59. Find $L\left\{\frac{\cos at - \cos bt}{t}\right\}$

[KNR 07 June 09] [CLT 09 Apr 12] [CUSAT 06 Nov 13] [CUSAT 12 Nov 15] [CLT 14 Apr 17] [KNR 07 Jan 18] [CLT 09 Apr 19]

Ans:

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\, ds \quad \text{where} \quad F(s) = L\{f(t)\}$$

$$L\left\{\frac{\cos at - \cos bt}{t}\right\} = \int_s^\infty L\{\cos at - \cos bt\}\, ds$$

$$= \int_s^\infty \left[\frac{s}{s^2 + a^2} - \frac{s}{s^2 + b^2}\right] ds$$

$$= \frac{1}{2}\left[\log(s^2 + a^2) - \log(s^2 + b^2)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2 + a^2}{s^2 + b^2}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{1 + \frac{a^2}{s^2}}{1 + \frac{b^2}{s^2}}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log(1) - \log\left(\frac{s^2 + a^2}{s^2 + b^2}\right)\right]$$

$$= -\frac{1}{2}\log\left(\frac{s^2 + a^2}{s^2 + b^2}\right) = \frac{1}{2}\log\left(\frac{s^2 + b^2}{s^2 + a^2}\right)$$

60. Find $L\left\{\frac{\cos 2t - \cos 3t}{t}\right\}$

[CLT 09 Apr 18] [CLT 14 Apr 18] [CLT 14 Apr 19]

Ans:

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\, ds \quad \text{where} \quad F(s) = L\{f(t)\}$$

$$L\left\{\frac{\cos 2t - \cos 3t}{t}\right\} = \int_s^\infty L\{\cos 2t - \cos 3t\}\, ds$$

$$= \int_s^\infty \left[\frac{s}{s^2 + 4} - \frac{s}{s^2 + 9}\right] ds$$

$$= \frac{1}{2}\left[\log(s^2 + 4) - \log(s^2 + 9)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2 + 4}{s^2 + 9}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{1 + \frac{4}{s^2}}{1 + \frac{9}{s^2}}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log(1) - \log\left(\frac{s^2 + 4}{s^2 + 9}\right)\right]$$

$$= -\frac{1}{2}\log\left(\frac{s^2 + 4}{s^2 + 9}\right) = \frac{1}{2}\log\left(\frac{s^2 + 9}{s^2 + 4}\right)$$