

# **KSU CET**

**S1 & S2 Notes**

2019 Scheme



22/8/2019

## 1. INTERFERENCE

### Interference:

Interference can be defined as the interaction between two or more waves, of the same or very close frequencies emitted from coherent sources, where the wavefronts are combined according to the principle of superposition.

In optics, the interference means the superposition of two or more waves which results in a new wave pattern.

The resulting variation in the disturbances produced by the waves is called the interference pattern.

### Superposition:

Superposition is the vectorial addition of waves (electric field wave).

### Coherent sources:

Interference pattern will be observed only if the sources are coherent. Two sources of light are said to be coherent, if they emit waves of the same frequency (or wavelength), nearly the same amplitude and maintain a constant phase difference between them. No source is perfectly coherent.

Examples of coherent source: Lasers, Naillamp, ...

We cannot have perfectly coherent waves because of the following reasons:

- Uncertainty principle:

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{4\pi}$$

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2m} \quad (\text{E-energy})$$

$$\Delta \varphi \geq \frac{\hbar}{2m} \cdot \frac{1}{\Delta t}$$

- Maintaining a constant phase difference.

Maintaining a constant phase difference is difficult because wave is not continuous.

### Coherence

Coherence is a property of waves that helps in getting stationary interference, i.e., the interference which is temporally and spatially constant. The coherence of a wave depends on the characteristics of its source.

#### 1. Temporal coherence (related to time)

- A measure of the correlation between the phase of a wave (light) at different points along the direction of wave propagation.
- If the phase difference of the wave crossing the two points lying along the direction of wave propagation is independent of time, then the wave is

said to have temporal coherence.

- Also known as longitudinal coherence.

- screen should be placed at a distance less than coherent plane.

## 2. Spatial coherence (related to space)

- A measure of the correlation between the phases of a wave (light) at different points transverse to the direction of propagation.

- If the phase difference of the waves crossing the two points lying on a plane perpendicular to the direction of wave propagation is independent of time, then the wave is said to have spatial coherence.

- Also known as lateral coherence.

### Coherence Time and Coherence Length

For a single frequency wave, the time interval over which the phase remains constant is called the coherence time. It is generally represented by  $\Delta t$ .

The distance travelled by the light pulses during the coherence time is known as coherence length. It is represented by  $\Delta L$ . The coherence length is also called the spatial interval, which is the length over which the phase of the wave remains constant.

$$\Delta L = c \Delta t$$

constructive Interference: When two waves are in phase.

When two waves are not displaced with respect to each other or when they are displaced through an integral number of wavelengths, constructive interference takes place. Bright bands are observed at those points.

Path difference,  $\alpha = n\lambda$

Destructive Interference

When two waves are displaced with respect to each other by an odd number of half-wavelengths, destructive interference results. Dark bands of light are observed at those points.

Path difference,  $\alpha = (2n+1)\frac{\lambda}{2}$

Partially destructive or constructive Interference

Partial interference occurs when two waves have the same frequency and wavelength and are added together, but the waves are added so the crests and troughs don't line up. This is between constructive and destructive interference.

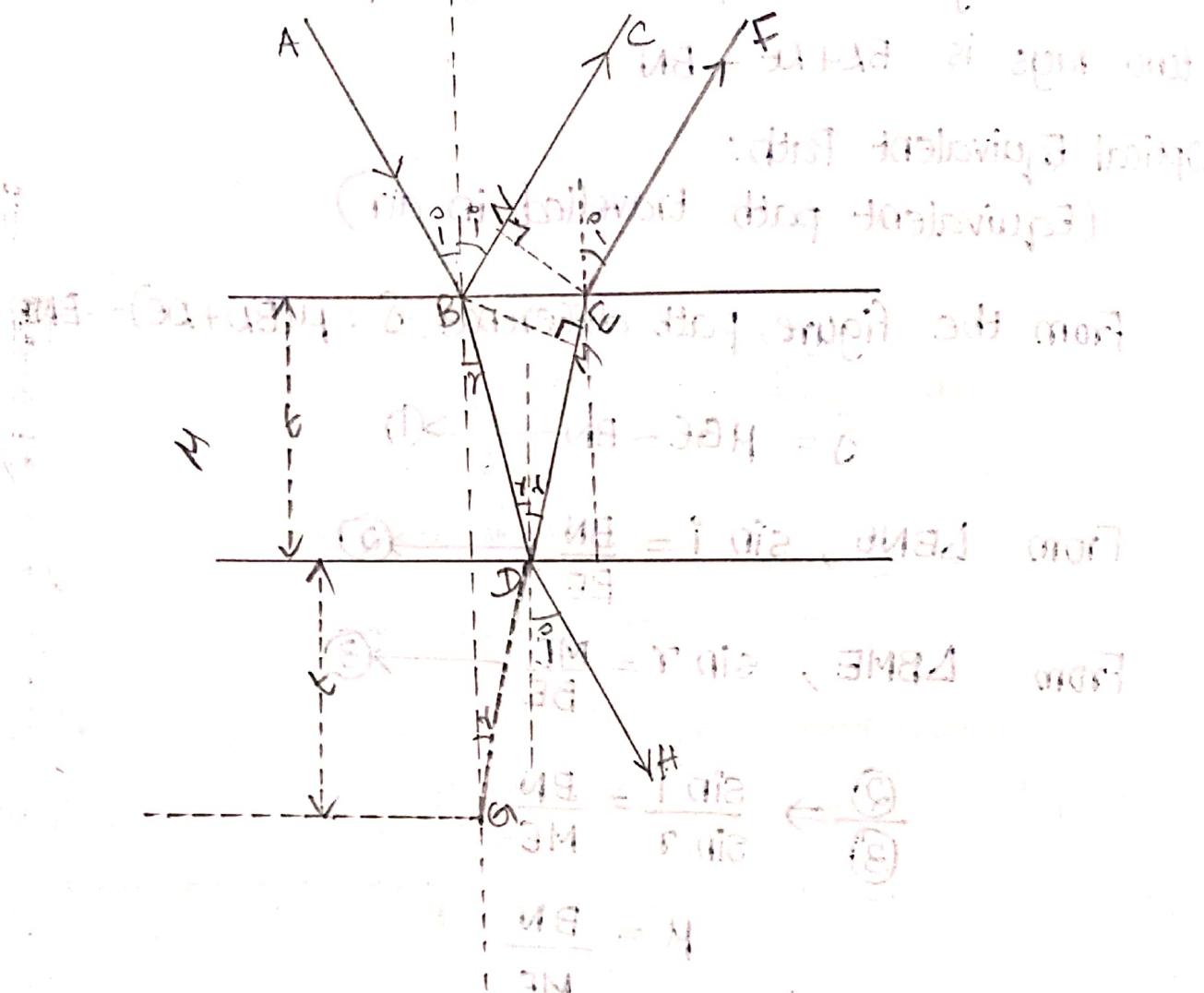
Note:

There is no production or redistribution of energy only.

29/8/2021

## Interference In Thin Films - Reflected System

Consider a transparent thin film of refractive index ' $\mu$ ' and thickness 't'.



Consider a ray of light AB falling on the upper surface of the film at an angle ' $i$ '. A part of the ray gets reflected and the other part gets refracted along BD at an angle ' $r$ '. At point D, the wave BD is again partly reflected from the second surface along DE and partly emerges out along DH. Thus, interference occurs between reflected waves BE and EF. and also between the

transmitted waves

Geometrical Path difference:

The geometric path difference between the two rays is  $BA + AE - BN$ .

Optical Equivalent Path:  
(Equivalent path travelled in air)

From the figure, path difference,  $\delta = \mu(BD + DE) - BN$

$$\delta = HGE - BN \rightarrow ①$$

$$\text{From } \Delta BNE, \sin i = \frac{BN}{BE} \rightarrow ②$$

$$\text{From } \Delta BME, \sin r = \frac{ME}{BE} \rightarrow ③$$

$$\frac{②}{③} \Rightarrow \frac{\sin i}{\sin r} = \frac{BN}{ME}$$

$$\mu = \frac{BN}{ME}$$

$$BN = \mu ME \rightarrow ④$$

Sub. ④ in ①, the path difference  $\delta = \mu GE - \mu ME$

$$\therefore \delta = \mu(GE - ME)$$

$$\text{Multiplying by } \cos r, \delta = \mu GM \rightarrow ⑤$$

$$\text{From } \Delta BMG, \cos r = \frac{GM}{BG} = \frac{GM}{2t}$$

$$GM = 2t \cos r \rightarrow ⑥$$

$$\text{sub: } ⑥ \text{ in } ⑤ \Rightarrow \delta = \mu(2t \cos r)$$

$$\boxed{\delta = 2\mu t \cos r}$$

This is called cosine law.

According to electromagnetic theory, a phase reversal will occur when light gets reflected from a denser medium.

$\therefore BN$  has to be replaced with  $BN + \frac{\lambda}{2}$ .

$\therefore$  The actual path difference is

$$\boxed{\delta = 2\mu t \cos r - \frac{\lambda}{2}}$$

30/8/2019 If  $\delta = n\lambda$ , constructive interference will take place and the film will appear bright.

$$\text{i.e., } 2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = n\lambda + \frac{\lambda}{2}$$

$$2\mu t \cos r = (2n+1) \frac{\lambda}{2}$$

(condition for bright)

When  $\delta = (2n+1) \frac{\lambda}{2}$ , destructive interference will take place and the film will appear dark.

$$2\mu t \cos r - \frac{\lambda}{2} = (2n+1) \frac{\lambda}{2}$$

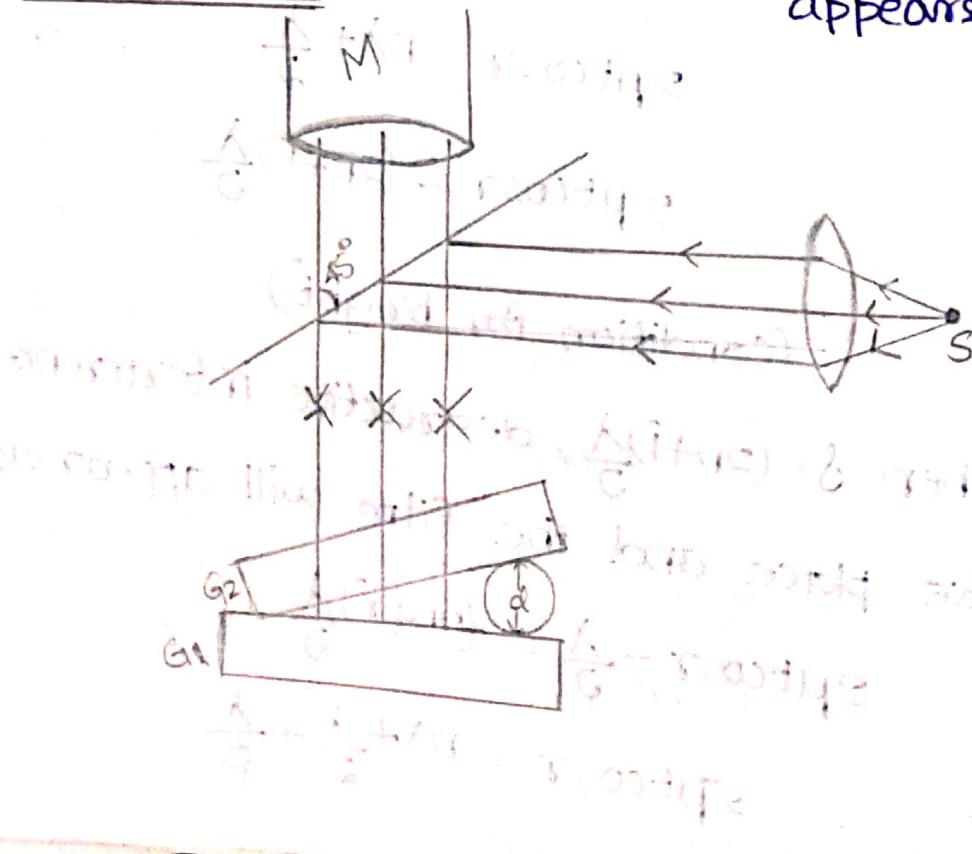
$$2\mu t \cos r = n\lambda + \frac{\lambda}{2} + \frac{\lambda}{2}$$

$$2\mu t \cos r = (n+1)\lambda$$

When  $2\mu t \cos r$  is an integral multiple of  $\lambda$ , the film will appear dark.

Q: Why do soap bubbles and oil slicks appear coloured?

The condition for destructive interference is  $2\mu t \cos r = n\lambda$ . So, the thickness of soap bubble may vary from point to point. Wavelength  $\lambda$  and angle  $r$  also changes. White light consists of a range of wavelengths and for specific values of 't' and 'r', waves of only certain wavelengths (colours) constructively interfere. Therefore, only those colours are present in reflected light. Other wavelengths interfere destructively and hence absent. Hence, the film at a particular point appears coloured.



For dark band,  $2\mu t \cos r = n\lambda$

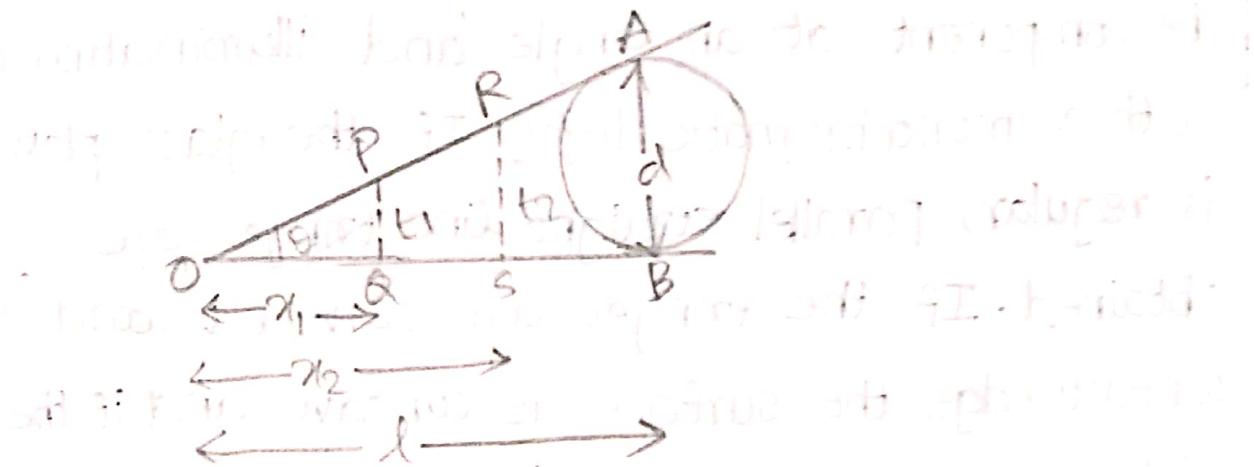
when incident normally,  $i=0$  and  $r$  tends to zero.

$$2\mu t = n\lambda$$

For air,  $\mu = 1$

$$2t = n\lambda$$

$$2t = \delta \rightarrow ①$$



Let PQ be the position of  $m^{\text{th}}$  dark fringe

and RS corresponds to  $n^{\text{th}}$  dark fringe.

$$t_1 = x_1 \tan \theta$$

$$t_2 = x_2 \tan \theta$$

$$t_2 - t_1 = (x_2 - x_1) \tan \theta$$

$$\tan \theta = \frac{t_2 - t_1}{x_2 - x_1} = \frac{n\lambda - m\lambda}{2(x_2 - x_1)} = \frac{(n-m)\lambda}{(x_2 - x_1)2}$$

$$\frac{x_2 - x_1}{n-m} = \beta \text{ (band width)}$$

$$\therefore \tan \theta = \frac{\lambda}{2\beta}$$

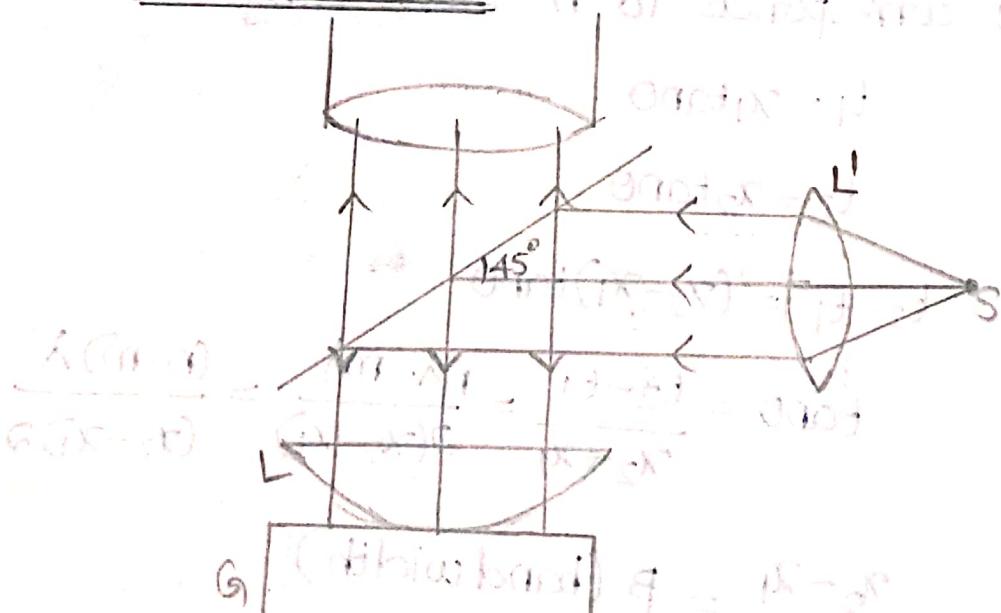
$$d = l \tan \theta = \frac{\lambda l}{2B}$$

Q: How will you check the planeness of a glass plate using interferometric technique?

The planeness of a surface can be quickly inspected visually by keeping an optical flat on the component at an angle and illuminating it with a monochromatic light. If the glass plate is regular, parallel straight line fringes are obtained. If the fringes are curved towards the contact edge, the surface is concave and if the fringes curve away, it is convex.

19/9/2019

### NEWTON'S RINGS



When a plano-convex lens is placed over a flat glass plate, then a thin air layer is formed between glass plate and a convex lens.

Interference is obtained from bottom surface of lens and top surface of glass.

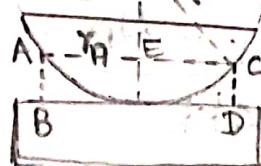
For destructive superposition,

$$2\mu t \cos r = n\lambda$$

$$r \rightarrow 0, \cos r = 1, \mu = 1$$

$$2t = n\lambda$$

As we move to centre, the rings will appear closer and closer. This is because the thickness is not uniform throughout.



Let  $AB=t$

Consider  $\triangle OEC$ , also being a right-angled triangle

$$OE^2 + EC^2 = OC^2$$

$$(R-t)^2 + r_n^2 = R^2$$

$$R^2 - 2Rt + t^2 + r_0^2 = R^2$$

$$t^2 + r_0^2 = 2Rt$$

$\therefore t$  is very small,  $t^2$  can be neglected.

$$r_0^2 = 2Rt$$

$$2t = \frac{r_0^2}{R}$$

$$r_0^2 = nRA$$

$$r_0 = \sqrt{nRA}$$

Equation to Find  $\lambda$  in Lab

$$D_n = 2\sqrt{nRA}$$

$$D_n^2 = 4nRA$$

$$D_{n+k} = 2\sqrt{(n+k)RA}$$

$$D_{n+k}^2 - D_n^2 = 4(n+k)RA - 4nRA$$

$$D_{n+k}^2 - D_n^2 = 4kRA$$

$$\lambda = \frac{D_{n+k}^2 - D_n^2}{4kR}$$

Q: Explain how will you find refractive index of a transparent liquid using Newton's rings experiment.

$$f_1 = f_2 + f_3 - f_4$$

Refractive index,  $\mu = \frac{\text{velocity of light in air}}{\text{velocity of light in medium}}$

Let  $\lambda$  be wavelength in air and  $\lambda'$  be

wavelength in medium.

$$\mu = \frac{c}{\lambda} = \frac{c}{\lambda'} \cdot \frac{\lambda'}{\lambda}$$

$$\mu = \frac{\lambda}{\lambda'}$$

$$\lambda = \frac{A_{D+K}^2 - A_D^2}{4kR}$$

$$\lambda' = \frac{\lambda}{\mu}$$

$$\lambda' = \frac{A_{D+K}^{1.2} - A_D^{1.2}}{4kR}$$

$$\frac{\lambda}{\mu} = \frac{A_{D+K}^{1.2} - A_D^{1.2}}{4kR}$$

$$\frac{A_{D+K}^2 - A_D^2}{4kR \mu} = \frac{A_{D+K}^{1.2} - A_D^{1.2}}{4kR}$$

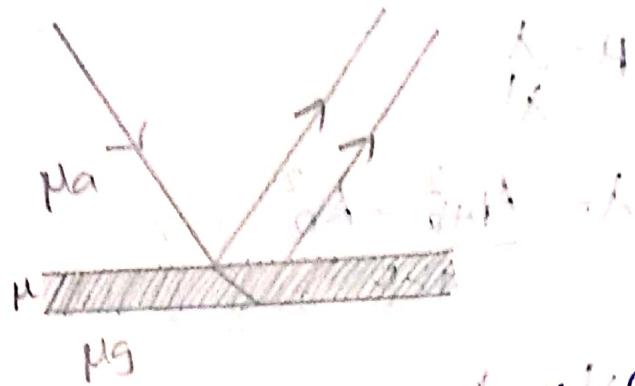
$$\mu = \frac{A_{D+K}^2 - A_D^2}{A_{D+K}^{1.2} - A_D^{1.2}}$$

$$= \frac{\Delta \lambda}{\Delta \lambda'}$$

$$= \frac{\Delta \lambda}{4\Delta \lambda'}$$

## ANTIREFLECTION COATINGS

Anti reflection coatings are thin transparent coatings of optical thickness of one-quarter wavelength given on a surface in order to suppress reflections from the surface.



Let 't' be the thickness of the film and ' $\mu'$ ' be the refractive index of the film material.  
The two waves interfere destructively,

$$\therefore 2\mu t \cos r = (2n+1) \frac{\lambda}{2}$$

$$2\mu t = (2n+1) \frac{\lambda}{2}$$

Since, the thickness of the film is minimum,

$$n=0$$

$$2\mu t = \frac{\lambda}{2}$$

$$\mu = \frac{\lambda}{4t}$$

$$t = \frac{\lambda}{4\mu}$$

20/9/2019

## DIFFRACTION

Diffraction phenomenon is a common characteristic of all kinds of waves. When waves pass near an obstacle, they tend to bend around the edges of the obstacle.

The phenomenon of bending of light around the edges of an obstacle or the encroachment of light within the geometrical shadow is known as diffraction.

### Fresnel and Fraunhofer Diffraction

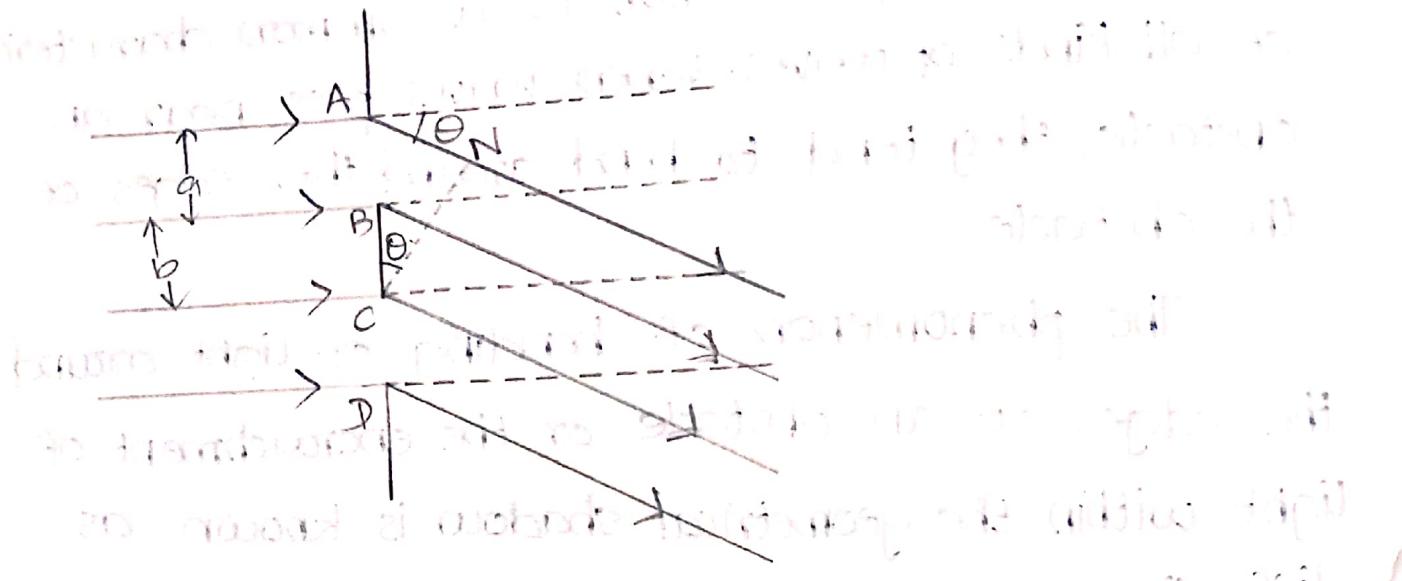
#### Fresnel Diffraction

- Source of light and the screen are at finite distance from the aperture.
- Rays are not parallel.
- Incident wave front is not planar.
- The phase of secondary wavelets is not the same at all points in the plane of the obstacle.
- It is experimentally simple but the analysis proves to be very complex.

#### Fraunhofer Diffraction

- The source of light and the screen are at infinite distances from the aperture.
- Rays are parallel.
- Incident wave front as such is plane.
- The secondary wavelets, which originate from the unblocked portions of the wavefront, are in same phase.
- This is simple to handle mathematically because the rays are parallel.

## GRATING EQUATION:



$$AN = (a+b)\sin\theta$$

$$(a+b)\sin\theta = n\lambda \quad (\text{constructive interference})$$

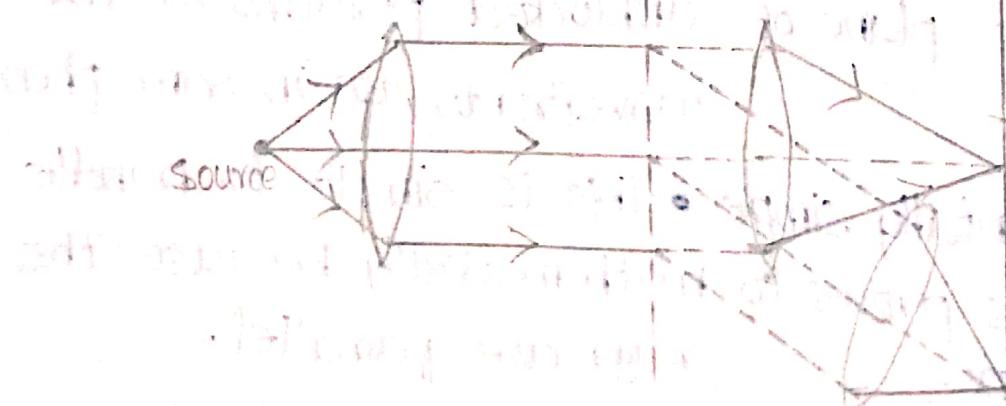
$(a+b)$  is called grating element.

$\sin\theta = \frac{1}{(a+b)}n\lambda$

$$\frac{1}{a+b} = N = \text{No. of grating per unit (lines)}$$

∴  $\sin\theta = Nn\lambda$

This is called grating equation.



## RESOLVING POWER:

Resolving power is the capability of an optical instrument to produce clearly separate images of two objects situated very close to each other.

### Rayleigh's Criterion:

The theory of optical instruments is based on the laws of geometrical optics and rectilinear propagation of light. These laws are only approximately true. While a beam of light from a point object passes through the objective of a telescope, the lens acts like a circular aperture and produces a diffraction pattern instead of a point image. This diffraction pattern is known as Airy's disc. If there are two close point objects then two diffraction patterns are produced, which may overlap on each other and it may be difficult to distinguish them as separate. To obtain the measure of the resolving power of an objective lens, Rayleigh suggested a criterion known as Rayleigh criterion.

According to Rayleigh criterion, in order to see two spectral lines as separate, the  $n^{\text{th}}$  principal maximum of the higher wavelength should fall atleast on the first secondary minimum after the  $n^{\text{th}}$  principal maximum of the lower wavelength. This is equivalent to the condition that the distance between the centers of the patterns shall be equal to the radius of the Airy's disc. This is called as Rayleigh's limit of resolution.



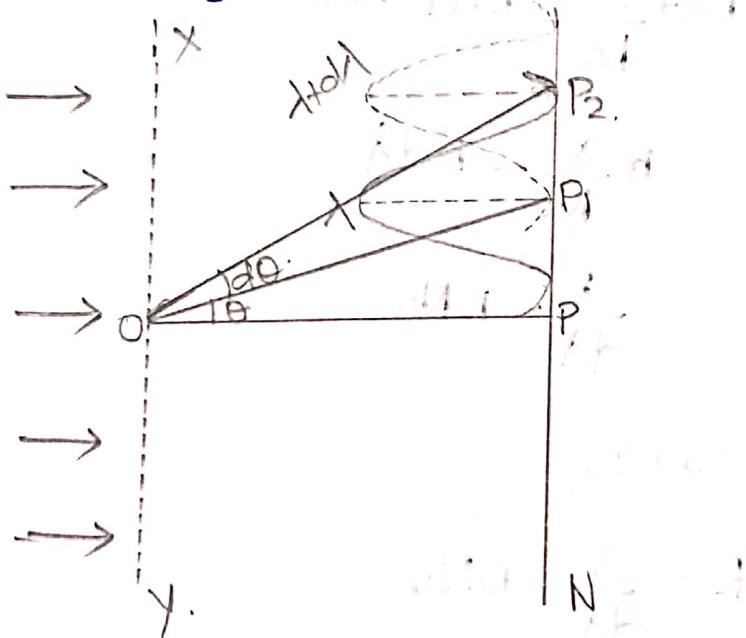
With Aperture Unresolved, the two patterns overlap significantly and cannot be distinguished as separate. Just resolved, the two patterns are well separated by a distinct gap. If the aperture is increased further, the patterns will merge again and become unresolved.

## Resolving Power of Grating

The spectral resolving power of a grating is the smallest wavelength interval ( $d\lambda$ ) that can be detected by it. It is the ability of grating to distinctly separate two very close spectral lines.

$$\text{Resolving power, } R = \frac{\lambda}{d\lambda},$$

where  $d\lambda$  is the difference of two wavelengths which a grating can resolve.



Consider a parallel beam of light of wavelengths  $\lambda$  and  $\lambda + d\lambda$  incident normally on the plane transmission grating having grating element (a+b) and total number of lines 'N'.

The two wavelengths  $\lambda$  and  $(\lambda + d\lambda)$  will be

just resolved if the following two equations are satisfied.

For the principal maximum,

$$(a+b)\sin\theta = n(\lambda + d\lambda)$$

For secondary minimum,

$$(a+b)\sin\theta = n\lambda + \frac{\lambda}{N_0}$$

where,  $\frac{\lambda}{N_0}$  - path difference between secondary wavelets from two successive slits-

$$\therefore n\lambda + \frac{\lambda}{N_0} = n\lambda + nd\lambda$$

$$\therefore \frac{\lambda}{N_0} = nd\lambda$$

$$\frac{\lambda}{d\lambda} = nN_0$$

Resolving power,

$$R = \frac{\lambda}{d\lambda} = nN_0$$

Thus, the resolving power of a grating depends on the total number of lines and the order of the spectrum.

## Dispersive Power of Grating

Dispersive power is the change in the angle of diffraction per unit change in wavelength.

The diffraction of the  $n^{\text{th}}$  order principal maximum,

$$(a+b)\sin\theta = n\lambda$$

Differentiating,

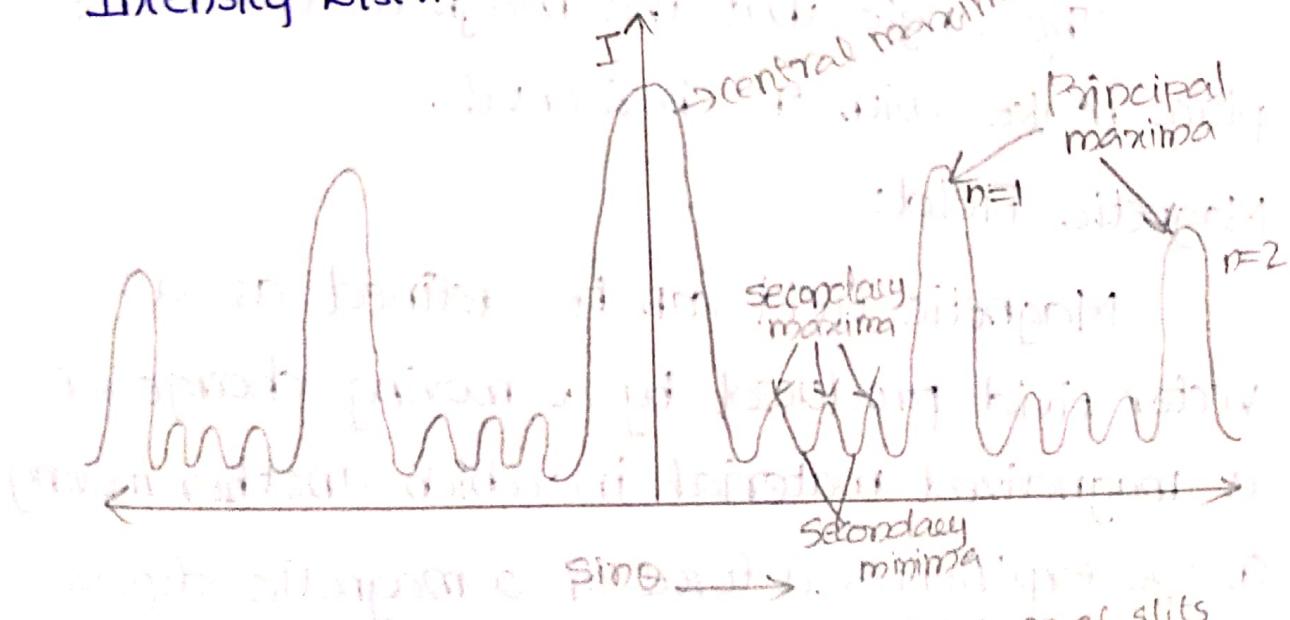
$$(a+b)\cos\theta = n \frac{d\lambda}{d\theta}$$

$$\therefore \frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

$$\boxed{\frac{d\theta}{d\lambda} = \frac{Nn}{\cos\theta}}$$

where,  $N = \frac{1}{ab}$

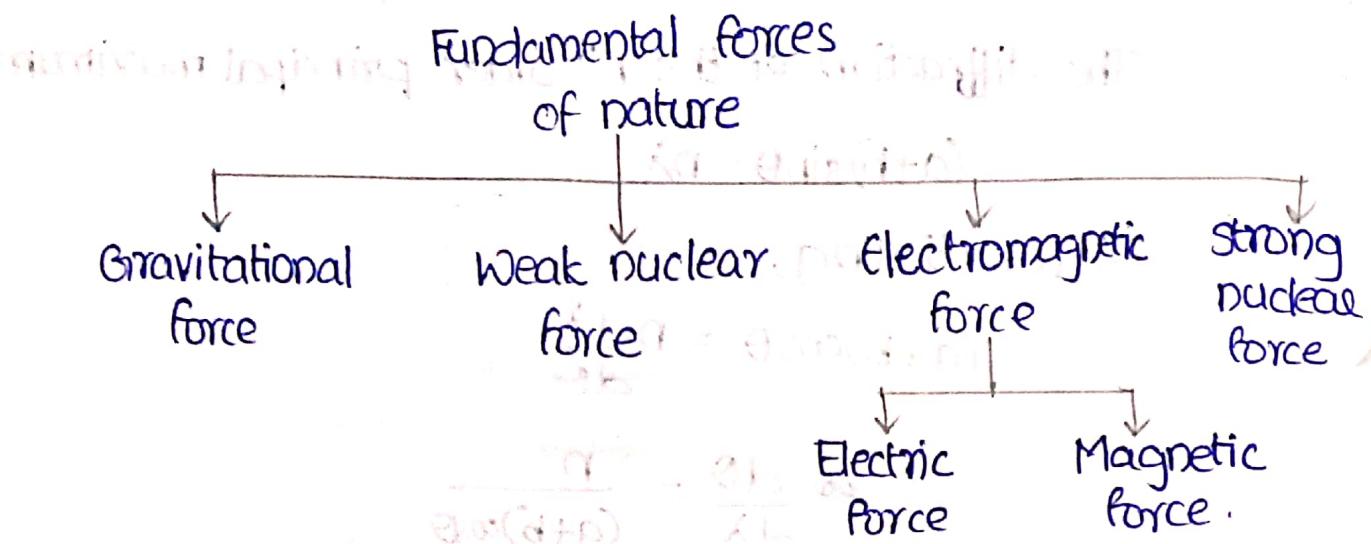
Intensity Distribution Due to  $N$  slits.



$$\begin{aligned} N &= \text{total no. of slits} \\ N_0 &= \text{no. of slits} \\ (N_0 - 1) &\Rightarrow \text{minima} \\ (N_0 - 2) &\Rightarrow \text{maxima} \end{aligned}$$

# MAGNETISM AND ELECTROMAGNETIC THEORY

## Magnetism



Magnetic declination:

Angle between magnetic and geometric axes which varies with places.

Magnetic inclination:

The angle that the magnetic field at a place makes with the horizontal.

Magnetic Field:

Magnetic field can be defined as a vector field produced by a moving charge or a magnetized material in which another moving charge experiences a force or a magnetic dipole experiences a torque.

The strength of magnetic field is quantitatively expressed by the vector quantity, magnetic induction (Magnetic Flux Density),  $B$ .

## Magnetic Flux Density / Magnetic Induction

Magnetic induction at a point is defined as the flux passing through unit area around the point. Its unit is  $\text{Wb/m}^2$  or Tesla.

## Magnetizing field ( $H$ ):

The magnetic field in which a material is kept is called magnetizing field,  $H$ . It represents the strength of the magnetic field. Units of  $H$  are  $\text{A/m}$  in SI system.  $B = \mu_0 H$

## Magnetization, $M$ :

It is defined as the magnetic dipole moment per unit volume developed inside a solid. Its unit is  $\text{A/m}$ .

$$M = \chi H$$

where,  $\chi$  is the proportionality constant and is known as magnetic susceptibility.

Magnetic susceptibility ( $\chi$ )  
It is the ratio of magnetization  $M$  to the magnetizing field  $H$ .

$$\text{Susceptibility, } \chi = \frac{M}{H}$$

Magnetic Permeability

The ability of material to support the formation of magnetic field within it is given by permeability ( $\mu$ ). Its value denotes the ease with which the magnetic field line pass through the material.

Magnetic permeability,  $\mu$  of a material is defined as the ratio of resultant magnetic field ' $B$ ' inside the material to the magnetizing field ' $H$ '.

$$\mu = \frac{B}{H}$$

Its unit is henry/metre ( $H/m$ ).

$$\text{Relative permeability, } \mu_r = \frac{\mu}{\mu_0}$$

$$B_o \propto H$$

$$B_o = \mu_0 H$$

$$B \propto H$$

$$B = \mu H$$

$$\frac{B}{B_0} = \frac{H}{H_0} = \mu_r$$

Relation connecting  $B$ ,  $H$  and  $\mu_r$

$$B = \mu_0 H r H$$

$$B = \mu_0 (H + M)$$

$$B = \mu_0 H \left(1 + \frac{M}{H}\right)$$

$$B = \mu_0 H \left(1 + \chi\right)$$

$$\chi = \mu_r - 1$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T/m}$$

## GAUSS' LAW FOR MAGNETIC FLUX DENSITY

Magnetic flux through a small elemental area,  $d\phi_B = \vec{B} \cdot d\vec{s}$

$$\text{Total magnetic flux, } \phi_B = \int \vec{B} \cdot d\vec{s}$$

Magnetic flux is scalar quantity and its unit is weber (wb).

$$1 \text{ wb} = 1 \text{ Tm}^2 = (\text{Nm})/\text{A}$$

Gauss' law in magnetism states that the net magnetic flux through any closed surface is zero.

$$\oint \vec{B} \cdot d\vec{s} = 0$$

This means it is impossible to isolate the magnetic poles and the simplest magnetic structure that can exist is a magnetic dipole.

From the equation, it is clear that the magnetic field lines always form closed loops

### DIFFERENTIATION OF SCALARS AND VECTORS.

Sir Hamilton introduced a differential operator  $\nabla$  (del) which is capable of differentiating both scalar and vector at equal ease. It is symbolically represented as,

$$\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$$

Three simple operations - gradient, divergence and curl - can be performed using  $\nabla$  operator.

The differentiation of a scalar function appears in the form of gradient and that of vector function as divergence and curl.

#### 1. GRADIENT:

If  $\phi(x, y, z)$  is a scalar function, then the product of  $\nabla$  and  $\phi$  is called gradient of scalar function.

$$\text{grad } \phi = \nabla \cdot \phi = \hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z}$$

- $\nabla \cdot \phi$  is a vector quantity function.
- Gradient of a scalar function gives the unique direction in which the function changes most rapidly.

e.g., If  $\phi$  is temperature, then gradient of  $\phi$  represents the temperature gradient. It gives the net rate of change of temperature with position coordinates and its direction gives the direction in which this change is maximum.

### Physical Significance:

- Its magnitude is equal to max. rate of rise of scalar field.
- Directed along the direction in which max. change occurs.
- Gradient represent the combination of rate of change in  $x, y, z$  at a given point in three or higher dimensional curve.

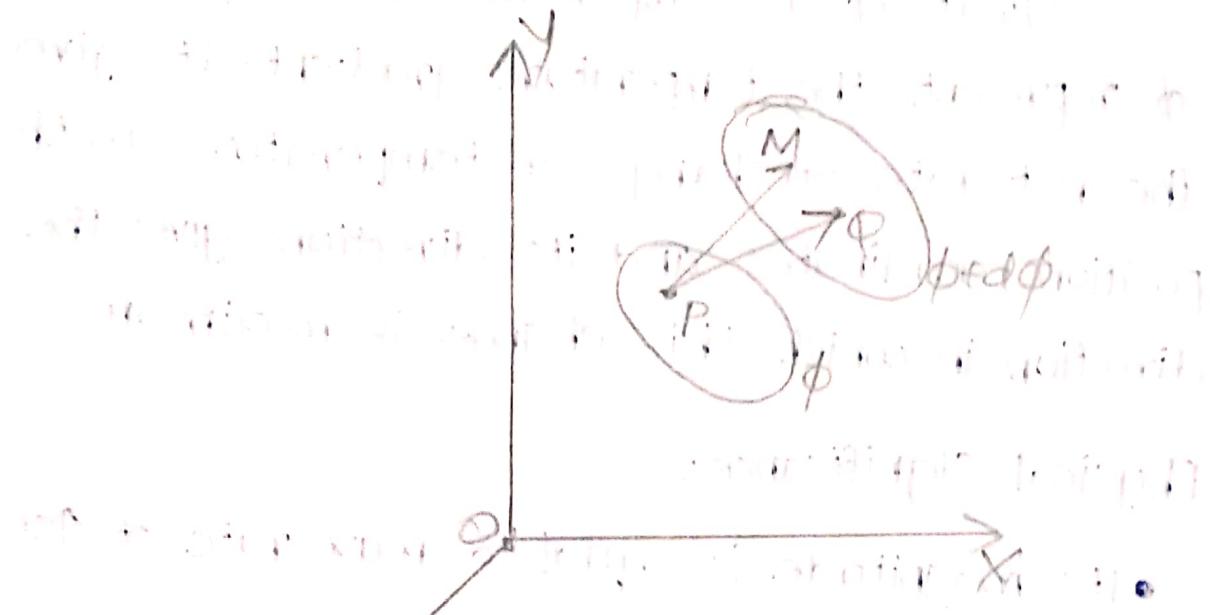
Consider a scalar function  $\phi(\vec{r})$ ,

$$\text{where, } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

If a surface  $\phi(x, y, z) = c$  drawn through any point such that the function has same value everywhere on the surface, it is called a level surface of the function  $\phi$ .

Let  $P(\vec{r})$  be a point on one level surface

and  $Q(\vec{r} + d\vec{r})$  a point on neighbouring level surface.



$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz$$

$$\nabla \phi = \hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z}$$

$$d\vec{r} = \hat{i} dx + \hat{j} dy + \hat{k} dz$$

$$\nabla \phi \cdot d\vec{r} = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz$$

$$\nabla\phi \cdot d\mathbf{r} = d\phi$$

\* If  $\mathbf{Q}$  is a point on the same level surface,  
 $d\phi = 0$ .

$$\nabla\phi \cdot d\mathbf{r} = 0$$

•  $\Rightarrow \nabla\phi$  is perpendicular to level surface.

Consider a line PM  $\perp$  level surface.

$$d\phi = \nabla\phi \cdot ds = |\nabla\phi| ds \cos 0$$

This shows that  $\nabla\phi$  is along  $ds$  or along PM  
(outward normal to surface).

•  $\nabla\phi$  is the most rapid rate of variation  
of  $\phi$  in magnitude and direction (along normal)

$$\bullet \int_A^B \vec{V} \cdot d\vec{r} = \int_A^B \nabla\phi \cdot d\vec{r} = \int_A^B d\phi = \phi_B - \phi_A$$

(independent of path)

$\therefore$  closed line integral of the gradient of a scalar field is zero.

$$\oint \nabla\phi \cdot d\mathbf{r} = \oint d\phi = 0$$

## 2. DIVERGENCE

The dot product of vector differential operator

$\nabla$  with a ~~vector function~~ is known as the divergence  
of the vector function.

$$\text{Let } \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{\nabla} \cdot \vec{A} = \left( \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \cdot (A_x \hat{i} + A_y \hat{j} + A_z \hat{k})$$

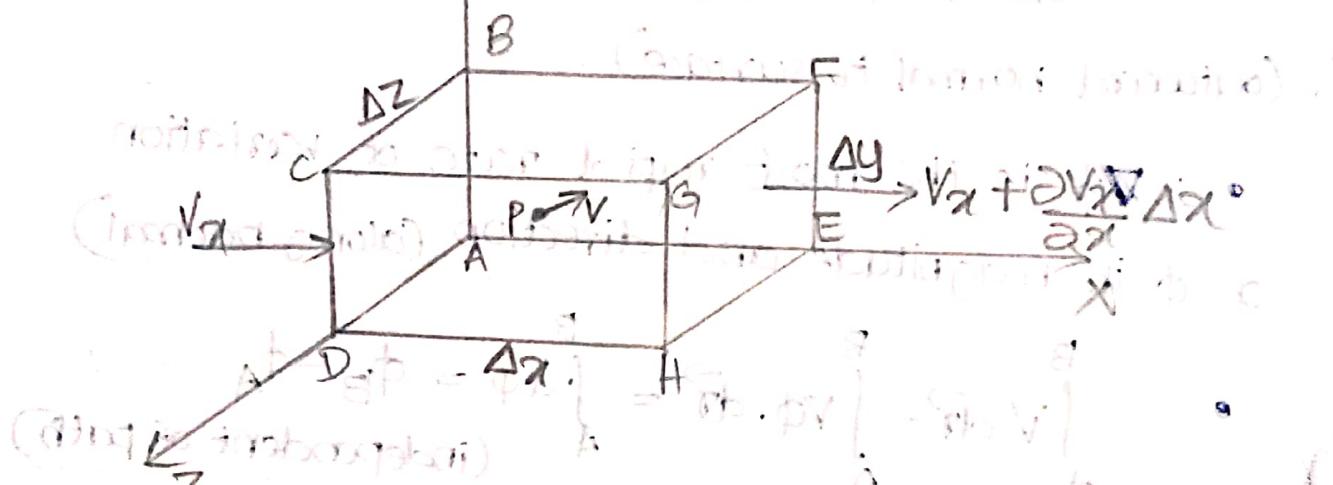
$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Physical significance:

$\nabla \cdot \vec{A}$  is zero if  $A_x, A_y, A_z$  are constant.

Outflow (flux) =  $\text{Area} \cdot \vec{v}_x = \Phi_x$

Net flux in the direction of flow along  $x$ -axis



Along  $x$ -direction, inflow =  $v_x \times \text{Area of ABCD}$

$$= v_x \Delta y \Delta z$$

$$\text{Outflow} = \left( v_x + \frac{\partial v_x}{\partial x} \Delta x \right) \Delta y \Delta z$$

Net outflow along  $x$ -axis.

Outflow rate = Outflow - Inflow

$$\text{Outflow rate} = \frac{\partial v_x}{\partial x} \Delta x \Delta y \Delta z$$

constant outflow rate = 0

Along  $y$ -axis,

$$\text{Net outflow} = \frac{\partial v_y}{\partial y} \Delta x \Delta y \Delta z$$

Along  $z$ -axis,

$$\text{Net outflow} = \frac{\partial v_z}{\partial z} \Delta x \Delta y \Delta z$$

$$\text{Total outflow} = \Delta x \Delta y \Delta z \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$

$$= (\vec{\nabla} \cdot \vec{v}) \Delta x \Delta y \Delta z \text{ for volume } V$$

Thus, divergence of a vector function at a point is the net outflow per unit volume in unit time at that point.

Physical significance:

- Valid at a point.
- gives the rate per unit volume at which a physical quantity is emanating from that point.
- Divergence of a vector field is:
  - \* positive — at source point
  - \* negative — at sink point
  - \* zero — where there is neither sink nor source.

In fluid dynamics,

$$\nabla \cdot (PV) = -\frac{\partial P}{\partial t}$$

- \* Divergence (+ve) - fluid expands, density decreases.
  - \* Divergence (-ve) - fluid contracts, density increases.
  - \* Divergence (0) - Incompressible fluid ( $\nabla \cdot \vec{V} = 0$ )
- A vector such as  $\vec{V}$  whose divergence is zero is called solenoidal.

### 3. CURL:

The cross product of differential operator  $\nabla$  with a vector point function is known as 'curl of the function'.

Let  $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{i} \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{j} \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right)$$

$$+ \hat{k} \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

### Physical Significance Of Curl

- Measure of circulation.
- There are vector fields which cannot be expressed as the gradient of a scalar point functions. For such fields, line integral along a

closed path is not zero. The term, 'curl of a vector field' concerns such fields only.

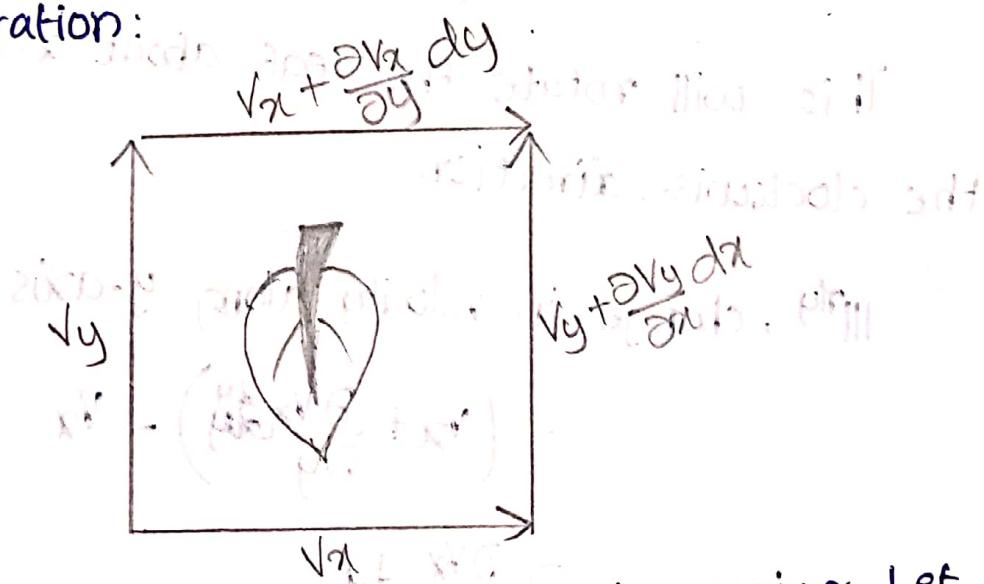
- curl of the vector field: value of the maximum line integral of a vector field per unit area at a certain point.

- Curl A or rot A is associated with some sort of rotation or circulation.

- When curl A is non-zero, the vector field must have circulation.

- When curl ~~A~~  $|\nabla \times A| = 0$ , there will be no circulation and the vector field is called as 'irrotational' or non-curl field.

#### Illustration:



width of the river: X-axis

flow of water: Y-axis

depth of river: Z-axis

Let us visualize a floating leaf flowing down the stream.

Rate of rise of  $y$ -component of velocity in  $x$ -dir.

$$= \frac{\partial v_y}{\partial x} dx$$

Increased velocity in unit length on right side

of the leaf  $= v_y + \frac{\partial v_y}{\partial x} dx$

Change in velocity along  $x$ -axis,

$$= \left( v_y + \frac{\partial v_y}{\partial x} dx \right) - v_y$$

$$= \frac{\partial v_y}{\partial x} dx$$

This will rotate the leaf about  $z$ -axis in the clockwise direction.

III<sup>ly</sup>, change in velocity along  $y$ -axis

$$= \left( v_x + \frac{\partial v_x}{\partial y} dy \right) - v_x$$

$$= \frac{\partial v_x}{\partial y} dy$$

This will rotate the leaf about  $z$ -axis in the anticlockwise direction.

$$\begin{aligned}
 \text{curl } &= V_x dx + \left( V_y + \frac{\partial V_y}{\partial x} dx \right) dy - V_y dy - \left( V_x + \frac{\partial V_x}{\partial y} dy \right) dx \\
 &= V_x dx + V_y dy + \frac{\partial V_y}{\partial x} dx dy - V_y dy - V_x dx - \frac{\partial V_x}{\partial y} dy dx \\
 &= \left( \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right) dy dx.
 \end{aligned}$$

If the curl of a vector is not zero, it is rotational; otherwise irrotational.

Gauss' divergence Theorem

$$\int_S \vec{A} \cdot d\vec{s} = \int_V \nabla \cdot \vec{A} dV$$

The surface integral of a vector function  $\vec{A}$  taken over a closed surface  $S$  is equal to the volume integral of the divergence of the vector function taken over the volume  $V$  bounded by the surface.

Stokes' Theorem:

$$\int_S \text{curl } \vec{A} \cdot d\vec{s} = \oint_C \vec{A} \cdot d\vec{l}$$

surface integral of the curl of a vector-function  $\vec{A}$  taken over a closed

function A taken over a surface is equal to the line integral of the vector function taken over the boundary of the surface.

### EQUATION OF CONTINUITY,

Equation of continuity is based on the principle of conservation of charge.

• Unbalanced generation of charge

• Unbalanced disappearance of charge

$$\nabla \cdot A = \rho$$

A current which has no longitudinal positive and negative charges is called unbalanced current. It is due to difference between two balanced currents of similar sign and two unbalanced currents without either positive or negative sign.

$$\nabla \cdot A = \rho$$

→ If there is no unbalance in the current then it is balanced current.

14/10/2019

## QUANTUM MECHANICS

### Classical Mechanics.

The three laws of conservation, namely conservation of linear momentum, angular momentum and energy formed the basis for classical mechanics. Classical mechanics was developed assuming that particles are localized.

### Quantum Theory

In 1900, Max Planck put forward a revolutionary hypothesis that the molecules in a source emit energy not continuously but in small discrete packets called quanta. Magnitude of each packet is given by  $E = h\nu$ .

In 1905, Einstein used this theory to explain photoelectric effect. In 1913, Bohr made use of Planck's hypothesis to explain stability of atom.

### Blackbody Radiation.

A perfect emitter or absorber of radiation is called a blackbody. When a perfect blackbody is heated, it emits radiation at all frequencies and thus, it is a good radiator as well as a good

absorber. The radiation emitted by a perfect blackbody is called blackbody radiation. It depends only on the temperature of body not on the shape and size composition of the body.

### de-Broglie Hypothesis

In 1924, Louis de Broglie proposed that matter has dual character just like radiation. He reasoned out that nature exhibits a great amount of symmetry. Therefore, if a light wave can act as a wave sometimes and as a particle at other times, then particles such as electrons should also act as waves at times. This is known as de Broglie hypothesis.

According to de-Broglie, any moving particle is associated with a wave. The waves associated with particles are known as de Broglie waves or matter waves.

### Wavelength of a Photon.

$$E = h\nu$$

$$E = mc^2$$

$$h\nu = mc^2$$

$$\frac{hc}{\lambda} = mc^2$$

$$\lambda = \frac{h}{mc}$$

where, m - mass of photon

c - velocity of photon

$$\lambda = \frac{h}{P}$$

P - momentum of photon.

In case of a material body,

$$\lambda = \frac{h}{mv}$$

Note:

We cannot measure the wavelength of macroscopic particles. Wave nature of microscopic particles is significant.

wavelength of an Electron

If an electron is accelerated through a potential of 'V' volt, then,

$$\frac{1}{2}mv^2 = eV \text{ Joules}$$

$$v = \sqrt{\frac{2eV}{m}}$$

$$\lambda = \frac{h}{P} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2eV}{m}}}$$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{h}{\sqrt{2mk}}, K - \text{kinetic energy}$$

$$\lambda = \frac{150}{\sqrt{V}} \text{ Å}^{\circ}, V - \text{potential}$$

de-Broglie Waves are Insignificant in Case of Macro bodies.

de-Broglie wavelength of waves is given by,

$$\lambda = \frac{h}{mv}$$

As the mass 'm' of the body increases, the wavelength tends to be insignificant. Therefore, the wavelength associated with macroparticles become insignificant in comparison to the size of the bodies themselves even at very low velocities. Because of the smaller magnitude of Planck's constant 'h', the wavelength  $\lambda$  will be significant only in case of micro-particles.

For example, consider a cricket ball of mass 500g flying with a velocity of 50 km/hr, then,

$$\lambda = \frac{6.62 \times 10^{-34} \text{ Js}}{0.5 \text{ kg} \times 13.9 \text{ m/s}} = 10^{-34} \text{ m} = 10^{-24} \text{ Å}^{\circ}.$$

This wavelength is insignificant in comparison to the size of the ball.

### Heisenberg Uncertainty Principle.

In 1926, Werner Heisenberg formulated a coherent mathematical structure, later known as matrix mechanics for the quantum theory of atoms. At around the same time, Schrodinger viewed the quantum as the apparent atomicity of matter disguised under the underlying continuum. Schrodinger's theory came to be known as wave mechanics.

The uncertainty principle states that it is impossible to know both the exact position and exact momentum of an object at the same time.

$$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2}$$

#### Applications:

1. Absence of electrons inside a nucleus.
2. Uncertainty in the frequency of light emitted by an atom - Natural Line Broadening.
3. Radius of Bohr's First Orbit

## Wave Function and its Physical Significance

The waves are described by a certain quantity that varies with position and time.

This quantity  $\psi$  is called a wave function.

$\psi$  is a mathematical function which describes

the state of a particle or a system. It is a function of position co-ordinates and time.

It is, in general, a complex function. It represents the state of a quantum mechanical system.

### Physical Significance:

$\psi$  has no direct physical significance,

as it is not an observable quantity. However,

the value of the wave function is related to the probability of finding the particle at a given

place at a given time.

$\psi^* \psi$  or  $|\psi|^2$  represents the probability density or the probability of finding the particle in unit volume.

$\psi^* \psi d\tau$  represents the probability of finding the particle in ' $d\tau$ ' volume.

If a particle exists in a given region of space, the total probability of finding the particle in that region is one.

$$\text{i.e., } \int_{-\infty}^{\infty} \psi^* \psi d\tau = 1$$

This is called normalisation condition and a wave function that obeys this equation is said to be normalised.

Conditions to be satisfied by  $\psi$ -function

An acceptable wave function  $\psi$  must be normalized and fulfill the following requirements:

- $\psi$  function must be single-valued:  
A function related to a physical quantity cannot have more than one value at that point. If it has more than one value at a point, it means that there is more than one value of probability of finding the particle at that point.

- $\psi$  function must be finite:

If  $\psi$  is infinite, it would imply an infinitely large probability of finding the particle at that point. This would violate the uncertainty principle.

- $\psi$  function must be continuous.

Since  $\psi$  is related to a physical quantity, it cannot have a discontinuity at any point.

Therefore, the wave function  $\psi$  and its space derivatives  $\frac{\partial\psi}{\partial x}$ ,  $\frac{\partial\psi}{\partial y}$  and  $\frac{\partial\psi}{\partial z}$  should be continuous across any boundary.

- $\psi$  must be normalisable.

28/10/2019

## SCHRÖDINGER WAVE EQUATION

This is the fundamental equation in quantum mechanics. This equation defines the wave properties of electrons and also predicts their particle-like behaviour.

### Time Dependent Schrödinger Wave Equation

consider a particle moving in positive  $x$ -direction. Differential equation of wave associated with that particle is

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{V^2} \frac{\partial^2 \psi}{\partial t^2}$$

Solution is  $\psi = ae^{i(Kx - \omega t)}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h/p} = \frac{2\pi p}{h} = \frac{P}{\hbar}$$

$$\omega = 2\pi\nu = \frac{2\pi E}{\hbar} = \frac{E}{\hbar}$$

$$\psi = ae^{i\left(\frac{Px}{\hbar} - \frac{Et}{\hbar}\right)}$$

$$\boxed{\psi = ae^{\frac{i}{\hbar}(Px - Et)}} \rightarrow \textcircled{1}$$

This is the wave equation.

Differentiating  $\psi$  twice with respect to  $x$ ,

$$\frac{\partial \psi}{\partial x} = ae^{\frac{i}{\hbar}(Px - Et)} \times \frac{i}{\hbar} P$$

$$\frac{\partial^2 \psi}{\partial x^2} = ae^{\frac{i}{\hbar}(Px - Et)} \times \frac{i}{\hbar} P \times \frac{i}{\hbar} P$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\psi \frac{P^2}{\hbar^2}$$

$$\boxed{P^2 \psi = -\hbar^2 \frac{\partial^2 \psi}{\partial x^2}} \rightarrow \textcircled{2}$$

Differentiating  $\textcircled{1}$  with respect to  $t$ ,

$$\frac{\partial \psi}{\partial t} = ae^{\frac{i}{\hbar}(Px - Et)} \times \frac{-i}{\hbar} E$$

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar} E \psi$$

$$E \psi = -\frac{\hbar}{i} \frac{\partial \psi}{\partial t}$$

$$E \psi = i \hbar \frac{\partial \psi}{\partial t} \quad \rightarrow \textcircled{3}$$

Total energy of the particle is given by

$$E = KE + PE$$

$$E = \frac{P^2}{2m} + V \quad \rightarrow \textcircled{4}$$

Multiplying  $\textcircled{4}$  by  $\psi$ ,

$$E \psi = \frac{P^2}{2m} \psi + V \psi$$

$$i \hbar \frac{\partial \psi}{\partial t} = \frac{1}{2m} \left( -\hbar^2 \frac{\partial^2 \psi}{\partial x^2} \right) + V \psi$$

$$i \hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V \psi$$

This equation is known as time dependent

Schrödinger wave equation.

It can be written as

$$\left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V \right] \psi = i \hbar \frac{\partial \psi}{\partial t}$$

when extended to three-dimensional case,

$$-\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi + V\psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + V \right] \psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$H\psi = i\hbar \frac{\partial \psi}{\partial t}$$

where,  $H$  is called Hamiltonian operator and  $\psi$  is a function of position coordinate and time.

### Time Independent Schrödinger Wave Equation

In a number of cases the potential energy  $V$  of a particle does not depend on time; it varies with the position of the particle only and the field is said to be stationary. The wave function can be written as a product of function of  $x$ ,  $\psi(x)$  and a function of  $t$ ,  $\phi(t)$ .

$$\Psi(x, t) = \psi(x)\phi(t) \quad \rightarrow ①$$

Differentiating with respect to  $x$ ,

$$\frac{\partial \Psi}{\partial x} = \phi \frac{\partial \psi}{\partial x}$$

$$\frac{\partial^2 \Psi}{\partial x^2} = \phi \frac{\partial^2 \phi}{\partial x^2} \longrightarrow \textcircled{2}$$

Differentiating ① with respect to t,

$$\frac{\partial \Psi}{\partial t} = \phi \frac{\partial \phi}{\partial t} \longrightarrow \textcircled{3}$$

Sub. ② and ③ in time dependent Schrödinger equation,

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$$-\frac{\hbar^2}{2m} \phi \frac{\partial^2 \phi}{\partial x^2} + V \phi \phi = i\hbar \phi \frac{\partial \phi}{\partial t}$$

→ ÷ing throughout by  $\phi \phi$ ,

$$-\frac{\hbar^2}{2m} \frac{1}{\phi} \frac{\partial^2 \phi}{\partial x^2} + V = \frac{i\hbar}{\phi} \frac{\partial \phi}{\partial t} \longrightarrow \textcircled{4}$$

Here, LHS is a function of x only and RHS is a function of t only. So, both LHS and RHS should be equal to the same separation constant. Let the separation constant be K.

$$-\frac{\hbar^2}{2m} \frac{1}{\Psi} \frac{\partial^2 \Psi}{\partial x^2} + V = K \longrightarrow \textcircled{4a}$$

$$i\hbar \frac{\partial}{\partial t} \frac{1}{\phi} \frac{\partial \phi}{\partial t} = k \rightarrow ④b$$

$$\frac{\partial \phi}{\phi} = \frac{k}{i\hbar} dt = -\frac{i^2 k}{i\hbar} dt = -\frac{i}{\hbar} k dt$$

Integrating,

$$\int \frac{\partial \phi}{\phi} = \int -\frac{i}{\hbar} k dt$$

$$\ln \phi = -\frac{i}{\hbar} kt$$

$$\boxed{\phi = e^{-\frac{i}{\hbar} kt}} \rightarrow ⑤$$

Sub. ⑤ in ①,

$$\Psi(x, t) = \psi(x) \cdot e^{-\frac{i}{\hbar} kt}$$

$$\text{Differentiating, } \frac{\partial \Psi}{\partial t} = \psi e^{\frac{i}{\hbar} kt} x - \frac{i}{\hbar} k \psi$$

$$\frac{\partial \Psi}{\partial t} = \frac{-i}{\hbar} k \Psi$$

$$k\Psi = -\frac{\hbar}{i} \frac{\partial \Psi}{\partial t} = i\hbar \frac{\partial \Psi}{\partial t}$$

$$k\Psi = E\Psi$$

$$k = E$$

$$\therefore \textcircled{A} \Rightarrow -\frac{\hbar^2}{2m} \frac{1}{\Phi} \frac{\partial^2 \Phi}{\partial x^2} + V = E$$

Multiplying by  $\Phi$ ,

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Phi}{\partial x^2} + V\Phi = E\Phi$$

$$\boxed{\frac{\partial^2 \Phi}{\partial x^2} + \frac{2m(E-V)}{\hbar^2} \Phi = 0}$$

This equation involves only space coordinates and is called the time-independent Schrödinger wave equation.

Note: (not in syllabus)

### OPERATORS

An operator is an entity which when applied to a function, transforms it into a new function.

The operator  $\hat{A}$  is called a linear operator

if it satisfies the following conditions:

- $\hat{A}(\Phi_1 + \Phi_2) = \hat{A}\Phi_1 + \hat{A}\Phi_2$

- $\hat{A}(c\Phi) = c\hat{A}\Phi$

Only linear operators are used in quantum mechanics.

## Important Operators:

The most important operators of quantum mechanics are position operator, momentum operator, energy operator and Hamiltonian operator.

- $E\psi = i\hbar \frac{\partial \psi}{\partial t}$

Here,  $i\hbar \frac{\partial}{\partial t} \Rightarrow$  Energy operator.

- $P\psi = -i\hbar \frac{\partial \psi}{\partial x}$

$-i\hbar \frac{\partial}{\partial x} \Rightarrow$  Momentum operator

OR  $-i\hbar \nabla$

- $\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Rightarrow$  Kinetic Energy operator.

- $\left[ \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V \right] \Rightarrow \hat{H}$  operator.

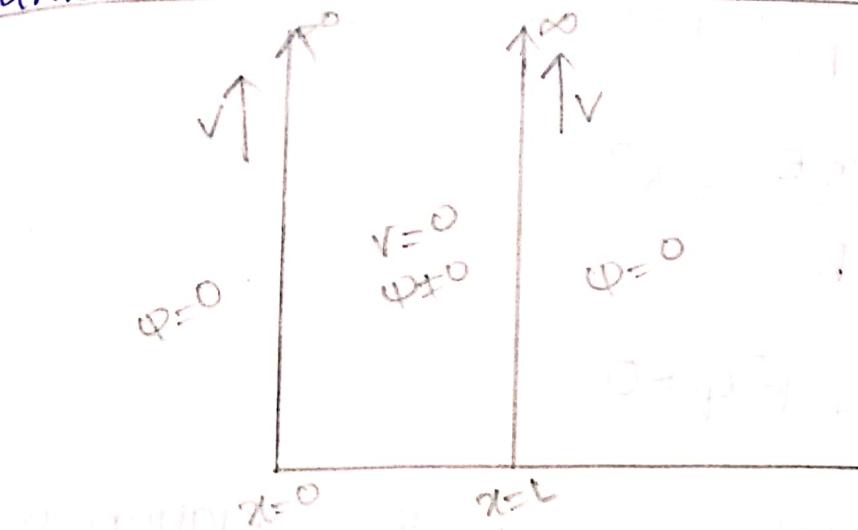
- $\hat{A}\psi = \lambda\psi$

$\lambda$  is called Eigen value and this equation is called Eigen value equation.

$\psi$  - Eigen state / Eigen function

## Applications Of Schrodinger Equation In Quantum Mechanics

# 1. Particles in one-dimensional box or potential well.



A potential well is a potential energy function  $V(x)$  that has a minimum. Consider a particle confined to the region  $0 < x < L$ . It can move freely within the region  $0 < x < L$  but subject to strong forces at  $x=0$  and  $x=L$ . Therefore, it can never cross to the right of the region  $x > L$  or to the left of  $0$ . It means that  $V=0$  in the region  $0 < x < L$  and rises to infinity ( $V=\infty$ ) at  $x=0$  and  $x=L$ . This situation is called a one-dimensional potential box.

For a particle trapped in one-dimensional box,  $V=0$ .

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m(E-V)}{\hbar^2} \psi = 0$$

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{2mE}{\hbar^2} \Phi = 0 \longrightarrow \textcircled{1}$$

$$\text{Let } \frac{2mE}{\hbar^2} = k^2$$

$$\frac{\partial^2 \Phi}{\partial x^2} + k^2 \Phi = 0$$

The general solution of the equation is

$$\Phi(x) = Ae^{ikx} + Be^{-ikx} \longrightarrow \textcircled{2}$$

$$\text{At } x=0; \Phi=0$$

$$0 = Ae^{ik0} + Be^{-ik0}$$

$$0 = A + B$$

$$B = -A \longrightarrow \textcircled{3}$$

Sub. \textcircled{3} in \textcircled{2},

$$\Phi = Ae^{ikx} - Ae^{-ikx}$$

$$\Phi = A[e^{ikx} - e^{-ikx}]$$

$$e^{i\theta} - e^{-i\theta} = 2i\sin\theta$$

$$\Phi = 2Ai \sin kx$$

$$\boxed{\Phi = C \sin kx}$$

where,  $C = 2Ai$

When,  $\varphi = 0$ ;  $x = L$

$$0 = C \sin kL$$

$C \neq 0$

$$\therefore \sin kL = 0$$

$$kL = n\pi$$

$$k = \frac{n\pi}{L}$$

$$\therefore \varphi = C \sin \frac{n\pi x}{L}$$

'C' can be obtained from the normalization condition.

$$\int_0^L \psi^* \psi dx = 1$$

$$\int_0^L C^2 \sin^2 \frac{n\pi x}{L} dx = 1$$

$$C^2 \int_0^L \sin^2 \frac{n\pi x}{L} dx = 1$$

$$\frac{C^2}{2} \int_0^L \left[ 1 - \cos \frac{2n\pi x}{L} \right] dx = 1$$

$$\frac{C^2}{2} \left[ x - \left( \sin \frac{2n\pi x}{L} \right) \times \frac{1}{2n\pi} \right]_0^L = 1$$

$$\frac{C^2}{2} \left[ L - \frac{L}{2\pi n} \sin 2n\pi - 0 \right] = 1$$

$\sin 2n\pi = 0$

$$\frac{C^2}{2} L = 1$$

$$C^2 = \frac{2}{L}$$

$$C = \sqrt{\frac{2}{L}}$$

$\therefore \Phi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$ , where, L - width of one-dimensional box.

This is the wave function.

Energy:

$$k^2 = \frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{L^2}$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$n \neq 0$

This is the energy of the particle trapped inside the potential well.

Energy levels:

In quantum mechanics, the particle cannot have zero energy.

The minimum possible energy is

$$E_1 = \frac{\pi^2 \hbar^2}{2mL^2} \quad (n=1)$$

This energy is called zero-point energy. The zero-point energy is a consequence of the uncertainty principle. If the energy of the particle is zero, its momentum also would be zero and the uncertainty principle requires that the wavelength  $\lambda$  be infinite. In such case, the particle cannot be confined to the box. Therefore, the particle must have a certain minimum amount of kinetic energy.

$$\text{For } n=2, E_2 = \frac{4\pi^2 \hbar^2}{2mL^2} = 4E_1$$

$$\text{For } n=3, E_3 = 9E_1$$

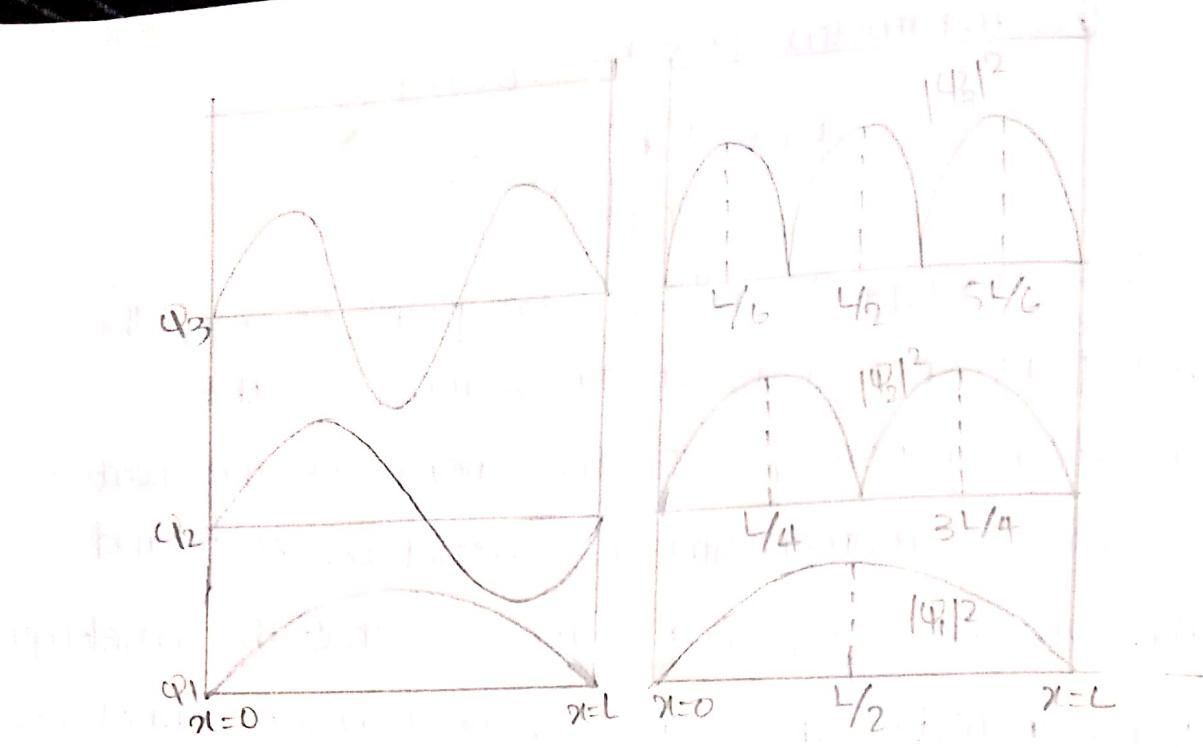
$$\text{For } n=4, E_4 = 16E_1$$

These are called the energy eigenvalues.

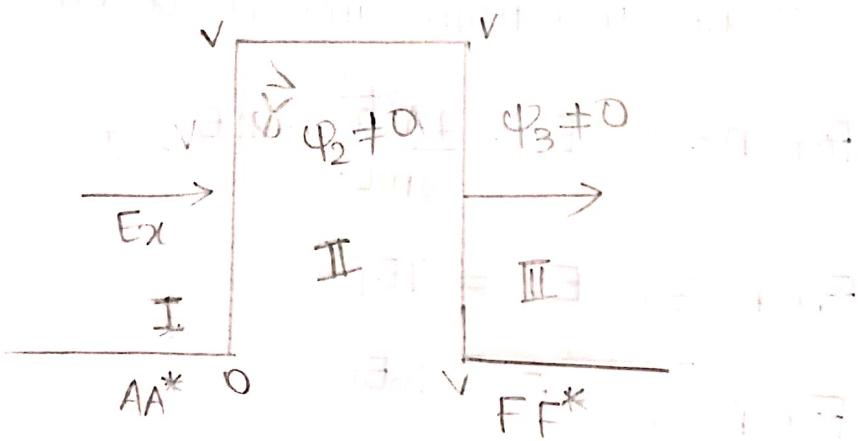
$$9E_1$$

$$4E_1$$

$$E = \frac{n^2 \hbar^2}{2mL^2}$$



## 2. Quantum Mechanical tunnelling



Even if  $E < V$ , according to quantum mechanics, there is a finite chance for the electron to leak to the other side of the barrier, i.e., there is a finite probability of finding particles in region I and region II. The electron tunneled through the barrier and hence in quantum mechanics,

the phenomenon of penetration of a barrier by a quantum particle is called tunneling.

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2mE}{\hbar^2} \psi = 0$$

Region I:  $V=0$

$$\frac{\partial^2 \psi_1}{\partial x^2} + k_1^2 \psi_1 = 0$$

$$\psi_1 = A e^{ik_1 x} + B e^{-ik_1 x} \quad k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Region II:

$$E < V$$

$$\frac{\partial^2 \psi_2}{\partial x^2} + \frac{2m(E-V)}{\hbar^2} \psi_2 = 0$$

$$\frac{\partial^2 \psi_2}{\partial x^2} - \frac{2m(V-E)}{\hbar^2} \psi_2 = 0$$

$$\psi_2 = C e^{-k_2 x} + D e^{k_2 x}$$

$$\psi_2 \neq 0$$

This does not represent a moving particle.

Region III:  $V=0$

$$\frac{\partial^2 \psi_3}{\partial x^2} + \frac{2mE}{\hbar^2} \psi_3 = 0$$

$$\varphi_3 = F e^{ik_1 x} + G e^{-ik_1 x} \quad (k_1 = k_3)$$

In region III, no particles are reflected since there is no boundary. Hence,  $G$  can be neglected.

$$\therefore \varphi_3 = F e^{ik_1 x}$$

$$\varphi_3 \neq 0$$

### Transmission Probability

Transmission probability is the ratio of flux of particles emerging to flux of particles incidenting.

Transmission probability,

$$T = \frac{F F^*}{A A^*} = \frac{|F|^2}{|A|^2}$$

$$T \propto e^{-2k_2 L}$$

Examples of tunnelling are Josephson tunnelling, Zener breakdown, etc.

### HEISENBERG's UNCERTAINTY PRINCIPLE

Contributions of Heisenberg:

1. Matrix Formulation

## 2. Heisenberg principle

It states that certain pairs of quantities cannot be determined simultaneously and accurately. It is an intrinsic property.

e.g., i) Position and momentum

$$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2}$$

If it is impossible to determine the position and momentum of a subatomic particle simultaneously.

$\Delta x$  and  $\Delta p_x$  are called canonically conjugate variables.

2) Energy and time

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

3) Angular displacement and angular momentum

$$\Delta \theta \cdot \Delta J \geq \frac{\hbar}{2}$$

## Applications of Uncertainty Principle

1. Absence of electrons inside nucleus.

Order of radius of a nucleus is about  $10^{-15}$  m. Therefore, for an electron to be confined

within such a nucleus, the uncertainty in its position may not exceed  $10^{-15}$  m, i.e.,

$$\Delta x \leq 10^{-15} \text{ m}$$

Let  $\Delta x = 5 \times 10^{-15} \text{ m}$

$$\Delta x \Delta p_x = \frac{\hbar}{4\pi} = 0.527 \times 10^{-34}$$

$$\Delta p_x = \frac{0.527 \times 10^{-34}}{5 \times 10^{-15}}$$

Velocity obtained is to maximum bar,  
 $= 0.1054 \times 10^9 \text{ m/s}$

Principle of Relativistic equation for energy is

$$E^2 = p^2 c^2 + m_0^2 c^4$$

where,  $m_0^2 c^4$  is the rest mass (which is very small and hence can be neglected).

$$E = p c$$

$$E = 1.054 \times 10^{20} \times 3 \times 10^8$$

$$\text{Significant value} = \frac{3.162 \times 10^{12}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 1.97 \times 10^7 \text{ eV}$$

$$\approx 20 \text{ MeV}$$

Experiments indicate that the electrons associated even with unstable atoms never have more than a fraction of this energy and thus, we can conclude that electrons cannot be present within nuclei.

## 2. Natural Line Broadening

(Uncertainty in the frequency of light emitted by an atom).

Consider the radiation of light from an "excited" atom. Such an atom remains in an excited state for about  $10^{-8}$  s.

$$\text{i.e., } \Delta t = 10^{-8} \text{ s.}$$

Uncertainty in measurement of energy is

$$\Delta E = \frac{h}{4\pi\Delta t}$$

$$= \frac{0.527 \times 10^{-34}}{10^{-8}}$$

$$= 0.527 \times 10^{-26} \text{ J}$$

$$E = h\nu$$

$$\Delta E = h\Delta\nu$$

$$h\Delta\nu = 0.527 \times 10^{-26}$$

Uncertainty in frequency,  $\Delta\nu = \frac{0.527 \times 10^{-26}}{6.625 \times 10^{-34}}$

$$= 7.95 \times 10^6 \text{ Hz}$$

$$\approx 8 \text{ MHz}$$

This is the irreducible limit to the accuracy with which we can determine the frequency of radiation emitted by an atom. That means the emitted spectral line is not sharp but has some width. This broadening of spectral line which cannot be reduced further is known as natural line broadening.

Q: Determine the de-Broglie wavelength of an electron having kinetic energy 1 eV.

$$eV = 1 \text{ eV}$$

$$V = 1 \text{ V}$$

$$\lambda = \sqrt{\frac{150}{V}}$$

$$= \sqrt{150}$$

$$= 12.27 \text{ Å}$$

Q: In jumping from an excited to a stationary state, an electron takes  $10^{-8}$  s. What is the uncertainty in the energy and frequency of emitted radiation?

$$\Delta E \cdot \Delta t = \frac{h}{4\pi}$$

$$\Delta E = \frac{0.527 \times 10^{-34}}{10^{-8}} = 0.527 \times 10^{-26} \text{ J}$$

$$\Delta E = h \Delta \nu$$

$$\Delta \nu = \frac{\Delta E}{h}$$

$$= \frac{0.527 \times 10^{-26}}{6.625 \times 10^{-34}}$$

$$= 7.95 \times 10^6 \text{ Hz}$$

$$\approx 8 \text{ MHz}$$

Q: Compare the uncertainties in the velocities of an electron and a proton confined to a box of  $10 \text{ Å}$ .

$$\Delta x \cdot \Delta p = \frac{h}{4\pi}$$

$$m \Delta x \Delta v = \frac{h}{4\pi}$$

$$\Delta V_e = \frac{h}{4\pi m_e \Delta x}$$

$$= \frac{0.527 \times 10^{-34}}{9.1 \times 10^{-31} \times 10 \times 10^{-10}} = 5.79 \times 10^4 \text{ ms}^{-1}$$

$$\Delta V_p = \frac{h}{4\pi m_p \Delta x}$$

$$= \frac{0.527 \times 10^{-34}}{1.67 \times 10^{-27} \times 10 \times 10^{-10}} = 31.5 \text{ ms}^{-1}$$

$$\frac{\Delta V_e}{\Delta V_p} = 1836$$

Q: A particle is moving in 1D potential box of infinite height and width  $25 \text{ Å}$ . Calculate the probability of finding the particle within an interval of  $5 \text{ Å}$  at the centre of the box when it is in a state of least energy.

$$\text{At centre, } x = \frac{L}{2} = \frac{25}{2} \text{ Å}$$

$$\Psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

$$= \sqrt{\frac{2}{25}} \sin \frac{\pi}{25}$$

$$= \sqrt{\frac{2}{25}}$$

$$\text{Probability} = |\Psi|^2 dx$$

$$= \frac{2}{25} \times 5 \\ = \underline{\underline{0.4}}$$

Q: Calculate first four energy values of an electron in a one-dimensional box of width  $L = 1\text{ nm}$  in eV. Also compute the probability for finding the particle between zero and  $\frac{L}{4}$  in the ground state.

$$E_1 = \frac{n^2 \pi^2 \hbar^2}{2m L^2}$$

$$= \frac{l^2 \pi^2 \hbar^2}{4\pi^2 \times 2m L^2}$$

$$= \frac{(6.625 \times 10^{-34})^2}{1.6 \times 10^{19} \times 8 \times 9.1 \times 10^{-31} \times 10^{-20}} \text{ eV}$$

$$= \underline{\underline{37.6 \text{ eV}}}$$

$$E_2 = 4E_1 = 37.6 \times 4 = 150.4 \text{ eV}$$

$$E_3 = 9E_1 = 37.6 \times 9 = 338.4 \text{ eV}$$

$$E_4 = 16E_1 = 37.6 \times 16 = \underline{\underline{601.6 \text{ eV}}}$$

$$\text{Probability, } |\psi|^2 = \int \psi^* \psi dx$$

$$= C^2 \int_0^L \sin^2 \frac{n\pi x}{L} dx$$

$$= \frac{C^2}{2} \left[ x - \left( \sin \frac{2n\pi x}{L} \right) \times \frac{L}{2\pi n} \right]_0^L$$

desirable area is  $\pi L^2 / 4$

$$= \frac{2}{\sqrt{\pi}} \left[ \frac{L^2}{4} - \sin \frac{2n\pi L}{L} \times \frac{L}{2\pi n} \right]$$

Probability of getting at least one defect =  $1 - \left[ \frac{1}{4} - \frac{1}{2\pi} \right]$

Since there are 4 defects in 1 bar, probability of getting at least one defect =  $0.09$

$$\frac{\sin nx}{x} = \beta$$

$$\frac{\sin nx}{x}$$

$$V_{d1} = \frac{(1 - \sin nx_1)}{x_1}$$

$$V_{d2} = \frac{1}{x_2}$$

$$V_{d3} = 0.09 \rightarrow P(x_3 < 0.09) = 0.9$$

$$V_{d4} = 0.09 \rightarrow P(x_4 < 0.09) = 0.9$$

$$V_{d5} = 0.09 \rightarrow P(x_5 < 0.09) = 0.9$$

## SUPERCONDUCTIVITY

### Superconductivity:

It is the phenomenon in which electrical resistance of materials suddenly disappears below a certain temperature. It is a state of matter exhibited usually at very low temperatures where the resistivity of the material drops to zero and becomes perfect diamagnetic.

Superconducting state is influenced by temperature, current and magnetic field. The low resistance for flow of current in metals is due to the scattering of the free electrons by vibrating ions of the lattice.

### Critical Temperature:

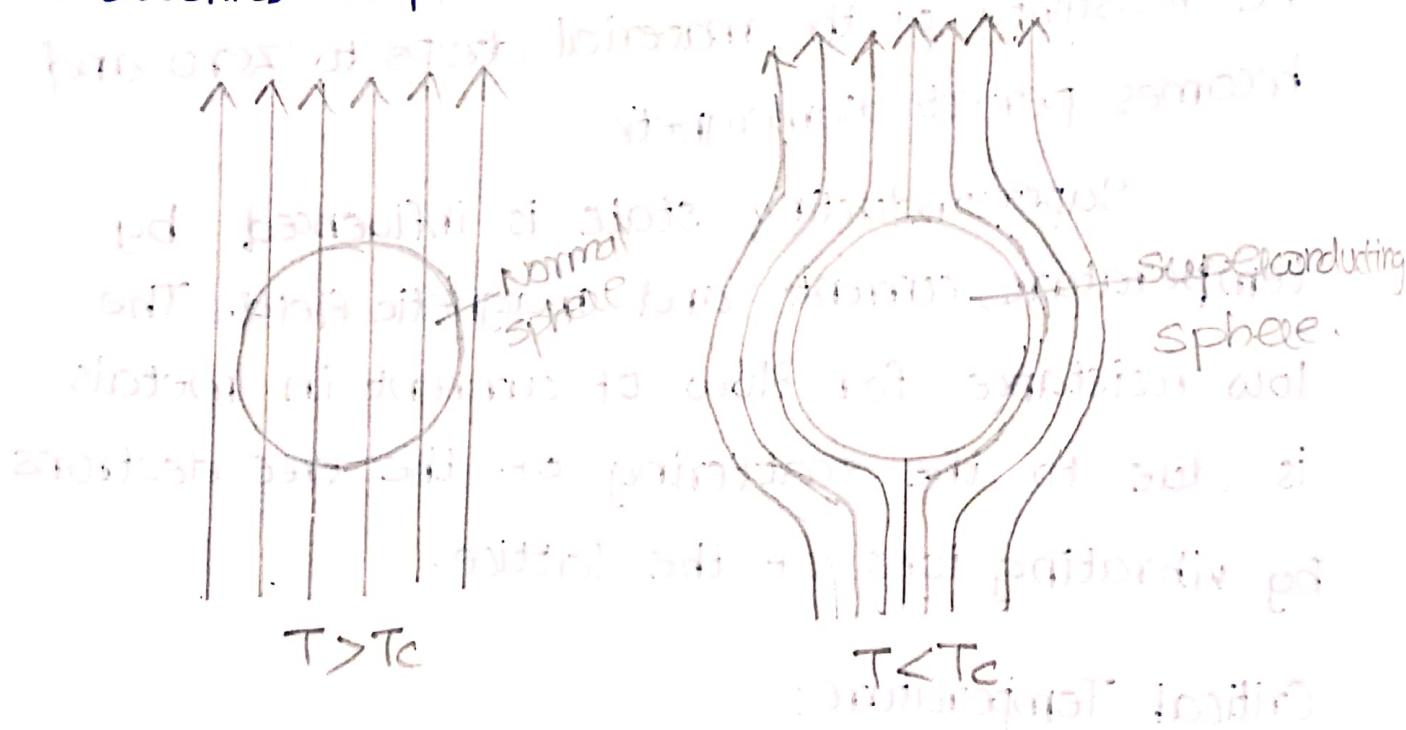
The temperature at which a normal material abruptly changes into a superconductor is called critical temperature or transition temperature ( $T_c$ ).

### MEISSNER EFFECT:

"The expulsion of magnetic flux when a material becomes superconducting in a magnetic

field is called Meissner effect".

In 1933, W. Hans Meissner and Robert Ochsenfeld found that when superconductors are cooled below their critical temperature in the presence of a magnetic field, the magnetic flux is expelled from the interior of the specimen and the superconductor becomes a perfect diamagnet.



They further showed that the effect is reversible. When the temperature is raised from below  $T_c$ , the flux suddenly penetrates the specimen at  $T=T_c$  and the material returns to the normal state.

Diagram showing flux expulsion from a superconductor

Magnetic induction inside the specimen,

$$B = \mu_0(H+M) = \mu_0(1+\chi)H$$

if magnetic volume,

In superconducting state; i.e., at  $T < T_c$ ,

$$B=0$$

$$\therefore \mu_0(H+M)=0$$

$$M=-H$$

$$\text{Susceptibility, } \chi = \frac{M}{H} = -1$$

This means, the superconductor is getting

magnetised in a direction opposite to the applied field and its susceptibility is  $-1$ . Thus, the substance in the superconducting state is a perfect diamagnet.

Note:

- Meissner effect contradicts the fundamental principles of electromagnetism.
- Meissner effect shows that in the superconductor, not only  $\frac{dB}{dt}=0$  but also  $B=0$ . Thus, two mutually independent properties, namely zero resistivity and perfect diamagnetism are the essential properties that characterize

the superconducting state.

### Applications:

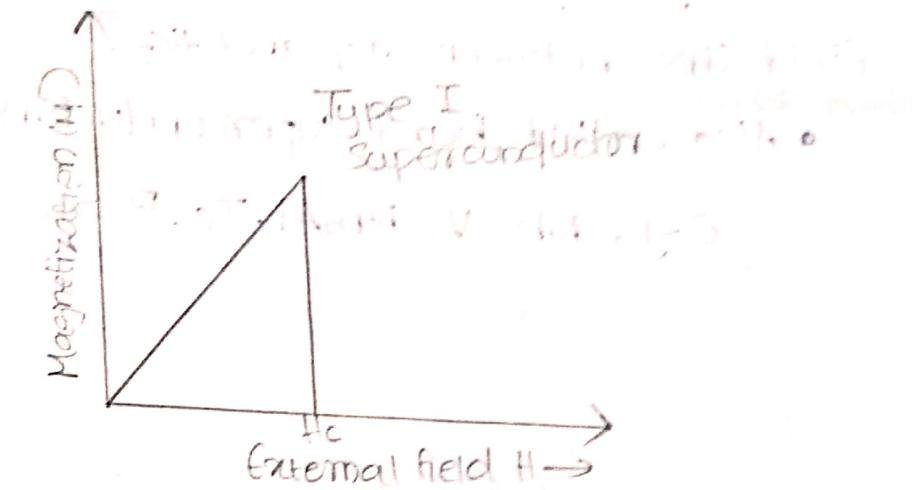
1. Used to prove whether a particular material is a superconductor or not.
2. Levitation effect (a smaller magnet repelled by a bigger superconductor hovers in air) is utilized in the operation of Maglev trains.

### TYPE-I AND TYPE-II SUPERCONDUCTORS

#### Type-I Superconductors:

- The transition from superconducting state to normal state in the presence of magnetic field occurs sharply at the critical value of  $H_c$ .
- Type I superconductors are perfectly diamagnetic below  $H_c$  and completely expel the magnetic field from the interior of the superconducting phase.
- Magnetization of the material grows in proportion to the external field up to the critical field strength  $H_c$  and then it suddenly drops to zero at the transition to the normal state.
- Poor carriers of electric current (can flow only in surface layer) e.g., Al, Pb and In

- Also called soft superconductors.

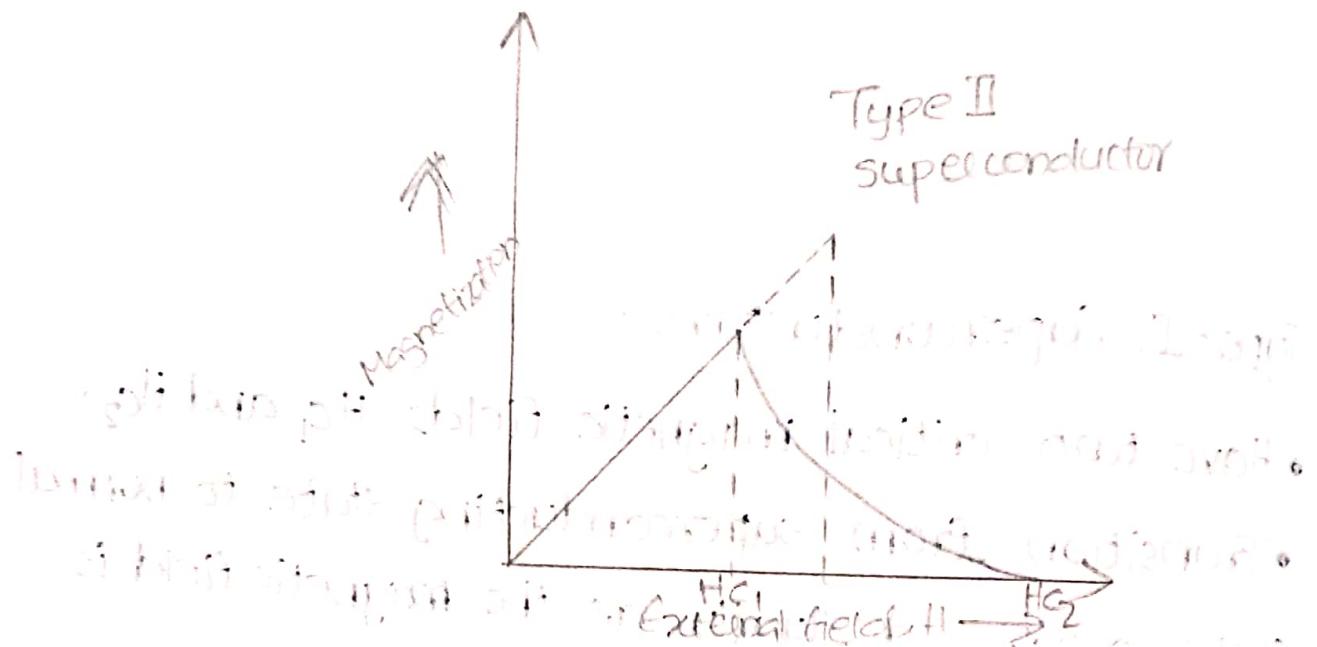


## Type-II Superconductors:

- Have two critical magnetic fields,  $H_{c1}$  and  $H_{c2}$ .
- Transition from superconducting state to normal state occurs gradually as the magnetic field is increased from  $H_{c1}$  to  $H_{c2}$ .
- Magnetization of the material grows in proportion to the external field up to the lower critical field  $H_{c1}$ . The external magnetic flux is expelled from the interior of the material till then. At  $H_{c1}$ , the magnetic field lines begin penetrating the material.
- Magnetic flux through the material increases as the magnetic field is raised further. At the upper critical field  $H_{c2}$ , the magnetization disappears totally and the external field completely penetrates and destroys the superconductivity.

- carry larger currents when the magnetic field lies between  $H_c$  and  $H_{c2}$ .
- Also called hard superconductors.

e.g., Nb, V,  $Nb_3Al$ , Tc, B, Si, ...



### Type-I

- Exhibit complete Meissner effect.
- Show perfect diamagnetic behaviour.
- Have only one critical magnetic field,  $H_c$ .
- No mixed state or intermediate state in case of these materials.

### Type-II

- Do not exhibit complete Meissner effect.
- Do not show perfect diamagnetic behaviour.
- Have two critical magnetic fields,  $H_{c1}$  and  $H_{c2}$ .
- Mixed state or intermediate state is present in these materials.

- Material loses magnetization abruptly.
- Highest value of  $H_c$  is about  $0.1 \text{ Wb/m}^2$ .
- Known as soft superconductors.
- Lead, tin, mercury are examples.
- Material loses magnetization gradually.
- Upper critical field is of the order of  $30 \text{ Wb/m}^2$ .
- Known as hard superconductors.
- $\text{Nb-Sn}, \text{Nb-Ti}, \text{Nb-Zr}, \text{V}-\text{Ga}$  are examples.

### Critical Magnetic Field

The minimum magnetic field which is necessary to regain the normal resistivity is called the critical magnetic field,  $H_c$ .

When the applied magnetic field exceeds the critical value  $H_c$ , the superconducting state is destroyed and the material goes into normal state.

As the magnetic field increases, the critical temperature decreases. The magnetic field approaches zero as  $T$  approaches  $T_c$ .

Critical magnetic field at temperature  $T$ ,

$$H_c(T) = H_c(0) \left[ 1 - \frac{T^2}{T_c^2} \right]$$

$H_c(0)$   $\Rightarrow$  critical magnetic field.

## BCS Theory of Superconductivity

In 1957, the American physicists, J. Bardeen, L.N. Cooper and J.R. Schrieffer developed the quantum theory of superconductivity, which came to be known as BCS theory. This theory assumes interaction of two electrons through quantized lattice vibrations. It successfully explained the effects like zero resistivity, Meissner effect etc.

The two principal features of BCS theory are:

- (i) cooper pairs, electron pairs are formed and propagate throughout the lattice.
- (ii) cooper pairs propagate without resistance as the electrons move in resonance with phonons.

BCS theory provides two important results, namely the existence of energy gap and the flux quantization.

- The e's of a cooper pair have a lower energy than two unpaired electrons. The theory predicted the existence of an energy gap between the ground state and first excited state. The energy gap represents the energy required to

break up a cooper pair. Hence, larger energy gaps correspond to more stable superconductors.

According to BCS theory, the energy gap at

OK is

$$E_g(0) = 2\Delta = 3.52 kT_c$$

(Theory from text)

## HIGH-TEMPERATURE SUPERCONDUCTORS

### Properties:

- Brittle in nature.
- Properties of normal state of these materials are highly anisotropic.
- The Hall coefficient (ratio of induced  $E$  to pdt of current density and applied  $B$ ) is positive indicating that the charge carriers are holes.
- behaviour cannot be explained by BCS theory.
- Isotope effect is absent in these materials.
- Magnetic properties of these materials are highly anisotropic.
- Type-II superconductors.
  - critical magnetic field tends to be higher for materials with a high  $T_c$ .

Year	Compound	Transition temperature (Tc)
1973	Film of Niobium and Ge	23 K
1974	Ba-Pb-Bi-O <sub>3</sub>	38 K
1987 (Paul Chu)	YBa <sub>2</sub> Cu <sub>3</sub> O <sub>7</sub> (YBCO)	90 K
1987 (Michel)	Bi-Sr-Ca-Cu-O (BSCCO)	107 K
1988 (Parkin)	Tl-Ba-Ca-Cu-O (TBCCO)	125 K
2000	Hg <sub>12</sub> Tl <sub>3</sub> Ba <sub>20</sub> Ca <sub>30</sub> Cu <sub>45</sub> O <sub>125</sub>	138 K

## APPLICATION OF SUPERCONDUCTORS

### (i) Power transmission:

If the electrical grid is made of superconducting wires rather than Al, there would be no need to transform the electricity to a higher voltage and then back down again.

### (ii) Transformers and electrical machines:

The transformers and electrical machines with superconducting coils generate greatly stronger magnetic fields than magnetic circuits that use ferromagnetic materials produce. The normal eddy current losses and hysteresis

losses will not be present in superconducting devices and hence the size of motors and generators will be drastically reduced.

### (iii) Diagnostic Equipments:

• High magnetic fields are essential in many areas of research and diagnostic equipments in medicine.

• Superconducting solenoids produce very strong magnetic field. They are small in size and are less cumbersome.

• They do not need either large power supplies or the means of removing heat. The low power requirement and simple cooling technique leads to a large saving in cost.

• Superconducting solenoids have improved the MRI, as they can be smaller and more efficient than an equivalent conventional magnet.

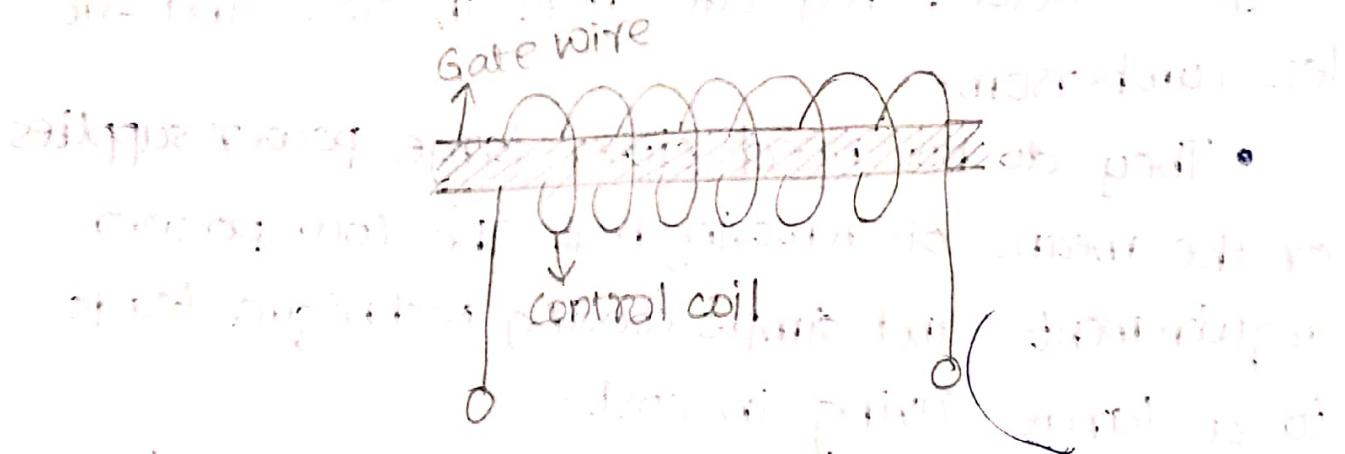
### (iv) Electronic switches:

Type II superconductors can be used as very fast electronic switches. This has allowed building a 4-bit computer microchip operating at about 500 times the speed of

earlier processors.

#### (V) Cryotrons:

The application of a magnetic field greater than its critical magnetic field changes the superconducting state of a superconducting material to normal state and removal of the field brings the material back from normal state to the superconducting state. This fact is used in developing cryotron switches.



#### (vi) Maglev Trains:

- Most spectacular application.
- Coaches of train do not slide over steel rails but float on a four inch air cushion above the track using superconducting magnets.
- This eliminates friction and energy loss as heat allowing the train to reach high speeds of the order of 500 km/hr.

- Levitation Effect is utilized.
- make train travel much faster, smoother and more efficient due to lack of friction between tracks and train.

#### (vii) SQUIDS:

A superconducting quantum interference device (SQUID) is a device used to measure extremely weak magnetic flux. The heart of a SQUID is a superconducting ring, which contains one or more Josephson junctions (superconductor-insulator-superconductor layer) ~~constit~~. Because of their extreme sensitivity, SQUIDS find applications in many fields, engineering, medicine and many other fields.

(viii) Very large scale superconducting magnets are used in magneto hydrodynamic (MHD) power plants, controlled fusion and energy storage.

(ix) Superconductors are used to perform logic and store function in computers.

(x) Large scale magnets have applications for sea and land transportation.

## NANOTECHNOLOGY.

### Nanoscale.

It means a billionth ( $10^{-9}$ ) part of a unit.

1 nm is one thousand millionth of a metre.

$$1 \text{ nm} = 10^{-9} \text{ m} = 10^3 \mu\text{m} = 10 \text{ A}^\circ$$

A single human hair is 80,000 nm wide.

A red blood cell is 7,000 nm wide.

Smallest thing that a human naked eye can resolve is 10,000 nm.

### Significance of Nanoscale.

Properties of many of the properties of solids depend on the size of the solid. The averaging of properties no longer works as the particle becomes smaller and smaller. Hence, the properties of materials change drastically in nanometer range.

Two principal factors cause the properties of nanomaterials to differ significantly from other materials; statistical mechanical effects and quantum mechanical effects. These factors can enhance properties such as reactivity, strength and

## Surface to Volume ratio:

Nanomaterials have a relatively larger surface area when compared to the same mass of material produced in the larger form. This can make nanomaterials more chemically reactive and affect their strength or electrical properties.

$$\text{Surface area/Volume} = \frac{4\pi R^2}{4\pi R^3/3} = \frac{3}{R}$$

Surface to volume ratio increases as radius  $R \downarrow$  SES.

## Quantum Confinement:

Quantum confinement is the restricted motion of randomly moving electron in specific energy levels, when the dimension of a material approaches the de-Broglie wavelength of electron.