$\lim_{t\to a} r(t) = \langle \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t)\rangle$

$r'(t) = \langle f'(t), g'(t), h'(t)\rangle dt$

$\frac{d}{du}[u(t)\times v(t)] = u'(t)\times v(t) + u(t)\times v'(t)$

$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\, dt$

If $\lim_{(a,b)} f(x,y) = L$ then $\forall \varepsilon\ \exists \delta > 0$
s.t if $(x,y)\in D$ and $0 < \sqrt{(x-a)^2+(y-b)^2} < \delta$
then $|f(x,y) - L| < \varepsilon$

Test limit along many paths, $x=0, y=x^2, x=y^2; y=0$ etc

Equation of plane tangent to surface $z = f(x,y)$:

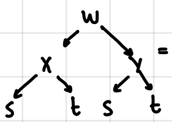
$z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

The linear approximation of $f(x,y)$ at a point is the equation of the plane tangent to the surface

Point estimate: $L(a,b) = f_x(x_0,y_0)(a-x_0) + f_y(x_0,y_0)(b-y_0) + f(x_0,y_0)$

Differential: $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \dots$

Chain rule: construct a diagram



$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s}$

$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$

$\frac{dx}{dy} = -\frac{\partial F/\partial x}{\partial F/\partial y} = -\frac{F_x}{F_y}$; $\frac{\partial z}{\partial x} = -\frac{\partial F/\partial x}{\partial F/\partial z}$ $\frac{\partial z}{\partial y} = \frac{\partial F/\partial x}{\partial F/\partial z}$

$D_u f(x,y) = f_x(x,y)a + f_y(x,y)b \Rightarrow \nabla f = \langle f_x, f_y\rangle$

make sure to make the vector into a unit vector if it isn't already

f increases the fastest in the direction of the gradient vector and the maximum rate of change is $|\nabla f(a,b)|$.

Tangent Plane to a level surface

$F_x(x_0,y_0,z_0)(x-x_0) + F_y(x_0,y_0,z_0)(y-y_0) + F_z(x_0,y_0,z_0)(z-z_0) = 0$

Minimum and Maximum:

$D = D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - [f_{xy}(a,b)]^2$

If $D>0$ and $f_{xx}(a,b)>0$, then $f(a,b)$ is a min

If $D>0$ and $f_{xx}(a,b)<0$, then $f(a,b)$ is a saddle point

If $D=0$, test gives no information

Global minimum and maximum

1. Find values of f at critical points of in D
2. Find the extreme values of f on the boundary of D
3. The largest values from step 1 and 2 is the absolute maximum value.

$\nabla f(x_0,y_0,z_0) = \lambda\nabla g(x_0,y_0,z_0)$
where $g(x,y,z) = k$.

$\nabla f(x_0,y_0,z_0) = \lambda\nabla g(x_0,y_0,z_0) + \mu\nabla h(x_0,y_0,z_0)$
where $g(x,y,z) = k$ and $h(x,y,z) = c$

$\iint_D f(x,y)\,dA = \int_a^b\int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx = \int_c^d\int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$

$r^2 = x^2+y^2$ $x = r\cos\theta$ $y = r\sin\theta$; $R = \{(r,\theta) \mid a\le r\le b, \alpha\le\theta\le\beta\}$

$\iint_R f(x,y)\,dA = \int_\alpha^\beta\int_a^b f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$ DONT FORGET r

$A(S) = \iint_D \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\, dA$

Over Box $= [a,b]\times[c,d]\times[r,s]$

$\iiint_B f(x,y,z)\,dV = \int_r^s\int_c^d\int_a^b f(x,y,z)\,dx\,dy\,dz$

$\int_{\text{domain}}\int_{\text{Range}}\int_{\text{lower equation}}^{\text{upper equation}} (\text{Integrand})\, dV$

$m = \iiint_E \rho(x,y,z)\,dV$

Moments:

$M_{yz} = \iiint_E x\rho(x,y,z)\,dV$ $M_{xz} = \iiint_E y\rho(x,y,z)\,dV$

$M_{xy} = \iiint_E z\rho(x,y,z)\,dV$

center of mass: $\left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m}\right)$

moments of inertia

$I_x = \iiint_E (y^2+z^2)\rho(x,y,z)\,dV$

$I_y = \iiint_E (x^2+z^2)\rho(x,y,z)\,dV$

$I_z = \iiint_E (x^2+y^2)\rho(x,y,z)\,dV$

$r = \langle x,y,z\rangle$, $r_0 = \langle x_0,y_0,z_0\rangle$, $a = \langle a_1,a_2,a_3\rangle$, $b = \langle b_1,b_2,b_3\rangle$ } Parametric equation of plane

$\Rightarrow x = x_0+ua_1+vb_1$, $y = y_0+ua_2+vb_2$, $z = z_0+ua_3+vb_3$

$x = x$, $y = f(x)\cos\theta$, $z = f(x)\sin\theta$ } equation of revolution

Tangent plane equation: $\langle r_u\times r_v\rangle\cdot\langle x-x_0, y-y_0, z-z_0\rangle$

Surface Area: $A(S) = \iint_D |r_u\times r_v|\,dA$ or $\iint_D \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dA$

surface integral: $\iint_S f(x,y,z)\,dS = \iint_D f(r(u,v))|r_u\times r_v|\,dA$

Cylindrical Coordinates

$\text{div}\,F = \nabla\cdot F$

$\text{curl}\,F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f(x) & f(y) & f(z)\end{vmatrix}$

$x = r\cos\theta$, $y = r\sin\theta$, $z = z$

$r^2 = x^2+y^2$, $\tan\theta = \frac{y}{x}$, $z = z$

$\iiint_E f(x,y,z)\,dV = \int_\alpha^\beta\int_{h_1(\theta)}^{h_2(\theta)}\int_{u_1(r\cos\theta, r\sin\theta)}^{u_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)\, r\,dz\,dr\,d\theta$

$\int_C F\cdot dr = \iint_S \text{curl}\,F\cdot dS = \iint_S \text{curl}\,F\cdot n\,dS$

$\iint_S F\,dS = \iiint_E \text{div}\,F\cdot dV$

Spherical Coordinate

$x = \rho\sin\phi\cos\theta$ $y = \rho\sin\phi\sin\theta$ $z = \rho\cos\phi$; $\rho^2 = x^2+y^2+z^2$

ρ is radius of sphere

$E := \{(\rho,\theta,\phi) \mid a\le\rho\le b, \alpha\le\theta\le\beta, c\le\phi\le d\}$ $\beta-\alpha\le 2\pi$, $d-c\le\pi$

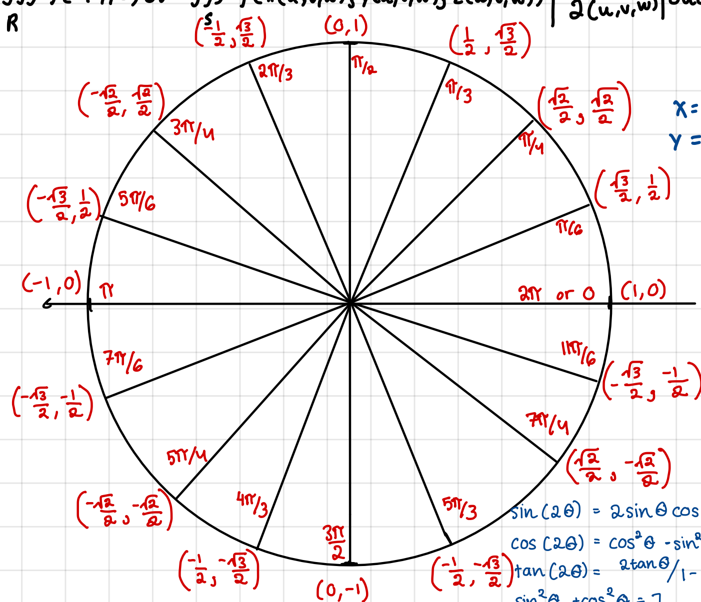
$\iiint_E f(x,y,z)\,dV = \int_c^d\int_\alpha^\beta\int_a^b f(\rho\sin\phi\cos\theta, \rho\sin\phi\cos\theta, \rho\cos\phi)\rho^2\sin\phi\, d\rho\, d\theta\, d\phi$

Jacobian:

$\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix}$ in $\mathbb{R}^2$ $\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w}\end{vmatrix}$ in $\mathbb{R}^3$

Change of variables:
find u and y s.t integral is easier to find
Transform the region on the xy plane to the uy plane
Find the new boundaries of integration
solve this integral

$\iiint_R f(x,y,z)\,dV = \iiint f(x(u,v,w), y(u,v,w), z(u,v,w))\left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| du\,dv\,dw$



$x = \cos\theta$
$y = \sin\theta$

Surface Integral: $\iint_D f(x,y,g(x,y))\sqrt{\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2+1}\,dA$

$n = \frac{r_u\times r_v}{|r_u\times r_v|}$

Surface Integral of F across S: $\iint_S F\cdot n\,dS = \iint_S F\cdot dS = \iint_D F\cdot(r_u\times r_v)\,dA$

Surface Integral of F across S if S is function of x and y:
$\iint_S F\cdot dS = \iint_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right) dA$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\tan(2\theta) = 2\tan\theta / 1 - \tan^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2 x = \frac{\cos 2x + 1}{2}$

$\int_C f(x,y)\,ds = \int_a^b f(x(t), y(t))\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\, dt$ (or with variable of parameterization)

$\int_C f(x,y)\,dx = \int_a^b f(x(t), y(t))\cdot x'(t)\,dt$

$\int_C f(x,y)\,dy = \int_a^b f(x(t), y(t))\cdot y'(t)\,dt$

$r(t) = (1+t)r_0 + tr_1$ $0\le t\le 1$

$m = \int_C \rho(x,y)\,ds$

$\bar{x} = \frac{1}{m}\int_C x\rho(x,y)\,ds$

$\bar{y} = \frac{1}{m}\int_C y\rho(x,y)\,ds$

$\int_C f(x,y,z)\,ds = \int_a^b f(x(t), y(t), z(t))\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\, dt$

$\int_C P\,dx + Q\,dy = \iint_S \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA$

$\int_C F\cdot dr = \int_a^b F(r(t))\cdot r'(t)\,dt$

If $F(x,y) = P(x,y)i + Q(x,y)j$ and $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ then $\exists f$ s.t $\nabla f = F$ ⟹ for 3d if curl F = 0 then F is conservative

and therefore $\int_C F\cdot dr = \int_C \nabla f\cdot dr = f(r(b)) - f(r(a))$

$\frac{d}{dx}(c) = 0$ $\frac{d}{dx}[f(x)+g(x)] = f'(x) + g'(x)$ $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + g'(x)f(x)$

$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ $\frac{d}{dx}cf(x) = cf'(x)$ $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$ $\frac{d}{dx}x^n = nx^{n-1}$

$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$ $\frac{d}{dx}(a^x) = a^x\ln a$ $\frac{d}{dx}\ln|x| = \frac{1}{x}$

$\frac{d}{dx}(\log_a x) = \frac{1}{x\ln a}$ $\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\cos x = -\sin x$ $\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\csc x = -\csc x\cot x$ $\frac{d}{dx}\sec x = \sec x\tan x$ $\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}\cos^{-1}x = \frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$ $\frac{d}{dx}\csc^{-1}x = \frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$ $\frac{d}{dx}\cot^{-1}x = -\frac{1}{1+x^2}$