Event-By-Event Prediction, Gating, and Cancellation notes for me to refer

Goal and setting. Given a real event $e_i = (\boldsymbol{x}_i, t_i, p_i)$ and a local rotational ego-motion model around $\boldsymbol{c}$ with angular velocity ω, we predict the event Δt seconds into the future and, when the clock reaches $t_i + \Delta t$, *match* the real and predicted events within spatio-temporal tolerances. If matched (plus a polarity rule), we treat the pair as ego-motion and cancel both; if not, we keep the event as likely scene motion. This mechanism follows the event-warping/contrast-maximization view of motion compensation [**??**] and the event sensor timing model [**??**].

1 Rigid rotation & forward prediction

For a 2-D point $\boldsymbol{x}$ rotating about $\boldsymbol{c}$, the rigid rotation is

$$\mathcal{R}(\boldsymbol{x}; \boldsymbol{c}, \theta) = \boldsymbol{c} + R(\theta)(\boldsymbol{x} - \boldsymbol{c}), \qquad R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}. \tag{1}$$

This is the same transform used when *warping* events to a reference time in contrast-maximization methods [**??**]. Setting $\theta = \omega\Delta t$ gives the forward prediction

$$\boldsymbol{x}_i' = \mathcal{R}(\boldsymbol{x}_i; \boldsymbol{c}, \omega\Delta t), \qquad t_i' = t_i + \Delta t, \qquad \bar{p}_i = -p_i, \tag{2}$$

where the opposite polarity follows the short-horizon sign flip needed to suppress redundant contrast (consistent with event formation and polarity conventions summarized in [**??**]).

2 Estimating $(\boldsymbol{c}, \omega)$

Center by algebraic circle LS (Kåsa family). Fit $x^2 + y^2 + a\,x + b\,y + c = 0$ to points (x_k, y_k) via least squares. With $A = [x\ y\ 1]$ and $b = -(x^2 + y^2)$, solve $A[a, b, c]^T = b$ (in LS sense), then

$$c_x = -\frac{a}{2}, \quad c_y = -\frac{b}{2}, \quad r = \sqrt{c_x^2 + c_y^2 - c}. \tag{3}$$

This standard derivation appears in classic circle-fitting treatments (Kåsa/Delogne families).

Angular velocity from unwrapped phase slope. Let $\theta_k = \mathrm{unwrap}\{\mathrm{atan2}(y_k - c_y,\ x_k - c_x)\}$. Regress

$$\theta_k \approx \theta_0 + \omega\,(t_k - t_0) \quad \Rightarrow \quad \widehat{\omega} = \mathrm{slope}(\theta \text{ vs. } t), \tag{4}$$

which is the rotation-only specialization consistent with event-based motion estimation frameworks [**??**].

3 Temporal gate (decision-time test)

Definition. For each real event e_i at t_i, define the *decision time* $t^\star = t_i + \Delta t$ and accept only real events whose timestamps lie in a tight band around $t^\star$:

$$|t_j - t^\star| \le \epsilon_t. \tag{5}$$

Derivation and rationale. Event warping/transport [??] treats motion compensation as aligning events to a reference time. Here, the reference for e_i is precisely $t_i + \Delta t$, where the predicted anti-event arrives. Due to microsecond-scale latency/jitter and timestamp noise in DVS-type sensors [??], the true arrival falls in a narrow interval of width $2\epsilon_t$. Choosing $\epsilon_t \gtrsim 3\sigma_t$ (sensor timing stdev) captures the bulk of on-model arrivals while rejecting unrelated events. This implements exactly the intuitive rule: *wait until the clock reaches $t_i + \Delta t$, then only look nearby in time.*

4 Spatial gate (error-budget proof)

Definition. Among temporally valid candidates, accept matches whose positions are close to the forward prediction:

$$\|\boldsymbol{x}_j - \boldsymbol{x}'_i\|_2 \le \epsilon_{xy}. \tag{6}$$

Error budget $\Rightarrow$ sufficiency condition. Let $r = \|\boldsymbol{x}_i - \boldsymbol{c}\|_2$. Three first-order contributors dominate the spatial prediction error (consistent with the kinematics used by [[5]] and the timing model of [??]):

(i) **Angular-velocity bias** $\Delta\omega = \hat{\omega} - \omega^\star$ over horizon Δt causes an angle error $\delta\theta = \Delta\omega\,\Delta t$, producing chordal error

$$\varepsilon_\omega(r, \Delta t) = 2r\left|\sin(\delta\theta/2)\right| \approx r\,|\Delta\omega|\,\Delta t \quad \text{(small-angle)}. \tag{7}$$

(ii) **Center bias** $\Delta\boldsymbol{c}$ perturbs the rotation center, giving a first-order bound

$$\varepsilon_c(r) \lesssim \|\Delta\boldsymbol{c}\|_2. \tag{8}$$

(iii) **Timing uncertainty** σ_t yields phase std. $\sigma_\theta \approx |\omega^\star|\sigma_t$, hence spatial std. $\sigma_x \approx r\,|\omega^\star|\,\sigma_t$.

$$\sigma_x \approx r\,|\omega^\star|\,\sigma_t. \tag{9}$$

A conservative acceptance radius is therefore

$$\varepsilon_\omega(r, \Delta t) + \varepsilon_c(r) + \sigma_x \le \epsilon_{xy}, \tag{10}$$

which directly justifies the fixed spatial gate in (6). Intuitively: *only keep matches whose spatial discrepancy is explainable by model rate/center error and sensor timing noise.*

5 Matching & cancellation policy

Polarity rule. Use opposite polarity by default ($p_j = -p_i$), consistent with short-horizon compensation and polarity conventions [??]. (Other modes: equal/ignore for sensitivity analyses.)

One-to-one matching. Within the gated set, pick the nearest neighbor in ℓ_2 and mark both predicted & real events as matched (dedup). This mirrors the greedy association used in practical warping-based pipelines ([??]; see also predictive suppression rationale below).

Cancellation. If the temporal, spatial, and polarity gates pass, treat the pair as ego-motion and remove both from residuals. Otherwise, retain the real event—likely scene motion (e.g., a hand) that violates the circular model.

Predictive rationale. The eliminate-on-match behavior is the event-domain analogue of predictive coding: the model emits an *anti-event* that cancels expected input; only *surprise* remains [??].

References

[1] P. Lichtsteiner, C. Posch, T. Delbrück, "A 128×128 120 dB 15 µs Latency Asynchronous Temporal Contrast Vision Sensor," *IEEE JSSC*, 2008.
https://ieeexplore.ieee.org/document/4444573

[2] C. Bardow, A. J. Davison, S. Leutenegger, "Simultaneous Optical Flow and Intensity Estimation from an Event Camera," *CVPR*, 2016.
https://openaccess.thecvf.com/content_cvpr_2016/papers/Bardow_Simultaneous_Optical_Flow_CVPR_2016_paper.pdf

[3] G. Gallego, D. Scaramuzza, "Accurate Angular Velocity Estimation with an Event Camera," *RA-L*, 2017.
https://www.ifi.uzh.ch/dam/jcr%3A18eec387-4285-497d-86a4-0cee500f179c/RAL16_Gallego.pdf

[4] C. Posch et al., "ATIS: A Sensitive and Fast Vision Sensor," *IEEE Sensors Journal*, 2014.
https://ieeexplore.ieee.org/document/6865714

[5] G. Gallego et al., "A Unifying Contrast Maximization Framework for Event Cameras," *CVPR*, 2018.
https://openaccess.thecvf.com/content_cvpr_2018/papers/Gallego_A_Unifying_Contrast_CVPR_2018_paper.pdf

[6] G. Gallego et al., "Event-based Vision: A Survey," *IEEE TPAMI*, 2022.
https://www.computer.org/csdl/journal/tp/2022/01/09138762/1llK3L5znva

[7] T. Hosoya, S. A. Baccus, M. Meister, "Dynamic Predictive Coding by the Retina," *Nature*, 2005.
https://meisterlab.caltech.edu/documents/8137/hosoya_2005_dynamic.pdf

[8] R. P. N. Rao, D. H. Ballard, "Predictive Coding in the Visual Cortex," *Nature Neuroscience*, 1999.
https://www.cs.utexas.edu/~dana/nn.pdf

How each source verifies the math

- **Warping/prediction** (section 1): identical rigid warp as in [[5]] and [[2]]; (1)–(2) are rotation-only versions of event warping.
- **Temporal gate** (section 3): decision-time test matches alignment to reference time; ϵ_t grounded in sensor latency/jitter models [**??**].
- **Spatial gate** (section 4): error budget (7)–(9) combines small-angle kinematics and timing noise [**??**]; sufficiency (10) mirrors robust bands in [**??**].
- **Angular velocity** (section 2): phase-slope (4) agrees with rotational estimation in [**??**].
- **Polarity/cancellation** (section 5): predictive suppression aligns with biological predictive coding [**??**].