Constructive Deconstruction: Evaluating Industrial Policies in Strategic Interactions and Production Networks*

Ko Sugiura†

November 1, 2023
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Abstract

Over the past few decades, industrial policies have gained renewed momentum. In tandem with this resurgence, the literature has documented that production processes in many developed economies rely on production networks between industries, each of which features oligopolistic competition. In this paper, I contemplate the economic impact of an industrial policy through the lens of a general equilibrium multisector model of heterogenous oligopolistic firms with a production network. For the identification, I develop a new procedure that first deconstructs the policy parameter into firm-level variables — firm-level sufficient statistics — as well as sector-level variables, and then identifies these building blocks before finally reconstructing the policy parameter of interest. Using my framework, I study the impact of one part of the U.S. CHIPS and Science Act, which was passed into law in 2022, on gross domestic product (GDP). My estimation predicts that accounting for firms' strategic interactions nearly doubles the magnitude of the estimate of the policy effect, highlighting the policy relevance of strategic interactions.

Keywords: Policy evaluations, Industrial policies, Strategic interactions, Production networks, Identification

JEL Codes: E61, E65, F13, F41, L13, L16

*I thank Nathan Canen, Hiroyuki Kasahara, Bent Sørensen and Kei-Mu Yi for useful comments.
†Department of Economics, University of Houston, Science Building 3581 Cullen Boulevard Suite 230, Houston, 77204-5019, Texas, United States.

1 Introduction

Over the past few decades, industrial policies — policies that are purposefully targeted at particular industries — have regained its popularity as policy tools for a range of purposes.[1] In the U.S., for example, as a protectionist trade measure, the Trump administration raised tariffs on selected products.[2] National security concerns have led to the CHIPS and Science Act of 2022, which is projected to make nearly 53 billion U.S. dollars of investment in semiconductor industry.[3]

Equally important for the policymaker are ex ante evaluations of the impacts of these policies on economic outcomes such as gross domestic product (GDP). Despite the rich stockpile of empirical reduce-form estimates on the industrial policies targeted at certain sectors,[4] they cannot generally be transferred to the case of other sectors because of the substantial heterogeneity of the economy in two dimensions. First, there has been a growing recognition in the literature that oligopolistic competition between heterogenous firms is the key to replicate a number of salient empirical regularities: e.g., incomplete pass-through of a price shock (Atkeson and Burstein 2008) and market power (De Loecker et al. 2020, 2021). Second, sectors' heterogenous roles on production networks have sparked an extensive research on their implications for business cycle (Acemoglu et al. 2012) and misallocation (Baqaee and Farhi 2020; Bigio and La'O 2020), for instance. However, even in the realm of structural modeling, no existing research has included both of these two features at once, impeding a precise ex ante evaluation of an industrial policy.

In this paper, I develop a structural framework for policy evaluation, accounting for firm's oligopolistic competitions and production networks between industries. I define the policy effect as the change in GDP due to an industrial policy.[5] The key mechanism of our model is that the production network compounds not only the firms' markup responses with respect to firms' own choices but also those with respect to competitors' (strategic complementarities), with the latter absent in monopolistic models. That is, failure to account for strategic interactions can lead to a both quantitatively and qualitatively inaccurate prediction. Since moreover in strategic interaction models, individual firms have the potential to exert nonnegligible influence over sectoral outcomes, the policy parameter cannot be characterized by aggregate variables alone, invalidating the aggregate sufficient statistics approach, a method typically used in the

[1] Examples include ... See Rodrik (2008), Juhász et al. (2023) and Juhász and Steinwender (2023) for comprehensive overview of industrial policies.

[2] This is what is called the U.S.-China trade war.

[3] part of Bidenomics.

[4] Citation...

[5] My framework can also be used to analyze other policy parameters. See Remark 5.2.

recent macroeconomics literature.[6] This paper exploits the widely-used firm-level data, and identifies the policy effect in terms of individual firm's responses, what I call firm-level sufficient statistics. To examine the effect of one portion of the CHIPS Act, I undertake a hypothetical policy experiment of increasing the subsidy on semiconductor industry, and compare the estimate based on my model to that based on monopolistic competition. I find that the former estimate nearly doubles the latter in magnitude, echoing the empirical relevance of accounting for strategic competitions in the presence of a production network.

My model builds on Liu (2019) to study a general equilibrium multisector model of a production network, but differs by assuming that each sector is populated by heterogenous oligopolistic firms, thereby firm-level markups being endogenously variable. The government helps firms to purchase sectoral intermediate goods through an ad-valorem subsidy specific to the purchaser sector. The policy effect is defined as the change in GDP due to the a shift in the level of sector-specific subsidy (i.e., an industrial policy). To keep track of the endogenously variable markups, I restrict the sectoral aggregators to be a demand system that is homothetic with single aggregator (HSA; Matsuyama and Ushchev 2017). One benefit of this specification is that firms' interactions are summarized by the single aggregator.[7] Still, the fact that individual firms are nonnegligible to the aggregate hampers the identification of the policy effect by the aggregate sufficient statistics. This paper proposes an alternative approach that recovers the policy parameter in terms of firm-level responses.

The identification analysis of this paper consists of three layers: namely, top, middle and bottom layers. In the top layer, the object of interest is written as an integral of the responsiveness of GDP with respect to a policy change. The middle layer further deconstructs the responsiveness of GDP into the comparative statics of firm- and sector-level variables. These comparative statics are then recovered by solving the systems of equations that are derived from the firm's optimization problems, taking the firm-level output quantity and price, and elasticities of firm-level demand and production functions as given. The bottom layer obtains these conditioning variables — firm-level sufficient statistics — by leveraging the control function approach of the industrial organization literature. In doing so, I restrict the class of demand and production functions and the way in which firms' productivities enter the individual firm's decision. I show that these assumptions are flexible enough to accommodate the specifications that are commonly used in the macroeconomics literature. My identification approach is constructive, so that a

[6] See Chetty (2009) and Kleven (2021) for a general idea of the sufficient statistics approach. See, e.g., Arkolakis et al. (2012), Adão et al. (2017), Arkolakis et al. (2019) and Adão et al. (2020) for the applications in the context of macroeconomics.

[7] A similar insight is exploited by Amiti et al. (2014) and Arkolakis et al. (2019).

nonparametric estimator for the policy effect can be obtained by reading off these procedure from the bottom to the top. Unlike the calibration-type approach, my estimation does not require any external information (e.g., parameter estimates from the preceding research) and thus can be performed in a self-contained fashion.

Finally, I bring my framework to the U.S. firm-level data to evaluate the economic impacts of the CHIPS and Science Act, which selectively promotes the semiconductor industry and was put into law in 2022. As one part of this policy, I consider a hypothetical policy experiment of shifting the ad-valorem subsidy on the Computer and Electronic Products industry from the 2021 level, which is 14.89%, to an alternative level of 16.00% — equivalent to 0.56 billion U.S. dollars. The estimate accounting for strategic interactions as well as the production network predicts that GDP fall by 1.34 billion dollars, while the estimate based on monopolistic competition under the production network suggests a much smaller decrease by 0.71 billion dollars. Comparing these two estimates underlines the policy relevance of correctly accounting for the firm's strategic interactions.

To better understand the mechanism behind this, I look at the responsiveness of GDP at the 2021 level with an industry-level breakdown. First, I decompose the responsiveness of sectoral GDP decomposing into four components: namely, i) the changes in output quantities (quantity effects), ii) the associated changes in output prices (price effects), iii) the changes in input costs due to changes in input quantities (switching effects) and iv) the changes in input costs due to changes in input prices (wealth effects).[8] An important insight here is that in the networked economy, output of one sector may be used as input in all sectors, so that the output price change in one sector directly affects the input price of all sectors. My estimation suggests that for many sectors in oligopolistic competition, even if they produce more of their products, input prices do not decrease so much as output price do, leaving them with a higher input cost. (Negative contributions of the switching effects dominate.)

Second, to study the tension between these four forces from the angle of path-through coefficients. We define macro complementarity as the sector-level policy-cost pass-through. We define as the sector-level cost-price pass-through. I theoretically find that the sector-level cost-price pass-through coefficient takes the form of a weighted sum of firms' strategic complementarities in the sector, which in turn is compounded along the production network to give the sector-level policy-cost pass-through coefficient. I refer to the former as micro complementarity and the latter macro complementarity.

[8]In a networked economy, output of one sector is an input for other sectors, thereby the change in the output price in one sector inducing the changes in input prices in other sectors.

Related literature

I see this paper contributing to four important strands of the literature. First, the framework put forth in this paper is directly related to the literature on *ex ante* counterfactual predictions of economic shocks (e.g., trade costs, productivity) such as Arkolakis et al. (2012), Melitz and Redding (2015), Adão et al. (2017), Feenstra (2018) and Adão et al. (2020). My framework, though, marks distinction in two ways. First, these preceding papers are based on perfectly competitive or monopolistic firms, while ours explicitly accounts for firms' strategic interactions. Second, the existing literature is mostly concerned with directly expressing an aggregate outcome in terms of aggregate variables (aggregate sufficient statistics). On the contrary, my approach first deconstructs the object of interest into firm-level variables as well as sector-level variables, and then show that these can be recovered from the observables, before the researcher can re-construct the same objective outcome (firm-level sufficient statistics).

Second, while there have been rich volume of empirical literature examining the effects of an industrial policy (e.g., Aghion et al. 2015; Juhász 2018; Kalouptsidi 2018; Criscuolo et al. 2019),[9] a much less effort has been devoted to the theoretical work. This paper adds to this strand of the literature in two important ways. First, optimal industrial policy design in a multi-sector environment is explored in Itskhoki and Moll (2019) and Liu (2019) for exogenous market distortions; in Lashkaripour and Lugovskyy (2023) for endogenous but constant markups; and in Bartelme et al. (2021) for endogenously varying market distortions. In my model, the market distortions arise from oligopolistic competitions, and thus can endogenously vary according to the strategic interactions.[10,11] Second, my paper intersects with the treatment effect literature. Among many others, Criscuolo et al. (2019) discuss "reduced-form" causal effects of an industrial policy.[12] The causal interpretation of their policy parameter, however, is limited to those units who have experienced (exogenous) changes in the eligibility of receiving the policy. From the perspective of a policymaker who considers well-bing of a society as a whole, such a locally tailored notion of "causal effect" might not be of central interest. In the spirit of the *policy relevant treatment*

[9] For more through literature review, see Lane (2021) and references therein.

[10] The model entertained in my papers bears some resemblance to those studied in the literature on welfare loss due to misallocation in the presence of production networks such as Jones (2011, 2013), Baqaee and Farhi (2020, 2022) and Bigio and La'O (2020). These works are principally interested in characterizing the welfare loss: they start from an efficient economy — i.e., they assume away from an initial state of the market distortions — and then focus on the consequence of adding a policy as a source of distortion. my paper admits market distortions in the initial state of the economy, including policy itself, and then contemplates a welfare-improving policy prescription.

[11] Grassi (2017) also studies the case of oligopoly, but his focus is on positive analysis under a parametric specification of production and demand functions. my paper is concerned with evaluating the policy effects with a minimal set of parametric assumptions.

[12] Juhász (2018) use

effects (PRTE; Heckman and Vytlacil 2001, 2005, 2007), this paper contemplates an alternative policy parameter that is both economically interesting and causal in the sense of Marshall (1890).

Third, this paper contributes to the literature documenting empirical relevance of endogenous firms' markups such as Atkeson and Burstein (2008), Amiti et al. (2014), Edmond et al. (2015), Arkolakis et al. (2019), Gaubert and Itskhoki (2020), and De Loecker et al. (2021). I connect this line of research to the literature on sectoral comovements of prices and quantities (e.g., Basu 1995; Huang and Liu 2004; Huang et al. 2004; Huang 2006; Nakamura and Steinsson 2010; La'O and Tahbaz-Salehi 2022; Rubbo 2023) by introducing production networks across sectors. Specifically, I show that the sectoral comovements are traced out by the combination of the within-sector interactions summarizing firms' strategic complementarities (I refer to it as *micro complementarities*) and the between-sector interactions compounding the micro complementarities along the production network (I call it *macro complementarities*).[13]

Lastly, outside the domain of the macroeconomics literature, my method is tightly linked to the industrial organization literature on identification of firms' production functions. In particular, the so called control function approach (e.g., Olley and Pakes 1996; Levinsohn and Petrin 2003) has customary assumed a perfect competition (e.g., Ackerberg et al. 2015; Gandhi et al. 2019), or a monopolistic competition (e.g., Kasahara and Sugita 2020). My paper extends their method to strategic interactions by adapting the notion of sufficient statistics for competitors' decisions and productivities.[14]

2 Overview

In this section, I provide an overview of this paper using a simple input-output accounting framework and a duopoly model. This section serves two purposes. First, this section illustrates the implication of featuring both a production network and an oligopolistic competition in policy analysis, and contrasts it to the cases without either of these mechanisms. To this end, I first consider the standard input-output accounting framework with sector-level markups,[15] and then provide a microfoundation to study the firm-level endogenous markup adjustment. Second, this section explains the identification challenge in this environment and briefly sketches the approach pushed forward in this paper.

[13]These terminologies draw from Klenow and Willis (2016) and Alvarez et al. (2023).

[14]Doraszelski and Jaumandreu (2019), Brand (2020) and Bond et al. (2021) draw attention to the risk of simply applying the standard control function approach to the case of oligopolistic competitions, but they do not provide a methodology to deal with the strategic interactions in recovering the firm's production function.

[15]I consider a single country, closed-economy version of Timmer et al. (2015).

2.1 Setup

Consider an economy consisting of two sectors, indexed by $i \in \{1, 2\}$. Each sector's sales (measured in appropriate monetary unit) is denoted by x_i. Assume that for each industry i, the sector's sales (x_i) is different from the expenditure ($\tilde{x}_i$) by the rate of μ_i: i.e., $\tilde{x}_i = \frac{1}{\mu_i} x_i$. I consider the case of $\mu_i > 1$, in which the distortion can be interpreted as a sector-level markup (a microfoundation is provided in Section 2.3). Let the expenditure for final consumption of sector i's product be denoted by y_i. The share of sector j's good in sector i's expenditure represented by $\omega_{i,j}$ for $i, j \in \{1, 2\}$ (Table 3 (b)). I use an array $\Omega := [\omega_{i,j}]_{i,j \in \{1,2\}}$ to keep track of the input-output structure.

Let X and Y be vectors stacking x_i's and y_i's, respectively, i.e., $X := [x_1 \ x_2]'$ and $Y := [y_1 \ y_2]'$, and let M be a 2×2 diagonal matrix with typical diagonal element being the sectoral markup and zero otherwise. The value-added, denoted by VA, is given by

$$VA = (1 - M^{-1})X, \tag{1}$$

where I is an identity matrix.

Figure 1: Input-Output Table

Seller \ Purchaser	Sector 1	Sector 2	Final Consumption	Total Sales
Sector 1	$\omega_{1,1}\tilde{x}_1$	$\omega_{2,1}\tilde{x}_2$	y_1	x_1
Sector 2	$\omega_{1,2}\tilde{x}_1$	$\omega_{2,2}\tilde{x}_2$	y_2	x_2
Value-added (VA)	$(1 - \frac{1}{\mu_1})x_1$	$(1 - \frac{1}{\mu_2})x_2$		
Total Expenditure	$\tilde{x}_1$	$\tilde{x}_2$		

Note: This figure represents an input-output table corresponding to (35).

The (nominal) gross domestic product is given by the total value-added: i.e., $GDP = (VA)'\iota$, where ι is a 2×1 vector of ones.[16] Let τ_1 denote a policy specific to sector 1,[17] and suppose that a policymaker hopes to learn the change in GDP as a result of changing the value of this policy from the current level τ_1^0 to an alternative level τ_1^1, which is denoted by $\Delta GDP(\tau_1^0, \tau_1^1)$. Observe here that the object of policy

[16] Although this section focuses on nominal GDP for ease of exposition, the central tenet is carried over to the case of real GDP, which is contemplated in Section 3.

[17] As far as the discussion of this section is concerned, the policy tool τ_1 can be left unspecified, while it could represent a variety of policy, such as sales tax and input subsidy.

interest can be decomposed as:

$$\Delta GDP(\tau_1^0, \tau_1^1) = \int_{\tau_1^0}^{\tau_1^1} \frac{dGDP(\tau_1)}{d\tau_1} d\tau_1, \tag{2}$$

where $GDP(\tau_1)$ stands for GDP as a function of τ_1.[18]

Next, I show how the difference in the market structure, in conjunction with the production network, lends itself as a difference in the policy parameter $\Delta GDP(\tau_1^0, \tau_1^1)$. To this end, I introduce two notions: macro and micro complementarities.

2.2 Macro Complementarities

To begin with, observe that $GDP = (VA)'\iota$, and thus $\frac{dGDP}{d\tau_1} = \left(\frac{dVA}{d\tau_1}\right)'\iota$, where ι is a 2×1 vector of ones.[19] For ease of exposition, I assume that the final consumption Y is invariant to the policy change: i.e., $\frac{dY}{d\tau_1} = 0$. Totally differentiating (1) yields

$$\frac{dVA}{d\tau_1} = \underbrace{-\frac{dM^{-1}}{d\tau_1}X}_{\text{direct effect of the changes in markups}} + \underbrace{(I - M^{-1})\frac{dX}{d\tau_1}}_{\text{indirect effect of the changes in markups}}, \tag{3}$$

where $\frac{dM^{-1}}{d\tau_1} = -M^{-1}\frac{dM}{d\tau_1}M^{-1}$ and $\frac{dX}{d\tau_1} = \sum_{n=1}^{\infty}\sum_{l=0}^{n-1}(\Omega M^{-1})^{l+1}\frac{dM}{d\tau_1}M^{-1}(\Omega M^{-1})^{n-l-1}Y$, with $\frac{dM}{d\tau_1} = \text{diag}([\frac{d\mu_1}{d\tau_1}\ \frac{d\mu_2}{d\tau_1}])$.[20] In (3), the first term indicates the direct effects of the changes in markups, and the second term dictates the impacts of the changes in markups that come through changes in sales. Notice that the marginal changes in sectors' sales $\frac{dX}{d\tau_1}$ are proportional to the final consumption augmented by the elasticities of markups with the ratio assigned to the sector's location on the production network.[21] Moreover, this term traces out the comovements of sectoral sales dx_i and dx_j with $i \neq j \in \{1, 2\}$, which I call the *macro complementarities*.

To delve deeper into the macro complementarities, the next subsection contemplates a microfoundation of the responses of the sectoral markups $\frac{dM}{d\tau_1}$ through the lens of a duopoly model.[22]

[18] It is assumed that GDP is continuously differentiable in τ_1.

[19] For an arbitrary vector $U = [u_1\ u_2]$, I define $\frac{dU}{d\tau_1} := [\frac{du_1}{d\tau_1}\ \frac{du_2}{d\tau_1}]$.

[20] For an arbitrary vector U, the operator diag(U) gives a diagonal matrix whose typical diagonal element is an element of U.

[21] Specifically, the premultiplying term $(\Omega M^{-1})^{l+1}$ captures the sector's intermediate sales to all industries used as intermediate inputs in the $(l+1)$th round of production process (upstreamness), while the postmultiplying term $(\Omega M^{-1})^{n-l-1}$ corresponds to the sector's intermediate purchases from all industries used as intermediate inputs in the $(n-l-1)$th round of production process (downstreamness). See Antràs et al. (2012) and Antràs and Chor (2019).

[22] Grassi and Sauvagnat (2019) consider a setup similar to (35) with assuming that the markups are exogenous.

2.3 Micro Complementarities

I employ a variant of Melitz and Ottaviano (2008). Suppose that each industry i is populated by two firms $k \in \{1, 2\}$ (i.e., a duopoly), each producing a single differentiated product under a constant marginal cost mc_{ik}. The firms engage in a Cournot competition of complete information. Firms' products are aggregated into a single homogenous sectoral good Q_i according to a quadratic production function:

$$Q_i = q_{i0} + a(q_{i1} + q_{i2}) - \frac{b}{2}(q_{i1}^2 + q_{i2}^2) - \frac{1}{2}c(q_{i1} + q_{i2})^2, \tag{4}$$

where q_{i0} is an outside good, q_{ik} is meant to be the demand of firm k's product for $k \in \{1, 2\}$, and a, b and c are demand parameters.[23] Assuming positive demand for each product, the inverse demand function faced by firm $k \in \{1, 2\}$ is given by $p_{ik} = a - bq_{ik} - c(q_{i1} + q_{i2})$.

I define markup as the ratio of price to marginal cost. That is, a firm-level markup is given by $\mu_{ik} := \frac{p_{ik}}{mc_{ik}}$ and the sector-level markup $\mu_i := \frac{P_i}{mc_{i1}+mc_{i2}+1}$, where $P_i = \frac{1}{2}(p_{i1} + p_{i2})$ is the industry's price index. Total differentiation yields

$$\frac{d\mu_i}{d\tau_1} = \underbrace{\kappa_{i1}\frac{\partial \mu_{i1}(\cdot)}{\partial q_{i1}}\frac{dq_{i1}}{d\tau_1} + \kappa_{i2}\frac{\partial \mu_{i2}(\cdot)}{\partial q_{i2}}\frac{dq_{i2}}{d\tau_1}}_{\text{changes in markups with respect to own choices}} + \underbrace{\kappa_{i1}\frac{\partial \mu_{i1}(\cdot)}{\partial q_{i2}}\frac{dq_{i2}}{d\tau_1} + \kappa_{i2}\frac{\partial \mu_{i2}(\cdot)}{\partial q_{i1}}\frac{dq_{i1}}{d\tau_1}}_{\text{changes in markups with respect to competitors' choices}}, \tag{5}$$

where $\mu_{ik}(\cdot)$ is the firm k's markup function and $\kappa_{ik} = \frac{mc_{ik}}{mc_{i1}+mc_{i2}+1}$ for each $k \in \{1, 2\}$. The first two terms of (5) account for the contributions from the firms' markup elasticities with respect to own choices. The third and fourth terms capture the effects of the firms' markup elasticities with respect to competitors' choices that come through the strategic interaction (i.e., the strategic complementarity). That is, $\frac{d\mu_i}{d\tau_1}$ involves a weighted average of (functions of) strategic complementarities; I refer to this weighted average the *micro complementarity*. It is worth stressing that when the market is monopolistically competitive, the latter is dropped from (5). Acknowledging this, I can write as

$$\frac{dM}{d\tau_1} = \bar{M} + \tilde{M}, \tag{6}$$

where $\bar{M}$ and $\tilde{M}$ are diagonal matrices with the (i, i) entry being equal the first two terms and the other two terms of (5), respectively.

[23] The outside good x_{i0} is used as a numeriare good and produced in a perfectly competitive fashion under constant returns to scale at unit cost. Labor is assumed to be the sole factor of the production. The demand parameters a, b and c are all assumed to be positive. See Melitz and Ottaviano (2008) for the detail.

To summarize, firms' markup elasticities with respect to own choices and competitors' (strategic complementarities) add up to sectors' micro complementarities as given in (6), which in turn accrue through the production network, yielding sectors' macro complementarities according to (3).

Lastly, substituting (3) and (6) into (2) yields

$$\begin{aligned}
&\Delta GDP(\tau_1^0, \tau_1^1) \\
&= \underbrace{\int_{\tau_1^0}^{\tau_1^1} \left[\left\{ \bar{M} M^{-2} \sum_{n=0}^{\infty} (\Omega M^{-1})^n - (I - M^{-1}) \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} (\Omega M^{-1})^{l+1} \bar{M} M^{-1} (\Omega M^{-1})^{n-l-1} \right\} Y \right]' \iota \, d\tau_1}_{\text{the impact of the policy reform coming through the changes in firms' markups due to own choices}} \\
&\quad + \underbrace{\int_{\tau_1^0}^{\tau_1^1} \left[\left\{ \tilde{M} M^{-2} \sum_{n=0}^{\infty} (\Omega M^{-1})^n - (I - M^{-1}) \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} (\Omega M^{-1})^{l+1} \tilde{M} M^{-1} (\Omega M^{-1})^{n-l-1} \right\} Y \right]' \iota \, d\tau_1}_{\text{the impact of the policy reform coming through the changes in firms' markups due to competitors' choices}} .
\end{aligned} \tag{7}$$

The first term represents the change in GDP as a direct consequence of the firms' own choices. The second term stands for the change in GDP induced by the competitors' choices. When sectors are monopolistically competitive, the second term of (7) is dropped. In the absence of production networks, (7) holds with replacing Ω by an identity matrix I. Hence, failure to account for either of a production network or an oligopolistic competition generates a prediction qualitatively different from the one given by (7).

Quantitatively, the consequence of embracing strategic interactions is ambiguous as the sign of the second term of (5) depends on all firm's strategic complementarities, which can either positive or negative. That is, the (integrand of the) second term of (7) may act either in a way that fortifies or counteracts the (integrand of the) first term.

2.4 Idea of Identification Strategy

The starting point of our identification analysis is (7) (the top layer). The difficulty arises from the fact that the integrands cannot simply be written in terms of sectoral aggregates only, as it consists of a finite number of firms.[24] In the example above, there are only two firms in each sector. This means that each firm is the crucial competitor to the other, so that both firms are not negligible in determining the

[24] In the case of a monopolistic competition, the literature typically assumes that each market is populated by a mass of continuum of infinitesimally small small firms. In such a setup, individual firms are negligible relative to the sectoral aggregate. See Gaubert and Itskhoki (2020).

sectoral aggregate.

To get a better sense of how it matters, suppose for a moment that the two firms are equally productive and thus equally competitive. In this case, each firm accounts for the half of the sectoral aggregate. Ignoring the contribution of either of them substantially yields a sectoral outcome that substantially differs from the true one. Hence the existing approach that relies on the assumption of infinitesimally small firms cannot be applied in our framework.

The idea here is to recover the firm-level responses at the expense of additional assumptions. Observe first that the integrands of (7) are expressed in terms of the comparative statics as shown in (5). These comparative statics are pinned down by the system of equations that result from the underlying firm's optimization problems, taking the firm-level quantity and price, and derivatives of firm-level production and demand functions (and thus the firm's markup elasticities $\bar{M}$ and $\tilde{M}$) as given (the middle layer). These firm-level conditioning variables can be identified by applying the control function approach of the industrial organization literature to the case of an oligopolistic competition (the bottom layer). To do this, though, requires additional assumptions. Continuing the setup sketched above, this subsection further illustrates the idea of the bottom layer.

Now, I assume that *i)* the sectoral aggregators take the form of a demand system that is homothetic with a single aggregator, *ii)* the firm-level production functions exhibit constant returns to scale with Hicks-neutral productivity and *iii)* competitors' productivities enter firm's decision only through a single aggregate. The plausibility of these assumptions can immediately be verified as shown below.[25]

First, it is known that (4) falls into the class of HSA demand system.[26] Second, suppose that the firm-level production in sector i is given by $q_{ik} = z_{ik} f_i(m_{ik,1}, m_{ik,2})$, where $f_i(\cdot)$ is constant returns to scale with z_{ik} and $m_{ik,j}$ representing firm k's productivity and input demand for sector j's good, respectively.[27] It is assumed that firms first decide the output quantity under a Cournot duopoly, followed by the decision about input quantity of sectoral goods. Under this setting,[28] the firm k's marginal cost can be written as $mc_{ik} = z_{ik}^{-1} mc_i$, where mc_i is a constant common to all firms in sector i.[29] The Cournot-Nash equilibrium quantity is given by $q^*_{ik} = K_i z_{ik}^{-1} + \bar{H}_i$, where K_i and $\bar{H}_i = H_i(z_{i1}, z_{i2})$ are

[25] In Section 3, I show that these assumptions are flexible enough to encompass the specifications that are commonly used in the macroeconomics literature.

[26] See Arkolakis et al. (2019).

[27] The demand for sectoral goods aligns with the production network Ω.

[28] It is implicitly assumed that sectoral input goods are variable in the firm's production, and the input markets are perfectly competitive, both of which are standard in the literature (e.g., Grassi 2017; Kasahara and Sugita 2020).

[29] See Section 3, and also Grassi (2017).

sector-specific constants, with $H_i(\cdot)$ being a sector-specific function of firms' productivities.[30] The latter can be interpreted as representing the market competitiveness. Lastly, firm's input decision is thus constrained by the following production possibility frontier:

$$z_{ik} f_i(\ell_{ik}, m_{ik}) = q^*_{ik} = K_i z_{ik}^{-1} + \bar{H}_i,$$

from which it follows that there exits a function $\mathcal{M}_i$ such that $z_{ik} = \mathcal{M}_i(\ell_{ik}, m_{ik}; \bar{H}_i)$. Accounting for the firm's strategic interaction through $\bar{H}_i$, this expression conforms to the control function of the industrial organization literature, so that the techniques developed in those literature can readily be applied.

3 Model

This section spells out a general framework for a closed-economy, multi-sector model of oligopolistic competitions of heterogenous firms under production networks. The model is akin to Liu (2019), who considers the optimal policy in the presence of a production network when there are exogenous market distortions, but I depart from his setup by replacing the exogenous wedges with endogenously variable firms' markups. In my model, the markups can arise from oligopolistic competitions among a finite number of heterogenous firms and the non-CES specification of the residual inverse demand functions faced by the firms.[31] I restrict my attention to the short-run policy effects abstracting away from the entry and exit decisions (extensive margins) as postulated in Mayer et al. (2021) and Wang and Werning (2022).[32]

The model is static and there is no uncertainty. The economy consists of a representative household, a government and N production sectors, indexed by $i \in \mathbf{N} := \{1, \ldots, N\}$. Each sector i is populated by a finite number N_i of oligopolistic firms, indexed by $k \in \mathbf{N}_i := \{1, \ldots, N_i\}$, each of which produces a single differentiated good. There is a sectoral aggregator that aggregates the firms' products in the same sector into a single intermediate good à la Bigio and La'O (2020). Sectoral goods are further combined to produce a final consumption good. Both the economy-wide and sectoral aggregators operate in perfectly

[30] Since the information structure is complete, $H_i(z_{i1}, z_{i2})$ is a constant and common to all firms.

[31] Arkolakis et al. (2019) consider a model of variable markups under a monopolistic competition with a flexible class of non-CES demand functions. my paper adds an additional source of endogenous markups, strategic interactions.

[32] The short-run scope can be rationalized by acknowledging that firms' entry and exit decisions generally take considerable amount of cost and time. Technically, accommodating the endogenous choice of entry and exit requires another layer of fixed point problem concerning the free entry condition, which in general is very hard to solve (Wang and Werning 2022). In particular, given that the number of firms in my setup is finite, it is even not possible to consider differentiation of the free entry condition. Extending the theory to the long-run analysis is left for future works.

competitive markets.

The transaction of sectoral intermediate goods by firms shapes the input-output linkages, denoted by $\Omega := [\omega_{i,j}]_{i,j=1}^{N}$ with $\omega_{i,j}$ being the share of sector j's intermediate good in sector i's expenditure.[33]

3.1 Market Imperfections and Policy Interventions

Oligopolistic competitions in the output markets open up the possibility of the firms' earning nonnegative economic profits. Let Π_i be the total profit earned by firms in sector i. This can be interpreted as deadweight loss due to the firm's market power in sector i. The total deadweight loss, denoted by Π, is obtained by adding this up over all sectors: i.e., $\Pi := \sum_{i=1}^{N} \Pi_i$. I assume that all firms are owned by the household, so that Π is rebated back to the household's income as dividends.

The presence of the deadweight loss gives the government incentive to put forth welfare-improving interventions. Let $\boldsymbol{\tau}^0$ and $\boldsymbol{\tau}^1$ denote the policy regime currently in place, and an alternative policy regime, respectively. Suppose that the policymaker is interested in the policy impacts on some policy parameters of shifting from $\boldsymbol{\tau}^0$ to $\boldsymbol{\tau}^1$.[34] In what follows, I consider the government that manipulates sector specific policy instruments: i.e., $\boldsymbol{\tau} := \{\tau_i\}_{i=1}^{N}$, where τ_i is understood as an ad-valorem subsidy on sector i's purchase of intermediate sectoral goods if it is positive, and a tax otherwise.[35] Suppose that a policymaker wants to learn the impact on real gross domestic product (GDP) of subsidizing semiconductor industry.

To make the model amenable to causal effect analysis, I impose the following policy invariance assumptions.

Assumption 3.1 (Policy Invariance of N, N_i, $\boldsymbol{\omega}_L$ and Ω)**.** *Throughout the policy reform from $\boldsymbol{\tau}^0$ to $\boldsymbol{\tau}^1$, i) the number of sectors N, ii) the number of firms in each sector N_i, and iii) the shape of the input-output linkages $\boldsymbol{\omega}_L$ and Ω do not change.*

Assumption 3.1 (i) is consistent to the focus of this study on ad-valorem subsidies, excluding other competition interventions. Invariance condition (ii) assumes away endogenous entry and exit in response to the policy change, in line with the short-run scope of my analysis. Part (iii) states that the input-

[33] Analogously, I write $\boldsymbol{\omega}_L := [\omega_{i,L}]_{i=1}^{N}$ with $\omega_{i,L}$ indicating the labor share of sector i's expenditure.

[34] That the government is interested in changing the policy scheme implies that the currently policy regime $\boldsymbol{\tau}^0$ is not yet optimized, and rather play part of the market distortions as in Bigio and La'O (2020). This treatment of the policy variable is conceptually distinct from Liu (2019).

[35] I abstract from other measures of pro- and anti-competitive policies such as antitrust regulation and antidumping policies.

output linkages $\boldsymbol{\omega}_L$ and Ω do not reshape in reaction to the policy reform. This again accords with the scope of my analysis and also resonates the existing literature that assumes the production network to be stable over a period of time (e.g., Baqaee and Farhi 2020).

[36] The government's policy expenditure is assumed to be financed to the consumer by way of a lump-sum transfer.

3.2 Household

Consider a representative household that consumes a final consumption good, and inelastically supplies labor across sectors. The household derives utility only from consumption of the final good, with the utility function being the standard.

Assumption 3.2 (Utility Function)**.** *The consumer's utility function is strictly monotonic, and continuously differentiable in the final consumption good.*

Assumption 3.2 means that there exists a one-to-one mapping between utility level and consumption of the final good. Based on this preference, the household chooses the utility-maximizing quantity of the final consumption good subject to the binding budget constraint:

$$C = WL + \Pi - T, \tag{8}$$

where Π is total firm's profit, and T indicates the tax payment to the government in the form of a lump-sum transfer. I let the price index of the final consumption good be the numeraire.

3.3 Technologies

Economy-wide and sectoral aggregations. The economy-wide aggregator collects sectoral intermediate goods to produce a final consumption good Y using the production function $\mathcal{F}$:

$$Y = \mathcal{F}(\{X_i\}_{i=1}^N), \tag{9}$$

where $\mathcal{F} : \mathbb{R}_+^N \to \mathbb{R}_+$, and X_i represents the sector i's intermediate good used for the production of the final consumption good. In each sector $i \in \mathbf{N}$, firm-level products are aggregated into a single sectoral

[36] From the onset of Bidenomics, the administration has implemented a variety of pro-competitive measures (see, e.g., The White House 2023b). Investigating the impacts of these policies is beyond the scope of this paper and left for future works.

good Q_i according to:

$$Q_i = F_i(\{q_{ik}\}_{k=1}^{N_i}), \tag{10}$$

where $F_i : \mathbb{R}_+^{N_i} \to \mathbb{R}_+$ represents the sector-specific aggregator that collects firms' products in sector i, and q_{ik} denotes the quantity of firm k's product.[37]

This aggregator satisfies the following standard assumptions.

Assumption 3.3 (Economy-Wide & Sectoral Aggregators)**.** *i) The economy-wide aggregation function $\mathcal{F}$ is increasing and concave in each of its arguments. ii) For each $i \in \mathbf{N}$, the sectoral aggregator F_i is a) twice continuously differentiable, and b) increasing and concave in each of its arguments.*

Notice Assumption 3.3 does not require the sectoral aggregator F_i to exhibit constant return to scale unlike Liu (2019) and Bigio and La'O (2020). Under this assumption, both the economy-wide and sectoral aggregators operate in perfectly competitive markets. The price index of sector i's good P_i is defined through the sectoral cost-minimization problem.[38]

Sectoral aggregator serves two purposes. First, it is a useful modeling device that allows us to unite firms' differentiated goods into a single homogenous good (**?**La'O and Tahbaz-Salehi 2022). The economic content of this aggregation is that every buyer of goods from sector i purchases the same bundle of the goods produced by the firms in that sector (Liu 2019). Second, from the perspective of an individual firm, the sectoral aggregator acts as a "demand function," through which the firm's strategic interaction is mediated. To begin with, I assume away from the unobservable demand-side heterogeneity. Given Assumption 3.4, it is tantamount to replacing the expenditure share function Ψ_{ik} in (11) by Ψ_i.

In order to make the model amenable to empirical analysis while maintaining flexibility, I restrict the sectoral aggregator to take the form of a *homothetic demand system with a single aggregator* (HSA; Matsuyama and Ushchev 2017).

Assumption 3.4 (HSA Inverse Demand Function)**.** *In each sector $i \in \mathbf{N}$, the sectoral aggregator F_i exhibits a HSA inverse demand function: i.e., the inverse demand function faced by firm $k \in \mathbf{N}_i$ is given*

[37] To economize on notation, I use the same notation q_{ik} to mean the demand for firm k's good and firm k's output quantity. By doing this, I implicitly apply the market clearing condition for individual firms' products, as the sectoral aggregator is the only purchaser of firms' products.

[38] See the unit cost condition (63) in Appendix C.2.

by:

$$p_{ik} = \frac{\Phi_i}{q_{ik}} \Psi_i\left(\frac{q_{ik}}{A_i(\mathbf{q}_i)}; \mathcal{I}_i\right) \qquad \textit{with} \qquad \sum_{k'=1}^{N_i} \Psi_i\left(\frac{q_{ik'}}{A_i(\mathbf{q}_i)}; \mathcal{I}_i\right) = 1, \tag{11}$$

where Φ_i is a constant indicating the expenditure by sector i's aggregator, Ψ_i represents the share of firm k's good in the expenditure of sector i's aggregator, and $A_i(\mathbf{q}_i)$ denotes the aggregate quantity index capturing interactions between firms' choices with $\mathbf{q}_i := \{q_{ik'}\}_{k'=1}^{N_i}$.

From an individual firm's perspective, the quantity index $A_i(\mathbf{q}_i)$ in (11) summarizes the firms' interactions in sector i, and this is the only channel through which other firms' choices matter to the firm's own decision.[39] Put differently, Assumption 3.4 rules out the possibility that any other firm's quantity enters the firm's inverse demand independently of $A_i(\mathbf{q}_i)$. In this sense, $A_i(\mathbf{q}_i)$ acts as a sufficient statistic for other firms' choices as in Amiti et al. (2014) and Arkolakis et al. (2019).

Assumption 3.4 means that unobservable heterogeneity in the sectoral aggregator is equally imposed on all firms products.[40] Nevertheless notice that this assumption does not implies that the inverse demand function is common to all firms, because the quantity index function $A_i(\cdot)$ is allowed to be asymmetric in its arguments.

The HSA specification (11) is broad enough to accommodate a wide variety of aggregators including those that are commonly used in the international trade literature; e.g., the constant elasticity of substitution (CES), the symmetric translog (Feenstra and Weinstein 2017), the constant response demand (Mrázová and Neary 2017, 2019), and the flexible class of non-CES homothetic aggregators explored in Kimball (1995), Burstein and Gopinath (2014) and Arkolakis et al. (2019).[41]

Example 3.1 (CES aggregator)**.** *The CES aggregator is routinely assumed in the bulk of the macroeconomic literature on international pricing (Atkeson and Burstein 2008; Amiti et al. 2014; Gaubert and Itskhoki 2020). Consider the CES aggregator in sector i:*

$$F_i(\{q_{ik}\}_{k=1}^{N_i}) := \left(\sum_{k=1}^{N_i} \delta_{ik}^{\sigma_i} q_{ik}^{\frac{\sigma_i - 1}{\sigma_i}}\right)^{\frac{\sigma_i}{\sigma_i - 1}},$$

[39]Intuitively, instead of keeping track of every single one of other firms' choices, the firm only needs to look at this aggregate quantity.

[40]This assumption is adopted only to simplify identification and estimation, and can be relaxed at the cost of additional technicality.

[41]See also Matsuyama and Ushchev (2017), Kasahara and Sugita (2020) and Matsuyama (2023) for other instances.

where σ_i represents the elasticity of substitution specific to sector i, and δ_{ik} is a demand shifter specific to firm k's product. Associated to this is the residual inverse demand curve faced by firm k:

$$p_{ik} = \frac{\delta_{ik} q_{ik}^{-\frac{1}{\sigma_i}}}{\sum_{k'=1}^{N_i} \delta_{ik'} q_{ik'}^{-\frac{1}{\sigma_i}}} R_i, \tag{12}$$

where R_i is the total expenditure to sector i's good. Suppose $\delta_i = \delta_{ik} = \delta_{ik'}$ for all $k, k' \in \mathbf{N}_i$. Acknowledging that $R_i = \Phi_i$ and letting $A_i(\mathbf{q}_i) := (\sum_{k'=1}^{N_i} \delta_i q_{ik'}^{-\frac{1}{\sigma_i}})^{\frac{\sigma_i}{\sigma_i - 1}}$, Assumption 3.4 is satisfied with $\Psi_i(x; \mathcal{I}_i) := \delta_i x^{\frac{\sigma_i - 1}{\sigma_i}}$ for all $x \in \mathscr{S}_i$.

Firm-level production. The firm-level production process combines labor and material inputs, where the latter is a composite of sectoral intermediate goods along the production network. It is assumed that all inputs are variable; i.e., firms do not incur fixed costs. To focus on the short-run behavior, I do not model the entry decisions; instead I assume that each sector is populated by an exogenously fixed number of firms that are heterogenous in productivities.

In the output market of each sector, firms engage in a Cournot competition of complete information, while they are perfectly competitive in input markets. Thus, each firm first chooses its output quantity so as to maximize its profits in the Cournot-quantity competition, followed by the input decisions based on cost-minimization problems under the constraint of output quantity.

The production technology for firm k in sector i is described by:

$$q_{ik} = z_{ik} f_i(\ell_{ik}, m_{ik}) \qquad \textit{with} \qquad m_{ik} = \mathcal{G}_i(\{m_{ik,j}\}_{j=1}^N), \tag{13}$$

where q_{ik}, ℓ_{ik} and m_{ik} are, respectively, the quantity of gross output, labor input and material input, z_{ik} denotes the firm's Hicks-neutral productivity, $m_{ik,j}$ is the input demand for sector j's intermediate good, and $f_i : \mathbb{R}_+^2 \to \mathbb{R}_+$ and $\mathcal{G}_i : \mathbb{R}_+^N \to \mathbb{R}_+$ represent the production technologies specific to the sector.[42] Note that $\mathcal{G}_i$ reflects the input-output linkages Ω.[43]

Example 3.2 (Nested Cobb-Douglas Production Function)**.** *The specification* (13) *includes the nested*

[42] I abstract away the capital accumulation in order to stick to a static environment. When bringing my model to data, I interpret the firm's productivity z_{ik} as overall production capacity including capital assets. See Appendix B.3.3.

[43] Under the specification (13), it holds that for each $i \in \mathbf{N}$, $\omega_{i,L} + \sum_{j=1}^N \omega_{i,j} = 1$.

Cobb-Douglas production function (e.g., Bigio and La'O 2020):

$$q_{ik} = z_{ik}\ell_{ik}^{\alpha_i} m_{ik}^{1-\alpha_i} \qquad \text{with} \qquad m_{ik} = \prod_{j=1}^{N} m_{ik,j}^{\gamma_{i,j}}, \tag{14}$$

where α_i stands for labor share specific to the sector, and $\gamma_{i,j}$ is the share of sector j's good in the material input used by sector i with $\sum_{j=1}^{N}\gamma_{i,j} = 1$. In this setup, $\omega_{i,L} = \alpha_i$ and $\omega_{i,j} = (1-\alpha_i)\gamma_{i,j}$.[44]

Notice that the both aggregators f_i and $\mathcal{G}_i$ are only traced by sector index i, meaning that firms in the same sector i have access to the same production technologies up to the idiosyncratic heterogenous productivity z_{ik}. This also implies that producer-side heterogeneity pertaining to product differentiation (e.g., quality) is encoded in the productivity term z_{ik}.[45]

Assumption 3.5 (Firm-Level Production Functions)**.** *For each sector $i \in \mathbf{N}$, both aggregators f_i and $\mathcal{G}_i$ i) display constant returns to scale, ii) are twice continuously differentiable in all arguments, iii) are increasing and concave in each of its arguments, and iv) satisfy $f_i(0,0) = 0$ and $\mathcal{G}_i(\mathbf{0}) = 0$. Moreover, v) for each firm $k \in \mathbf{N}_i$ in sector i, it holds that $\big(\frac{\partial f_i(\cdot)}{\partial \ell_{ik}}\big)^2 \frac{\partial^2 f_i(\cdot)}{\partial m_{ik}^2} + \big(\frac{\partial f_i(\cdot)}{\partial m_{ik}}\big)^2 \frac{\partial^2 f_i(\cdot)}{\partial \ell_{ik}^2} - 2\frac{\partial f_i(\cdot)}{\partial \ell_{ik}}\frac{\partial f_i(\cdot)}{\partial m_{ik}}\frac{\partial^2 f_i(\cdot)}{\partial \ell_{ik}\partial m_{ik}} < 0$ for all $(\ell_{ik}, m_{ik}) \in \mathbb{R}_+^2$.*

Assumptions 3.5 (i) – (iv) jointly state that the aggregators f_i and $\mathcal{G}_i$ are neoclassical, an assumption employed in Bigio and La'O (2020).[46] Assumption (v) guarantees an interior solution for the firm's cost minimization problem.

Importantly, when a firm decides quantity of output, it also takes into account its input decision in forward-looking way. Thus, the firm's decision problem proceeds backward. First, taking the quantities of output and material input, and sectoral price indices as given, the firm's optimal demand for sectoral intermediate goods are given by:

$$\{m_{ik,j}^*\}_{j=1}^N \in \underset{\{m_{ik,j}\}_{j=1}^N}{\arg\min} \quad \sum_{j=1}^{N}(1-\tau_i)P_j m_{ik,j} \qquad s.t. \quad \mathcal{G}_i(\{m_{ik,j}\}_{j=1}^N) \geq \bar{m}_{ik}, \tag{15}$$

[44] Note that (14) can equivalently be written as $q_{ik} = (\check{z}_{ik}\ell_{ik})^{\alpha_i} m_{ik}^{1-\alpha_i}$ with $\check{z}_{ik} = z_{ik}^{1/\alpha_i}$. That is, the Hicks-neutral productivity z_{ik} is observationally equivalent to a labor-augmenting productivity $\check{z}_{ik}$ (e.g., Grassi 2017).

[45] In my setup, differentiated goods are produced by heterogenous firms, so that the level at which product differentiation is defined is the same as that at which firm heterogeneity is defined. Thus the notion of firm coincides with that of variety.

[46] Although Assumption 3.5 (i) might appear to be restrictive at the first glance, a number of applied researches have found that the constant returns to scale serves a good approximation (e.g., Basu and Fernald (1997), Syverson (2004), Foster et al. (2008), and Bloom et al. (2012)). The CRS production functions are customary assumed by recent works on firm-level macroeconomic models: for example, (Atkeson and Burstein 2008) in an oligopolistic competition model of international trade, Baqaee and Farhi (2022) in a multi-country model of international trade in the presence of production networks.

where $m^*_{ik,j}$ denotes the optimal level of purchase of sector j's good, and $\bar{m}_{ik}$ indicates the level of material input corresponding to a given quantity of output. Note that the unit cost condition associated to (15) defines the cost index of material input P_i^M gross of the policy $\boldsymbol{\tau}$.

Second, taking the output quantity and input prices as given, the optimal input quantities for firm k in sector i are given by:

$$\{\ell^*_{ik}, m^*_{ik}\} \quad \in \quad \underset{\ell_{ik}, m_{ik}}{\arg\min} \quad W\ell_{ik} + P_i^M m_{ik} \qquad s.t. \quad z_{ik} f_i(\ell_{ik}, m_{ik}) \geq \bar{q}_{ik}, \tag{16}$$

where W denotes the wage[47] and $\bar{q}_{ik}$ is a given level of output quantity.

Remark 3.1. *Input decisions* (15) *and* (16) *are separated purely for the expositional purpose. These two problems can be collapsed to a single cost-minimization problem, in which labor input and demand for sectoral goods are chosen simultaneously.*

Third, taking the competitors' quantity choices and aggregate variables as given, firm k in sector i chooses the quantity of output $q_{ik} \in \mathscr{S}_i := \mathbb{R}_+ \cup \{+\infty\}$ to maximize its profit.[48] Let $\pi_{ik}(\cdot, \cdot; \mathcal{I}_i) : \mathscr{S}_i \times \mathscr{S}_i^{N_i - 1} \to \mathbb{R}$ represent the firm k's profit function that maps its own quantity choice q_{ik} and competitors' choices $\mathbf{q}_{i,-k} := \{q_{ik'}\}_{k' \neq k}$ to the profit under the information set $\mathcal{I}_i$:

$$\mathcal{I}_i := \{Y, \{X_j\}_{j=1}^N, \{Q_j\}_{j \neq i}, W, \{P_j\}_{j \neq i}, \boldsymbol{\omega}_L, \Omega, \boldsymbol{\tau}\}.$$

The construction of $\mathcal{I}_i$ reflects the fact that when firms in sector i make quantity decisions, they take these aggregate variables as fixed while internalizing the possibility of Q_i and P_i varying as a result of their own decisions.[49] Hence, for each $i \in \mathbf{N}$, the Cournot-Nash equilibrium quantities $\mathbf{q}^*_i := \{q^*_{ik}\}_{k=1}^{N_i}$ must satisfy the following system of equations:

$$q^*_{ik} \quad = \quad \underset{q}{\arg\max} \quad \pi_{ik}(q, \mathbf{q}_{i,-k}; \mathcal{I}_i) \qquad\qquad \forall k \in \mathbf{N}_i. \tag{17}$$

I impose the following regularity conditions.

Assumption 3.6 (Regularity Conditions of the Profit Function)**.** *For each sector $i \in \mathbf{N}$, (i) $\mathscr{S}_i$ is a nonempty, compact and convex set. Moreover, for each firm $k \in \mathbf{N}_i$ in sector i, (ii) $\pi_{ik}(q_{ik}, \mathbf{q}_{i,-k}; \mathcal{I}_i)$ is*

[47] Since labor force is assumed to be frictionlessly mobile across sectors, the wage W is common for all sectors.

[48] The firm's profit here is defined as revenue minus variable costs.

[49] Note that as seen in (20), the government spending G can be dropped under (8), (18) and (19).

a continuous function both in q_{ik} and $\mathbf{q}_{i,-k}$, and (iii) $\pi_{ik}(q_{ik}, \mathbf{q}_{i,-k}; \mathcal{I}_i)$ is quansi-concave in q_{ik}.

Under Assumption 3.6, the existence of the Cournot-Nash equilibria in each sector immediately follows from the Debreu-Glicksberg-Fan theorem (Debreu 1952; Fan 1952; Glicksberg 1952).

3.4 Government

The government sets the level of subsidies $\boldsymbol{\tau}$ under the balanced budget. The government expenditure consists of two components. First, government purchases the final consumption good, which can be conceived as public spending G. The second element refers to the total policy expenditure S_i in sector i. The residual between these two spendings is charged on the representative consumer in the form of a lump-sum tax T. Hence the government's budget constraint is

$$G + \sum_{i=1}^{N} S_i = T \qquad \textit{where} \qquad S_i := \sum_{k=1}^{N_i} \sum_{j=1}^{N} \tau_i P_j m_{ik,j}. \tag{18}$$

3.5 Equilibria

3.5.1 Market Clearing

Since the final consumption good is either consumed by the household or purchased by the government., the market clearing condition for the final consumption good reads

$$Y = C + G. \tag{19}$$

Substituting (8) and (18) into (19), it follows

$$Y = WL + \Pi - \sum_{i=1}^{N} S_i, \tag{20}$$

which is nothing but the income accounting identity of GDP.

Sectoral intermediate good is used either for producing the final consumption good or as input in individual-firm's production: for each $j \in \mathbf{N}$

$$Q_j = X_j + \sum_{i=1}^{N} \sum_{k=1}^{N_i} m_{ik,j}. \tag{21}$$

Labor L is assumed to be in inelastic supply and fully employed, and is frictionlessly mobile across sectors and firms, thus satisfying:

$$L = \sum_{i=1}^{N} \sum_{k=1}^{N_i} \ell_{ik}. \tag{22}$$

3.5.2 Equilibria Defined

I assume that subsidies $\boldsymbol{\tau}$ are exogenously determined (by the government),[50] and the network structures $\boldsymbol{\omega}_L$ and Ω is invariant to a policy shift (Assumption 3.1), while other aggregate variables are endogenously determined in equilibrium.

Defining the equilibria in this model amounts to finding a fixed point in the endogenous aggregate variables.

Definition 3.1 (General Equilibria). *Given the realization of firms' productivities $\{\{z_{ik}\}_{k=1}^{N_i}\}_{i=1}^{N}$, sector-input-specific subsidies $\boldsymbol{\tau}$ and the input-output linkages $\boldsymbol{\omega}_L$ and Ω, the general equilibria of this model are defined as fixed points that solve the following problems:*

Sectoral equilibria: *For each sector i, given the information set $\mathcal{I}_i$, the solution to the quantity-setting game* (17) *yields a vector of sectoral Cournot-Nash equilibrium quantities $\{q^*_{ik}\}_{k=1}^{N}$, followed by the cost-minimization problems* (15) *and* (16) *to derive the optimal labor and material inputs $\{\ell^*_{ik}, m^*_{ik}\}_{k=1}^{N_i}$, and input demand for sectoral intermediate goods $\{\{m^*_{ik,j}\}_{j=1}^{N}\}_{k=1}^{N_i}$.*

Aggregate equilibria: *Given a collection of sectoral equilibrium quantities $\{q^*_{ik}, \ell^*_{ik}, m^*_{ik}, \{m^*_{ik,j}\}_{j=1}^{N}\}_{i,k}$, an aggregate equilibrium is referenced by the set of aggregate quantities $\{Y^*, \{X^*_j, Q^*_j\}_{j=1}^{N}\}$ together with the set of aggregate prices $\{W^*, \{P^*_j\}_{j=1}^{N}\}$, such that i) the household maximizes its utility subject to* (8)*, ii) the income accounting identity* (20) *holds, and iii) the market clearing conditions for composite intermediate goods* (21)*, and labor* (22) *are satisfied.*[51]

3.6 The Object of Interest

In line with my running example, suppose that the policymaker is interested in the impacts on real GDP of increasing subsidy on semiconductor industry (see Section 3.1). To learn such policy effects, formal

[50] I abstract from issues of endogenous policies such as Grossman and Helpman (1994).

[51] The market clearing condition for individual firms' products is straightforward as firm-level products are only used by the sectoral aggregator. Thus it is already implicitly applied in the exposition.

policy analysis requires holding all else unaltered. This inherently involves a counterfactual question of what would have happen if only the policy instrument in question were to be changed.

Let $Y^{\boldsymbol{\tau}}$ be the country's GDP in equilibrium under policy regime $\boldsymbol{\tau}$. From (20) and (22), it follows that

$$Y^{\tau} = \sum_{i=1}^{N} Y_i(\boldsymbol{\tau}) \qquad \textit{where} \qquad Y_i(\boldsymbol{\tau}) := \sum_{k=1}^{N_i} \left(W^* \ell_{ik}^* + \pi_{ik}^* - \sum_{j=1}^{N} \tau_i P_j^* m_{ik,j}^* \right), \tag{23}$$

where π_{ik} stands for the firm k's profit. In (23), $Y_i(\boldsymbol{\tau})$ can be viewed as sectoral i's GDP, with each of its summands corresponding to individual firm's contribution.[52]

Now since the policymaker is concerned with the policy effect on Y^{τ} of shifting from $\boldsymbol{\tau}^0$ to $\boldsymbol{\tau}^1$, the object of interest $\Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1)$ is defined as:

$$\Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) := \sum_{i=1}^{N} Y_i(\boldsymbol{\tau}^1) - \sum_{i=1}^{N} Y_i(\boldsymbol{\tau}^0).^{53} \tag{24}$$

The policy parameter (24) directly compares the country's GDP under $\boldsymbol{\tau}^0$ to that under $\boldsymbol{\tau}^1$. A virtue of this parameter is that it is a causal parameter in the sense of Marshall (1890): i.e., a *ceteris paribus* change in an outcome variable across different policy regimes.

Remark 3.2. *The growth rate* $\%\Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1)$ *of the kind studied in Arkolakis et al. (2012) and Adão et al. (2017) can be obtained as* $\%\Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) := \frac{1}{Y^{\tau^0}} \Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1)$.

Letting $n \in \mathbf{N}$ index semiconductor industry, I consider a situation where the policymaker counterfactually shifts the policy variable from τ_n^0 to τ_n^1, while all other policy variables are fixed constant.[54] For now, suppose that the policymaker is interested particularly in the policy change within the historically observed support of the subsidies, denoted by $\mathscr{T} := \times_{i=1}^{N} \mathscr{T}_i$ with $\mathscr{T}_i$ representing the observed support of the subsidies on sector i. The following assumption excludes the scenario where the new policy is such a policy that has never been experimented before.

Assumption 3.7 (Support Condition)**.** $[\tau_n^0, \tau_n^1] \subseteq \mathscr{T}_n$

[52] Each summand can be rearranged as: $W^* \ell_{ik}^* + \pi_{ik}^* - \sum_{j=1}^{N} \tau_i P_j^* m_{ik,j}^* = p_{ik} q_{ik} - \sum_{j=1}^{N} P_j^* m_{ik,j}^*$, which is the value added gross of the firm's markup.

[53] This shares the spirit with the *policy relevant treatment effect* (PRTE; Heckman and Vytlacil 2001, 2005, 2007). In particular, under Assumption 3.1, my policy parameter (24) is essentially equivalent to: $\frac{1}{N} \sum_{i=1}^{N} Y_i(\boldsymbol{\tau}^1) - \frac{1}{N} \sum_{i=1}^{N} Y_i(\boldsymbol{\tau}^0)$. This expression allows for the interpretation as *the average treatment effect (ATE) of the policy change on sectoral GDP* (e.g., Baier and Bergstrand 2007, 2009).

[54] That is, $\tau_{n'}^0 = \tau_{n'}^1$ for all $n' \neq n$.

4 Data

This section briefly describes the dataset used in my empirical analysis and the procedures in which I construct the empirical counterparts to the variables in my framework. The details are provided in Appendix B.

Our dataset spans between 2000 and 2021, but I do not exploit its time-series feature; rather I regard it as a collection of snapshots of the same economy with varying levels of subsidies. In this way, I can construct a "repeated samples" with variations in policy variables. I assume that the realizations are generated from an equilibrium.

4.1 Wage and Price Indices

Data on wage and labor hour worked is taken from the U.S. Bureau of Labor Statistics (BLS) through the Federal Reserve Bank of St. Louis (FRED) at annual frequency. Consistent with my conceptual framework, I use average hourly earnings of all employees as my data counterpart of the wage W^*.[55] I obtain data on sectoral price index P_i^* from the GDP by industry data at the Bureau of Economic Analysis (BEA), wherein the industries in the BEA data are used as the empirical counterparts of sectors of my framework.

4.2 Input-Output Tables

Following Baqaee and Farhi (2020), I adopt the annual U.S. input-output data from the BEA, omitting the government, noncomparable imports and second-hand scrap industries. The data contains industrial output and inputs for 66 industries, and covers from 1995 to 2015. I further follow Gutiérrez and Philippon (2017) in segmenting the industries into a coarser categories, leaving us with 38 industries.

Each input-output account comes with two distinct tables: namely, the use and supply table. The use table reports the amounts of commodities used by each industry as intermediate inputs and by final user, and the value added by each industry. The value-added section of the use table includes compensation of employees and taxes on products less subsidies for each purchaser industry. Each cell in the supply table indicates the amount of each commodity produced by each industry.

To transform the use table into the industry-by-industry format, I make the following assumption:

[55] Recall that labor is assumed to be frictionless mobile across sectors, which implies that the wage is the same everywhere in the economy.

each product has its own specific sales structure, irrespective of the industry where it is produced (Assumption B.1). Here the sales structure refers to the shares of the respective intermediate and final users in the sales of a commodity. Under this assumption, I can convert the commodity-by-industry use table to the industry-by-industry table, thereby conforming to my conceptual model of the production network Ω (see Appendix B.2.1 for detail). Using the compensation of employees, I can also construct data for $\boldsymbol{\omega}_L$.[56] The transformed input-output table can further be used to back out data for $\boldsymbol{\tau}$ as a value-added net subsidy, which is understood as an amalgamate of sales and input subsidies.

4.3 Compustat Data

The dataset for firm-level variables is Compustat, which is assembled by S&P and provided by the Wharton Research Data Services (WRDS). The Compustat data records information about firm-level financial statements, such as sales, input expenditure, capital stock information, and detailed industry activity classifications, from 1950 to 2016. From this data, in conjunction with the data on aggregate variables, I first construct measurements for firm-level revenue r^*_{ik}, labor ℓ^*_{ik} and material m^*_{ik} inputs. I follow De Loecker et al. (2020) in eliminating outliers.

Since, however, the dataset does not offer further breakdown of material input, I need to apportion the expenditure on material input to generate separate information about the demand for sectoral intermediate goods. This requires an explicit functional-form assumption on the material input aggregator $\mathcal{G}_i$ in (13). In this paper, I employ a Cobb-Douglas production function:

$$m_{ik} = \prod_{j=1}^{N} m_{ik,j}^{\gamma_{i,j}}, \tag{25}$$

where $m_{ik,j}$ is sector j's intermediate good demanded by firm k in sector i and $\gamma_{i,j}$ denotes the input share of sector j's intermediate good with $\sum_{j=1}^{N} \gamma_{i,j} = 1$. A virtue of this specification is that the production network across sectoral intermediate goods $\{\omega_{i,j}\}_{j=1}^{N}$ are directly reflected in the output elasticity parameters $\{\gamma_{i,j}\}_{j=1}^{N}$, which are constant.[57] This property is plausible in light of the particular focus of this paper on the short-run effects of the policies.[58] Under this specification, the demand for

[56] Throughout the transformation, the value-added section of the use table remains intact.

[57] The Cobb-Douglass production function has traditionally been used in a wide range of the macroeconomics literature: e.g., the real business cycle theory (Long and Plosser 1983; Horvath 1998, 2000), and international trade (Caliendo and Parro 2015; Grassi 2017; Bigio and La'O 2020). The recent literature has emphasized the importance of an endogenous input-output structure of the economy and employed a CES aggregator (e.g., Atalay 2017; Baqaee and Farhi 2019; Caliendo et al. 2022).

[58] See Assumption 3.1.

sectoral intermediate goods $\{m^*_{ik,j}\}_{j=1}^N$ are given by

$$m^*_{ik,j} = \gamma_{i,j} \frac{P_i^{M*}}{(1-\tau_i)P_j^*} m^*_{ik}, \tag{26}$$

where $P_i^{M*} m^*_{ik}$ indicates the expenditure on material input gross of subsidies, which can be obtained in the data (see Fact B.5).[59]

I admit the possibility that the data on firm-level revenues and costs are subject to measurement errors.[60] Importantly, the Compustat data does not provide information about output quantity and price. To recover these variables from the observables that are possibly prone to measurement errors, I leverage a methodology that has recently been developed in the industrial organization literature (see Section 5.2).

5 Identification and Estimation

This section discusses identification of the object of interest (24) based on the model laid out in Section 3 and the dataset described in Section 4. The identification results are constructive, which naturally validates the use of nonparametric plug-in estimators.

5.1 Identification Strategy

Under Assumptions 3.1 and 3.7, the object of interest (24) is equivalently rewritten as:

$$\Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) = \sum_{i=1}^N Y_i(\boldsymbol{\tau}^1) - \sum_{i=1}^N Y_i(\boldsymbol{\tau}^0) = \sum_{i=1}^N \int_{\boldsymbol{\tau}^0}^{\boldsymbol{\tau}^1} \frac{dY_i(s)}{ds} ds. \tag{27}$$

Our identification argument builds on (27), and aims to identify the integrand $\frac{dY_i(s)}{ds}$ for all $s \in [\boldsymbol{\tau}^0, \boldsymbol{\tau}^1]$. Total differentiation of (23) at $\boldsymbol{\tau} \in [\boldsymbol{\tau}^0, \boldsymbol{\tau}^1]$ delivers

$$\left.\frac{dY_i(s)}{ds}\right|_{s=\boldsymbol{\tau}} = \sum_{k=1}^{N_i} \left\{ \frac{d(W^* \ell^*_{ik})}{d\tau_n} - \sum_{j=1}^N P_j^* m^*_{ik,j} \mathbb{1}_{\{i=n\}} - \sum_{j=1}^N \tau_i \frac{d(P_j^* m^*_{ik,j})}{d\tau_n} + \frac{d\pi^*_{ik}}{d\tau_n} \right\}, \tag{28}$$

where $\mathbb{1}_{\{i=n\}}$ is an indicator function that takes one if $i = n$, and zero otherwise, and $\frac{d\pi^*_{ik}}{d\tau_n} = \frac{dp^*_{ik}}{d\tau_n} q^*_{ik} + p^*_{ik} \frac{dq^*_{ik}}{d\tau_n} - \frac{d(W^*\ell^*_{ik} + P_i^{M*} m^*_{ik})}{d\tau_n}$. Viewing (23) from the household's perspective (i.e., the income accounting

[59] In Appendix B.3.2, I further derive an explicit expression for P_i^{M*}.

[60] I assume additive separability in terms of log variables.

identity), the first term (28) accounts for the policy effect on labor income, the second and third term represent the shift in policy expenditure due to the policy change itself (the direct effects), and due to the firms' input reallocations (indirect effects), respectively. The last term indicates the change in dividend (i.e., firm's profit) in response to the policy shock.

The existing approach to recover (28) is to characterize its left hand side in terms of aggregate variables that are directly observed in data (e.g., Arkolakis et al. 2012, 2019; Adão et al. 2020). Their aggregation results crucially hinge on the modeling assumption of a mass of continuum of firms. Under this assumption, individual firms are infinitesimally small and thus inconsequential to the aggregate variables owing to the law of large numbers (Gaubert and Itskhoki 2020). By contrast, my framework embraces only a finite number of firms, in which case firm-level idiosyncrasies are not washed away even in the aggregate. my approach is rather to recover each of the firm-level responses in the right hand side of (28). In doing so, I apply the control function approach that has developed in the industrial organization literature. As a byproduct, the characterization result of this paper does not rely on the approximation of (28) around the economy with no pre-existing policies (i.e., $\boldsymbol{\tau}^0 = \mathbf{0}$) as employed in Liu (2019) and Baqaee and Farhi (2022).

Remark 5.1. *(i) The idea behind* (28) *resembles the so called exact hat algebra (Dekle et al. 2007, 2008), a method that is routinely used to generate a counterfactual prediction in the literature (e.g., Caliendo and Parro 2015; Adão et al. 2017, 2020).*[61] *The premise of the exact hat algebra is that all endogenous equilibrium variables are observable. This requirement, however, is not fulfilled in my case as firm-level quantity q^*_{ik} and price p^*_{ik} are not available in data (see Section 4). In Section 5.2, I provide a path forward to move on in the presence of these unobservable endogenous variables. (ii) The left hand side of* (28) *alone may be of limited practical relevance because it only measures the impact of an infinitesimally small policy reform around $\boldsymbol{\tau}^0$. My target parameter* (24)*, on the contrary, can be used to analyze a large policy reform from $\boldsymbol{\tau}^0$ to $\boldsymbol{\tau}^1$.*[62] *(iii) While useful as an approximation around the equilibrium in response to a small shock, the common practice of setting $\boldsymbol{\tau}^0 = \mathbf{0}$ (e.g., Liu 2019; Baqaee and Farhi 2022) is rarely feasible in empirical research because in most cases it is that $\mathbf{0} \notin \mathscr{T}$.*

[61] See Costinot and Rodríguez-Clare (2014) for the outline of the method.
[62] See also Kleven (2021) for discussion.

5.2 Identification

Identifying (28) consists in two layers: namely, outer and inner layer. The outer layer delivers comparative statics by solving systems of equations, conditional on the values of firm-level variables, and derivatives of the firm-level production and inverse demand functions (Appendix C.3).[63] Notice here that a) firm-level quantity and price are not observed in my dataset (see Section 4), and b) derivatives of the firm-level production and inverse demand functions are not known by definition (see Section 3). These are recovered in the inner layer with the aid of techniques of the industrial organization literature. This subsection describes how I address these issues in turn.

First, to recover firm-level price and quantity from the revenue and cost data, I exploit the firm's optimization conditions for the input choices and apply the method developed in Kasahara and Sugita (2020).[64] Applying their method in my context, however, requires an additional assumption because when firms decide their output quantities in the strategic interactions, they foresee the competitors' output and input choices as well as their own input choice, letting the strategic interactions effectively carrying over input decisions, a feature absent in Kasahara and Sugita (2020).[65]

To insulate the input decisions from the strategic interactions, I push forward the insight that under the specification of the HSA demand system (11), competitors' choices matter only through a single aggregator.[66] Let $\mathscr{L}_i$ and $\mathscr{M}_i$ be the observed supports of labor and material inputs, respectively. The following assumption extends the so called scaler unobservability assumption and its strict monotonicity in firm's productivity (see, e.g., Ackerberg et al. 2015; Gandhi et al. 2019) at the cost of limiting the path in which competitors' productivities enter the firm's quantity decision.

Assumption 5.1. *For each $i \in \mathbf{N}$, (i) there exist functions $\chi_i : \mathbb{R}_+ \times \mathbb{R} \to \mathcal{S}_i$ and $H_i : \mathbb{R}_+^{N_i} \to \mathbb{R}$ such*

[63] This incurs several regularities conditions to ensure the existence of inverse matrices. For the possible concern regarding this, see Section ...

[64] It has long been recognized that the use of the quantity measure of revenue data — revenue data deflated by price index — as a proxy for quantity data induces the so called omitted price bias (Klette and Griliches 1996), and masks the demand-side heterogeneity encoded in firm-specific price variables. See, e.g., Klette and Griliches (1996), Doraszelski and Jaumandreu (2019), Flynn et al. (2019), Bond et al. (2021) and Kirov et al. (2022) as well as Kasahara and Sugita (2020) for the detail.

[65] The host of the literature on the identification of production functions assumes away strategic interactions. For example, in the context of the control function approach, Ackerberg et al. (2015) and Gandhi et al. (2019) assume perfectly competitive markets, and Kasahara and Sugita (2020) focuses on monopolistic competitions. Doraszelski and Jaumandreu (2019) and Brand (2020) point out that the canonical scalar unobservability assumption eliminates the possibility of strategic interactions, and examine the extent to which the estimates are biased if the standard approach is mistakenly used. Matzkin (2008) considers the identification of a system of equation permitting strategic interactions, but requires linear separability in excluded regressors, which may not be supported on the theoretical ground in my context.

[66] In general, this idea extends beyond the HSA demand system insofar as the competitors' decisions are encapsulated in a single aggregator. See Appendix C.1.

that (i) $q^*_{ik} = \chi_i(z_{ik}, H_i(\mathbf{z}_i))$ *and (ii)* $\frac{\partial \chi_i(\cdot)}{\partial z_{ik}} \neq 1$.

Under Assumption 5.1, it can be shown that there exist some functions $\mathcal{H}_i : \mathbb{R}^{N_i}_{+} \to \mathbb{R}$ and $\mathcal{M}_i : \mathscr{L}_i \times \mathscr{M}_i \times \mathbb{R} \to \mathscr{Z}_i$ such that

$$z_{ik} = \mathcal{M}_i(\ell_{ik}, m_{ik}, \mathcal{H}_i(\mathbf{z}_i); \mathcal{I}_i) \qquad \forall k \in \mathbf{N}_i. \tag{29}$$

In this sense, Assumptions 5.1 (i) and (ii) correspond, respectively, to the so-called scalar unobservability assumption and the strict monotonicity (e.g., Ackerberg et al. 2015; Gandhi et al. 2019). The equation (29) allows the econometrician to control for the unobservable productivity in terms of the observable labor and material inputs. The literature resort to the timing assumption to derive the control function, while the expression (29) stems only from the constraint faced by the cost-minimizing firm.

The equation admits an interpretation analogous to the quantity index $A_i(\cdot)$ in Assumption 3.4: i.e., $\mathcal{H}_i(\mathbf{z}_i)$ is a sufficient statistic for the competitors' productivities, and it can most naturally be understood as a measure of the overall competitiveness of the market.[67] Given that the information structure of the oligopolistic competition is complete, its value is known to all firms in the same sector but not necessarily known to the econometrician.

Assumption 5.1, together with Assumption 3.4, permits a variety of specifications both for sector- and firm-level production functions. Continuing Examples 3.1 and 3.2, I demonstrate that these assumptions are satisfied in a model widely used in the international trade literature.

Example 5.1 (Duopoly with a CES Sectoral Aggregator)**.** *Consider the setup outlined in Examples 3.1 and 3.2. In view of Assumption* **??**, *it is posited that* $\delta_{i1} = \delta_{i2} = \delta_i$. *To make my claim as transparent as possible, I focus on the case of duopoly, i.e.,* $k \in \{1, 2\}$. *It can be shown that the Cournot-Nash equilibrium prices* $\mathbf{p}^*_i := \{p^*_{i1}, p^*_{i2}\}$ *satisfy the following system of equations:* $p^*_{ik} = \frac{\sigma}{(\sigma-1)(1-s_{ik}(\mathbf{p}^*_i))} mc_i(z_{ik})$, *with* $s_{ik}(\mathbf{p}^*_i) := \frac{\delta_i^\sigma {p^*_{ik}}^{1-\sigma}}{\delta_i^\sigma {p^*_{i1}}^{1-\sigma} + \delta_i^\sigma {p^*_{i2}}^{1-\sigma}}$ *where* $mc_i(z_{ik}) := z_{ik}^{-1} mc_i$ *is the firm k's marginal cost that depends on the firm's productivity.*[68] *Solving this yields* $q^*_{ik} = \frac{\sigma-1}{\sigma} R_i mc_i^{-\sigma} \mathcal{H}_i(\mathbf{z}_i) z^\sigma_{ik}$. *where I let* $\mathcal{H}_i(\mathbf{z}_i) := \frac{\delta_i^2 mc_i(z_{i1})^{\frac{1-\sigma}{\sigma}} mc_i(z_{i2})^{\frac{1-\sigma}{\sigma}}}{(\delta_i mc_i(z_{i1})^{\frac{1-\sigma}{\sigma}} + \delta_i mc_i(z_{i2})^{\frac{1-\sigma}{\sigma}})^{\frac{\sigma^2-\sigma+2}{\sigma}}}$. *This conforms to Assumption 5.1 as long as* $\sigma \neq 1$.

Taking this expression as given, the input decision is constrained by the production possibility frontier at output level q^*_{ik}: $z_{ik} \ell_{ik}^{\alpha_i} m_{ik}^{1-\alpha_i} = \frac{\sigma-1}{\sigma} R_i mc_i^{-\sigma} \mathcal{H}_i(\mathbf{z}_i) z^\sigma_{ik}$. *Upon solving this for* z_{ik}, *I obtain* $z_{ik} =$

[67] Since $\mathcal{H}_i(\cdot)$ is only indexed by sector i, it could in principle be absorbed by the subscript of $\mathcal{M}_i$. Nevertheless, I prefer to leave it explicitly to emphasize the existence of strategic interactions.

[68] mc_i represents part of the marginal cost common across firms in the same sector and it is given by $mc_i = \alpha_i^{-\alpha_i}(1 - \alpha_i)^{1-\alpha_i} W^{\alpha_i} (P^M_i)^{1-\alpha_i}$.

$\{\frac{\sigma-1}{\sigma}R_i mc_i^{-\sigma}\mathcal{H}_i(\mathbf{z}_i)\ell_{ik}^{-\alpha_i}m_{ik}^{-(1-\alpha_i)}\}^{\frac{1}{1-\sigma}}$. *Thus there exists a function* $\mathcal{M}_i$ *such that* $z_{ik} = \mathcal{M}_i(\ell_{ik}, m_{ik}, \mathcal{H}_i(\mathbf{z}_i); \mathcal{I}_i)$, *yielding the equation* (29).

Second, to recover the first- and second-order derivatives of both the firm-level production function and residual inverse demand functions faced by firms, I exploit the information about the firm's production function. Under the Hicks-neutral productivity specification (14) and CRS assumption (Assumption 3.5), the derivatives of the production functions are pinned down by the markup-augmented cost share, and labor and material inputs through a method developed by Gandhi et al. (2019). Moreover, combining the HSA specification (11) and the identified firm-level quantities and prices, I can also recover the derivatives of residual inverse demand functions faced by firms, as studied in Kasahara and Sugita (2020).

Theorem 5.1 (Identification of the Object of Interest)**.** *Suppose that Assumption 5.1 holds. Then, the object of interest* (24) *is identified from the observables.*

Proof. See Appendix C.5. □

A version of Theorem 5.1 remains valid for the case of a monopolistic competition.

Corollary 5.1. *Suppose that firms operate in a monopolistic competition in the output market. Then, the object of interest* (24) *is identified from the observables.*

Remark 5.2. *Although this paper focuses on the difference in GDP with respect to a policy change* (24) *as a principal object of policy interest, my framework recovers all firm-level responses — the finest ingredients of the model, and thus can be applied to study other policy parameter. First, the volume of unilateral trade flow from sector* j *to* i *is given by* $U_{i,j} = \sum_{k=1}^{N_i} m_{ik,j}$, *so that its response to a policy change is:* $\frac{dU_{i,j}}{d\tau_n} = \sum_{k=1}^{N_i}\frac{dm_{ik,j}}{d\tau_n}$, *where* $\frac{dm_{ik,j}}{d\tau_n}$ *is identified in my framework. Moreover, the volume of bilateral trade flow between sector* i *and* j, *denoted by* $B_{i,j}$, *can be analyzed similarly because of* $B_{i,j} = U_{i,j} + U_{j,i}$.[69] *Second, the difference in consumption before and after a policy change can be analyzed if the government spending being fixed. When* G *is fixed, totally differentiating* (19) *yields* $\frac{dY}{d\tau_n} = \frac{dC}{d\tau_n}$, *where the identification of the left hand side is established in Section 5. Third, another interesting policy objects are sector-level distributional outcomes such as the cross-sectional distribution of the changes in sectoral GDP due to a policy reform.*

[69] For the inference of dyadic variable such as unilateral and bilateral trade flows, I recommend to use a network-robust standard error proposed by Canen and Sugiura (2023).

5.3 Estimation

Since the identification results demonstrated above are constructive, I build on the analogy principle to obtain a nonparametric estimator for the policy effect (24).[70] I first nonparametrically estimate the values of the firm-level quantity and price, and the first- and second-order derivatives of the firm's production function, and then guided by the theory, I combine these to derive the nonparametric estimator for (24). Given that the object of interest is assumed to be continuous with respect to exogenous variables, the resulting estimator must be consistent. The accuracy of my estimator is verified through a number of simulation studies in Appendix E.

As stated in Section 4, I acknowledge the possibility that the data on firm-level revenues and costs are contaminated by measurement errors. To purge of the measurement errors, my estimation of the firm-level quantity and price follows the convention of the industrial organization literature in applying a polynomial regression of degree two. In estimating the firm's production elasticities, I follow the specification suggested in Gandhi et al. (2019). See Appendix D for the detail.

6 Empirical Application: The CHIPS and Science Act of 2022

In this section, I bring my framework to the real-world data described in Section 4. As a policy narrative, I contemplate the recent episode of the CHIPS and Science Act (CHIPS), which was passed into law in 2022 and is planned to invest nearly 53 billion U.S. dollars in U.S. semiconductor manufacturing, research and development, and workforce (The White House 2023a). This policy also includes a 25 percent tax credit for manufacturing investment, which is projected to provide up to 24.25 billion U.S. dollars for the next 10 years (Congressional Budget Office 2022). In my model, this tax credit can be analyzed as an additional subsidy targeted at the Computer and Electronic Product Manufacturing industry (Appendix B.2.2), which is indexed by n. Simply dividing the estimated 24.25 billion U.S. dollars by 10 years implies 2.43 billion U.S. dollars per year. This corresponds to raising the subsidy to 19.23%.[71] In our dataset, the historically observed support of subsidy on this industry is between 3.51% and 16.26%.[72]

However, analyzing the whole part of this policy requires the researcher to send the value of the subsidy

[70] Our approach takes a stance on estimation, rather than calibration. See Hansen and Heckman (1996) for the discussion concerning the pros and cons of these two methods. See also Matzkin (2013) for nonparametric estimation.

[71] The total amount of value-added tax in 2021 is 8.44 billion U.S. dollar, and the total expense on material input is 56.53 billion U.S. dollar. Hence, $(8.44 + 2.43)/56.53 \times 100 = 19.23\%$. See Appendix B.2.2.

[72] In the dataset, the semiconductor subsidy was 3.51% in 2007 and 16.26% in 2019. In terms of our notation in Section 3, it is represented as $\mathscr{T}_n = [0.0351, 0.1626]$.

τ_n to outside the observed support, while our identification result hinges on the "within the observed support" assumption (Assumption 3.7). Extending our analysis to outside the support is possible at the cost of additional assumptions as explored in Canen and Song (2022). But this goes beyond the scope of this paper, and left for future work. Instead, the exercise of this section focuses on a part of the CHIPS subsidy. Specifically, I consider a hypothetical policy scenario of increasing the subsidy on the semiconductor industry from the 2021 level of 14.94% to an alternative ratio of 16.00% — equivalent to 0.56 billion U.S. dollars.[73] This accounts for approximately one-fourth of the per-year tax credit.[74] Note that this policy scenario satisfies Assumption 3.7. It is assumed that the semiconductor industry is the only industry that is directly targeted during this policy reform.

The goal of this section is to estimate the change in GDP due to this counterfactual industrial policy, and moreover to analyze the mechanism behind the estimated policy effect. In Section 6.1, I first calculate the estimate of the policy effect (24). To shed light on the policy-relevance of accounting for strategic interactions, I carry out the estimation for both the monopolistic and oligopolistic cases.[75] In Section 6.2, I taking advantage of the structural construction of my framework to provide a breakdown of the gains and losses of the policy reform into sector-level price and quantity effects. To understand the determination of these effects, I further delve into the comovement of sectoral price and material cost indices.

6.1 The Policy Effect: Change in GDP

Based on (27), I estimate the change in GDP due to the policy reform from $\tau_n^0 = 0.1494$ to $\tau_n^0 = 0.1600$. An advantage of my approach is that the responsiveness of GDP can be traced out as a (possibly nonlinear) function of the subsidy over $[\tau_n^0, \tau_n^1]$. For computation purpose, I segment this interval into a fixed number of segments, and calculate the estimate according to:

$$\widehat{\Delta Y}(\tau_n^0, \tau_n^1) \approx \sum_{v=0}^{\bar{v}-1} \sum_{i=1}^{N} \widehat{\frac{dY_i(s)}{ds}\bigg|}_{s=\tau_n^0+v\Delta\tau} \times \Delta\tau, \tag{30a}$$

[73] To make our analysis as close to as the reality, we set the currently policy regime to the latest year available, which is year 2021. In terms of our model, this policy reform can be expressed by letting $\tau_n^0 = 0.1494$ and $\tau_n^1 = 0.1600$.

[74] Observe that $\frac{16-14.94}{19.23-14.94} = 0.2471$. One way to interpret this policy scenario is that it takes time to put the whole part of the CHIPS Act into effect, and what can realize in a short run is only a part of it. This view is consistent with the short-run perspective of this paper.

[75] In view of Corollary 5.1, these two cases are analyzed in a unified framework.

where $\Delta\tau := \frac{\tau_n^1 - \tau_n^0}{\bar{v}}$ with $\bar{v}$ being the number of bins equally segmenting the interval $[\tau_n^0, \tau_n^1]$. In this analysis, I set $\bar{v} = 10$. To highlight the consequence of ignoring the nonlinearity, I also estimate the policy effect assuming the following approximation:

$$\widehat{\Delta Y}(\tau_n^0, \tau_n^1) \approx \sum_{i=1}^{N} \left.\widehat{\frac{dY_i(s)}{ds}}\right|_{s=\tau_n^0} \times (\tau_n^1 - \tau_n^0), \tag{30b}$$

i.e., the estimate is computed by assuming that the responsiveness of GDP is constant throughout the course of the policy change at the level of the current policy regime.

Table 5 compares the estimates for the policy effect based on (30a) and (30b) in both cases of monopolistic and oligopolistic competitions. Two things stand out about this table. First, the estimate (30a) under an oligopolistic competition is almost twice as large in magnitude as that under a monopolistic competition. This reflects the impact of the policy reform coming through the strategic interactions as studied in Section 2. The substantial discrepancy between these two estimates highlights the policy-relevance of strategic interactions. Second, whichever the type of the market competition is, the estimates based on (30b) is quantitatively significantly different from that those based on (30a).[76] This underlines the substantial degree of nonlinearity in the responsiveness of GDP as a function of the subsidy, which is visualized in Figure 2.[77] The nonlinearity essentially arises from the fact that the firms' reactions depend on their quantity and price, and their production elasticities, each of which in turn depends on the value of the underlying subsidy.

Lastly, it is clear in Figure 2 (b) that there is a steady upward trend from 15.60% and 16.00%, and it thus might appear to be tempting to argue that further increasing the subsidy by, say 2%, will eventually revert the policy effect to be positive. However, my identification result builds on Assumption 3.7, which restricts an alternative policy to stay inside the observed support of the policy variable. Establishing the identification for a policy that sends the policy variable to outside the observed support in general requires additional invariance conditions as studied in Canen and Song (2022).

[76] For the case of a monopolistic competition, the estimates are different qualitatively as well.

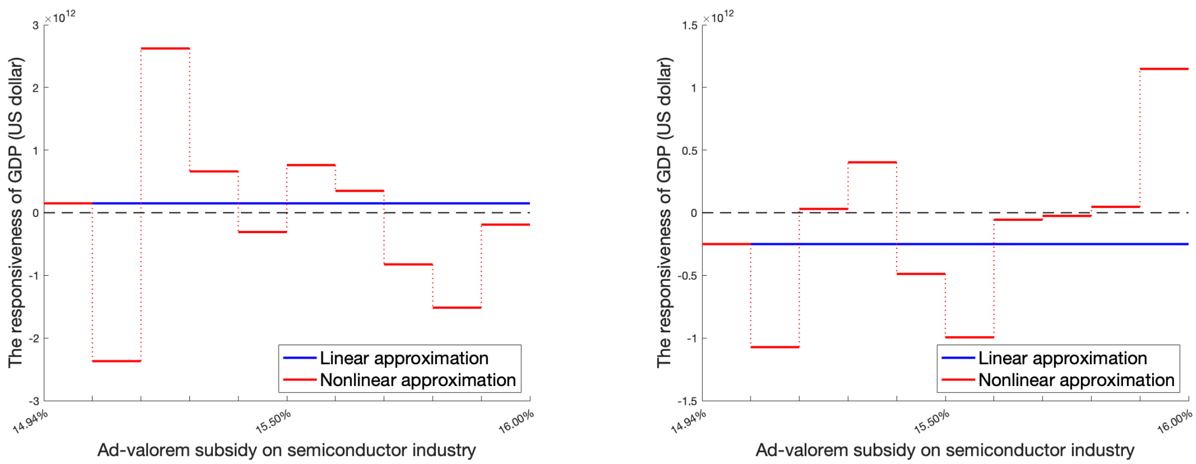
[77] Figure 2 compares the values of the total derivatives of Y with respect to semiconductor subsidy τ_n over the course of the policy reform from τ_n^0 to τ_n^1. Note that $\frac{dY}{ds} = \sum_{i=1}^{N} \frac{dY_i(s)}{ds}$, and thus the area surrounded by the blue/red line and the broken line indicating zero represents the policy effect of interest (24).

Table 1: The estimates of the object of interest

(billion U.S. dollars)	Monopolistic competition	Oligopolistic competition
Estimates based on (30a)	-0.71	-1.34
Estimates based on (30b)	1.76	-2.93

Note: This table compares the estimates for the object of interest (24) based on the benchmark and my method. The estimates are measured in unit of billion U.S. dollars.

Figure 2: The total derivative of Y with respect to τ_n



(a) Monopolistic Competition (b) Oligopolistic Competition

Note: This figure illustrates the estimates of the total derivative of (economy-wide) GDP with respect of the semiconductor subsidy between $\tau_n = 14.94\%$ and 16.00%. Panel (a) shows the result for the case of monopolistic competition, and panel (b) for the case of oligopolistic competition. The red one represents the estimates based on the nonlinear approximation (30a). The blue line indicates the estimates based on the linear approximation (30b). The broken line stands for zero. Hence, the part surrounded by the broken line and those (solid and dotted) red lines above it measures the total increment of GDP over the course of the policy change, while the other part gives the total decrement in GDP. The difference of these two areas delivers the estimated value of the policy effect according to (30a). Similarly, the area surrounded by the broken line and blue line gives the estimated value of the policy effect according to (30b).

6.2 Mechanism

To study the mechanism behind the results obtained in Section 6.1, I investigate the determination of the integrand of (27), i.e., the responsiveness of sectoral GDP.

6.2.1 Responsiveness of sectoral GDP

Design. I anchor my interpretation of the responsiveness of sectoral GDP around (28):

$$\left.\frac{dY_i(s)}{ds}\right|_{s=\tau_n} = \sum_{k=1}^{N_i}\Bigg(\underbrace{\frac{dp^*_{ik}}{d\tau_n}q^*_{ik}}_{\text{price effects}} + \underbrace{p^*_{ik}\frac{dq^*_{ik}}{d\tau_n}}_{\text{quantity effects}} \Bigg) - \sum_{k=1}^{N_i}\sum_{j=1}^{N}\Bigg(\underbrace{\frac{dP^*_j}{d\tau_n}m^*_{ik,j}}_{\text{wealth effects}} + \underbrace{P^*_j\frac{dm^*_{ik,j}}{d\tau_n}}_{\text{switching effects}} \Bigg), \tag{31}$$

which states that the marginal effect of a policy change consists of changes in revenue and expenditure on material input net of subsidies. The former is broken down into the price and quantity effects. When a firm produces more of its output, the price effect dictates the loss due to the increased supply in light of the law of demand. Under oligopolistic competitions, this downward pressure depends not only on the increase in firm's own quantity but also on a change in every other firm's output quantity through the cross-price elasticities of demand. The quantity effects are proportional to the given level of the firm's output price. The other component of (31) can similarly be decomposed into two parts: the wealth and switching effects. The wealth effects are changes in firm's "budget" as a result of changes in sectoral price indices. The switching effects are changes in sectoral composition of firm's input purchase, holding the price level constant.

Result. Table 2 reports the rankings of the top and bottom five industries in terms of gains/losses on sectoral GDP for monopolistic and oligopolistic competition.

From this table, it can be seen that the sectoral distributional consequence — which sector wins and which sectors lose — depends on the tension between two types of price and quantity effects defined in (31). To build intuition about this, suppose that all firms in a sector increase their production of output (positive quantity effects). By the law of demand, this lowers the output prices (negative price effects). These two effects induce another set of price and quantity effects. First, to produce more of their goods, the firms increase the purchase of input goods (negative switching effects). Second, since their products are sold at lower prices and used as input by other sectors according to the production network, they expect to see the reduction in prices of other sectoral goods, which in turn lower their input cots (positive wealth effect). The total effect depends on which of these price and quantity effects are dominant.

Take Computer and Electronic Products industry for an example. Under a monopolistic competition, the positive components (the quantity and wealth effects) jointly dominate the negative parts (the price and switching effects). Meanwhile, when the markets are oligopolistic, the positive quantity effects are almost exactly offset by the negative price effects, while the positive wealth effects are surpassed by the negative switching effects, leaving the firms with higher input cost. Loosely speaking, the input costs do not fall so much as the semiconductor firms have expected. This echoes the insight gleaned in Section 2.3 that the network compounds the firms' strategic complementarities, amplifying or reducing the policy effects across industries.

Next, to explore the determination of this tension with a particular focus on the price and wealth effects, I turn to the comovement of sectoral price indices and material cost indices.

6.2.2 Macro and Micro Complementarities

Here, I derive two "reduced-form" equations of comparative statics.[78] These two equations jointly envision the process in which the within-sector overall strategic complementarities (micro complementarities) are compounded through the production network into the between-sector complementarities (macro complementarities).[79] It is these two complementarities that dictate the comovement of sectoral price and material cost indices.

Key equations. For the sake of focus, it is assumed for a moment that the wage is invariant to the policy change, thereby being dropped from the equations below. First, totally differentiating the firm's profit-maximization and cost-minimization problems delivers

$$\frac{dP_i^*}{d\tau_n} = \bar{\lambda}_{i\cdot}^{M} \frac{dP_i^{M*}}{d\tau_n}, \tag{32}$$

where $\bar{\lambda}_{i\cdot}^{M}$ is the coefficient representing the pass-through coefficient of a change in the material input cost to the sectoral price index. It involves the all firms' strategic complementarities with respect to every other firms in the same sector. Then, I call $\bar{\lambda}_{i\cdot}^{M}$ the *micro complementarities*.

[78] These thee types of comparative statics are determined in a simultaneous system of equations. A fuller account can be found in Appendix C.3. After substituting one another, the system can be transformed into a set of reduced-form equations. The associated "reduced-form coefficients" can be obtained as soon as the firms' production functions and demand functions faced by firms are estimated through the method described in Section 5.3.

[79] I borrow terminologies from Klenow and Willis (2016) and Alvarez et al. (2023).

Table 2: Responsiveness of Sectoral GDP (in Billion U.S. Dollar)

(a) Monopolistic Competition (with the Production Network)

Industry	Total Effects	Effects on Revenue		Effects on Material Cost	
		p.effect	q.effect	w.effect	s.effect
Wholesale trade	2679.40	3129.08	-14997.04	2900.08	-17447.44
Computer and electronic products	196.76	-538.04	1098.37	-152.90	516.47
Hospitals and nursing	87.26	-13.15	77.68	31.92	-54.64
Food services and drinking places	79.37	-27.08	117.76	19.42	-8.11
Construction	77.40	-25.43	121.94	8.12	11.00
	⋮				
	⋮				
Paper products, printing, and related activities	-202.47	162.66	-659.94	169.49	-464.30
Broadcasting and telecommunications	-369.96	1079.64	-1948.26	597.88	-1096.54
Petroleum and coal products	-551.58	740.38	-462.15	2091.71	-1261.90
Motor vehicles, bodies and trailers, and parts	-720.69	626.73	-2963.23	687.48	-2303.29
Retail trade	-725.91	2993.65	-8432.83	2989.46	-7702.73
Total	150.74				

(b) Oligopolistic Competition (with the Production Network)

Industry	Total Effects	Effects on Revenue		Effects on Material Cost	
		p.effect	q.effect	w.effect	s.effect
Accommodation	0.73	-2.15	3.38	-1.28	1.77
Wood products	0.59	0.83	-1.26	-0.47	-0.56
Plastics, rubber and mineral products	0.47	-6.35	6.26	-4.89	4.32
Railroad and truck transportation	0.44	-1.29	1.53	-1.35	1.15
Hospitals and nursing	0.21	-0.25	0.25	0.19	-0.41
	⋮				
	⋮				
Retail trade	-13.00	-67.09	68.81	-53.39	68.12
Wholesale trade	-14.28	-70.76	71.60	-78.28	93.40
Miscellaneous manufacturing	-44.50	43.98	-125.57	0.66	-37.75
Petroleum and coal products	-58.79	-186.41	187.48	-104.18	164.04
Computer and electronic products	-94.70	-251.29	252.58	-59.75	155.74
Total	-250.23				

Note: This table reports the estimates for top and bottom five firms in terms of the total effects (i.e., the change in sectoral GDP in the order of million dollars). Panel (a) shows the results for Specification I, while Panel (b) illustrates the estimates for Specification II. Since the network spillover effects are by construction absent in Specification I, results for other industries are omitted in Panel (a). In each of the panels, the total effects are broken down to the effects on revenue and material input cost. They are further decomposed into four effects according to in (31): namely, *p.effect* stands for the price effects, *q.effect* the quantity effects, *w.effect* the wealth effects and *s.effect* the switching effects. Notice that the total effects are given by the effects on revenue *minus* the effects on material cost (see (31)).

Second, from the cost-minimization problem for the material input aggregator, I have

$$\frac{dP_i^{M*}}{d\tau_n} = -h_{i,n}\frac{P_n^{M*}}{1-\tau_n}, \tag{33}$$

where $h_{i,n}$ indicates the (i,n) entry of $(I-\Gamma)^{-1}$, with $\Gamma := \left[\gamma_{i,j}\frac{P_i^{M*}}{P_j^*}\bar{\lambda}_{j\cdot}^M\right]_{i,j=1}^N$. Note that the array of the output elasticities $[\gamma_{i,j}]_{i,j=1}^N$ reflects the input-output structure Ω (Fact B.5). Hence the matrix $(I-\Gamma)^{-1}$ can be considered a version of Leontief inverse matrix that compounds the sectors' micro complementarities $\bar{\lambda}_{j\cdot}^M$. In (33), $h_{i,n}$ captures a policy-cost pass through, and represents the rate at which the change in a sectoral cost index $\frac{dP_i^{M*}}{d\tau_n}$ moves in the same direction as the direct effect of the subsidy $-\frac{P_n^{M*}}{1-\tau_n}$. I call this comovement of sectoral price indices the *macro complementarities*. For example, if this coefficient is positive, in which case the sector i is said to be macro complement, an increase in the subsidy for sector n leads to a lower material input cost for sector i.

Reading off these "reduced-form" equations in reverse order, i.e., from (33) to (32), I can proceed as if the material cost indices were determined first, followed by the adjustments of the sectoral price indices.

Result. Table 3 reports the responses of sectoral price indices and material cost indices along with the coefficients indicating macro and micro complementarities for the top and bottom five industries listed in Table 2. Note that Table 3 shows the estimates containing the general equilibrium effects, so that the numbers do not exactly follow (32) and (33), and any residuals should be interpreted as the effect of the change in wage.[80] In this empirical analysis, I obtain $-\frac{P_n^{M*}}{1-\tau_n} = -827.92$.

The semiconductor industry in monopolistic competition is qualitatively the same as that in oligopolistic competition in the sense of being macro and micro complement. That is, the industry is macro complement to itself, and thus the material cost index decreases according to (33). Since the industry is also micro complement, the cost reduction is translated via (32) into the decrease in the output price.

However, they differ quantitatively. For the case of oligopolistic competition, the industry is less micro complement, so that the pass-through of the cost change is smaller. This means that the firms in oligopolistic competition is more profit-making than in monopolistic competition, if the cost change were to be the same. The table also shows, however, that the response of the material input cost differs substantially across two types of competitions. In view of (33), this can be traced back to the significant difference in the sector's macro complementarity, resonates the observation made in Section 2; i.e., when

[80] The general equilibrium effects are not dominant, accounting only for 10 to 20% in each of (32) and (33).

the markets feature strategic interactions, the production network compounds not only the firm's markup responses with respect to own choice, but also with respect to competitors'.[81] That is, the difference in the wealth effects in Table 2 arises from the difference in the macro complementarity, while the one in the price effects reflects the combination of macro and micro complementarities.

Table 3: The Changes in Sectoral Price Indices and Material Cost Indices

(a) Monopolistic Competition (with the Production Network)

Industry (i)	$h_{i,n}$	$\frac{dP_i^{M*}}{d\tau_n}$	$\bar{\lambda}_{i\cdot}^M$	$\frac{dP_i^*}{d\tau_n}$
Wholesale trade	-1.11	6567.20	0.63	4013.97
Computer and electronic products	4.12	-2268.93	0.24	-667.05
Hospitals and nursing	-0.97	3312.98	0.31	-285.32
Food services and drinking places	-0.63	2460.67	0.11	-360.25
Construction	0.11	101.80	0.40	-92.43
⋮				
Paper products, printing, and related activities	-1.19	3964.86	0.23	700.26
Broadcasting and telecommunications	0.42	4140.84	0.16	567.66
Petroleum and coal products	0.00	471.62	0.05	28.53
Motor vehicles, bodies and trailers, and parts	-0.60	1560.55	0.60	618.57
Retail trade	-1.46	7218.48	0.22	1372.51

(b) Oligopolistic Competition (with the Production Network)

Industry (i)	$h_{i,n}$	$\frac{dP_i^{M*}}{d\tau_n}$	$\bar{\lambda}_{i\cdot}^M$	$\frac{dP_i^*}{d\tau_n}$
Accommodation	0.12	-110.80	0.11	-10.58
Wood products	0.06	-50.19	-0.21	11.59
Plastics, rubber and mineral products	0.16	-140.44	0.06	-9.21
Railroad and truck transportation	0.12	-112.13	0.07	-8.94
Hospitals and nursing	-0.03	20.10	0.04	-0.91
⋮				
Retail trade	0.15	-128.92	0.08	-11.20
Wholesale trade	0.20	-177.26	0.11	-19.16
Miscellaneous manufacturing	-0.05	60.67	4.86	203.32
Petroleum and coal products	0.03	-23.49	0.49	-11.57
Computer and electronic products	1.67	-1391.01	0.11	-153.40

Note: This table displays the estimates for the elements of (32) and (33) for those industries listed in Table 2. Panel (a) shows the results for monopolistic competition and panel (b) for oligopolistic competition.

[81] This observation appears clearer in the Wholesale Trade and Retail Trade industries, both of which are macro substitute in monopolistic competition, while being macro complement in oligopolistic competition.

7 Conclusions

Industrial policies have been and will continue to be an important policy tool for policymakers to achieve ranging policy goals. Meanwhile, the economic impact of an industrial policy has been relatively unexplored. This paper studies the effect of an industrial policy on an aggregate outcome in the presence of strategic interactions and production networks. To this end, I develop a general equilibrium multisector model of heterogenous oligopolistic firms with a production network. For the identification, I develop a new, multi-layered identification procedure that first deconstructs the policy parameter into sectoral aggregate variables as well as firm-level variables — firm-level sufficient statistics, and recovers the latter by using the control function approach of the industrial organization literature before finally reconstructing the original policy parameter. To accommodate the firm's strategic interactions, I restrict the classes of the firm's inverse demand and production function and the path through witch the other firm's productivities enters the firm's production decision. I show that these assumptions are general enough to encompass many specifications that are commonly used in the macroeconomics literature. Given that all firm-level responses — the finest ingredient of the model — are identified, my method can be used to study a variety of policy parameters such as GDP, consumption, intersectoral trade flow, and sectoral distributional outcomes. Moreover, since my approach is constructive, a nonparametric estimator for the policy effect can thus be obtained by reading off this procedure in reverse without adapting any external information (e.g., parameter estimates from the preceding research).

Our estimates, based on the U.S. firm-level data, suggest that accounting for the firm's strategic interactions doubles the magnitude of the policy effect of an additional subsidy on semiconductor industry relative to the case where firms are monopolistic. This is because when a strategic interactions are present, the production network compounds not only firm-level markup responses with respect to the firm's own choices but also with respect to competitors', whereas the latter is absent in monopolistic competition. This additional wedge in the network spillovers manifests itself as the differences in the comovements of sectoral price indices and material cost indices, or pass-through coefficients.

My framework can be by no means a panacea for any policy evaluation problems. As alluded by Baqaee and Farhi (2020), my analysis is susceptible to errors to the extent that my main data, Compustat data, is incomplete and non-representative, and requires substantial imputation.[82] Besides data limitation, there are three directions on theoretical study for future works. First, this paper abstracts away from

[82]See also Covarrubias et al. (2020).

the firm's entry and exit problem over the course of policy reform, restricting the scope of analysis to short-run policy effects. Accommodating the long-run perspective inserts an additional layer into our framework: namely, the free-entry condition. Deriving the comparative statics, however, is nontrivial in our setup as the number of firms is finite and thus the standard notion of derivatives cannot be well-defined. Third, the identification analysis of this paper assumes that the economy features a single equilibria, and the same equilibria realizes under a different policy, and the policy reform is restricted to be within the historically observed support. These limitations can be simultaneously addressed at the cost of additional assumptions concerning the equilibrium selection probability as studied in Canen and Song (2022). Third, my model is static, and thus silent about policy implications of capital accumulation, which is usually at the center of industrial debate. An extension to a dynamic environment requires not only an explicit consideration of the firm's own future choices but also competitors' future choices. This convoluted forward-looking nature opens up another source of multiplicity of equilibria.

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A Insights

A.1 Review of Input-Output Table

Consider an economy consisting of three industries: namely, Agriculture (A), Broadcasting (B), and Computers (C) industries, indexed by $i = 1, 2$, and 3, respectively. Each industry's sales (measured in appropriate monetary unit) is denoted by x_i for $i \in \{1, 2, 3\}$. When there are no market distortions, each industry's sales is equivalent to the industry's expenditure, and it derives from final consumption (F) by consumers and intermediate use by all firms. The expenditure for final consumption is indicated by y_i.

The share of sector j's good in sector i's expenditure represented by $\omega_{i,j}$ for $i, j \in \{1, 2, 3\}$. I use an array $\Omega := [\omega_{i,j}]_{i,j\in\{1,2,3\}}$ to keep track of the input-output structure.[83]. For instance, the Agriculture industry's sales consists of $\omega_{1,1}$ of the Agriculture industry's expenditure (x_1), $\omega_{2,1}$ of the Broadcasting industry's expenditure (x_2), $\omega_{3,1}$ of the Computer industry's expenditure (x_3), and the final consumption (y_1): $x_1 = \omega_{1,1}x_1 + \omega_{2,1}x_2 + \omega_{3,1}x_3 + y_1$ (see Table 3 (a)). Stacking this expression for all sectors into a matrix form, the sectoral expenditure, sales and final consumption satisfy the following relationship: $X = \Omega X + Y$, where X and Y are vectors stacking x_i's and y_i's, respectively, i.e., $X := [x_1\ x_2\ x_3]'$ and $Y := [y_1\ y_2\ y_3]'$. Under a regularity condition,[84] this can be written as

$$X = \underbrace{Y}_{\text{final demand}} + \underbrace{\Omega(I - \Omega)^{-1}Y}_{\text{intermediate demand}}.^{85} \tag{34}$$

This expression decomposes the industries' sales into the demand of goods for final consumption and for intermediate use, with the latter proportional to the final consumption.

Next, I introduce market distortions in this accounting framework. I assume that for each industry $i \in \{1, 2, 3\}$, the industry's sales (x_i) is different from the expenditure ($\tilde{x}_i$) by the rate of μ_i: i.e., $\tilde{x}_i = \frac{1}{\mu_i}x_i$. I consider the case of $\mu_i > 0$, in which the distortion can be interpreted as a sector-level markup (a microfoundation is provided in Section 2.3). Let M be a 3×3 diagonal matrix with typical diagonal element being the sectoral markup and zero otherwise. Then under an additional regularity condition, a version of (34) for this case is given by

$$X = \underbrace{Y}_{\text{final demand}} + \underbrace{\Omega M^{-1}(I - \Omega M^{-1})^{-1}Y}_{\text{intermediate demand}}, \tag{35}$$

where ΩM^{-1} is interpreted as a markup-augmented input-output linkage (Table 3 (b)).[86]

Derivation of (34) **and** (35)**.** For the sake of exposition, I start by assuming that the Leontief inverse matrix exists.

Assumption A.1. $(I - \Omega)^{-1}$ *exists.*

[83]Note that for each $i \in \{1, 2, 3\}$, $\sum_{j\in\{1,2,3\}} \omega_{i,j} = 1$

[84]See Appendix A.

[85]See Appendix A.

[86]See Appendix A.

Figure 3: Input-Output Table

Seller \ Purchaser	A	B	C	F	Total Sales
A	$\omega_{1,1}x_1$	$\omega_{2,1}x_2$	$\omega_{3,1}x_3$	y_1	x_1
B	$\omega_{1,2}x_1$	$\omega_{2,2}x_2$	$\omega_{3,2}x_3$	y_2	x_2
C	$\omega_{1,3}x_1$	$\omega_{2,3}x_2$	$\omega_{3,3}x_3$	y_3	x_3
Total Expenditure	x_1	x_2	x_3		

(a) without market distortions

Seller \ Purchaser	A	B	C	F	Total Sales
A	$\omega_{1,1}\tilde{x}_1$	$\omega_{2,1}\tilde{x}_2$	$\omega_{3,1}\tilde{x}_3$	y_1	x_1
B	$\omega_{1,2}\tilde{x}_1$	$\omega_{2,2}\tilde{x}_2$	$\omega_{3,2}\tilde{x}_3$	y_2	x_2
C	$\omega_{1,3}\tilde{x}_1$	$\omega_{2,3}\tilde{x}_2$	$\omega_{3,3}\tilde{x}_3$	y_3	x_3
Total Expenditure	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$		

(b) with market distortions

Note: Panel (a) of this figure represents an input-output table corresponding to (34), i.e., when there are no market distortions. Panel (b) describes the one for the case of (35), i.e., when there are market distortions (e.g., sector-specific markups).

This assumption is always satisfied (Carvalho and Tahbaz-Salehi 2019). Note that Assumption A.1 implies:

$$(I-\Omega)^{-1} = \sum_{n=0}^{\infty} \Omega^n. \tag{36}$$

It can be shown that the (i, j) entry of the right hand side is

$$\omega_{i,j} + \sum_{k\in\{1,2,3\}} \omega_{i,k}\omega_{k,j} + \sum_{k\in\{1,2,3\}} \sum_{l\in\{1,2,3\}} \omega_{i,k}\omega_{k,l}\omega_{k,j} + \dots .$$

This dictates how important industry j is for industry i as a direct and indirect input supplier (Carvalho and Tahbaz-Salehi 2019).

Under Assumption A.1, it follows that

$$\begin{aligned}(I-\Omega)(I-\Omega)^{-1} &= I\\ \therefore (I-\Omega)^{-1} &= I + \Omega(I-\Omega)^{-1}.\end{aligned}$$

Hence I have (34):

$$\begin{aligned}X &= \Omega X + Y\\ \therefore (I-\Omega)X &= Y\\ \therefore X &= Y + \Omega(I-\Omega)^{-1}Y.\end{aligned}$$

Here I note that it follows from (36) that $\Omega(I-\Omega)^{-1} = \sum_{n=1}^{\infty} \Omega^n$.

To prove (35), I need to strengthen Assumption A.1.

Assumption A.2. $(I - \Omega M^{-1})^{-1}$ *exists.*

With Assumption A.2, I obtain (35) in a manner analogous to (34).

Derivation of (??). To derive (??), I utilize the following fact.

Fact A.1. *(i) For a square matrix B and for any integer $n \geq 1$, $dB^n = \sum_{l=0}^{n-1} B^l (dB) B^{n-l-1}$. (ii) For a square matrix B, $dB^{-1} = -B^{-1}(dB)B^{-1}$.*

Observe that again by (36) it holds that $\Omega M^{-1}(I - \Omega M^{-1})^{-1} = \sum_{n=1}^{\infty} (\Omega M^{-1})^n$. Since the final consumption (Y) is assumed to be invariant to the shock, total differentiation of (35) yilds

$$
\begin{aligned}
dX &= d\{\Omega M^{-1}(I - \Omega M^{-1})\}Y + \{\Omega M^{-1}(I - \Omega M^{-1})\} \underbrace{dY}_{=0} \\
&= \sum_{n=1}^{\infty} d(\Omega M^{-1})^n Y \\
&= \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} (\Omega M^{-1})^l d(\Omega M^{-1}) M^{-1} (\Omega M^{-1})^{n-l-1} Y \\
&= -\sum_{n=1}^{\infty} \sum_{l=0}^{n-1} (\Omega M^{-1})^{l+1} (dM) M^{-1} (\Omega M^{-1})^{n-l-1} Y,
\end{aligned}
$$

where the third and fourth equalities are the consequence of Facts A.1 (i) and (ii), respectively.

B Detail of Data

This section provides the detailed account of the data source used in my paper, and how I construct the empirical counterparts of the variables.

B.1 Aggregate-Level Data

Data on wage-related concepts are obtained from the U.S. Bureau of Labor Statistics (BLS) through the Federal Reserve Bank of St. Louis (FRED) at annual frequency. In my model, labor is assumed to be frictionlessly mobile across sectors so that the wage W is common for all sectors. Thus I use "average hourly earnings of all employees, total private" as the empirical counterpart of my wage. In addition, I also obtain the measures of total number of employees (*All Employees, Total Private*) and of total hours worked per year (*Hours of Wage and Salary Workers on Nonfarm Payrolls*), from which I can compute the average hours worked per employee per year (see Appendix B.3). Note that both the total number of employees and total hours worked exclude farms mainly because of the peculiarities of the structure of the agricultural industry and characteristics of its workers: e.g., various definitions of agriculture, farms, famers and farmworkers; considerable seasonal fluctuation in the employment (Daberkow and Whitener

1986). In this sense, the corresponding data for farms industry in my dataset should be considered being inputed by the average of other sectors.

Sectoral price index data is available at the Bureau of Economic Analysis (BEA). I use *U.Chain-Type Price Indexes for Gross Output by Industry — Detail Level (A)* as the data.

These are summarized in the following fact.

Fact B.1 (Wage & Sectional Price Index)**.** *The wage W and sectoral price indices $\{P_i\}_{i=1}^N$ are directly observed in the data.*

B.2 Sector-Level Data: Industry Economic Accounts (IEA)

Our analysis involves two types of sector-level data: namely, the input-output table and sector-input-specific tax/subsidy, both of which come from the input-output accounts data of the Bureau of Economic Analysis (BEA). In line with the global economic accounting standards, such as the System of National Accounts 2008 (UN 2008), the BEA input-output table consists in two tables: the use and supply table.

The use table shows the uses of commodities (goods and services) by industries as intermediate inputs and by final users, with the columns indicating the industries and final users and the rows representing commodities. This table reports three pieces of information: intermediate inputs, final demand and value added. Each cell in the intermediate input section records the amount of a commodity purchased by each industry as an intermediate input, valued at producer' or purchasers' prices.[87] The final demand section accounts for expenditure-side components of GDP. The value-added part bridges the difference between an industry's total output and the its total cost for intermediate inputs. I will further elaborate on this part in the upcoming section (Appendix B.2.2).

The supply table shows total supply of commodities by industries, with the columns indicating the industries and the rows representing commodities. This table comprises domestic output and imports. Each cell of the domestic output section presents the total amount of each commodity supplied domestically by each industry, valued at the basic prices. The import section records the total amount of each commodity imported from foreign countries, valued at the importers' customs frontier price (i.e., the c.i.f. valuation).[88]

Segmentation. my analysis is based on the BEA's industry classification at the summary level, which is roughly equivalent to the three-digit NAICS (North American Industry Classification System). I make two modifications in conjunction with the availability of Compustat data . First, I omit several industries and products from my analysis. Following Bigio and La'O (2020), I exclude finance, insurance, real estate,

[87] Typically, the IEA is valued at either of the producers', basic, or purchasers' prices. The producers' prices are the total amount of monetary units received from the purchasers for a unit of a good and service that is sold. The basic prices mean the total amount retained by the producer for a unit of a good and service. This price plays a pivotal role in the producer's decision making about production and sales. The purchasers' prices refer to the total amount payed by the purchasers for a unit of a good and service that they purchase. This is the key for the purchasers to make their purchasing decisions. By definition, the basic prices are equal to the producers' prices minus taxes payable for a unit of a good and service plus any subsidy receivable for a unit of a good and service; and the purchasers' prices are equivalent to the sum of the producers' prices and any wholesale, retail or transportation markups charged by intermediaries between producers and purchasers. See BEA (2009) and Young et al. (2015) for the detail.

[88] The importers' customs frontier price is calculated as the cost of the product at foreign port value plus insurance and freight charges to move the product to the domestic port. See Young et al. (2015) for the detail.

rental and leasing (FIRE) sectors from my analysis.In the BEA's input-output table, These sectors are indexed by 521CI, 523, 524, 525, HS, ORE, and 532RL. I also follow Baqaee and Farhi (2020) in dropping two product categories: namely, Scrap, used and secondhand goods and Noncomparable imports and rest-of-the-world adjustment. These are indexed by "Used" and "Others," respectively. I again follow Baqaee and Farhi (2020) in removing the government sectors, which are reported with the indices 81, GFGD, GFGN, GFE, GSLG, and GSLE. Second, drawing on Gutiérrez and Philippon (2017), I merge some of the BEA's industries. This manipulation makes sure that each industry has a good coverage of Compustat firms (Gutiérrez and Philippon 2017). In my context, this also helps us focus on modestly imperfectly competitive markets. After all, I are left with 38 industries (Table 4).

Table 4: Mapping of BEA Industry Codes to Segments

BEA code	Industry	Mapped segment
111CA	Farms	Farms, forestry, fishing, and related activities
113FF	Forestry, fishing, and related activities	Farms, forestry, fishing, and related activities
211	Oil and gas extraction	Oil and gas extraction
212	Mining, except oil and gas	Mining, except oil and gas
213	Support activities for mining	Support activities for mining
22	Utilities	Utilities
23	Construction	Construction
311FT	Food and beverage and tobacco products	Food and beverage and tobacco products
313TT	Textile mills and textile product mills	Textile and apparel products
315AL	Apparel and leather and allied products	Textile and apparel products
321	Wood products	Wood products
322	Paper products	Paper products, printing, and related activities
323	Printing and related support activities	Paper products, printing, and related activities
324	Petroleum and coal products	Petroleum and coal products
325	Chemical products	Chemical products
326	Plastics and rubber products	Plastics, rubber and mineral products
327	Nonmetallic mineral products	Plastics, rubber and mineral products
331	Primary metals	Primary metals
332	Fabricated metal products	Fabricated metal products
333	Machinery	Machinery
334	Computer and electronic products	Computer and electronic products
335	Electrical equipment, appliances, and components	Electrical equipment, appliances, and components
3361MV	Motor vehicles, bodies and trailers, and parts	Motor vehicles, bodies and trailers, and parts
33640T	Other transportation equipment	Motor vehicles, bodies and trailers, and parts
337	Furniture and related products	Furniture and related products
339	Miscellaneous manufacturing	Miscellaneous manufacturing
42	Wholesale trade	Wholesale trade
441	Motor vehicle and parts dealers	Retail trade
445	Food and beverage stores	Retail trade
452	General merchandise stores	Retail trade
4A0	Other retail	Retail trade
481	Air transportation	Air transportation
482	Rail transportation	Railroad and truck transportation
483	Water transportation	Other transportation
484	Truck transportation	Railroad and truck transportation
485	Transit and ground passenger transportation	Other transportation
486	Pipeline transportation	Other transportation
4870S	Other transportation and support activities	Other transportation
493	Warehousing and storage	Other transportation
511	Publishing industries, except internet (includes software)	Publishing industries

BEA code	Industry	Mapped segment
512	Motion picture and sound recording industries	Motion picture and sound recording industries
513	Broadcasting and telecommunications	Broadcasting and telecommunications
514	Data processing, internet publishing, and other information services	Information and data processing services
521CI	Federal Reserve banks, credit intermediation, and related activities	Omitted
523	Securities, commodity contracts, and investments	Omitted
524	Insurance carriers and related activities	Omitted
525	Funds, trusts, and other financial vehicles	Omitted
HS	Housing	Omitted
ORE	Other real estate	Omitted
532RL	Rental and leasing services and lessors of intangible assets	Omitted
5411	Legal services	Professional services
54120P	Miscellaneous professional, scientific, and technical services	Professional services
5415	Computer systems design and related services	Professional services
55	Management of companies and enterprises	Omitted
561	Administrative and support services	Administrative and waste management
562	Waste management and remediation services	Administrative and waste management
61	Educational services	Educational services
621	Ambulatory health care services	Health care services
622	Hospitals	Hospitals and nursing
623	Nursing and residential care facilities	Hospitals and nursing
624	Social assistance	Health care services
711AS	Performing arts, spectator sports, museums, and related activities	Arts
713	Amusements, gambling, and recreation industries	Arts
721	Accommodation	Accommodation
722	Food services and drinking places	Food services and drinking places
81	Other services, except government	Omitted
GFGD	Federal general government (defense)	Omitted
GFGN	Federal general government (nondefense)	Omitted
GFE	Federal government enterprises	Omitted
GSLG	State and local general government	Omitted
GSLE	State and local government enterprises	Omitted
Used	Scrap, used and secondhand goods	Omitted
Other	Noncomparable imports and rest-of-the-world adjustment	Omitted

Note: This table shows the correspondence between the BEA's industry classification (at summary level) and my segmentation, which draws heavily on Gutiérrez and Philippon (2017). The first two columns ("BEA code" and "Industry") list the BEA codes and the corresponding industries as used in the BEA's input-output table. The third column ("Mapped segment") indicates the names of the segments I define.

B.2.1 Transformation to Symmetric Input-Output Tables

Although the use table comes very close to an empirical counterpart of the production network of my model, it cannot be directly used in my empirical analysis as it only shows the uses of each commodity by each industry, not the uses of each industrial product by each industry. This is because the BEA's accounting system allows for each industry to produce multiple commodities (e.g., secondary production), contradicting to my conceptualization. Hence I first need to convert the use table to a symmetric industry-by-industry input output table by transferring inputs and output over the rows in the use and supply table, respectively.[89] This reattribution of the commodities supplied will leave us with the industry-by-industry use table, which is my input-output table. This is accompanied by the transformed supply table, whose off-diagonal elements are all zero.[90] To do this, I impose an assumption about how each commodity is used.

Assumption B.1 (Fixed Product Sales Structures, (Eurostat 2008))**.** *Each product has its own specific sales structure, irrespective of the industry where it is produced.*

The term "sales structure" here refers to the shares of the respective intermediate and final users in the sales of a commodity. Under Assumption B.1, each commodity is used at the constant rates regardless of in which industry it is produced. For example, a unit of an manufacturing product supplied by agriculture industry will be transferred from the use of manufacturing product to that of agricultural products in the use table in the same proportion to the use of manufacturing products.[91] Note that the value added part remains intact throughout this manipulation. Recorded in each cell of the intermediate inputs section of the resulting industry-by-industry table is the empirical counterpart of my $\sum_{k=1}^{N_i}(1-\tau_{i,j})P_i m_{ik,j}$, and each cell of the compensation of employee corresponds to $\sum_{k=1}^{N_i} W\ell_{ik}$. These are the data that is used four constructing the production network in my empirical analysis as shown in the following fact.

Fact B.2. *Under Assumption B.1, the input-output linkages $\boldsymbol{\omega}_L$ and Ω are recovered from the observables.*

Proof. By Shephard lemma,[92] it holds that for each $i, j \in \mathbf{N}$, the cost-based intermediate expenditure shares $\omega_{i,j}$ satisfies

$$\begin{aligned}\omega_{i,j} &= \frac{\sum_{k=1}^{N_i}(1-\tau_{i,j})P_j m_{ik,j}}{\sum_{k=1}^{N_i}\left\{\sum_{j'=1}^{N}(1-\tau_{i,j'})P_{j'}m_{ik,j'} + W\ell_{ik}\right\}} \\ &= \frac{\sum_{k=1}^{N_i}(1-\tau_{i,j})P_j m_{ik,j}}{\sum_{j'=1}^{N}\sum_{k=1}^{N_i}(1-\tau_{i,j'})P_{j'}m_{ik,j'} + \sum_{k=1}^{N_i} W\ell_{ik}}. \end{aligned} \tag{37}$$

Also, for each $i \in \mathbf{N}$, cost-based equilibrium factor expenditure shares $\omega_{i,L}$ satisfies:

$$\begin{aligned}\omega_{i,L} &= \frac{\sum_{k=1}^{N_i} W\ell_{ik}}{\sum_{k=1}^{N_i}\left\{\sum_{j'=1}^{N}(1-\tau_{i,j'})P_{j'}m_{ik,j'} + W\ell_{ik}\right\}} \\ &= \frac{\sum_{k=1}^{N_i} W\ell_{ik}}{\sum_{j'=1}^{N}\sum_{k=1}^{N_i}(1-\tau_{i,j'})P_{j'}m_{ik,j'} + \sum_{k=1}^{N_i} W\ell_{ik}}. \end{aligned}$$

[89] For example, if there is a non-zero entry in the cell of the supply table whose column is agriculture and whose row is manufacturing products, it is recorded in the use table as the supply of manufacturing products, the largest component of which should be accounted for by the supply from manufacturing industry. Now my goal is to modify this attribution in a way that the supply of manufacturing products by agriculture industry is treated as agricultural products. To this end, I need to subtract the contributions of agriculture industry from the use of manufacturing products, and transfer them to the agricultural commodities, thereby changing the classification of the row from commodity to industry.

[90] There is another approach to transform the use table to a symmetric commodity-by-commodity table. In such a case, sectors of my conceptual model corresponds to commodities in the data. See Eurostat (2008) for the detail.

[91] Related to this assumption is the fixed industry sales structure assumption, in which . However, it is Assumption B.1 that is widely used by statistical offices for various reasons. See Eurostat (2008) for the detail.

[92] See Liu (2019), Baqaee and Farhi (2020) and Bigio and La'O (2020) for application and reference.

Since $\left\{\sum_{k=1}^{N_i}(1-\tau_{i,j})P_j m_{ik,j}\right\}_{i,j=1}^{N}$ and $\{\sum_{k=1}^{N_i} W\ell_{ik}\}_{i=1}^{N}$ are directly observed in the transformed industry-by-industry input-output table, I can immediately recover $\boldsymbol{\omega}_L$ and Ω, as desired. $\square$

Figure compares the input-output table based on the use table and transformed industry-by-industry input-output table.

B.2.2 Sectoral Tax/Subsidy

Given that the use table has been transformed into a symmetric industry-by-industry input-output table, I can proceed to back out the tax/subsidy from the transformed table. In this step, I exploit the feature of the use table that reports value added at basic and purchasers' prices. The value added measured at basic prices is composed of i) compensation of employees (V001), ii) gross operating surplus (V003) and iii) other taxes on production (T00OTOP) less subsidies (T00OSUB). The value added at producers' prices further entails iv) taxes on products (T00TOP) and imports less subsidies (T00SUB).[93] According to BEA (2009), the tax-related components of (iii) and (iv) jointly include, among many others, sales and excise taxes, customs duties, property taxes, motor vehicle licenses, severance taxes, other taxes and special assessments as well as commodity taxes, while the subsidy-related components refer to monetary grants paid by government agencies to private business and to government enterprises at another level of government.[94] I consider the sum of (iii) and (iv) to be the empirical counter part of the policy expenditure in my model. This choice is motivated by the mapping between the BEA's data construction and my conceptualization. First, the construction of data states:

$$
\begin{aligned}
&Profits_i = (Revenue_{ik} + TaxSubsidy1) - (LaborCost_{ik} + MaterialCost_{ik} + TaxSubsidy2) \\
&\therefore \underbrace{Revenue - MaterialCost_i}_{\text{Value-added}} = \underbrace{Profits_i}_{\text{Gross operating profits}} + \underbrace{LaborCost_i}_{\text{Compensation of employees}} - \underbrace{(TaxSubsidy1 - TaxSubsidy2)}_{\text{Value-added taxes less subsidies}},
\end{aligned}
\tag{38}
$$

where $TaxSubsidy1$ is taxes less subsidies on revenues, and $TaxSubsidy2$ those on input costs. Notice that the value-added taxes less subsidies $(TaxSubsidy1 - TaxSubsidy2)$ are available in the data.

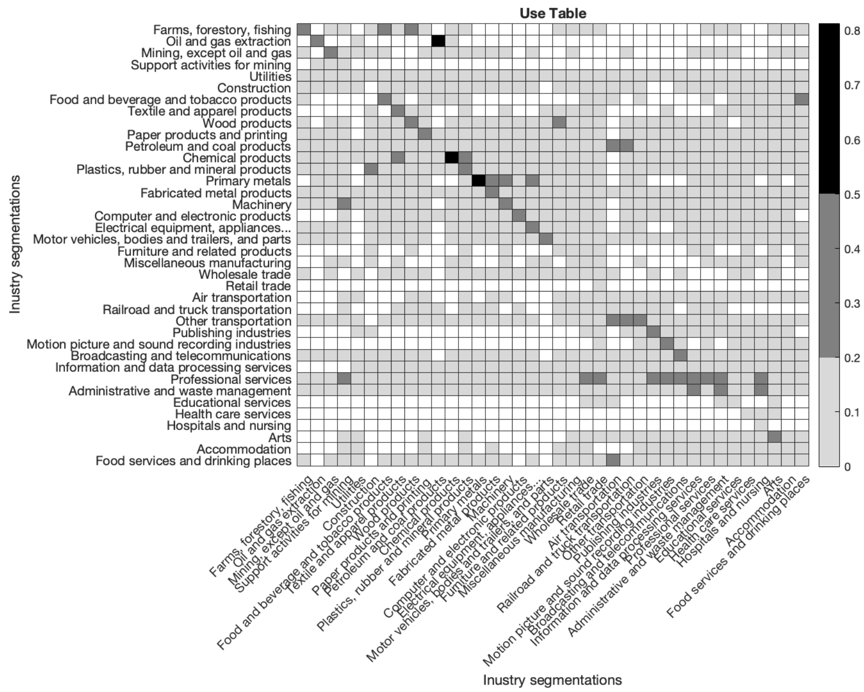
To back out tax/subsidy data from this table, I need to restrict the scope of analysis to sector-specific tax/subsidy.

Assumption B.2. *Taxes and subsidies are specific to sectors: i.e.,* $\boldsymbol{\tau} := \{\tau_i\}_{i=1}^{N}$.

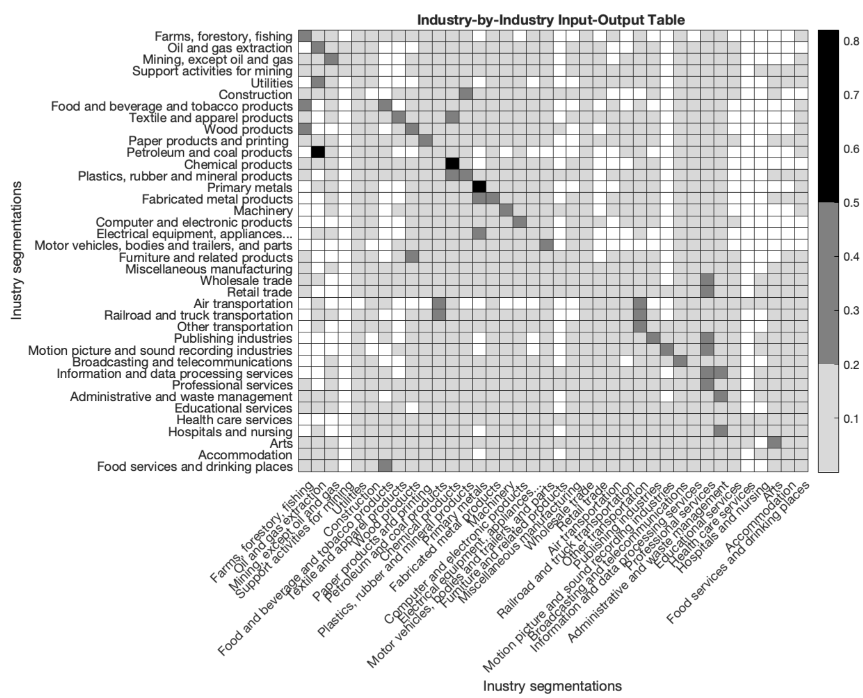
[93]By construction, the sum of the latter across all industries has to coincide with GDP for the economy.

[94]In BEA (2009), compensation of employees is defined to be ""

Table 5: Comparison of Input-Output Tables



(a) Use table



(b) Transformed industry-by-industry table

Note: This figure illustrates the input-output table in terms of cost share of sectoral goods. Panel (a) shows the use table that is provided by BEA, while panel (b) reports the transformed industry-by-industry table. White cells indicate zero, while light, medium and dark grey cells represent the low ($0 \sim 0.2$), medium ($0.2 \sim 0.5$) and high ($0.5 \sim 1.0$) cost shares, respectively.

Under this assumption, the theoretical counterpart of the data construction (38) is

$$\sum_{k=1}^{N_i} \pi^*_{ik} = \sum_{k=1}^{N_i} p^*_{ik} q^*_{ik} - \left\{ W^* \ell^*_{ik} + (1-\tau_i) \sum_{j=1}^{N} P_i^{M*} m^*_{ik,j} \right\}$$
$$\therefore \underbrace{\sum_{k=1}^{N_i} p^*_{ik} q^*_{ik} - \sum_{j=1}^{N} P_i^{M*} m^*_{ik,j}}_{\text{Value-added}} = \underbrace{\sum_{k=1}^{N_i} \pi^*_{ik}}_{\text{Gross operating profits}} + \underbrace{W^* \ell^*_{ik}}_{\text{Compensation of employees}} - \underbrace{\tau_i \sum_{j=1}^{N} P_i^{M*} m^*_{ik,j}}_{\text{Value-added taxes less subsidies}} . \tag{39}$$

On the basis of this formulation, I can back out ad-valorem taxes/subsidy from the constructed input-output table. This is summarized in the following fact.

Fact B.3. *Under Assumptions B.1 and B.2, sector-specific subsidies $\boldsymbol{\tau} := \{\tau_i\}_{i=1}^N$ are recovered from the observables.*

Proof. For each sector (industry) $i \in \mathbf{N}$, I have

$$(1-\tau_i) \sum_{j=1}^{N} \sum_{k=1}^{N_i} P_j^* m^*_{ik,j} = \sum_{j=1}^{N} IntermExpend_{i,j}, \tag{40}$$

where $IntermExpend_{i,j}$ means the sector i's total expenditure on sector j, which is observed in the (i,j) entry of the industry-by-industry input-output table constructed in Appendix B.2.1. Meanwhile, comparing (38) to (39), I obtain

$$\tau_i \sum_{j=1}^{N} \sum_{k=1}^{N_i} P_j^* m^*_{ik,j} = VAT_i, \tag{41}$$

where VAT_i stands for the sector i's value-added taxes less subsidies, reported in the BEA use table.

Rearranging (40) and (41), I can recover the data for sector-specific taxes/subsidies:

$$\tau_i = \frac{VAT_i}{VAT_i + \sum_{j=1}^{N} IntermExpend_{i,j}}.$$

□

Remark B.1. *Operationalizing the ad-valorem taxes/subsidies in this way, its conceptual definition should be interpreted as an overall extent of wedges that promotes or demotes the purchase of input goods.*

B.3 Firm-Level Data: Compustat Data

The data source for firm-level data is the Compustat data provided by the Wharton Research Data Services (WRDS). This database provides detailed information about a firm's fundamentals, based on financial accounts. Though the coverage is limited to publicly traded firms, they tend to be much larger than private firms and thus account for the dominant part of the industry dynamics (Grullon et al. 2019).

For the analysis of my paper, I use the following items: Sales (SALES), Costs of Goods Sold (COGS), Selling, General & Administrative Expense (SGA) and Number of Employees (EMP).

I basically follow De Loecker et al. (2020) and De Loecker et al. (2021) in constructing the empirical counterparts of the variables of my model. That is, SALES corresponds to the firm's revenue, COGS to the firm's variable costs, and SGA to the firm's fixed costs. Although my model abstracts away from fixed entry costs, I need to apportion labor and material inputs between the variable and fixed costs to recover labor and material inputs. To this end, De Loecker et al. (2020) rely on a parametric assumption, while my framework does not impose any particular functional form restriction on the firm-level production. I instead use the direct measurement of the number of employees (EMP), and assume that the cost shares of labor and material are constant for both fixed and variable costs.

Assumption B.3 (Constant Cost Share)**.** *For each sector $i \in \mathbf{N}$ and each firm $k \in \mathbf{N}_i$, $VariableLaborCost_{ik} : VariableMaterialCost_{ik} = FixedLaborCost_{ik} : FixedMaterialCost_{ik} = \delta_{ik} : 1 - \delta_{ik}$, where $\delta_{ik} \in [0,1]$ is a constant specific to firm k.*

This assumption states that my empirical measurement of the variable costs $COGS_{ik}$ and fixed costs SGA_{ik} are made up of the same proportion of labor and material inputs.

B.3.1 Labor & Material Inputs

As in De Loecker et al. (2021), my construction starts from combining $COGS_{ik}$ and SGA_{ik} to compute the total costs. The firm k's total costs is given by:

$$\begin{aligned}
TotalCosts_{ik} &= TotalLaborCost_{ik} + TotalMaterialCost_{ik} \\
&= VariableLaborCost_{ik} + FixedLaborCost_{ik} + VariableMaterialCost_{ik} + FixedMaterialCost_{ik} \\
&= \underbrace{VariableLaborCost_{ik} + VariableMaterialCost_{ik}}_{COGS_{ik}} + \underbrace{FixedLaborCost_{ik} + FixedMaterialCost_{ik}}_{SGA_{ik}} \\
&= COGS_{ik} + SGA_{ik}. \qquad (42)
\end{aligned}$$

Since both $Cogs_{ik}$ and SGA_{ik} are observed in the data, I can compute the firm k's total expense ($TotalCost_{ik}$).

The total expenditure on labor input is

$$\begin{aligned}
TotalLaborCosts_{ik} &= VariableLaborCosts_{ik} + FixedLaborCosts_{ik} \\
&= W \times AverageHoursWorked \times \underbrace{Employees_{ik}}_{EMP_{ik}} \\
&= W \times \frac{TotalHours}{TotalEmployees} \times EMP_{ik}. \qquad (43)
\end{aligned}$$

From Fact B.1, the wage W is directly observed in data. I can also observe both $TotalHours$ and $TotalEmployees$ in the BEA data. Moreover the Compustat data provide information about the number of employees (EMP_{ik}). Hence I can calculate the firm k's total labor expense ($TotalLaborCosts_{ik}$). Then, the total expenditure on material input is obtained by

$$TotalMaterialCosts_{ik} = TotalCosts_{ik} - TotalLaborCosts_{ik}. \tag{44}$$

Next, I invoke Assumption B.3 to derive,

$$\therefore VariableLaborCost_{ik} = \frac{1-\delta_{ik}}{\delta_{ik}} VariableMaterialCost_{ik}, \tag{45}$$

and

$$\therefore FixedLaborCost_{ik} = \frac{1-\delta_{ik}}{\delta_{ik}} FixedMaterialCost_{ik}, \tag{46}$$

From (45) and (46), I have

$$\begin{aligned}
VariableMaterialCost_{ik} + FixedMaterialCost_{ik} &= TotalMaterialCost_{ik} \\
\therefore \frac{\delta_{ik}}{1-\delta_{ik}}(VariableLaborCost_{ik} + FixedLaborCost_{ik}) &= TotalMaterialCost_{ik} \\
\therefore \frac{\delta_{ik}}{1-\delta_{ik}} TotalLaborCost_{ik} &= TotalMaterialCost_{ik},
\end{aligned}$$

so that

$$\delta_{ik} = \frac{TotalMaterialCost_{ik}}{TotalLaborCost_{ik} + TotalMaterialCost_{ik}}, \tag{47}$$

where both $TotalLaborCost_{ik}$ and $TotalMaterialCost_{ik}$ can be calculated according to (43) and (44), respectively.

Once again by Assumption B.3,

$$VariableMaterialCost_{ik} = \frac{1-\delta_{ik}}{\delta_{ik}} VariableLaborCost_{ik},$$

so that I have

$$\begin{aligned}
VariableLaborCost_{ik} + VariableMaterialCost_{ik} &= COGS_{ik} \\
\therefore VariableLaborCost_{ik} &= \delta_{ik} COGS_{ik},
\end{aligned}$$

and

$$VariableMaterialCost_{ik} = (1-\delta_{ik}) COGS_{ik},$$

Since δ_{ik} is given by (47), I can recover $VariableLaborCost_{ik}$ (the empirical counterpart of $W^* \ell^*_{ik}$) and $VariableMaterialCost_{ik}$ (the empirical counterpart of $P_i^{M^*} m^*_{ik}$) from data. In view of Fact B.1, I can divide the former by the wage W^*, and the latter by the sectoral cost index $P_i^{M^*}$ to obtain the firm's labor ℓ^*_{ik} and material input m^*_{ik}. These are summarized in the following fact.

Fact B.4 (Labor & Material Inputs)**.** *Under Assumption B.3, the firm-level labor input ℓ^*_{ik} and material input m^*_{ik} are recovered from the data.*

B.3.2 Recovering Derived Demand for Sectoral Intermediate Goods

Since I lack separate data on the firm-level input demand for sectoral intermediate goods, I have to divid the firm's expenditure on material input in a way that is consistent with the configuration of the input-output linkage. To this end, I make additional assumptions on the form of aggregator function $\mathcal{G}_i$ in (13). Specifically, I assume that the material input m_{ik} aggregates sectoral intermediate goods according to the Cobb-Douglas production function:[95]

Assumption B.4. *The material input m_{ik} comprises sectoral intermediate goods according to the Cobb-Douglas production function:*

$$m_{ik} = \prod_{j=1}^{N} m_{ik,j}^{\gamma_{i,j}}, \tag{48}$$

where $m_{ik,j}$ is sector j's intermediate good demanded by firm k in sector i and $\gamma_{i,j}$ denotes the input share of sector j's intermediate good with $\sum_{j=1}^{N} \gamma_{i,j} = 1$.

Here it is implicitly assumed that the input share is the same within sector i. The producer price index for material input P_i^M is defined through the following cost minimization problem:

$$\begin{aligned} P_i^M :=& \min_{\{m^\circ_{ik,j}\}_{j=1}^N} \sum_{j=1}^{N} (1-\tau_i) P_j m^\circ_{ik,j} \\ & s.t. \quad \prod_{j=1}^{N} (m^\circ_{ik,j})^{\gamma_{i,j}} \geq 1. \end{aligned} \tag{49}$$

In recovering the input demand for sectoral intermediate goods, I make use of the following fact.

Under Assumption B.4, with the aid of the formulation (49), I can recover both the cost index of material input and the input demand for sectoral intermediate goods from the observables.

Fact B.5 (Identification of $\gamma_{i,j}$, P_i^M & $m_{ik,j}$)**.** *Suppose that Assumptions B.2 and B.4 holds. Then, i) for each sector $i = \{1, \ldots, N\}$, the input shares $\{\gamma_{i,j}\}_{j=1}^N$, and the cost index for material input P_i^M are identified from the observables; and ii) for each sector $i = \{1, \ldots, N\}$ and for each firm $k \in \mathbf{N}_i$, the input demand for composite intermediate goods $\{m_{ik,j}\}_{j=1}^N$ are identified from the observables.*

Proof. (i) From the first order conditions for the cost minimization, I have

$$(1-\tau_i) P_{j'} m_{ik,j'} = \frac{\gamma_{i,j'}}{\gamma_{i,j}} (1-\tau_i) P_j m_{ik,j},$$

[95] In principle, this assumption is necessitated in order to compensate the shortcoming of the dataset at hand. This assumption could be relaxed to the extent which allows us to recover the material input and demand for sectoral intermediate goods. Also this assumption could even be omitted if detailed data on firm-to-firm trade is available such as [reference...].

Substituting this into (37) leads to

$$\omega_{i,j} = \frac{\sum_{k=1}^{N_i}(1-\tau_i)P_j m_{ik,j}}{\frac{1}{\gamma_{i,j}}\sum_{k=1}^{N_i}(1-\tau_i)P_j m_{ik,j} + \sum_{k=1}^{N_i} W\ell_{ik}},$$

where I note $\sum_{j'=1}^{N}\gamma_{i,j'} = 1$ by assumption. Rearranging this, I arrive at

$$\begin{aligned}
\gamma_{i,j} &= \frac{\sum_{k=1}^{N_i}(1-\tau_i)P_j m_{ik,j}}{\frac{1}{\omega_{i,j}}\sum_{k=1}^{N_i}(1-\tau_i)P_j m_{ik,j} - \sum_{k=1}^{N_i} W\ell_{ik}} \\
&= \frac{\sum_{k=1}^{N_i}(1-\tau_i)P_j m_{ik,j}}{\sum_{j'=1}^{N}\sum_{k=1}^{N_i}(1-\tau_i)P_{j'} m_{ik,j'}} \\
&= \frac{\omega_{i,j}}{\sum_{j'=1}^{N}\omega_{i,j'}}.
\end{aligned}$$

Since terms in the right hand side $\{\omega_{i,j'}\}_{j'=1}^{N}$ are observed in the data (see Appendix B.2.1), the parameter $\gamma_{i,j}$ can thus be identified for all $i \in \mathbf{N}$.

From (49), the cost index for material input P_i^M is given by:

$$P_i^M = \prod_{j=1}^{N}\frac{1}{\gamma_{i,j}^{\gamma_{i,j}}}\{(1-\tau_i)P_j\}^{\gamma_{i,j}}. \tag{50}$$

Given that $\{\gamma_{i,j}\}_{j=1}^{N}$ are identified above, P_i^M is also identified.

(ii) Now, using again the first order condition for the cost minimization problem, I have

$$(1-\tau_i)P_j = \nu_{ik}\gamma_{i,j}\frac{m_{ik}}{m_{ik,j}},$$

where ν_{ik} is the marginal cost of constructing additional unit of material input (De Loecker and Warzynski 2012; De Loecker et al. 2016, 2020), which is P_i^M. Hence,

$$m_{ik,j} = \gamma_{i,j}\frac{P_i^M}{(1-\tau_i)P_j}m_{ik}, \tag{51}$$

from which $m_{ik,j}$, the input demand for sector j's composite intermediate good from sector i, is identified. This completes the poof. □

B.3.3 Treatment of Capital

Our theoretical framework is static and abstract away from capital accumulation over periods of time. In reality, however, capital plays a great important role in firm's production and input decisions. As a matter of fact, various information about capital is reported in my data source. To make my conceptual framework consistent to the empirical measurement, I impose the following assumption.

Assumption B.5 (Capital Endowment)**.** *For each sector $i \in \mathbf{N}$, i) each firm $k \in \mathbf{N}_i$ is endowed with capital stock before input decisions are made; and ii) capital stock enters the firm-level production function* *in a Hicks-neutral fashion.*

Assumption B.5 (i) states that firms do not choose but are given capital, and this capital endowment is independent of labor and material inputs. Note that the capital endowment can still be a function of the firm's productivity. Assumption B.5 (ii) means that the capital enters the production function in a multiplicative way. Under these two requirements, the firm's capital and productivity are nor discernible. This implies that the productivity in my model should be understood as a composite of these two components, or overall capability of production. For example, a "productive" firm in my model is so either because it has an efficient technology of production or because it is endowed with massive capital assets such as a large factory. Whichever the case is, capital endowment is treated as part of the unobservable firm-level productivity.

C Identification: Proofs of Theorems

In this section, theoretical results displayed in Section 5 are derived in a more general setup under a milder setting than the main text. Specifically, I allow for a sector-input-specific subsidy as in Liu (2019), and identify the firm-level quantity and price without imposing the HSA demand system (Assumption 3.4), followed by the identification of the residual inverse demand curve under Assumption 3.4. Accordingly, this section considers a policymaker who has control over sector-input-specific subsidies $\boldsymbol{\tau} \coloneqq \{\tau_{i,j}\}_{i,j=1}^{N}$ and wants to evaluate the effect of a particular subsidy $\tau_{n,n'}$ on the country's GDP.

To investigate the behavior of $Y_i(\boldsymbol{\tau})$ in response to a change in $\tau_{n,n'}$, I assume that it is totally differentiable in terms of $\tau_{n,n'}$.

Assumption C.1 (Total Differentiability)**.** *For each sector $i \in \mathbf{N}$, $Y_i(\boldsymbol{\tau})$ is totally differentiable with respect to $\tau_{n,n'}$.*

Under this assumption, taking total derivatives of (23) with respect to $\tau_{n,n'}$ yields

$$\left.\frac{dY_i(s)}{ds}\right|_{s=\tau_{n,n'}} = \sum_{k=1}^{N_i}\left(\underbrace{\frac{dp^*_{ik}}{d\tau_{n,n'}}q^*_{ik}}_{\text{price effects}} + \underbrace{p^*_{ik}\frac{dq^*_{ik}}{d\tau_{n,n'}}}_{\text{quantity effects}}\right) - \sum_{k=1}^{N_i}\sum_{j=1}^{N}\left(\underbrace{\frac{dP^*_j}{d\tau_{n,n'}}m^*_{ik,j}}_{\text{Ialth effects}} + \underbrace{P^*_j\frac{dm^*_{ik,j}}{d\tau_{n,n'}}}_{\text{switching effects}}\right). \tag{52}$$

Clearly, the object of interest is characterized by the eight variables appearing in the right hand side of (52): namely, p^*_{ik}, q^*_{ik}, $m^*_{ik,j}$, $\frac{dp^*_{ik}}{d\tau_{n,n'}}$, $\frac{dq^*_{ik}}{d\tau_{n,n'}}$, $\frac{dm^*_{ik,j}}{d\tau_{n,n'}}$, P^*_i, and $\frac{dP^*_i}{d\tau_{n,n'}}$. The goal of my analysis therefore boils down to identifying the values of these variables.

C.1 Recovering the Values of Firm-Level Quantity and Price

In this subsection, I derive the identification of the firm-level quantity and prices under a set of slightly milder conditions than described in the main text.

C.1.1 Identification of the Values of Markup

It can be shown that under the assumptions imposed in the main text (summarized below for the ease of exposition), I can immediately recover the firm-level markups from the observables.[96]

Assumption C.2 (Input Markets)**.** *(i) The input markets are perfectly competitive. (ii) All inputs are variable.*

This assumption is maintained in Section 3.3.

Fact C.1. *Suppose that Assumptions 3.5 and C.2 and hold. For each sector $i \in \mathbf{N}$ and each firm $k \in \mathbf{N}_i$, the value of the firm-level markup μ^*_{ik} can be recovered from the data.*

[96] See (Syverson 2019), De Loecker et al. (2020) and Kasahara and Sugita (2020) for discussion.

Proof. Observe that under Assumption C.2, the firm's markup μ_{ik} can be expressed as:

$$\begin{aligned}
\mu_{ik}^* &:= \frac{p_{ik}^*}{MC_{ik}^*} \\
&= \frac{p_{ik}^*}{AC_{ik}^*}\frac{AC_{ik}^*}{MC_{ik}^*} \\
&= \frac{p_{ik}^* q_{ik}^*}{AC_{ik}^* q_{ik}^*}\frac{AC_{ik}^*}{MC_{ik}^*} \\
&= \frac{Revenue_{ik}^*}{TC_{ik}^*}\frac{AC_{ik}^*}{MC_{ik}^*},
\end{aligned}$$

where MC_{ik}^*, AC_{ik}^*, and TC_{ik}^* represent the equilibrium values of the marginal, average, and total costs, respectively. Note here that $\frac{AC_{ik}^*}{MC_{ik}^*}$ is the elasticity of cost with respect to quantity (Syverson 2019), which in my case equals one due to Assumption 3.5 (i). Hence, I have

$$\mu_{ik}^* = \frac{Revenue_{ik}^*}{TC_{ik}^*},$$

i.e., the value of the firm's markup equals to the ratio of revenue to total costs, both of which are observed in the data. Thus, the value of the firm-level markup μ_{ik}^* is identified from the observables, as desired. $\square$

C.1.2 Identification of the Values of Quantity and Price

The following assumption is milder than Assumption **??** and encompasses the HSA demand system required in Assumption 3.4. Yet I maintain Assumption **??**. Let $\mathscr{R}_i$, $\mathscr{L}_i$ and $\mathscr{M}_i$ be the observed supports of revenue r_{ik}, labor input ℓ_{ik} and material input m_{ik}, respectively.

Assumption C.3 (Residual Inverse Demand Function). *For each sector $i \in \mathbf{N}$,*

(i) there exist some functions $H_{1,i}, H_{2,i} : \mathbb{R}_+^{N_i} \to \mathbb{R}$ such that for each firm $k \in \mathbf{N}_i$, there exists a function $\psi_i : \mathscr{S}_i \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}_+$ such that $p_{ik} = \psi_i(q_{ik}, H_{1,i}(\mathbf{q}_i), H_{2,i}(\mathbf{q}_i); \mathcal{I}_i)$;

(ii) there exist some functions $\mathcal{H}_{1,i}, \mathcal{H}_{2,i} : \mathbb{R}_i^{N_i} \to \mathbb{R}$ such that a) there exists a function $\chi_i : \mathbb{R}\times\mathbb{R}\times\mathbb{R} \to \mathscr{S}_i$ such that $q_{ik}^* = \chi_i(z_{ik}, \mathcal{H}_{1,i}(\mathbf{z}_i), \mathcal{H}_{2,i}(\mathbf{z}_i); \mathcal{I}_i)$ for all $k \in \mathbf{N}_i$; and b) there exists a function $\mathcal{M}_i : \mathscr{L}_i \times \mathscr{M}_i \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ such that $z_{ik} = \mathcal{M}_i(\ell_{ik}, m_{ik}, \mathcal{H}_{1,i}(\mathbf{z}_i), \mathcal{H}_{2,i}(\mathbf{z}_i); \mathcal{I}_i)$ for all $k \in \mathbf{N}_i$.

Assumption C.3 (i) and (ii), respectively, states that the other players' choices and productivities matter only through some transformations that are common across firms in the same sector: i.e., these jointly constitute sufficient statistics for competitors' quantity decisions and productivities. In particular, the assumption (i) embeds the HSA demand system described in Assumption 3.4 in that $H_{1,i}(\cdot)$ and $H_{2,i}(\cdot)$ corresponding to the quantity index $A_i(\cdot)$ in (11) but is not necessarily constrained by Assumption 3.4.[97] This assumption moreover includes the case of a homothetic demand system with direct implicit additivity (HDIA) and a homothetic demand system with indirect implicit demand system (HIIA), proposed in

[97]Either of $H_{1,i}(\cdot)$ and $H_{2,i}(\cdot)$ needs to be "shut down" adequately.

(Matsuyama and Ushchev 2017). Note here that Assumption C.3 (i) does not require homotheticity of the demand system.

Remark C.1. *In principle, Assumption C.3 can be extended to an arbitrary number of aggregator functions $H(\cdot)$ and $\mathcal{H}(\cdot)$ insofar as they are all common across firms in the same sector.*

To facilitate exposition, I introduce a tilde notation to denote the logarithm of each variable. For instance, I write the logarithms of firm's revenue, labor and material inputs, and productivity as $\tilde{r}_{ik}$, $\tilde{\ell}_{ik}$, $\tilde{m}_{ik}$ and $\tilde{z}_{ik}$, respectively. Also, the logarithms of firm's output quantity and price are expressed as:

$$\tilde{q}_{ik} := \ln q_{ik} = \tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}; \tilde{z}_{ik}), \tag{53}$$

and

$$\tilde{p}_{ik} := \ln p_{ik} = \tilde{\psi}_i(\tilde{q}_{ik}, \tilde{H}_{1,i}(\tilde{\mathbf{q}}_i), \tilde{H}_{2,i}(\tilde{\mathbf{q}}_i); \mathcal{I}_i), \tag{54}$$

where $\tilde{f}_i(\cdot) := (\ln \circ f_i \circ \exp)(\cdot)$, $\tilde{\psi}_{ik}(\cdot) := (\ln \circ \psi_{ik} \circ \exp)(\cdot)$, and $\tilde{H}_{1,i}(\cdot) := (\ln \circ H_i \circ \exp)(\cdot)$ with $\tilde{H}_{2,i}(\cdot)$ being analogously defined. Correspondingly, the observed supports for r_{ik}, ℓ_{ik} and m_{ik} are denoted by $\tilde{\mathscr{R}}_i$, $\tilde{\mathscr{L}}_i$ and $\tilde{\mathscr{M}}_i$, respectively. In what follows, I let the aggregator functions $H_{1,i}$, $H_{2,i}$ and the information set $\mathcal{I}_i$ be absorbed in the sector index i for the sake of brevity.

Let $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$, respectively, denote the equilibrium values of the first-order derivatives of the log-production function with respect to log-labor and log-material: i.e.,

$$\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} := \left. \frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \right|_{(\tilde{\ell}_{ik}, \tilde{m}_{ik}) = (\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik})},$$

and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$ is analogously defined.

It can easily be shown that $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$ are identified from the data.

Proposition C.1. *Suppose that Assumptions 3.5 and C.2 hold. Then, the equilibrium values of the derivative of the production function with respect to labor and material can be recovered from the observables.*

Proof. Under Assumptions 3.5 and C.2, the firm's input cost minimization problem is well-defined and has interior solutions only. For a given level of output $\tilde{q}^*_{ik}$, the Lagrange function associated to the firm's cost minimizing problem in terms of the logarithm variables reads:

$$\tilde{\mathcal{L}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}, \xi_{ik}) := \exp\{\tilde{W} + \tilde{\ell}_{ik}\} + \exp\{\tilde{P}^M_i + \tilde{m}_{ik}\} - \xi_{ik}\Big(\exp\{\tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}; \tilde{z}_{ik})\} - \exp\{\tilde{q}^*_{ik}\} \Big),$$

where ξ_{ik} represents the Lagrange multiplier indicating the marginal cost of producing an additional unit of output at the given level $\tilde{q}^*_{ik}$ (De Loecker and Warzynski 2012; De Loecker et al. 2016, 2020). The first order conditions at $\tilde{q}^*_{ik}$ are given by

$$[\tilde{\ell}_{ik}] : \exp\{\tilde{W} + \tilde{\ell}^*_{ik}\} - \xi_{ik}\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} \exp\{\tilde{f}_i(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik}; \tilde{z}_{ik})\} = 0 \tag{55}$$

$$[\tilde{m}_{ik}] : \exp\{\tilde{P}^M_i + \tilde{m}^*_{ik}\} - \xi_{ik}\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}} \exp\{\tilde{f}_i(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik}; \tilde{z}_{ik})\} = 0, \tag{56}$$

where $\tilde{\ell}^*_{ik}$ and $\tilde{m}^*_{ik}$, respectively, are labor and material inputs corresponding to the given q^*_{ik}. Taking the ratio between (55) and (56), I have

$$\frac{\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}}{\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}} = \frac{\exp\{\tilde{W} + \tilde{\ell}^*_{ik}\}}{\exp\{\tilde{P}^M_i + \tilde{m}^*_{ik}\}}. \tag{57}$$

Here, due to Assumption 3.5 (i),

$$\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} + \frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}} = 1,$$

so that (57) gives

$$\begin{aligned}
\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} &= \frac{\exp\{\tilde{W} + \tilde{\ell}^*_{ik}\}}{\exp\{\tilde{W} + \tilde{\ell}^*_{ik}\} + \exp\{\tilde{P}^M_i + \tilde{m}^*_{ik}\}} \\
\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}} &= \frac{\exp\{\tilde{P}^M_i + \tilde{m}^*_{ik}\}}{\exp\{\tilde{W} + \tilde{\ell}^*_{ik}\} + \exp\{\tilde{P}^M_i + \tilde{m}^*_{ik}\}}.
\end{aligned}$$

Since both $\exp\{\tilde{W} + \tilde{\ell}^*_{ik}\}$ and $\exp\{\tilde{P}^M_i + \tilde{m}^*_{ik}\}$ are available in the data, I thus can identify $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$ from the observables, as claimed. $\square$

Next, I closely follows Kasahara and Sugita (2020) in identifying the equilibrium values of firm-level output quantity and price and thus the notations are intentionally set closed to theirs.

To begin with, I admit a measurement error in the observed log-revenue:[98]

$$\begin{aligned}
\tilde{r}_{ik} &= \tilde{\psi}_i(\tilde{q}_{ik}) + \tilde{q}_{ik} + \tilde{\eta}_{ik} \\
&= \tilde{\varphi}_i(\tilde{q}_{ik}) + \tilde{\eta}_{ik} \\
&= \tilde{\varphi}_i(\tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}, \tilde{\mathcal{M}}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})) + \tilde{\eta}_{ik} \\
&= \tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}) + \tilde{\eta}_{ik},
\end{aligned}$$

where $\tilde{\varphi}_i(\tilde{q}_{ik}) \coloneqq \tilde{\psi}_i(\tilde{q}_{ik}) + \tilde{q}_{ik}$, and $\tilde{\phi}_i(\cdot)$ is the nonparametric component of the revenue function in terms of labor and material inputs satisfying $\tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}) = \tilde{\varphi}_i(\tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}, \tilde{\mathcal{M}}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}))$. The additive separability

[98] The measurement error is supposed to capture the variation in revenue that cannot be explained by firm-level input variables nor aggregate variables. This can be conceived as i) a shock to the firm's production that is unanticipated to the firm and hits after the firm's decision has been made, ii) the coding error in the measurement used by the econometrician to observe the revenue.

of the log measurement error $\tilde{\eta}_{ik}$ is chosen to conform to the bulk of the literature on identification and estimation of production functions.[99]

Towards identification, it is posited that the econometrician has knowledge about the following conditions.

Assumption C.4. *(i) Strict Exogeneity.* $E[\tilde{\eta}_{ik}|\tilde{\ell}_{ik}, \tilde{m}_{ik}] = 0$. *(ii) Continuous Differentiability.* $\phi_i(\cdot)$ *is at least first differentiable in each of its argument. (iii) Normalization. For each* $i \in \mathbf{N}$ *and each* $k \in \mathbf{N}_i$*, there exists a pair of labor and material inputs* $(\tilde{\ell}^{\circ}_{ik}, \tilde{m}^{\circ}_{ik}) \in \mathscr{L}_i \times \mathscr{M}_i$ *such that* $f_i(\tilde{\ell}^{\circ}_{ik}, \tilde{m}^{\circ}_{ik}; z_{ik}) = 0$.

Lemma C.1. *Suppose that Assumptions 3.5, C.2, and C.4 hold. Then, the logarithms of the firm-level output quantity* $\tilde{q}^*_{ik}$ *and price* $\tilde{p}^*_{ik}$ *can be identified from the observables.*

Proof.

Step 1:

The first step identifies the firm's revenue free of the measurement errors $\bar{\tilde{r}}_{ik}$ in terms of $(\tilde{\ell}_{ik}, \tilde{m}_{ik})$, eliminating the measurement error $\tilde{\eta}_{ik}$. From Assumption C.4, I can identify $\tilde{\phi}_i(\cdot)$, $\bar{\tilde{r}}_{ik}$ and $\tilde{\varepsilon}_{ik}$ according to

$$\begin{aligned}
&\tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}) = E[\tilde{r}_{ik}|\tilde{x}_{ik}];\\
&\bar{\tilde{r}}_{ik} = \tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}); \text{and}\\
&\tilde{\eta}_{ik} = \tilde{r}_{ik} - \bar{\tilde{r}}_{ik}.
\end{aligned}$$

Step 2:

Next, I aim to identify the derivative of the inverse of the revenue function $\tilde{\varphi}_i$. By definition, it is true that

$$\tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}, \tilde{\mathcal{M}}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})) = \tilde{\varphi}_i^{-1}(\bar{\tilde{r}}_{ik}), \tag{58}$$

where I know from the identification result above that $\bar{\tilde{r}}_{ik} = \ln K_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})$. Taking derivatives of (58) with respect to $\tilde{\ell}_{ik}$ and $\tilde{m}_{ik}$ derivers

$$\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{\ell}_{ik}} + \frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}(\cdot)_i}{\partial \tilde{\ell}_{ik}} = \frac{\partial \tilde{\varphi}_i^{-1}(\cdot)}{\partial \bar{\tilde{r}}_{ik}} \frac{\partial \tilde{\phi}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \tag{59}$$

$$\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{m}_{ik}} + \frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)}{\partial \tilde{m}_{ik}} = \frac{\partial \tilde{\varphi}_i^{-1}(\cdot)}{\partial \bar{\tilde{r}}_{ik}} \frac{\partial \tilde{\phi}_i(\cdot)}{\partial \tilde{m}_{ik}} \tag{60}$$

for all $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$. Here notice that $\frac{d\tilde{\varphi}_i^{-1}(\cdot)}{d\bar{\tilde{r}}_{ik}} = \left(\frac{d\tilde{\varphi}_i(\cdot)}{d\tilde{q}_{ik}}\right)^{-1}$, with the right hand side being the firm's markup (Kasahara and Sugita 2020). Owing to Fact C.1, the equilibrium firm's markup (in log)

[99] This specification is equivalent to assume that the error terms enter in a multiplicative way the system of structural equations in terms of the original variables. The additive separability of the measurement errors in terms of the logarithm variables are canonically employed in the literature (Olley and Pakes 1996; Levinsohn and Petrin 2003; Ackerberg et al. 2015; Gandhi et al. 2019).

$\tilde{\mu}_{ik}$ is obtained by

$$\tilde{\mu}_{ik} = \bar{\tilde{r}}_{ik} - \tilde{TC}_{ik}(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik}),$$

where $\tilde{TC}_{ik}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) := \ln[\exp\{\tilde{W} + \tilde{\ell}_{ik}\} + \exp\{\tilde{P}^M_i + \tilde{m}_{ik}\}]$.

Thus, $\frac{d\tilde{\varphi}_i^{-1}(\cdot)}{d\bar{\tilde{r}}_{ik}}$ is identified as

$$\frac{\partial \tilde{\varphi}_i^{-1}(\cdot)}{\partial \bar{\tilde{r}}_{ik}} = \tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}) - \ln[\exp\{\tilde{W} + \tilde{\ell}_{ik}\} + \exp\{\tilde{P}^M_i + \tilde{m}_{ik}\}].$$

Since the values of $\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{m}_{ik}}$ are identified in Proposition C.1, I can also identify $\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)}{\partial \tilde{m}_{ik}}$, respecitively, through (59) and (60):

$$\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)}{\partial \tilde{\ell}_{ik}} = \frac{\partial \tilde{\varphi}_i^{-1}(\cdot)}{\partial \bar{\tilde{r}}_{ik}} \frac{\partial \tilde{\phi}_i(\cdot)}{\partial \tilde{\ell}_{ik}} - \frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{\ell}_{ik}}, \tag{61}$$

and

$$\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)}{\partial \tilde{m}_{ik}} = \frac{\partial \tilde{\varphi}_i^{-1}(\cdot)}{\partial \bar{\tilde{r}}_{ik}} \frac{\partial \tilde{\phi}_i(\cdot)}{\partial \tilde{m}_{ik}} - \frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{m}_{ik}}. \tag{62}$$

Step 3: The final step recovers the realized value of firm-level output quantity by means of integration:

$$\begin{aligned}\tilde{q}^*_{ik} &= \tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}, \tilde{z}_{ik}) \\ &= \int_{\tilde{\ell}^\circ_{ik}}^{\tilde{\ell}_{ik}} \left(\frac{\partial \tilde{f}_i}{\partial \tilde{\ell}_{ik}} + \frac{\partial \tilde{f}_i}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i}{\partial \tilde{\ell}_{ik}}\right)(s, \tilde{m}_{ik}) ds + \int_{\tilde{m}^\circ_{ik}}^{\tilde{m}_{ik}} \left(\frac{\partial \tilde{f}_i}{\partial \tilde{m}_{ik}} + \frac{\partial \tilde{f}_i}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i}{\partial \tilde{m}_{ik}}\right)(\tilde{\ell}^\circ_{ik}, s) ds,\end{aligned}$$

where I note that the value of $\tilde{f}_i(\tilde{\ell}^\circ_{ik}, \tilde{m}^\circ_{ik}, \tilde{z}_{ik})$ is known to the econometrician in light of Assumption C.4 (iii).

Lastly, I can also recover the realized value of the firm-level output price $\tilde{p}^*_{ik}$ through:

$$\tilde{p}^*_{ik} = \bar{\tilde{r}}_{ik} - \tilde{q}^*_{ik}.$$

This completes the proof. □

Remark C.2. *(i) Lemma C.1 rests on the identifiability of the value of the firm-level markup* μ_{ik} *(Fact C.1).Kasahara and Sugita (2020) instead exploit the panel structure of their dataset to first identify the firm's productivity from the observables. my framework, on the contrary, is static in nature, which prohibits the use of panel data. In this light, the use of Fact C.1 can be considered a compromise between the data availability and the model assumptions. (ii) Notice that I are not concerned with identifying the firm's productivity per se, and thus the proof of Lemma C.1 does not invoke the feature of the Hicks-neutral productivity in the firm-level production function* (14)*: i.e., the lemma goes through the case of non-Hicks-neutral productivity as studied Demirer (2022) and Pan (2022). Under Hicks-neutrality, it holds* $\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{z}_{ik}} = 1$.

Having Lemma C.1 established, the firm-level quantity and price can immediately be recovered by reverting (53) and (54).

Proposition C.2. *Suppose that the assumptions required in Lemma C.1 hold. Then the equilibrium values of the firm-level quantity q^*_{ik} and price p^*_{ik} are identified from the observables.*

C.2 Recovering Demand Function (Sectoral Aggregator)

I consider recovering the inverse demand function To begin with, each sectoral aggregator transforms firm-level products into a single sectoral good through based on the cost minimization problem. This defines the following unit cost condition: for each $i = 1, \dots, N$,

$$\begin{aligned} P_i \coloneqq \min_{\{e^\circ_{ik}\}_{i=1}^{N}} & \sum_{k=1}^{N_i} p_{ik} e^\circ_{ik} \\ s.t. \quad & F_i(\{e^\circ_{ik}\}_{k=1}^{N_i}) \geq 1, \end{aligned} \tag{63}$$

where p_{ik} is the price of a product set by firm k in sector i.

By solving this, it follows that there exists a mapping $\mathcal{P}_i : \mathscr{S}_i^{N_i} \to \mathbb{R}_+$ such that

$$P_i = \mathcal{P}_i(\mathbf{q}_i; \mathcal{I}_i). \tag{64}$$

C.2.1 HSA Demand System

With my notation, the HSA demand system in Assumption **??** can be expressed as follow.

First, by definition

$$\Phi_i \coloneqq \sum_{k=1}^{N_i} p^*_{ik} q^*_{ik},$$

where p^*_{ik} and q^*_{ik} are the equilibrium (realized) values of firm-level price and quantity. Then I can take

$$\Phi_i = \sum_{k=1}^{N_i} \varphi_i(q^*_{ik}), \tag{65}$$

where $r_{ik} = \varphi_i(q_{ik})$ with $\varphi_i(\cdot) \coloneqq (\exp \circ \tilde{\varphi}_i \circ \ln)(\cdot)$.

Next, the residual inverse demand function faced by firm k in sector i takes the form of

$$p_{ik} = \frac{\Phi_i}{q_{ik}} \Psi_i\left(\frac{q_{ik}}{A_i(\mathbf{q}_i)}\right), \tag{66}$$

where

$$\Psi_i(q_{ik}) = \frac{\varphi_i(q_{ik})}{\Phi_i}, \tag{67}$$

with

$$\sum_{k=1}^{N_i} \Psi_i \left(\frac{q_{ik}}{A_i(\mathbf{q}_i)} \right) = 1. \tag{68}$$

C.2.2 Proof

I first identify the quantity index $A_i(\cdot)$ over the entire support $\mathscr{S}_i^{N_i}$. This is shown in Kasahara and Sugita (2020).

Lemma C.2 (Identification of A_i; Kasahara and Sugita (2020))**.** *Suppose that the same assumptions in Lemma C.1 are satisfied. Assume moreover that Assumption 3.4 holds with* (65) – (68)*. Then, the quantity index $A_i(\mathbf{q}_i)$ is identified.*

Under Lemma C.2, the quantity index $A_i(\cdot)$ is nonparametrically identified as a function of $\mathbf{q}_i$, so that its derivatives can also be nonparametrically identified.

Corollary C.1 (Identification of $\frac{\partial A_i(\cdot)}{\partial q_{ik}}$ and $\frac{\partial^2 A_i(\cdot)}{\partial q_{ik} q_{ik'}}$)**.** *Suppose that the same assumptions required in Lemma C.2 hold. Then, for each $i \in \mathbf{N}$, i) $\frac{\partial A_i(\cdot)}{\partial q_{ik'}}$ and ii) $\frac{\partial^2 A_i(\cdot)}{\partial q_{ik} q_{ik'}}$ are identified for all $k, k' \in \mathbf{N}_i$.*

The identified quantity index $A_i(\cdot)$ can be combined once again with (65) – (68) to recover the residual inverse demand functions faced by firms under Assumption 3.4.

Proposition C.3. *Suppose that the same assumptions required in Lemma C.2 hold. Then, the residual inverse demand functions $\psi_i(\cdot)$ can be identified from the observables.*

For each sector $i \in \mathbf{N}$ and for each firm $k \in \mathbf{N}_i$, let $mr_{ik} : \mathscr{S}_i \times \mathscr{S}_i^{N_i - 1} \to \mathbb{R}$ be the marginal revenue function; that is, $mr_{ik}(q_{ik}, \mathbf{q}_{i,-k}; \mathcal{I}_i) \coloneqq \frac{\partial \psi_i(\cdot)}{\partial q_{ik}} q_{ik} + p_{ik}$. Given Lemma C.2, it is immediate to show that for each $k \in \mathbf{N}_i$, $mr_{ik}(\cdot)$ and its partial derivatives $\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}}$ for each $k' \in \mathbf{N}_i$ is identified.

Lemma C.3 (Identification of Marginal Revenue Function)**.** *Suppose that the assumptions required in Lemma C.2 are satisfied. Then, i) the firm-level marginal revenue function $mr_{ik}(\cdot)$ and ii) its partial derivatives $\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}}$ for each $k' \in \mathbf{N}_i$ are identified.*

I can further recover the sectoral aggregator $F_i(\cdot)$, the partial derivatives of $F_i(\cdot)$ with respect to q_{ik} (denoted by $\frac{\partial F_i(\cdot)}{\partial q_{ik}}$) and the partial derivatives of $\mathcal{P}_i(\cdot)$ with respect to q_{ik} (denoted by $\frac{\mathcal{P}_i(\cdot)}{\partial q_{ik}}$) for all $k \in \mathbf{N}_i$ are identified under an additional normalization condition.

Assumption C.5 (Normalization of HSA Demand System)**.** *There exists a collection of constants $\{c_{ik}\}_{k=1}^{N_i}$ such that $F_i(\{c_{ik}\}_{k=1}^{N_i}) = 1$.*

Lemma C.4 (Identification of Sectoral Aggregators)**.** *Suppose that the assumptions required in Lemma C.2 are satisfied. Assume moreover that Assumption C.5 holds. Then, i) the sectoral aggregator $F_i(\cdot)$, and ii) the derivatives $\frac{\partial F_i(\cdot)}{\partial q_{ik}}$ and $\frac{\mathcal{P}_i(\cdot)}{\partial q_{ik}}$ for each $k' \in \mathbf{N}_i$, are identified as a function of $\mathbf{q}_i$. In particular, evaluated at the realized values, it holds that $\frac{\partial F_i(\cdot)^*}{\partial q_{ik}} = \frac{p_{ik}^*}{P_i^*}$ and $\frac{\mathcal{P}_i(\cdot)^*}{\partial q_{ik}} = -\frac{p_{ik}^*}{Q_i^*}$.*

Proof. i) By Proposition 1 (i) and Remark 3 (self-duality) of Matsuyama and Ushchev (2017), there exists a unique monotone, convex, continuous and homothetic rational preference over the support of q associated to the HSA inverse demand system (66) – (68). Clearly, this preference corresponds to the sectoral aggregator F_i. Moreover, a variant of Proposition 1 (ii) of Matsuyama and Ushchev (2017) implies that Q_i can be expressed as[100]

$$\ln F_i(\mathbf{q}_i) = \ln A_i(\mathbf{q}_i) + \sum_{k=1}^{N_i} \int_{c_{ik}}^{q_{ik}/A_i(\mathbf{q}_i)} \frac{\Psi_i(\zeta)}{\zeta} d\zeta, \tag{69}$$

where $\{c_{ik}\}_{k=1}^{N_i}$ satisfy Assumption C.5.

Since, by Lemma C.2, $A_i(\cdot)$ is identified, it remains to prove that for each $k \in \mathbf{N}$, $\frac{\Psi_i(\zeta)}{\zeta}$ is identified for all $\zeta \in [c_{ik}, \frac{q_{ik}}{A_i(\mathbf{q}_i)}]$.

Observe that φ_i in (67) is obtained by taking the continuous transformation and inverse of $\tilde{\varphi}_i^{-1}$, which is identified in the proof of Lemma C.1. Moreover, notice that for the realized values $\{q_{ik}^*\}_{k=1}^{N_i}$, I can recover Φ_i using (65): i.e.,

$$\Phi_i = \sum_{k=1}^{N_i} \varphi_i(q_{ik}^*),$$

where I emphasize that Φ_i is a constant that firms take as given. Then the identification of $\frac{\Psi_i(\zeta)}{\zeta}$, for $\zeta \in [c_{ik}, \frac{q_{ik}}{A_i(\mathbf{q}_i)}]$, comes directly from its construction (67).

Hence, I can identify $F_i(\cdot)$ as a function of $\mathbf{q}_i$.

ii) Taking partial derivatives of (69) with respect to q_{ik}: for all $\mathbf{q}_i \in \mathscr{S}_i^{N_i}$,

$$\frac{\frac{\partial F_i(\cdot)}{\partial q_{ik}}}{F_i(\mathbf{q}_i)} = \frac{\frac{\partial A_i(\cdot)}{\partial q_{ik}}}{A_i(\mathbf{q}_i)} + \frac{1}{q_{ik}} \Psi_i\Big(\frac{q_{ik}}{A_i}\Big) - \bigg(\sum_{k'=1}^{N_i} \Psi_i\Big(\frac{q_{ik'}}{A_i}\Big)\bigg) \frac{1}{A_i(\mathbf{q}_i)} \frac{\partial A_i(\cdot)}{\partial q_{ik}},$$

so that by construction

$$\begin{aligned}
\frac{\partial F_i(\cdot)}{\partial q_{ik}} &= F_i(\mathbf{q}_i) \bigg\{ \bigg(1 - \sum_{k'=1}^{N_i} \Psi_i\Big(\frac{q_{ik'}}{A_i}\Big)\bigg)\bigg\} \frac{1}{A_i} \frac{\partial A_i}{\partial q_{ik}} + \frac{1}{q_{ik}} \Psi_i\Big(\frac{q_{ik}}{A_i(\mathbf{q}_i)}\Big) \\
&= F_i(\mathbf{q}_i) \frac{1}{q_{ik}} \Psi_i\Big(\frac{q_{ik}}{A_i(\mathbf{q}_i)}\Big) \\
&= F_i(\mathbf{q}_i) \frac{1}{q_{ik}} \frac{\varphi(\frac{q_{ik}}{A_i(\mathbf{q}_i)})}{\Phi_i} \\
&= \frac{F_i(\mathbf{q}_i)}{\Phi_i} \frac{1}{q_{ik}} \varphi\Big(\frac{q_{ik}}{A_i(\mathbf{q}_i)}\Big),
\end{aligned}$$

where the second equality follows from (68), and the last equation is a consequence of (65).

[100] See also Kasahara and Sugita (2020).

Moreover, it hods by (65) that

$$\mathcal{P}_i(\mathbf{q}_i)F_i(\mathbf{q}_i) = \Phi_i.$$

Then, taking the partial derivatives of the both hand sides with respect to q_{ik}, I obtain

$$\begin{aligned}
&\frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik}} F_i(\mathbf{q}_i) + \mathcal{P}_i(\mathbf{q}_i)\frac{\partial \mathcal{F}_i(\cdot)}{\partial q_{ik}} = 0 \\
&\therefore \frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik}} = -\frac{P_i}{Q_i}\frac{\partial \mathcal{F}_i(\cdot)}{\partial q_{ik}}.
\end{aligned}$$

This identifies $\frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik}}$ as a function of $\mathbf{q}_i$.

iii) For the realized values $\mathbf{q}_i^*$, if follows from (i) and (ii) of this lemma that

$$\begin{aligned}
\frac{\partial F_i(\cdot)^*}{\partial q_{ik}} &= \frac{F_i(\mathbf{q}_i^*)}{\Phi_i}\frac{1}{q_{ik}^*}\varphi\Big(\frac{q_{ik}^*}{A_i(\mathbf{q}_i^*)}\Big) \\
&= \frac{Q_i^*}{P_i^* Q_i^*}\frac{1}{q_{ik}^*} r_{ik}^* \\
&= \frac{p_{ik}^*}{P_i^*},
\end{aligned}$$

and, thus

$$\begin{aligned}
\frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik}} &= -\frac{P_i^*}{Q_i^*}\frac{p_{ik}^*}{P_i^*} \\
&= -\frac{p_{ik}^*}{Q_i^*}.
\end{aligned}$$

This completes the proof. □

C.3 Recovering Comparative Statics

This section explores the identification of the comparative statics. I first identify the comparative statics up to the total derivative of wage using the profit-maximization and cost-minimization problems. Then I invoke the labor market clearing condition (22) to identify the policy impact on wage, leading to the full identification of those comparative statics that have been identified up to the change in wage in the previous stage (Appendices C.3.1 – C.3.2), which in turn is followed by the identification of changes in input demand for sectoral intermediate goods (Appendix C.3.3).

C.3.1 Profit Maximization

In each sector $i \in \mathbf{N}$, for the equilibrium wage W^*, the material price index P_i^{M*} and for each firm's optimal quantity q_{ik}^*, there exist a pair of labor and material inputs that satisfies the following one-step profit maximization problem:

$$
\begin{aligned}
(\bar{\ell}_{ik}^*, \bar{m}_{ik}^*) \quad &\in \quad \underset{\ell_{ik}, m_{ik}}{\arg\max} \left\{ p_{ik}^* q_{ik}^* - (W^* \ell_{ik} + P_i^{M*} m_{ik}) \right\} \\
& \qquad s.t. \quad q_{ik}^* = f_i(\ell_{ik}, m_{ik}; z_{ik}).
\end{aligned}
$$

The first order conditions with respect to labor and material inputs are given, respectively, by:

$$
[\ell_{ik}] : mr_{ik}(\cdot)^* \left. \frac{\partial f_i(\cdot)}{\partial \ell_{ik}} \right|_{(\ell_{ik}, m_{ik}) = (\bar{\ell}_{ik}^*, \bar{m}_{ik}^*)} = W^* \tag{70}
$$

$$
[m_{ik}] : mr_{ik}(\cdot)^* \left. \frac{\partial f_i(\cdot)}{\partial m_{ik}} \right|_{(\ell_{ik}, m_{ik}) = (\bar{\ell}_{ik}^*, \bar{m}_{ik}^*)} = P_i^{M*}, \tag{71}
$$

where $mr_{ik}(\mathbf{q}_i)$ is the firm k's marginal revenue function, and I denote $mr_{ik}(\cdot)^* \coloneqq mr_{ik}(\mathbf{q}_i^*)$.

Taking total derivatives of the both hand sides of (70) and (71) in terms of τ_n yields, respectively,

$$
\left(\sum_{k'=1}^{N_i} \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq_{ik'}^*}{d\tau_{n,n'}} \right) \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} + mr_{ik}^*(\cdot) \left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} \frac{d\bar{\ell}_{ik}^*}{d\tau_{n,n'}} + \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \frac{d\bar{m}_{ik}^*}{d\tau_{n,n'}} \right) = \frac{dW^*}{d\tau_{n,n'}} \tag{72}
$$

$$
\left(\sum_{k'=1}^{N_i} \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq_{ik'}^*}{d\tau_{n,n'}} \right) \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} + mr_{ik}(\cdot)^* \left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} m_{ik}} \frac{d\bar{\ell}_{ik}^*}{d\tau_{n,n'}} + \frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2} \frac{d\bar{m}_{ik}^*}{d\tau_{n,n'}} \right) = \frac{dP_i^{M*}}{d\tau_{n,n'}}, \tag{73}
$$

where

$$
\frac{dq_{ik}^*}{d\tau_{n,n'}} = \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \frac{d\bar{\ell}_{ik}^*}{d\tau_{n,n'}} + \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \frac{d\bar{m}_{ik}^*}{d\tau_{n,n'}}.
$$

Here, remember that firms only choose their output quantities through the profit maximization, while input decisions are made in a way that minimizes total cots. Thus the "optimal" labor $\bar{\ell}_{ik}^*$ and material inputs $\bar{m}_{ik}^*$ chosen above are not necessarily the same ones as actually chosen by the firm. Rather, $\bar{\ell}_{ik}^*$ and material inputs $\bar{m}_{ik}^*$ should be understood as a combination of inputs that only pins down the change in the firm's output quantity, whose corresponding production possibility frontier is in turn used to determine the optimal input choices in the subsequent cost minimization problem (see Section **??**).

From (72) and (73), it follows that, in equilibrium,

$$
\begin{aligned}
&\left(\sum_{k'=1}^{N_i} \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq_{ik'}^*}{d\tau_{n,n'}} \right) \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \bar{\ell}_{ik}^* + mr_{ik}(\cdot)^* \left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} \bar{\ell}_{ik}^* \frac{d\bar{\ell}_{ik}^*}{d\tau_{n,n'}} + \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \bar{\ell}_{ik}^* \frac{d\bar{m}_{ik}^*}{d\tau_{n,n'}} \right) \\
&+ \left(\sum_{k'=1}^{N_i} \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq_{ik'}^*}{d\tau_{n,n'}} \right) \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \bar{m}_{ik}^* + mr_{ik}(\cdot)^* \left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} m_{ik}} \bar{m}_{ik}^* \frac{d\bar{\ell}_{ik}^*}{d\tau_{n,n'}} + \frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2} \bar{m}_{ik}^* \frac{d\bar{m}_{ik}^*}{d\tau_{n,n'}} \right) \\
&= \frac{dW^*}{d\tau_{n,n'}} \bar{\ell}_{ik}^* + \frac{dP_i^{M*}}{d\tau_{n,n'}} \bar{m}_{ik}^*
\end{aligned}
$$

$$
\begin{aligned}
\therefore &\left(\sum_{k'=1}^{N_i}\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}}\frac{dq_{ik'}^*}{d\tau_{n,n'}}\right)\left(\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\bar{\ell}_{ik}^*+\frac{\partial f_i(\cdot)^*}{\partial m_{ik}}\bar{m}_{ik}^*\right)\\
&+mr_{ik}(\cdot)^*\left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2}\bar{\ell}_{ik}^*+\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\bar{m}_{ik}^*\right)\frac{d\bar{\ell}_{ik}^*}{d\tau_{n,n'}}+mr_{ik}(\cdot)^*\left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\bar{\ell}_{ik}^*+\frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2}\bar{m}_{ik}^*\right)\frac{d\bar{m}_{ik}^*}{d\tau_{n,n'}}\\
&=\frac{dW^*}{d\tau_{n,n'}}\bar{\ell}_{ik}^*+\frac{dP_i^{M*}}{d\tau_{n,n'}}\bar{m}_{ik}^*\\
\therefore\ & q_{ik}^*\sum_{k'=1}^{N_i}\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}}\frac{dq_{ik'}^*}{d\tau_{n,n'}}=\frac{dW^*}{d\tau_{n,n'}}\bar{\ell}_{ik}^*+\frac{dP_i^{M*}}{d\tau_{n,n'}}\bar{m}_{ik}^*\\
\therefore\ & \sum_{k'=1}^{N_i}\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}}\frac{dq_{ik'}^*}{d\tau_{n,n'}}=\frac{1}{q_{ik}^*}\left(\frac{dW^*}{d\tau_{n,n'}}\bar{\ell}_{ik}^*+\frac{dP_i^{M*}}{d\tau_{n,n'}}\bar{m}_{ik}^*\right), \qquad (74)
\end{aligned}
$$

where the third implication is a consequence of Assumption 3.5 (i). The expression (74) holds for each firm in the same sector, thereby constituting a system of N_i equations in N_i unknowns (i.e., total derivatives of the optimal quantities with respect to subsidy):

$$
\underbrace{\begin{bmatrix}
\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{iN_i}}\\
\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{iN_i}}\\
\vdots & \vdots & \ddots & \vdots\\
\frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}}
\end{bmatrix}}_{=:\Lambda_{i,1}}
\begin{bmatrix}
\frac{dq_{i1}^*}{d\tau_{n,n'}}\\
\frac{dq_{i2}^*}{d\tau_{n,n'}}\\
\vdots\\
\frac{dq_{iN_i}^*}{d\tau_{n,n'}}
\end{bmatrix}
=
\underbrace{\begin{bmatrix}
\frac{\bar{\ell}_{i1}^*}{q_{i1}^*} & \frac{\bar{m}_{i1}^*}{q_{i1}^*}\\
\frac{\bar{\ell}_{i2}^*}{q_{i2}^*} & \frac{\bar{m}_{i2}^*}{q_{i2}^*}\\
\vdots & \vdots\\
\frac{\bar{\ell}_{iN_i}^*}{q_{iN_i}^*} & \frac{\bar{m}_{iN_i}^*}{q_{iN_i}^*}
\end{bmatrix}}_{=:\Lambda_{i,2}}
\begin{bmatrix}
\frac{dW^*}{d\tau_{n,n'}}\\
\frac{dP_i^{M*}}{d\tau_{n,n'}}
\end{bmatrix}. \qquad (75)
$$

In order to ensure that this system can be solved for the total derivatives of quantity with respect to subsidy, I impose an assumption that the premultiplying term of the left hand side is invertible.

Assumption C.6. *For each sector $i \in \mathbf{N}$, the matrix*

$$
\Lambda_{i,1} \coloneqq \begin{bmatrix}
\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{iN_i}}\\
\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{iN_i}}\\
\vdots & \vdots & \ddots & \vdots\\
\frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}}
\end{bmatrix}
$$

is nonsingular.

Assumption C.6 requires that the column vectors of $\Lambda_{i,1}$ are linearly independent. This assumption trivially holds in monopolistic competition as $\Lambda_{i,1}$ simplifies to a diagonal matrix. The economic content of this assumption in the case of oligopolistic competitions directly pertains to firms' strategic complementarities.

Example C.1 (Duopoly)**.** *For simplicity, consider a case of duopoly, wherein firm 1 and 2 are engaged in a competition over quantity. It generally holds that $|\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}}| \geq |\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}}|$. But, it is also true that $|\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}}| \leq |\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}}|$. Hence there is no such a constant that makes the column vectors $\Lambda_{i,1}$*

linearly dependent. In this sense, Assumption C.6 excludes a situation where the firm's own strategic complementarity is exactly the same as the competitor's.

Under Assumption C.6, the system of equations (75) can be solved for $\{\frac{dq^*_{ik}}{d\tau_{n,n'}}\}_{k=1}^{N_i}$:

$$\begin{bmatrix} \frac{dq^*_{i1}}{d\tau_{n,n'}} \\ \frac{dq^*_{i2}}{d\tau_{n,n'}} \\ \vdots \\ \frac{dq^*_{iN_i}}{d\tau_{n,n'}} \end{bmatrix} = \Lambda_{i,1}^{-1}\Lambda_{i,2} \begin{bmatrix} \frac{dW^*}{d\tau_n} \\ \frac{dP_i^{M*}}{d\tau_{n,n'}} \end{bmatrix}.$$

In this expression, $\Lambda_{i,1}^{-1}$ captures the strategic interactions between firms through changes in marginal revenues. Moreover, it can also be seen, from this expression, that $\{\frac{dq^*_{ik}}{d\tau_{n,n'}}\}_{k=1}^{N_i}$ depends on the levels of firm's current production $\Lambda_{i,2}$ as well as the responsiveness of the wage and material cost index.

Fact C.2. *Suppose that Proposition C.2 and Lemma C.3 hold. Then, for each sector $i \in \mathbf{N}$, the matrix $\Lambda_{i,1}^{-1}\Lambda_{i,2}$ in* (76) *is identified.*

Proof. First, $\{q^*_{ik}\}_{k=1}^{N_i}$ are identified by Proposition C.2. Next, it follows from Lemma C.3 that $\{\frac{\partial mr_{ik}}{\partial q^*_{ik'}}\}_{k,k'}$ are identified. Hence, the matrix $\Lambda_{i,1}^{-1}\Lambda_{i,2}$ in (76) is identified, as desired. ☐

Letting $\lambda^{-1}_{ik,k'}$ be the (k,k') entry of the matrix $\Lambda_{i,1}^{-1}$, I can write

$$\begin{aligned} \frac{dq^*_{ik}}{d\tau_{n,n'}} &= \left(\sum_{k'=1}^{N_i} \lambda^{-1}_{ik,k'} \frac{\bar{\ell}^*_{ik'}}{q^*_{ik'}}\right) \frac{dW^*}{d\tau_{n,n'}} + \left(\sum_{k'=1}^{N_i} \lambda^{-1}_{ik,k'} \frac{\bar{m}^*_{ik'}}{q^*_{ik'}}\right) \frac{dP_i^{M*}}{d\tau_{n,n'}} \\ &= \bar{\lambda}^L_{ik} \frac{dW^*}{d\tau_{n,n'}} + \bar{\lambda}^M_{ik} \frac{dP_i^{M*}}{d\tau_{n,n'}}, \end{aligned} \tag{76}$$

where $\bar{\lambda}^L_{ik} := \sum_{k'=1}^{N_i} \lambda^{-1}_{ik,k'} \frac{\bar{\ell}^*_{ik'}}{q^*_{ik'}}$ and $\bar{\lambda}^M_{ik} := \sum_{k'=1}^{N_i} \lambda^{-1}_{ik,k'} \frac{\bar{m}^*_{ik'}}{q^*_{ik'}}$ correspond to the kth element of the first and second column of the matrix $\Lambda_{i,1}^{-1}\Lambda_{i,2}$, respectively.

Note that $\bar{\lambda}^L_{ik}$ and $\bar{\lambda}^M_{ik}$, respectively, can be understood as a measure of the sensitivity (elasticity) of the sector's overall strategic complementarity to a change in firm k's output quantity, with the weight assigned to the ratio between output and input quantities.[101] These measures capture the extent of influence that each firm exerts in strategic interactions. Intuitively, (76) states that the policy shocks coming through the changes in the labor wage and material input cost affect the firm's quantity adjustment decision in proportion to the "market share" encoded in the weighted elasticities $\bar{\lambda}^L_{ik}$ and $\bar{\lambda}^M_{ik}$ of the sectoral strategic complementarity. I call these measures the *indices of firm's contribution to sectoral strategic*

[101] Observe that for a square matrix $\mathcal{O}$, the inverse matrix $\mathcal{O}^{-1}$ is given by $\mathcal{O}^{-1} = \frac{\mathrm{adj}(\mathcal{O})}{|\mathcal{O}|}$, where $\mathrm{adj}(\mathcal{O})$ is the adjoint matrix of $\mathcal{O}$, i.e., the transpose of the cofactor matrix. The cofactor matrix C of $\mathcal{O}$ is defined as $C := [c_{a,b}]_{a,b}$, where $c_{a,b} := (-1)^{a+b}|M_{a,b}|$, with $M_{a,b}$ representing the minor matrix of $\mathcal{O}$ that can be created by eliminating the a-th row and b-th column from the matrix $\mathcal{O}$. In my context, the k'-th column of the cofactor matrix of $\Lambda_{i,1}$ excludes $\{\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}}\}_{k=1}^{N_i}$, all of which are in turn ruled out from the k'-th row of the adjoint matrix. Since the determinant involves the effect of all firms' quantity changes, the weighted sum along each row of $\Lambda_{i,1}^{-1}$ reflects the contribution of the changes in firm k''s output quantity.

complementarity. These indices tell us the extent to which the market competition is affected by the change in firm k's quantity,[102] and are similar in spirit to the index of competitor price changes of Amiti et al. (2019). While their index compares the firm's contribution to the rest of the market, my indices $\bar{\lambda}^L_{ik}$ and $\bar{\lambda}^M_{ik}$ compares the rest of the market to the entire market, backing out the firm's share. This observation is best illustrated in the example of duopoly (see Example **??**), and becomes acute in the case of monopolistic competitions.

Example C.2 (Monopolistic Competition)**.** *I consider the same setup as Example* **??***, but depart by assuming that both firms are monopolistic. In this case,*

$$\Lambda_{i,1}^{-1} = \begin{bmatrix} (\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}})^{-1} & 0 \\ 0 & (\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}})^{-1} \end{bmatrix}.$$

Then two measures of the firm 1's contribution to the overall sectoral strategic complementarity are given by $\bar{\lambda}^L_{i1} = (\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}})^{-1}\frac{\ell^*_{i1}}{q^*_{i1}}$ and $\bar{\lambda}^M_{i1} = (\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}})^{-1}\frac{m^*_{i1}}{q^*_{i1}}$, both of which are typically negative.[103] *Provided that both $\bar{\lambda}^L_{i1}$ and $\bar{\lambda}^M_{i1}$ are negative,* (76) *implies that when the wage and material cost index become higher in reaction to a policy change, firm 1 decreases its output quantity. An analogous argument applies to firm 2. When the firms are oligopolistic as in Example* **??***, the signs of $\bar{\lambda}^L_{i1}$ and $\bar{\lambda}^M_{i1}$ are ambiguous because they are determined in relation to the strategic complementarities.*

Totally differentiating (64) yields

$$\frac{dP^*_i}{d\tau_{n,n'}} = \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} \frac{dq^*_{ik'}}{d\tau_{n,n'}}. \tag{77}$$

Upon substituting (76) into (77), I can write

$$\begin{aligned} \frac{dP^*_i}{d\tau_{n,n'}} &= \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} \left(\bar{\lambda}^L_{ik'} \frac{dW^*}{d\tau_{n,n'}} + \bar{\lambda}^M_{ik'} \frac{dP^{M*}_i}{d\tau_{n,n'}} \right) \\ &= \left(\sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} \bar{\lambda}^L_{ik'} \right) \frac{dW^*}{d\tau_{n,n'}} + \left(\sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} \bar{\lambda}^M_{ik'} \right) \frac{dP^{M*}_i}{d\tau_{n,n'}} \\ &= \bar{\lambda}^L_{i\cdot} \frac{dW^*}{d\tau_{n,n'}} + \bar{\lambda}^M_{i\cdot} \frac{dP^{M*}_i}{d\tau_{n,n'}}, \end{aligned} \tag{78}$$

where $\bar{\lambda}^L_{i\cdot} := \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} \bar{\lambda}^L_{ik'}$ and $\bar{\lambda}^M_{i\cdot} := \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} \bar{\lambda}^M_{ik'}$. These are a weighted sum of the elasticities of sectoral price index with respect to firms' quantities, with the weight assigned to a firm's index of strategic complementarity in that sector. From the expression (78), $\bar{\lambda}^L_{i\cdot}$ and $\bar{\lambda}^M_{i\cdot}$ can be interpreted as representing a pass-through of a change in the wage and material input cost to the sectoral price index, respectively.

[102] That these indices are negative means the presence of the firm drugs the sectoral strategic complementarity in the direction of strategic substitutability, and vice verse.

[103] Precisely, the sign depends on the demand side parameters. For instance, when the sectoral aggregator takes the form of a CES production function as in Example 3.1, these indices are negative as long as $\sigma_i > 2$.

Example C.3 (Monopolistic Competition)**.** *Continuing Example C.2 and assuming that* $\bar{\lambda}^L_{i1}$*,* $\bar{\lambda}^L_{i2}$*,* $\bar{\lambda}^M_{i1}$ *and* $\bar{\lambda}^M_{i2}$ *have all turned out to be negative, I can proceed to calculate* $\bar{\lambda}^L_{i\cdot}$ *and* $\bar{\lambda}^M_{i\cdot}$*. Due to the law of demand (i.e.,* $\frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} < 0$ *for all* $k' \in \mathbf{N}_i$*), these are both positive. In light of* (78)*, this in turn implies a higher sectoral price index in response to higher wage and material cost index, which accords with a lower output quantity seen in Example C.2.*

Fact C.3. *Suppose that Proposition C.2 and Lemma C.4 hold. Then, for each sector* $i \in \mathbf{N}$*,* $\bar{\lambda}^L_{i\cdot}$ *and* $\bar{\lambda}^M_{i\cdot}$ *are identified.*

Proof. First, $\mathbf{q}^*_i$ and $\mathbf{p}^*_i$ are identified by Proposition C.2. Next, it can immediately be seen from Fact C.2 that $\lambda_{ik,1}$ and $\lambda_{ik,2}$ are identified. Moreover, in view of Lemma C.4, $\frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik}}$ can be expressed in terms of $\mathbf{p}^*_i$ and Q^*_i. Hence, $\bar{\lambda}^L_{i\cdot}$ and $\bar{\lambda}^M_{i\cdot}$ are identified. □

Meanwhile, taking total derivatives of (50), it holds that for a given n and n',

$$\frac{dP^{M*}_i}{d\tau_{n,n'}} = -\sum_{j=1}^{N} \frac{\gamma_{i,j}}{1-\tau_{i,j}} P^{M*}_i \mathbb{1}_{\{i=n,j=n'\}} + \sum_{j=1}^{N} \gamma_{i,j} \frac{P^{M*}_i}{P^*_j} \frac{dP^*_j}{d\tau_{n,n'}}, \tag{79}$$

where $\mathbb{1}_{\{i=n,j=n'\}}$ takes one if $i = n$ and $j = n'$, and zero otherwise.

Substituting (78) for $\{\frac{dP^*_j}{d\tau_{n,n'}}\}_{j=1}^N$ into (79), I arrive at

$$\frac{dP^{M*}_i}{d\tau_{n,n'}} = -\sum_{j=1}^{N} \frac{\gamma_{i,j}}{1-\tau_{i,j}} P^{M*}_i \mathbb{1}_{\{i=n,j=n'\}} + \left(\sum_{j=1}^{N} \gamma_{i,j} \frac{P^{M*}_i}{P^*_j} \bar{\lambda}^L_{j\cdot}\right) \frac{dW^*}{d\tau_{n,n'}} + \left(\sum_{j=1}^{N} \gamma_{i,j} \frac{P^{M*}_i}{P^*_j} \bar{\lambda}^M_{j\cdot}\right) \frac{dP^{M*}_j}{d\tau_{n,n'}}. \tag{80}$$

Denoting $\Gamma_1 := \left[\gamma_{i,j} \frac{P^{M*}_i}{P^*_j} \bar{\lambda}^L_{j\cdot}\right]_{i,j=1}^N$ and $\Gamma_2 := \left[\gamma_{i,j} \frac{P^{M*}_i}{P^*_j} \bar{\lambda}^M_{j\cdot}\right]_{i,j=1}^N$, and letting $\iota := [1, 1, \ldots, 1]'$ be a $N \times 1$ vector of ones, I stack (80) over sectors to obtain the following system of equations:

$$\begin{bmatrix} \frac{dP^{M*}_1}{d\tau_{n,n'}} \\ \vdots \\ \frac{dP^{M*}_N}{d\tau_{n,n'}} \end{bmatrix} = -\begin{bmatrix} \sum_{j=1}^N \frac{\gamma_{1,j}}{1-\tau_{1,j}} P^{M*}_1 \mathbb{1}_{\{1=n,j=n'\}} \\ \vdots \\ \sum_{j=1}^N \frac{\gamma_{N,j}}{1-\tau_{N,j}} P^{M*}_N \mathbb{1}_{\{N=n,j=n'\}} \end{bmatrix} + \Gamma_1 \iota \frac{dW^*}{d\tau_{n,n'}} + \Gamma_2 \begin{bmatrix} \frac{dP^{M*}_1}{d\tau_{n,n'}} \\ \vdots \\ \frac{dP^{M*}_N}{d\tau_{n,n'}} \end{bmatrix}$$

$$\therefore (I - \Gamma_2) \begin{bmatrix} \frac{dP^{M*}_1}{d\tau_{n,n'}} \\ \vdots \\ \frac{dP^{M*}_N}{d\tau_{n,n'}} \end{bmatrix} = -\begin{bmatrix} \sum_{j=1}^N \frac{\gamma_{1,j}}{1-\tau_{1,j}} P^{M*}_1 \mathbb{1}_{\{1=n,j=n'\}} \\ \vdots \\ \sum_{j=1}^N \frac{\gamma_{N,j}}{1-\tau_{N,j}} P^{M*}_N \mathbb{1}_{\{N=n,j=n'\}} \end{bmatrix} + \Gamma_1 \iota \frac{dW^*}{d\tau_{n,n'}} \tag{81}$$

where I represents an $N \times N$ identity matrix.

Fact C.4. *The matrices* Γ_1 *and* Γ_2 *in* (81) *is identified.*

Proof. In view of Fact B.5, $\{\gamma_{i,j}\}_{i,j}$ and $\{P^{M*}_i\}_{i=1}^N$ are identified from the observables. Moreover, $\{\bar{\lambda}^L_{j\cdot}\}_{j=1}^N$ and $\{\bar{\lambda}^M_{j\cdot}\}_{j=1}^N$ are identified due to Fact C.3. Hence, both Γ_1 and Γ_2 in (81) are identified. □

To uniquely solve (81) for $\{\frac{dP^{M*}_j}{d\tau_{n,n'}}\}_{j=1}^N$, I need an additional regularity condition.

Assumption C.7. *The matrix* $(I-\Gamma_2)$ *is nonsingular.*

This assumption guarantees that $(I-\Gamma_2)$ is invertible. Under Assumption C.7, it follows from (81) that

$$\begin{bmatrix} \frac{dP_1^{M*}}{d\tau_{n,n'}} \\ \vdots \\ \frac{dP_N^{M*}}{d\tau_{n,n'}} \end{bmatrix} = (I-\Gamma_2)^{-1} \begin{bmatrix} -\sum_{j=1}^N \frac{\gamma_{1,j}}{1-\tau_{1,j}} P_1^{M*} \mathbb{1}_{\{1=n,j=n'\}} \\ \vdots \\ -\sum_{j=1}^N \frac{\gamma_{N,j}}{1-\tau_{N,j}} P_N^{M*} \mathbb{1}_{\{N=n,j=n'\}} \end{bmatrix} + (I-\Gamma_2)^{-1}\Gamma_1 \iota \frac{dW^*}{d\tau_{n,n'}}. \tag{82}$$

Observe here that Γ_2 is a version of the adjacency matrix capturing the input-output linkages among sectors (see Fact B.5). Hence, $(I-\Gamma_2)^{-1}$ can be conceived as a type of the Leontief inverse matrix, augmented by the source sector's strategic interactions (i.e., market distortion). For some $i \neq n$, the (i,n) entry of this strategic-complementarity-adjusted Leontief inverse can be written as a geometric sum:

$$\gamma_{i,n}\frac{P_i^{M*}}{P_n^*}\bar{\lambda}_{n\cdot}^M + \sum_{j=1}^N \gamma_{i,j}\gamma_{j,n}\frac{P_i^{M*}}{P_j^*}\frac{P_j^{M*}}{P_n^*}\bar{\lambda}_{j\cdot,}^M\bar{\lambda}_{n\cdot}^M + \sum_{j=1}^N\sum_{j'=1}^N \gamma_{i,j}\gamma_{j,j'}\gamma_{j',n}\frac{P_i^{M*}}{P_j^*}\frac{P_j^{M*}}{P_{j'}^*}\frac{P_{j'}^{M*}}{P_n^*}\bar{\lambda}_{j\cdot}^M\bar{\lambda}_{j'\cdot}^M\bar{\lambda}_{n\cdot}^M + \ldots. \tag{83}$$

This infinite sum expression embodies the so called "strategic complementarities" in firm's price setting.(e.g., Nakamura and Steinsson 2010; La'O and Tahbaz-Salehi 2022).[104] To gain some intuition for this, suppose that sector i uses sector n's ($n \neq i$) intermediate good directly and indirectly along the production network. For the sake of brevity, assume in addition that $\bar{\lambda}_{j\cdot,} > 0$ for all $j \in \mathbf{N}$. When sector n is subsidized, the reduced input cost stimulates the production in that sector, leading to a lower sectoral output price index of sector n according to (78). The pass-through ratio is given by $\bar{\lambda}_{n\cdot}^M$. This change in the sector n's output price index affects the cost index of sector i through multiple channels. The first term of (83) stands for the first-order spillover effect: the lower price index of sector n directly reduces the sector i's input cost. The second term captures the second-order spillover effect coming via a third sector j. The output price index of sector j decreases as firms in sector j can produce more of their goods by taking advantage of cheaper input costs. This effect is encapsulated in $\bar{\lambda}_{j\cdot}$. This chain of reductions in input cost takes place along the network. I call this comovement of sectoral cost indices the *macro complementarities*.

In general, the sign and magnitude of the macro complementarities are ambiguous, because they are mediated by the source sector firm's strategic complementarities, encoded in $\bar{\lambda}_{j,\cdot}$, which I call the *micro complementarities*.

Example C.4. *Consider an economy consisting on three sectors, i.e., sector 1, 2 and 3. Suppose that the overall strategic complementarity in sector 2 is such that* $\bar{\lambda}_{2\cdot}^M < 0$*, and that in sector 3 is* $\bar{\lambda}_{3\cdot}^M > 0$*. Sector 1 purchases input goods from sector 3 directly and indirectly through sector 2. Assume that sector 3 is subsidized. In this case, the corresponding expression for* (83) *from the sector 1's viewpoint is given*

[104] The quotation marks are attached to emphasize that in my model firms are not explicitly engaged in strategic interactions across sectors.

by

$$\gamma_{1,3}\frac{P_1^{M*}}{P_3^*}\bar{\lambda}_{3\cdot}^M + \gamma_{1,2}\gamma_{2,3}\frac{P_1^{M*}}{P_2^*}\frac{P_2^{M*}}{P_3^*}\bar{\lambda}_{2\cdot}^M\bar{\lambda}_{3\cdot}^M.$$

The first term represents the first-order spillover effect from the subsidized sector. This induces a positive correlation, as discussed above. The second term dictates the second-order spillover effect coming through sector 2. On the one hand, the input cost for sector 2 decreases owing to lower sectoral intermediate good from sector 3. The sectoral price index of sector 2, however, will go up because the competition in sector 2 is such that $\bar{\lambda}_{2\cdot}^M < 0$*. (This is especially the case when the firms' products are strategic complement of one another.) Thus, the presence of sector 2, through a higher price index of sector 2's intermediate good, partially undermines or may even revert the positive spillover effect from the subsidized sector.*

Remark C.3. *The literature on New Keynesian models , such as Nakamura and Steinsson (2010) and La'O and Tahbaz-Salehi (2022) use the strategic complementarities in firm's price setting to refer to the relationship between sectoral output indices. A similar observation can be obtained for sectoral output indices by substituting* (79) *into* (78) *to cancel* $\{\frac{dP_j^*}{d\tau_{n,n'}}\}_{j=1}^N$*. The intuition retains the same as described above.*

Lemma C.5 (Identification of $\frac{dP_i^{M*}}{d\tau_{n,n'}}$)**.** *Suppose that Assumptions C.6 and C.7 hold. Then, the value of* $\frac{dP_i^{M*}}{d\tau_{n,n'}}$ *is uniquely identified up to* $\frac{dW^*}{d\tau_{n,n'}}$*.*

Proof. In view of Fact B.5, $\{\gamma_{i,j}\}_{i,j}$ and $\{P_i^{M*}\}_{i=1}^N$ in (82) are identified from the observables. Moreover, by Fact C.4, Γ_1 and Γ_2 are also identified from the observables. Thus I can uniquely identify $\{\frac{dP_i^{M*}}{d\tau_{n,n'}}\}_{i=1}^N$ up to $\frac{dW^*}{d\tau_{n,n'}}$ through (82), as claimed. □

Lemma C.6 (Identification of $\frac{dP_i^*}{d\tau_{n,n'}}$)**.** *Suppose that the assumptions required in Lemma C.5 are satisfied. Then, the value of* $\frac{dP_i^*}{d\tau_{n,n'}}$ *is identified up to* $\frac{dW^*}{d\tau_{n,n'}}$*.*

Proof. In light of Lemma C.5, I identify $\{\frac{dP_i^{M*}}{d\tau_{n,n'}}\}_{i=1}^N$ up to $\frac{dW^*}{d\tau_{n,n'}}$. Substituting these into (78), I can identify $\{\frac{dP_i^*}{d\tau_{n,n'}}\}_{i=1}^N$ up to $\frac{dW^*}{d\tau_{n,n'}}$ as

$$\frac{dP_i^*}{d\tau_{n,n'}} = \bar{\lambda}_{i\cdot}^L\frac{dW^*}{d\tau_{n,n'}} + \bar{\lambda}_{i\cdot}^M\frac{dP_i^{M*}}{d\tau_{n,n'}},$$

where the identification of $\bar{\lambda}_{i\cdot}^L$ and $\bar{\lambda}_{i\cdot}^M$ follows from Fact C.3. This proves the claim. □

Lemma C.7 (Identification of $\frac{dq_{ik}^*}{d\tau_{n,n'}}$)**.** *Suppose that the assumptions required in Proposition C.2 and Lemma C.3 are satisfied. Assume moreover that Assumptions C.6 and C.7 hold. Then, the value of* $\frac{dq_{ik}^*}{d\tau_{n,n'}}$ *is identified up to* $\frac{dW^*}{d\tau_{n,n'}}$*.*

Proof. In (76), $\Lambda_{i,1}^{-1}\Lambda_{i,2}$ is identified by Fact C.2, and $\frac{dP_i^{M*}}{d\tau_{n,n'}}$ is identified up to $\frac{dW^*}{d\tau_{n,n'}}$ by Lemma C.5. Thus, I can identify the value of $\frac{dq_{ik}^*}{d\tau_{n,n'}}$ up to $\frac{dW^*}{d\tau_{n,n'}}$, completing the proof. □

C.3.2 Cost Minimization 1: Input Decision

The increment (or decrement) of the output quantity in reaction to the policy change, $\frac{dq_{ik}^*}{d\tau_n}$, pins down a new production possibility frontier, along which the quantities of labor and material inputs adjust.

Firm k's cost minimization problem in sector i is formulated as: for given W, P_i^M and q_{ik}^*,

$$\begin{aligned}(\ell_{ik}^*, m_{ik}^*) \quad &\in \quad \arg\min_{\ell_{ik}, m_{ik}} W\ell_{ik} + P_i^M m_{ik} \\ & \qquad s.t. \quad f_i(\ell_{ik}, m_{ik}; z_{ik}) \geq q_{ik}^*.\end{aligned}$$

The associated Lagrange function is

$$\mathcal{L}_i(\ell_{ik}, m_{ik}, \xi_{ik}) \coloneqq W\ell_{ik} + P_i^M m_{ik} - \xi_{ik}\Big(f_i(\ell_{ik}, m_{ik}; z_{ik}) - q_{ik}^*\Big).$$

In equilibrium, the first order conditions are satisfied at $(\ell_{ik}, m_{ik}) = (\ell_{ik}^*, m_{ik}^*)$:

$$\begin{aligned}&[\ell_{ik}] : W^* = \xi_{ik}^* \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \\ &[m_{ik}] : P_i^{M*} = \xi_{ik}^* \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \\ &[\xi_{ik}] : f_i(\ell_{ik}^*, m_{ik}^*; z_{ik}) = q_{ik}^*,\end{aligned}$$

where ξ_{ik}^* is the marginal cost of production at the given quantity q_{ik}^*. Note that under Assumption 3.5 (i), ξ_{ik}^* equals the average cost: i.e., $\xi_{ik}^* = \frac{TC_{ik}^*}{q_{ik}^*}$ where $TC_{ik}^* \coloneqq TC_{ik}(W, P_i^M, q_{ik})\big|_{(W,P_i^M,q_{ik})=(W^*,P_i^{M*},q_{ik}^*)}$ with $TC_{ik}(\cdot)$ denoting, with a slight abuse of notation, the firm's total cost function. (see also Fact C.1).

Fact C.5 (Identification of λ_{ik}^*)**.** *Suppose that Proposition C.2 holds. Then ξ_{ik}^* is identified.*

Proof. Applying Proposition C.2, q_{ik}^* is identified. Since TC_{ik}^* is directly observed in data, I can thus identify ξ_{ik}^*, as desired. □

Remark C.4. *Two sets of "optimal" labor and material inputs $(\bar{\ell}_{ik}^*, \bar{m}_{ik}^*)$ and (ℓ_{ik}^*, m_{ik}^*) need to be distinguished. They reside on the same production possibility frontier, but do not necessarily coincide. It is the latter that minimizes the total cost of producing q_{ik}^*.*

Totally differentiating the first order conditions, one obtains

$$\frac{dW^*}{d\tau_{n,n'}} = \frac{d\xi_{ik}^*}{d\tau_{n,n'}}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} + \xi_{ik}^*\left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2}\frac{d\ell_{ik}^*}{d\tau_{n,n'}} + \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\frac{dm_{ik}^*}{d\tau_{n,n'}}\right) \tag{84}$$

$$\frac{dP_i^{M*}}{d\tau_{n,n'}} = \frac{d\xi_{ik}^*}{d\tau_{n,,n'}}\frac{\partial f_i(\cdot)^*}{\partial m_{ik}} + \xi_{ik}^*\left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} m_{ik}}\frac{d\ell_{ik}^*}{d\tau_{n,n'}} + \frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2}\frac{dm_{ik}^*}{d\tau_{n,n'}}\right) \tag{85}$$

$$\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\frac{d\ell_{ik}^*}{d\tau_{n,n'}} + \frac{\partial f_i(\cdot)^*}{\partial m_{ik}}\frac{dm_{ik}^*}{d\tau_{n,n'}} = \frac{dq_{ik}^*}{d\tau_{n,n'}}. \tag{86}$$

Observe here that

$$
\begin{aligned}
\frac{d\xi_{ik}^*}{d\tau_{n,n'}} &= \frac{d(TC_{ik}^*/q_{ik}^*)}{d\tau_{n,n'}} \\
&= \frac{1}{q_{ik}^*}\frac{dTC_{ik}^*}{dq_{ik}^*} - TC_{ik}\frac{1}{(q_{ik}^*)^2}\frac{dq_{ik}^*}{d\tau_{n,n'}} \\
&= \frac{1}{q_{ik}^*}\left(\frac{\partial TC_{ik}(\cdot)^*}{\partial W}\frac{dW^*}{d\tau_{n,n'}} + \frac{\partial TC_{ik}(\cdot)^*}{\partial P_i^M}\frac{dP_i^{M*}}{d\tau_{n,n'}} + \frac{\partial TC_{ik}(\cdot)^*}{\partial q_{ik}}\frac{dq_{ik}^*}{d\tau_{n,n'}}\right) - \frac{1}{q_{ik}^*}\frac{TC_{ik}^*}{q_{ik}^*}\frac{dq_{ik}^*}{d\tau_{n,n'}} \\
&= \frac{1}{q_{ik}^*}\left(\ell_{ik}^*\frac{dW^*}{d\tau_{n,n'}} + m_{ik}^*\frac{dP_i^{M*}}{d\tau_{n,n'}} + \frac{\partial TC_{ik}(\cdot)^*}{\partial q_{ik}}\frac{dq_{ik}^*}{d\tau_{n,n'}}\right) - \frac{1}{q_{ik}^*}\frac{TC_{ik}^*}{q_{ik}^*}\frac{dq_{ik}^*}{d\tau_{n,n'}} \\
&= \frac{1}{q_{ik}^*}\left(\ell_{ik}^*\frac{dW^*}{d\tau_{n,n'}} + m_{ik}^*\frac{dP_i^{M*}}{d\tau_{n,n'}} + \xi_{ik}^*\frac{dq_{ik}^*}{d\tau_{n,n'}}\right) - \frac{1}{q_{ik}^*}\xi_{ik}^*\frac{dq_{ik}^*}{d\tau_{n,n'}} \\
&= \frac{\ell_{ik}^*}{q_{ik}^*}\frac{dW^*}{d\tau_{n,n'}} + \frac{m_{ik}^*}{q_{ik}^*}\frac{dP_i^{M*}}{d\tau_{n,n'}}.
\end{aligned}
\tag{87}
$$

where the fourth equality is due to the Shephard lemma, and the fifth one follows from the fact that under Assumption 3.5 (i), the marginal cost equals average cost.

From (84) and (87),

$$
\begin{aligned}
&\frac{dW^*}{d\tau_{n,n'}} = \left(\frac{\ell_{ik}^*}{q_{ik}^*}\frac{dW^*}{d\tau_{n,n'}} + \frac{m_{ik}^*}{q_{ik}^*}\frac{dP_i^{M*}}{d\tau_{n,n'}}\right)\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} + \xi_{ik}^*\left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2}\frac{d\ell_{ik}^*}{d\tau_{n,n'}} + \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\frac{dm_{ik}^*}{d\tau_{n,n'}}\right) \\
&\therefore \xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2}\frac{d\ell_{ik}^*}{d\tau_{n,n'}} + \xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\frac{dm_{ik}^*}{d\tau_{n,n'}} = \left(1 - \frac{\ell_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\right)\frac{dW^*}{d\tau_{n,n'}} - \frac{m_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\frac{dP_i^{M*}}{d\tau_{n,n'}}.
\end{aligned}
\tag{88}
$$

From (85) and (87),

$$
\begin{aligned}
&\frac{dP_i^{M*}}{d\tau_{n,n'}} = \left(\frac{\ell_{ik}^*}{q_{ik}^*}\frac{dW^*}{d\tau_{n,n'}} + \frac{m_{ik}^*}{q_{ik}^*}\frac{dP_i^{M*}}{d\tau_{n,n'}}\right)\frac{\partial f_i(\cdot)^*}{\partial m_{ik}} + \xi_{ik}^*\left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} m_{ik}}\frac{d\ell_{ik}^*}{d\tau_{n,n'}} + \frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2}\frac{dm_{ik}^*}{d\tau_{n,n'}}\right) \\
&\therefore \xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\frac{d\ell_{ik}^*}{d\tau_{n,n'}} + \xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2}\frac{dm_{ik}^*}{d\tau_{n,n'}} = -\frac{\ell_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial m_{ik}}\frac{dW^*}{d\tau_{n,n'}} + \left(1 - \frac{m_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial m_{ik}}\right)\frac{dP_i^{M*}}{d\tau_{n,n'}}.
\end{aligned}
\tag{89}
$$

The set of equations (86), (88) and (89), coupled with (76), can be summarized into a matrix form:

$$
\begin{bmatrix}
\xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} & \xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}} \\
\xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}} & \xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2} \\
\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \frac{\partial f_i(\cdot)^*}{\partial m_{ik}}
\end{bmatrix}
\begin{bmatrix}
\frac{d\ell_{ik}^*}{d\tau_{n,n'}} \\
\frac{dm_{ik}^*}{d\tau_{n,n'}}
\end{bmatrix}
=
\begin{bmatrix}
1 - \frac{\ell_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & -\frac{m_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \\
-\frac{\ell_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial m_{ik}} & 1 - \frac{m_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \\
\bar{\lambda}_{ik}^L & \bar{\lambda}_{ik}^M
\end{bmatrix}
\begin{bmatrix}
\frac{dW^*}{d\tau_{n,n'}} \\
\frac{dP_i^{M*}}{d\tau_{n,n'}}
\end{bmatrix}.
\tag{90}
$$

Notice that under Assumption 3.5 (i), (88) and (89) are essentially identical. Hence, the system of equations (90) simplifies to:

$$
\begin{bmatrix}
\xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} & \xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}} \\
\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \frac{\partial f_i(\cdot)^*}{\partial m_{ik}}
\end{bmatrix}
\begin{bmatrix}
\frac{d\ell_{ik}^*}{d\tau_n} \\
\frac{dm_{ik}^*}{d\tau_n}
\end{bmatrix}
=
\begin{bmatrix}
1 - \frac{\ell_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & -\frac{m_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \\
\bar{\lambda}_{ik}^L & \bar{\lambda}_{ik}^M
\end{bmatrix}
\begin{bmatrix}
\frac{dW^*}{d\tau_{n,n'}} \\
\frac{dP_i^{M*}}{d\tau_{n,n'}}
\end{bmatrix}.
\tag{91}
$$

It is immediate to show that (91) can be solved for $\frac{d\ell_{ik}^*}{d\tau_n}$ and $\frac{dm_{ik}^*}{d\tau_n}$ as soon as acknowledging the following fact.

Fact C.6. *Suppose that Assumption 3.5 holds. Then, the matrix*

$$\begin{bmatrix} \xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} & \xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \\ \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \end{bmatrix}$$

is nonsingular, i.e., invertible.

Proof. By Assumption 3.5 (i), it holds that for each firm k, traced by $z_{ik} \in \mathscr{Z}_i$,

$$\frac{\partial f_i(\cdot)}{\partial \ell_{ik}} \ell_{ik} + \frac{\partial f_i(\cdot)}{\partial m_{ik}} m_{ik} = q_{ik}$$

and

$$\frac{\partial^2 f_i(\cdot)}{\partial \ell_{ik}^2} \ell_{ik} + \frac{\partial^2 f_i(\cdot)}{\partial \ell_{ik} \partial m_{ik}} m_{ik} = 0, \tag{92}$$

for any $(q_{ik}, \ell_{ik}, m_{ik}) \in \{(q, \ell, m) \in \mathscr{S}_i \times \mathscr{L}_i \times \mathscr{M}_i \mid q = f_i(\ell, m, z_{ik})\}$.

Then the determinant of the matrix in question is given by

$$\begin{aligned} \begin{vmatrix} \xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} & -\xi_{ik}^* \frac{\partial f_i^2(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \\ \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \end{vmatrix} &= \begin{vmatrix} -\xi_{ik}^* \frac{m_{ik}^*}{\ell_{ik}^*} \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial \ell_{ik}} & \xi_{ik}^* \frac{\partial f_i^2(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \\ \frac{q_{ik}^*}{\ell_{ik}^*} - \frac{m_{ik}^*}{\ell_{ik}^*} \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} & \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \end{vmatrix} \\ &= -\xi_{ik}^* \frac{m_{ik}^*}{\ell_{ik}^*} \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial \ell_{ik}} - \xi_{ik}^* \left(\frac{q_{ik}^*}{\ell_{ik}^*} - \frac{m_{ik}^*}{\ell_{ik}^*} \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \right) \frac{\partial f_i^2(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \\ &= -\xi_{ik}^* \frac{q_{ik}^*}{\ell_{ik}^*} \frac{\partial f_i^2(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \\ &< 0, \end{aligned}$$

where the last strict inequality is a consequence of Assumptions 3.5. This means that the matrix is nonsingular, as claimed. □

In light of Fact C.6, the system of equations (91) can be uniquely solved for $\frac{d\ell_{ik}^*}{d\tau_n}$ and $\frac{dm_{ik}^*}{d\tau_n}$. Towards the identification of $\frac{d\ell_{ik}^*}{d\tau_{n,n'}}$ and $\frac{dm_{ik}^*}{d\tau_{n,n'}}$, I need to recover the first- and second-order partial derivatives of the firm-level production function. my approach heavily draws from Gandhi et al. (2019), and exploits the Hicks-neutral productivity of the firm-level production function as assumed in (13). For the ease of reference, this is summarized below.

Assumption C.8 (Hicks-neutral Productivity Shocks)**.** *For each $i \in \mathbf{N}$ and for each $k \in \mathbf{N}_i$, the firm-level productivity shifter z_{ik} is Hicks-neutral.*

The detail of the identification argument is relegated to Appendix C.4. Provided that the first- and second-order derivatives of the firm-level production functions are recovered, I are ready to identify the changes in labor and material inputs in response to changes in subsidies.

Lemma C.8 (Identification of $\frac{d\ell^*_{ik}}{d\tau_{n,n'}}$ and $\frac{dm^*_{ik}}{d\tau_{n,n'}}$). *Suppose that the assumptions required in Lemma C.7 are satisfied. Assume moreover that Assumption C.8 holds. Then, the values of $\frac{d\ell^*_{ik}}{d\tau_n}$ and $\frac{dm^*_{ik}}{d\tau_n}$ are uniquely identified up to $\frac{dW^*}{d\tau_{n,n'}}$.*

Proof. Using Fact C.6, I can write (91) uniquely as

$$\begin{bmatrix} \frac{d\ell^*_{ik}}{d\tau_{n,n'}} \\ \frac{dm^*_{ik}}{d\tau_{n,n'}} \end{bmatrix} = \begin{bmatrix} \xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell^2_{ik}} & \xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}} \\ \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \end{bmatrix}^{-1} \begin{bmatrix} 1-\frac{\ell^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & -\frac{m^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \\ \bar{\lambda}^L_{ik} & \bar{\lambda}^M_{ik} \end{bmatrix} \begin{bmatrix} \frac{dW^*}{d\tau_{n,n'}} \\ \frac{dP^{M*}_i}{d\tau_{n,n'}} \end{bmatrix}$$
$$= -\left(\xi^*_{ik}\frac{q^*_{ik}}{\ell^*_{ik}}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\right)^{-1} \begin{bmatrix} \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} & -\xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}} \\ -\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell^2_{ik}} \end{bmatrix} \begin{bmatrix} 1-\frac{\ell^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & -\frac{m^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \\ \bar{\lambda}^L_{ik} & \bar{\lambda}^M_{ik} \end{bmatrix} \begin{bmatrix} \frac{dW^*}{d\tau_{n,n'}} \\ \frac{dP^{M*}_i}{d\tau_{n,n'}} \end{bmatrix}. \tag{93}$$

First, q^*_{ik} and ξ^*_{ik} are identified by Proposition C.2 and Fact C.5, respectively. Next, the partial derivatives of the production function are identified by Lemma C.11 in Appendix C.4. Finally, the total derivatives $\frac{dP^{M*}_i}{d\tau_{n,n'}}$ and $\frac{dq^*_{ik}}{d\tau_{n,n'}}$ are identified up to $\frac{dW^*}{d\tau_{n,n'}}$ through Lemmas C.5 and C.7, respectively. Hence, I also can uniquely identify $\frac{d\ell^*_{ik}}{d\tau_{n,n'}}$ and $\frac{dm^*_{ik}}{d\tau_{n,n'}}$ up to $\frac{dW^*}{d\tau_{n,n'}}$, as desired. □

Remark C.5. *It is worth noticing that* (93) *can be decomposed into two terms as follows:*

$$\begin{bmatrix} \frac{d\ell^*_{ik}}{d\tau_{n,n'}} \\ \frac{dm^*_{ik}}{d\tau_{n,n'}} \end{bmatrix} = \underbrace{-\left(\xi^*_{ik}\frac{q^*_{ik}}{\ell^*_{ik}}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\right)^{-1} \begin{bmatrix} \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} & -\xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}} \\ -\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell^2_{ik}} \end{bmatrix} \begin{bmatrix} 1-\frac{\ell^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & -\frac{m^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \\ \bar{\lambda}^L_{ik} & \bar{\lambda}^M_{ik} \end{bmatrix}}_{\text{firm } k\text{'s input elasticities}} \underbrace{\begin{bmatrix} \frac{dW^*}{d\tau_{n,n'}} \\ \frac{dP^{M*}_i}{d\tau_{n,n'}} \end{bmatrix}}_{\text{policy shocks}}.$$

The leading three terms jointly account for the responsiveness of the firm's labor and material input decisions to the changes in wage and the cost index due to a policy shift, which are given by the last term. The former can be identified and thus estimated independently the latter. That is, once the former is obtained, (93) *can be viewed as a "reduced-form" relationship between the changes of labor and material inputs and the those of wage and material cost index.*

The comparative statics in this section so far have been identified up to $\frac{dW^*}{d\tau_{n,n'}}$. Next, to attain the full identification of the comparative statics, I aim to identify $\frac{dW^*}{d\tau_{n,n'}}$ from the observables by making use of the labor market clearing condition (22). First, let

$$D_{ik} = \begin{bmatrix} d_{ik,11} & d_{ik,12} \\ d_{ik,21} & d_{ik,22} \end{bmatrix}$$

be the 2×2 matrix expressing the firm's input elasticities' part of (93): i.e.,

$$D_{ik} := -\left(\xi^*_{ik}\frac{q^*_{ik}}{\ell^*_{ik}}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\right)^{-1} \begin{bmatrix} \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} & -\xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}} \\ -\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell^2_{ik}} \end{bmatrix} \begin{bmatrix} 1-\frac{\ell^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & -\frac{m^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \\ \bar{\lambda}^L_{ik} & \bar{\lambda}^M_{ik}. \end{bmatrix}$$

Then, I can write (93) as

$$\frac{d\ell_{ik}^*}{d\tau_{n,n'}} = d_{ik,11}\frac{dW^*}{d\tau_{n,n'}} + d_{ik,12}\frac{dP_i^{M*}}{d\tau_{n,n'}}, \tag{94}$$

$$\frac{dm_{ik}^*}{d\tau_{n,n'}} = d_{ik,21}\frac{dW^*}{d\tau_{n,n'}} + d_{ik,22}\frac{dP_i^{M*}}{d\tau_{n,n'}}. \tag{95}$$

Next, observe that from (82), I can write

$$\frac{dP_i^{M*}}{d\tau_{n,n'}} = \vartheta_{i,1} + \vartheta_{i,2}\frac{dW^*}{d\tau_{n,n'}}, \tag{96}$$

where $\vartheta_{i,1}$ and $\vartheta_{i,2}$ are the i-th element of $-(I-\Gamma_2)^{-1}[\frac{\gamma_{1,n'}}{1-\tau_{1,n'}}P_1^{M*}\mathbb{1}_{\{n=1\}},\ldots,\frac{\gamma_{N,n'}}{1-\tau_{N,n'}}P_N^{M*}\mathbb{1}_{\{n=N\}}]'$ and $(I-\Gamma_2)^{-1}\Gamma_1\iota$, respectively.

Therefore, upon substituting (96) into (94), I arrive at

$$\begin{aligned}\frac{d\ell_{ik}^*}{d\tau_{n,n'}} &= d_{ik,11}\frac{dW^*}{d\tau_{n,n'}} + d_{ik,12}\left(\vartheta_{i,1} + \vartheta_{i,2}\frac{dW^*}{d\tau_{n,n'}}\right)\\ &= \vartheta_{i,1}d_{ik,12} + (d_{ik,11} + \vartheta_{i,2}d_{ik,12})\frac{dW^*}{d\tau_{n,n'}}.\end{aligned} \tag{97}$$

To ensure the point identification, I maintain the following regularity condition.

Assumption C.9 (Regularity Condition)**.** $\sum_{i=1}^N\sum_{k=1}^{N_i}(d_{ik,11} + \vartheta_{i,2}d_{ik,12}) \neq 0$.

The implication of this assumption is studied in Remark C.6.

Lemma C.9 (Identification of $\frac{dW^*}{d\tau_{n,n'}}$)**.** *Suppose that the assumptions required in Lemma C.8 are satisfied. Assume moreover that Assumption C.9 holds. Then, the value of* $\frac{dW^*}{d\tau_{n,n'}}$ *is identified.*

Proof. Totally differentiating the labor market clearing condition (22), I have

$$\frac{dL}{d\tau_{n,n'}} = \sum_{i=1}^N\sum_{k=1}^{N_i}\frac{d\ell_{ik}^*}{d\tau_{n,n'}}.$$

Since here labor supply is inelastic, it then must be $\frac{dL}{d\tau_{n,n'}} = 0$, so that

$$0 = \sum_{i=1}^N\sum_{k=1}^{N_i}\frac{d\ell_{ik}^*}{d\tau_{n,n'}}. \tag{98}$$

Substituting (97) for $\frac{d\ell_{ik}^*}{d\tau_{n,n'}}$ into (98) leads us to

$$0 = \sum_{i=1}^N\sum_{k=1}^{N_i}\left\{\vartheta_{i,1}d_{ik,12} + (d_{ik,11} + \vartheta_{i,2}d_{ik,12})\frac{dW^*}{d\tau_{n,n'}}\right\}, \tag{99}$$

which, under Assumption C.9, can be rearranged to

$$\frac{dW^*}{d\tau_{n,n'}} = -\frac{\sum_{i=1}^{N}\sum_{k=1}^{N_i}\vartheta_{i,1}d_{ik,12}}{\sum_{i=1}^{N}\sum_{k=1}^{N_i}(d_{ik,11}+\vartheta_{i,2}d_{ik,12})}.$$

Given that $\vartheta_{i,1}$, $\vartheta_{i,2}$, $d_{ik,11}$, and $d_{ik,12}$ are all identified, this expression identifies the value of $\frac{dW^*}{d\tau_{n,n'}}$, proving the claim. $\square$

Remark C.6. *Since* (99) *is essentially an identity (i.e., the labor market clearing condition), when Assumption C.9 is violated, it should also holds that*

$$\sum_{i=1}^{N}\sum_{k=1}^{N_i}\vartheta_{i,1}d_{ik,12} = 0$$
$$\therefore \sum_{i=1}^{N}\vartheta_{i,1}\sum_{k=1}^{N_i}d_{ik,12} = 0,$$

where the left hand side allows for an interpretation as an weighted average of an within-sector competitiveness measure $\sum_{k=1}^{N_i}d_{ik,12}$ *weighted by the location* $\vartheta_{i,1}$ *of that sector on the production network. Hence, this indicates asymmetry either among firms or sectors.*

Proposition C.4 (Full Identification of the Comparative Statics)**.** *Suppose that the assumptions required in Lemma C.9 are satisfied. Then all the relevant comparative statics are fully identified from the observables.*

Proof. Under the maintained assumptions, I can invoke Lemmas C.5, C.6, C.7 and C.8 to identify, respectively, $\frac{dP_i^{M*}}{d\tau_{n,n'}}$, $\frac{dP_i^*}{d\tau_{n,n'}}$, $\frac{dq_{ik}^*}{d\tau_{n,n'}}$, $\frac{d\ell_{ik}^*}{d\tau_{n,n'}}$ and $\frac{dm_{ik}^*}{d\tau_{n,n'}}$ up to $\frac{dW^*}{d\tau_{n,n'}}$. Meanwhile, it is possible to recover $\frac{dW^*}{d\tau_{n,n'}}$ from observables as studied in Lemma C.9. Thus, I can identify all the relevant comparative statics, such as $\frac{dP_i^{M*}}{d\tau_{n,n'}}$, $\frac{dP_i^*}{d\tau_{n,n'}}$, $\frac{dq_{ik}^*}{d\tau_{n,n'}}$, $\frac{d\ell_{ik}^*}{d\tau_{n,n'}}$ and $\frac{dm_{ik}^*}{d\tau_{n,n'}}$, from observables, as claimed. $\square$

Observe that as far as the structure of the demand function is concerned, both perfectly competitive markets and monopolistic markets can be viewed as special cases of oligopolistic markets. Notice moreover that an economy without the production network can be embedded into the current framework as an extreme scenario, where the off-diagonal elements of the input-output matrix are set to zero . These insights take us to the following corollary.

Corollary C.2. *Suppose that the assumptions required in Lemma C.9 are satisfied. Then, i) if the market is perfectly competitive, a version of Proposition C.4 holds with letting* $\frac{\partial\psi_{ik}(\cdot)}{\partial q_{ik'}} = 0$ *for all* $k, k' \in \mathbf{N}_i$ *with the sectoral equilibrium concepts appropriately modified; ii) if the market is monopolistically competitive, a version of Proposition C.4 holds with letting* $\frac{\partial\psi_{ik}(\cdot)}{\partial q_{ik'}} = 0$ *for all* $k' \neq k \in \mathbf{N}_i$ *with the sectoral equilibrium concepts appropriately modified; and iii) if the sectoral network is absent, a version of Proposition C.4 holds with letting* $\gamma_{i,j} = 0$ *for all* $i \neq j \in \mathbf{N}$.

C.3.3 Cost Minimization 2: Derived Demand for Sectoral Goods

Next, when the change in material input $\frac{dm^*_{ik}}{d\tau_n}$ is determined, the derived demand for sectoral goods are in turn adjusted so as to minimize the expenditure for purchase of those goods. Totally differentiating (51), I have

$$\frac{dm^*_{ik,j}}{d\tau_{n,n'}} = \left(\frac{1}{1-\tau_{n,n'}} \mathbb{1}_{\{i=n,j=n'\}} + \frac{1}{P^{M*}_i} \frac{dP^{M*}_i}{d\tau_{n,n'}} - \frac{1}{P^*_j} \frac{dP^*_j}{d\tau_{n,n'}} + \frac{1}{m^*_{ik}} \frac{dm^*_{ik}}{d\tau_{n,n'}} \right) m^*_{ik,j}, \tag{100}$$

where $\mathbb{1}_{\{i=n,j=n'\}}$ is an indicator function that takes one if $i = n$ and $j = n'$, and zero otherwise.

Proposition C.5 (Identification of $\frac{dm^*_{ik,j}}{d\tau_{n,n'}}$)**.** *Suppose that the assumptions required in Proposition C.4 are satisfied. Assume moreover that Assumption B.4 holds. Then for each $i \in \mathbf{N}$ and for each $k \in \mathbf{N}_i$, $\{\frac{dm^*_{ik,j}}{d\tau_{n,n'}}\}_{j=1}^{N}$ are identified from the observables.*

Proof. First, in view of Facts B.4 and B.5, $m^*_{ik,j}$ and P^{M*}_i are obtained from the data, respectively. Next, owing to Proposition C.4, the total derivatives $\frac{dP^{M*}_i}{d\tau_{n,n'}}$, $\frac{dP^*_i}{d\tau_{n,n'}}$ and $\frac{dm^*_{ik}}{d\tau_{n,n'}}$ are all identified from the observables. Hence, $\frac{dm^*_{ik,j}}{d\tau_{n,n'}}$ is identified through (100), as desired. $\square$

C.4 Recovering the Second-Order Partial Derivatives of the Firm-Level Production Functions

The goal of this section is to identify the second order derivatives of f_i with respect to ℓ_{ik} and m_{ik}. First of all, observe that under Assumption C.8, there exits a function $g_i : \mathscr{L}_i \times \mathscr{M}_i \to \mathbb{R}$ such that

$$f_i(\ell_{ik}, m_{ik}; z_{ik}) = z_{ik} g_i(\ell_{ik}, m_{ik}), \tag{101}$$

for all $(\ell_{ik}, m_{ik}, z_{ik}) \in \mathscr{L}_i \times \mathscr{M}_i \times \mathscr{Z}_i$. I define $\tilde{g}_i : \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i \to \mathbb{R}$ such that

$$\tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}; \tilde{z}_{ik}) = \tilde{z}_{ik} + \tilde{g}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}). \tag{102}$$

Our identification strategy is based on the following relationships between the partial derivatives of $\tilde{g}_i$ and those of f_i.

Fact C.7. *Under Assumption C.8, it holds that for all $(\ell_{ik}, m_{ik}, z_{ik}) \in \mathscr{L}_i \times \mathscr{M}_i \times \mathscr{Z}_i$,*

(i) $\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{\ell}_{ik}} = \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}$ *and* $\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{m}_{ik}} = \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}}$*;*

(ii) $\frac{\partial f_i(\cdot)}{\partial \ell_{ik}} = \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \frac{f_i(\cdot)}{\ell_{ik}}$ *and* $\frac{\partial f_i(\cdot)}{\partial m_{ik}} = \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} \frac{f_i(\cdot)}{m_{ik}}$*;*

(iii) $\frac{\partial^2 f_i(\cdot)}{\partial \ell^2_{ik}} = \frac{f_i(\cdot)}{\ell^2_{ik}} \left\{ \frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}^2_{ik}} + \left(\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \right)^2 + \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \right\}$, $\frac{\partial^2 f_i(\cdot)}{\partial m^2_{ik}} = \frac{f_i(\cdot)}{m^2_{ik}} \left\{ \frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{m}^2_{ik}} + \left(\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} \right)^2 + \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} \right\}$ *and* $\frac{\partial^2 f_i(\cdot)}{\partial \ell_{ik} \partial m_{ik}} = \frac{f_i(\cdot)}{\ell_{ik} m_{ik}} \left(\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik} \partial \tilde{m}_{ik}} + \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} \right)$,

where $f_i(\cdot) := f_i(\ell_{ik}, m_{ik}; z_{ik})$ and $\tilde{g}_i(\cdot) := \tilde{g}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})$.

Proof. (i) This immediately follows from taking (partial) derivatives of the both hand sides of (102) with respect to ℓ_{ik} and m_{ik}, respectively.

(ii) First, by definition

$$g_i(\ell_{ik}, m_{ik}) = \exp\left\{\tilde{g}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})\right\},$$

so that the partial derivative with respect to ℓ_{ik} reads

$$\begin{aligned}\frac{\partial g_i(\cdot)}{\partial \ell_{ik}} &= \exp\left\{\tilde{g}_i(\cdot)\right\}\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}\frac{d\ln \ell_{ik}}{d\ell_{ik}} \\ &= \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}\frac{g_i(\cdot)}{\ell_{ik}}.\end{aligned}$$

Similarly, it holds that

$$\frac{\partial g_i(\cdot)}{\partial m_{ik}} = \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}}\frac{g_i(\cdot)}{m_{ik}}.$$

Now, it follows from (101) that $\frac{\partial f_i(\cdot)}{\partial \ell_{ik}} = z_{ik}\frac{\partial g_i(\cdot)}{\partial \ell_{ik}}$ and $\frac{\partial f_i(\cdot)}{\partial m_{ik}} = z_{ik}\frac{\partial g_i(\cdot)}{\partial m_{ik}}$. Thus I have

$$\begin{aligned}\frac{\partial f_i(\cdot)}{\partial \ell_{ik}} &= z_{ik}\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}\frac{g_i(\cdot)}{\ell_{ik}} \\ &= \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}\frac{f_i(\cdot)}{\ell_{ik}},\end{aligned}$$

and

$$\frac{\partial f_i(\cdot)}{\partial m_{ik}} = \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}}\frac{f_i(\cdot)}{m_{ik}},$$

(iii) Taking the (partial) derivatives of the result of Part (ii),

$$\begin{aligned}\frac{\partial^2 f_i(\cdot)}{\partial \ell_{ik}^2} &= \frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}^2}\frac{f_i(\cdot)}{\ell_{ik}^2} + \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}\frac{\partial f_i(\cdot)}{\partial \ell_{ik}}\frac{1}{\ell_{ik}} - \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}\frac{f_i(\cdot)}{\ell_{ik}^2} \\ &= \frac{f_i(\cdot)}{\ell_{ik}^2}\left\{\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}^2} + \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}\frac{\ell_{ik}}{f_i(\cdot)}\frac{\partial f_i(\cdot)}{\partial \ell_{ik}} - \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}\right\} \\ &= \frac{f_i(\cdot)}{\ell_{ik}^2}\left\{\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}^2} + \left(\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}\right)^2 - \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}\right\},\end{aligned}$$

where the last equality is again due to Part (ii) of this fact.

An analogous argument applies to $\frac{\partial^2 f_i(\cdot)}{\partial m_{ik}^2}$ as well.

Next, differentiating Part (ii) also yields that

$$\frac{\partial^2 f_i(\cdot)}{\partial \ell_{ik}\partial m_{ik}} = \frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}\partial \tilde{m}_{ik}}\frac{f_i(\cdot)}{\ell_{ik}m_{ik}} + \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}\frac{\partial f_i(\cdot)}{\partial m_{ik}}\frac{1}{\ell_{ik}}$$

$$
\begin{aligned}
&= \frac{f_i(\cdot)}{\ell_{ik} m_{ik}} \left\{ \frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik} \partial \tilde{m}_{ik}} + \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \frac{m_{ik}}{f_i(\cdot)} \frac{\partial f_i(\cdot)}{\partial m_{ik}} \right\} \\
&= \frac{f_i(\cdot)}{\ell_{ik} m_{ik}} \left\{ \frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik} \partial \tilde{m}_{ik}} + \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} \right\},
\end{aligned}
$$

where I once again use Part (ii) to derive the last equality. This completes the proof. □

The identification results of Gandhi et al. (2019) rest on Fact C.7 (i). I further leverage insights from Fact C.7 (ii) and (iii). In particular, observe that looking at (ii) in equilibrium,

$$
\begin{aligned}
\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} &= \frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} \frac{f_i(\ell_{ik}^*, m_{ik}^*)}{\ell_{ik}^*} \\
&= \frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} \frac{q_{ik}^*}{\ell_{ik}^*},
\end{aligned}
$$

where the second equality follows from Proposition C.2. Likewise,

$$
\frac{\partial f_i(\cdot)^*}{\partial m_{ik}} = \frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{m}_{ik}} \frac{q_{ik}^*}{m_{ik}^*}.
$$

Moreover, invoking (iii) in equilibrium, I have

$$
\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} = \frac{q_{ik}^*}{(\ell_{ik}^*)^2} \left\{ \frac{\partial^2 \tilde{g}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}^2} + \left(\frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} \right)^2 - \frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} \right\}, \tag{103}
$$

and also

$$
\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} = \frac{q_{ik}^*}{\ell_{ik}^* m_{ik}^*} \left\{ \frac{\partial^2 \tilde{g}_i(\cdot)^*}{\partial \tilde{\ell}_{ik} \partial \tilde{m}_{ik}} + \left(\frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} \right) \left(\frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{m}_{ik}} \right) \right\}. \tag{104}
$$

Since q_{ik}^* can be identified from Proposition C.2, it remains to identify the values of the second-order derivatives of $\tilde{g}_i(\cdot)$ with respect to $\tilde{\ell}_{ik}$ and $\tilde{m}_{ik}$. To this end, I follow Gandhi et al. (2019) in nonparametrically identifying the first-oder partial derivatives of $\tilde{g}(\cdot)$ as a function of $\tilde{\ell}_{ik}$ and $\tilde{m}_{ik}$.

Remark C.7. *Although the equilibrium values $\frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$ can be recovered from the observables under Assumption 3.5 (i) (see Proposition C.1), I still need to identify $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}}$ as a function of $\tilde{\ell}_{ik}$ and $\tilde{m}_{ik}$ over the entire support $\tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$, so that the second-order derivatives of $\tilde{g}_i(\cdot)$ can be derived.*

The identification equations for the second-order derivatives are based on the one-step profit maximization set out in Appendix C.3.1. Under Assumption C.8, multiplying (72) by ℓ_{ik} and dividing by $p_{ik} q_{ik}$ lead to

$$
\begin{aligned}
\frac{mr_{ik}}{p_{ik}} \frac{\partial f_i(\cdot)}{\partial \ell_{ik}} \frac{\ell_{ik}}{q_{ik}} &= \frac{W \ell_{ik}}{p_{ik} q_{ik}} \\
\therefore \frac{1}{\mu_{ik}} \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} &= s_{ik}^{\ell},
\end{aligned}
$$

where $s_{ik}^{\ell} := \frac{W\ell_{ik}}{p_{ik}q_{ik}}$ is the labor cost relative to the revenue. Moreover, I use the fact that the marginal revenue equals to the marginal cost in equilibrium, thereby implying $\mu_{ik} := \frac{p_{ik}}{mc_{ik}} = \frac{p_{ik}}{mr_{ik}}$. Taking the logarithm of this expression, I have

$$\ln s_{ik}^{\ell} = \ln \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} - \ln \mu_{ik}. \tag{105}$$

However, in general this relationship cannot be directly fed into the data when the output market is imperfectly competitive, because firm-level markup have to be identified and thus estimated simultaneously (Kasahara and Sugita 2020). Nevertheless, I emphasize that under Assumption 3.5 (i), μ_{ik} is recovered in advance of solving (105) for the first-order derivative of $\tilde{g}_i$ with respect to $\tilde{\ell}_{ik}$ (Fact C.1). Taking stock of this, I adopt the same empirical specification as Gandhi et al. (2019):

$$\tilde{s}_{ik}^{\ell,\tilde{\mu}} = \ln \mathcal{E}_i^{\ell} + \ln \frac{\partial \tilde{g}_i}{\partial \tilde{\ell}_{ik}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) - \tilde{\varepsilon}_{ik}^{\ell}, \tag{106}$$

where $\tilde{s}_{ik}^{\ell,\tilde{\mu}} := \ln s_{ik}^{\ell} + \ln \mu_{ik}$ can readily be calculated from the data, and $\tilde{\varepsilon}_{ik}^{\ell}$ is a measurement error with $E[\tilde{\varepsilon}_{ik}^{\ell} \mid \tilde{\ell}_{ik}, \tilde{m}_{ik}] = 0$. The measurement error $\tilde{\varepsilon}_{ik}^{\ell}$ captures any unmodeled, non-systematic noise both in s_{ik}^{ℓ} and μ_{ik}, and is associated with the constant $\mathcal{E}_i^{\ell}$ through $\mathcal{E}_i^{\ell} = E[\exp\{\tilde{\varepsilon}_{ik}^{\ell}\}]$. Inclusion of the mean $\mathcal{E}_i^{\ell}$ is based on the suggestion made in Gandhi et al. (2019).

Our identification result is based on Gandhi et al. (2019), which is summarized in the following lemma for the sake of completion.

Lemma C.10 (Theorem 2 of Gandhi et al. (2019))**.** *Suppose that Assumptions 3.5 and C.8 hold. Then, the share regression* (106) *identifies both the labor elasticity and material elasticity of the log-production function for all* $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$.

Proof. First, I start by writing (106) as

$$\tilde{s}_{ik}^{\ell,\tilde{\mu}} = \ln D_{ik}^{\ell}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) - \tilde{\varepsilon}_{ik}^{\ell}, \tag{107}$$

where $\ln D_{ik}^{\ell}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) := \ln \mathcal{E}_i^{\ell} + \ln \frac{\partial \tilde{g}_i}{\partial \tilde{\ell}_{ik}}(\tilde{\ell}_{ik}, \tilde{m}_{ik})$. I can nonparametrically identify $\ln D_{ik}^{\ell}(\tilde{\ell}_{ik}, \tilde{m}_{ik})$ according to

$$\ln D_{ik}^{\ell}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) = E\big[\tilde{s}_{ik}^{\ell,\tilde{\mu}} | \tilde{\ell}_{ik}, \tilde{m}_{ik}\big]$$

for all $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$. The error term $\tilde{\varepsilon}_{ik}^{\ell}$ is identified through the specification (107):

$$\tilde{\varepsilon}_{ik}^{\ell} = \ln D_{ik}^{\ell}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) - \tilde{s}_{ik}^{\ell,\tilde{\mu}} \tag{108}$$

which in turn identifies the mean $\mathcal{E}_i^{\ell}$:

$$\mathcal{E}_i^{\ell} = E\big[\exp\{\tilde{\varepsilon}_{ik}^{\ell}\}\big] \tag{109}$$

Next, plug these back into the the definition of $\ln D_{ik}^{\ell}$, I identify the log-labor input elasticity of the log-production function:

$$\ln \frac{\partial \tilde{g}_i}{\partial \tilde{\ell}_{ik}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) = \ln D^{\ell}_{ik}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) - \ln \mathcal{E}^{\ell}_i$$
$$= \ln \frac{D^{\ell}_{ik}(\tilde{\ell}_{ik}, \tilde{m}_{ik})}{\mathcal{E}^{\ell}_i},$$

yielding

$$\frac{\partial \tilde{g}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})}{\partial \tilde{\ell}_{ik}} = \frac{D^{\ell}_{ik}(\tilde{\ell}_{ik}, \tilde{m}_{ik})}{\mathcal{E}^{\ell}_i} \tag{110}$$

for all $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$. The exact same argument holds for the log-material input elasticity of the log-production function $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}}$, completing the proof. □

Remark C.8. *Lemma C.10 identifies the log-production function for the entire support $\tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$ beyond the subspace spanned by the equilibrium relations (Gandhi et al. 2019; Pan 2022). Thus from this result I can also identify partial derivatives of $\tilde{g}_i$ of arbitrary order, as exemplified in Corollary C.3.*

Corollary C.3. *The second-order derivatives of log-production function with respect to log-labor and log-material inputs, i.e., $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}^2_{ik}}$, $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{m}^2_{ik}}$, and $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik} \tilde{m}_{ik}}$, are nonparametrically identified for all $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$.*

Now I prove that it is possible to identify the values of the second-order derivative of the production function corresponding to the equilibrium labor and material inputs.

Lemma C.11. *Suppose that the assumptions required in Proposition C.2 and Lemma C.10 are satisfied. The values of the second-order derivatives of the production function at equilibrium are identified from the observables.*

Proof. Using Fact C.7 (iii) at the equilibrium (observed) labor ℓ^*_{ik} and material m^*_{ik} inputs, I obtain (103) and (104). Here, q^*_{ik} can be recovered in view of Proposition C.2. Moreover, Lemma C.10 identifies the value of $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}}$ at the equilibrium values of inputs $(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik})$ are identified, while Corollary C.3 informs us of the equilibrium values of $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}^2_{ik}}$ and $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik} \partial \tilde{m}_{ik}}$. The equilibrium value of $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{m}^2_{ik}}$ can be retained through a similar argument. Hence, by tracing (103) and (104), I can recover the values of the second-order derivatives of the production function at equilibrium, as claimed. □

Remark C.9. *Lemma C.11 only identifies the values of the second-order derivatives of the firm-level production function at the equilibrium level of labor and material inputs, while being silent about the values at different (counterfactual) values of these inputs. This is because I lack the identification of the production function $f_i(\cdot)$ over the entire support; my approach instead rests on the knowledge about the value of equilibrium quantity, given by Proposition C.2. The punchline is that as far as the identification of* (28) *is concerned, the knowledge about the entire production function is not needed, which obviates additional assumptions.*

C.5 Identification of the Object of Interest

Theorem C.1 (Identification of $\frac{dY_i(s)}{ds}$)**.** *Suppose that the assumptions required in Proposition C.5 are satisfied. Then, the value of* $\frac{dY_i(s)}{ds}$ *is identified from the observables for all* $s \in [\tau^0_{n,n'}, \tau^1_{n,n'}] \subseteq \mathscr{T}$*.*

Proof. Provided that Proposition C.5 holds, the value of $\frac{dY_i(s)}{ds}$ evaluated at a given point in $[\tau^0_{n,n'}, \tau^1_{n,n'}]$ is identified according to (52). I can repeat the same argument for each point in the region $[\tau^0_{n,n'}, \tau^1_{n,n'}] \subseteq \mathscr{T}$, thereby recovering the function $\frac{dY_i(s)}{ds}$ for all $s \in [\tau^0_{n,n'}, \tau^1_{n,n'}] \subseteq \mathscr{T}$. □

Corollary C.4 (Identification of the Object of Interest)**.** *Suppose that the assumptions required in Theorem C.1 are satisfied. Then, the object of interest* (24) *is identified from the observables.*

Proof. In light of (27), I can write

$$\sum_{i=1}^{N} Y_i(\boldsymbol{\tau}^1) - \sum_{i=1}^{N} Y_i(\boldsymbol{\tau}^0) = \sum_{i=1}^{N} \int_{\tau^0_{n,n'}}^{\tau^1_{n,n'}} \frac{dY_i(s)}{ds} ds.$$

Here, it holds from Theorem C.1 that for each $i \in \mathbf{N}$, the function $\frac{dY_i(s)}{ds}$ is identified over $[\tau^0_{n,n'}, \tau^1_{n,n'}] \subseteq \mathscr{T}$. Therefore, by integrating the function $\frac{dY_i(s)}{ds}$ over this region, and adding it up over all sectors, I can recover the left hand side (i.e., the object of interest (24)), as desired. □

Proof of Theorem 5.1. The argument expanded so far continues to hold when sector-input-specific subsidy $\tau_{n,n'}$ is replaced by sector-specific one τ_n. For example, the expression (79) now reads:

$$\frac{dP_i^{M*}}{d\tau_n} = -\frac{1}{1-\tau_n} P_i^{M*} \mathbb{1}_{\{i=n\}} + \sum_{j=1}^{N} \gamma_{i,j} \frac{P_i^{M*}}{P_j^*} \frac{dP_j^*}{d\tau_{n,n'}},$$

where $\mathbb{1}_{\{i=n\}}$ equals one if $i = n$, and zero otherwise. It is immediate to show a version of the result of Corollary C.4 for this case. This observation establishes the theorem.

□

D Estimation Strategies

D.1 Firm-Level Quantities & Prices

To estimate $\tilde{\phi}_i(\cdot)$ in Step 1 of Lemma C.1, I consider the second-order polynomial regression specification:[105] namely,

$$\begin{aligned}\tilde{r}_{ik} &= b_{i,0} + b_{i,1}\tilde{\ell}_{ik} + b_{i,2}\tilde{m}_{ik} + b_{i,3}\tilde{\ell}^2_{ik} + b_{i,4}\tilde{m}^2_{ik} + b_{i,5}\tilde{\ell}_{ik}\tilde{m}_{ik} + \tilde{\eta}_{ik} \\ &= \tilde{x}_{ik}\boldsymbol{b}_i + \tilde{\eta}_{ik}, \end{aligned} \tag{111}$$

[105]Since the identification argument exploits the first-order derivatives of the function $\tilde{\phi}_i(\cdot)$, the specification has to be an order of no less than one. my choice of the second-order approximation gives a margin of flexible fit for the derivatives.

where $\tilde{x}_{ik} \coloneqq [\tilde{\ell}_{ik}, \tilde{m}_{ik}, \tilde{\ell}_{ik}^2, \tilde{m}_{ik}^2, \tilde{\ell}_{ik}\tilde{m}_{ik}]'$ and $\mathbf{b}_i \coloneqq [b_{i,0}, b_{i,1}, b_{i,2}, b_{i,3}, b_{i,4}, b_{i,5}]'$. Stacking in matrix form, I obtain

$$\tilde{\mathbf{r}}_i = \tilde{\mathbf{x}}_i \mathbf{b}_i + \tilde{\boldsymbol{\eta}}_i,$$

where $\tilde{\mathbf{r}}_i \coloneqq [\tilde{r}_{i1}, \dots, \tilde{r}_{iN_i}]'$, and and thus the ordinary least square (OLS) estimator is given by

$$\hat{\mathbf{b}}_i = (\tilde{\mathbf{x}}_i' \tilde{\mathbf{x}}_i)^{-1} \tilde{\mathbf{x}}_i' \tilde{\mathbf{r}}_i.$$

Hence, the fitted value of the log-revenue $\tilde{r}_{ik}$ is

$$\hat{\tilde{\phi}}_i(\tilde{x}_{ik}) \coloneqq \tilde{x}_{ik} \hat{\mathbf{b}}_i.$$

Moreover, given the estimator $\hat{\mathbf{b}}_i$, the specification (111) naturally gives rise to the estimator for the first-order partial derivatives of $\tilde{\phi}_i(\cdot)$ with respect to $\tilde{\ell}_{ik}$ and $\tilde{m}_{ik}$:

$$\begin{aligned}
\widehat{\frac{\partial \tilde{\phi}_i}{\partial \tilde{\ell}_{ik}}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) &\coloneqq \hat{b}_{i,1} + 2\hat{b}_{i,3}\tilde{\ell}_{ik} + \hat{b}_{i,5}\tilde{m}_{ik} \\
\widehat{\frac{\partial \tilde{\phi}_i}{\partial \tilde{m}_{ik}}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) &\coloneqq \hat{b}_{i,2} + 2\hat{b}_{i,4}\tilde{m}_{ik} + \hat{b}_{i,5}\tilde{\ell}_{ik}.
\end{aligned}$$

D.2 Second-Order Derivatives of the Firm-Level Production Function

As proposed in Gandhi et al. (2019), my nonparametric estimators are based on approximating the share regression (107) by a complete polynomial of degree two, and starts from solving the following least square formula:

$$\hat{\boldsymbol{\zeta}} \in \arg\min_{\boldsymbol{\zeta}^\circ} \sum_{k=1}^{N_i} \left\{ \tilde{s}_{ik}^{\ell,\tilde{\mu}} - \ln\left\{ \zeta_{i,0}^\circ + \zeta_{i,1}^\circ \tilde{\ell}_{ik} + \zeta_{i,2}^\circ \tilde{m}_{ik} + \zeta_{i,3}^\circ \tilde{\ell}_{ik}^2 + \zeta_{i,4}^\circ \tilde{m}_{ik}^2 + \zeta_{i,5}^\circ \tilde{\ell}_{ik}\tilde{m}_{ik} \right\} \right\}^2.$$

The solution to this minimization problem $\hat{\boldsymbol{\zeta}}$ gives rise to an estimator for $D_{ik}^\ell(\cdot)$:

$$\widehat{D}_{ik}^\ell(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \coloneqq \hat{\zeta}_{i,0} + \hat{\zeta}_{i,1}\tilde{\ell}_{ik} + \hat{\zeta}_{i,2}\tilde{m}_{ik} + \hat{\zeta}_{i,3}\tilde{\ell}_{ik}^2 + \hat{\zeta}_{i,4}\tilde{m}_{ik}^2 + \hat{\zeta}_{i,5}\tilde{\ell}_{ik} m_{ik}.$$

This, in conjunction (108) and (109), motivates the plug-in estimators for ε_{ik} and $\mathcal{E}_i$:

$$\hat{\varepsilon}_{ik}^\ell \coloneqq \ln \widehat{D}_{ik}^\ell(\tilde{\ell}_{ik}, \tilde{m}_{ik}) - \tilde{s}_{ik}^{\ell,\tilde{\mu}},$$

and

$$\widehat{\mathcal{E}}_i^\ell \coloneqq \frac{1}{N_i} \sum_{k=1}^{N_i} \exp\{\hat{\varepsilon}_{ik}\},$$

respectively. Based on (110), the estimator for the first-order derivative of the log-production function with respect to log-labor input is thus given by

$$\begin{aligned}\widehat{\frac{\partial \tilde{g}_i}{\partial \tilde{\ell}_{ik}}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) &:= \frac{\widehat{D}^{\ell}_{ik}(\tilde{\ell}_{ik}, \tilde{m}_{ik})}{\widehat{\mathcal{E}}^{\ell}_i} \\ &= \frac{1}{\widehat{\mathcal{E}}^{\ell}_i}\Big(\hat{\zeta}_{i,0} + \hat{\zeta}_{i,1}\tilde{\ell}_{ik} + \hat{\zeta}_{i,2}\tilde{m}_{ik} + \hat{\zeta}_{i,3}\tilde{\ell}^2_{ik} + \hat{\zeta}_{i,4}\tilde{m}^2_{ik} + \hat{\zeta}_{i,5}\tilde{\ell}_{ik}\tilde{m}_{ik}\Big).\end{aligned}$$

From this, I can also define the estimators for the second-order derivatives of log-production function with respect to log-labor and log-material inputs:

$$\begin{aligned}\widehat{\frac{\partial^2 \tilde{g}_i}{\partial \tilde{\ell}^2_{ik}}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) &:= \frac{1}{\widehat{\mathcal{E}}^{\ell}_i}\Big\{(\hat{\zeta}_{i,1} + 2\hat{\zeta}_{i,3})\tilde{\ell}_{ik} + \hat{\zeta}_{i,5}\tilde{m}_{ik}\Big\}, \\ \widehat{\frac{\partial^2 \tilde{g}_i}{\partial \tilde{\ell}_{ik}\tilde{m}_{ik}}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) &:= \frac{1}{\widehat{\mathcal{E}}^{\ell}_i}\Big\{(\hat{\zeta}_{i,2} + 2\hat{\zeta}_{i,4})\tilde{m}_{ik} + \hat{\zeta}_{i,5}\tilde{\ell}_{ik}\Big\}.\end{aligned}$$

Note that $\widehat{\frac{\partial^2 \tilde{g}_i}{\partial \tilde{m}^2_{ik}}}(\tilde{\ell}_{ik}, \tilde{m}_{ik})$ can be analogously defined by applying the same argument as above to the share regression with respect to material input $\tilde{m}_{ik}$.

Guided by the identification result (Lemma C.11), the estimates for the equilibrium values of the second-order derivatives of the production functions are given by

$$\widehat{\frac{\partial^2 f(\ell^*_{ik}, m^*_{ik})}{\partial \ell^2_{ik}}} = \frac{q^*_{ik}}{(\ell^*_{ik})^2}\left\{\widehat{\frac{\partial^2 \tilde{g}_i}{\partial \tilde{\ell}^2_{ik}}}(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik}) + \left(\widehat{\frac{\partial \tilde{g}_i}{\partial \tilde{\ell}_{ik}}}(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik})\right)^2 - \widehat{\frac{\partial \tilde{g}_i}{\partial \tilde{\ell}_{ik}}}(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik})\right\},$$

and

$$\widehat{\frac{\partial^2 f(\ell^*_{ik}, m^*_{ik})}{\partial \ell_{ik}\partial m_{ik}}} = \frac{q^*_{ik}}{\ell^*_{ik}m^*_{ik}}\left\{\widehat{\frac{\partial^2 \tilde{g}_i}{\partial \tilde{\ell}_{ik}\partial \tilde{m}_{ik}}}(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik}) + \widehat{\frac{\partial \tilde{g}_i}{\partial \tilde{\ell}_{ik}}}(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik})\widehat{\frac{\partial \tilde{g}_i}{\partial \tilde{m}_{ik}}}(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik})\right\}.$$

The estimates $\widehat{\frac{\partial^2 f(\ell^*_{ik}, m^*_{ik})}{\partial m^2_{ik}}}$ is also obtained in an analogous manner.

Remark D.1. *In general, it is not possible to obtain estimates of* $\frac{\partial^2 f(\ell_{ik}, m_{ik})}{\partial \ell^2_{ik}}$ *and* $\frac{\partial^2 f(\ell_{ik}, m_{ik})}{\partial \ell_{ik}\partial m_{ik}}$ *for arbitrary values of* ℓ_{ik} *and* m_{ik}*, as they are not identified for every pair of points* (ℓ_{ik}, m_{ik}) *in* $\mathscr{L}_i \times \mathscr{M}_i$*. Nevertheless, Lemma C.11 implies that there is still a hope of estimating the values of these functions on the point* (ℓ^*_{ik}, m^*_{ik})*.*

D.3 First- and Second-Order Derivatives of the Quantity Index

To begin with, it holds from (67) and (68) that

$$\sum_{k=1}^{N_i} \frac{1}{\Phi_i} \exp\left\{\tilde{\varphi}_i\left(\ln \frac{q_{ik}}{A_i(\mathbf{q}_i)}\right)\right\} = 1. \tag{112}$$

Let $x_{ik} \coloneqq \frac{q_{ik}}{A_i(\mathbf{q}_i)}$ and $\tilde{x}_{ik} \coloneqq \ln x_{ik}$. Taking derivatives of (112) with respect to $\bar{k} \in \mathbf{N}_i$,

$$\frac{1}{\Phi_i}\exp\{\tilde{\varphi}_i(\tilde{x}_{i\bar{k}})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{i\bar{k}}}\frac{d\ln x_{i\bar{k}}}{dx_{i\bar{k}}}\frac{\partial\frac{q_{i\bar{k}}}{A_i}}{\partial q_{i\bar{k}}} + \sum_{k\neq\bar{k}}\frac{1}{\Phi_i}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}}\frac{d\ln x_{ik}}{dx_{ik}}\frac{\partial\frac{q_{ik}}{A_i}}{\partial q_{i\bar{k}}} = 0.$$

Since here

$$\begin{aligned}\frac{\partial\frac{q_{i\bar{k}}}{A_i}}{\partial q_{i\bar{k}}} &= A_i^{-1} - q_{i\bar{k}}A_i^{-2}\frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}}\\ &= \frac{1}{A_i}\left(1 - \frac{q_{i\bar{k}}}{A_i}\frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}}\right),\end{aligned} \tag{113}$$

and

$$\frac{\partial\frac{q_{ik}}{A_i}}{\partial q_{i\bar{k}}} = -\frac{1}{A_i}\frac{q_{ik}}{A_i}\frac{\partial A_i(\cdot)}{\partial q_{ik}}, \tag{114}$$

I then have

$$\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{i\bar{k}}}\frac{1}{q_{i\bar{k}}}\left(1 - \frac{q_{i\bar{k}}}{A_i}\frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}}\right) + \sum_{k\neq\bar{k}}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}}\frac{1}{A_i}\frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}} = 0$$

$$\therefore \exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{i\bar{k}}}\frac{1}{q_{i\bar{k}}} = \frac{1}{A_i}\frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}}\sum_{k=1}^{N_i}\exp\{\tilde{x}_{ik}\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}} \tag{115}$$

$$\therefore \frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}} = \frac{A_i}{q_{i\bar{k}}}\frac{\exp\{\tilde{\varphi}_i(\tilde{x}_{i\bar{k}})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{i\bar{k}}}}{\sum_{k=1}^{N_i}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}}}. \tag{116}$$

Substituting (116) back into (113) and (114), I obtain

$$\frac{\partial\frac{q_{i\bar{k}}}{A_i}}{\partial q_{i\bar{k}}} = \frac{1}{A_i}\left(1 - \frac{\exp\{\tilde{\varphi}_i(\tilde{x}_{i\bar{k}})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{i\bar{k}}}}{\sum_{k=1}^{N_i}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}}}\right), \tag{117}$$

and

$$\frac{\partial\frac{q_{ik}}{A_i}}{\partial q_{i\bar{k}}} = -\frac{1}{A_i}\frac{\exp\{\tilde{\varphi}_i(\tilde{x}_{i\bar{k}})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{i\bar{k}}}}{\sum_{k=1}^{N_i}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}}}. \tag{118}$$

Next, I aim to derive analytical expressions for $\frac{\partial^2 A_i(\cdot)}{\partial q_{i\bar{k}}^2}$ and $\frac{\partial^2 A_i(\cdot)}{\partial q_{i\bar{k}}q_{i\bar{k}'}}$ for $\bar{k}, \bar{k}' \in \mathbf{N}_i$, in the sequel. As a starting point, I rewrite (115) as

$$\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{i\bar{k}}} = \frac{q_{i\bar{k}}}{A_i}\frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}}\sum_{k=1}^{N_i}\exp\{\tilde{x}_{ik}\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}}. \tag{119}$$

Let $lhs_{i\bar{k}}(\mathbf{q}_i)$ and $rhs_{i\bar{k}}(\mathbf{q}_i)$ denote the left and right hand sides of this equation, respectively.

Taking derivatives of these with respect to $q_{i\bar{k}}$ delivers

$$\begin{aligned}\frac{\partial lhs_i(\cdot)}{\partial q_{i\bar{k}}} &= \exp\{\tilde{\varphi}_i(\tilde{x}_{i\bar{k}})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{i\bar{k}}}\frac{d\ln x_{i\bar{k}}}{dx_{i\bar{k}}}\frac{\partial\frac{q_{i\bar{k}}}{A_i}}{\partial q_{i\bar{k}}}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{i\bar{k}}} + \exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial^2\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{i\bar{k}}^2}\frac{d\ln x_{i\bar{k}}}{dx_{i\bar{k}}}\frac{\partial\frac{q_{i\bar{k}}}{A_i}}{\partial q_{i\bar{k}}} \\ &= \exp\{\tilde{\varphi}_i(\tilde{x}_{i\bar{k}})\}\frac{A_i}{q_{i\bar{k}}}\frac{\partial\frac{q_{i\bar{k}}}{A_i}}{\partial q_{i\bar{k}}}\left\{\left(\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{i\bar{k}}}\right)^2 + \frac{\partial^2\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{i\bar{k}}^2}\right\},\end{aligned} \tag{120}$$

and

$$\begin{aligned}\frac{\partial rhs_i(\cdot)}{\partial q_{i\bar{k}}} =& \frac{\partial\frac{q_{i\bar{k}}}{A_i}}{\partial q_{i\bar{k}}}\frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}}\sum_{k=1}^{N_i}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}} \\ &+ \frac{q_{i\bar{k}}}{A_i}\frac{\partial^2 A_i(\cdot)}{\partial q_{i\bar{k}}^2}\sum_{k=1}^{N_i}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}} \\ &+ \frac{q_{i\bar{k}}}{A_i}\frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}}\frac{\partial}{\partial q_{i\bar{k}}}\sum_{k=1}^{N_i}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}} \\ =& \frac{\partial\frac{q_{i\bar{k}}}{A_i}}{\partial q_{i\bar{k}}}\frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}}\sum_{k=1}^{N_i}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}} \\ &+ \frac{q_{i\bar{k}}}{A_i}\frac{\partial^2 A_i(\cdot)}{\partial q_{i\bar{k}}^2}\sum_{k=1}^{N_i}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}} \\ &+ \frac{q_{i\bar{k}}}{A_i}\frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}}\sum_{k=1}^{N_i}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{A_i}{q_{ik}}\frac{\partial\frac{q_{ik}}{A_i}}{\partial q_{i\bar{k}}}\left\{\left(\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}}\right)^2 + \frac{\partial^2\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}^2}\right\}.\end{aligned} \tag{121}$$

Clearly, taking derivative of the both hand sides of (119) with respect to $q_{i\bar{k}}$ is tantamount to equating (120) to (121). After some algebra, I arrive at

$$\begin{aligned}\frac{\partial^2 A_i(\cdot)}{\partial q_{i\bar{k}}^2} =& -\frac{A_i}{q_{i\bar{k}}}\frac{\partial\frac{q_{i\bar{k}}}{A_i}}{\partial q_{i\bar{k}}}\frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}} \\ &- \frac{A_i}{q_{i\bar{k}}}\left(\sum_{k=1}^{N_i}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}}\right)^{-1} \\ &\times\left[\frac{q_{i\bar{k}}}{A_i}\frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}}\sum_{k=1}^{N_i}\left\{\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\right\}\frac{A_i}{q_{ik}}\frac{\partial\frac{q_{ik}}{A_i}}{\partial q_{i\bar{k}}}\left\{\left(\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}}\right)^2 + \frac{\partial^2\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}^2}\right\}\right. \\ &\left. - \exp\{\tilde{\varphi}_i(\tilde{x}_{i\bar{k}})\}\frac{A_i}{q_{i\bar{k}}}\frac{\partial\frac{q_{i\bar{k}}}{A_i}}{\partial q_{i\bar{k}}}\left\{\left(\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{i\bar{k}}}\right)^2 + \frac{\partial^2\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{i\bar{k}}^2}\right\}\right].\end{aligned}$$

Analogously, I can obtain

$$\begin{aligned}\frac{\partial^2 A_i(\cdot)}{\partial q_{i\bar{k}}\partial q_{i\bar{k}'}} =& -\frac{A_i}{q_{i\bar{k}}}\frac{\partial\frac{q_{i\bar{k}}}{A_i}}{\partial q_{i\bar{k}'}}\frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}} \\ &- \frac{A_i}{q_{i\bar{k}}}\left(\sum_{k=1}^{N_i}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{\partial\tilde{\varphi}_i(\cdot)}{\partial\tilde{x}_{ik}}\right)^{-1}\end{aligned}$$

$$
\times \left[\frac{q_{i\bar{k}}}{A_i} \frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}} \sum_{k=1}^{N_i} \left\{ \exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\} \right\} \frac{A_i}{q_{ik}} \frac{\partial \frac{q_{ik}}{A_i}}{\partial q_{i\bar{k}'}} \left\{ \left(\frac{\partial \tilde{\varphi}_i(\cdot)}{\partial \tilde{x}_{ik}} \right)^2 + \frac{\partial^2 \tilde{\varphi}_i(\cdot)}{\partial \tilde{x}_{ik}^2} \right\} \right.
$$

$$
\left. - \exp\{\tilde{\varphi}_i(\tilde{x}_{i\bar{k}})\} \frac{A_i}{q_{i\bar{k}}} \frac{\partial \frac{q_{i\bar{k}}}{A_i}}{\partial q_{i\bar{k}'}} \left\{ \left(\frac{\partial \tilde{\varphi}_i(\cdot)}{\partial \tilde{x}_{i\bar{k}}} \right)^2 + \frac{\partial^2 \tilde{\varphi}_i(\cdot)}{\partial \tilde{x}_{i\bar{k}}^2} \right\} \right].
$$

Note that $\frac{\partial A_i(\cdot)}{\partial q_{i\bar{k}}}$, $\frac{\partial \frac{q_{i\bar{k}}}{A_i}}{\partial q_{ik}}$ and $\frac{\partial \frac{q_{i\bar{k}}}{A_i}}{\partial q_{i\bar{k}'}}$ are already obtained in (116), (117) and (118), respectively.[106]

E Validation of the Estimation Procedure: Simulation Study

This section verifies the validity of the estimation strategy described in Section D through numerical simulations under a parametric specification that is widely used in the literature. Using the parametric model, I first generate simulation data for firm-level revenues, labor and material inputs, productivity, prices, quantity, and other aggregate variables.[107] Next I repeat the same simulation with a different choice of the policy value, and then calculate the change in GDP to measure the policy effects (the estimates based on this method is referred to as the "simulation-based estimates"). Now, the question is if the researcher can correctly estimate the policy effects without relying on the knowledge about the underlying parametric model. To highlight this, I also compute the policy effects using the results developed in Sections 5 and D (the estimates obtained by this approach is called the "theory-based estimates").

To simplify the comparison, it is assumed that under the current policy regime, no subsidies are imposed, i.e., $\tau_i^0 = 0$ for all $i \in \{1, 2, 3\}$. In estimating the policy effects, moreover, the researcher are not allowed to use the realization of productivity, prices and quantity as these are not observed in the real data either (see Section 4).

E.1 Setup

This subsection sets out the parametric form assumptions for the data generating process of this simulation. See Grassi (2017) for the detail of the theoretical properties.

The sectoral aggregator is assumed to be a constant elasticity of substitution (CES) production function:

$$
Q_i = \left(\sum_{k=1}^{N_i} \delta_{ik} q_{ik}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},
$$

where σ is elasticity of substitution and δ_{ik} stands for a demand shifter.

In each sector i, individual firm k transforms labor ℓ_{ik} and material m_{ik} into output q_{ik} using a Cobb-Douglas production function:

$$
q_{ik} = z_{ik} \ell_{ik}^{\alpha} m_{ik}^{1-\alpha},
$$

[106] Index needs to be relabeled appropriately.

[107] These data can be viewed either as the "true data" that realize from the data generating process, or the values that have been computed under the parameter values so calibrated.

where the output elasticity represents α and z_{ik} is productivity.

Material input is composed of sectoral intermediate goods according to the Cobb-Douglas production:

$$m_{ik} = \prod_{j=1}^{N} m_{ik,j}^{\gamma_{i,j}},$$

where $\gamma_{i,j}$ corresponds to the input share of sector j's intermediate good, reflecting the production network Ω.

E.2 Simulation Design

For ease of comparison, I assume that there are only three sectors in the economy (i.e., $N = 3$), each of which is populated by the identical set of firms with the number of firms being 50: i.e., $N_i = 50$ for all $i \in \{1, 2, 3\}$.

E.2.1 Production Networks & Subsidy

I consider two specifications for the production networks Ω. Specification 1 refers to a simple case in which no sectors are linked with each other (Panel (a) of Table 6). Firms in this specification produce output combining labor force and its own sectoral good only. Specification 2, on the other hand, allows firms to trade with other sectors (Panel (b) of Table 6). Firms in sector 1 under this specification purchase intermediate good from sector 2 with the cost share being 20%, while the remaining portion is accounted for by their own sector's good. Sector 3 is assumed to be symmetric to sector 1; i.e., the cost share of sector 2's good in firms in sector 3 is 20% and that of sector 3's good is 80%. Sector 2 purchases both from sector 1 and 3, each accounting for 20% of the firms production costs in the sector.

The focus of this experiment is on the effect of subsidizing purchase from a particular sector by 0.1%, i.e., $\tau_i^1 = 0.001$ for some $i \in \{1, 2, 3\}$. Assume that *ex ante* the economy features no subsidy (i.e., $\tau_n^0 = 0$ for all $n \in \{1, 2, 3\}$). Since firms are symmetric in Specification 1, I only look at the scenario where purchase of intermediate goods from sector 1 is subsidized. In Specification 2, however, the production network among sectors displays asymmetric pattern so that it is natural to expect to see differential effects across sectors depending on which sector is subsidized. I thus look into the effects of subsidizing sector 1 or 3 (Scenario 1) and sector 2 (Scenario 2) separately.[108]

Table 6: Adjacency Matrix for Specification 1 and 2

(a) Specification 1

$$\Omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) Specification 2

$$\Omega = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.2 & 0.6 & 0.2 \\ 0 & 0.2 & 0.8 \end{bmatrix}$$

Note: This table shows the adjacency matrix for Specification 1 (Panel (a)) and 2 (Panel (b)). The ij entry of the matrix Ω, $\omega_{i,j}$, indicates the share of sector j's intermediate good in the expenditure of firms in sector i.

[108] In Scenario 1, I can assume, with out loss of generality, that sector 1 is the sector that is subsidized.

E.2.2 Other Parameter Values

Parameter values are chosen in such a way that the Cournot-Nash equilibrium is well-defined. First, firms' heterogenous productivities are drawn from a log normal distribution: $z_{ik} \sim log(N(0, 0.02))$. I set $\alpha = 0.6$, $\sigma = 1.1$ (i.e., firms' products are substitutable) and $\delta_{ik} = (1/N_i)^{1/\sigma_i} =$ for all $i \in \{1, 2, 3\}$ and $k \in \{1, \ldots, N_i\}$.

The researcher has access to firm-level revenue, labor and material inputs, as well as aggregate variables; no access to firm-level productivities, prices and quantities. Consistent with my framework, the observed revenue is contaminated with a measurement error η_{ik}.[109]

Lastly I fix the wage rage at $W = 1$ throughout the simulation study, meaning that I focus on a partial equilibrium exercise. Also, I abstract from the firm's entry problem.

E.3 Results

[109] The measurement error is assumed to enter in a linear, additive fashion in logs; i.e., $\log r_{ik} = \log \bar{r}_{ik} + \log \eta_{ik}$, where $\bar{r}_{ik}$ and $\bar{r}_{ik}$ are the observed and true (simulated) revenue, respectively, with $E[\log \eta_{ik} \mid \ell_{ik}, m_{ik}] = 0$. See Section C.1.2.

Table 7: Simulation Results: The Policy Effects

	Model (I)	Model (II)	Model (III)	Model (IV)
Sector 1				
The effects on revenue				
price effect	-5889.9971	-7433.5995	-5876.1507	-7517.3979
	(-5492.7537)	(-7050.4060)	(-5680.9199)	(-7291.9330)
quantity effect	6478.9280	8176.8724	5876.1507	7517.3979
	(6042.0290)	(7755.4466)	(5680.9199)	(7291.9330)
The effects on input cost				
wealth effect	-235.5826	-245.0903	-209.3945	-220.1727
	(-199.7365)	(-210.5693)	(-202.4459)	(-213.4256)
switching effect	516.5769	665.5503	478.9621	625.8674
	(519.7847)	(667.1325)	(506.5706)	(650.1730)
The total effects	307.9366	322.8128	-269.5676	-405.6946
	(229.2271)	(248.4774)	(-304.1247)	(-436.7474)
Sector 2				
The effects on revenue				
price effect	0.0000	-1120.1258	0.0000	-1020.2217
	(0.0000)	(-931.2232)	(0.0000)	(-963.1243)
quantity effect	(-0.0000)	(1232.1253)	(-0.0000)	(1020.2217)
	(-0.0000)	(1024.3456)	(-0.0000)	(963.1243)
The effects on input cost				
wealth effect	(0.0000)	(-101.7693)	(0.0000)	(-89.0697)
	(0.0000)	(-84.6398)	(0.0000)	(-85.7879)
switching effect	(0.0000)	(99.5871)	(0.0000)	(84.3267)
	(-0.0000)	(88.0634)	(-0.0000)	(85.8254)
The total effects	(0.0000)	(114.1816)	(0.0000)	(4.7430)
	(-0.0000)	(89.6987)	(0.0000)	(-0.0375)
Sector 3				
The effects on revenue				
price effect	0.0000	-122.8816	0.0000	-99.8181
	(0.0000)	(-88.4324)	(0.0000)	(-91.4618)
quantity effect	-0.0000	135.1683	-0.0000	99.8181
	(-0.0000)	(97.2756)	(-0.0000)	(91.4618)
The effects on input cost				
wealth effect	0.0000	-11.1644	0.0000	-8.7145
	(0.0000)	(-8.0392)	(0.0000)	(-8.1482)
switching effect	0.0000	11.0586	0.0000	8.3437
	(-0.0000)	(8.3611)	(-0.0000)	(8.1485)
The total effects	0.0000	12.3926	0.0000	0.3709
	(-0.0000)	(8.5213)	(0.0000)	(-0.0003)
The change in GDP	307.9366	449.3870	-269.5676	-400.5808
	(229.2271)	(346.6975)	(-304.1247)	(-436.7853)

Note:

F Empirical Applications

F.1 Revisiting Liu (2019)

In this subsection, I revisit Liu (2019), who characterize the policy effects on output in terms of observable variables for the multi-sector economy with exogenous market distortions under production networks. To put my framework into perspective, I look to GDP net of firms' profits, defined as $Y^{Net} := Y - \Pi$.[110] In what follows, I refer to it simply as net GDP. It follows from (18) and (20) that $Y^{Net} = WL - \sum_{i=1}^{N} \sum_{k=1}^{N_i} \sum_{j=1}^{N} \tau_i P_j m_{ik,j}$.[111] Total differentiation delivers

$$\left.\frac{dY^{Net}}{d\tau_n}\right|_{\boldsymbol{\tau}=\boldsymbol{\tau}^0} = \left.\frac{d(WL)}{d\tau_n}\right|_{\boldsymbol{\tau}=\boldsymbol{\tau}^0} - \sum_{k=1}^{N_i} \sum_{j=1}^{N} P_j m_{nk,j} - \left.\sum_{i=1}^{N} \sum_{k=1}^{N_i} \sum_{j=1}^{N} \tau_i \left(\frac{dP_j}{d\tau_n} m_{ik,j} + P_j \frac{dm_{ik,j}}{d\tau_n} \right)\right|_{\boldsymbol{\tau}=\boldsymbol{\tau}^0}. \quad (122)$$

In (122), the first term of the right hand side is the policy effect on the household income, and the second and third terms, respectively, represent the direct and indirect effects on the policy expenditure. While Liu (2019) characterizes the first term in terms of aggregate observables, he sidesteps the identification of the third term by focusing on a special case where $\boldsymbol{\tau}^0 = \mathbf{0}$, so that (122) simplifies to

$$\left.\frac{dY^{Net}}{d\tau_n}\right|_{\boldsymbol{\tau}=0} = \left.\frac{d(WL)}{d\tau_n}\right|_{\boldsymbol{\tau}=0} - \sum_{k=1}^{N_i} \sum_{j=1}^{N} P_j m_{nk,j}. \quad (123)$$

By contrast, my framework explicitly recovers the third term of (122) without the need for restricting the value of $\boldsymbol{\tau}$ to zero.[112]

To study the consequence of simply applying the approach of Liu (2019), Figure compares the estimates of net GDP based on (122) and (123).

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[110] In Liu (2019), the market distortions are thrown out from the economy and thus the "output" in his paper corresponds to the GDP net of firms' profits in my framework. Bigio and La'O (2020) consider the setup where part of the firms' profits are rebated back to the household's budget while the remaining portion are thrown out.

[111] That is, Y^{Net} equals to the household's labor income net of policy expenditure.

[112] Note that in this section it is assumed that $\frac{dW}{d\tau_n} = 0$ and my model posits that the labor supply is inelastic. Hence $\left.\frac{d(WL)}{d\tau_n}\right|_{\boldsymbol{\tau}=\boldsymbol{\tau}^0} = 0$ for any value of $\boldsymbol{\tau}^0$.

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