Real-time Pedestrian Modelling: Implementing the Ensemble Kalman Filter for an Agent-Based Model

Keiran Suchak — 200888140

June 13, 2019

Abstract

This is the abstract.

Contents

List of Figures

List of Tables

Notes

- What is data assimilation?
- What is the Ensemble Kalman Filter?
- What are Agent-Based Models?
- How do people model pedestrians?
- What happens when an agent in one of the ensemble member models tries to leave the station early? Because we're only doing state estimation (meaning just the agent coordinates), the agent won't get reactivated.
- Errors should converge on something relating to twice the exit distance.
- Signposting at the beginning of each chapter
- Don't need to review calibration too much
- Don't need to talk about different types of enkf's too much
- Don't need to talk about DA with CA too much

Chapter 1

Introduction

A better understanding of how people move around their environment is of great utility to both academics and policy-makers. Such knowledge can be made use of in the contexts of urban planning, event management and emergency response, particularly when considering urban environments. Furthermore, this may also be of use to those interested in the social issues of mobility, inclusivity and accessibility of opportunities.

When considering such concepts, investigators often make use of modelling techniques. At their most fundamental, models represent our understanding of the system that we are studying — an understanding that may not be perfect (Stanislaw 1986). There exist modelling techniques for the simulation of how pedestrians move around urban spaces. However, these methods exist largely in isolation of the real-world — that is to say that whilst the simulations aim to reflect the real-world, there is no method by which we can incorporate up-to-date observations into these models to stop their divergence from reality.

Simulating pedestrian behaviour is often undertaken at the micro-scale, with such models typically aiming to model at the individual level or on a

spatially fine-grained grid (Burstedde et al. 2001). One of the most prevalent simulation methods in this field is that of Agent-Based Modelling. Such methods consist of two key components: agents and environments. In an Agent-Based Model, we prescribe sets of rules by which individuals interact with each other and their local environments; as interactions take place on the micro-scale, we typically observe the emergence of structure at the macro-scale such as crowding (Batty et al. 2003) or lane formation (Liu et al. 2014). The evaluation of these rules is often not deterministic and instead introduces some element of randomness; these stochastic elements aim to emulate the variability of human behaviour. The introduction of such randomness in conjunction with an imperfect understanding of the phenomena at play, however, typically result in simulation runs diverging from the real system.

In constructing their models, agent-based modellers undertake a development process that involves model verification, validation and calibration. We can take these to mean the following:

- **Model verification**: the process of ensuring that the implementation is an accurate representation of the model (Xiang et al. 2005).

- **Model validation**: the process of ensuring that the chosen model is an accurate representation of the phenomenon that we wish to study (Crooks et al. 2008).

- **Model calibration**: the process of searching for model parameter values such that we can achieve validation (Thiele et al. 2014).

Beyond this, modellers also make efforts to ensure that the initial model conditions are realistic by setting them based on historical data.

The practices of validation, calibration and setting initial model states based on historical data are appropriate for offline evaluations such as testing designs of new buildings or experimenting with different individual behaviours; however, when aiming to simulate events in real-time, this simply delays the inevitable divergence of the model from the real system. Furthermore, model parameters may be transient and thus require to be updated as time passes and the dynamics evolve.

Given the apparently inevitable divergence of stochastic simulations from the real systems that they aim to model, one may alternatively turn to big data. Data is now being generated in higher volumes and at greater velocity than ever before (Chen et al. 2014); however, there also exist issues with observation data from such systems. Whilst models typically allow us to simulate a whole system, observations are typically sparse in either time or space (or both); this is to say that observations rarely provide complete coverage of the events. We therefore seek a solution whereby we can integrate up-to-date observations into our models as the models continue to simulate the system.

One of the methods by which we can combine knowledge represented by our model with observations as they become available is through data assimilation techniques, which are most commonly used in the field of numerical weather prediction (Kalnay 2003). Such techniques are typically made up of two steps:

1. **Predict**: Run the model forward, estimating the state of the system, until new observations become available.

2. **Update**: Upon receipt of new observations, combine the model's estimate of the system state with the new data.

These steps are repeated iteratively in a cycle. It is important to note that just as the there is error associated with the model, we also acknowledge that there is observational error associated with the data. The aim of incorporating the observations into the model is to improve the model accuracy with respect to the true system state.

A large volume of work exists in which such techniques are applied to meteorological systems where the models used are based on differential equations. Significantly less work exists in which data assimilation methods are applied to agent-based models — in particular pedestrian models. This dissertation therefore aims to expand on the pre-existing work by implementing a data assimilation scheme known as the Ensemble Kalman Filter in conjunction with a relatively simple agent-based model of pedestrians crossing a two-dimensional station from one side to the other.

1.1 Aims and Objectives

With the above in mind, this dissertation aims to determine whether the Ensemble Kalman Filter method of data assimilation can be used to improve the accuracy with which an Agent-Based Model simulates pedestrian movements given synthetic real-time data. In order to achieve this, the following objectives are set out:

1. Develop a general Ensemble Kalman Filter computational class.

2. Apply the Ensemble Kalman Filter to the pedestrian model.

3. Compare the accuracy with which the Ensemble Kalman Filter implementation of the model simulates the pedestrian movement against the based model without data assimilation.

The remainder of this dissertation will provide an overview of some of the work that has been undertaken thus far on implementing data assimilation schemes with agent based models (Chapter 2), elaborate on how data assimilation works — particularly focussing on the aforementioned Ensemble Kalman Filter (Chapter 3) — and examine the effectiveness of this method in improving the accuracy of the agent-based model (Chapter 4).

Chapter 2

Literature Review

As touched upon in Chapter 1, the process of developing an agent-based model typically involves some form of model calibration. Model calibration is the procedure of fine-tuning the model that we are using such that it best fits the particular situation that we are seeking to model (Crooks & Heppenstall 2012). There are a large number of different manners in which we can calibrate agent-based models (Thiele et al. 2014). These approaches typically involve making use of real-world data to estimate the parameters and initial state of the model; this is, however, undertaken once prior to running the model.

In some situations, we aim to simulate events in real-time (or close to real-time). In such situations, we are often able to observe the evolution of the real-world system which we seek to model and consequently may wish to use this information to recalibrate the model. This would, however, require that we stop the simulation, undertake calibration, and restart the model. We therefore seek an approach that allows us to incorporate observations of the system whilst simulating the system — data assimilation.

This Chapter will therefore seek to provide a basic overview of data as-

similation, along with some coverage of the attempts that have been made to implement such techniques to agent-based models that simulate urban systems.

2.1 Data Assimilation

- Origins of filtering with Wiener filter, 1950. Only applies for stationary signals.
- Kalman-Bucy 1961
- Stratonovich 1968
- Jazwinski 1970
-

The process of data assimilation involves making use of observations along with prior knowledge (which, in our case, is encoded in a model) to produce increasingly accurate estimates of variables of interest. Such a process can be achieved through a Bayesian filtering approach (Bar-Shalom et al. 2004, Jazwinski 1970).

Under such a framework, the updating of the model state is undertaken on the basis of Bayes Rule (for which a derivation is provided in Appendix B):

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{2.1}$$

Bayes Rule is made up of four components:

1. $P(A)$: The probability of A, known as the **Prior**.
2. $P(A|B)$: The probability of A given B, known as the **Posterior**.

3. $P(B|A)$: The probability of B given A, known as the **Likelihood**.

4. $P(B)$: The probability of B, known as the **Marginal Likelihood**.

When applying this notation to the problem at hand, the components become:

1. **Prior**, $P(\mathbf{x})$: The probability distribution representing the prior state of the model.

2. **Posterior**, $P(\mathbf{x}|\mathbf{d})$: The probability distribution representing the updated state of the model in light of the observed data, that is to say the probability of the model state given the data.

3. **Likelihood**, $P(\mathbf{d}|\mathbf{x})$: The probability distribution of the observed data given the model state.

4. **Marginal Likelihood**, $P(\mathbf{d})$: The probability distribution representing the observed data.

With the above notation, Bayes Rule becomes:

$$P(\mathbf{x}|\mathbf{d}) = \frac{P(\mathbf{d}|\mathbf{x})\,P(\mathbf{x})}{P(\mathbf{d})} \tag{2.2}$$

The aim of a data assimilation scheme therefore becomes to provide an update to the state in the form of the posterior, $P(\mathbf{x}|\mathbf{d})$, given new observations, $P(\mathbf{d})$.

There exist a number of different schemes for tackling this problem which are often divided into two groups:

1. **Sequential**: What does this mean and some examples, Kalman Filter (and variations thereof), Particle Filter.

2. **Variational**: What does this mean and some examples, 3D-VAR, 4D-VAR.

Of the work that currently exists wherein investigators attempt to apply data assimilation schemes to agent-based models, most make use of sequential schemes.

2.2 Application of Sequential Data Assimilation to Agent-Based Models

2.2.1 Particle Filter-based Approaches

One of earliest pieces of work undertaken on the application of data assimilation schemes to agent-based models of urban environments was by Wang & Hu (2013). In this work, they simulate a smart office environment with people in it — a scenario that is becoming increasingly common with the advent of the Internet of Things (Zanella et al. 2014). The aim of the work was to make use of real-time data in conjunction with the agent-based simulation to provide more accurate estimates of the occupancy of the environment. This was achieved using the Particle Filter method of data assimilation; the method was chosen as it did not require the system to be Gaussian. The particle filter method operates by holding an ensemble of realisations of the simulation, each of which are evolved forward over time between observations; when observations are received, the particle states are weighted and the new state is obtained by sampling from these weighted particles. The observations used were synthetic data generated by the agent-based model, aiming to emulate motion sensors which would provide a binary response of whether a person was present in a given location.

The work undertaken consisted of two experiments — firstly simulating single agent in the environment, then going on to simulate two agents in the same environment. In the case if the first experiment, the agent was simulated with two different routing behaviours; for the first routing behaviour, the agent move forward sequentially through a series of waypoints, whilst for the second routing behaviour, the agent moves through a series of waypoints before turning back to return to its initial position. In this experiment, it was found that the simulation error decreased when the agent was detected at each of the sensors for both routing behaviours with error growing between detections; this is as expected — it confirms that the simulation becomes more accurate with the addition of further information regarding the system. In the case of the second routing behaviour, the simulation error also grew following the agent's turn to head back to its origin. In the second experiment, they aimed to simulate two agents in the same environment, with the two agents maintaining spatial separation. This simulation was run a number of times for different numbers of particles with a view to establishing a relationship between the number of particles and simulation accuracy. It was found that as the number of particles was increased (through 400, 800, 1200 and 1600), the simulation error decreased. It was also found, however, that the experiments with fewer particles (400 and 800) struggled to converge, with the smaller number of particles unable to provide sufficient coverage of the state space. It was therefore noted that as the number of agents was increased, the method was likely to struggle.

This final issue may be solved by an increase in the number of particles; however, this comes with an attached increase in the computational cost (both in terms of compute time and space). The implementation of the particle filter requires that a realisation of the model be kept for each particle,

resulting in growing memory requirements as the number of particles are increased. Furthermore, each particle is required to evolve the model for each time-step, resulting in an increasing computational cost.

There are two subsequent pieces of research which have sought to build directly upon the above initial investigation. The first of these was undertaken by Rai & Hu (2013), using the same simulation model and method of data assimilation but aiming to add a further layer which estimated behavioural patterns in the system. This was achieved using a hidden Markov model which was trained on the historical data of the system. The information encoded in the hidden Markov model is incorporated into the particle filter at the sampling step; the hidden Markov model is first used to identify the types of behaviour being exhibited by the system and particles then sample from different types, subsequently engaging the simulation with said behaviour. The remaining steps of the filtering process remained the same. The aim of applying this approach was to further improve the accuracy of simulations through the identification of pedestrian behaviours. In order to do so, the behaviours were divided into the following categories:

- **Outside**: All agents wait outside of a conference room.
- **In Conference**: All agents are attending meeting in the conference room.
- **Few Entering**: A small number of agents entering the conference room.
- **High Entering**: A large number of agents entering the conference room.
- **Few Leaving**: A small number of agent leaving the conference room.

- **High Leaving**: A large number of agent leaving the conference room.

These categorisations, particularly the *entering* and *leaving* states, present a problem. When entering the conference room, agents are more likely to be categorised into three groups — early few entering, high entering and late few entering — this re-categorisation helps to describe the status of the agents that are not in the process of entering:

- **Early few entering**: A small number of agents entering the conference room early before a meeting, with the other agents outside the conference room.

- **High entering**: A large number of agents entering the conference room.

- **Late few entering**: A small number of agents entering the conference room late after the meeting has started, with the other agents inside the conference room.

The investigation then attempts to determine the accuracy with which the model is able to identify the correct behaviour for agents, with accuracy being defined as

$$\frac{1}{T}\sum_{k=1}^{T} S_t^k - S_t^{real} \tag{2.3}$$

where T is taken to be the total number of simulation steps and S is the behaviour pattern state. It is unclear, however, how this calculation is undertaken given that the behavioural states are categories and not numerical; furthermore, the meaning behind the state notation is explained — it appears that k is a time-step index, however it does not make sense to compare the behavioural state at each time state to a static "real" behavioural state and the latter would likely be a transient property. Some indication is given that

a numerical encoding of the categories has been used for visualisation purposes, however this should not be used for arithmetic purposes, nor should any ordinality be inferred from it. Indeed, the results presented take the form of accuracy percentages, suggesting that a more conventional accuracy score has been used, such as

$$\frac{1}{T}\sum_{k=1}^{T}\mathbb{1}\left(\hat{S}_k = S_k^{real}\right) \tag{2.4}$$

where the indicator function, $\mathbb{1}(\ldots)$, returns 1 when the condition is fulfilled, else 0. The results suggest that the model developed accurately identifies the behavioural states except for states that occur infrequently; the model performs particularly well when identifying states in which agents are static, i.e. *outside* and *in conference*. Such supplementary information could improve the performance of data driven simulations, likely helping the process of data assimilation for parameter estimation. It is worth considering, however, that the addition of this further layer to the assimilation process would also result in a further increase in the computational cost.

The second investigation (Wang & Hu 2015) based on the original work by Wang & Hu (2013) more closely reflected the original work. The research was concerned with the same smart environment equipped with sensors around which agents move. The agents' behaviours were comprised of a combination of agent-repulsion and destination-attraction with constant agent movement speed over time. As in previous work, a particle filter was applied as the data assimilation scheme — a method that involves resampling. The authors propose three different methods for resampling (standard resampling, component set resampling and mixed component set resampling). Given these resampling schemes, they undertake four experiments.

The first of these experiments seeks to address the use of component set

resampling. This is achieved by testing the implementation for increasing numbers of agent with component set resampling, and comparing against the corresponding results when using standard resampling. It was found that, when using standard resampling, the number of particles in the ensemble that matched with observations decreased as the number of agents increased; the use of mixed component resampling was found to reduce the rate at which this occurred.

The second of the experiments aims to assess the effectiveness of the particle filter using the standard resampling scheme when simulating one agent. For this experiment, the agent was imparted with two different behaviours as in the original investigation. The effectiveness of the data assimilation scheme was assess by measuring the average distance per particle between the simulated agent and the real agent. It was once again found that the simulation error fell with each observation. Furthermore, it was noted that there were two situations that caused an increase in the simulation error:

1. The agent turning back on itself in an area where no sensors are present.

2. The agent approaches a 3-way intersection, where the agent is offered discretely different option of direction in which to travel; the particles, therefore, struggle to converge on the true state.

The third experiment aims to asses the effectiveness of standard resampling when applying the scheme to a system containing two agents. This was achieved by comparing the average error per agent per particle for different numbers of particles (1200 particles, 1600 particles and 2000 particles). It was found that as the number of particles increased, the simulation error decreased.

The final experiment aimed to similarly assess the effectiveness mixed

component set resampling for a system containing multiple agents, first starting with two agents. It was found that for each of the options for number of particles, the implementation of mixed component set resampling reduced the simulation error. Furthermore, when considering systems containing more agents (four or six), it was also found that the implementation of mixed component set resampling improved the simulation accuracy; however, as the number of agents in the system increased, this improvement reduced. This was attributed to situations when agents would crowd together, thus causing difficulties for the particles to distinguish different agents from binary sensors.

2.2.2 Kalman Filter-based Approaches

Other investigation have sought to apply different data assimilation schemes.

2.2.3 Other Approaches

2.2.4 Wang 2017

Maritime piracy is posing a genuine threat to maritime transport. The main purpose of simulation is to predict behaviours of many actual systems, and it has been successfully applied in many fields. But the application of simulation in the maritime domain is still scarce. The rapid development of network and measurement technologies brings about higher accuracy and better availability of online measurements. This makes the simulation paradigm names as dynamic data driven simulation increasingly popular. It can assimilation the online measurements into the running simulation models and ensure much more accurate prediction of the complex systems under study. In this paper, we study how to utilise the online measurements in the agent based

simulation of the maritime pirate activity. A new random finite set based data assimilation algorithm is proposed to overcome the limitations of the conventional vectors based data assimilation algorithms. The random finite set based general data model, measurement model, and simulation model are introduced to support the proposed algorithm. The details of the proposed algorithm are presented in the context of agent based simulation of maritime pirate activity. Two groups of experiments are used to practically prove the effectiveness and superiority of the proposed algorithm.

Whilst this dissertation focuses on the application of data assimilation methods to agent-based models, there also exists a body of work that makes use of the same methods in conjunction with cellular automata (Li et al. 2017, 2012). Cellular automata can be thought of as a specific case of agent-based models in which the environment is characterised by a discrete grid, with the occupancy of each grid cell being updated over time based on the state of its neighbouring cells.

2.3 Summary

Drawing all of the information together.

Issues to consider:

- What happens when there is incomplete information? i.e. data is not provided as frequently as we would like.
- What happens when agent are given discrete choices that alter their potential future states in a non-linear fashion?
- Can better information regarding agent behaviour be used to improve data assimilation?

Chapter 3

Method

As summarised in Chapter 2, there exist a number of different data assimilation schemes, many of which are used extensively in fields such as numerical weather prediction, ... and ... Such methods, however, have been sparsely used in the field of real-time urban simulation and as such this investigation attempts to build upon the existing work by implementing the Ensemble Kalman Filter in conjunction with a pedestrian agent-based model. This chapter therefore seeks to outline the method used in this investigation — the Ensemble Kalman Filter. As shall be explained, the Ensemble Kalman Filter is an approximation of the Kalman Filter, and as such some attention will first be given to the original Kalman Filter, followed by an explanation of the Ensemble Kalman Filter along with the innovations that it incorporates.

3.1 Kalman Filter

One of the earliest forms of Bayesian filtering is known as Wiener filtering (Wiener 1950), which is used in the field of signal processing. The subsequent development of the Kalman Filter (Kalman 1960, Kalman & Bucy

1961), however, is the foundation on which this work is based. The Kalman Filter is a sequential data assimilation scheme which updates the state and covariance matrix, weighted based on the uncertainty in the model forecasts and the observation uncertainty with a view to minimising the posterior mean squared error.

- When is it bad?
- What can we do to improve it?

1. Predict
2. Update

3.2 Ensemble Kalman Filter

Problems with the Kalman Filter:

- it assumes Gaussian PDFs
- it assumes linear model
- Cost of evolving covariance matrix

In order to address some of these problems, the Ensemble Kalman Filter was developed (Evensen 2003, 2009), which acts as an approximation of the Kalman Filter. This approximation is achieved through a Monte Carlo approach of using an ensemble of sample state vectors to represent the state distribution; this development mirrors the recent incorporation of Monte Carlo methods in the field of Bayesian statistics (Wikle & Berliner 2007). As such, the state is represented as follows:

$$\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_N] = [\mathbf{x}_i], \quad \forall i \in (1, N), \tag{3.1}$$

where the state ensemble matrix, $\mathbf{X}$, consists of N state vectors, $\mathbf{x}_i$. The mean state vector, $\bar{\mathbf{x}}$, can be found by averaging over the ensemble:

$$\bar{\mathbf{x}} = \sum_{i=1}^{N} \mathbf{x}_i. \tag{3.2}$$

Similarly, the observations are represented as follows:

$$\mathbf{D} = [\mathbf{d}_1, \ldots, \mathbf{d}_N] = [\mathbf{d}_i], \quad \forall i \in (1, N), \tag{3.3}$$

with each member of the data ensemble matrix, $\mathbf{D}$, being the sum of the original observation $\mathbf{d}$, and a random vector, ϵ_i:

$$\mathbf{d}_i = \mathbf{d} + \epsilon, \quad \forall i \in (1, N). \tag{3.4}$$

The random vector is drawn from an unbiased normal distribution:

$$\epsilon \sim \mathcal{N}(0, \mathbf{R}). \tag{3.5}$$

As with the model state, the mean data vector, $\bar{\mathbf{d}}$, can be found by averaging over the ensemble:

$$\bar{\mathbf{d}} = \sum_{i=1}^{N} \mathbf{d}_i. \tag{3.6}$$

Given that the noise added to the original data vector is unbiased, we should expect that

$$\lim_{N \to \infty} \bar{\mathbf{d}} = \mathbf{d} \tag{3.7}$$

Given the above framework, the data assimilation cycle is made up of:

1. Predict

2. Update:

$$\hat{\mathbf{X}} = \mathbf{X} + \mathbf{K}\left(\mathbf{D} - \mathbf{H}\mathbf{X}\right), \tag{3.8}$$

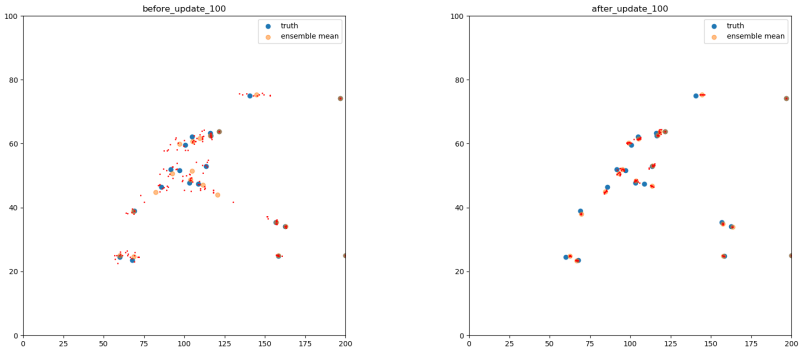
with the Kalman Gain Matrix being given by

$$\mathbf{K} = \mathbf{Q}\mathbf{H}^T\left(\mathbf{H}\mathbf{Q}\mathbf{H}^T + \mathbf{R}\right)^{-1}. \tag{3.9}$$

Chapter 4

Results

State parameters that were given, i.e. assimilation period, ensemble size.



(a) Before update.

(b) After update.

Figure 4.1: Effect of Kalman Filter update on state of ABM.

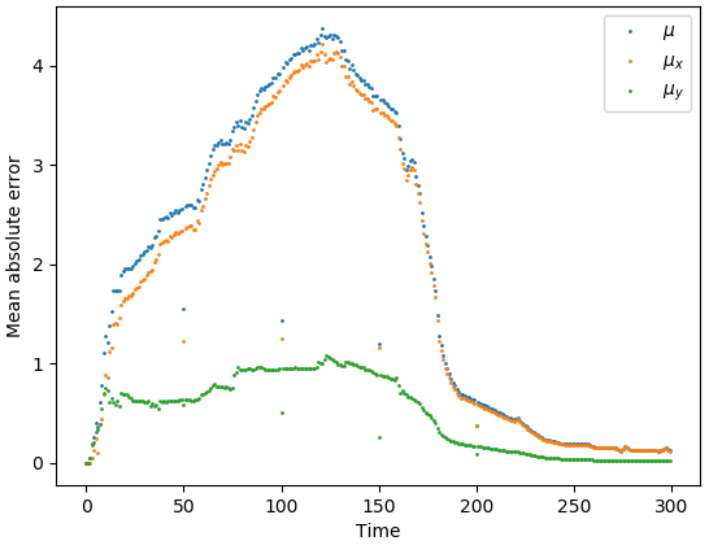


Figure 4.2: Variation of errors with simulation time.

Chapter 5

Conclusion

This is the conclusion.

5.1 Future Work

Talk about the different types of EnKF and the implications for ensemble size (Keller et al. 2018).

- Damping: counteract filter divergence
- Localisation: reduce the effect of spurious correlations
- Hybrid EnKF: Covariance matrix is made up of the weighted sum of the usual covariance matrix and a separate static covariance matrix that encodes prior underlying knowledge about the system
- Dual EnKF: Split the state vector into state and parameters. At assimilation: update parameters, recalculate forecast, update state
- Normal Score EnKF: Developed to handle non-Gaussian PDFs in EnKF. At assimilation: transform state, parameters and measurements into Z-

scores, perform EnKF update based on transformed values, transform back from Z-scores

- Iterative EnKF

Problems to consider:

- What happens when agents leave the system — does the filter recognise this correctly?
- What happens when we are provided with aggregated information?
- What happens when we are provided with different levels of information for different agents?
- Can the filter tell agents apart?
- We have artificially told the filter information about agents' entrance and exits - what happens if it doesn't know these? Do we then need to do some assimilation for data assimilation?

Ideas for transfer:

- Explore impact of different filter parameters on filter performance
- Include derivation of multivariate Kalman filter
- Exploration of complexity of code (time and space)
- Multithreading to deal with computational cost - identifying cut-off point below which it is better to use serial computation.

Appendix A

Code Documentation

This is where I explain the design choices for how the enkf was coded.

- options for generating observations:
  - external
    - observations come from a previous model run as synthetic data
    - therefore they should be independent
    - the problem here is that the ensemble members probably have been set with the wrong entrances and exits or agents
    - this is likely more realistic, because when modelling people we ultimately won't know where they intend to go
    - but we may be able to overcome this using parameter estimation DA.
  - internal
    - observations come from the `base_model`
    - all ensemble members are deep copies of the `base_model`

* consequently, they have the same parameters and initial conditions
* problem here is that it's not very realistic
* but we're going with it for this particular version

Appendix B

Bayes Rule

The joint probability — $P(A, B)$ — is the probability of event A occurring and event B occurring. The conditional probability — $P(A|B)$ — is the probability of event A occurring given that event B occurs. The joint and conditional probabilities are linked by the following relationship:

$$P(A, B) = P(A)P(B|A). \tag{B.1}$$

Similarly, we have

$$P(B, A) = P(B)P(A|B). \tag{B.2}$$

Given that the joint probabilities of A, B and B, A are equal:

$$P(A, B) = P(B, A),$$

we have that

$$P(A)P(B|A) = P(B)P(A|B). \tag{B.3}$$

From this, we can derive the typical form of Bayes Rule:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} \tag{B.4}$$

Bibliography

Bar-Shalom, Y., Li, X. R. & Kirubarajan, T. (2004), *Estimation with applications to tracking and navigation: theory algorithms and software*, John Wiley & Sons.

Batty, M., DeSyllas, J. & Duxbury, E. (2003), 'The discrete dynamics of small-scale spatial events: agent-based models of mobility in carnivals and street parades', *International Journal of Geographical Information Science* **17**(7), 673–697.

Burstedde, C., Klauck, K., Schadschneider, A. & Zittartz, J. (2001), 'Simulation of pedestrian dynamics using a two-dimensional cellular automaton', *Physica A: Statistical Mechanics and its Applications* **295**(3-4), 507–525.

Chen, M., Mao, S. & Liu, Y. (2014), 'Big data: A survey', *Mobile networks and applications* **19**(2), 171–209.

Crooks, A., Castle, C. & Batty, M. (2008), 'Key challenges in agent-based modelling for geo-spatial simulation', *Computers, Environment and Urban Systems* **32**(6), 417–430.

Crooks, A. T. & Heppenstall, A. J. (2012), Introduction to agent-based modelling, *in* 'Agent-based models of geographical systems', Springer, pp. 85–105.

Evensen, G. (2003), 'The ensemble kalman filter: Theoretical formulation and practical implementation', *Ocean dynamics* **53**(4), 343–367.

Evensen, G. (2009), 'The ensemble kalman filter for combined state and parameter estimation', *IEEE Control Systems Magazine* **29**(3), 83–104.

Jazwinski, A. H. (1970), 'Mathematics in science and engineering', *Stochastic processes and filtering theory* **64**.

Kalman, R. E. (1960), 'A new approach to linear filtering and prediction problems', *Journal of basic Engineering* **82**(1), 35–45.

Kalman, R. E. & Bucy, R. S. (1961), 'New results in linear filtering and prediction theory', *Journal of basic engineering* **83**(1), 95–108.

Kalnay, E. (2003), *Atmospheric modeling, data assimilation and predictability*, Cambridge university press.

Keller, J., Hendricks Franssen, H.-J. & Marquart, G. (2018), 'Comparing seven variants of the ensemble kalman filter: How many synthetic experiments are needed?', *Water Resources Research* **54**(9), 6299–6318.

Li, X., Lu, H., Zhou, Y., Hu, T., Liang, L., Liu, X., Hu, G. & Yu, L. (2017), 'Exploring the performance of spatio-temporal assimilation in an urban cellular automata model', *International Journal of Geographical Information Science* **31**(11), 2195–2215.

Li, X., Zhang, Y., Liu, X. & Chen, Y. (2012), 'Assimilating process context information of cellular automata into change detection for monitoring land use changes', *International Journal of Geographical Information Science* **26**(9), 1667–1687.

Liu, S., Lo, S., Ma, J. & Wang, W. (2014), 'An agent-based microscopic pedestrian flow simulation model for pedestrian traffic problems', *IEEE Transactions on Intelligent Transportation Systems* **15**(3), 992–1001.

Rai, S. & Hu, X. (2013), Behavior pattern detection for data assimilation in agent-based simulation of smart environments, *in* '2013 IEEE/WIC/ACM International Joint Conferences on Web Intelligence (WI) and Intelligent Agent Technologies (IAT)', Vol. 2, IEEE, pp. 171–178.

Stanislaw, H. (1986), 'Tests of computer simulation validity: what do they measure?', *Simulation & Games* **17**(2), 173–191.

Thiele, J. C., Kurth, W. & Grimm, V. (2014), 'Facilitating parameter estimation and sensitivity analysis of agent-based models: A cookbook using netlogo and r', *Journal of Artificial Societies and Social Simulation* **17**(3), 11.

Wang, M. & Hu, X. (2013), Data assimilation in agent based simulation of smart environment, *in* 'Proceedings of the 1st ACM SIGSIM Conference on Principles of Advanced Discrete Simulation', ACM, pp. 379–384.

Wang, M. & Hu, X. (2015), 'Data assimilation in agent based simulation of smart environments using particle filters', *Simulation Modelling Practice and Theory* **56**, 36–54.

Wiener, N. (1950), *Extrapolation, interpolation, and smoothing of stationary time series: with engineering applications*, Technology Press.

Wikle, C. K. & Berliner, L. M. (2007), 'A bayesian tutorial for data assimilation', *Physica D: Nonlinear Phenomena* **230**(1-2), 1–16.

Xiang, X., Kennedy, R., Madey, G. & Cabaniss, S. (2005), Verification and validation of agent-based scientific simulation models, *in* 'Agent-directed simulation conference', Vol. 47, p. 55.

Zanella, A., Bui, N., Castellani, A., Vangelista, L. & Zorzi, M. (2014), 'Internet of things for smart cities', *IEEE Internet of Things journal* **1**(1), 22–32.