

04/08/2021
Wednesday

MODULE - 4

RECTILINEAR TRANSLATION

• Velocity $v = \frac{dx}{dt}$

• acceleration $a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$

EQUATION OF KINEMATICS

• $v = u + at$

• $v^2 = u^2 + 2as$

• $s = ut + \frac{1}{2}at^2$

$a = \text{acceleration } (\text{m/s}^2)$

$t = \text{time}$ $\frac{\text{odx GXP}}{\text{GXP}} = \frac{6 \times 2}{3} = 30 \text{ m/s}$

$v = \text{final velocity } (\text{m/s})$

$u = \text{initial velocity } (\text{m/s})$

$s = \text{distance}$

(a) for a freely falling body

$$v = u + gt$$

$$v^2 = u^2 + 2gh$$

$$h = ut + \frac{1}{2}gt^2$$

(b) When a particle moves upward

$$v = u - gt$$

$$v^2 = u^2 - 2agh$$

$$h = ut - \frac{1}{2}gt^2$$

VELOCITY-TIME CURVE

* axis \rightarrow time \rightarrow velocity

area under v-t curve \rightarrow displacement

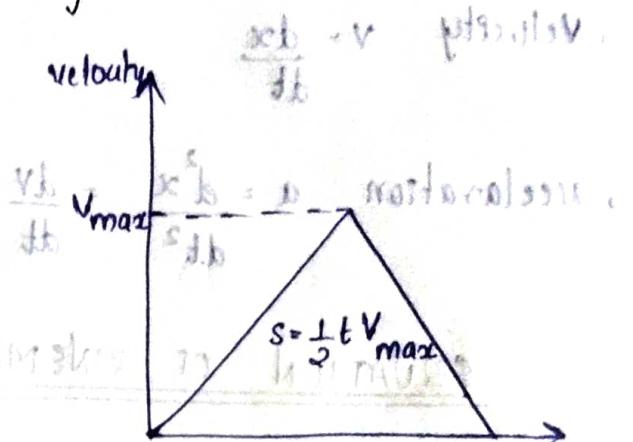
slope of v-t curve \rightarrow acceleration

① A train travels b/w 2 stopping stations, 7m apart in 14 min. Assuming that its motion is one of uniform acceleration for part of the journey and uniform retardation for the rest, prove that the greatest speed on the journey is 60 km/hr.

$$S = 7 \text{ m} \quad u = 0$$

$$t = 14 \text{ min} = \left(\frac{14}{60}\right) \text{ hr}$$

$$S = \frac{1}{2} t V_{\max}$$



$$V_{\max} = \frac{S \times 2}{t} = \frac{7 \times 2 \times 60}{14} = \underline{\underline{60 \text{ km/hr}}}$$

- Q5/08 ② A car travelling at 40 kmph sights a distant signal at 150m and comes uniformly to rest at the signal. It remains at rest for 20 As allowed by the signal, it uniformly accelerates and attains 40 kmph in 250m. Calculate the time lost due to signal.

From velocity-time graph, (a)

$$150 = \frac{1}{2} \times t_1 \times 11.11 \quad \text{fp-U = v}$$

$$t_1 = \frac{300}{11.11} = 27 \text{ s} \quad \text{fp-U = v}$$

$$t_2 = 20 \text{ s} \quad \text{fp-U = v}$$

$$t_3 = \frac{250}{11.11} = 45 \text{ s} \quad \text{fp-U = v}$$

$$\text{Total time of travel} = t_1 + t_2 + t_3 = 27 + 20 + 45 = \underline{\underline{92 \text{ s}}}$$

Time required to cover a distance of $(150+250) = 400 \text{ m}$ with a uniform velocity of 11.11 m/s

$$T = \frac{400}{11.11} = \underline{\underline{36 \text{ s}}}$$

Time lost due to signal

$$(t_1 + t_2 + t_3) - T = 9.2 - 3.6 = \underline{\underline{5.6\text{s}}}$$

③ The motion of a particle along a straight line is defined as $s = 25t + 5t^2 - 2t^3$, where s is in metres and t in second.

Find (i) velocity and acceleration at the start.

(ii) the time the particle reaches maximum velocity

(iii) the maximum velocity of the particle

$$s = 25t + 5t^2 - 2t^3$$

$$\text{Velocity } v = \frac{ds}{dt} = 25 + 10t - 6t^2 \leftarrow \frac{d}{dt} - 6t^2 = 0$$

$$\text{Acceleration } a = \frac{dv}{dt} = 10 - 12t$$

2 marks = v (refer to QMP)

(i) At $t=0$,

$$v = 25 + 0 - 0 = \underline{\underline{25 \text{ m/s}}} \quad \leftarrow \quad s + \frac{1}{2}at^2 = v$$

$$a = 10 - 0 = \underline{\underline{10 \text{ m/s}^2}} \quad \leftarrow \quad \frac{d}{dt} = 10 - 12t = 0$$

(ii) At maximum velocity, $\frac{dv}{dt} = 0$ i.e., $a = 0$

$$10 - 12t = 0$$

$$12t = 10 \Rightarrow t = \frac{10}{12} = \underline{\underline{0.83\text{s}}}$$

(iii) The maximum velocity of the particle at $t = 0.83\text{s}$

$$\therefore v_{\max} = 25 + 10 \times 0.83 - 6 \times 0.83^2 \times 0.83 = 25 + 8.3 - 4.13 = \underline{\underline{29.17\text{ m/s}}}$$

$$= 25 + 8.3 - 4.13 = \frac{25 + 4.17}{\frac{1}{6}} = \underline{\underline{V = 29.17\text{ m/s}}}$$

$$2 + \frac{1}{6} - \frac{1}{2} \cdot F = \frac{26}{\frac{1}{6}} = V$$

④ The displacement of a particle is given by $s = t^3 - 3t^2 + 2t + 5$. Find the time at which the acceleration is zero and the time at which velocity is 2m/s.

$$s = t^3 - 3t^2 + 2t + 5$$

(i) $\frac{dv}{dt} = a = 0$; i.e. the acceleration is zero when

$$a = \frac{d^2s}{dt^2} = \frac{d}{dt}(3t^2 - 6t + 2) = 6t - 6$$

$$a = 0$$

$$0 = 6t - 6 \Rightarrow 6t = 6 \Rightarrow t = 1\text{s}$$

(ii) Time at which $v = 2\text{m/s}$

$$v = 3t^2 - 6t + 2$$

$$2 = 3t^2 - 6t + 2 \Rightarrow 3t^2 - 6t = 0 \Rightarrow t = 0 \text{ or } 2\text{s}$$

$$3t^2 = 6t \Rightarrow t = 2\text{s}$$

⑤ A point is moving in a straight line with acceleration given by $a = 15t - 20$. It passes through a reference point at $t=0$ and another point 30m away after an interval of 5 seconds. Calculate the displacement, velocity and acceleration of the point after a further interval of 5 seconds.

$$a = 15t - 20; \text{ at } t=0, s=0, \text{ at } t=5, s=30\text{m}$$

$$a = \frac{dv}{dt} = 15t - 20 \Rightarrow v = 15t^2 - 20t + C$$

$$v = \int \frac{dv}{dt} dt = \frac{15t^2 - 20t}{2} + C$$

$$v = \frac{ds}{dt} = 7.5t^2 - 20t + C$$

$$S = \int (7.5t^2 - 20t + C_1) dt$$

initial position & initial velocity

$$= \frac{7.5t^3}{3} - \frac{20t^2}{2} + C_1 t + C_2$$

with $t=0$ when $S=0$

$$= 2.5t^3 - 10t^2 + C_1 t + C_2$$

now set to find

At $t=0, S=0,$

$$0 = 0 - 0 + 0 + C_2$$

$$\therefore C_2 = 0$$

At $t=5, S=30m$

$$30 = 2.5 \times 5^3 - 10 \times 25 + C_1 \times 5 + 0$$

$$30 = 2.5 \times 125 - 10 \times 25 + C_1 \times 5$$

(1) \rightarrow (2) \rightarrow (3) \rightarrow (4)

$$30 = 312.5 - 250 + 5C_1$$

$$30 = 62.5 + 5C_1$$

$$5C_1 = 32.5$$

$$S = \frac{1}{6} t^3 + d_1 t = x$$

$$5C_1 = 32.5$$

$$C_1 = 6.5$$

$$S = \frac{1}{6} t^3 + 6.5 t$$

$$S = 2.5t^3 - 10t^2 - 6.5t$$

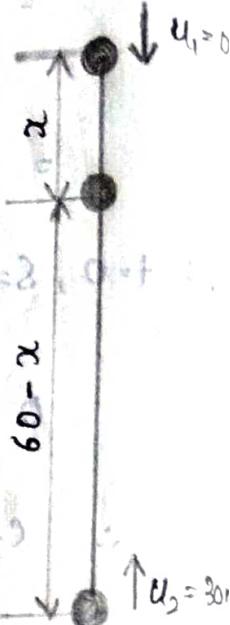
$$= 2.5 \times 10^3 - 10 \times 10^2 - 6.5 \times 10$$

$$= 2500 - 1000 - 65 = \underline{\underline{1435m}}$$

$$\text{Velocity } V = 7.5t^2 - 20t - 6.5 = 7.5 \times 10^2 - 20 \times 10 - 6.5 = \underline{\underline{543.5m/s}}$$

$$\text{Acceleration } a = 15t - 20 = 15 \times 10 - 20 = \underline{\underline{130m/s^2}}$$

⑥ A stone is dropped from the top of a tower, 60m high. At the same time another stone is thrown upwards from the foot of the tower with a velocity of 30 m/s. When and where does the two stones cross each other?



Height of tower $h = 60\text{m}$

$$U_1 = 0, \quad U_2 = 30 \text{ m/s}$$

$$t_1 = t_2 = t$$

Let x be the distance from the top of the tower where the two stones cross each other.

$$x = u_1 t + \frac{1}{2} g t^2 = 0 + \frac{1}{2} g t^2 \quad \text{--- ①}$$

$$60 - x = u_2 t - \frac{1}{2} g t^2 + ex_1 + ex_0 - \textcircled{2}$$

Adding equations (1) and (2)

$$G_0 = U_2 \times t \quad \text{Pd + U26 - 261g} \quad 08$$

$$t = \frac{60}{80} = \underline{\underline{0.75}} \quad \text{PZ + 0.6d} = 0.8$$

$$x = u_1 t + \frac{1}{2} g t^2$$

$$201 = 0 + \frac{1}{2} \times 9.81 \times 2^2 = 9.81 \times 2^2 = \underline{\underline{19.62 \text{ m}}}$$

The two stones will cross each other at a distance of 19.67 m from the top of the tower after 2 seconds.

KINETICS

Three approaches to solutions of problems in kinetics

1. Direct application of Newton's Second Law.
2. Use of Work-energy Principle.
3. Solution by Impulse and Momentum.

DIRECT APPLICATION OF NEWTON'S SECOND LAW

• Force is directly proportional to the product of mass and acceleration.

• Newton's law reduces to $F = m \times a$.

• Whenever a system of force acts on a body,

Resultant/Net force = (mass) \times (acceleration in the direction of resultant of force)

Q) A block weighing 1000N rest on a horizontal plane. Find the magnitude of the force required to give the block an acceleration of 2.5 m/s^2 to the right. The coefficient of friction b/w the block and the plane is 0.25.

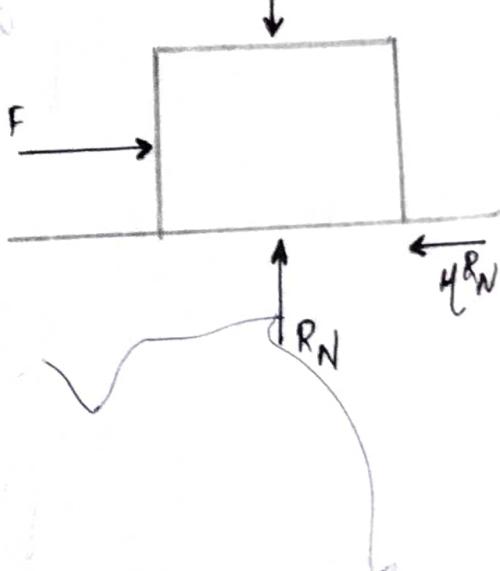
$$W = 1000 \text{ N}, a = 2.5 \text{ m/s}^2, \mu = 0.25$$

Since there is no motion in the vertical direction,

Net force in the vertical direction = 0

$$R_N - W = 0$$

$$R_N = W = 1000 \text{ N}$$



Net force in the horizontal direction = $mx\alpha$

$$F - \mu R_N = mx\alpha$$

$$F = 0.25 \times 1000 \times \frac{1000}{9.81} \times 2.5 = 254.84$$

and borrow 2' road to horizontal to vertical.

$$F = 250 - 254.84$$

Resultant force = $254.84 + 250$

$$\underline{\underline{F_3 = 504.84}}$$

Ans(2)

Q A body of mass 50 kg slides down a rough inclined plane whose inclination to the horizontal is 30° . If $\mu = 0.4$, find a?

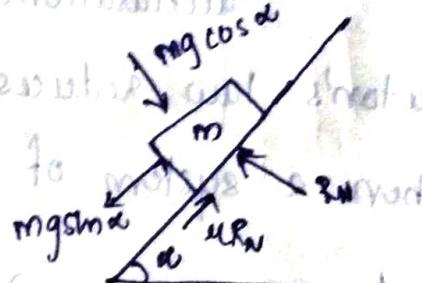
$$m = 50 \text{ kg}$$

$$\alpha = 30^\circ$$

$$\mu = 0.4$$

No motion in \perp direction of inclined plane

$$R_N - mg \cos \alpha = 0$$



(not to scale)

$$R_N = mg \cos \alpha$$

Net force along inclined plane

$$mg \sin \alpha - \mu R_N = ma \Rightarrow mg \sin \alpha - \mu mg \cos \alpha = ma$$

$$a = g \sin \alpha - \mu g \cos \alpha$$

$$= 9.81 \sin 30 - 0.4 \times 9.81 \times \cos 30$$

$$= \underline{\underline{1.51 \text{ m/s}^2}}$$

Ans(3)

$$O = 0.5 - \frac{1.51}{9.81}$$

$$1000 \times 0.5 = 500$$

Two blocks A and B are held stationary 10m apart on a 20° incline as shown. The coefficient of friction $\mu_A = 0.3$ while it is 0.2 b/w plane B $\mu_B = 0.2$. If blocks are released simultaneously, calculate the time taken & distance travelled by each block before they are at verge of collision.

Consider motion of block A

$$\boxed{\begin{aligned} \text{Net force} &= m \times a \\ m_A g \sin \theta - \mu R_{NA} &= m_A a_A \end{aligned}}$$

$$m_A g \sin \theta - \mu m_A g \cos \theta = m_A a_A$$

$$250 \sin 20 - 0.3 R_{NA} = \frac{250}{9.81} a_A$$

$$250 \sin 20 - 0.3 \times 250 \cos 20 = \frac{250}{9.81} a_A$$

$$a_A = \underline{0.59 \text{ m/s}^2}$$

Consider the motion of block B,

Net force $\omega = \text{mass} \times \text{acceleration}$

$$\boxed{\begin{aligned} m_B g \sin \theta - \mu R_{NB} &= m_B \times a_B \\ 500 \sin 20 - 0.2 \times R_{NB} &= \frac{500}{9.81} \times a_B \end{aligned}} \Rightarrow m_B g \sin \theta - \mu m_B g \cos \theta = m_B \times a_B$$

$$\Rightarrow \underline{a_B = 1.51 \text{ m/s}^2}$$

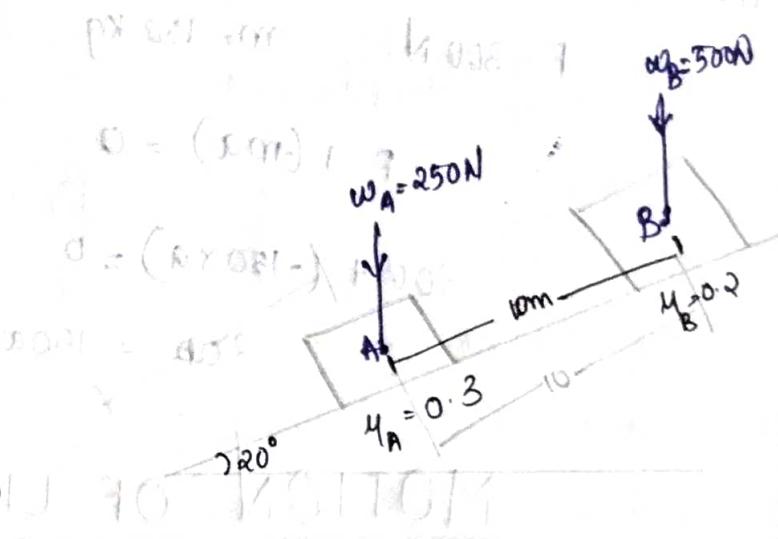
Let x be the distance travelled by A in t seconds, then the distance travelled by B in the same t second will be $(10+x)$

$$S_A = x = u_A t + \frac{1}{2} a_A t^2 \rightarrow x = 0 + \frac{1}{2} \times 0.59 \times t^2$$

$$S_B = 10 + x = 0 + \frac{1}{2} \times 1.51 \times t^2 \Rightarrow t = \underline{4.66 \text{ s}}$$

$$x = \frac{1}{2} \times 0.59 \times (4.66)^2 = \underline{6.41 \text{ m}}$$

$$\left[\frac{t+1}{t} \right] \omega = \underline{\frac{g}{2}}$$



D'ALEMBERT'S PRINCIPLE :- application of Newton's law

The resultant of a system of forces acting on a body in motion is in dynamic equilibrium with the inertia force.

- (10) A force of 300 N acts on a body of mass 150 kg. Calculate the acceleration of the body using D'Alembert's principle.

$$F = 300 \text{ N} \quad m = 150 \text{ kg}$$

$$F + (-ma) = 0$$

$$300 + (-150 \times a) = 0$$

$$300 = 150a \Rightarrow a = 2 \text{ m/s}^2$$

$$F + (-ma) = 0$$

$(-ma)$ = inertia force

MOTION OF LIFT

- application of newton's second law.

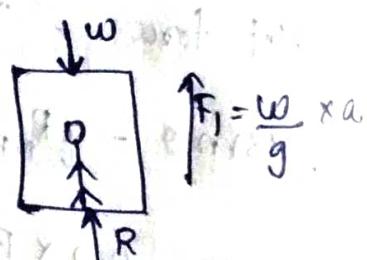
(a) Lift moving downwards

- $a \rightarrow$ downwards ; inertia of force \rightarrow upwards

$$R + F_I - W = 0$$
$$R = W - F_I$$

$$R = W - \frac{W}{g} a$$

$$R = W \left[1 - \frac{a}{g} \right]$$

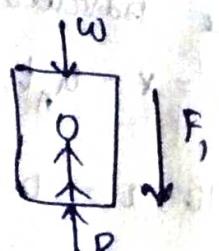


(b) Lift moving upwards

$$R - W - F_I = 0$$

$$R = W + F_I$$

$$R = W \left[1 + \frac{a}{g} \right]$$



NOTES:-

- A lift moves with uniform velocity, the acceleration of lift is 0.
- When lift moves down with acceleration, man exerts less force on the floor of lift, and when lift moves up with acceleration, man exerts more force on the floor of lift.
- Direction of normal force is opposite to direction of acceleration
→ when lift accelerates
- Direction of normal force is same to direction of acceleration
→ when lift decelerates.

(ii) A lift has an upward a. is 1.2 m/s^2 , what a force will a man weighing 750 N exert on floor of lift? What force would be exert if the lift had an acceleration of 1.2 m/s^2 downwards? What upward a would cause his weight to exert a force of 900N on the floor?

[KTU Jan 2016, June 2016, May 2019]

case (i)

when the lift moves upward $a = 1.2 \text{ m/s}^2$ $w = 750 \text{ N}$

$$R = w \left[1 + \frac{a}{g} \right] = 750 \left[1 + \frac{1.2}{9.81} \right] = 841.74 \text{ N}$$

case (ii)

when lift moving downwards

$$R = w \left[1 - \frac{a}{g} \right] = 750 \left[1 - \frac{1.2}{9.81} \right] = 658.26 \text{ N}$$

case (iii)

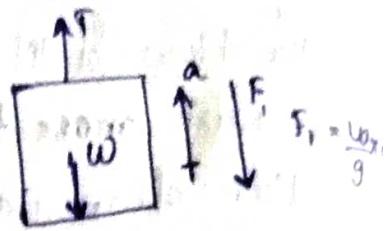
$w = 750 \text{ N}$ $R = 900 \text{ N}$

$$\text{when lift moves up } R = w \left[1 + \frac{a}{g} \right] \rightarrow 900 = 750 \left[1 + \frac{a}{9.8} \right]$$

$$\frac{900}{750} = 1 + \frac{a}{9.8} \rightarrow a = 1.96 \text{ m/s}^2$$

(12) An elevator of total weight 5000N starts to move upwards with a constant acc. of 1 m/s^2 . Find the force in the cable during the acceleration motion. Also find the force at the deceleration elevator under the feet of a man weighing 600N when the elevator moves up with a uniform retardation of 1 m/s^2 .

case (i) Elevator moves upwards with acceleration



$$T - w - F_i = 0$$

$$T = w + \frac{w}{g} a$$

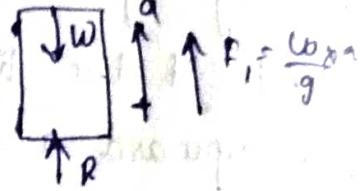
$$T = w \left[1 + \frac{a}{g} \right] = 5000 \left[1 + \frac{1}{9.81} \right] = 5509.6 \text{ N}$$

case (ii) Elevator moves up with uniform deceleration; man's wt. force is upwards $w=600\text{N}$

$$R + F_i - w = 0$$

$$R = w - F_i = w - \frac{w}{g} a$$

$$= w \left[1 - \frac{a}{g} \right] = 600 \left[1 - \frac{1}{9.81} \right] = 538.84 \text{ N}$$



13/08/2021

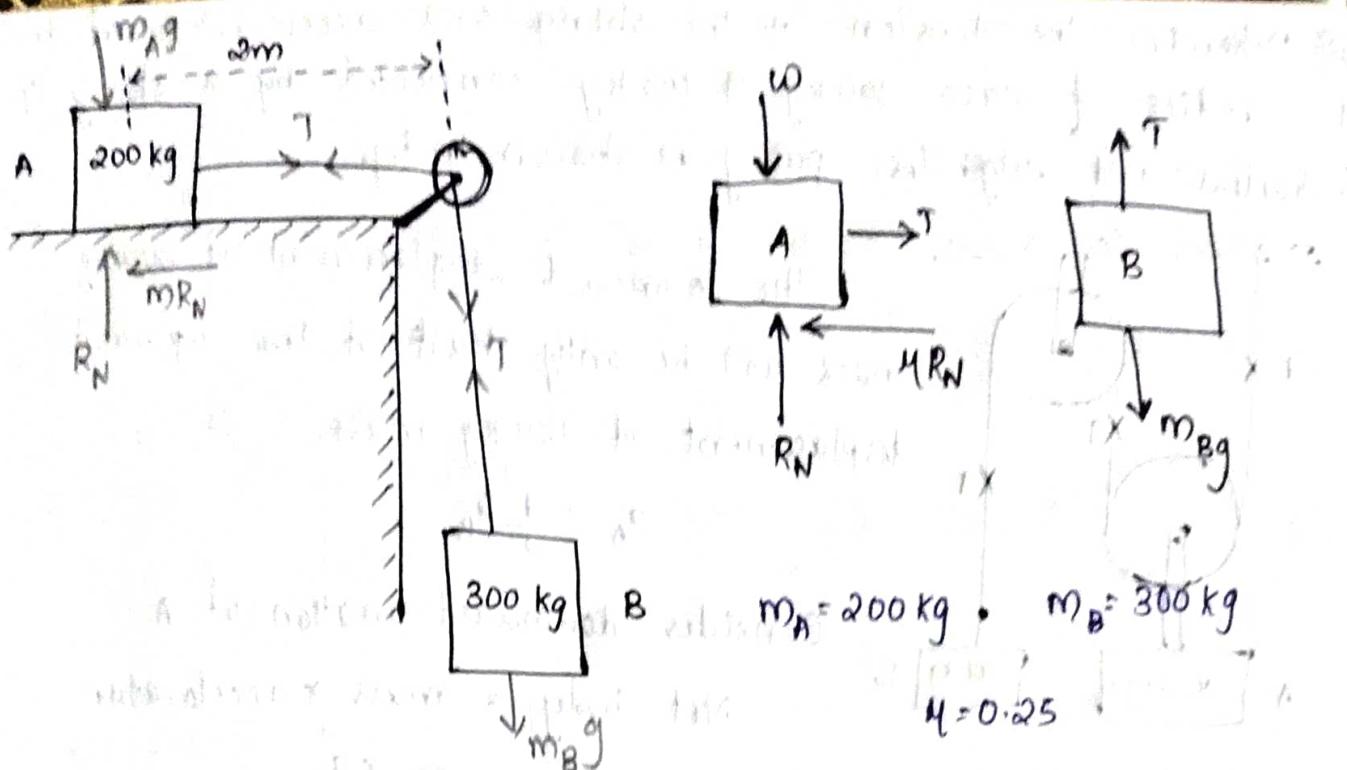
MOTION OF CONNECTED BODIES

Consider motion of each body separately and apply Newton's Law of motion and find acceleration of the body and tension in the string.

- Force in the direction of motion = +ve

- Force in the opp. direction of motion = -ve.

(13) Two blocks are joined by an inextensible string as shown in the fig. If the system is released from rest, determine the velocity of block after it has moved 2m. Assume the coefficient of friction b/w block and plane is 0.25. The pulley is weightless and frictionless.



Let T be the tension in the string, since $x_A = x_B$

Consider the motion of block A,

$$\text{Net force} = \text{mass} \times \text{acceleration}$$

$$T - \mu R_N = m_A \times a$$

$$T - 0.25 \times 200 \times 9.81 = 200 \times a \quad \text{--- (i)}$$

Consider the vertical motion of block B,

$$\text{Net force} = \text{mass} \times \text{acceleration}$$

$$m_B g - T = m_B \times a$$

$$300 \times 9.81 - T = 300 \times a \quad \text{--- (ii)}$$

Adding (i) + (ii)

$$300 \times 9.81 - T + T - 0.25 \times 200 \times 9.81 = 200a + 300a$$

$$300 \times 9.81 - 200 \times 9.81 = 500a$$

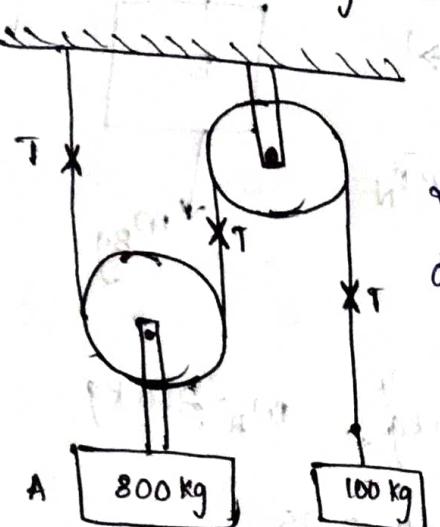
$$a = 4.905 \text{ m/s}^2$$

$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2 \times 4.905 \times 2$$

$$v = 4.43 \text{ m/s}$$

(14) Determine the tension in the string and acceleration of the two bodies of mass 800 kg & 100 kg connected by a string & frictionless and weightless pulley as shown in fig.



The downward displacement of 800 kg mass will be only half of the upward displacement of 100 kg mass.

$$a_A = \frac{1}{2} a_B$$

Consider downward motion of A

Net force = mass \times acceleration

$$m_A g - 2T = m_A \times a_A$$

$$300 \times 9.81 - 2T = 300 \times a_A$$

$$2943 - T = 150 a_A \quad (i)$$

Consider upward motion of B

Net force = mass \times acceleration

$$T - m_B g = m_B a_B$$

$$T - 100 \times 9.81 = 100 \times 2 \times a_A$$

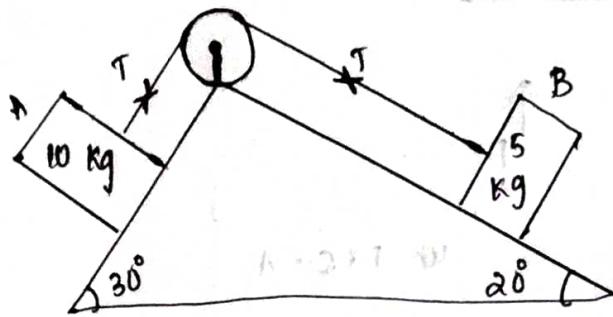
$$T - 981 = 200 a_A \quad (ii)$$

(i) + (ii)

$$1471.5 - T + T - 981 = 350 a_A$$

$$490.5 = 350 a_A \Rightarrow a_A = 1.40 \text{ m/s}^2$$

(15) Two smooth inclined planes whose inclinations with horizontal are 30° and 20° are placed back to back. Two bodies, of mass 10kg and 5kg are placed on them & are connected by a string as shown in fig. Calculate the T in the string and acceleration of the bodies.



The downward displacement

of body A will be equal to upward placement of B

$$a_A = a_B = a$$

Consider motion of A

$$\text{Net force} = m_A \times a_A$$

$$m_A g \sin \theta - T = m_A \times a_A$$

$$10 \times 9.81 \times \sin 30^\circ - T = 10 \times a \Rightarrow 9.81 \times 5 - T = 10a \quad (i)$$

Consider motion of B

$$T - m_B g \sin \theta = m_B \times a_B$$

$$T - 5 \times 9.81 \times 0.34 = 5a_B \quad (ii)$$

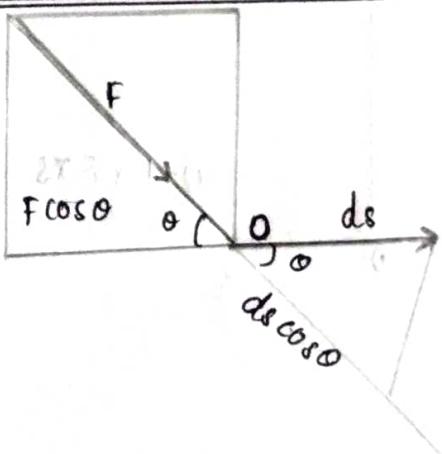
(i) + (ii)

$$9.81 \times 5 - T + T - 5 \times 9.81 \times 0.34 = 15a \quad \text{or } 15a = 15a \Rightarrow a = 2.16 \text{ m/s}^2$$

WORK-ENERGY EQ. IN RECTILINEAR TRANSLATION

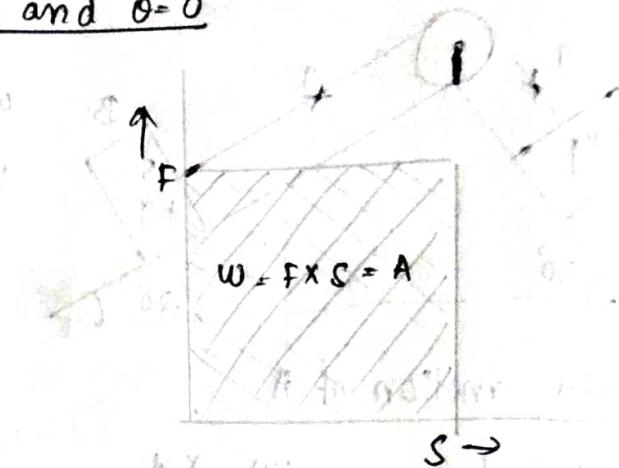
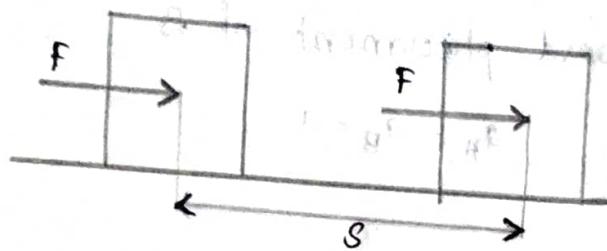
WORK

- Force \rightarrow body moved along the line of action.
- Work done, $dW = F ds \cos \theta$

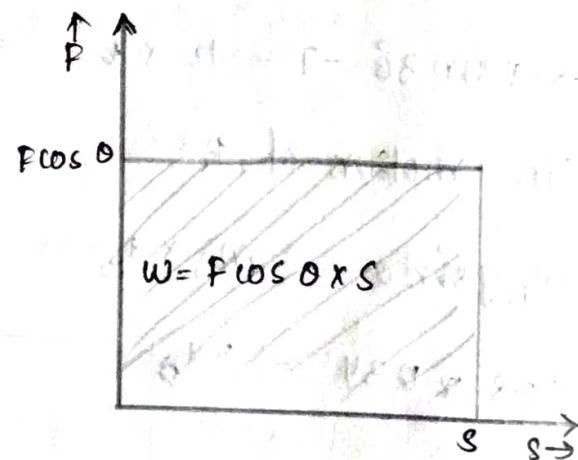
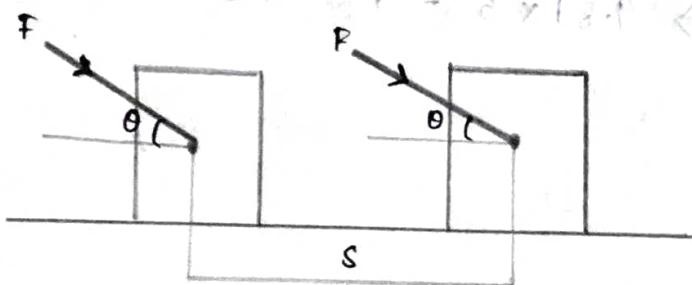


- Work is positive :- force or component of force in same direction of displacement
- Work is negative :- opp. direction
- Unit :- Joule [Newton-metre (N.m)]

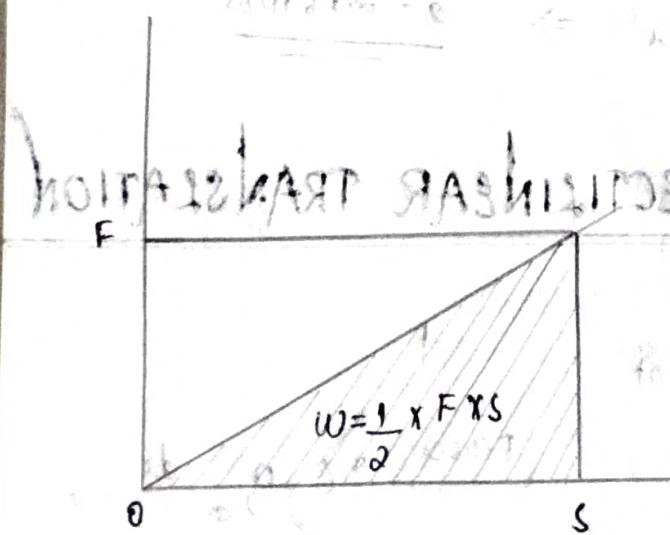
Case (i) When force F is constant and $\theta = 0$



(ii) When F is constant and is inclined at θ with direction of motion.



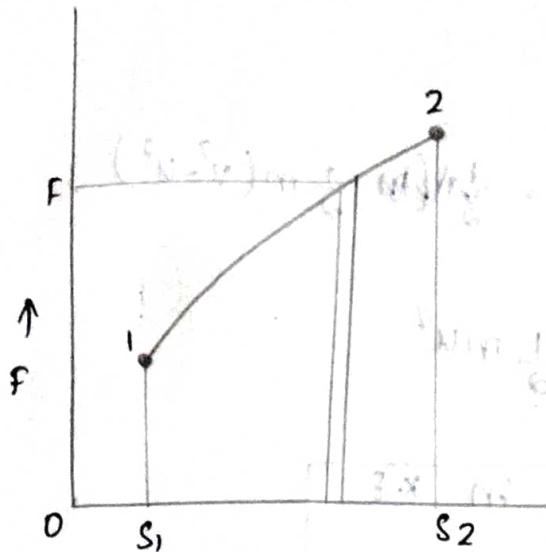
(iii) When the force F varies linearly & $\theta = 0$



work done W = (average force during the displacement) \times displacement

$$W = \frac{(0+F)}{2} \times S = \frac{1}{2} \times F \times S$$

case (iv): Work done by a variable force, $F = f(s)$



Work done W = area under the curve 1-2 of force displacement diagram

$$W = \int_{s_1}^{s_2} F \times ds$$

ENERGY

- capacity to do work
- unit = N-m (or J) (unit of work or energy - derived units)
- $K.E. = \frac{1}{2}mv^2$ (unit of kinetic energy = kg m²/s²)
- $P.E. = mgh$ (unit of potential energy = J)

WORK - ENERGY PRINCIPLE

The work done by a system of forces acting on a body during a displacement is equal to the change in kinetic energy of the body during the same displacement.

$$\text{Resultant Force} = m \times a$$

$$a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = \frac{dv}{ds} \times v$$

$$a = v \frac{dv}{ds}$$

$$F = m \times a$$

$$= m \times v \frac{dv}{ds}$$

$$F \times ds = m v dv$$

Integrating on both sides

$$\int_0^s F \cdot ds = \int_u^v m v \, dv$$

$$F \times s = m \left[\frac{v^2}{2} \right]_u^v = \frac{1}{2} m (v^2 - u^2)$$

$$s = \frac{1}{2} m v^2 - \frac{1}{2} m u^2$$

Work done = change in K.E

IMPULSE - MOMENTUM PRINCIPLE

- derived from Newton's Second Law
- momentum = product of mass & velocity. ($m \times v$)
- Impulsive force = large force acts over a short period of time
- The impulsive force F acting over a time interval t_1 to t_2 is defined by the integral.
- If F is the resultant force acting on a body of mass m , then Newton's second law

$$\text{Force } F = m a$$

$$a = \frac{dv}{dt}$$

$$F = m \cdot \frac{dv}{dt} = \frac{d(mv)}{dt}$$

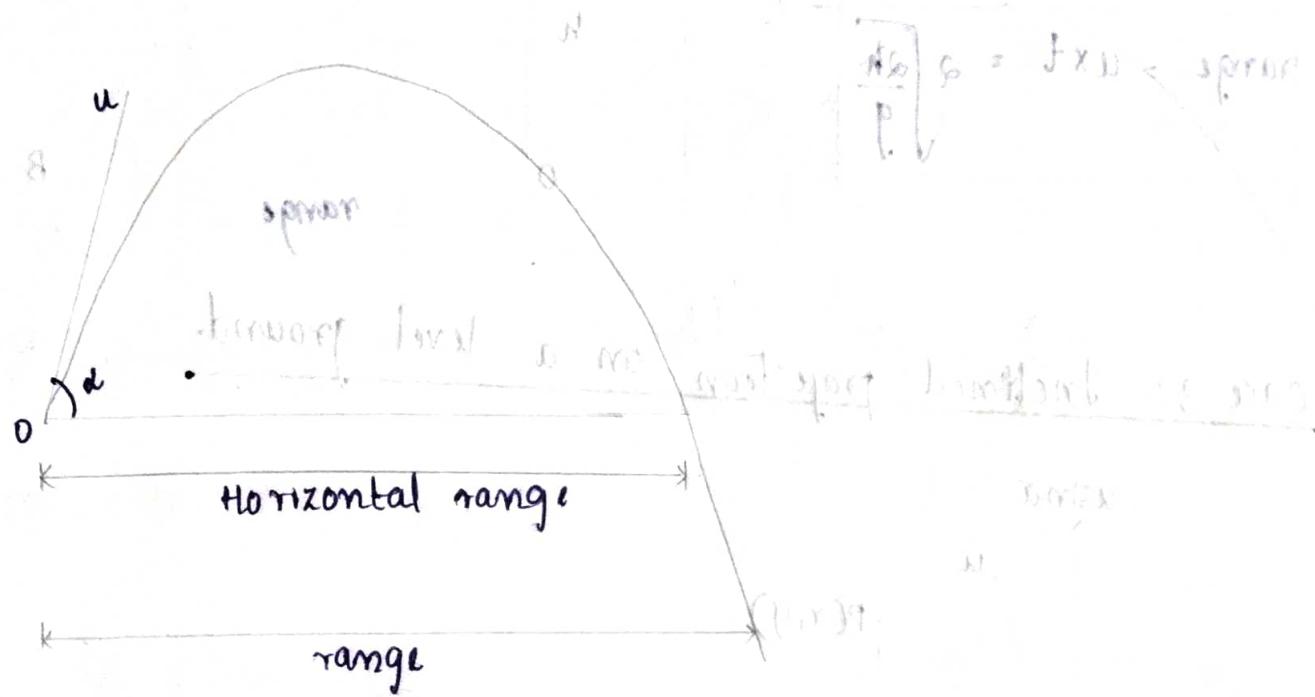
$$F dt = d(mv) \Rightarrow \int F dt = \int dm v = m dv$$

$$\int_{t_1}^{t_2} F \cdot dt = m [v]_{v_1}^{v_2} = m(v_2 - v_1)$$

$$F(t_2 - t_1) = m(v_2 - v_1) \rightarrow F t = m(v_2 - v_1)$$

Impulse = final momentum - initial momentum

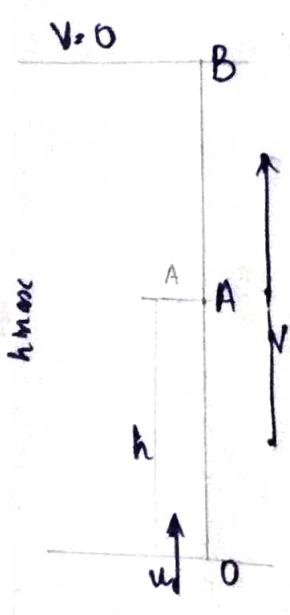
MOTION OF PROJECTILE



Case I :- Motion of a particle projected vertically into space

$$\alpha = 90^\circ$$

when $h = h_{\text{max}}$, $v = 0$



$$h_{\text{max}} = \frac{u^2}{2g}$$

$$\text{Time to attain max height } t_1 = \frac{u}{g}$$

$$\text{time of flight } 2t_1 = \frac{2u}{g} = T$$

$$T = \sqrt{\frac{2h_{\text{max}}}{g}}$$

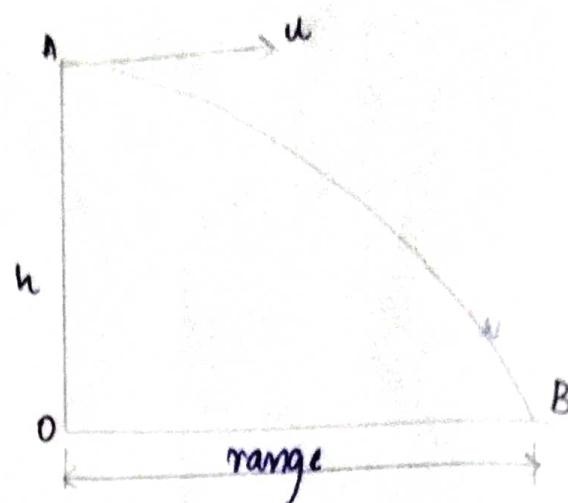
$$\text{Range} = 0$$

Case 2 :- motion of a plane thrown horizontally into space

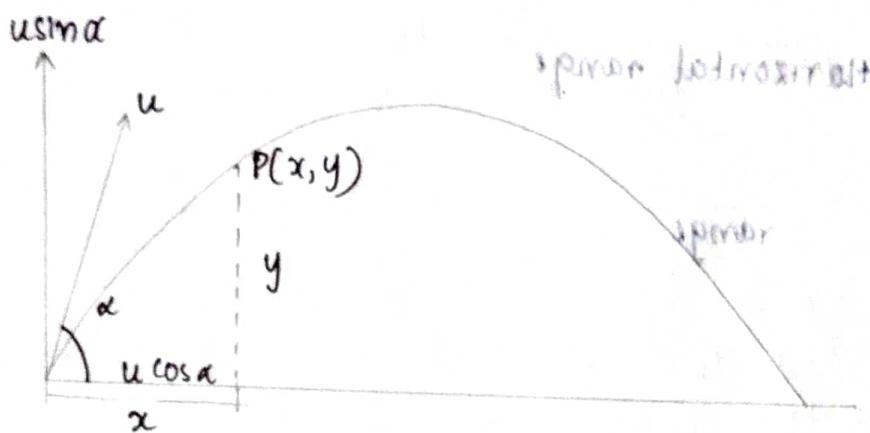
Time of flight

$$T = t = \sqrt{\frac{2h}{g}}$$

$$\text{range} = uxt = u \sqrt{\frac{2h}{g}}$$



case 3 :- Inclined projection on a level ground.



Prove that the trajectory of an inclined projection on a level ground is a parabola.

$$x = (u \cos \alpha) \times t$$

$$t = \frac{x}{u \cos \alpha}$$

$$h = (u \sin \alpha) \times t - \frac{1}{2} g t^2$$

$$h = (u \sin \alpha) \times \frac{x}{u \cos \alpha} - \frac{1}{2} g \left(\frac{x}{u \cos \alpha}\right)^2$$

express

$$y = u \sin \alpha \times \frac{xt}{u \cos \alpha} = \frac{1}{2} g \left(\frac{x}{u \cos \alpha} \right)^2$$

$$y = x \frac{\sin \alpha}{\cos \alpha} - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \alpha}$$

$$y = x \tan \alpha - \frac{1}{2} \frac{gx^2}{u^2 \cos^2 \alpha}$$

equation of a parabola.

EQUATIONS OF A PROJECTILE MOTION

Maximum height

$$h_{\max} = \frac{u^2 \sin^2 \alpha}{2g}$$

Time to attain h_{\max}

$$T = \frac{2u \sin \alpha}{g}$$

Horizontal range

$$R = \frac{u^2 \sin 2\alpha}{g}$$

(1) $u = 100 \text{ m/s}$ $\alpha = 30^\circ$

Find the horizontal range, h_{\max} attained by bullet & time of flight.

$$R = \frac{u^2 \sin 2\alpha}{g} = \frac{100 \times 100}{9.8} \sin 60^\circ$$

$$= \frac{100 \times 100}{9.8} \times \frac{\sqrt{3}}{2} = \underline{\underline{882.79 \text{ m}}}$$

$$h_{\max} = \frac{u^2 \sin^2 \alpha}{2g} = \frac{100 \times 100 \times \frac{1}{2} \times \frac{1}{2}}{2 \times 10} = \underline{\underline{127.55 \text{ m}}}$$

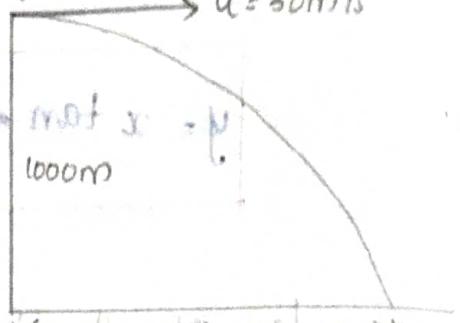
$$T = \frac{2us \sin \alpha}{g} = \frac{2 \times 100 \times \frac{1}{2}}{9.8} = \frac{10.20 s}{\cancel{2}}$$

- (17) A pilot flying his bomb at a height of 1000m with uniform horizontal velocity of 30 m/s wants to drop a target on the ground. At what distance from the target, he should release the bomb? $u = 30 \text{ m/s}$

$$h = ut + \frac{1}{2} g t^2$$

$$1000 = 0 + \frac{1}{2} g t^2$$

$$t = \sqrt{\frac{2000}{g}} = 14.295$$



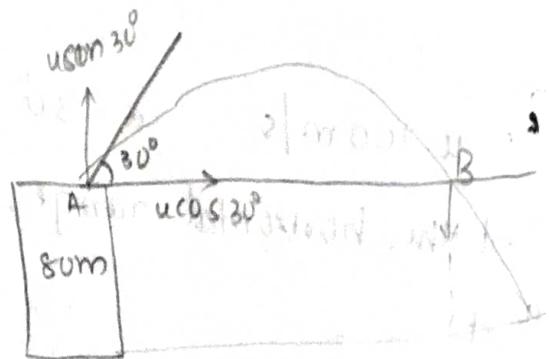
\because the horizontal velocity remains constant, the horizontal distance moved in 14.29 seconds is $v_x t$

$$x = 30 \times 14.29 = 428.7 \text{ m}$$

PRACTISE PROBLEMS

- (18) A stone is thrown upwards at an angle of 30° to the horizontal from a point P on a tower of height 80m and it strikes the ground. The initial velocity of stone = 100 m/s. Calculate (a) time of flight of stone (b) The greatest elevation above the ground reached by the stone.

$$u = 100 \text{ m/s} \quad h = 80 \text{ m} \quad \alpha = 30^\circ$$



(a) t

$$y = (us \sin \alpha)t - \frac{1}{2} g t^2$$

$$-80 = 100 \sin 30 t - \frac{1}{2} \times 9.8 t^2$$

$$4.905 t^2 - 50t - 80 = 0 \Rightarrow t = \frac{110.59 \text{ s}}{8.7}$$

(b)

$$H = \frac{v^2 \sin^2 \alpha}{2g} = \frac{100 \times 100 \times \sin^2 30}{2 \times 9.8} = 127.42$$

$$\text{Greatest elevation} = 127.42 + 80 = 207.42 \text{ m}$$

(19) A cricket ball thrown by a fielder from a height of 2m at an angle 45° to the horizontal with an initial velocity 25 m/s hit the wickets at the height of 0.6m from ground. How far was the fielder from wickets?

$$u_y = u \sin 45^\circ = 25 \times 0.707 = 17.68 \text{ m/s}$$

$$u_x = u \cos 45^\circ = 25 \times 0.707 = 17.68 \text{ m/s}$$



$$s_y = u_y t - \frac{1}{2} g t^2$$

$$(0.6 - 2) = 17.68 t - \frac{1}{2} \times 9.8 \times t^2$$

$$-1.4 = 17.68 t - 4.9 t^2 \Rightarrow 4.9 t^2 - 17.68 t - 1.4 = 0$$

$$\underline{\underline{t = 3.68 \text{ s}}}$$

$$\text{Range} = u \cos 45^\circ \times 3.68 = \underline{\underline{65.05 \text{ m}}}$$