

Real-time Pedestrian Modelling: Implementing  
the Ensemble Kalman Filter for an  
Agent-Based Model

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# Abstract

This is the abstract.

# Contents

<b>Abstract</b>	<b>2</b>
<b>List of Figures</b>	<b>5</b>
<b>List of Tables</b>	<b>6</b>
<b>1 Introduction</b>	<b>8</b>
1.1 Aims and Objectives . . . . .	11
<b>2 Literature Review</b>	<b>13</b>
2.1 Data Assimilation . . . . .	14
2.2 Application of Data Assimilation to Agent-Based Models . . .	16
2.2.1 Wang 2015 . . . . .	20
2.2.2 Ward 2016 . . . . .	21
2.2.3 Wang 2017 . . . . .	21
2.3 Summary . . . . .	23
<b>3 Method</b>	<b>24</b>
3.1 Kalman Filter . . . . .	24
3.2 Ensemble Kalman Filter . . . . .	25
<b>4 Results</b>	<b>27</b>

<b>5 Conclusion</b>	<b>29</b>
5.1 Future Work . . . . .	29
<b>A Code Documentation</b>	<b>31</b>
<b>B Bayes Rule</b>	<b>33</b>
<b>Bibliography</b>	<b>34</b>

# List of Figures

4.1	Effect of Kalman Filter update on state of ABM. . . . .	27
4.2	Variation of errors with simulation time. . . . .	28

# List of Tables

# Notes

- What is data assimilation?
- What is the Ensemble Kalman Filter?
- What are Agent-Based Models?
- How do people model pedestrians?
- What happens when an agent in one of the ensemble member models tries to leave the station early? Because we're only doing state estimation (meaning just the agent coordinates), the agent won't get reactivated.
- Errors should converge on something relating to twice the exit distance.
- Signposting at the beginning of each chapter
- Don't need to review calibration too much
- Don't need to talk about different types of enkf's too much
- Don't need to talk about DA with CA too much

# Chapter 1

## Introduction

A better understanding of how people move around their environment is of great utility to both academics and policy-makers. Such knowledge can be made use of in the contexts of urban planning, event management and emergency response, particularly when considering urban environments. Furthermore, this may also be of use to those interested in the social issues of mobility, inclusivity and accessibility of opportunities.

When considering such concepts, investigators often make use of modelling techniques. At their most fundamental, models represent our understanding of the system that we are studying — an understanding that may not be perfect (Stanislaw 1986). There exist modelling techniques for the simulation of how pedestrians move around urban spaces. However, these methods exist largely in isolation of the real-world — that is to say that whilst the simulations aim to reflect the real-world, there is no method by which we can incorporate up-to-date observations into these models to stop their divergence from reality.

Simulating pedestrian behaviour is often undertaken at the micro-scale, with such models typically aiming to model at the individual level or on a



spatially fine-grained grid (Burstedde et al. 2001). One of the most prevalent simulation methods in this field is that of Agent-Based Modelling. Such methods consist of two key components: agents and environments. In an Agent-Based Model, we prescribe sets of rules by which individuals interact with each other and their local environments; as interactions take place on the micro-scale, we typically observe the emergence of structure at the macro-scale such as crowding (Batty et al. 2003) or lane formation (Liu et al. 2014). The evaluation of these rules is often not deterministic and instead introduces some element of randomness; these stochastic elements aim to emulate the variability of human behaviour. The introduction of such randomness in conjunction with an imperfect understanding of the phenomena at play, however, typically result in simulation runs diverging from the real system.

In constructing their models, agent-based modellers undertake a development process that involves model verification, validation and calibration. We can take these to mean the following:

- **Model verification:** the process of ensuring that the implementation is an accurate representation of the model (Xiang et al. 2005).
- **Model validation:** the process of ensuring that the chosen model is an accurate representation of the phenomenon that we wish to study (Crooks et al. 2008).
- **Model calibration:** the process of searching for model parameter values such that we can achieve validation (Thiele et al. 2014).

Beyond this, modellers also make efforts to ensure that the initial model conditions are realistic by setting them based on historical data.

The practices of validation, calibration and setting initial model states based on historical data are appropriate for offline evaluations such as testing designs of new buildings or experimenting with different individual behaviours; however, when aiming to simulate events in real-time, this simply delays the inevitable divergence of the model from the real system. Furthermore, model parameters may be transient and thus require to be updated as time passes and the dynamics evolve.

Given the apparently inevitable divergence of stochastic simulations from the real systems that they aim to model, one may alternatively turn to big data. Data is now being generated in higher volumes and at greater velocity than ever before (Chen et al. 2014); however, there also exist issues with observation data from such systems. Whilst models typically allow us to simulate a whole system, observations are typically sparse in either time or space (or both); this is to say that observations rarely provide complete coverage of the events. We therefore seek a solution whereby we can integrate up-to-date observations into our models as the models continue to simulate the system.

One of the methods by which we can combine knowledge represented by our model with observations as they become available is through data assimilation techniques, which are most commonly used in the field of numerical weather prediction (Kalnay 2003). Such techniques are typically made up of two steps:

1. **Predict:** Run the model forward, estimating the state of the system, until new observations become available.
2. **Update:** Upon receipt of new observations, combine the model's estimate of the system state with the new data.

These steps are repeated iteratively in a cycle. It is important to note that just as there is error associated with the model, we also acknowledge that there is observational error associated with the data. The aim of incorporating the observations into the model is to improve the model accuracy with respect to the true system state.

A large volume of work exists in which such techniques are applied to meteorological systems where the models used are based on differential equations. Significantly less work exists in which data assimilation methods are applied to agent-based models — in particular pedestrian models. This dissertation therefore aims to expand on the pre-existing work by implementing a data assimilation scheme known as the Ensemble Kalman Filter in conjunction with a relatively simple agent-based model of pedestrians crossing a two-dimensional station from one side to the other.

## 1.1 Aims and Objectives

With the above in mind, this dissertation aims to determine whether the Ensemble Kalman Filter method of data assimilation can be used to improve the accuracy with which an Agent-Based Model simulates pedestrian movements given synthetic real-time data. In order to achieve this, the following objectives are set out:

1. Develop a general Ensemble Kalman Filter computational class.
2. Apply the Ensemble Kalman Filter to the pedestrian model.
3. Compare the accuracy with which the Ensemble Kalman Filter implementation of the model simulates the pedestrian movement against the based model without data assimilation.

The remainder of this dissertation will provide an overview of some of the work that has been undertaken thus far on implementing data assimilation schemes with agent based models (Chapter 2), elaborate on how data assimilation works — particularly focussing on the aforementioned Ensemble Kalman Filter (Chapter 3) — and examine the effectiveness of this method in improving the accuracy of the agent-based model (Chapter 4).

# Chapter 2

## Literature Review

As touched upon in Chapter 1, the process of developing an agent-based model typically involves some form of model calibration. Model calibration is the procedure of fine-tuning the model that we are using such that it best fits the particular situation that we are seeking to model (Crooks & Heppenstall 2012). There are a large number of different manners in which we can calibrate agent-based models (Thiele et al. 2014). These approaches typically involve making use of real-world data to estimate the parameters and initial state of the model; this is, however, undertaken once prior to running the model.

In some situations, we aim to simulate events in real-time (or close to real-time). In such situations, we are often able to observe the evolution of the real-world system which we seek to model and consequently may wish to use this information to recalibrate the model. This would, however, require that we stop the simulation, undertake calibration, and restart the model. We therefore seek an approach that allows us to incorporate observations of the system whilst simulating the system — data assimilation.

This Chapter will therefore seek to provide a basic overview of data as-

simulation, along with some coverage of the attempts that have been made to implement such techniques to agent-based models that simulate urban systems.

## 2.1 Data Assimilation

- Origins of filtering with Wiener filter, 1950. Only applies for stationary signals.
- Kalman-Bucy 1961
- Stratonovich 1968
- Jazwinski 1970
- 

The process of data assimilation involves making use of observations along with prior knowledge (which, in our case, is encoded in a model) to produce increasingly accurate estimates of variables of interest. Such a process can be achieved through a Bayesian filtering approach (Bar-Shalom et al. 2004, Jazwinski 1970).

Under such a framework, the updating of the model state is undertaken on the basis of Bayes Rule (for which a derivation is provided in Appendix B):

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \quad (2.1)$$

Bayes Rule is made up of four components:

1.  $P(A)$ : The probability of  $A$ , known as the **Prior**.
2.  $P(A|B)$ : The probability of  $A$  given  $B$ , known as the **Posterior**.

3.  $P(B|A)$ : The probability of  $B$  given  $A$ , known as the **Likelihood**.
4.  $P(B)$ : The probability of  $B$ , known as the **Marginal Likelihood**.

When applying this notation to the problem at hand, the components become:

1. **Prior**,  $P(\mathbf{x})$ : The probability distribution representing the prior state of the model.
2. **Posterior**,  $P(\mathbf{x}|\mathbf{d})$ : The probability distribution representing the updated state of the model in light of the observed data, that is to say the probability of the model state given the data.
3. **Likelihood**,  $P(\mathbf{d}|\mathbf{x})$ : The probability distribution of the observed data given the model state.
4. **Marginal Likelihood**,  $P(\mathbf{d})$ : The probability distribution representing the observed data.

With the above notation, Bayes Rule becomes:

$$P(\mathbf{x}|\mathbf{d}) = \frac{P(\mathbf{d}|\mathbf{x}) P(\mathbf{x})}{P(\mathbf{d})} \quad (2.2)$$

The aim of a data assimilation scheme therefore becomes to provide an update to the state in the form of the posterior,  $P(\mathbf{x}|\mathbf{d})$ , given new observations,  $P(\mathbf{d})$ .

There exist a number of different schemes for tackling this problem which are often divided into two groups:

1. **Sequential**: What does this mean and some examples, Kalman Filter (and variations thereof), Particle Filter.

2. **Variational:** What does this mean and some examples, 3D-VAR, 4D-VAR.

Of the work that currently exists wherein investigators attempt to apply data assimilation schemes to agent-based models, most make use of sequential schemes.

## 2.2 Application of Sequential Data Assimilation to Agent-Based Models

One of earliest pieces of work undertaken on the application of data assimilation schemes to agent-based models of urban environments was by Wang & Hu (2013). In this work, they simulate a smart office environment with people in it — a scenario that is becoming increasingly common with the advent of the Internet of Things (Zanella et al. 2014). The aim of the work was to make use of real-time data in conjunction with the agent-based simulation to provide more accurate estimates of the occupancy of the environment. This was achieved using the Particle Filter method of data assimilation; the method was chosen as it did not require the system to be Gaussian. The particle filter method operates by holding an ensemble of realisations of the simulation, each of which are evolved forward over time between observations; when observations are received, the particle states are weighted and the new state is obtained by sampling from these weighted particles. The observations used were synthetic data generated by the agent-based model, aiming to emulate motion sensors which would provide a binary response of whether a person was present in a given location.

The work undertaken consisted of two experiments — firstly simulating single agent in the environment, then going on to simulate two agents in the



same environment. In the case of the first experiment, the agent was simulated with two different routing behaviours; for the first routing behaviour, the agent moves forward sequentially through a series of waypoints, whilst for the second routing behaviour, the agent moves through a series of waypoints before turning back to return to its initial position. In this experiment, it was found that the simulation error decreased when the agent was detected at each of the sensors for both routing behaviours with error growing between detections; this is as expected — it confirms that the simulation becomes more accurate with the addition of further information regarding the system. In the case of the second routing behaviour, the simulation error also grew following the agent’s turn to head back to its origin. In the second experiment, they aimed to simulate two agents in the same environment, with the two agents maintaining spatial separation. This simulation was run a number of times for different numbers of particles with a view to establishing a relationship between the number of particles and simulation accuracy. It was found that as the number of particles was increased (through 400, 800, 1200 and 1600), the simulation error decreased. It was also found, however, that the experiments with fewer particles (400 and 800) struggled to converge, with the smaller number of particles unable to provide sufficient coverage of the state space. It was therefore noted that as the number of agents was increased, the method was likely to struggle.

This final issue may be solved by an increase in the number of particles; however, this comes with an attached increase in the computational cost (both in terms of compute time and space). The implementation of the particle filter requires that a realisation of the model be kept for each particle, resulting in growing memory requirements as the number of particles are increased. Furthermore, each particle is required to evolve the model for

each time-step, resulting in an increasing computational cost.

There are two subsequent pieces of research which have sought to build directly upon the above initial investigation. The first of these was undertaken by Rai & Hu (2013), using the same simulation model and method of data assimilation but aiming to add a further layer which estimated behavioural patterns in the system. This was achieved using a hidden Markov model which was trained on the historical data of the system. The information encoded in the hidden Markov model is incorporated into the particle filter at the sampling step; the hidden Markov model is first used to identify the types of behaviour being exhibited by the system and particles then sample from different types, subsequently engaging the simulation with said behaviour. The remaining steps of the filtering process remained the same. The aim of applying this approach was to further improve the accuracy of simulations through the identification of pedestrian behaviours. In order to do so, the behaviours were divided into the following categories:

- **Outside:** All agents wait outside of a conference room.
- **In Conference:** All agents are attending meeting in the conference room.
- **Few Entering:** A small number of agents entering the conference room.
- **High Entering:** A large number of agents entering the conference room.
- **Few Leaving:** A small number of agent leaving the conference room.
- **High Leaving:** A large number of agent leaving the conference room.

These categorisations, particularly the *entering* and *leaving* states, present a problem. When entering the conference room, agents are more likely to be categorised into three groups — early few entering, high entering and late few entering — this re-categorisation helps to describe the status of the agents that are not in the process of entering:

- **Early few entering:** A small number of agents entering the conference room early before a meeting, with the other agents outside the conference room.
- **High entering:** A large number of agents entering the conference room.
- **Late few entering:** A small number of agents entering the conference room late after the meeting has started, with the other agents inside the conference room.

The investigation then attempts to determine the accuracy with which the model is able to identify the correct behaviour for agents, with accuracy being defined as

$$\frac{\sum_{k=1}^T S_t^k - S_t^{real}}{T} \quad (2.3)$$

where  $T$  is taken to be the total number of simulation steps and  $S$  is the behaviour pattern state. It is unclear, however, how this calculation is undertaken given that the behavioural states are categories and not numerical; furthermore, the meaning behind the state notation is explained — it appears that  $k$  is a time-step index, however it does not make sense to compare the behavioural state at each time state to a static “real” behavioural state and the latter would likely be a transient property. Some indication is given that a numerical encoding of the categories has been used for visualisation purposes, however this should not be used for arithmetic purposes, nor should

any ordinality be inferred from it. Indeed, the results presented take the form of accuracy percentages, suggesting that a more conventional accuracy score has been used, such as

$$\frac{1}{T} \sum_{k=1}^T \mathbb{1} \left( \hat{S}_k = S_k^{real} \right) \quad (2.4)$$

where the indicator function,  $\mathbb{1}(\dots)$ , returns 1 when the condition is fulfilled, else 0. The results suggest that the model developed accurately identifies the behavioural states except for states that occur infrequently; the model performs particularly well when identifying states in which agents are static, i.e. *outside* and *in conference*. Such supplementary information could improve the performance of data driven simulations, likely helping the process of data assimilation for parameter estimation.

### 2.2.1 Wang 2015

Agent-based simulations are useful for studying people’s movement and to help making decisions in situations like emergency evacuation in smart environments. These agent-based simulations are typically used as offline tools and do not assimilate real time data from the environment. With more and more smart buildings equipped with sensor devices, it is possible to utilise real time sensor data to dynamically inform the simulations to improve simulation results. In this paper, we propose a method to assimilate real time sensor data in agent-based simulations of smart environments based on particle filters. The data assimilation aims to estimate the system state, i.e. people’s location information in real time, and use the estimated states to provide initial conditions for more accurate simulation/prediction of the system dynamics in the future. We develop a particle filter-based data assimilation framework and propose a new resampling method named as component set

resampling to improve data assimilation for multiple agents. The proposed framework and method are demonstrated and evaluated through experiments using a sparsely populated smart environment.

### **2.2.2 Ward 2016**

A widespread approach to investigating the dynamical behaviour of complex social systems is via agent-based models. In this paper, we describe how such models can be dynamically calibrated using the ensemble kalman filter, a standard method of data assimilation. Our goal is twofold. First, we want to present the enfk in a simple setting for the benefit of abm practitioners who are unfamiliar with it. Second, we want to illustrate to data assimilation experts the value of using such methods in the context of abms of complex social systems and the new challenges these types of model present. We work towards these goals within the context of a simple question of practical value: how many people are there in Leeds (or any other major city) right now? We build a hierarchy of exemplar models that we use to demonstrate how to apply the enfk and calibrate these using open data of football counts in Leeds.

### **2.2.3 Wang 2017**

Maritime piracy is posing a genuine threat to maritime transport. The main purpose of simulation is to predict behaviours of many actual systems, and it has been successfully applied in many fields. But the application of simulation in the maritime domain is still scarce. The rapid development of network and measurement technologies brings about higher accuracy and better availability of online measurements. This makes the simulation paradigm names as dynamic data driven simulation increasingly popular. It can assimila-

tion the online measurements into the running simulation models and ensure much more accurate prediction of the complex systems under study. In this paper, we study how to utilise the online measurements in the agent based simulation of the maritime pirate activity. A new random finite set based data assimilation algorithm is proposed to overcome the limitations of the conventional vectors based data assimilation algorithms. The random finite set based general data model, measurement model, and simulation model are introduced to support the proposed algorithm. The details of the proposed algorithm are presented in the context of agent based simulation of maritime pirate activity. Two groups of experiments are used to practically prove the effectiveness and superiority of the proposed algorithm.

- Ward et al. (2016) — model of pedestrians on Briggate, a 1-D strip along which pedestrians walk, using enkf for both state and parameter calibration.
- Wang & Hu (2013, 2015) — agents occupying a smart environment/building with a view to modelling population density, using particle filter.
- Rai & Hu (2013) — agents occupying a smart office/building, using particle filter, extends Wang & Hu (2013, 2015) by incorporating a Hidden Markov Model for behaviour pattern detection.
- Wang et al. (2017) — model of maritime pirates, using random finite set based data assimilation instead of kf or pf — why? does this have any relevance?

Whilst this dissertation focuses on the application of data assimilation methods to agent-based models, there also exists a body of work that makes use of the same methods in conjunction with cellular automata (Li et al.

2017, 2012). Cellular automata can be thought of as a specific case of agent-based models in which the environment is characterised by a discrete grid, with the occupancy of each grid cell being updated over time based on the state of its neighbouring cells.

## **2.3 Summary**

Drawing all of the information together.

# Chapter 3

## Method

As summarised in Chapter 2, there exist a number of different data assimilation schemes, many of which are used extensively in fields such as numerical weather prediction, ... and ... Such methods, however, have been sparsely used in the field of real-time urban simulation and as such this investigation attempts to build upon the existing work by implementing the Ensemble Kalman Filter in conjunction with a pedestrian agent-based model. This chapter therefore seeks to outline the method used in this investigation — the Ensemble Kalman Filter. As shall be explained, the Ensemble Kalman Filter is an approximation of the Kalman Filter, and as such some attention will first be given to the original Kalman Filter, followed by an explanation of the Ensemble Kalman Filter along with the innovations that it incorporates.

### 3.1 Kalman Filter

One of the earliest forms of Bayesian filtering is known as Wiener filtering (Wiener 1950), which is used in the field of signal processing. The subsequent development of the Kalman Filter (Kalman 1960, Kalman & Bucy



1961), however, is the foundation on which this work is based. The Kalman Filter is a sequential data assimilation scheme which updates the state and covariance matrix, weighted based on the uncertainty in the model forecasts and the observation uncertainty with a view to minimising the posterior mean squared error.

- When is it bad?
- What can we do to improve it?

1. Predict
2. Update

## 3.2 Ensemble Kalman Filter

Problems with the Kalman Filter:

- it assumes Gaussian PDFs
- it assumes linear model
- Cost of evolving covariance matrix

In order to address some of these problems, the Ensemble Kalman Filter was developed (Evensen 2003, 2009), which acts as an approximation of the Kalman Filter. This approximation is achieved through a Monte Carlo approach of using an ensemble of sample state vectors to represent the state distribution; this development mirrors the recent incorporation of Monte Carlo methods in the field of Bayesian statistics (Wikle & Berliner 2007). As such, the state is represented as follows:

$$\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_N] = [\mathbf{x}_i], \quad \forall i \in (1, N), \quad (3.1)$$

where the state ensemble matrix,  $\mathbf{X}$ , consists of  $N$  state vectors,  $\mathbf{x}_i$ . The mean state vector,  $\bar{\mathbf{x}}$ , can be found by averaging over the ensemble:

$$\bar{\mathbf{x}} = \sum_{i=1}^N \mathbf{x}_i. \quad (3.2)$$

Similarly, the observations are represented as follows:

$$\mathbf{D} = [\mathbf{d}_1, \dots, \mathbf{d}_N] = [\mathbf{d}_i], \quad \forall i \in (1, N), \quad (3.3)$$

with each member of the data ensemble matrix,  $\mathbf{D}$ , being the sum of the original observation  $\mathbf{d}$ , and a random vector,  $\epsilon_i$ :

$$\mathbf{d}_i = \mathbf{d} + \epsilon, \quad \forall i \in (1, N). \quad (3.4)$$

The random vector is drawn from an unbiased normal distribution:

$$\epsilon \sim \mathcal{N}(0, \mathbf{R}). \quad (3.5)$$

As with the model state, the mean data vector,  $\bar{\mathbf{d}}$ , can be found by averaging over the ensemble:

$$\bar{\mathbf{d}} = \sum_{i=1}^N \mathbf{d}_i. \quad (3.6)$$

Given that the noise added to the original data vector is unbiased, we should expect that

$$\lim_{N \rightarrow \infty} \bar{\mathbf{d}} = \mathbf{d} \quad (3.7)$$

Given the above framework, the data assimilation cycle is made up of:

1. Predict
2. Update:

$$\hat{\mathbf{X}} = \mathbf{X} + \mathbf{K}(\mathbf{D} - \mathbf{H}\mathbf{X}), \quad (3.8)$$

with the Kalman Gain Matrix being given by

$$\mathbf{K} = \mathbf{Q}\mathbf{H}^T(\mathbf{H}\mathbf{Q}\mathbf{H}^T + \mathbf{R})^{-1}. \quad (3.9)$$

# Chapter 4

## Results

State parameters that were given, i.e. assimilation period, ensemble size.

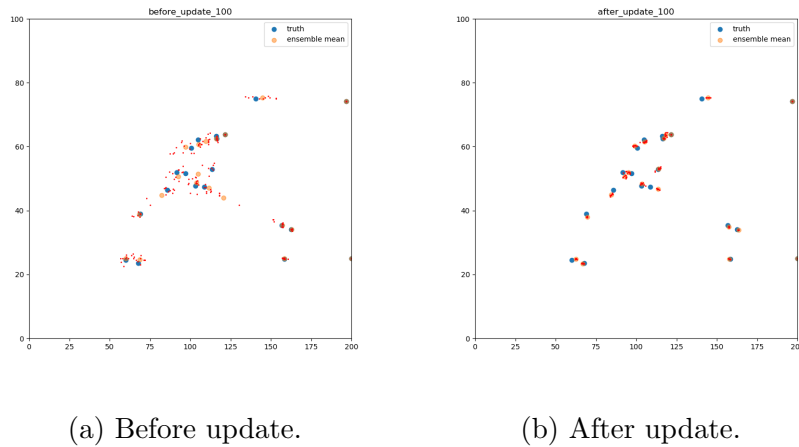


Figure 4.1: Effect of Kalman Filter update on state of ABM.

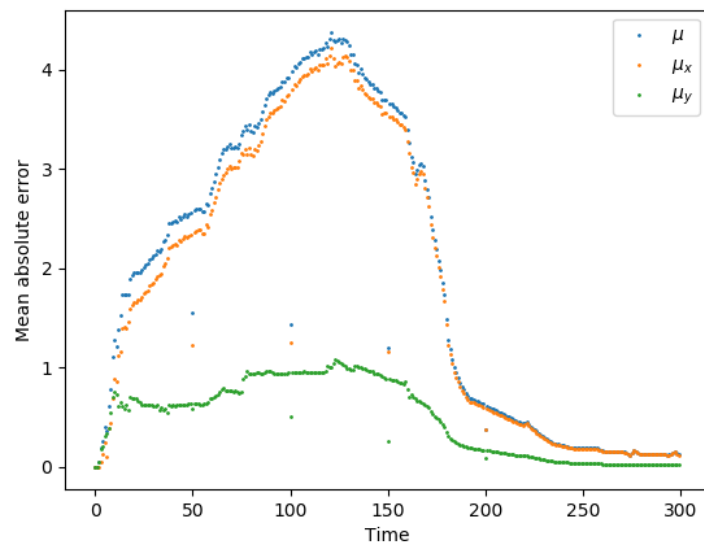


Figure 4.2: Variation of errors with simulation time.

# Chapter 5

## Conclusion

This is the conclusion.

### 5.1 Future Work

Talk about the different types of EnKF and the implications for ensemble size (Keller et al. 2018).

- Damping: counteract filter divergence
- Localisation: reduce the effect of spurious correlations
- Hybrid EnKF: Covariance matrix is made up of the weighted sum of the usual covariance matrix and a separate static covariance matrix that encodes prior underlying knowledge about the system
- Dual EnKF: Split the state vector into state and parameters. At assimilation: update parameters, recalculate forecast, update state
- Normal Score EnKF: Developed to handle non-Gaussian PDFs in EnKF. At assimilation: transform state, parameters and measurements into Z-

scores, perform EnKF update based on transformed values, transform back from Z-scores

- Iterative EnKF

Problems to consider:

- What happens when agents leave the system — does the filter recognise this correctly?
- What happens when we are provided with aggregated information?
- What happens when we are provided with different levels of information for different agents?
- Can the filter tell agents apart?
- We have artificially told the filter information about agents' entrance and exits - what happens if it doesn't know these?

Ideas for transfer:

- Derivation of multivariate Kalman filter
- Exploration of complexity of code (time and space)
- Multithreading to deal with computational cost - identifying cut-off point below which it is better to use serial computation.

# Appendix A

## Code Documentation

This is where I explain the design choices for how the enkf was coded.

- options for generating observations:
  - external
    - \* observations come from a previous model run as synthetic data
    - \* therefore they should be independent
    - \* the problem here is that the ensemble members probably have been set with the wrong entrances and exits or agents
    - \* this is likely more realistic, because when modelling people we ultimately won't know where they intend to go
    - \* but we may be able to overcome this using parameter estimation DA.
  - internal
    - \* observations come from the `base_model`
    - \* all ensemble members are deep copies of the `base_model`

- \* consequently, they have the same parameters and initial conditions
- \* problem here is that it's not very realistic
- \* but we're going with it for this particular version



# Appendix B

## Bayes Rule

The joint probability —  $P(A, B)$  — is the probability of event  $A$  occurring and event  $B$  occurring. The conditional probability —  $P(A|B)$  — is the probability of event  $A$  occurring given that event  $B$  occurs. The joint and conditional probabilities are linked by the following relationship:

$$P(A, B) = P(A)P(B|A). \quad (\text{B.1})$$

Similarly, we have

$$P(B, A) = P(B)P(A|B). \quad (\text{B.2})$$

Given that the joint probabilities of  $A, B$  and  $B, A$  are equal:

$$P(A, B) = P(B, A),$$

we have that

$$P(A)P(B|A) = P(B)P(A|B). \quad (\text{B.3})$$

From this, we can derive the typical form of Bayes Rule:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} \quad (\text{B.4})$$

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