \text{where} \quad s = 2 \qquad ----- \quad (1)$$

$$L\{\sin t\} = \frac{1}{s^2+1}$$

$$L\{t\sin t\} = (-1)\,\frac{d}{ds}\left(\frac{1}{s^2+1}\right)$$

$$= -\left[\frac{(s^2+1)(0) - 1\,(2s)}{(s^2+1)^2}\right] = \frac{2s}{(s^2+1)^2}$$

$$\therefore (1) \Rightarrow \quad \int_0^\infty t\,e^{-2t}\sin t\,dt = \left[\frac{2s}{(s^2+1)^2}\right]_{(s=2)}$$

$$= \frac{4}{25}$$