assigned on his short puts at expiration. The outcome is that he may end up with an unhedged stock position on Monday morning after expiration. If the stock should open on a gap, he could have a substantial loss that wipes out the profits of many reversals. This risk of stock closing at the strike may seem minute, but it is not. In the absence of any real buying or selling in the stock on expiration day, the process of discounting will force a stock that is near the strike virtually right onto the strike. Once it is near the strike, this risk materializes.

There are two basic scenarios that could occur to produce this unhedged stock position. First, suppose one decides that he will not get put and he exercises his calls. However, he was wrong and he *does* get put. He has bought double the amount of stock—once via call exercise and again via put assignment. Thus, he will be *long* on Monday morning. The other scenario produces the opposite effect. Suppose one decides that he *will* get put and he decides not to exercise his calls. If he is wrong in this case, he does not buy any stock—he didn't exercise nor did he get put. Consequently, he will be *short* stock on Monday morning.

If one is truly undecided about whether he will be assigned on his short puts, he might look at several clues. First, has any late news come out on Friday evening that might affect the market's opening or the stock's opening on Monday morning? If so, that should be factored into the decision regarding exercising the calls. Another clue arises from the price at which the stock was trading during the Friday expiration day, prior to the close. If the stock was below the strike for most of the day before closing at the strike, then there is a greater chance that the puts will be assigned. This is so because other arbitrageurs (discounters) have probably bought puts and bought stock during the day and will exercise to clean out their positions.

If there is still doubt, it may be wisest to exercise only half of the calls, hoping for a partial assignment on the puts (always a possibility). This halfway measure will normally result in some sort of unhedged stock position on Monday morning, but it will be smaller than the maximum exposure by at least half.

Another approach that the arbitrageur can take if the stock is near the strike of the reversal during the late trading of the options' life—during the last few days—is to roll the reversal to a later expiration or, failing that, to roll to another strike in the same expiration. First, let us consider rolling to another expiration. The arbitrageur knows the dollar price that equals his effective rate for a 3-month reversal. If the current options can be closed out and new options opened at the next expiration for at least the effective rate, then the reversal should be rolled. This is not a likely event, mostly due to the fact that the spread between the bid and asked prices on four separate options makes it difficult to attain the desired price. *Note:* This entire four-way order can be entered as a spread order; it is not necessary to attempt to "leg" the spread.

The second action—rolling to another strike in the same expiration month—may be more available. Suppose that one has the July 45 reversal in place (long July 45 call and short July 45 put). If the underlying stock is near 45, he might place an order to the exchange floor as a three-way spread: Sell the July 45 call (closing), buy the July 45 put (closing), and sell the July 40 call (opening) for a net credit of 5 points. This action costs the arbitrageur nothing except a small transaction charge, since he is receiving a 5-point credit for moving the strike by 5 points. Once this is accomplished, he will have moved the strike approximately 5 points away and will thus have avoided the problem of the stock closing at the strike.

Overall, these four risks are significant, and reversal arbitrageurs should take care that they do not fall prey to them. The careless arbitrageur uses effective rates too close to current market rates, establishes reversals with puts in-the-money, and routinely accepts the risk of acquiring an unhedged stock position on the morning after expiration. He will probably sustain a large loss at some time. Since many reversal arbitrageurs work with small capital and/or have convinced their backers that it is a riskless strategy, such a loss may have the effect of putting them out of business. That is an unnecessary risk to take. There are countermeasures, as described above, that can reduce the effects of the four risks.

Let us consider the risks for conversion traders more briefly. The risk of stock closing near the strike is just as bad for the conversion as it is for the reversal. The same techniques for handling those risks apply equally well to conversions as to reversals. The other risks are similar to reversal risks, but there are slight nuances.

The conversion arbitrage suffers if there is a dividend cut. There is little the arbitrageur can do to predict this except to be aware of the fundamentals of the company before entering into the conversion. Alternatively, he might avoid conversions in which the dividend makes up a major part of the profit of the arbitrage.

Another risk occurs if there is an early assignment on the calls before the ex-dividend date and the dividend is not received. Moreover, an early assignment leaves the arbitrageur with long puts, albeit fractional ones since they are surely deeply out-of-the-money. Again, the policy of establishing conversions in which the dividend is not a major factor would help to ease the consequences of early assignment.

The final risk is that interest rates increase during the time the conversion is in place. This makes the carrying costs larger than anticipated and might cause a loss. The best way to hedge this initially is to allow a margin for error. Thus, if the prevailing interest rate is 12%, one might only establish reversals that would break even if rates rose to 14%. If rates do not rise that far on average, a profit will result. The arbitrageur can attempt to hedge this risk by *shorting* interest-bearing paper that matures at approximately the same time as the conversions. For example, if one has $5 million worth of 3-month conversions established at an effective rate of 14% and he shorts 3-month paper at 12½%, he locks in a profit of 1½%. This is not common practice for conversion arbitrageurs, but it does hedge the effect of rising interest rates.

SUMMARY OF CONVERSION ARBITRAGE

The practice of conversion and reversal arbitrage in the listed option markets helps to keep put and call prices in line. If arbitrageurs are active in a particular option, the prices of the put and call will relate to the underlying price in line with the formulae given earlier. Note that this is also a valid reason why puts tend to sell at a lower price than calls do. The cost of money is the determining factor in the difference between put and call prices. In essence, the "cost" (although it may sometimes be a credit) is subtracted from the theoretical put price. Refer again to the formula given above for the profit potential of a conversion. Assume that things are in perfect alignment. Then the formula would read:

Put price = Striking price + Call price – Underlying price – Fixed cost

Furthermore, if the underlying is at the striking price, the formula reduces to:

Put price = Call price – Fixed cost

So, whenever the fixed cost, which is equal to the carrying charge less the dividends, is greater than zero (and it usually is), the put will sell for less than the call if the underlying is at the striking price. Only in the case of a large-dividend-paying stock, when the fixed cost becomes negative (that is, it is not a cost, but a credit), does the reverse hold true. This is supportive evidence for statements made earlier that at-the-money calls sell for more than at-the-money puts, all other things being equal. The reader can see quite clearly that it has nothing to do with supply and demand for the puts and calls, a fallacy that is sometimes proffered. This same sort of analysis can be used to prove the broader statement that calls have a greater time value premium than puts do, except in the case of a large-dividend-paying stock.

One final word of advice should be offered to the public customer. He may sometimes be able to find conversions or reversals, by using the simplistic formula, that appear to have profit potentials that exceed commission costs. Such positions do exist from time to time, but the rate of return to the public customer will almost assuredly be less than the short-term cost of money. If it were not, arbitrageurs would be onto the position very quickly. The public option trader may not actually be thinking in terms of comparing the profit potential of a position with what he could get by placing the money into a bank, but he must do so to convince himself that he cannot feasibly attempt conversion or reversal arbitrages.

THE "INTEREST PLAY"

In the preceding discussion of reversal arbitrage, it is apparent that a substantial portion of the arbitrageur's profits may be due to the interest earned on the credit of the position. Another type of position is used by many arbitrageurs to take advantage of this interest earned. The arbitrageur sells the underlying short and simultaneously buys an in-the-money call that is trading slightly over parity. The actual amount over parity that the arbitrageur can afford to pay for the call is determined by the interest that he will earn from his short sale and the dividend payout before expiration. He does not use a put in this type of position. In fact, this "interest play" strategy is merely a reversal arbitrage without the short put. This slight variation has a residual benefit for the arbitrageur: If the underlying stock should drop dramatically in price, he could make large profits because he is short the underlying stock. In any case, *he will make his interest credit less the amount of time value premium paid for the call less any dividends lost.*

Example 1: XYZ is sold short at 60, and a January 50 call is bought for 10.25 points. Assume that the prevailing interest rate is 1% per month and that the position is established one month prior to expiration. XYZ pays no dividend. The total credit brought in from the trades is $4,975, so the arbitrageur will earn $49.75 in interest over the course of 1 month. If the stock is above 50 at expiration, he will exercise his call to buy stock at 50 and close the position. His loss on the security trades will be $25—the amount of time value premium paid for the call option. (He makes 10 points by selling stock at 60 and buying at 50, but loses 10.25 points on the exercised call.) His overall profit is thus $24.75.

Example 2: A real-life example may point out the effect of interest rates even more dramatically. In early 1979, IBM April 240 calls with about six weeks of life remaining were over 60 points in-the-money. IBM was not going to be ex-dividend in that time. Normally, such a deeply in-the-money option would be trading at parity or even a discount when the time remaining to expiration is so short. However, these calls were trading 3.50 points over parity because of the prevailing high interest rates at the time. IBM was at 300, the April 240 calls were trading at 63.50, and the prevailing interest rate was approximately 1% per month. The credit from selling the stock and buying the call was $23,700, so the arbitrageur earned $365.50 in interest for 1½ months, and lost $350—the 3.50 points of time value premium that he paid for the call. This still left enough room for a profit.

In Chapter 1, it was stated that interest rates affect option prices. The above examples of the "interest play" strategy quite clearly show why. As interest rates rise, the arbitrageur can afford to pay more for the long call in this strategy, thus causing the call price to increase in times of high interest rates. If call prices are higher, so will put prices be, as the relationships necessary for conversion and reversal arbitrage are preserved. Similarly, if interest rates decline, the arbitrageur will make lower bids, and call and put prices will be lower. They are active enough to give truth to the theory that option prices are directly related to interest rates.

THE BOX SPREAD

An arbitrage consists of simultaneously buying and selling the same security or equivalent securities at different prices. For example, the reversal consists of selling a put and simultaneously shorting stock and buying a call. The reader will recall that the short stock/long call position was called a synthetic put. That is, shorting the stock and buying a call is equivalent to buying a put. The reversal arbitrage therefore consists of selling a (listed) put and simultaneously buying a (synthetic) put. In a similar manner, the conversion is merely the purchase of a (listed) put and the simultaneous sale of a (synthetic) put. Many equivalent strategies can be combined for arbitrage purposes. One of the more common ones is the box spread.

Recall that it was shown that a bull spread or a bear spread could be constructed with either puts or calls. Thus, if one were to simultaneously buy a (call) bull spread and buy a (put) bear spread, he could have an arbitrage. In essence, he is merely buying and selling equivalent spreads. If the price differentials work out correctly, a risk-free arbitrage may be possible.

Example: The following prices exist:

XYZ common, 55
XYZ January 50 call, 7
XYZ January 50 put, 1
XYZ January 60 call, 2
XYZ January 60 put, 5.50

The arbitrageur could establish the box spread in this example by executing the following transactions:

Buy a call bull spread:		
Buy XYZ January 50 call	7 debit	
Sell XYZ January 60 call	2 credit	
Net call cost		5 debit

Buy a put bear spread:		
Buy XYZ January 60 put	5.50 debit	
Sell XYZ January 50 put	1 credit	
Net put cost		4.50 debit
Total cost of position		9.50 debit

No matter where XYZ is at January expiration, this position will be worth 10 points. *The arbitrageur has locked in a risk-free profit* of 50 cents, since he "bought" the box spread for 9.50 points and will be able to "sell" it for 10 points at expiration. To verify this, evaluate the position at expiration, first with XYZ above 60, then with XYZ between 50 and 60, and finally with XYZ below 50. If XYZ is above 60 at expiration, the puts will expire worthless and the call bull spread will be at its maximum potential of 10 points, the difference between the striking prices. Thus, the position can be liquidated for 10 points if XYZ is above 60 at expiration. Now assume that XYZ is between 50 and 60 at expiration. In that case, the out-of-the-money, written options would expire worthless—the January 60 call and the January 50 put. This would leave a long, in-the-money combination consisting of a January 50 call and a January 60 put. These two options must have a total value of 10 points at expiration with XYZ between 50 and 60. (For example, the arbitrageur could exercise his call to buy stock at 50 and exercise his put to sell stock at 60.) Finally, assume that XYZ is below 50 at expiration. The calls would expire worthless if that were true, but the remaining put spread—actually a bear spread in the puts—would be at its maximum potential of 10 points. Again, the box spread could be liquidated for 10 points.

The arbitrageur must pay a cost to carry the position, however. In the prior example, if interest rates were 6% and he had to hold the box for 3 months, it would cost him an additional 14 cents ($.06 \times 9.50 \times 3/12$). This still leaves room for a profit.

In essence, a bull spread (using calls) was purchased while a bear spread (using puts) was bought. The box spread was described in these terms only to illustrate the fact that the arbitrageur is buying and selling equivalent positions. The arbitrageur who is utilizing the box spread should not think in terms of bull or bear spread, however. Rather, he should be concerned with "buying" the entire box spread at a cost of less than the differential between the two striking prices. By "buying" the box spread, it is meant that both the call spread portion and the put spread portion are debit spreads. *Whenever the arbitrageur observes that a call spread and a put spread using the same strikes and that are both debit spreads can be bought for less than the difference in the strikes plus carrying costs, he should execute the arbitrage.*

* * *

Obviously, there is a companion strategy to the one just described. It might sometimes be possible for the arbitrageur to "sell" both spreads. That is, he would establish a credit call spread and a credit put spread, using the same strikes. *If this credit were greater than the difference in the striking prices, a risk-free profit would be locked in.*

Example: Assume that a different set of prices exists:

XYZ common, 75
XYZ April 70 call, 8.50
XYZ April 70 put, 1
XYZ April 80 call, 3
XYZ April 80 put, 6

By executing the following transactions, the box spread could be "sold":

Sell a call (bear) spread:		
Buy April 80 call	3 debit	
Sell April 70 call	8.50 credit	
Net credit on calls		5.50 credits
Sell a put (bull) spread:		
Buy April 70 put	1 debit	
Sell April 80 put	6 credit	
Net credit on puts		5 credit
Total credit of position		10.50 credit

In this case, no matter where XYZ is at expiration, the position can be bought back for 10 points. This means that the arbitrageur has locked in risk-free profit of 50 cents. To verify this statement, first assume that XYZ is above 80 at April expiration. The puts will expire worthless, and the call spread will have widened to 10 points—the cost to buy it back. Alternatively, if XYZ were between 70 and 80 at April expiration, the long, out-of-the-money options would expire worthless and the in-the-money combination would cost 10 points to buy back. (For example, the arbitrageur could let himself be put at 80, buying stock there, and called at 70, selling the stock there—a net "cost" to liquidate of 10 points.) Finally, if XYZ were below 70 at expiration, the calls would expire worthless and the put spread would have widened to 10 points. It could then be closed out at a cost of 10 points. In each case, the arbitrageur is able to liquidate the box spread by buying it back at 10.

In this sale of a box spread, he would earn interest on the credit received while he holds the position.

There is an additional factor in the profitability of the box spread. Since the sale of a box generates a credit, the arbitrageur who sells a box will earn a small amount of money from that sale. Conversely, the purchaser of a box spread will have a charge for carrying cost. Since profit margins may be small in a box arbitrage, these carrying costs can have a definite effect. As a result, boxes may actually be sold for 5 points, even though the striking prices are 5 points apart, and the arbitrageur can still make money because of the interest earned.

These box spreads are not easy to find. If one does appear, the act of doing the arbitrage will soon make the arbitrage impossible. In fact, this is true of any type of arbitrage; it cannot be executed indefinitely because the mere act of arbitraging will force the prices back into line. Occasionally, the arbitrageur will be able to find the option quotes to his liking, especially in volatile markets, and can establish a risk-free arbitrage with the box spread. It can be evaluated at a glance. Only two questions need to be answered:

1. If one were to establish a debit call spread and a debit put spread, using the same strikes, would the total cost be *less than* the difference in the striking prices plus carrying costs? If the answer is yes, an arbitrage exists.
2. Alternatively, if one were to sell both spread—establishing a credit call spread and a credit put spread—would the total credit received plus interest earned be *greater than* the difference in the striking prices? If the answer is yes, an arbitrage exists.

There are some risks to box arbitrage. Many of them are the same as those risks faced by the arbitrageur doing conversions or reversals. First, there is risk that the stock might close at either of the two strikes. This presents the arbitrageur with the same dilemma regarding whether or not to exercise his long options, since he is not sure whether he will be assigned. Additionally, early assignment may change the profitability: Assignment of a short put will incur large carrying costs on the resulting long stock; assignment of a short call will inevitably come just before an ex-dividend date, costing the arbitrageur the amount of the dividend.

There are not many opportunities to actually transact box arbitrage, but the fact that such arbitrage exists can help to keep markets in line. For example, if an underlying stock begins to move quickly and order flow increases dramatically, the specialist or market-markers in that stock's options may be so inundated with orders that they cannot be sure that their markets are correct. They can use the principles of box arbitrage to keep prices in line. The most active options would be the ones at strikes nearest to the current stock price. The specialist can quickly add up the markets of the call and put at the nearest strike above the stock price and add to that the markets of the options at the strike just below. The sum of the four should add up to a price that surrounds the difference in the strikes. If the strikes are 5 points apart, then the sum of the four markets should be something like 4.50 bid, 5.50 asked. If, instead, the four markets add up to a price that allows box arbitrage to be established, then the specialist will adjust his markets.

VARIATIONS ON EQUIVALENCE ARBITRAGE

Other variations of arbitrage on equivalent positions are possible, although they are relatively complicated and probably not worth the arbitrageur's time to analyze. For example, one could buy a butterfly spread with calls and simultaneously sell a butterfly spread using puts. A listed straddle could be sold and a synthetic straddle could be bought—short stock and long 2 calls. Inversely, a listed straddle could be bought against a ratio write—long stock and short 2 calls. The only time the arbitrageur should even consider anything like this is when there are more sizable markets in certain of the puts and calls than there are in others. If this were the case, he might be able to take an ordinary box spread, conversion, or reversal and add to it, keeping the arbitrage intact by ensuring that he is, in fact, buying and selling equivalent positions.

THE EFFECTS OF ARBITRAGE

The arbitrage process serves a useful purpose in the listed options market, because it may provide a secondary market where one might not otherwise exist. Normally, public interest in an in-the-money option dwindles as the option becomes deeply in-the-money or when the time remaining until expiration is very short. There would be few public buyers of these options. In fact, public selling pressure might increase, because the public would rather liquidate in-the-money options held long than exercise them. The few public buyers of such options might be writers who are closing out. However, if the writer is covered, especially where call options are concerned, he might decide to be assigned rather than close out his option. This means that the public seller is creating a rather larger supply that is not offset by a public demand. The market created by the arbitrageur, especially in the basic put or call arbitrage, essentially creates the demand. Without these arbitrageurs, there could conceivably be no buyers at all for those options that are short-lived and in-the-money, after public writers have finished closing out their positions.

Equivalence arbitrage—conversion, reversals, and box spreads—helps to keep the relative prices of puts and calls in line with each other and with the underlying stock price. This creates a more efficient and rational market for the public to operate in. The arbitrageur would help eliminate, for example, the case in which a public customer buys a call, sees the stock go up, but cannot find anyone to sell his call to at higher prices. If the call were too cheap, arbitrageurs would do reversals, which involve call purchases, and would therefore provide a market to sell into.

Questions have been raised as to whether option trading affects stock prices, especially at or just before an expiration. If the amount of arbitrage in a certain issue becomes very large, it could appear to temporarily affect the price of the stock itself. For example, take the call arbitrage. This involves the sale of stock in the market. The corresponding stock purchase, via the call exercise, is not executed on the exchange. Thus, as far as the stock market is concerned, there may appear to be an inordinate amount of selling in the stock. If large numbers of basic call arbitrages are taking place, they might thus hold the price of the stock down until the calls expire.

The put arbitrage has an opposite effect. This arbitrage involves buying stock in the market. The offsetting stock sale via the put exercise takes place off the exchange. If a large amount of put arbitrage is being done, there may appear to be an inordinate amount of buying in the stock. Such action might temporarily hold the stock price up.

In a vast majority of cases, however, the arbitrage has no visible effect on the underlying stock price, because the amount of arbitrage being done is very small in comparison to the total number of trades in a given stock. Even if the open interest in a particular option is large, allowing for plenty of option volume by the arbitrageurs, the actual act of doing the arbitrage will force the prices of the stock and option back into line, thus destroying the arbitrage.

Rather elaborate studies, including doctoral theses, have been written that try to prove or disprove the theory that option trading affects stock prices. Nothing has been proven conclusively, and it may never be, because of the complexity of the task. Logic would seem to dictate that arbitrage could temporarily affect a stock's movement if it has discount, in-the-money options shortly before expiration. However, one would have to reasonably conclude that the size of these arbitrages could almost never be large enough to overcome a directional trend in the underlying stock itself. Thus, in the absence of a definite direction in the stock, arbitrage might help to perpetuate the inertia; but if there were truly a preponderance of investors wanting to buy or sell the stock, these investors would totally dominate any arbitrage that might be in progress.

RISK ARBITRAGE USING OPTIONS

Risk arbitrage is a strategy that is well described by its name. It is basically an arbitrage—the same or equivalent securities are bought and sold. However, *there is generally risk because the arbitrage usually depends on a future event occurring in order for the arbitrage to be successful*. One form of risk arbitrage was described earlier concerning the speculation on the size of a special dividend that an underlying stock might pay. That arbitrage consisted of buying the stock and buying the put, when the put's time value premium is less than the amount of the projected special dividend. The risk lies in the arbitrageur's speculation on the size of the anticipated special dividend.

MERGERS

Risk arbitrage is an age-old type of arbitrage in the stock market. *Generally, it concerns speculation on whether a proposed merger or acquisition will actually go through as proposed.*

Example: XYZ, which is selling for $50 per share, offers to buy out LMN and is offering to swap one share of its (XYZ's) stock for every two shares of LMN. This would mean that LMN should be worth $25 per share if the acquisition goes through as proposed. On the day the takeover is proposed, LMN stock would probably rise to about $22 per share. It would not trade all the way up to 25 until the takeover was approved by the shareholders of LMN stock. The arbitrageur who feels that this takeover will be approved can take action. He would sell short XYZ and, for every share that he is short, he would buy 2 shares of LMN stock. If the merger goes through, he will profit. The reason that he shorts XYZ as well as buying LMN is to protect himself in case the market price of XYZ drops before the acquisition is approved. In essence, he has sold XYZ and also bought the equivalent of XYZ (two shares of LMN will be equal to one share of XYZ if the takeover goes through). This, then, is clearly an arbitrage. However, it is a risk arbitrage because, if the stockholders of LMN reject the offer, he will surely lose money. His profit potential is equal to the remaining differential between the current market price of LMN (22) and the takeover price (25). If the proposed acquisition goes through, the differential disappears, and the arbitrageur has his profit.

The greatest risk in a merger is that it is canceled. If that happens, stock being acquired (LMN) will fall in price, returning to its pre-takeover levels. In addition, the acquiring stock (XYZ) will probably rise. Thus, the risk arbitrageur can lose money on both sides of his trade. *If either or both of the stocks involved in the proposed takeover have options, the arbitrageur may be able to work options into his strategy.*

In merger situations, since large moves can occur in both stocks (they move in concert), option purchases are the preferable option strategy. If the acquiring company (XYZ) has in-the-money puts, then the purchase of those puts may be used instead of selling XYZ short. The advantage is that if XYZ rallies dramatically during the time it takes for the merger to take effect, then the arbitrageur's profits will be increased.

Example: As above, assume that XYZ is at 50 and is acquiring LMN in a 2-for-1 stock deal. LMN is at 22. Suppose that XYZ rallies to 60 by the time the deal closes. This would pull LMN up to a price of 30. If one had been short 100 XYZ at 50 and long 200 LMN at 22, then his profit would be \$600—a \$1,600 gain on the 200 long LMN minus a \$1,000 loss on the XYZ short sale.

Compare that result to a similar strategy substituting a long put for the short XYZ stock. Assume that he buys 200 LMN as before, but now buys an XYZ put. If one could buy an XYZ July 55 put with little time premium, say at 5.50 points, then he would have nearly the same dollars of profit if the merger should go through with XYZ below 55.

However, when XYZ rallies to 60, his profit increases. He would still make the \$1,600 on LMN as it rose from 22 to 30, but now would only lose \$550 on the XYZ put—a total profit of \$1,050 as compared to \$600 with an all-stock position.

The disadvantage to substituting long puts for short stock is that the arbitrageur does not receive credit for the short sale and, therefore, does not earn money at the carrying rate. This might not be as large a disadvantage as it initially seems, however, since it is often the case that it is very expensive—even impossible—to borrow the acquiring stock in order to short it. If the stock borrow costs are very large or if no stock can be located for borrowing, the purchase of an in-the-money put is a viable alternative. The purchase of an in-the-money put is preferable to an at- or out-of-the-money put, because the amount of time value premium paid for the latter would take too much of the profitability away from the arbitrage if XYZ stayed unchanged or declined. This strategy may also save money if the merger falls apart and XYZ rises. The loss on the long put may well be less than the loss would be on short XYZ stock. It should be noted, however, that if the stock is hard to borrow, that fact will be built into the (in-the-money) put prices, and they may therefore be more expensive than they normally would.

Note also that one could sell the XYZ July 55 call short as well as buy the put. This would, of course, be synthetic short stock and is a pure substitute for shorting the stock. The use of this synthetic short is recommended only when the arbitrageur cannot borrow the acquiring stock. If this is his purpose, he should use the in-the-money put and out-of-the-money call, since if he were assigned on the call, he could not borrow the stock to deliver it as a short sale. The use of an out-of-the-money call lessens the chance of eventual assignment.

The companion strategy is to buy an in-the-money call instead of buying the company being acquired (LMN). This has advantages if the stock falls too far, either because the merger falls apart or because the stocks in the merger decline too far. Additionally, the cost of carrying the long LMN stock is eliminated, although that is generally built into the cost of the long calls. The larger amount of time value premium in calls as compared to puts makes this strategy often less attractive than that of buying the puts as a substitute for the short sale.

One might also consider selling options instead of buying them. Generally this is an inferior strategy, but in certain instances it makes sense. The reason that option sales are inferior is that they do not limit one's risk in the risk arbitrage, but they cut off the profit. For example, if one sells puts on the company being acquired (LMN), he has a bullish situation. However, if the company being acquired (XYZ) rallies too far, there will be a loss, because the short puts will stop making money as soon as LMN rises through the strike. This is especially disconcerting if a takeover bidding war should develop for LMN. The arbitrageur who is long LMN will participate nicely as LMN rises heavily in price during the bidding war. However, the put seller will not participate to nearly the same extent.

The sale of in-the-money calls as a substitute for shorting the acquiring company (XYZ) can be beneficial at certain times. It is necessary to have a plus tick in order to sell stock short. When many arbitrageurs are trying to sell a stock short at the same time, it may be difficult to sell such stock short. Morever, natural owners of XYZ may see the arbitrageurs holding the price down and decide to sell their long stock rather than suffer through a possible decline in the stock's price while the merger is in progress. Additionally, buyers of XYZ will become very timid, lowering their bids for the same reasons. All of this may add up to a situation in which it is very difficult to sell the stock short, even if it can be borrowed. The sale of an in-the-money call can overcome this difficulty. The call should be deeply in-the-money and not be too long-term, for the arbitrageur does not want to see XYZ decline below the strike of the call. If that happened, he would no longer be hedged; the other side of the arbitrage—the long LMN stock—would continue to decline, but he would not have any remaining short against the long LMN.

LIMITS ON THE MERGER

There is another type of merger for stock that is more difficult to arbitrage, but options may prove useful. In some merger situations, the acquiring company (XYZ) promises to give the shareholders of the company being acquired (LMN) an amount of stock equal to a set dollar price. This amount of stock would be paid even if the acquiring company rose or fell moderately in price. If XYZ falls too far, however, it cannot pay out an extraordinarily increased number of shares to LMN shareholders, so XYZ puts a limit on the maximum number of shares that it will pay for each share of LMN stock. Thus, the shareholders of XYZ are guaranteed that there will be some downside buffer in terms of dilution of their company in case XYZ declines, as is often the case for an acquiring company. However, if XYZ declines too far, then LMN shareholders will receive less. In return for getting this downside guarantee, XYZ will usually also stipulate that there is a *minimum* amount of shares that they will pay to LMN shareholders, even if XYZ stock rises tremendously. Thus, if XYZ should rise tremendously in price, then LMN shareholders will do even better than they had anticipated. An example will demonstrate this type of merger accord.

Example: Assume that XYZ is at 50 and it intends to acquire LMN for a stated price of $25 per share, as in the previous example. However, instead of merely saying that it will exchange two shares of LMN for one share of XYZ, the company says that it wants the offer to be worth $25 per share to LMN shareholders as long as XYZ is between 45 and 55. Given this information, we can determine the maximum and minimum number of shares that LMN shareholders will receive: The maximum is the stated price, 25, divided by the lower limit, 45, or 0.556 shares; the minimum is 25 divided by the higher limit, 55, or 0.455.

This type of merger is usually stated in terms of how many shares of XYZ will be issued, rather than in terms of the price range that XYZ will be able to move in. In either case, one can be derived from the other, so that the manner in which the merger deal is stated is merely a convention. In this case, for example, the merger might be stated as being worth $25 per share, with each share of LMN being worth at least 0.455 shares of XYZ and at most 0.556 shares of XYZ. Note that these ratios make the deal worth 25 as long as XYZ is between 45 and 55: 45 times 0.556 equals 25, as does 0.455 times 55.

If the acquiring stock, XYZ, is between 45 and 55 at the time the merger is completed, then the number of shares of XYZ that each LMN shareholder will receive is determined in a preset manner. Usually, at the time the merger is announced, XYZ will say that its price on the closing date of the merger will be used to establish the proper ratio. As a slight alternative, sometimes the acquiring company will state that the price to be used in determining the final ratio is to be an average of the closing prices of the stock over a stated period of time. This stated period of time might be something like the 10 days prior to the closing of the merger.

Example: Suppose that the closing price of XYZ on the day that the merger closes is to be the price used in the ratio. Furthermore, suppose that XYZ closes at 51 on that day. It is within the prestated range, so a calculation must be done in order to determine how many shares of XYZ each LMN shareholder will get. This ratio is determined by dividing the stated price, 25, by the price in question, 51. This would give a final ratio of 0.490196. The final ratio is usually computed to a rather large number of decimal points in order to assure that LMN shareholders get as close to $25 per share as possible.

The above two examples explain how this type of merger works. A merger of this type is said to have "hooks"—the prices at which the ratio steadies. This makes it difficult to arbitrage. As long as XYZ roams around in the 45 to 55 range, the arbitrageur does not want to short XYZ as part of his arbitrage, because the price of XYZ does not affect the price he will eventually receive for LMN—25. Rather, he would buy LMN and wait until the deal is near closing before actually shorting XYZ. By waiting, he will know approximately how many shares of XYZ to short for each share of LMN that he owns. The reason that he must short XYZ at the end of the merger is that there is usually a period of time before the physical stock is reorganized from LMN into XYZ. During that time, if he were long LMN, he would be at risk if he did not short XYZ against it.

Problems arise if XYZ begins to fall below 45 well before the closing of the merger, the lower "hook" in the merger. If it should remain below 45, then one should set up the arbitrage as being short 0.556 shares of XYZ for each share of LMN that is held long. As long as XYZ remains below 45 until the merger closes, this is the proper ratio. However, if, after establishing that ratio, XYZ rallies back above 45, the arbitrageur can suffer damaging losses. XYZ may continue to rise in price, creating a loss on the short side. However, LMN will not follow it, because the merger is structured so that LMN is worth 25 unless XYZ rises too far. Thus, the long side stops following as the short side moves higher.

On the other hand, no such problem exists if XYZ rises too far from its original price of 50, going above the upper "hook" of 55. In that case, the arbitrageur would already be long the LMN and would not yet have shorted XYZ, since the merger was not yet closing. LMN would merely follow XYZ higher after the latter had crossed 55.

This is not an uncommon dilemma. Recall that it was shown that the acquiring stock will often fall in price immediately after a merger is announced. Thus, XYZ may fall close to, or below, the lower "hook." Some arbitrageurs attempt to hedge themselves by shorting a little XYZ as it begins to fall near 45 and then completing the short if it drops well below 45. The problem with handling the situation in this way is that one ends up with an inexact ratio. Essentially, he is forcing himself to predict the movements of XYZ.

If the acquiring stock drops below the lower "hook," there may be an opportunity to establish a hedge without these risks if that stock has listed options. The idea is to buy puts on the acquiring company, and for those puts to have a striking price nearly equal to the price of the lower "hook." The proper amount of the company being acquired (LMN) is then purchased to complete the arbitrage. If the acquiring company subsequently rallies back into the stated price range, the puts will not lose money past the striking price and the problems described in the preceding paragraph will have been overcome.

Example: A merger is announced as described in the preceding example: XYZ is to acquire LMN at a stated value of $25 per share, with the stipulation that each share of LMN will be worth at least 0.455 shares of XYZ and at most 0.556 shares. These share ratios equate to prices of 45 and 55 on XYZ.

Suppose that XYZ drops immediately in price after the merger is announced, and it falls to 40. Furthermore, suppose that the merger is expected to close sometime during July and that there are XYZ August 45 puts trading at 5.50. This represents only 50 cents time value premium. The arbitrageur could then set up the arbitrage by buying 10,000 LMN and buying 56 of those puts. Smaller investors might buy 1,000 LMN and buy 6 puts. Either of these is in approximately the proper ratio of 1 LMN to 0.556 XYZ.

TENDER OFFERS

Another type of corporate takeover that falls under the broad category of risk arbitrage is the tender offer. In a tender offer, the acquiring company normally offers to exchange cash for shares of the company to be acquired. Sometimes the offer is for all of the shares of the company being acquired; sometimes it is for a fractional portion of shares. In the latter case, it is important to know what is intended to be done with the remaining shares. These might be exchanged for shares of the acquiring company, or they might be exchanged for other securities (bonds, most likely), or perhaps there is no plan for exchanging them at all. In some cases, a company tenders for part of its own stock, so that it is in effect both the acquirer and the acquiree. Thus, tender offers can be complicated to arbitrage properly. The use of options can lessen the risks.

In the case in which the acquiring company is making a cash tender for all the shares (called an "any and all" offer), the main use of options is the purchase of puts as protection. One would buy puts on the company being acquired at the same time that he bought shares of that company. If the deal fell apart for some reason, the puts could prevent a disastrous loss as the acquiring stock dropped. The arbitrageur must be judicious in buying these puts. If they are too expensive or too far out-of-the-money, or if the acquiring company might not really drop very far if the deal falls apart, then the purchase of puts is a waste. However, if there is substantial downside risk, the put purchase may be useful.

Selling options in an "any and all" deal often seems like easy money, but there may be risks. If the deal is completed, the company being acquired will disappear and its options would be delisted. Therefore, it may often seem reasonable to sell out-of-the-money puts on the acquiring company. If the deal is completed, these expire worthless at the closing of the merger. However, if the deal falls through, these puts will soar in price and cause a large loss. On the other hand, it may also seem like easy money to sell naked calls with a striking price higher than the price being offered for the stock. Again, if the deal goes through, these will be delisted and expire worthless. The risk in this situation is that another company bids a higher price for the company on which the calls were written. If this happens, there might suddenly be a large upward jump in price, and the written calls could suffer a large loss.

Options can play a more meaningful role in the tender offer that is for only part of the stock, especially when it is expected that the remaining stock might fall substantially in price after the partial tender offer is completed. An example of a partial tender offer might help to establish the scenario.

Example: XYZ proposes to buy back part of its own stock. It has offered to pay $70 per share for half the company. There are no plans to do anything further. Based on the fundamentals of the company, it is expected that the remaining stock will sell for approximately $40 per share. Thus, the average share of XYZ is worth 55 if the tender offer is completed (one-half can be sold at 70, and the other half will be worth 40). XYZ stock might sell for $52 or $53 per share until the tender is completed. On the day after the tender offer expires, XYZ stock will drop immediately to the $40 per share level.

There are two ways to make money in this situation. One is to buy XYZ at the current price, say 52, and tender it. The remaining portion would be sold at the lower price, say 40, when XYZ reopened after the tender expired. This method would yield a profit of $3 per share if exactly 50% of the shares are accepted at 70 in the tender offer. In reality, a slightly higher percentage of shares is usually accepted, because a few people make mistakes and don't tender. Thus, one's average net price might be $56 per share, for a $4 profit from this method. The risk in this situation is that XYZ opens substantially below 40 after the tender at 70 is completed.

Theoretically, the other way to trade this tender offer might be to sell XYZ short at 52 and cover it at 40 when it reopens after the tender offer expires. Unfortunately, this method cannot be effected because there will not be any XYZ stock to borrow in order to sell it short. All owners will tender the stock rather than loan it to arbitrageurs. Arbitrageurs understand this, and they also understand the risk they take if they try to short stock at the last minute: They might be forced to buy back the stock for cash, or they may be forced to give the equivalent of $70 per share for half the stock to the person who bought the stock from them. For some reason, many individual investors believe that they can "get away" with this strategy. They short stock, figuring that their brokerage firm will find some way to borrow it for them. Unfortunately, this usually costs the customer a lot of money.

The use of calls does not provide a more viable way of attempting to capitalize on the drop of XYZ from 52 to 40. In-the-money call options on XYZ will normally be selling at parity just before the tender offer expires. If one sells the call as a substitute for the short sale, he will probably receive an assignment notice on the day after the tender offer expires, and therefore find himself with the same problems the short seller has.

The only safe way to play for this drop is to buy puts on XYZ. These puts will be very expensive. In fact, with XYZ at 52 before the tender offer expires, if the consensus opinion is that XYZ will trade at 40 after the offer expires, then puts with a 50 strike will sell for at least $10. This large price reflects the expected drop in price of XYZ. Thus, it is not beneficial to buy these puts as downside speculation unless one expects the stock to drop further than to the $40 level. There is, however, an opportunity for arbitrage by buying XYZ stock and also buying the expensive puts.

Before giving an example of that arbitrage, a word about short tendering is in order. Short tendering is against the law. It comes about when one tenders stock into a tender offer when he does not really own that stock. There are complex definitions regarding what constitutes ownership of stock during a tender offer. One must be net long all the stock that he tenders on the day the tender offer expires. Thus, he cannot tender the stock on the day before the offer expires, and then short the stock on the next day (even if he could borrow the stock). In addition, one must subtract the number of shares covered by certain calls written against his position: Any calls with a strike price less than the tender offer price must be subtracted. Thus, if he is long 1,000 shares and has written 10 in-the-money calls, he cannot tender any shares. The novice and experienced investor alike must be aware of these definitions and should not violate the short tender rules.

Let us now look at an arbitrage consisting of buying stock and buying the expensive puts.

Example: XYZ is at 52. As before, there is a tender offer for half the stock at 70, with no plans for the remainder. The July 55 puts sell for 15, and the July 50 puts sell for 10. It is common that both puts would be predicting the same price in the after-market: 40.

If one buys 200 shares of XYZ at 52 and buys one July 50 put at 10, he has a locked-in profit as long as the tender offer is completed. He only buys one put because he is assuming that 100 shares will be accepted by the company and only 100 shares will be returned to him. Once the 100 shares have been returned, he can exercise the put to close out his position.

The following table summarizes these results:

Initial purchase	
Buy 200 XYZ at 52	$10,400 debit
Buy 1 July 50 put at 10	1,000 debit
Total Cost	$11,400 debit
Closing sale	
Sell 100 XYZ at 70 via tender	7,000 credit
Sell 100 XYZ at 50 via put exercise	5,000 credit
Total proceeds	$12,000 credit
Total profit: $600	

This strategy eliminates the risk of loss if XYZ opens substantially below 40 after the tender offer. The downside price is locked in by the puts.

If more than 50% of XYZ should be accepted in the tender offer, then a larger profit will result. Also, if XYZ should subsequently trade at a high enough price so that the July 50 put has some time value premium, then a larger profit would result as well. (The arbitrageur would not exercise the put, but would sell the stock and the put separately in that case.)

Partial tender offers can be quite varied. The type described in the above example is called a "two-tier" offer because the tender offer price is substantially different from the remaining price. In some partial tenders, the remainder of the stock is slated for purchase at substantially the same price, perhaps through a cash merger. The above strategy would not be applicable in that case, since such an offer would more closely resemble the "any and all" offer. In other types of partial tenders, debt securities of the acquiring company may be issued after the partial cash tender. The net price of these debt securities may be different from the tender offer price. If they are, the above strategy might work.

In summary, then, one should look at tender offers carefully. One should be careful not to take extraordinary option risk in an "any and all" tender. Conversely, one should look to take advantage of any "two-tier" situation in a partial tender offer by buying stock and buying puts.

PROFITABILITY

Since the potential profits in risk arbitrage situations may be quite large, perhaps 3 or 4 points per 100 shares, the public can participate in this strategy. Commission charges will make the risk arbitrage less profitable for a public customer than it would be for an arbitrageur. The profit potential is often large enough, however, to make this type of risk arbitrage viable even for the public customer.

In summary, the risk arbitrageur may be able to use options in his strategy, either as a replacement for the actual stock position or as protection for the stock position. Although the public cannot normally participate in arbitrage strategies because of the small profit potential, risk arbitrages may often offer exceptions. The profit potential can be large enough to overcome the commission burden for the public customer.

PAIRS TRADING

A stock trading strategy that has gained some adherents in recent years is pairs trading. Simplistically, this strategy involves trading pairs of stocks—one held long, the other short. Thus, it is a hedged strategy, but it is not bona fide arbitrage. The two stocks' price movements are related historically. The pairs trader would establish the position when one stock was expensive with respect to the other one, historically. Then, when the stocks return to their historical relationship, a profit would result. In reality, some fairly complicated computer programs search out the appropriate pairs.

The interest on the short sale offsets the cost of carry of the stock purchased. Therefore, the pairs trader doesn't have any expense except the possible differential in dividend payout.

The bane of pairs trading is a possible escalation of the stock sold short without any corresponding rise in price of the stock held long. A takeover attempt might cause this to happen. Of course, pairs traders will attempt to research the situation to ensure that they don't often sell short stocks that are perceived to be takeover candidates.

Pairs traders can use options to potentially reduce their risk if there are in-the-money options on both stocks. One would buy an in-the-money put instead of selling one stock short, and would buy an in-the-money call on the other stock instead of buying the stock itself. In this option combination, traders are paying very little time value premium, so their profit potential is approximately the same as with the pairs trading strategy using stocks. (One would, however, have a debit, since both options are purchased; so there would be a cost of carry in the option strategy.)

If the stocks return to their historical relationship, the option strategy will reflect the same profit as the stock strategy, less any loss of time value premium. One added advantage of the option strategy, however, is that if a takeover occurs, the put has limited liability, and the trader's loss would be less.

Another advantage of the option strategy is that if both stocks should experience large moves, it could make money even if the pair doesn't return to historical norms. This would happen, for example, if both stocks dropped a great deal: The call has limited loss, while the put's profits would continue to accrue. Similarly, to the upside, a large move by both stocks would make the put worthless, but the call would keep making money. In both cases, the option strategy could profit even if the pair of stocks didn't perform as predicted.

This type of strategy—buying in-the-money options as substitutes for both sides of a spread or hedge strategy—is discussed in more detail in Chapter 31 on index spreading and Chapter 35 on futures spreads.

FACILITATION (BLOCK POSITIONING)

Facilitation is the process whereby a trader seeks to aid in making markets for the purchase or sale of large blocks of stock. This is not really an arbitrage, and its description is thus deferred to Chapter 28.

SUMMARY

Arbitrage involving options can be very profitable, but unless the profit potential is sufficiently large, it is generally a strategy that is for professional traders who are exchange members—who pay little or no commissions. Various forms of arbitrage are possible, ranging from riskless (discount, dividend, conversion and reversal arbitrage, interest plays, boxes, and equivalence arbitrage). While they may entail some risk (underlying expiring right at the striking price, for example), their risk is small. There are also forms of arbitrage that involve much more risk—and therefore more profit potential—involving mergers, takeovers, and tender offers. Regardless, one must be sure in any of the arbitrage situations, that the underlying he is trading matches the terms of the options in his position. Otherwise it will not be a riskless arbitrage; in fact, risk could be quite large.

CHAPTER 28

Mathematical Applications

In previous chapters, many references have been made to the possibility of applying mathematical techniques to option strategies. Those techniques are developed in this chapter. Although the average investor—public, institutional, or floor trader—normally has a limited grasp of advanced mathematics, the information in this chapter should still prove useful. It will allow the investor to see what sorts of strategy decisions could be aided by the use of mathematics. It will allow the investor to evaluate techniques of an information service. Additionally, if the investor is contemplating hiring someone knowledgeable in mathematics to do work for him, the information to be presented may be useful as a focal point for the work. The investor who does have a knowledge of mathematics and also has access to a computer will be able to directly use the techniques in this chapter.

THE BLACK–SCHOLES MODEL

Since an option's price is the function of stock price, striking price, volatility, time to expiration, and short-term interest rates, *it is logical that a formula could be drawn up to calculate option prices* from these variables. Many models have been conceived since listed options began trading in 1973. Many of these have been attempts to improve on one of the first models introduced, the *Black–Scholes model.* This model was introduced in early 1973, very near the time when listed options began trading. It was made public at that time and, as a result, gained a rather large number of adherents. The formula is rather easy to use in that the equations are short and the number of variables is small.

The actual formula is:

$$\text{Theoretical option price} = pN(d_1) - se^{-rt}N(d_2)$$

$$\text{where } d_1 = \frac{\ln\left(\frac{p}{s}\right) + \left(r + \frac{v^2}{2}\right)t}{v\sqrt{t}}$$

$$d_2 = d_1 - v\sqrt{t}$$

The variables are:

p = stock price
s = striking price
t = time remaining until expiration, expressed as a percent of a year
r = current risk–free interest rate
v = volatility measured by annual standard deviation
ln = natural logarithm
$N(x)$ = cumulative normal density function

An important by-product of the model is the exact calculation of the delta—that is, the amount by which the option price can be expected to change for a small change in the stock price. The delta was described in Chapter 3 on call buying, and is more formally known as the *hedge ratio.*

$$\text{Delta} = N(d_1)$$

The formula is so simple to use that it can fit quite easily on most programmable calculators. In fact, some of these calculators can be observed on the exchange floors as the more theoretical floor traders attempt to monitor the present value of option premiums. Of course, a computer can handle the calculations easily and with great speed. A large number of Black–Scholes computations can be performed in a very short period of time.

The cumulative normal distribution function can be found in tabular form in most statistical books. However, for computation purposes, it would be wasteful to repeatedly look up values in a table. Since the normal curve is a smooth curve (it is the "bell-shaped" curve used most commonly to describe population distributions), the cumulative distribution can be approximated by a formula:

$$x = 1 - z(1.330274y^5 - 1.821256y^4 + 1.781478y^3 - .356538y^2 + .3193815y)$$

$$\text{where } y = \frac{1}{1 + .2316419\,|\sigma|} \qquad \textit{and} \qquad z = .3989423e^{-\sigma^2/2}$$

$$\text{Then } N(\sigma) = x \text{ if } \sigma > 0 \qquad \text{or} \qquad N(\sigma) = 1 - x \text{ if } \sigma < 0$$

This approximation is quite accurate for option pricing purposes, since one is not really interested in thousandths of a point where option prices are concerned.

Example: Suppose that XYZ is trading at 45 and we are interested in evaluating the July 50 call, which has 60 days remaining until expiration. Furthermore, assume that the volatility of XYZ is 30% and that the risk-free interest rate is currently 10%. The theoretical value calculation is shown in detail, in order that those readers who wish to program the model will have something to compare their calculations against.

Initially, determine t, d_1, and d_2, by referring to the formulae on the previous page:

$$t = 60/365 = .16438 \text{ years}$$

$$d_1 = \frac{\ln(45/50) + (.1 + .3 \times .3/2) \times .16438}{.3 \times \sqrt{.16438}}$$

$$= \frac{-.10536 + (.145 \times .16438)}{.3 \times .40544} = -.67025$$

$$d_2 = -.67025 - .3\sqrt{.16438} = -.67025 - (.3 \times .40544) = -.79189$$

Now calculate the cumulative normal distribution function for d_1 and d_2 by referring to the above formulae:

$$d_1 = -.67025$$

$$y = \frac{1}{1 + (.2316419)\ \ |-.67025|} = \frac{1}{1.15526} = .86561$$

$$z = .3989423e^{-(-.67025)\times(-.67025)/2}$$

$$= .3989423e^{-0.22462} = .31868$$

There are too many calculations involved in the computation of the fifth-order polynomial to display them here. Only the result is given:

$$x = .74865$$

Since we are determining the cumulative normal distribution of a negative number, the distribution is determined by subtracting x from 1.

$$N(d_1) = N(-.67025) = 1 - x = 1 - .74865 = .25134$$

In a similar manner, which requires computing new values for x, y, and z,

$$N(d_2) = N(-.79189) = 1 - .78579 = .21421$$

Now, returning to the formula for theoretical option price, we can complete the calculation of the July 50 call's theoretical value, called value here for short:

$$\begin{aligned} \text{value} &= 45 \times N(d_1) - 50 \times e^{-.1 \times .16438} \times N(d_2) \\ &= 45 \times .25134 - 50 \times .9837 \times .21421 \\ &= .7746 \end{aligned}$$

Thus, the theoretical value of the July 50 call is just slightly over ¾ of a point. Note that the delta of the call was calculated along the way as $N(d_1)$ and is equal to just over .25. That is, the July 50 call will change price about ¼ as fast as the stock for a small price change by the stock.

This example should answer many of the questions that readers of the previous editions have posed. The reader interested in a more in-depth description of the model, possibly including the actual derivation, should refer to the article "Fact and Fantasy in the Use of Options."[1] One of the less obvious relationships in the model is that call option prices will increase (and put option prices will decrease) as the risk-free interest rate increases. It may also be observed that the model correctly preserves relationships such as increased volatility, higher stock prices, or more time to expiration, which all imply higher option prices.

CHARACTERISTICS OF THE MODEL

Several aspects of this model are worth further discussion. First, the reader will notice that *the model does not include dividends paid by the common stock*. As has been demonstrated, dividends act as a negative effect on call prices. Thus, direct application of the model will tend to give inflated call prices, especially on stocks that pay relatively large dividends. There are ways of handling this. Fisher Black, one of the coauthors of the model, suggested the following method: Adjust the stock price to be used in the formula by subtracting, from the current stock price, the present worth of the dividends likely to be paid before maturity. Then calculate the option price. Second, assume that the option expires just prior to the last ex-dividend date preceding actual option expiration. Again adjust the stock price and calculate the option price. Use the higher of the two option prices calculated as the theoretical price.

Another, less exact, method is to apply a weighting factor to call prices. The weighting factor would be based on the dividend payment, with a heavier weight being applied to call options on high-yielding stock. It should be pointed out that, in many of the applications that are going to be prescribed, it is not necessary to know the exact theoretical price

[1]Fisher Black, *Financial Analysts Journal*, July–August 1975, pp. 36–70.

of the call. Therefore, the dividend "correction" might not have to be applied for certain strategy decisions.

The model is based on a lognormal distribution of stock prices. Even though the normal distribution is part of the model, the inclusion of the exponential functions makes the distribution lognormal. For those less familiar with statistics, a normal distribution has a bell-shaped curve. This is the most familiar mathematical distribution. The problem with using a normal distribution is that it allows for negative stock prices, an impossible occurrence. Therefore, the lognormal distribution is generally used for stock prices, because it implies that the stock price can have a range only between zero and infinity. Furthermore, the upward (bullish) bias of the lognormal distribution appears to be logically correct, since a stock can drop only 100% but can rise in price by more than 100%. Many option pricing models that antedate the Black–Scholes model have attempted to use empirical distributions. An empirical distribution has a different shape than either the normal or the lognormal distribution. Reasonable empirical distributions for stock prices do not differ tremendously from the lognormal distribution, although they often assume that a stock has a greater probability of remaining stable than does the lognormal distribution. Critics of the Black–Scholes model claim that, largely because it uses the lognormal distribution, the model tends to overprice in-the-money calls and underprice out-of-the-money calls. This criticism is true in some cases, but does not materially subtract from many applications of the model in strategy decisions. True, if one is going to buy or sell calls solely on the basis of their computed value, this would create a large problem. However, if strategy decisions are to be made based on other factors that outweigh the overpriced/underpriced criteria, small differentials will not matter.

The computation of volatility is always a difficult problem for mathematical application. In the Black-Scholes model, volatility is defined as the annual standard deviation of the stock price. This is the regular statistical definition of standard deviation:

$$\sigma^2 = \frac{\sum_{i=1}^{n} (P_i - P)^2}{n - 1}$$

$$v = \sigma / P$$

where

P = average stock price of all P_i's

P_i = daily stock price

n = number of days observed

v = volatility

When volatility is computed using past stock prices, it is called a historical volatility. *The volatilities of stocks tend to change over time.* Certain predictable factors, such as a large stock split increasing the float of the stock, can reduce the volatility. The entry of a company into a more speculative area of business may increase the volatility. Other, less well-defined factors can alter the volatility as well. Since the volatility is a very crucial element of the pricing model, it is important that the modeler use a reasonable estimate of the current volatility. *It has become apparent that an annual standard deviation is not accurate, because it encompasses too long a period of time.* Recent efforts by many modelers have suggested that one should perhaps weight the recent stock price action more heavily than older price action in arriving at a current volatility. This is a possible approach, but the computation of such factors may introduce as much error as using the annual standard deviation does. The problem of accurately computing the volatility is critical, because the model is so sensitive to it.

Computing Lognormal Historical Volatility. The above calculation does not give the proper input for the Black–Scholes model because the model assumes that the *logarithms* of changes in price are normally distributed, not the prices themselves. That is, the term P_i in the above formula should be changed.

Example: XYZ closed at 51 today and at 50 yesterday. Thus, its percentage change for the day is 51/50 = 1.02. The natural logarithm of 1.02 is then based on the volatility formula:

$$\ln({}^{51}\!/_{50}) = \ln(1.02) = 0.0198$$

This is similar to saying that arithmetically the stock was up 2% today, but on a lognormal basis, it was only up 1.98%

If the stock is down, this method will yield a negative number. Suppose that on the following day, XYZ declined from 51 back to 50. The number to use in the volatility formula would then be:

$$\ln({}^{50}\!/_{51}) = \ln(0.9804) = -0.0198$$

A new equation can now be formulated using this concept. It will yield volatilities that are consistent with the Black–Scholes model:

$$v = \sqrt{\frac{\sum_{i=1}^{n} (X_i - X)^2}{n-1}}$$

where $X_i = \ln(P_i/P_{i-1})$; P_i = closing price on day i and $\bar{X}$ = the average of the X_i's over the desired number of days.

So to compute a 10-day historical volatility, one would need 11 observations. In the following example, do not be concerned with the complete details if you do not plan to compute the volatilities yourself; they are provided for the mathematician or programmer who needs to check his work:

Day	XYZ Stock	P_i/P_{i-1}	$X_i = \ln(P_i/P_{i-1})$	$(X_i - \bar{X})^2$
1	153.875			
2	153.625	.9984	–.0016	.000020
3	151	.9829	–.0172	.000405
4	146	.9669	–.0337	.001336
5	144.125	.9872	–.0129	.000250
6	147.25	1.0217	.0215	.000345
7	146.25	.9932	–.0068	.000094
8	149.5	1.0222	.0220	.000365
9	152.5	1.0201	.0199	.000289
10	158.625	1.0402	.0394	.001332
11	158.375	0.9984	–.0016	.000020
			AVG: 0.0028825	Σ: 0.004455

The average of the lns (4th column) over the 10 days is 0.00288.

The difference of each ln from the mean, squared, is then summed (5th column). For example, for day 1 the term is $(-.0016 - .00288)^2 = .00002$. This is the top number in the far right-hand column. This process can be computed for each number in the "ln" column. The sum of all these terms is 0.004455.

$$\text{Now } \upsilon = \sqrt{(.004455/9)} = 0.022249$$

This is a 10-day volatility. To convert it into an annual volatility, we need to multiply by the square root of the number of trading days in a year. Since there are approximately 260 trading days in a year, the final volatility would be:

$$\upsilon = 0.022249 \times \sqrt{(260)} = 0.3587$$

Thus, one could say that the volatility of XYZ is 36% on an annualized basis.

This is then the proper way to calculate historical volatility. Obviously, the strategist can calculate 10-, 20-, and 50-day and annual volatilities if he wishes—or any other number for that matter. In certain cases, one can discern valuable information about a stock or future and its options by seeing how the various volatilities compare with one another.

COMPUTING COMPOSITE IMPLIED VOLATILITY

There is, in fact, a way in which *the strategist can let the market compute the volatility for him.* This is called using the *implied volatility*; that is, the volatility that the market itself is implying. This concept makes the assumption that, for options with striking prices close to the current stock price and for options with relatively large trading volume, the market is fairly priced. This is something like an efficient market hypothesis. If there is enough trading interest in an option that is close to the money, that option will generally be fairly priced. Once this assumption has been made, a corollary arises: *If the actual price of an option is the fair price, it can be fixed in the Black–Scholes equation while letting volatility be the unknown variable.* The volatility can be determined by iteration. In fact, this process of iterating to compute the volatility can be done for each option on a particular underlying stock. This might result in several different volatilities for the stock. If one weights these various results by volume of trading and by distance in- or out-of-the-money, a single volatility can be derived for the underlying stock. This volatility is based on the closing price of all the options on the underlying stock for that given day.

Example: XYZ is at 33 and the closing prices are given in Table 28-1. Each option has a different implied volatility, as computed by determining what volatility in the Black–Scholes model would result in the closing price for each option: That is, if .34 were used as the volatility, the model would give 4½ as the price of the January 30 call. In order to rationally combine these volatilities, weighting factors must be applied before a volatility for XYZ stock itself can be arrived at.

The weighting factors for volume are easy to compute. The factor for each option is merely that option's daily volume divided by the total option volume on all XYZ options (Table 28-2). The weighting functions for distance from the striking price should probably not be linear. For example, if one option is 2 points out-of-the-money and another is 4 points out-of-the-money, the former option should not necessarily get twice as much weight as the latter. Once an option is too far in- or out-of-the-money, it should not be given much or any weight at all, regardless of its trading volume. Any parabolic function of the following form should suffice:

$$\text{Weighting factor} \begin{cases} = \dfrac{(x-a)^2}{a^2} & \text{if } x \text{ is less than } a \\ = 0 & \text{if } x \text{ is greater than } a \end{cases}$$

TABLE 28-1.
Implied volatilities, closing price, and volume.

Option	Option Price	Volume	Implied Volatility
January 30	4.50	50	.34
January 35	1.50	90	.28
April 35	2.50	55	.30
April 40	1.50	5	.38
		200	

TABLE 28-2.
Volume weighting factors.

Option	Volume	Volume Weighting Factor
January 30	50	.25 (50/200)
January 35	90	.45 (90/200)
April 35	55	.275 (55/200)
April 40	5	.025 (5/200)

where x is the percentage distance between stock price and strike price and a is the maximum percentage distance at which the modeler wants to give any weight at all to the option's implied volatility.

Example: An investor decides that he wants to discard options from the weighting criterion that have striking prices more than 25% from the current stock price. The variable, a, would then be equal to .25. The weighting factors, with XYZ at 33, could thus be computed as shown in Table 28-3. To combine the weighting factors for both volume and distance from strike, the two factors are multiplied by the implied volatility for that option. These products are summed up for all the options in question. This sum is then divided by the products of the weighting factors, summed over all the options in question. As a formula, this would read:

$$\text{Implied volatility} = \frac{\sum(\text{Volume factor} \times \text{Distance factor} \times \text{Implied volatility})}{\sum(\text{Volume factor} \times \text{Distance factor})}$$

In our example, this would give an implied volatility for XYZ stock of 29.8% (Table 28-4). Note that the implied volatility, .298, is not equal to any of the individual option's implied volatilities. Rather, it is a composite figure that gives the most weight to the heavily

TABLE 28-3.
Distance weighting factors.

Option	Distance from Stock Price	Distance Weighting Factor
January 30	.091 (3/33)	.41
January 35	.061 (2/33)	.57
April 35	.061 (2/33)	.57
April 40	.212 (7/33)	.02

TABLE 28-4.
Option's implied volatility.

Option	Volume Factor	Distance Factor	Option's Implied Volatility
January 30	.25	.41	.34
January 35	.45	.57	.28
April 35	.275	.57	.30
April 40	.025	.02	.38

$$\text{Implied volatility} = \frac{.25 \times .41 \times .34 + .45 \times .57 \times .28 + .275 \times .57 \times .30 + .025 \times .02 \times .38}{.25 \times .41 + .45 \times .57 + .275 \times .57 + .025 \times .02}$$
$$= .298$$

traded, near-the-money options, and very little weight to the lightly traded (5 contracts), deeply out-of-the-money April 40 call. This implied volatility is still a form of standard deviation, and can thus be used whenever a standard deviation volatility is called for.

This method of computing volatility is quite accurate and proves to be sensitive to changes in the volatility of a stock. For example, as markets become bullish or bearish (generating large rallies or declines), most stocks will react in a volatile manner as well. Option premiums expand rather quickly, and this method of implied volatility is able to pick up the change quickly. One last bit of fine-tuning needs to be done before the final volatility of the stock is arrived at. On a day-to-day basis, the implied volatility for a stock—especially one whose options are not too active—may fluctuate more than the strategist would like. A smoothing effect can be obtained by taking a moving average of the last 20 or 30 days' implied volatilities. An alternative that does not require the saving of many previous days' worth of data is to use a momentum calculation on the implied volatility. For example, today's final volatility might be computed by adding 5% of today's implied volatility to 95% of yesterday's final volatility. This method requires saving only one previous piece of data—yesterday's final volatility—and still preserves a "smoothing" effect.

Once this implied volatility has been computed, it can then be used in the Black–Scholes model (or any other model) as the volatility variable. Thus one could compute the theoretical value of each option according to the Black–Scholes formula, utilizing the implied volatility for the stock. Since the implied volatility for the stock will most likely be somewhat different from the implied volatility of this particular option, there will be a discrepancy between the option's actual closing price and the theoretical price as computed by the model. This differential represents the amount by which the option is theoretically overpriced or underpriced, *compared to other options on that same stock.*

Computing a Volatility Skew

There is not a single, definitive way to calculate a single number for each stock each day that represents the skew in the options, but this is one acceptable way. Essentially, the process is this:

1. Calculate the individual implied volatility of each option.
2. Calculate the standard deviation of the series in step 1. It is not necessary to weight these individual implied volatilities as one does in the composite implied volatility calculation. Rather, merely compute the standard deviation of the set of implied volatilities. Also, note that one may want to eliminate options that are essentially trading with little or no time value premium from this standard deviation calculation, since they are not representative of the "normal" options on this stock.
3. Divide the result of step 2 by the composite implied volatility, computed as shown in the preceding section.

Example: XYZ is trading at 6.50. It has several listed options, with various individual implied volatilities.

Option	Implied Volatility
Mar 5 call	85.0%
June 5 call	77.5%
Mar 7.5 call	75.0%
June 7.5 call	70.0%

The standard deviation of these four numbers is 6.25. Note that this number does not take into account the price or the volume of the individual options. However, deeply in- or out-of-the-money options would not be included if their time value premium is extremely small.

Furthermore, assuming that the composite implied volatility of the above four options (which *does* use volume and distance in- or out-of-the-money), is 75.0%, the "skew factor" for this stock on this day would be:

$$\text{Skew factor} = 6.25 / 75.0 = 8.3\%$$

Similar skew factors would be computed for all stocks, and then ranked. Those with the highest skew factors are likely to have a distinct volatility skew. One would have to look at the implied volatilities of the individual options on any particular stock with a large skew factor to see what is causing the skew.

If an "event" (FDA hearing, lawsuit verdict, earnings report, etc.) is due, that might cause a horizontal skew—where the implied volatilities of the options expiring just after the anticipated event are more expensive than all other options. Conversely, in a bearish market, there might be a vertical skew, where options at lower strikes, for example, have higher implied volatilities than options at higher strikes. Strategies dealing with these skews will be discussed in the chapter on Volatility Trading Techniques.

Once the Composite Implied Volatility and the Volatility Skew Factor are computed, one should consider keeping a database of daily values for every stock, index, ETF, and futures contract. With this information, one would then be able to compute percentiles of implied volatility and skew, looking back over time. These are useful statistics to help one decide if a particular stock's options are indeed expensive or cheap, or if they are unusually skewed.

EXPECTED RETURN

Certain investors will enter positions only when the historical percentages are on their side. When one enters into a transaction, he normally has a belief as to the possibility of making a profit. For example, when he buys stock he may think that there is a "good chance" that there will be a rally or that earnings will increase. The investor may consciously or unconsciously evaluate the probabilities, but invariably, an investment is made based on a positive expectation of profit. Since options have fixed terms, they lend themselves to a more rigorous computation of expected profit than the aforementioned intuitive appraisal. This more rigorous approach consists of computing the expected return. *The expected return is nothing more than the return that the position should yield over a large number of cases.*

TABLE 28-5.
Calculation of expected returns.

Price of XYZ in 6 Months	Chance of XYZ Being at That Price
Below 30	20%
31	10%
32	10%
33	10%
34	10%
Above 35	40%
	100%

A simple example may help to explain the concept. The crucial variable in computing expected return is to outline what the chances are of the stock being at a certain price at some future time.

Example: XYZ is selling at 33, and an investor is interested in determining where XYZ will be in 6 months. Assume that there is a 20% chance of XYZ being below 30 in 6 months, and that there is a 40% chance that XYZ will be above 35 in 6 months. Finally, assume that XYZ has an equal 10% chance of being at 31, 32, 33, or 34 in 6 months. All other prices are ignored for simplification. Table 28-5 summarizes these assumptions.

Since the percentages total 100%, all the outcomes have theoretically been allowed for. Now suppose a February 30 call is trading at 4 and a February 35 call is trading at 2 points. A bull spread could be established by buying the February 30 and selling the February 35. This position would cost 2 points—that is, it is a 2-point debit. The spreader could make 3 points if XYZ were above 35 at expiration for a return of 150%, or he could lose 100% if XYZ were below 30 at expiration. The expected return for this spread can be computed by multiplying the outcome at expiration for each price by the probability of being at that price, and then summing the results. For example, if XYZ is below 30 at expiration, the spreader loses $200. It was assumed that there is a 20% chance of XYZ being below 30 at expiration, so the expected loss is 20% times $200, or $40. Table 28-6 shows the computation of the expected results at all the prices. The total expected profit is $100. This means that the expected return (profit divided by investment) is 50% ($100/$200). This appears to be an attractive spread, because the spreader could "expect" to make 50% on his money, less commissions.

What has really been calculated in this example is merely *the return that one would expect to make in the long run if he invested in the same position many times throughout history*. Saying that a particular position has an expected return of 8 or 9% is no different

TABLE 28-6.
Computation of expected profit.

XYZ Price at Expiration	Chance of Being at That Price (A)	Profit at That Price (B)	Expected Profit: (A) x (B)
Below 30	20%	–$200	–$ 40
31	10%	– 100	– 10
32	10%	0	0
33	10%	+ 100	+ 10
34	10%	+ 200	+ 20
Above 35	40%	+ 300	+ 120
		Total expected profit	$100

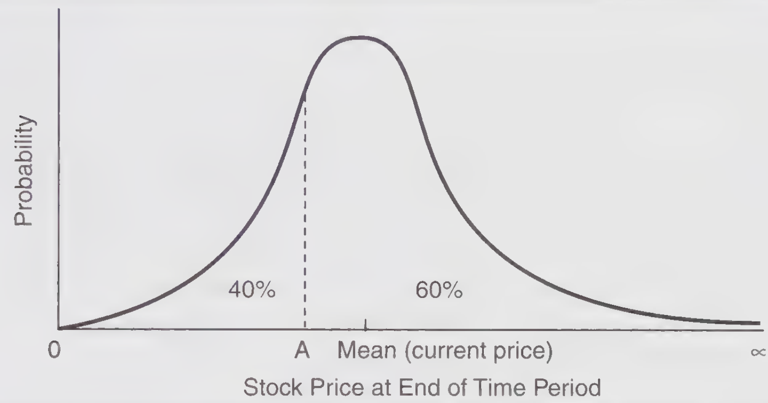
from saying that common stocks return 8 or 9% in the long run. Of course, in bull markets stock would do much better, and in bear markets much worse. In a similar manner, this example bull spread with an expected return of 50% may do as well as the maximum profit or as poorly as losing 100% in any one case. It is the total return on many cases that has the expected return of 50%. *Mathematical theory holds that, if one constantly invests in positions with positive expected returns, he should have a better chance of making money.*

As is readily observable, the selection of what percentages to assign to the possible outcomes in the stock price is a crucial choice. In the example above, if one altered his assumption slightly so that XYZ had a 30% chance of being below 30 and a 30% chance of being above 35 at expiration, the expected return would drop considerably, to 25%. Thus, *it is important to have a reasonably accurate and consistent method of assigning these percentages.* Furthermore, the example above was too simplistic, in that it did not allow for the stock to close at any fractional prices, such as 32.50. A correct expected return computation must take into account all possible outcomes for the stock.

Fortunately, there is a straightforward method of computing the expected percentage chance of a given stock being at a certain price at a certain point in time. This computation involves using the distribution of stock prices. As mentioned earlier, the Black–Scholes model assumes a lognormal distribution for stock prices, although many modelers today use nonstandard (empirical or heuristic) distributions. No matter what the distribution, *the area under the distribution curve between any two points gives the probability of being between those two points.*

Figure 28-1 is a graph of a typical lognormal distribution. The peak always lies at the "mean," or average, of the distribution. For stock price distributions, under the random walk assumption, *the "mean" is generally considered to be the current stock price.* The

FIGURE 28-1.
Typical lognormal distribution.



graph allows one to visualize the probability of being at any given price. Note that there is a fairly great chance that the stock will be relatively unchanged; there is no chance that the stock will be below zero; and there is a bullish bias to the graph—the stock could rise infinitely, although the chances of it doing so are extremely small.

The chance that XYZ will be below the mean at the end of the time period is 50% in a random walk distribution. This also means that 50% of the area under the graph lies to the left of the mean and 50% lies to the right of the mean. Note point A on the graph. Forty percent of the area under the distribution curve lies to the left of point A and 60% lies to the right of it. This means that there is a 40% chance that the stock will be below price A at the end of the time period and a 60% chance that the stock will be above price A. Consequently, the distribution curve can be used to determine the probabilities necessary for the expected return computation. The reader should take note of the fact that these probabilities apply to the *end of the time period*. They say nothing about the chances that XYZ might dip below price A at some time *during* the time period. To compute that percentage, an involved computation is necessary.

The height and width of the distribution graph are determined by the volatility of the underlying stock, when volatility is expressed as a standard deviation. This is consistent with the method of computing volatility described earlier in this chapter. Implied volatility can, of course, be used. Since the option modeler is generally interested in time periods other than one year, *the annual volatility must be converted into a volatility for the time period in question*. This is easily accomplished by the following formula:

$$v_t = v\sqrt{t}$$

where

υ = annual volatility
t = time, in years
υ_t = volatility for time, t.

As an example, a 3-month volatility would be equal to one-half of the annual volatility. In this case, t would equal .25 (one fourth of a year), so $\upsilon_{.25} = \upsilon\sqrt{.25} = .5\upsilon$.

The necessary groundwork has been laid for the computation of the probability necessary in the expected return calculation. The following formula gives the probability of a stock that is currently at price p being below some other price, q, at the end of the time period. The lognormal distribution is assumed.

Probability of stock being below price q at end of time period t:

$$P\ (\text{below}) = N\left(\frac{\ln\left(\frac{q}{p}\right)}{\upsilon_t}\right)$$

where

N = cumulative normal distribution
p = current price of the stock
q = price in question
ln = natural logarithm for the time period in question.

If one is interested in computing the probability of the stock being *above* the given price, the formula is

$$P\ (\text{above}) = 1 - P\ (\text{below})$$

With this formula, the computation of expected return is quickly accomplished with a computer. One merely has to start at some price—the lower strike in a bull spread, for example—and work his way up to a higher price—the high strike for a bull spread. At each price point in between, the outcome of the spread is multiplied by the probability of being at that price, and a running sum is kept.

Simplistically, the following iterative equation would be used.

$$P\ (\text{of being at price } x) = P\ (\text{below } x) - P\ (\text{below } y)$$

where y is close to but less than x in price. As an example:

$$P \text{ (of being at 32.4)} = P \text{ (below 32.4)} - P \text{ (below 32.3)}$$

Thus, once the low starting point is chosen and the probability of being below that price is determined, one can compute the probability of being at prices that are successively higher merely by iterating with the preceding formula. In reality, *one is using this information to integrate the distribution curve.* Any method of approximating the integral that is used in basic calculus, such as the Trapezoidal Rule or Simpson's Rule, would be applicable here for more accurate results, if they are desired.

A partial example of an expected return calculation follows.

Example: XYZ is currently at 33 and has an annual volatility of 25%. The previous bull spread is being established—buy the February 30 and sell the February 35 for a 2-point debit—and these are 6-month options. Table 28-7 gives the necessary components for computing the expected return. Column (A), the probability of being below price q, is computed according to the previously given formula, where $p = 33$ and $v_t = .177\,({}_t = .25\sqrt{1/2})$. The first stock price that needs to be looked at is 30, since all results for the bull spread are equal below that price—a 100% loss on the spread. The calculations would be performed for each tenth of a point up through a price of 35. The expected return is computed by multiplying the two right-hand columns, (B) and (C), and summing the results. Note that column (B) is determined by subtracting successive numbers in column (A). It would not be particularly enlightening to carry this example to completion, since the rest of the computations are similar and there is a large number of them.

TABLE 28-7.
Calculation of expected returns.

Price at Expiration (q)	(A) P (below q)	(B) P (of being at q)	(C) Profit on Spread
30	.295	.295	–$200
30.10	.301	.006	– 190
30.20	.308	.007	– 180
30.30	.316	.008	– 170
.	.	.	.
.	.	.	.
.	.	.	.

* * *

In theory, if one had the data and the computer power, he could evaluate a wide range of strategies every day and come up with the best positions on an expected return basis. He would probably get a few option buys (puts or calls), some bull spreads, some naked writes and ratio calendar spreads, fewer straddles and ratio writes, and a few covered call writes. This theory would be somewhat difficult to apply in practice, because of the massive numbers of calculations involved and also because of the accuracy of closing price data. It was mentioned previously that a computer will assume that "bad" closing prices are actually attainable. By a "bad" closing price, it is meant that the option did not trade simultaneously with the stock later in the day, and that the actual market for the option is somewhat different in price than is reflected by the closing price for the option. A daily contract volume "screen" will help alleviate this problem. For example, one may want to discard any option from his calculations if that option did not trade a predetermined, minimum number of contracts during the previous day. Data that give closing bids and offers for each option are more expensive but also more reliable, and would alleviate the problem of "bad" closing prices. In addition to a volume screen, another way of reducing the calculations required is to limit oneself to strategies in which one has interest, or which one is reasonably certain will fit in well with his investment objectives. Regardless of the limitations that one places upon the quantity of computations, some computer power is necessary to compute expected return. A sophisticated programmable calculator may be able to provide a real-time calculation, but could never be used to evaluate the entire option universe and come up with a ranking of the preferable situations each day. On-line computer systems are also available that can provide these types of calculations using up-to-the-minute prices. While real-time prices may occasionally be useful, it is not an absolute necessity to have them.

One other by-product of the expected return calculation is that it could be used as another model for predicting the theoretical value of an option. All one would have to do is compute the probabilities of the stock being at each successive price above the striking price of the option by expiration, and sum them up. The result would be the theoretical option value. These data are published by some services and generally give a different theoretical value than would the Black–Scholes model. The reason for the difference most readily lies in the inclusion of the risk-free interest rate in the Black–Scholes model and its omission in the expected return model.

APPLYING THE CALCULATIONS TO STRATEGY DECISIONS

CALL WRITING

One method of ranking covered call writes that was described in Chapter 2 was to rank all the writes that provided at least a minimal acceptable level of return by their probability of not losing money. If one were interested in safety, he might decide to use this approach. Suppose he decided that he would consider any write that provided an annualized total return (capital gains, dividends, and commissions) of at least 12%. This would eliminate many potential writes, but would leave him with a fairly large number of writing candidates each day. He knows the downside break-even point at expiration in each write. Therefore, the probability of the stock being below that break-even point at expiration can be computed quickly. His final list would *rank those writes with the least chance of being below the break-even point at expiration as the best writes.* Again, this ranking is based on an expected probability and is, of course, no guarantee that the stock will not, in reality, fall below the break-even point. However, over time, *a list of this sort should provide the most conservative covered writes.*

Example: XYZ is selling for 43 and a 6-month July 40 call is selling for 8 points. After including dividends and commission costs for a 500-share position, the downside break-even point at expiration is 36. If the annualized volatility of XYZ is 25%, the probability of making money at expiration can be computed. The 6-month volatility is 17.7% (25% times the square root of ½ year). The probability of being below 36 can be computed by using the formula given earlier in this section:

$$P\,(\text{below 36 in 6 months}) = N\left(\frac{\ln\left(\dfrac{36}{43}\right)}{.177}\right) = N\left(\frac{-.178}{.177}\right) = 0.158$$

The expected probability of XYZ being below 36 in 6 months is 15.8%. Therefore, this would be an attractive write on a conservative basis, because it has a large probability of making money (nearly 85% chance of not being below the break-even point at expiration). The return if exercised in this example is approximately 20% annualized, so it should be acceptable from a profit potential viewpoint as well. It is a relatively easy matter to perform a similar calculation, with the aid of a computer, on all covered writing candidates.

The ability to measure downside protection in terms of a common denominator—volatility—can be useful in other types of covered call writing analyses. The writer interested in writing out-of-the-money calls, which generally have higher profit potential, is still interested in having an idea of what his downside protection is. He might, for example, decide that he wants to invest in situations in which the probability of making money is at least 60%. This is not an unusually difficult requirement to fulfill, and will leave many attractive covered writes with a high profit potential to choose from. A downside requirement stated in terms of probability of success removes the necessity of having to impose arbitrary requirements. Typical arbitrary requirements would be including only calls that sell for one point or more, or stating that the downside protection must be a certain percentage of the stock price. These obviously cannot suffice for stocks with different volatilities. Rather, the downside protection criterion should be stated in terms of "probability of down protection" or, alternatively, in terms of the volatility itself. In this manner, a uniform comparison can be made between volatile and nonvolatile stocks.

CALL BUYING

The option buyer can also constructively use the measurement of volatility to aid him in his option buying decisions. In Chapter 3, it was shown that *evaluating the profitability of calls based on the volatility of the underlying stock is the correct way to analyze an option purchase.* One specific method of analysis is described. There are certain variables in this analysis that may be altered to fit the call buyer's individual preferences, but the general logic is applicable to all cases.

As a first step, *one should decide upon a uniform stock movement for ranking call purchases.* One might decide to rank all purchases by how they would perform if the underlying stock moved up in accordance with its volatility. The phrase "in accordance with its volatility" must be quantified. For example, one might decide to assume that every stock could move up one standard deviation, and then rank all call purchases on that basis. *The prospective call buyer must also fix the time period that he wants to use.* Generally, one looks at purchases to be held for 30 days, 60 days, and 90 days.

The exact steps to be followed in the analysis of profitability and risk can be listed as follows:

1. Specify the distance that underlying stock can move, up or down, in terms of its volatility.
2. Select the holding period over which the analysis is to take place.

PROFITABILITY

3. Calculate the stock price that the stock would move up to, when the foregoing assumptions are implemented.
4. Using a pricing model, such as the Black–Scholes model, estimate what the option price would become after the upward stock movement.

5. Calculate the percent profit, after deducting commissions.
6. Repeat steps 4 and 5 for each option on the stock.

A final ranking of all potential call buys can be obtained by performing steps 3 through 6 on all stocks, and ranking the purchases by their percentage reward.

RISK

7. Calculate the stock price that the stock could fall to, when the assumptions in steps 1 and 2 are applied.
8. With a model, price the option after the stock's decline.
9. Calculate the percentage loss after commissions.
10. Compute a reward/risk ratio: Divide the percentage profit from step 5 by the percentage risk from step 9.
11. Repeat steps 8 through 10 for each option on the stock.

A final ranking of less aggressive option purchases can be constructed by performing steps 7 through 11 on all stocks, and ranking the purchases by their reward/risk ratio.

The higher profitability list of option purchases will tend to be at- or slightly out-of-the-money calls. The less aggressive list, ranked by reward/risk potential, will tend to be in-the-money options.

Example: Steps 1 and 2: Suppose an investor wants to look at option purchases for a 90-day holding period, under the assumption that each stock could move up by one standard deviation in that time. (There is only about a 16% chance that a stock will move more than one standard deviation in one direction in a given time period. Therefore, in actual practice, one might want to use a smaller stock movement in his ranking calculations.) Furthermore, assume that the following data are known:

XYZ common, 41;
XYZ volatility, 30% annually;
XYZ January 40 call, 4; and
time to January expiration, 6 months

Step 3: Calculate upward stock potential. This is accomplished by the following formula:

$$q = pe^{av}t$$

where

p = current stock price
q = potential stock price
v_t = volatility for the time period, t
a = a constant (see below).

The constants, a and t, are fixed under the assumptions in steps 1 and 2. The first constant, a, is the number of standard deviations of movement to be allowed. In our example, a = 1. That is, the analysis is being made under the assumption that the stock could move up by one standard deviation. The second constant, t, is .25, since the analysis is for a 90-day holding period, which is 25% of a year. In this example:

$$v_t = v\sqrt{t} = .30\sqrt{.25} = .30 \times .50 = .15$$

so

$$q = 41e^{.15} = 41 \times 1.16 = 47.64$$

Thus, this stock would move up to approximately 47.64 if it moved one standard deviation in exactly 90 days.

Step 4: Using the Black–Scholes model, the XYZ January 40 call can be priced. It would be worth approximately 8.10 if XYZ were at 47.60 and there were 90 days' less life in the call.

Step 5: Calculate the profit potential. For this example, commissions will be ignored, but they should be included in a real-life situation.

$$\text{Percents profit} = \frac{8.10 - 4}{4} = \frac{4.10}{4} = 103\%$$

Thus, if XYZ stock moves up by one standard deviation over the next 90 days, this call would yield a projected profit of 103%. Recall again that there is only about a 16% chance of the stock actually moving at least this far. If all options on all stocks are ranked under this same assumption, however, a fair comparison of profitable options will be obtained.

Step 6 is omitted from this example. It would consist of performing a similar profit analysis (steps 4 and 5) on all other XYZ options, with the assumption that XYZ is at 47.60 after 90 days.

Step 7: Calculate the downside potential of XYZ. The formula for the downside potential of the stock is nearly the same as that used in step 3 for the upside potential:

$$
\begin{aligned}
q &= pe^{-av_t} \\
&= 41e^{-.15} = 41 \times .86 = 35.39
\end{aligned}
$$

XYZ would fall to approximately 35.39 in 90 days if it fell by one standard deviation. Note that the actual distances that XYZ could rise and fall are not the same. The upward potential was 6.60 points, while the downward potential is about 5.75 points. This difference is due to the use of the lognormal distribution.

Step 8: Using the Black–Scholes model, one could estimate that the XYZ January 40 call would be worth about 1.10 if XYZ were at 35.39 in 90 days.

Step 9: The risk potential in the January 40 call would be:

$$\text{Percent risk} = \frac{4 - 1.10}{4} = \frac{2.90}{4} = 73\%$$

Step 10: The reward/risk ratio is merely the percentage reward divided by the percentage risk:

$$\text{Reward/risk ratio} = \frac{103\%}{73\%} = 1.41$$

Step 11: This analysis would be repeated for all XYZ options, and then for all other optionable stocks. The less aggressive call purchases would be ranked by their reward/risk ratios, with higher ratios representing more attractive purchases. More aggressive purchases would be ranked by the potential rewards only (step 5).

This completes the call buying example. Before leaving this section, it should be noted that the assumption of ranking the purchases after one full standard deviation movement by the underlying stock is probably excessive. A more moderate assumption would be that the stock might be able to move .7 standard deviation. There is about a 25% expected chance that a stock could move up at least .7 standard deviation at the end of a fixed time period.

PRICING A PUT OPTION

Theoretical models for pricing put options have been derived; that is, ones that are separate from call pricing models. Black and Scholes presented such a model in their original paper. However, as has been demonstrated, there is a relationship between put and call prices in the listed option market due to the conversion and reversal strategies.

One could use the basic call pricing model for the purpose of predicting put prices if he assumes that arbitrageurs will efficiently influence the market via conversions.

Theoreticians will argue that such a method of pricing puts assumes that the arbitrage process is always present and works efficiently, and that this is not true. The "conversion efficiency" assumption could be a serious fault if one were trying to determine the exact overpriced or underpriced nature of the put option. However, if one is merely comparing various put strategies under constant assumptions, the arbitrage model for pricing puts works quite well.

The listed put's price can be estimated by using the call pricing model and the arbitrage formula. Recall that the arbitrageur must include the cost of carrying the position as well as the dividends to be received.

$$\begin{matrix}\text{Theoretical} \\ \text{put}\end{matrix} = \begin{matrix}\text{Theoretical} \\ \text{call price}\end{matrix} + \begin{matrix}\text{Strike} \\ \text{price}\end{matrix} - \begin{matrix}\text{Stock} \\ \text{price}\end{matrix} - \begin{matrix}\text{Carrying} \\ \text{cost}\end{matrix} + \text{Dividends}$$

The "theoretical call price" is obtained from the Black–Scholes model. The carrying cost is the cost of money (interest rate) times the striking price, multiplied by the time to expiration. Recall that this is the approximation formula for carrying cost (see Chapter 27 for comments on present value and compounding). Consequently, if XYZ were at 41 and a 6-month January 40 call option were valued at 4 points by the Black–Scholes model, the theoretical put price could be estimated. Assume that the cost of money interest rate is 10% annually, and that the stock will pay \$.50 in dividends in 6 months (t = ½ year).

$$\begin{aligned}\text{Theoretical put price} &= 4 + 40 - 41 - (.10 \times 40 \times \tfrac{1}{2}) + .50 \\ &= 3 - 2 + .50 \\ &= 1.50\end{aligned}$$

This means that if the call could be sold for 4 points, the arbitrageur would be willing to pay up to 1.50 points for the put to establish a conversion. The arbitrageur's price is used as the theoretical listed put price estimate.

PUT BUYING

Put option purchases can be ranked in a manner very similar to that described for call option buying. Reward opportunities occur when the stock falls in accordance with its volatility. An upward stock movement represents risk for the put buyer. All of the 11 steps in the previous section on call buying are applicable to put buying. The pricing of the put necessary for steps 4 and 8 is done in accordance with the arbitrage model just presented.

If an underlying stock does not have listed puts trading, the synthetic put can be considered. While all U.S.-listed stocks have both puts and calls at every strike, there are still situations with warrants, especially in foreign countries, that are applicable to the following discussion. Recall that synthetic puts are created for customers by some brokerage houses. The brokerage sells the stock short and buys a call. The customer can purchase the synthetic put for the amount of the risk involved, plus any dividends to be paid by the underlying stock. The synthetic put pricing formula that would be used in steps 4 and 8 of the option buying analysis is exactly the same as the arbitrage model for listed puts, except that the carrying costs are omitted:

$$\begin{matrix}\text{Theoretical synthetic} \\ \text{put price}\end{matrix} = \begin{matrix}\text{Theoretical} \\ \text{call price}\end{matrix} + \begin{matrix}\text{Strike} \\ \text{price}\end{matrix} - \begin{matrix}\text{Stock} \\ \text{price}\end{matrix} + \text{Dividends}$$

When the ranking analysis is performed, very few synthetic puts will appear as attractive put buys. This is because, when the customer buys a synthetic put, he must advance the full cost of the dividend, but receives no offsetting cost reduction for the credit being earned by the short stock position. Consequently, synthetic puts are always more expensive, on a relative basis, than are listed puts. However, if one is particularly bearish on a stock that has no listed puts, a synthetic put may still prove to be a worthwhile investment. The recommended analysis can give him a feeling for the reward and risk potential of the investment.

CALENDAR SPREADS

The pricing model can help in determining which neutral calendar spreads are most attractive. Recall that in a neutral calendar spread, one is selling a near-term call and buying a longer-term call, when the stock is relatively close to the striking price of the calls. The object of the spread is to capture the time decay differential between the two options. The neutral calendar spread is normally closed when the near-term option expires. The pricing model can aid the spreader by estimating what the profit potential of the spread is, as well as helping in the determination of the break-even points of the position at near-term expiration.

To determine the maximum profit potential of the spread, assume that the near-term call expires worthless and use the pricing model to estimate the value of the longer-term call with the stock exactly at the striking price. Since commission costs are relatively large in spread transactions, it would be best to have the computations include commissions. Calculating a second profit potential is sometimes useful as well—*the profit if unchanged.* To determine how much profit would be made if the stock were unchanged at near-term expiration, assume that the spread is closed with the near-term call equal to its intrinsic value (zero if the stock is currently below the strike, or the difference between the stock price and the strike if the stock is initially above the strike). Then use the pricing model to estimate the value of the longer-term call, which will then have three or six months of life remaining, with the stock unchanged. The resulting differential between the near-term call's intrinsic value and the estimated value of the longer-term call is an estimate of the price at which the spread could be liquidated. The profit, of course, is that differential minus the current (initial) differential, less commissions.

In the earlier discussion of calendar spreads, it was pointed out that there is both an upside break-even point and a downside break-even point at near-term expiration. These break-even points can be estimated with the use of the pricing model. One method of determination involves estimating the liquidating value of the spread at successive stock prices. When the liquidating value is found to be equal to the initial value, plus commissions, a break-even point has been located.

Example: If the spread in question is using options with a striking price of 30, one would begin his break-even point calculations at a price of 30. Estimate the liquidating value of the spread at 30, 29.90, 29.80, 29.70, and so forth until the break-even point is found. Once the downside break-even point has been determined in this manner, the iterations to locate the upside break-even point should begin again at the striking price. Thus, one would evaluate the liquidating value at 30, 30.10, 30.20, and so on. This is somewhat of a brute-force method, but with a computer it is fairly fast. The number of calculations can be reduced by adopting a more complicated iteration process.

A final useful piece of information can be obtained with the aid of the pricing model—*the theoretical value of the spread.* Recompute the estimated value of both the near-term and longer-term calls at the current time and stock price, using the implied volatility for the underlying stock. The resultant differential between the two estimated call prices may differ substantially from the actual differential, perhaps highlighting an attractive calendar spread situation. One would want to establish spreads in which the theoretical differential is *greater than* the actual differential (that is, he would want to buy a "cheap" calendar spread).

Once these pieces of information have been computed, the strategist can rank the spread possibilities by whatever criterion he finds most workable. *The logical method of ranking the spreads is by their return if unchanged.* The spreads with the highest return if unchanged at near-term expiration are those in which the stock price and striking price were close together initially, a basic requirement of the neutral calendar spread. More complicated ranking systems should try to include the theoretical value of the spread and possibly even the maximum potential of the spread. *A similar analysis can, of course, be worked out for put calendar spreads,* using the arbitrage pricing model for puts.

RATIO STRATEGIES

Ratio strategies involve selling naked options. Therefore, the strategist has potentially large risk, either to the upside or to the downside or both. He should attempt to get a feeling for how probable this risk is. The formulae for determining the probability of a stock being above or below a certain price at some time in the future can give him these probabilities. For example, in a straddle writing situation, the strategist would want to compute such arithmetic quantities as maximum profit potential, return if unchanged, collateral required at upside break-even point or at upside action point (recall that the collateral requirement increases for naked options on an adverse stock movement), and the break-even points themselves. The probabilities of being above the upper break-even point at expiration or below the lower break-even point should be computed as well. Moreover, an expected return analysis could be performed on the position to determine the general level of profitability of the position with respect to all other positions of the same type on other stocks. Such an expected return analysis need not assume that the position is held to expiration. Firm traders, paying little or no commissions, might be interested in seeing the expected results for a holding period as short as 30 days or less. Public customers might use a longer holding period, on the assumption that they would not trade the position as readily because of commission costs. Ratio positions should be ranked either by return if unchanged or by expected return.

The analyses described for calendar spreads and ratio positions should not be relied upon as gospel. In the proposed forms of analysis, one is projecting future option prices and stock prices under the assumption that the volatility of the underlying stock will remain the same. Although this may be true in some cases, there will also be many times when the volatility of the underlying stock will change during the life of the position. If the volatility decreases, the projected break-even points for a calendar spread will be too far away from the striking price. Thus, a loss would result at some prices where the spreader expected to make money. If the volatility increases, the expected return of a ratio position will drop, because the probabilities of the stock moving outside the profit range will increase, thereby increasing the probability of loss.

The effect of a changing volatility can be counteracted, in theory, by continuing to monitor the position daily after it has been established. In a straddle write, for example, if the stock begins to move dramatically, the expected return may become very low. If this happens, adjustments could be made to the position to improve it. Such monitoring is difficult to apply in practice for the public customer, because the commission costs involved in constant position adjustments would mount rapidly. There is no exact method that would allow for infrequent, periodic adjustments, but *by using a follow-up analysis the public customer may be able to get a better feeling for the timing of adjusting a position.* For example, suppose that one initially wrote a 5-point straddle when the stock was at 30. Sometime later, the stock is at 34. The expected return of writing a 5-point straddle with a strike of 30 when the stock is at 34 could be computed for the shorter time period remaining until expiration. If the expected return is negative, an adjustment needs to be made. Adopting this form of adjusting would keep the number of trades to a minimum, but would still allow the strategist to determine when his position has become improperly balanced. Of course, the current volatility would be used in making these determinations. Another follow-up monitoring technique, using the deltas of the options involved, is presented later in this chapter, and has been described several times previously.

FACILITATION OR INSTITUTIONAL BLOCK POSITIONING

In this and the following section, *the advantages of using the hedge ratio are outlined.* These strategies are primarily member firm, not public customer, strategies, since they are best applied in the absence of commission costs. An institutional block trader may be able to use options to help him in his positioning, particularly when he is trying to help a client in a stock transaction.

Suppose that a block trader wants to make a bid for stock to facilitate a customer's sell order. If he wants some sort of a hedge until he can sell the stock that he buys, and the stock has listed options, he can sell some options to hedge his stock position. To determine the quantity of options to sell, he can use the hedge ratio. The exact formula for the hedge ratio was given earlier in this chapter, in the section on the Black–Scholes pricing model. It is one of the components of the formula. Simply stated, the hedge ratio is merely the delta of the option—that is, the amount by which the option will change in price for small changes in the stock price. By selling the correct number of calls against his stock purchase, the block trader will have a neutral position. This position would, in theory, neither gain nor lose for small changes in the stock price. He is therefore buying himself time until he can unwind the position in the open market.

Example: A trader buys 10,000 shares of XYZ, and a January 30 call is trading with a hedge ratio of .50. To have a neutral position, the trader should sell options against 20,000 shares of stock (10,000 divided by .50 equals 20,000). Thus, he should sell 200 of the January 30's. If the hedge ratio is correct—largely a function of the volatility estimate of the underlying stock—the trader will have greatly eliminated risk or reward on the position for small stock movements. Of course, if the block trader wants to assume some risk, that is a different matter. However, for the purposes of this discussion, the assumption is made that the block trader merely wants to facilitate the trade in the most risk-free manner possible. In this sample position, if the stock moves up by 1 point, the option should move up by 50 cents. The trader would make $10,000 on his stock position and would lose

$10,000 on his 200 short options—he has no gain or loss. Once the trader has the neutral position established, he can then begin to concentrate on unwinding the position.

In actual practice, this hedge ratio may not work exactly, because it tends to change constantly as the stock price changes. If the trader finds the stock moving more than fractionally, he may have to add more calls or buy some in, to maintain a neutral hedge ratio. This would expose him to some risk, but the risk is substantially smaller than if he had not hedged at all. Of course, there would also be certain cases in which he would profit by the stock price change. For example, implied volatility could decrease, making the calls cheaper.

A similar hedge can be established by the block trader who sells stock to accommodate a customer buy order. He could buy calls in accordance with the hedge ratio, to set up a neutral position.

This process of facilitation is quite widely practiced, especially by brokerage houses that are trying to attract the business of the large institutional customer. Since the introduction of listed call options and their applications for facilitating orders, many quotes for large blocks of stock have improved considerably. The block trader (who works for the brokerage house) is willing to make a higher bid or a lower offer if he can use options to hedge his position. This facilitation with options results in a better market (higher bid or lower offer) from the point of view of the institutional customer. Without the availability of such listed options, the block trader would probably make a bid or an offer that was substantially away from the prevailing market price in order to work out of his stock-only position with a lessened degree of risk. This would obviously present a poorer market for the institutional customer.

THE NEUTRAL SPREAD

The hedge ratios (deltas) of two or more options may be used to determine a neutral spread. This strategy is especially useful to market-makers on the options exchanges who may want to reduce the risk of options bought or sold in the process of providing a public market. If the hedge ratios of two options are known, *the neutral ratio is determined by dividing the two hedge ratios.*

Example: An XYZ January 35 has a hedge ratio of .25, and an XYZ January 30 has a hedge ratio of .50, so a neutral ratio would be 2:1 (.50 divided by .25). That is, one would sell 2 January 35's against one long January 30, or, conversely, would buy 2 January 35's against one short January 30. Thus, a market-maker who has just bought 50 January 30's in an effort to provide a market for a public seller of that call could hedge his position by selling 100 January 35's. This should keep his risk small, for small stock price changes, until he can unwind the position. The ratio for the neutral spread is not as sensitive to the volatility estimate of the underlying stock as is the ratio concerning stock and options. This is because the same volatility estimate is applied to both options, and the resultant ratio for the spread would not tend to change greatly.

The risk trader can also use the neutral spread ratio to his advantage. This concept was illustrated several times in previous chapters describing ratio writing, ratio spreads, and straddle writes. Ratio spreads are quite popular with member firm traders and floor traders. Recall that a ratio spread consists of buying options at a certain strike, and selling more options further out-of-the-money. The hedge ratios can, of course, be used by the trader, or by a public customer, to initially establish a neutral position. Perhaps more important, the hedge ratio can also be used as a follow up action to keep the position neutral after the stock changes in price. This strategy is the "delta spread" described in Chapter 11.

The risk trader is not attempting to establish the spread with the idea of minimizing risk for small stock movements. Rather, he is looking to make a profit, but would prefer to remain as neutral as possible on the underlying stock. He is implementing a risk strategy that has a neutral outlook on the underlying stock. He is selling much more time value premium than he is buying.

Example: The purchase of 15 January 30 calls and the sale of 30 January 35 calls—a ratio call spread—may be a position taken for profit potential. It would be a neutral position if the deltas were .60 and .30, for example. This spread would do best if the stock were at exactly 35 at expiration. However, if the stock rose quickly before expiration, the spread ratio would decrease from 2:1 to perhaps 3:2. That is, the neutral ratio between the January 30 call and the January 35 call should be 3 short January 35's to 2 long January 30's. If the trader wants to balance his position, he could buy 5 more January 30's, giving him a total of 20 long versus the 30 short January 35's that he originally sold. Conversely, if the stock dropped in price, the neutral spread ratio might increase, indicating that more calls should be sold. For example, if this stock declines, the neutral ratio might be 3:1. In that case, 15 more January 35's could be sold, making the position short 45 calls versus 15 long calls, which would produce the neutral 3:1 ratio.

It would not be proper to adjust the ratio constantly, because the frequent whipsaw losses on trading movements would wipe out the profit potential of the position. However, the trader may want to pick out points, in advance, at which he wants to reevaluate his position before something drastic goes wrong. For example, if the foregoing spread were established with the stock at a price of 30, the spreader might want to readjust at 33 or 27, whichever comes first.

By monitoring the spread using the hedge ratio, the trader may also be able to discern whether he has established too bullish or too bearish a position.

Example: The trader starts with the example described above—long 15 January 30 calls and short 30 January 35 calls—when the hedge ratios were .60 and .30, respectively. Some later time, the stock falls to 27 and the trader needs to reevaluate his position. The hedge ratios may have become .42 for the January 30 and .14 for the January 35, indicating that a 3:1 ratio would be neutral (.42/.14 = 3). He now has a bullish position, because his 2:1 ratio is *less than* the neutral 3:1 ratio. It is not mandatory that the trader act on this information. He may actually be bullish on the stock at this point and decide to remain with his position. The usefulness of the hedge ratio is that it allows him to see that his position is bullish, so he can make a correct judgment. Without this knowledge, he might still think his position to be neutral, a critical mistake if he indeed wants to be neutral. If the trader's ratio is *greater than* the neutral ratio (2:1 vs. 3:2, for example), he is bearishly positioned.

As a final point, it should be noted that the ratio can be adjusted by buying or selling either option.

Example: If the stock falls and it is desired that the ratio be increased to 3:1, one might sell more January 35's or might decide to sell out some of his January 30's. A bullish adjustment could be made by buying on either side of the spread in a similar manner. In general, one should adjust by selling time premium or buying intrinsic value. That is, out-of-the-moneys are usually sold and in-the-moneys are usually bought, when adjusting.

AIDING IN FOLLOW-UP ACTION

The computer can also be an invaluable aid to the strategist in that it can help him monitor his positions. It is generally necessary for the strategist to have some way of inputting his positions into an inventory database and also to have some way of identifying different securities that are grouped within the same trading position. Once this has been done, *the computer can simultaneously read pricing data (either realtime or closing prices) and the inventory database to generate information concerning the current status of any position.*

A current mark to market (profit and loss) statement is of obvious use in that the trader can see how he is doing each day. The computer can also easily generate a set of warning flags that may be of interest to the trader, and could produce a list summarizing possible positions that need action. In most of the strategies that were described, it was shown that the strategist should avoid early assignment if at all possible. It is a simple matter for the computer to calculate the remaining time value premium of any short options, and to warn the trader if there is only a small amount of time value premium remaining, perhaps 0.10 or less. For similar reasons, the trader may want to have a daily list of positions that are nearing maturity, perhaps with less than 1 month of life remaining in the options. A flag indicating an approaching ex-dividend date might also be useful for this purpose.

If the trader inputs another piece of information into the database, the computer can help him in another follow-up action. In most strategies that were described, especially those involving uncovered options, the trader wants to take some sort of follow-up action based on the price movement of the underlying stock. If the stock rallies too far, he may want to cover short calls or buy other calls as protection. If the stock declines too far, similar maneuvers would apply to put options or to rolling down short calls. If the trader inputs the stock prices at which he would like to take action, the computer can monitor each day's closing price of the stock and generate a list of positions that have exceeded their upside or downside action points.

The computer can also do more sophisticated types of position monitoring. Recall that it was pointed out that the deltas of the options involved in a position can be compared to each other to tell whether the position is bullish or bearish. The Black–Scholes model can be used to calculate the deltas of the options in one's positions. Then the net position can be determined by the computer, thereby telling the trader whether his position has become "delta long" (bullish), "delta short" (bearish), or neutral. If he sees that a position is bearish and he does not want to be structured in that way, he can make bullish adjustments. The delta spread and neutral spread strategies very conveniently lend themselves to such types of follow-up action, although any of the more complicated straddle writing and protected straddle writing positions can be monitored usefully in this way as well.

The computation for determining whether a position is net short or net long generally involves calculating the "equivalent stock position" (ESP). If one owns 10 calls that have a delta of .45, his equivalent stock position from those calls is 10 x 100 shares per call × .45 = 450. That is, owning those 10 calls is equivalent to owning 450 shares of the underlying stock, according to the delta. All puts and calls can be reduced to an ESP and can then, of course, be combined with any actual long or short stock in the position to produce an ESP for the entire strategy. The resultant ESP for each of the trader's positions can be printed form the computer along with the items described above.

Further sophisticated measures can be taken. The computer can generate a table of results at expiration. If so desired, this could be presented as a graph, but that is not really necessary. A table suffices quite well, as shown by most of the examples in this book. Such a picture has meaning only if all options in the position expire at the same time. If they don't, one may instead want the computer to compose a table of results or a graph at *near-term expiration*. Thus, in a calendar spread, for example, one could see what sorts of profitability he would be looking at when it was time to remove the spread.

Finally, the computer can compute the expected return of a position already in place. This would give a more dynamic picture of the position, and this expected return is usually for a relatively short time period. That time period might be 30 days, or the time remaining until expiration, whichever is less. The expected return is calculated in much the same manner as the expected return computation described earlier in this chapter. First, one uses the stock's volatility to construct a range of prices over which to examine the position. Second, one uses the Black–Scholes model to calculate the values of the various options in the position at that future time and at the various stock prices. Some of the results should be displayed in table form by the computer program. The expected profit is computed, as described earlier, by summing the multiples of the probabilities of the stock being at each price by the result of the position at that price. The expected return is then computed by dividing the expected profit by the expected investment. Since margin computations can require involved computer programs, it is sufficient to omit this last step and merely observe the expected profit. The following example shows how a sample position might look as the computer displays the position itself, the ESP, the profit at expiration, and the expected profit in 30 days. A complex position is assumed, in order that the value of these analyses can be demonstrated.

Example: The following position exists when XYZ is at 31.75. It is essentially a backspread combined with a reverse ratio write. It resembles a long straddle in that there is increased profit potential in either direction if the stock moves far enough by expiration. Initially, the computer should display the position and the ESP.

Position		Delta	ESP	
Short	4,500 XYZ	1.00	Short	4,500 shares
Short	100 XYZ April 25 calls	0.89	Short	8,900 shares
Long	50 XYZ April 30 calls	0.76	Long	3,800 shares
Long	139 XYZ July 30 calls	0.74	Long	10,286 shares
Total ESP			Long	686 shares
Total money in position: $163,500 credit				

The advantage of using the ESP is that this fairly complex position is reduced to a single number. The entire position is equivalent to being long 686 shares of the common stock. Essentially, this is close to delta-neutral for such a large position. The next item that the computer should display is the total credit or debit in the position to date. With this information, an expiration picture can be drawn if it is applicable. In this position, since there is a mixture of April and July options, a strict expiration picture does not apply. Rather, the computer should draw a picture based on the position at April expiration or on a shorter time horizon.

Assume that April expiration is still some time away, so that the computer will instead draw the picture 30 days hence. In order to do so, the computer uses the stock's volatility to project stock prices 30 days in the future. Seven stock prices are shown in the next table; they represent points along the distribution curve of the stock, ranging from minus one and one-half standard deviations to plus one and one-half standard deviations of movement from the current stock price. While these seven points certainly do not comprise the entire spectrum of possible stock moves, they are a representative sample.

Stock Price in 30 Days	Standard Deviations	Expected Results
35.90	+1.5	+$15,847
34.10	+1.0	+ 12,355
32.90	+0.5	+ 10,097
31.75	0.0	+ 9,443
30.60	–0.5	+ 10,743
29.50	–1.0	+ 14,172
28.50	–1.5	+ 19,605
Expected profit		+$11,426

Obviously, this position has had some profitable adjustments made to it in the past. That is not important at this point, because the trader is interested only in the future. If the current mark to market of this position were in excess of $11,426, then he should consider removing the position, since it would be more profitable than the expected profit.

IMPLEMENTATION

Many of the analyses described in this chapter can be obtained from a reliable data service or brokerage firm. The strategist who plans to prepare his own analysis, either by himself or by contracting the programming work out, should be aware that computer programs should not be written in website languages such as Java Script, PHP, HTML, etc., because the mathematics are far too complicated. Languages such as Pascal, C, C++, Visual Basic, or any high-level structured programming language would suffice, although Java could be considered as well, keeping in mind that Java's main usage is not for computational purposes. Reliable option pricing data that include dividend information on the underlying stock are also necessary. The larger programmable calculators can handle calculations such as the Black–Scholes model, computing the hedge ratio, and determining the probability of a stock being above or below a certain price at some future time.

However, more involved calculations, such as computing the implied volatility or determining the expected return of a position, require the use of a computer.

SUMMARY

Two basic mathematical aids have been presented: the pricing model and the ability to predict the probability of a stock's movement. The hedge ratio and the expected return analysis are extensions of the basic aids. Any strategy can be evaluated with these tools. Such an analysis should be able to give the trader or strategist some idea of the relative attractiveness of establishing the position, and may also aid in making follow-up adjustments to the position. All the analyses rely heavily on one's estimate of the volatility of the underlying stock. Using the implied volatility seems to be one of the best ways to obtain an accurate, current volatility estimate, since it is derived from the prices in the market itself. The applications presented here are not all-inclusive. The strategist who is, or becomes, familiar with the advantages of rigorous mathematical analysis will be able to construct many other aids for his trading that utilize the basic mathematics described in this chapter.

PART V

Index Options and Futures

CHAPTER 29

Introduction to Index Option Products and Futures

Since their introduction in 1981, listed index options have proved to be very popular. Index options are options whose underlying security is not a single stock but rather an index composed of many stocks. These include options on index futures contracts. Most popular types of cash settlement options are options on indices or subindices. The strategies employed in trading these options are not substantially different from those used in trading stock options, with a few notable exceptions. However, the options themselves tend to be priced differently and to trade differently. It is these differences between stock options and index options on which we will predominantly concentrate.

Index products—cash-based options, futures-based options, and index futures—will be the main topic of discussion in this section of the book. We will look at how indices are constructed, how to use these products to speculate, how to hedge, and how to spread one index against another. Both futures and options will be used in these strategies. The discussion of other futures options—currencies, grains, bonds, etc—will be deferred to a later chapter.

In this chapter, we will be looking at introductory facts about index options and futures which differentiate them from the equity options that have encompassed the entire previous part of this book. First, however, we will take an in-depth look at stock indices and the methods of calculating them. Also in this chapter there will be a discussion of futures contracts and how trading them differs from trading stocks and stock options.

INDICES

Since many cash-based or futures options have an index of stocks underlying the option, it is useful to understand how indices are calculated, in order that one may be able to understand how an individual stock's movement within the index affects the overall value of the index. The indices on which options are traded are generally stock indices—that is, the items making up the index are stocks. There are two main ways of calculating a stock index: weighted by price or weighted by capital.

CAPITALIZATION-WEIGHTED INDICES

The capitalization of a stock is the total dollar value of its securities to current market prices: It is the multiple of the number of shares outstanding (the float) and the current stock price. In a capitalization-weighted index, the capitalizations of all stocks in the index are computed and added together to produce the total market value of the index. The price of each stock in the index is multiplied by the total number of shares of that stock that are outstanding (the "float"), and their sum is calculated. Finally, this total sum is divided by another number, termed the "divisor," to produce the final index value. An example will help to illustrate the concept of calculating the value of a capitalization-weighted index.

Example: Suppose that an index is composed of three stocks whose prices and floats are given in the following table. The multiple of price times float (capitalization) is also included in the table.

Stock	Price	Float	Capitalization
A	30	175,000,000	5,250,000,000
B	90	50,000,000	4,500,000,000
C	50	100,000,000	5,000,000,000
		Total capitalization:	14,750,000,000

Most indices use a divisor since it would be unwieldy to say that the index closed at 14,750,000,000 (for example, think of trying to quote the Dow-Jones Industrial Average as such a large figure). The divisor is generally an arbitrary number that is initially used to reduce the index value to a workable number. When an index is initiated, the divisor might be set so that the index starts out at an even number. Suppose that in the sample index above, we wanted the initial value—as represented by the given prices and floats—to be 100.00. Then we would set the initial divisor to 147,500,000. Thus the total capitalization of the index divided by the divisor would give a value of 100.00.

The divisor of an index can be changed to provide continuity for the index's value when changed occur in the individual components. Note that the divisor does not have to be changed when a stock splits, because the price is adjusted downward automatically by an amount equal to the increase in the float of the stock that is splitting. Notice that in the above example, if stock B should split 2-for-1 then its price would be 45 (90 ÷ 2) and its float would double to 100 million shares from 50 million. Thus, the capitalization of stock B remains the same: $4,500,000,000.

However, if a stock should alter its capitalization in a manner that is not reflected by an automatic adjustment in its price, then the divisor must be changed. For example, a company might issue more stock in a secondary offering—something that would not cause the exchange where the stock is listed to automatically reduce the price of the stock. To produce continuity in the value of the index between the day the secondary is issued and the day after it is issued, the divisor is changed to keep the index value the same. Consider the following example.

Example: Using the same sample index as before, suppose that the following prices exist at the closing one day:

Stock	Price	Float	Capitalization
A	40	175,000,000	7,000,000,000
B	80	50,000,000	4,000,000,000
C	60	100,000,000	6,000,000,000
	Total capitalization:		17,000,000,000
	Divisor: 147,500,000		
	Index value: 115.25		

Now suppose that stock A issues a 2-million-share secondary that evening, giving that stock a total float of 177 million shares. Such an action would change the value of the index as follows:

Stock	Price	Float	Capitalization
A	40	177,000,000	7,080,000,000
B	80	50,000,000	4,000,000,000
C	60	100,000,000	6,000,000,000
	Total capitalization:		17,080,000,000
	Divisor: 147,500,000		
	Index value: 115.80		

However, it makes no sense to change the value of the index from 115.25 to 115.80 when nothing actually changed in the marketplace. If investors deem it necessary to lower the price of stock A in the marketplace because of the secondary issue, so be it. But such a change in investor philosophy would be reflected in the price of the index as the stock drops. So, in order to keep the value of the index the same on the morning after the secondary is issued, the divisor must be changed to reflect the extra 2 million shares of stock A. The new divisor would be equal to the new total capitalization (17,080,000,000) divided by the old index value (115.2542373). This would give the new divisor:

$$\text{New divisor: } 148,194,117.6$$

As this example demonstrates, the divisor of a capitalization-weighted index can change quite often. Fortunately, there are organizations that are responsible for keeping the index current and for calculating the divisor every time it needs changing. Thus, an investor who needs to know the latest divisor can generally find it out by making a phone call or visiting a website. This is far easier than keeping track of everything by oneself.

In a capitalization-weighted index, the stocks with the largest market value have the most weight within the index. This means that indices that contain such largely capitalized stocks as Apple, Google, IBM, AT&T, General Electric, and Exxon will be dominated by those stocks. *In order to compute the percentage that a stock comprises of the index, it is merely necessary to divide that stock's capitalization by the total capitalization of the index.* Using the previous example, one can see how the percentage is computed.

Stock	Price	Float	Capitalization	Pct
A	40	177,000,000	7,080,000,000	41.5%
B	80	50,000,000	4,000,000,000	23.4%
C	60	100,000,000	6,000,000,000	35.1%
	Total capitalization:		17,080,000,000	100.0%

Another interesting statistic to know regarding any index is how many shares of each stock are in the index. In a capitalization-weighted index, *the number of shares of each stock is determined by dividing the stock's float by the divisor of the index*. In the same sample index, the following table shows how many shares of each stock are in the index.

Stock	Price	Float	Capitalization	Shares
A	40	177,000,000	7,080,000,000	1.20
B	80	50,000,000	4,000,000,000	0.34
C	60	100,000,000	6,000,000,000	0.68
		Total capitalization:	17,080,000,000	
		Divisor: 147,500,000		
		Index value: 115.80		

Thus, if stock A goes up by one point, then the value of the index would increase by 1.20 points since there are 1.2 shares of stock A in the index. One can see the value of computing such a statistic—it readily allows him to see how any individual stock's movement will affect the index movement during a trading day. This is especially useful when a stock is halted, but the index itself keeps trading.

Example: Suppose that, in the above index, stock C has halted trading. There are 0.68 shares of stock C in the index. Suppose that stock C is indicated 3 points lower, but that the index is currently trading unchanged from the previous night's close due to the fact that both stocks A and B are unchanged on the day. If one were to try to price the options on the index, he would be wrong to use the current price of the index since that will soon change when stock C opens. However, there is not really a problem since one can readily see that if stock C opens 3 points lower, then the index will drop by 2.04 points (3×0.68). Thus one should price the options as if the index were already trading about 2 points lower. This kind of anticipation depends, of course, on knowing the number of shares of stock C in the index.

A similar type of analysis is useful when trying to predict longer-term effects of a stock on an index. If you thought stock C had a chance of rallying 30 points, then one can see that this would cause the index to rise over 20 points. Given this type of relationship, there are sometimes option spreads between the stock's options and the index's options that will be profitable based on such an assumption.

It should also be noted that the number of shares of stock in a capitalization-weighted index does not change on a daily basis since it does not depend on the price of the stocks in the index. However, *the percent that each stock comprises of the index does change each day as prices change.* Thus, the number of shares is a more stable statistic to keep track of, and is also more directly usable to anticipate index value changes as stock prices change.

Capitalization-weighted indices are the most prevalent type, and most investors are familiar with several of them: the Standard and Poor's 500, the Standard and Poor's 400, the Standard and Poor's 100 (also called by its quote symbol, OEX), the New York Stock Exchange Index, and the American Stock Exchange Index.

PRICE-WEIGHTED INDICES

A price-weighted index contains an equal number of shares of each stock in the index. A price-weighted index is computed by adding together the prices of the various stocks in the index and then dividing that sum by the divisor to produce the index value. Again, the divisor is initially an arbitrary number that is used to produce a desired original index value-something like 100.00, for example. Let us use the same three stocks we were using above to construct an example of a price-weighted index. Assume the divisor at the time of this example is 1.65843.

Stock		Price
A		30
B		90
C		50
Price total:		170
	Divisor: 1.65843	
	Index value: 102.51	

Unlike the capitalization-weighted index, the divisor needs to be changed when a stock's price is adjusted by the exchange where it is listed (as in a stock split or stock dividend) but does not have to be adjusted when the company issues more stock. That is, the divisor in a price-weighted index is changed when the price of the stock is adjusted, but not when the stock's capitalization is changed.

If a certain stock issues new stock in a secondary offering, the exchange where its stock is listed will not automatically adjust the price of the stock downward. Hence, there is no change to the divisor of any price-weighted index containing that stock because the closing price of the stock was not adjusted by the exchange.

However, in the above example, if stock B should split 2-for-1, the exchange would change its closing from 90 to 45. Thus, the sum of the stocks in the price-weighted index would change without the market even being open. Consequently, the divisor would need to be changed to reflect the split. The following example sums up the situation after stock B splits 2-for-1.

Stock	Price
A	30
B	45
C	50
Price total:	125

Old divisor: 1.65843

Previous closing index value: 102.51

New divisor (i.e., the divisor necessary to keep the index value unchanged): 1.21943

The new divisor is calculated by dividing the new sum of the prices, 125, by the old closing price, 102.51. Thus the divisor is reduced in order to produce the same index value—102.51—even though the sum of the prices of the stocks in the index is now 125 instead of the previous 170. Note that the new divisor is not dependent on the old divisor.

Another statistic that we looked at with capitalization-weighted indices was the number of shares of each stock in the index. In a price-weighted index each stock in the index has the same number of shares and that number is equal to 1 divided by the divisor of the index. In the last example above, with the divisor equal to 1.21943, there would be 1/1.21943 or 0.82 shares of each stock in the index. Thus any stock that was up by 1 point during a given day would be contributing an upward movement of 0.82 points in the index. Before the split there were 0.60 (1/1.65843) shares of each stock.

Another way to look at the revision of a price-weighted index following a split by one of its stocks is the following: If one stock splits, then to reestablish the fact that there are an equal number of shares of each stock in the index, part of the extra (split) shares should be sold off and used to buy an equal number of shares of each of the remaining stocks. Note that before the split there were 0.60 shares of each stock, and 0.82 shares after. When stock B split 2-for-1, it increased its shares from 0.60 to 1.20, so to rebalance the index it was necessary to sell 0.38 shares of stock B and use the proceeds to buy 0.22 shares of each of stocks A and C.

A price-weighted index's divisor can be subject to fairly frequent revision, just as was the case with the capitalization-weighted index. These divisors are maintained by the organizations responsible for originating them, and they can be easily obtained just by calling the proper organization. The most popular price-weighted indices are the various Dow-Jones indices.

The stock with the most weight in a price-weighted index is the one with the highest price, which is substantially different from the capitalization-weighted index where the stock with the most weight is the one with the most market value. Thus, in the above examples, the stock with the greatest weight in the index would be stock B before the split and C after the split. Of course, the matter of a stock's volatility has something to do with which stock has the most weight in the change of the value of the index. Thus, if stock B was the highest-priced stock at $90 per share, but had a very low volatility, then its price changes would be small and it might consequently not have as great an influence on the changes in the price of the index as some lower-priced stock would.

In general, one is far less concerned with a stock's weight in a price-weighted index than he is in a capitalization-weighted index. That is, one might notice that four or five large stocks—IBM, AT&T, Exxon, Apple, and General Electric, for example—might make up over 30% of the S&P 100 even though they represent only 5% of the stocks in the index. However, the same five stocks in a price-weighted index of 100 stocks would probably account for very nearly 5% of the index because their prices are not substantially different from those of the other 95 stocks (even though their capitalizations are). So if one were to notice a large change in the price of IBM, one might figure that capitalization-weighted indices that contained that stock would also be showing somewhat unusual price changes in the same direction that IBM is moving. A price-weighted index that contained IBM would, of course, also be affected by IBM's price change, but not extraordinarily so since IBM would have far less weight in the price-weighted index.

SECTORS

Sector is a term used to refer to an index of stocks in which all the stocks are members of the same industry group. Examples of groups on which sectors have been created—and on which options have traded—are computers and technology, international oils, domestic oils, gold, transportation, airlines, and gaming and hotels. These indices are computed in the same ways as described above—either price-weighted or capitalization-weighted. They generally consist of fewer stocks than their major counterparts, however. Most sectors are comprised of between 20 and 30 stocks, since that is about all of the stocks in any one specific industry group. The large indices are usually referred to as "broad-based" indices, as opposed to the smaller sectors which are often referred to as "narrow-based" indices.

Options trade on these sectors. The intent of these options is to allow portfolio managers—who often are group-oriented—to be able to hedge off parts of their portfolio by industry group. The options on these sectors are usually cash-based options. Strategies will be discussed later, but there is not much difference in strategy between broad-based or narrow-based index options. One difference is that broad-based option writers receive more favorable margin treatment (that is, they are required to put up less collateral) than narrow-based option writers.

CASH-BASED OPTIONS

Now that the reader is familiar with indices, let us look at the most popular type of listed index option, the cash-based option.

Cash-based options do not have any physical entity underlying the option contract. Rather, if the option is exercised or assigned, the settlement is done with cash only—there is no equity involved. This type of option is generally issued on an index, such as the S&P 500, for which it would be virtually impossible to actually deliver the underlying securities in case of assignment or exercise.

Since many investors feel that it is easier to predict the market's movement rather than that of an individual stock, the cash-based index option has become very popular. Other indices that underlie cash-based option contracts are the New York Stock Exchange Index, the S&P 500 Index, the S&P 100 Index (OEX—an index introduced by the CBOE), the NASDAQ Index (NDX), the Dow-Jones 30 Industrials (DJX), and several other indices. In each of these cases there are too many stocks in the index, and too many varying quantities of each of the stocks, to be able to handle the physical delivery of each of the stocks in the case of exercise or assignment. Some cash-based options are based on subindices (that is, subgroups of the larger indices such as the transportation group).

EXERCISE AND ASSIGNMENT

It is important to understand the ramifications of exercise and assignment when dealing with this type of option. *When a cash-based option is exercised, the owner receives cash equal to the difference between the index's closing price and the strike price of the option.* The option writer who is assigned must pay out an equal amount. The following example shows how a call exercise might work. In this and the following examples we will use a fictional index ZYX (index symbols often end in X).

Example: Suppose an investor buys a ZYX September 160 call option. At a later date, the index has risen substantially in price and closes at 175.24 on a particular day. The investor decides it is time to take his profit by exercising his call option. Assuming the ZYX contract is worth \$100 per point, just as stock options are, he receives cash in the amount of \$100 times the difference between the index closing price and the strike price: $\$100 \times (175.24 - 160.00) = \$1{,}524$. He has no further position or rights—the option position disappears from his account by virtue of the exercise and he does not acquire any security by the exercise; he gets only cash.

An assignment would work in a similar manner, with the seller of an option having to pay out of his account cash equal to the difference between the index closing price and the option's striking price. As an example, suppose that a trader sells a put option on the ZYX Index—the October 165 put. Subsequently, the index drops in price, and one morning the writer of this put option finds that he has been assigned (as of the previous day, as is the case with stock options). If the index closed at 157.58 on the previous day, then the option writer's account will be debited an amount equal to $100 × (165.00 – 157.58) = $742.

EUROPEAN VERSUS AMERICAN EXERCISE

Before proceeding with more examples of index option exercise and the accompanying strategies, it is necessary to introduce two new definitions. *American exercise* means that an option may be exercised at any time; *European exercise* means that an option may be exercised only on its expiration day. Many of the cash-based index options have the European exercise feature. All stock options and some index options have the American exercise feature.

The European exercise feature was introduced because institutional investors who might tend to write calls against their portfolio of stocks wanted some assurance that their protection wouldn't be unexpectedly taken away from them. Thus several index option series became European exercise. Two major ones are the cash-based index options on the S&P 500 Index (SPX) and the cash-based options on the Dow-Jones 30 Industrials. OEX remains an American exercise.

In-the-money European put options will be cheaper than their American counterparts. This is because an arbitrageur would have to carry the position all the way to expiration; he could not exercise his puts and liquidate the position immediately. In fact, deeply in-the-money European puts will trade at a discount; the higher short-term interest rates are, the deeper the discount will be.

This can affect the full protective capability of long-term European puts. If a portfolio manager buys puts to protect his portfolio and the market crashes, the puts might be deeply in-the-money. If these puts have a European exercise feature, they would be selling at a deep discount and therefore would not have afforded all the price protection that the portfolio manager had been looking for.

American Exercise Consideration. The primary reason for the holder of an index option to exercise the option is to take his profit. One might think that, if the holder wanted to take a profit, he would merely sell his option in the open market. Of course, if he could, he would. However, many times the deeply in-the-money options sell at a substantial discount during the trading day. A deep discount is considered to be ½ to ¾ of a point, or more. Near the end of the day, these options tend to trade at only slight discounts. In either case, the holder of the option may decide to exercise rather than to sell at any discount. Of course, if one is the holder of a call option that is trading at a substantial discount in the morning of a particular day, and he decides to exercise, he may lose more by the end of the day (if the market trades down) than he would have if he had merely sold at the deep discount in the first place. In fact, some theoreticians feel that the "job" of a deeply in-the-money cash-based option during the trading day is to try to predict the market's close. This, of course, is not a "job" that can be consistently done with accuracy (if it could, the traders doing the predicting would be rich beyond their wildest dreams).

If the holder of a cash-based call option turned bearish, that would be another reason to exercise. That's right—if the holder of a cash-based call option is bearish, he should exercise because, by so doing, he liquidates his bullish position and takes his profit. This is somewhat opposite from an option that has a physical underlying security, such as a stock option. This presents an interesting scenario: If one turns bearish late in the day, even after the close, he might conceivably try to exercise his calls to liquidate his position. The exchanges recognize that such tactics might not be in everyone's best interest—for example, if one waited to see how the money supply numbers looked on a particular evening before exercising, he would definitely have an advantage over the writers of those same options. The writers could no longer viably hedge their positions after the market had closed. In order to prevent this, cash-based option exercise notices are only acceptable until 5 minutes after the options close trading on that exchange on any given trading day (except expiration, of course), in order to allow both holders and writers to be on somewhat equal footing.

There is one more fact regarding exercise of cash-based options that will interest brokerage customers, retail or institutional. Most brokerage firms will charge a commission for the cash-based option exercise or assignment. When index options were first traded, commissions were quite high. Currently, however, one should generally be paying a commission based upon the equivalent option price.

Example: In the previous example, one exercised a ZYX Sep 160 call at expiration when the index closed at 175.24. This is a differential of 15.24. One should pay a commission as if he had sold his long calls at a price of 15.24, not on anything more.

For writers of cash-based options, things are not so different from stock options. The writer is still warned of impending assignment by the fact that the option is trading at a discount. If it is not trading at a discount, it is probably not in danger of being assigned. Also, since there is no stock involved and therefore no dividends paid, the writer of a cash-based put option need only be concerned with whether the put is trading at a discount, not with whether it is trading at a discount to underlying price less the dividend, as is the case with stock options.

Traders doing spreads in cash-based options have special worries, however. *What may seem to be a limited-risk spread may acquire more risk than one initially perceived, due to early assignment of the short options in the spread.* Consider the following example.

Example: Suppose that an investor establishes a bearish call spread in ZYX options—he buys the November 160 call at a price of 1 and simultaneously sells the November 155 call at 3. His risk on the spread is \$300 plus commissions if he has to pay the maximum, limited debit of \$500 to buy back the spread, or so it appears. However, suppose that the index rises substantially in price and the spreader is assigned on the short side of his spread with the index at 175.24. He thus is charged a debit of \$2,024 to "cover" each short call via the assignment: \$100 times the in-the-money amount, 175.24 − 155.00, or 20.24. He receives this assignment notice in the morning before the next trading day begins. Note that he cannot merely exercise his long, since, if he did that, he would then receive the next night's closing price for his long. Under the worst scenario, suppose the market receives disappointing economic news the next day and opens sharply lower—with the index at 172. If he sells his long Nov 160 calls at parity (\$1,200), he will have paid a debit of \$824—larger than his initial, theoretically "limited" maximum debit of \$500. Thus he loses \$624 on this spread (\$824 less the initial credit of \$200)—over twice the theoretically limited loss of \$300.

If the market should open sharply lower and trade down, he could lose more money than he thought because his long position is now exposed—there is no longer a spread in place after the short option is assigned. Of course, this could work to his advantage if the market rallied the next day. The point is, however, that a spread in cash-based options acquires more risk than the difference in the strikes (the maximum risk in stock options) if the short option in the spread becomes a deeply in-the-money option, ripe for assignment.

NAKED MARGIN

When an index is designated as "broad-based," a lesser margin requirement applies to the writer of naked options. The SEC determines which indices are broad-based. A broad-based index receives more favorable margin treatment because the underlying index will not normally change in price as quickly as a stock or subindex. Thus, the naked writer theoretically has less of a risk with a naked broad-based index option.

The requirement for writing a broad-based index option naked is 15% of the index, plus the option premium, minus the amount, if any, that the option is out-of-the-money. There is a minimum requirement: for calls, it is 10% of the index value; for puts, it is 10% of the striking price. Both minima are in addition to the option premium.

Example: Suppose that the ZYX is at 168.00, with a Dec 170 call selling for 6 and a Dec 170 put selling for 5. The requirement for selling the call naked would be calculated as follows:

15% of index	$2,520
Plus call premium	+ 600
Less out-of-money amount	– 200
Naked call requirement	$2,920

The requirement for writing the Dec 170 put naked would be:

5% of index	$2,520
Plus put premium	+ 500
Naked put requirement	$3,020

Both of these requirements are above the minimum of 10% of the index.

Options on narrow-based indices are subject to the same naked requirements as stock options: 20% of the index plus the premium less an out-of-the-money amount, with a minimum requirement of 15% of the index.

Other margin requirements are similar to those for stock options. For example, if one wanted to write the Dec 170 straddle naked, using the same prices as in the last example, he would have a margin requirement equal to $3,020—the larger of the put or call requirement, just as he would for stock options. Spread requirements for index options work in exactly the same manner as they do for stock options. Those accounts that qualify for portfolio margin will have reduced requirements and not the requirements stated above.

FUTURES

We will now take a look at how futures contracts work. This section will be concerned only with cash-based index futures; futures for physical delivery are included in a later chapter. The ordinary stock investor might think that he will be able to employ index option strategies without getting involved in futures. While it may be *possible* to avoid futures, the strategist will realize that they are a necessary part of the entire index-trading strategy. Thus, in order to be completely prepared to hedge one's positions and to operate in an optimum manner, the use of index futures or index futures options is a necessary complement to nearly all index strategies.

A commodities futures contract is a standardized contract calling for the delivery of a specified quantity of a certain commodity at some future time. The older, more conventional types of commodities contracts were futures on grains, meats, and metals. In recent years, futures have expanded extensively and have encompassed financial securities—bonds, T-bills, currencies, etc. In recent decades, futures have been issued that are cash-based; that is, no actual commodity is deliverable. Rather, the contract settles for cash. Some of these cash-based futures contracts have stock market indices as their underlying 'commodity.' It is this latter type of future that will be the subject of the examples in this section, although the basic facts regarding futures are applicable to all futures contracts, cash-based or not.

Several types of traders or investors use futures contracts. One is the speculator: He is able to generate tremendous leverage with futures and may be able to capitalize on small swings in the price of the underlying commodity. Another is the true hedger: He is a dealer in the actual underlying commodity and uses futures to hedge his price risk. This is the more economic function of futures. Examples of hedges for physical commodities as well as stocks will be presented in later chapters. However, let us look at a simple example of how one might hedge a stock portfolio with stock index futures.

Example: Suppose that a stock mutual fund operates under the philosophy that investors cannot outperform a bullish market, so the best investment strategy when one is bullish is just to "buy the market." That is, this mutual fund actually buys all the stocks in the Standard & Poor's 500 Index and holds them.

If the manager of this fund turns bearish, he would want to sell out his positions. However, the commission costs and slippage (difference between bid and asked prices) for liquidating the entire portfolio would be large. Also, the act of selling so much stock might actually depress the market, thereby devaluing the remainder of his portfolio before he can sell it.

This manager might sell S&P 500 futures against his portfolio instead of selling his stocks. Such a futures contract would move up or down in line with the S&P 500 Index as it rises or falls. Suppose that he sold enough futures to hedge the entire dollar value of his stock portfolio. Then, even if the stock market declined, his futures contracts would decline also and would theoretically prevent him from having a loss. Of course, he couldn't make much of a gain if the market went up, since the futures would then lose money. What this money manager has accomplished is that he has effectively sold his stock portfolio without incurring stock commission costs (futures commissions are normally quite small).

If he turns bullish again at some later date, he can buy the futures back, and have his long stocks free to profit if the market rises. Again, he does not spend the stock commission nor does he have to go through the tedious process of placing 500 stock orders to "buy" the S&P 500—he merely places one order in futures contracts.

Futures contracts often trade at premiums to the underlying commodity, due to the fact that the investor who buys the future does not have to spend the money that one who buys all the stocks would have to spend. Thus, he saves the carrying costs but forsakes any dividends. This savings is reflected by the marketplace in that a premium is placed on the price of the futures contract. As a consequence, longer-term contracts trade at a larger premium than do near-term contracts, much as is the case with options. In most cases, however, the index futures trader is concerned with the nearest-term contract, and perhaps the next one out in time. Short-term interest rates affect the premium on the cash-based futures. The "fair value" calculations for such contracts are presented in the next chapter.

TERMS OF THE CONTRACT

There are cash-based index futures on several indices, although some of these futures contracts are not heavily traded. The most heavily traded contract is the e-mini future on the S&P 500 Index. This contract trades on the Chicago Mercantile Exchange. It has contracts that expire every 3 months (March, June, September, December) and a 1-point move in the futures contract is worth $50. There is no particular reason why a 1-point move is worth $50, that is merely how the contract is defined. The "big" S&P 500 futures contract is worth $250 per point of movement.

Example: A futures trader buys 1 March e-mini S&P 500 contract at 401.00 (the smallest unit of trading is 0.25 points). The contract rises in price to 403.50. The trader has a profit of 2.50 points, or $125 (2.50 points × $50 per point).

The terms of futures contracts can change as the exchanges on which they are traded attempt to adjust the contracts to be more competitive in the current trading environment. Consequently, the strategist should check with his broker to determine the exact terms of any contract before he begins trading it.

FUTURES TRADING

Futures generally trade in electronic markets. Previously, they traded in pits via an open outcry method, as stocks and options did. But in recent years, most futures pits have been replaced by electronic markets. This change has been very beneficial to the individual trader, since he can see the futures markets and the option markets on his trading screen—just as he can for stock and index options. This was not always the case.

MARGIN, LIMITS, AND QUOTES

Futures contracts are traded on margin and are marked to market every day. Generally, the amount of margin required is small in comparison to the total size of the contract, so that there is tremendous leverage in trading futures. Anyone trading the futures must deposit the initial margin amount in his account on the day he initiates the trade. Then at the end of each day, the amount of gain or loss on the contract is computed, and the account is credited if there is a gain or debited if there is a loss. In case of a loss, if there is not sufficient margin in the account, the trader must add more cash to his account to cover the loss. This daily margin computation is known as maintenance margin. Treasury bills or other securities are good collateral for the initial margin.

Example: The S&P 500 futures contract is a cash-based futures contract that trades on the Chicago Mercantile Exchange. Since the contract is settled in cash, there is no actual physical commodity underlying the contract. Rather, the contract is based on the value of the S&P 500 stock index. At the expiration of the contract, each open contract is marked to market at the closing price of the S&P 500 stock index and disappears. All contracts are settled for cash on their final day and then they no longer exist—they expire. The terms of the contract specify that each point of movement is worth \$250. Thus, if the S&P 500 Index itself is at 1405, then the S&P futures contract is a contract on \$250 × 1405, or \$351,250 worth of stocks comprising the index. Assume the initial margin for one of these contracts is \$30,000, although it may vary at specific brokerage houses.

Suppose that a trader buys one December S&P futures contract for his account sometime in October. With the underlying index at 1405.00, suppose he pays 1417.50 for the futures contract. The futures will trade at a premium or at a discount to the index itself, based on the level of risk-free interest rates. The reason regarding this will be discussed in a later section. Initially, the customer puts up \$30,000 as margin, and this may be in the form of T-bills. On the next day, however, the market declines and the futures close at 1406.00. This represents a loss of 11.50 points from the purchase price. At \$250 per point, the trader has a loss of \$2,875 (250 × 11.50) at this time. He is required to add \$2,875 in cash into the account, only if there is not already sufficient collateral in the account.

If he continued to hold the contract until expiration, this process of adding his daily gain or subtracting his daily loss from the account would recur each day. Finally, on the last day, the futures contract settles at the "a.m." settlement price if the S&P 500 Index (see next section) and the variation margin is calculated again at that price. Then the futures contract is expired, so it is "erased" from his account. He is then left with only the cash that he made or lost on the trade of his contract.

The leverage produced by small margin requirements (as a percent of the total value of the contract) is a major factor in making futures very volatile, in dollar terms. In the preceding example, a $30,000 margin investment controls $351,250 worth of stocks. Thus he is leveraged almost 12-to-1 in this trade. Due to their volatility, many futures contracts trade with a limit. That is, the price can only fluctuate a fixed amount above or below the previous day's closing price. This concept is intended to prevent traders with large positions from being able to manipulate the market drastically in either direction.

S&P and NYSE Expiration. S&P 500 futures expiration occurs in a somewhat complex way, compared to those indices whose options and futures expire at the last sale on the final day of trading. Some years ago, in order to attempt to reduce the volatility that index futures and options expiration was causing in the stock market, the NYSE and the Chicago Mercantile Exchange (where the S&P 500 futures trade) agreed to change the expiration of their index products from the end of trading on the last Friday to the morning of that day. The effect of expiration on the stock market is discussed in the next chapter.

As a result, the S&P futures and futures options settle in the following manner on their last day of trading. On expiration day—the third Friday of the month—the "final" price for purposes of settling futures and options is comprised of taking the opening trade of each stock and calculating an index price based on those opening prices. There is no actual trading in the futures and options on that last Friday; they cease trading at the close of trading on the previous day, Thursday.

The purpose of this change was to give the specialists on the New York Stock Exchange more time to line up the other side of trades to handle order imbalances. Under the new rules, index arbitrageurs are forced to enter their buy or sell orders as market on open orders on that last Friday before 9 am EST. The specialist can then take his time in opening the stock if he needs to; he can solicit orders if there is too much stock to buy or sell.

The effect of all this is that the "final" index price for settlement purposes is not known until all the stocks in the index have opened. It may take some time to open all 500 stocks in the S&P 500 Index if there is a volatile market that Friday morning (perhaps caused by news) or if there are severe order imbalances in many of the stocks (caused by index arbitrageurs). Index arbitrage is described in Chapter 30.

Limits. Originally, index futures traded without limits. However, the stock market crash of 1987 changed that. Certain parties felt that if the futures—which were leading the market down—had ceased trading for a while, the stock market could have stabilized. As a result, a series of trading limits now exists for stock index futures. These are designed to be "circuit breakers"—to prevent a stock market crash. They are not limits in the sense that other futures have limits, but they are similar.

The levels at which these circuit breakers occur may change from time to time, based on the volatility of the stock market and the price levels at which the S&P futures are trading. That is, if the S&P futures are trading at 1500, one can expect wider circuit breakers than if they are trading at 600. These circuit breakers only apply to downside moves by the stock market. The first in the series of circuit breakers usually halts trading for only 10 minutes. After that, if the market trades lower—usually something on the order of a 10% decline—then a longer circuit breaker is instituted for about 30 minutes or so. After that they could open again, and if they reached the next limit down—something probably on the order of 20%—then trading would be halted for a longer time (two hours or so—again, the details depend on the current regulations). If there is any time left in the trading day, they can open again, and trade down to a final limit, at which time trading would be halted for the day. They could not trade any lower that day, although they could trade lower the next day if need be.

There are similar limits imposed by the NYSE on its trading—based on the Dow-Jones Industrial Averages. Those limits don't necessarily line up exactly with the limits on the S&P futures. That fact might cause problems for hedgers should any of these severe downside limits actually occur.

There are actually other "circuit breakers" designed to prevent runaway stock markets, but they are not related to limits on futures trading. They will be described along with index arbitrage and program trading in Chapter 30.

Quotes. While stock and stock options are always quoted in pennies, or sometimes nickels, such is not the case with futures. Some futures trade in fractions, while others trade in cents. In the coming chapters, there will be many examples of the trading details of futures and options. However, the investor should familiarize himself with the details of an individual contract before beginning to trade it or its options. One's commodity broker can easily supply this information, or it can be obtained from the website of the exchange where the futures trade.

OPTIONS ON INDEX FUTURES

As we saw earlier, futures contracts allow a person dealing in a commodity to minimize profit fluctuations in his commodity. The mutual fund manager who sold the futures largely removed any possibility of further upside profit or downside loss. Options, however, allow a little more leeway than futures do. With the option, a person can lock in one side of his position, but can leave room for further profits if conditions improve. For example, the mutual fund manager might buy put options on the S&P 500 Index to hedge his downside risk, but still leave room for upside profits if the stock market rises. This is different from the sale of a futures contract, which locks in his profit, but does not leave any room for further profits if the market moves favorably.

Options trade on many types of futures contracts. The security underlying the option is the futures contract having the same expiration month, not the entity underlying the futures contract itself. Thus, if one exercises a listed futures option, he receives a futures contract position, not the physical commodity.

Example: A trader owns the ZYX futures December 165 call option (165 is the striking price). Assume the ZYX December future closed at 171.20. Both the calls and the futures are worth \$500 per point. If the call is exercised, the trader then owns one ZYX December (same expiration month as the option) futures contract at a price of 165. Since the current price is 171.20, there is a maintenance margin credit of \$3,100 in his account (500×6.20 points). Note that even though the option is an option on a future which is cash-based, the exercise may provide the holder of the option with a futures contract position, not with cash.

At the present time, there are futures options on all of the various index futures contracts.

EXPIRATION DATES

Futures options have specifics much the same as stock options do: expiration month (agreeing with the expiration months of the underlying futures contract), striking price, etc. If a trader buys a futures option, he must pay for it in full, just as with stock options. Margin requirements vary for naked futures options, but are generally more lenient than stock options. Often, the naked requirement is based on the futures margin, which is much less than the 20% of the underlying stock price as is the case with listed stock options.

When the futures option has a cash-based futures contract underlying it, the option and the futures generally expire on the same day. Thus, if one were to exercise a ZYX option on expiration day, one would receive the future in his account, which would in turn become cash because the future is cash-settled and expiring as well.

Example: Suppose that a trader owns a ZYX December 165 call option, in which the trading unit in both the underlying futures and the option is \$500 per point. He holds the option through the last day of trading. On that last day, the ZYX Index settles at 174.00. The call is automatically exercised, and the following sequence occurs:

1. Buy one ZYX future at 165.00 via the call exercise.
2. Mark the future at 174.00, the closing price. This is a variation margin profit of \$4,500 (174.00 – 165.00) × \$500
3. The option is removed from the account because it was exercised, and the future is removed as well because it expired.

Thus, the exercise of the option generates \$4,500 in cash into the account and leaves behind no futures or option contracts. We do not know if this represents a profit or loss for this call holder, since we do not know if his original cost was greater than \$4,500 or not.

It should be noted that futures option expiration dates, in general, are fairly complex. They are not normally the third Friday of the expiration month, as stock options are. *Index futures options* generally *do* expire on the third Friday of the expiration month, but many physical commodity options do not. These differences will be discussed in the later chapter on futures options.

OPTION PREMIUMS

The dollar amount of trading of a futures option contract is normally the same as that of the underlying future. That is, since the S&P 500 future is worth \$250 per point, so are the S&P 500 futures options.

Example: An investor buys an S&P 500 December 1410 call for 4.20 with the index at 1409.50. The cost of the call is \$1,050 (4.20 × 250). The call must be paid for in full, as with equity options.

An interesting fact about futures options is that the longer-term options have a "double premium" effect. The option itself has time value premium and its underlying security, the future, also has a premium over the physical commodity. This phenomenon can produce some rather startling prices when looking at calendar spreads.

Example: The ZYX Index is trading at 162.00 sometime during the month of January. Suppose that the March ZYX futures contract is trading at 163.50 and the June futures contract at 167.50. These prices are reasonable in that they represent a premium over the index itself which is 162.00. These premiums are related to the amount of time remaining until the expiration of the futures contract and the prevailing short-term interest rate.

Now, however, let us look at two options—the March 165 put and the June 165 put. The March 165 put might be trading at 3 with its underlying security, the March futures contract, trading at 163.50. The June 165 put option has as its underlying security the June futures contract. Since the June option has more time remaining until expiration, it will have more time value premium than a March option would. However, the underlying June future is trading at 167.50, so the June 165 put option is 2½ points out-of-the-money and therefore might be selling for 2½. This makes a very strange-looking calendar spread with the longer-term option selling at 2½ and the near-term option selling for 3. This is due to the fact, of course, that the two options have different underlying securities. One is in-the-money and the other is out-of-the-money. These two underlyings—the March and June futures—have a price differential of their own. So the option calendar spread is inverted due to this double premium effect.

FUTURES OPTION MARGIN

Most futures exchanges have gone to the form of option margin called SPAN, which stands for Standard Portfolio ANalysis of Risk. This form of margining is very fair and attempts to base the margin requirement of an option position on the probability of movement by the underlying futures contract, as well as on the potential change of implied volatility of the options in question.

The older method of margining option positions is known as the "customer margin" method. The customer margin method generally results in higher margin requirements. The reader is referred to the chapter on futures and futures options for an in-depth discussion of SPAN and other option margin requirements.

OTHER TERMS

Just as futures on differing physical commodities have differing terms, so do options on those futures. Some options have striking prices 5 points apart while others have strikes only 1 point apart, reflecting the volatility of the commodity. Specifically, for index futures options, the S&P 500 options have striking prices 5 points apart.

Futures option symbology is essentially the same as that for stock options now that the OSI has taken place. Numeric values are used for strikes and expiration dates. However, there is one major difference: the expiration month for most futures and options still adheres to the old futures method. Some vendors may translate for you, but many do not: F = January, G = February, H = March, J = April, K = May, M = June, N = July, Q = August, U = September, V = October, X = November, and Z = December.

There are position limits for some futures options, but they allow for very large positions. Check with your broker for exact limits in the various futures options.

One factor concerning the trading of futures options can be of major concern to many customers and salesmen. Salesmen who are registered to sell stocks are not necessarily registered to sell futures—an additional test must be passed in order to sell many types of futures options. Similarly, many customers—primarily institutions—have received approval from their constituents to trade in stock options, but would need further approvals to trade futures or futures options. Neither of these things should stand in the way of the strategist—if there are opportunities in futures options, then the customer should find a broker who can trade them. Also if the strategist finds that he requires certain approvals from within his own institution before he can trade futures, then he should obtain those approvals.

STANDARD OPTION STRATEGIES USING INDEX OPTIONS

Let us return now to a general discussion of index option strategies. These apply to all index options—whether on equity option exchanges or on futures exchanges. The stock option strategies described in all of the preceding chapters of this book can be established with index options as well. The concepts are normally the same for index options as they would be for stock options. If one buys a call at one strike and sells a call at a higher strike, that is a bullish spread; if one sells both a put and a call at the same strike (a straddle), that is a neutral strategy. One uses deltas to determine how many options to sell against the ones that he buys in order to establish a delta neutral strategy. Likewise, he uses the deltas to tell, along the way, how his position is progressing and how to adjust it to keep it delta neutral.

We will not describe these same strategies over again. They have already been described in detail. The risk of early assignment removing one side of a position can alter some strategies. In some cases there are particular advantages or disadvantages with index options and futures. Thus, we will briefly go over some major option strategies, giving details pertaining to their usage as index option strategies.

OPTION BUYING

The most common reason for wanting to buy index options is to take advantage of the diversification that they provide, in addition to the normal advantages that option purchasing provides: leverage and limited dollar risk. Many people feel that it is easier to predict the general market direction than it is to predict an individual stock's direction. This feeling, can, of course, be put to good advantage by buying index options. However, sometimes it is not better to buy the index options. In such cases, it may actually be smarter to purchase a package of individual stock options.

Due to a phenomenon known as volatility skewing, it is possible for index options to have implied volatilities that are out of line with projected index or stock price movements. This phenomenon is discussed in detail in the chapter on advanced concepts.

For example, suppose that index puts are expensive, as they became after the 1987 stock market crash. When this happens it may actually be more profitable for a trader who is bearish on the market to buy a package of equity puts instead of buying index puts. The equity puts are forced to reflect the probability of stock price movement because arbitrage strategies will keep them in line. They will therefore be less expensive than index puts when this type of volatility skewing is present. Index puts can remain expensive for several reasons—primarily excessive demand and inflated margin requirements. In such situations, it is theoretically correct to buy a group of puts on stock options. In fact, one might even hedge this purchase by selling out-of-the money, overpriced index puts.

SELLING INDEX OPTIONS

In earlier chapters, we saw that many mathematically attractive strategies involve the sale of naked options—ratio writes, straddles, ratio spreads, etc. Index options present an even stronger case for these strategies. Recall that the greatest risk in these strategies with naked options is that the underlying security might move a great distance, thereby exposing the position to great loss if the movement is in the direction in which the naked options lie. That is, if one is naked calls and the underlying security rises dramatically, perhaps on a takeover bid, then large losses—potentially unlimited in the absence of follow-up action—could occur.

The strategist would, of course, never let the loss run uncontrolled. He would attempt to take some follow-up action to limit the loss or to neutralize the position. However, even the best strategist cannot hedge his position if the movement in the underlying occurs while the market is closed. For example, if the underlying security is a stock, certain news items might cause a large gap to occur between the closing price of a stock and its next opening price. Such news might be related to a takeover of the company or to a drastically negative earnings report, for example.

Index options do not have this particular drawback. An index—especially a broad-based index—is not as likely to open on a wide gap as a stock is. An index cannot be the subject of a takeover attempt. It cannot be severely depressed by bad earnings on one of its components. Thus, index options are more viable candidates for strategies involving naked option writing than stock options are. Index futures and options may often open on small gaps of a point or so, due to emotion or possibly due to the fact that a market that opens earlier (T-Bond futures, for example) has already made a rather large move by the time the futures open. Such a small gap is normally not extremely damaging to the naked writer.

One cannot assume that an index can never gap open widely—if something drastic were to happen in the marketplace that caused opening gaps in many stocks, then a gap could appear in the index itself. The worst case of such a gap, percentage-wise, was the stock market crash in 1987 when the major indices such as OEX and S&P 500 opened down over 20 points. While that remains the worst down day in history, there have been many other severe market collapses, such as in the fall of 2008. Therefore, one cannot assume that naked option writing of index options is a low-risk strategy; however, it is generally less risky than naked option writing of equity options.

HANDLING EARLY ASSIGNMENT OF CASH-BASED OPTIONS

Most index options are European exercise—they cannot be exercised early. However, there are still a few cash-based American-style indices, and they could be a problem if early assignment were to occur. The greatest problem that a spreader of index options has is the possibility of early assignment. This removes his hedge on one side of his position, exposing him to much more risk than he had wanted or anticipated.

One can often obtain a clue before early assignment occurs by observing the price of the in-the-money options. If they are trading at a discount, one can expect assignment to be more likely.

Example: ZYX is trading at 357 a few days before expiration of the January options. The stock market rallies heavily near the close, and the January 340 calls are trading with a market of 16.50 to 17.00 after 4 p.m. EST. Since parity is 17 for these calls, it is likely that a writer will receive an assignment notice in the morning.

The strategist who observes this situation taking place must make a rather quick decision. Since the market has rallied heavily on the close, it is likely that arbitrageurs or institutional accounts who are long index options are going to exercise them. The cynic among us would even think that they might be short stocks as well which they plan to cover in the morning. Notice that the effect of hedged call option sellers (i.e., spreaders) receiving assignment notices will be to make them all long the market. The short side of their spread will have been removed via assignment, and they will be left with only the long side. Therefore, in order to liquidate or hedge, they will have to sell stocks or index futures and options in the morning. This would force the market down temporarily and would be a boon to anyone who was short overnight.

The spreader's first potential choice of action is to notice what is happening near the close of trading and to try to exercise his long calls since he expects assignment of his short calls. The assignment, of course, is not certain—he is merely projecting it. Therefore, he could outfox himself and end up being very short if he did not receive an assignment notice on his short calls.

Assuming the strategist did not anticipate assignment and therefore did not exercise his long calls, he has several choices after receiving an assignment notice the next morning. First, he could do nothing. This would be an overly aggressive bullish stance for someone who was previously in a hedged position, but it is sometimes done. The strategist who takes this aggressive tack is banking on the fact that the selling after the assignment will be temporary, and the market will rebound thereafter, giving him the opportunity to close out his remaining longs at favorable prices. This is an overly aggressive strategy and is not recommended.

The most prudent approach to take when one receives an early assignment on a cash-based option is to immediately try to do something to hedge the remaining position. The simplest thing to do is to buy or sell futures, depending on whether the assignment was on a put or call. If one was assigned on a put, a portion of the bullishness (short puts are bullish) of one's position has been removed. Therefore, one might buy futures to quickly add some bullishness to the remaining position. Generally, if one were assigned early on calls, part of the bearishness of his position would have been removed—short calls being bearish—and he might therefore sell futures to add bearishness to his remaining position. Once hedged, the position can be removed during that trading day, if desired, by trading out of the hedge established that morning.

One should receive this assignment notice early in the morning, so he can immediately hedge his position in the overnight markets. If he waits until the day session opens, he might use futures or options to hedge. One should be particularly careful about placing market orders in an opening option rotation, especially on index options after a severe downside move has occurred the previous day. Market-makers are very nervous and are not willing to sell puts as protection to the public in that situation. Consequently, puts are notoriously overpriced after a large down day in the stock market. One should refrain from buying put options in the opening rotation in such a case. In the future, it is possible that comparable situations may exist on the upside. To date, however, all gaps and severe mispricing anomalies have been on the bearish side of the market, the downside.

CONCLUSION

The introduction of index products has opened some new areas for option strategists. The ideas presented in this chapter form a foundation for exploring this new realm of option strategies. Many traders are reluctant to trade futures options because futures seem too foreign. Such should not be the case. By trading in futures options, one can avail himself of the same strategies available in stock option. Moreover, he may be able to take advantage of certain features of futures and futures options that are not available with stock options.

Trading in index options can be very profitable, but only if one understands the risks involved—especially the risk of early assignment in cash-based options. The advantages to being able to "trade the market" as opposed to trading one stock at a time are obvious: If one is right on the market, his index option strategies will be profitable. This is superior to stock-oriented buying whereby one might be right on the market, but not make any money because calls were bought on stocks that didn't follow the market.

The strategist should consider all of his alternatives when trading in these markets. If he is bullish, should he really be buying SPX calls? Maybe futures calls on the S&P 500 are better. In fact, perhaps all the calls are so expensive that the underlying futures are the best buy. The ideas presented in this chapter lay the groundwork for the strategist to explore these questions and make the best decision for his investment strategy.

Finally, keep in mind that the index futures and options comprise a very diverse set of securities. They can be put to work for the investor, the trader, and the strategist in a multitude of ways. The only practical limit is in the mind of the user of these derivative securities.

PUT-CALL RATIO

Generally, we have not been concerned with technical trading systems in this book. Not that they aren't important, they are just in another category of investments other than option strategies. However, the put-call ratio system is so closely related to options that its inclusion is worthwhile.

The put-call ratio is simply the number of puts traded divided by the number of calls traded. It can be computed daily, weekly, or over any other time period. It can be computed for stock options, index options, or futures options. Sometimes it is computed using open interest instead of volume. Another way to compute the put call ratio is to divide the *dollars* spent on puts (sum of each put price times its trading volume) divided by the *dollars* spent on calls (sum of each call price times its trading volume). This is called the *weighted* put-call ratio. If it is calculated daily, one usually averages several days' worth of figures to smooth out the fluctuations.

Example: The morning paper shows that yesterday the trading activity for SPX options was:

$$\text{Total SPX Call Volume: } 200{,}000 \text{ Contracts}$$
$$\text{Total SPX Put Volume: } 300{,}000 \text{ Contracts}$$

Therefore, the ratio is:

$$\text{Index Put-Call Ratio} = \frac{300{,}000}{200{,}000} = 1.50 \text{ for yesterday}$$

This technical indicator is a contrary one. The contrarian thinking is along these lines: if everyone is buying puts, then everyone must be bearish; if everyone is doing something, they can't all be right; therefore the contrarian must assume a bullish stance.

So, if the put-call ratio is high, too many traders are buying puts; a contrarian would interpret that as a bullish sign. Conversely, if the put-call ratio is low, too many traders are buying calls; the contrarian would consider that as a bearish indicator. The theory behind contrary systems is that the majority of traders are wrong at major turning points in the market.

Many different ratios can be computed. The *total* put-call ratio includes all listed index and equity (but not futures) options. The *equity-only* put-call ratio includes only stock options, and not index options or ETFs. In futures, one would compute separate ratios for each commodity—Gold, Sugar, Corn, etc.—and would not combine them into one. However, one *would* combine all the months on any one particular commodity—April Gold, June Gold, Sept Gold, etc., would all be combined to compute the Gold put-call ratio.

With the advent of ETFs that follow commodities, one can often use the put-call ratio of a commodity as a signal for trading the related ETF (Gold options' put-call ratio for trading GLD, for example).

Obviously, the more highly traded option contracts produce a more reliable put-call ratio: equity options and index options being very liquid. Gold futures options by themselves are not that active and may produce distorted results for a period of time.

The Ratio Itself. Traders and investors almost always buy more calls than puts where stock options are concerned. Therefore, the equity put-call ratio is normally a number far less than 1.00. If call buying is rampant, the equity put-call ratio may dip into the 0.30 range on a daily basis. Very bearish days may occasionally produce numbers of 1.00 or higher. An average day will generally produce a ratio of around 0.50.

Index options, however, produce much larger ratios. Many institutional and other investors are constantly looking to avail themselves of the protective capability of index puts. Therefore, far more index puts are purchased than are equity puts. An average day might produce readings of 2.00 for some indices.

Interpreting the Ratio. There are several philosophies as to how to interpret the ratio once it has been calculated. All philosophies are of the contrarian variety, so the general comments made earlier that high ratios are bullish and low ratios are bearish still hold true. However, quantifying just what is "high" and what is "low" leaves room for interpretation.

One school believes that absolute ratios should be used. An example might be: "if the 10-day moving average of the equity put-call ratio is over 0.60, that is a buy signal."

Unfortunately, applying absolute figures to any of the ratios can be counterproductive at times. If the market is in the grip of a prolonged bearish move, more and more puts will continue to be purchased, sending the ratio quite high before it can reverse and start coming back to normal levels. Therefore, it is better to look for the ratios to make a high or a low before calling a buy or sell signal. This is a more dynamic interpretation; it allows for buy and sell signals to come at different absolute levels of the put-call ratio.

In general, the put call ratio should be rising when the underlying market is falling and vice versa. If that is *not* the case, then it is likely that hedging activity is taking place, and the ratio no longer has value as a contrary indicator until the hedging activity ceases. So, for example, if the underlying market is rising, but so is the put-call ratio, then one should ignore any signals that occur. That is because it is not normal for the ratio to be rising—i.e., for there to be heavy put buying—during a rising market. Hence, the put buying in this case is probably for hedging purposes by traders who are actually long the underlying market.

In summary, the put-call ratio is an easily calculated one. Daily fluctuations can be smoothed out into 10-, 20-, or 50-day moving averages. The ratio should be interpreted bullishly when there is too much put buying and bearishly when there is too much call buying. The phrase "too much" is not easily interpreted, but looking for local maxima or local minima in the chart pattern is a reasonable way to approach the problem. When the put-call ratio moving average is increasing, a buy signal would not be given until the average rolls over and begins declining; a sell signal would be generated when the average which is declining bottoms out.

SUMMARY

There are several kinds of indices and several kinds of trading vehicles: cash-based options, futures options, and futures. These various underlying securities have differing terms in the way they trade and also in the way their options are designed. This variety creates many opportunities for astute option strategists.

CHAPTER 30

Stock Index Hedging Strategies

This chapter is devoted primarily to examining the various ways that one might hedge a portfolio of stocks with index products. This portfolio might be a small one owned by an individual investor or it might be as large as the entire S&P 500 Index. We explore this strategy from the various viewpoints of the individual investor, the institutional money manager, and the arbitrageur. This technique of hedging stocks with index products has become quite popular and has also drawn some attention because of the way it can cause short-term movements in the entire stock market. The reasons why these movements occur are also explained. Finally, we look at ways of simulating a broad-based index by buying a group of stocks whose performance is geared to that of the index itself.

MARKET BASKETS

One of the most popular strategies using index futures and options has been the technique of buying stocks whose performance simulates the performance of a broader index and hedging that purchase with the sale of overpriced futures or options based on that index. The group of stocks that is purchased is commonly known as a "market basket" of stocks. This chapter describes how these baskets can be used to trade against a very broad index, such as the S&P 500, or a far narrower index, such as the Dow-Jones 30 Industrials (DJX), or even one as small as just a few stocks—perhaps a portfolio held by any investor.

The key to determining whether it will be profitable to trade some derivative security—options or futures—against a set of stocks is generally the level of premium in the futures contract itself. That is, if the S&P 500 Index is at 405.00 and the futures are trading at 408.00, then there is a premium of 3.00—the futures contract is trading 3.00 points higher than the index itself. The absolute level of the premium is not what is important, but rather the relationship between the premium and the fair value of the future. We will look at how to determine fair value shortly.

The futures are the leaders among the derivative securities, especially the S&P 500 futures. Whenever these become overpriced, other derivative securities will generally follow suit. In this chapter, when S&P 500 futures are referred to, it should be understood that either the "big contract," worth \$250 per point, or the e-mini contract, worth \$50 per point, could be utilized.

The normal scenario is for most of the index derivative securities—options and futures—to follow the lead of the S&P 500 futures. When this happens, the only thing that is fairly priced is the index itself—that is, stocks. Consequently, the logical way to hedge the derivative security is to do it with stocks. If the index is small enough, such as the 30-stock DJX, then one might buy all 30 stocks and sell the futures when they are overpriced. This is a complete hedge and would, in fact, be an arbitrage. In the case of a larger index such as the S&P 500, it would be possible only for the most professional traders to buy all 500 stocks, so one might buy a smaller subset of the index in hopes that this smaller set of stocks will mirror the performance of the index well enough to simulate having bought the entire index. We take in-depth looks at both types of hedging.

Even if the investor is not planning to use these hedging strategies, it is important for him to understand how they work. These strategies have certain ramifications for the way the entire stock market moves. In order to anticipate these movements, a working knowledge of these hedging strategies is necessary. The first thing that one must know in order to implement any of these hedging strategies is how to determine the fair value of a futures contract.

FUTURES FAIR VALUE

The formula for calculating the fair value of the futures contract is extremely simple, although one of the factors is a little difficult to obtain. First, let's look at the simple futures fair value formula.

Simple Formula:

$$\text{Futures fair value} = \text{Index} \times [1 + \text{Time} \times (\text{Rate} - \text{Yield})]$$

where *index* is the current value of the index itself, *rate* is the current carrying rate (typically, the broker loan rate), *yield* is the combined annual yield of all the stocks in the index, and *time* is the time, in years, remaining until expiration of the contract.

Example: Suppose the ZYX Index is composed of 500 stocks and is trading at 160.00, the broker loan rate is 10%, the yield on the 500 stocks is 5%, and there are exactly 3 months remaining until expiration of the futures contract. The time is .25, expressed in years, so the formula becomes:

$$\begin{aligned}\text{Future fair value} &= 160.00 \times [1 + .25 \times (.10 - .05)] \\ &= 160.00 \times (1 + .0125) = 162.00\end{aligned}$$

Thus, the future should be trading at about 2 points above the value of the index itself. This premium of the future over the index represents the savings in not having to pay for and carry 500 stocks, less the loss of the dividends on the stocks (the future does not pay dividends). If the future should get very expensive—trading at 3.50 or 4 points over the index—then it would have to be considered very overvalued, and an arbitrageur could move in to take advantage of that fact. Similarly, if the future should trade cheaply, at less than a point over fair value, there might be an arbitrage available in that case as well.

The fair value is really only a function of four things: the value of the index itself, the time remaining until expiration, the current carrying rate, and the dividends being paid by the stocks in the index until expiration. Notice that "the dividends paid by the stock in the index until expiration" is not quite the same as the yield of the 500 stocks, which we used in the above simple formula. We will expand more on that difference shortly.

Before doing that, however, let us look at how changes in the variables in the formula affect the fair value of the futures contract. More important, we are interested in how changes in the variables affect the *premium* of the futures contract over the index value. This is what one is primarily concentrating on when trading market baskets.

As the index value itself rises, the fair value premium rises. For example, if 2 points is the fair premium when the index is at 160, as in the above example, then 4 points would be the fair premium if the index were at 320 and all the other variables remained the same. Conversely, as the index falls, the fair value of the premium shrinks.

The premium rises and falls in direct correlation with the carrying rate as well as with time remaining until expiration. Note that this statement is true for stock options also, and for the same reason: The carrying costs are greater when rates are higher, or when one must hold for a longer time, or both. In the above example, if one were to assume there were 6 months to expiration instead of 3, the fair value of the premium would increase to 4 points from 2 points. Similarly, if the time were decreased, the fair value would be smaller.

Some investors, primarily institutional investors, use the short-term T-bill rate rather than the carrying rate in order to determine the futures fair value. The reason they do that is to determine whether the money they have in cash is better off in T-bills or in an arbitrage strategy such as this. More will be said about this use of the T-bill rate later.

Dividends Have an Inverse Correlation to the Premium Value. An increase in the overall yield of the index will shrink the fair value of the futures contract. This is because the futures holder does not get the dividends and therefore the future is not as valuable because of the loss of dividends. Conversely, if dividend yields fall, then the fair value of the premium increases. This is not the whole story on dividends, however.

Recall that a few paragraphs ago, it was pointed out that the yield and the amount of dividends are not exactly the same thing. This is because stocks don't pay their dividends in a uniform manner. Rather than paying a continuous yield as bonds do, stocks normally pay their dividends in four lump sums a year. This means that the yield variable in the simple formula shown above should be replaced by the actual amount of dividends remaining until expiration. This fact makes the computation of the fair value of the index a little more difficult. In order to do it accurately, one must know the dividend amount and ex-dividend dates of each of the stocks in the index. Knowing all of this information is a far more formidable task than knowing the yield of the index, since the yield is published weekly in several places. In fact, the services of a computer are required in order to compute the actual dividend on the larger indices, where 100 or more stocks are involved.

As a result, the actual formula changes slightly from the simple formula shown above.

Actual formula:

$$\text{Futures fair value} = \text{Index} \times (1 + \text{Time} \times \text{Rate}) - \text{Dividends}$$

In this formula, *the dividends are taken to be the present worth of all the dividends remaining until expiration of the future.*

Example: In an example similar to the one given for the simple formula, suppose the ZYX Index is trading at 160.00, the broker loan rate is 10%, the present worth of the dividends remaining until expiration is $1.89, and there are exactly 3 months remaining until expiration of the futures contract. The time is .25, expressed in years, so the formula becomes:

$$\begin{aligned}\text{Future fair value} &= 160.00 \times (1 + .25 \times .10) - 1.89\\ &= 160.00 \times (1 + .025) - 1.89 = 162.11\end{aligned}$$

In order to compute the present worth of the dividends of an index, it is necessary to know the amount of each stock's dividend as well as the payment date of the dividend. To compute the index's dividend, one computes the present worth of each dividend and multiplies that result by that stock's divisor in the index in order to give the dividend the proper weight. The index's total dividend is the sum of each of these individual stock computations. Each stock's divisor is merely the float of the stock divided by the divisor of the index. In a price-weighted index, it is not necessary to adjust the present worth of each dividend—merely add them together and divide by the divisor. As an example, we will look at a hypothetical index composed of three stocks in order to see how one computes the present worth of an index's dividends.

Example: Suppose a capitalization-weighted index is composed of three stocks: AAA, BBB, and CCC. Furthermore, suppose that the amount of each of the individual stocks' dividends and the time remaining until the dividend is paid are given in the following table, as well as each stock's float. Finally, assume the divisor for the index is 150,000,000.

Stock	Dividend Amount	Days Until Dividend Payout	Float
AAA	1.00	35	50,000,000
BBB	0.25	60	35,000,000
CCC	0.60	8	120,000,000
Divisor: 150,000,000			

In order to compute the present worth of a future amount, one uses the formula:

$$\text{Present worth} = \frac{\text{Future amount}}{(1 + \text{Rate})^{\text{Time}}}$$

where rate is the current short-term rate and time is expressed in years.

Assume that the current interest rate is 10%. Then the present worth of AAA's dividend would be:

$$\begin{aligned}\text{Present worth AAA} &= \frac{1.00}{(1+.10)^{(35/360)}}\\ &= \frac{1.00}{(1.10)^{(.0972)}}\\ &= \frac{1.00}{1.0093}\\ &= 0.9908\end{aligned}$$

The present worth of the dividend is always less than the actual dividend. The present worth of an amount is the amount of money that would have to be invested today at the stated rate (10% in this example) to produce the future amount. That is, 99.08 cents invested at 10% would be worth exactly 1.00 in 35 days.

The present worth of the other two dividends is .2461 for BBB and .5987 for CCC. The reader should verify for himself that these are indeed the correct amounts. Notice that the present worth of a dividend is not much less than the actual value of the dividend. However, in a larger index, where one is dealing with several hundred dividends, the present worth may be significantly different from the actual sum of the dividends, especially if short-term rates are high.

Since we made the assumption that this is a capitalization-weighted index, each of these figures must be adjusted for the capitalization of the stock in order to give each present worth the proper weight within the index. Thus, for AAA, the adjusted dividend would be .9908 times 50,000,000 (AAA's float), divided by 150,000,000, the divisor of the index. This would result in an adjusted dividend of .3303 for AAA. When similar adjustments are made for BBB and CCC, their adjusted values become .0574 and .4790, respectively.

Thus, the present worth of the dividend for the index would be the sum of the three individual adjusted present worths, or .3303 + 0.574 + .4790 = \$0.8667.

Note: If the index were a price-weighted index, the index's dividend would be the sum of these three present worths (.9908 + .2461 + .5987), divided by the divisor of that index.

The above fair value formula can be applied to options as well. For example, the OEX Index does not have futures. However, the fair value calculations can be done in the same manner, and the synthetic index then constructed by using puts and calls can be compared with that fair value.

Example: Suppose that OEX is trading at 364.50 and a September OEX future—if one existed—would have a fair value of 367.10. That is, the future would command a premium of 2.60. Not only should a future trade with that theoretical premium, but so should the "synthetic OEX" composed of puts and calls at the same strike. Hence, the synthetic OEX constructed with options should trade at about 367.10 also.

That is, if the OEX Sep 365 call were selling for 4.60 and the Sep 365 put were selling for 2.50, then the synthetic OEX constructed by the use of these two options would be priced at 367.10. Recall that one determines the synthetic cost by adding the strike, 365, to the call price, 4.60, and then subtracting the put price: 365 + 4.60 – 2.50 = 367.0. This synthetic price of 367.10 is literally the same as the theoretical futures price of 367.10.

The same calculations can be applied to any index with listed options trading. Let us now return to the broader subject at hand—trading market baskets of stocks against futures.

PROGRAM TRADING

Two terms that conjure up images of the stock market crash in 1987 and other severe price drops are "program trading" and "index arbitrage." Neither one by itself should affect the stock market, since they are two-sided strategies—involving buying stocks and selling futures. This two-sided aspect should have little effect on the market, theoretically. However, in practice, it is often the case that trades are not executed simultaneously, and the stock market takes a jump or a dive.

Program trading is nothing more than trading futures against a general stock portfolio. Index arbitrage is trading futures against the exact stocks that comprise an index.

Later discussions will assume that one is trying to create or simulate the index itself in order to hedge it with futures. This is the arbitrage approach. However, there are many other types of stock positions that may be hedged with the futures. These might include a portfolio of one's own construction containing various stocks, or might include a group of stocks from which one wants to remove "market risk." Normally, one would not own the makeup of any index, but rather would have a unique combination of stocks in his portfolio. Such an investor may want to use futures to hedge what he does own.

One reason why an investor who owned stocks would want to sell index products against them might be that he has turned bearish and would prefer to sell futures rather than incur the costs involved with selling out his stock portfolio (and repurchasing it later). Commission charges are quite small on futures transactions as compared to an equal dollar amount of stock. By selling the futures on an index—say, the S&P 500—he removes the "market risk" from his portfolio (assuming the S&P 500 represents the "market"). What is left over after selling the futures is the "tracking error." The discrepancy between the movement of the general stock market and any individual portfolio is called "tracking error." This investor will still make money if his portfolio outperforms the S&P 500, but he will find that he did not completely eliminate his losses if his portfolio underperforms the index. Note that if the market goes up, the investor will not make any money except for possible tracking error in his favor.

REMOVING THE MARKET RISK FROM A PORTFOLIO

Stock portfolios are diverse in nature, not necessarily reflecting the composition of the index underlying the futures contracts. The characteristics of the individual stocks must be taken into account, for they may move more quickly or more slowly than "the market." Let us spend a moment to define this characteristic of stocks that is so important.

VOLATILITY VERSUS BETA

Recall that when we originally defined volatility for use in the Black–Scholes model, we stated that Beta was not acceptable because it was strictly a measure of the correlation of a stock's performance to that of the stock market and was not a measure of how fast the stock changed in price. Now we are concerned with how the stock's movement relates to the market's as a whole. This is the Beta.

Unfortunately, Beta is not as readily available to the option strategist as is volatility. Many option traders merely have to punch a button on their quote machines and they can receive estimates of volatility. However, Beta estimates are more difficult to obtain, and the ones that are available are often for very long time periods, such as several years. These long-term Betas cannot be used for the purposes of the index hedging discussed in this chapter. Therefore, if one does not have access to shorter-term Beta calculations, then he can approximate Beta by comparing an individual stock's volatility with the market's volatility.

Example: XYZ is a relatively volatile stock, having both an implied and historical volatility of 36%. The overall stock market has a volatility of 15%. Therefore, one could approximate the Beta of XYZ as:

$$\text{Beta approximation} = 36/15 = 2.40$$

There are certain situations in which this approximation would not work well, because the stock has little or no correlation to the overall stock market (e.g., gold or oil stocks). If one has a portfolio of stocks of that type, then he should make a serious attempt to attain their Betas, for the Beta estimate method just described will not be accurate. Such stocks may be volatile—that is, they change in price fairly rapidly—but they may go in totally different directions from the overall stock market: They would thus have high volatility, but low Beta. This is not conducive to the above short-cut for approximating Beta from volatility.

The remaining examples in this chapter use the terms *Beta* and *adjusted volatility* synonymously. Adjusted volatility is merely the approximation of Beta from volatility as described above: the stock's volatility divided by the market's volatility.

THE PORTFOLIO HEDGE

In attempting to hedge a diverse portfolio, it is necessary to use the Beta or adjusted volatility because one does not want to sell too many or too few futures. For example, if the portfolio were composed of nonvolatile stocks and one sold too many futures against it, one could lose money if the market rallied, even if his portfolio outperformed the market. This would happen because the general market, being more volatile, would rally further than the nonvolatile portfolio. Ideally, one should sell only enough futures so that there would be no gain or loss if the market rallied. There would be only tracking error.

Conversely, if one does not sell enough futures against a volatile portfolio, then there is risk of loss if the market declines, since the portfolio would decline faster than the market.

The Beta or adjusted volatility of each stock is used in order to determine the proper number of futures to sell against the portfolio. The dollar value (capitalization) of each stock in the index is adjusted by that stock's volatility to give an "adjusted capitalization" for each stock. Then, when all these are added together, one will have determined how much "adjusted capitalization" must be hedged with futures. The suggested method, described in the following example, uses an adjusted volatility for each stock.

The steps to follow in determining how many futures to sell against a diverse portfolio of stocks are as follows:

1. If you don't know the Beta, divide each stock's volatility by the market's (S&P 500) volatility. This is the stock's adjusted volatility.
2. Multiply the quantity of each stock owned by its price and then multiply by the adjusted volatility from step 1. This gives the adjusted capitalization of the stock in the portfolio.
3. Add the results from step 2 together for each stock to get the total adjusted capitalization of the portfolio.
4. Divide the sum from step 3 by the index price of the futures to be used and the unit of trading for the futures ($250 per point for the S&P 500 futures) to determine how many futures to sell.

Example: Suppose that one owns a portfolio of three diverse stocks: 3,000 GOGO, an over-the-counter technology stock; 5,000 UTIL, a major public utility stock; and 2,000 OIL, a large oil company. The owner of this portfolio has become bearish on the market and would like to sell futures against the portfolio. He needs to determine how many futures to sell.

The prices and volatilities of these stocks are given in the following table. Assume that the volatility of the fictional ZYX Index is 15%. This is the "market's volatility" that is divided into each stock's volatility to get its adjusted volatility (step 1, above).

Stock	Volatility	Adjusted Volatility (Step 1)	Price	Quantity Owned	Adjusted Capitalization (Step 2)
GOGO	.60	4.00	25	3,000	$300,000
UTIL	.12	0.80	60	5,000	240,000
OIL	.30	2.00	45	2,000	180,000
Total adjusted capitalization:					$720,000 (step 3)

Now suppose that the ZYX Index is trading at 178.65 and a 1-point move in the futures is worth \$500. Step 4 can now be calculated: $\$720,000 \div 500 \div 178.65$, or 8.06 futures contracts. Thus, the sale of 8 futures contracts would adequately hedge this diverse portfolio.

There is an important nuance in this simple example: *The price of the index should be used in all hedging calculations, as opposed to using the price of the future.* There are many examples of hedging portfolios and market baskets with futures or options in this chapter and the next. Regardless of the situation, the value of the index should always be used to determine how much stock to buy or how much of the derivative security to sell.

Note that the actual capitalization of the above example portfolio was only \$465,000 (\$75,000 for GOGO, \$300,000 for UTIL, and \$90,000 for OIL). However, the portfolio is more volatile than the general market because of the presence of the two higher-volatility stocks. It is thus necessary to hedge \$720,000 worth of "market," or adjusted capitalization, in order to compensate for the higher volatility of the portfolio.

A similar process can be used for far larger portfolios. The estimate of volatility is, of course, crucial in these calculations, but as long as one is consistent in the source from which he is extracting his volatilities, he should have a reasonable hedge. There is no way to judge the future performance of a portfolio of stocks versus the ZYX Index. Thus, one has to expect a rather large tracking error. In this type of hedge, one hopes to keep the tracking error down to a few *percent*, which could be several points in the futures contracts over a long enough period of time. Of course, the tracking error can work in one's favor also. The main point to recognize here is that the vast majority of the risk of owning the portfolios has been eliminated by selling the futures contracts. The upside profit potential of the portfolios has been eliminated as well, but the premise was that the investor was bearish on the market.

Note that if the futures are overpriced when one enacts his bearishly oriented portfolio hedge, he will gain an additional advantage. This will act to offset some negative tracking error, should such tracking error occur. However, there is no guarantee that overpriced futures will be available at the time that the investor or portfolio manager decides to turn bearish. It is better to sell the futures and establish the hedge at the time one turns bearish, rather than to wait and hope that they will acquire a large premium before one sells them.

HEDGING PORTFOLIOS WITH INDEX OPTIONS

As mentioned earlier, one could substitute options for futures wherever appropriate. If he were going to sell futures, he could sell calls and buy puts instead. In this section, we are also going to take a more sophisticated look at using index options against stock portfolios.

First, let us examine how the investor from the previous example might use index options to hedge his portfolio.

Example: Suppose that an investor owns the same portfolio as in the previous example: 3,000 GOGO, 5,000 UTIL, and 2,000 OIL. He decides to hedge with index UVX, which has options worth $100 per point. Assume that the volatility of the UVX is 15%. This investor would then compute his total adjusted capitalization in the same manner as in the previous example, again arriving at a figure of $720,000.

Suppose that the UVX Index is at 175.60. This investor would want to hedge his $720,000 of adjusted capitalization with 4,100 "shares" of UVX ($720,000 ÷ 175.60). Since a 1-point move in UVX options is worth $100, this means that one would sell 41 UVX calls and buy 41 UVX puts. He would probably use the 175 strike or possibly the 180 strike, since those strikes are the ones whereby the calls have the least chance for early assignment.

Where short options are involved, as with the calls in the above example, one must be aware of the possibility of early assignment exposing the portfolio. Consequently, if the marketplace has an equal premium on the futures and the "synthetic" UVX, one should sell the futures in that case, because there is no possibility of unwanted assignment. However, if the options represent a synthetic price that is more expensive than the futures, then using the options may be more attractive.

Example: Suppose that our same investor has decided to hedge his portfolio with its $720,000 of adjusted capitalization. He is indifferent as to whether to use the ZYX futures or the UVX options. He will use whichever one affords him the better opportunity. The following table depicts the prices of the securities that he is considering, as well as their fair values.

Security	Current Price	Fair Value	Index Price
ZYX Jun Future	180.50	180.65	178.65
UVX Jun 175 Call	5	5	175.60
UVX Jun 175 Put	2	2½	175.60
UVX Jun 180 Call	2½	2½	175.60
UVX Jun 180 Put	4½	5	175.60

This investor essentially has three choices: (1) to use the ZYX futures, (2) to use the UVX options with the 175 strike, or (3) to use the UVX options with the 180 strike. Notice that the ZYX future is trading 15 cents below its fair value (180.50 vs. 180.65). The UVX Index fair value, as shown by the fair values of the options, is 177.50. This can be computed by adding the call price to the strike and subtracting the put price. In the case of either strike, the fair values indicate a UVX Index fair value of 177.50.

However, the actual markets are slightly out of line. When using the actual prices, one sees that he can sell the UVX Index synthetically for 178.00 whether he uses the 175's or the 180's. Thus, by using the UVX options he can sell the UVX "future" synthetically for ½ point over fair value, while the ZYX futures would have to be sold at 15 cents under fair value. Thus, the options appear to be a better choice since 65 cents (the 50 cents that the UVX options are overvalued plus the 15 cents that the futures are undervalued) is probably enough of an edge to offset the possibility of early assignment.

With the futures having been eliminated as a possibility, the investor must now choose which strike to use. Since he will be selling calls and buying puts, and since either strike allows him to synthetically sell the UVX "future" at 178, he should choose the 180 strike. This should be his choice because the 180 calls are out-of-the-money and thus less likely to be the object of an early assignment.

HEDGING WITH INDEX PUTS

Let us now move on to discuss ways of hedging in which a complete hedge is not established, but rather some risk is taken. The main difference between options and futures is that futures lock in a price, while options lock in a worst-case price (at greater cost) but leave room for further profit potential. To see this, consider a long stock portfolio hedged by short futures. In this case, one eliminates his upside profit potential except for positive tracking error. However, if he buys put options instead, he expends money—thereby incurring a greater cost to himself than if he had used futures—but he still has profit potential if the market rallies.

One could hedge a long stock portfolio with options by either buying index puts or selling index calls. Buying the puts is generally the more attractive strategy, especially if the puts are cheap. In order to properly establish the hedge, it is not only necessary to adjust the dollars of stock in accordance with the Beta, but the deltas of the options must be taken into account as well. The following example will demonstrate the use of puts to hedge a portfolio of diverse stocks.

Example: Assume that an investor has the same portfolio of three stocks that was used in a previous example: 3,000 GOGO, 5,000 UTIL, and 2,000 OIL. He has become somewhat bearish on the market in general and would like to hedge some of his downside risk. However, he decides to use puts for the hedge just in case there is a further rally in the market.

The table from the earlier example is reprinted below, showing the adjusted volatilities and capitalizations for each stock in the portfolio. The total adjusted capitalization of the portfolio is $720,000, as before.

Stock	Volatility	Adjusted Volatility (Step 1)	Price	Quantity Owned	Adjusted Capitalization (Step 2)
GOGO	.60	4.00	25	3,000	$300,000
UTIL	.12	0.80	60	5,000	240,000
OIL	.30	2.00	45	2,000	180,000
Total adjusted capitalization:					$720,000 (step 3)

There are two ways that one might want to approach hedging this $720,000 portfolio with puts.

1. As disaster insurance: Buy enough (out-of-the-money) puts so that the portfolio would be 100% hedged below the striking price of the puts.
2. As a hedge against current market movements: Buy enough puts so that all current portfolio movements are hedged.

Example—Method 1: In this method, the portfolio manager is looking for disaster insurance. He is not so much concerned with hedging current market movements as he is with preventing a major loss if the market should collapse. The manager often uses an out-of-the-money put for disaster insurance.

Assume that he is going to use the UVX Index puts, which are worth $100 per point. The March 170 puts, trading at 1, are going to be used in the hedge. The index is currently at 178.00.

He would therefore divide his portfolio's adjusted capitalization ($720,000) by the value of the striking price of the puts to be used. In this case, the value of the striking price is $\$17{,}000\ (100 \times 170)$.

$$\text{Puts to buy} = \$720{,}000/\$17{,}000 = 42.3$$

Cost of 42 puts: $\$4{,}200$

Striking value of 42 puts: $\$714{,}000\ (42 \times \$17{,}000)$

The cost of buying 42 puts is $4,200. This can be thought of as an insurance premium, paid to buy $714,000 worth of insurance. He will have market risk on his portfolio between the current price of the index (178.00) and the striking price (170.00). The 42 puts would hedge a little of the drop in his portfolio during that 8-point drop in the index, but their full protective value would not be felt until they were in-the-money. It is not an exact hedge, of course, since the UVX Index may perform differently from the portfolio once UVX drops below 170. However, this put purchase will definitely remove a great deal of the market risk of further drops.

Example—Method 2: In this method, the portfolio manager is attempting to hedge the current value of his portfolio. He wants no further downside losses in his portfolio at all. He would generally buy at- or in-the-money puts in this case and would use the put's delta in order to construct a complete hedge.

Again assume that he is going to use the UVX Index puts, which are worth \$100 per point. In this case, however, with the index at 178.00, he is considering the March 180 puts, trading at 4.50, with a delta of –0.60 to be used in the hedge.

In this case, the number of puts is determined by using the same formula as in the above example and then also dividing by the absolute value of the delta:

$$\begin{aligned} \text{Puts to buy} &= \$720,00/(100 \times 180)/0.60 \\ &= 67 \\ \text{Cost of protection: } & 67 \times \$450 = \$30,150 \end{aligned}$$

In this case, the portfolio manager is spending much more for the puts, but for his additional expense, he acquires immediate protection for his portfolio. Furthermore, there is some intrinsic value to the puts he bought (2 points, or \$13,400 on 67 of them). If the UVX Index drops at all, these puts will immediately begin to hedge his entire portfolio against loss. Of course, if the market rises, he loses his much more expensive insurance cost.

When one uses options instead of futures to hedge his position, he must make adjustments when the deltas of the options change. This was not the case when futures were used; perhaps with futures, one might recalculate the adjusted capitalization of the portfolio occasionally, but that would not be expected to affect the quantity of futures to any great degree. With put options, however, the changing delta can make the position delta short when the market declines, or can make it delta long if the market rises. This situation is akin to being long a straddle—the position becomes delta short as the market declines and becomes delta long as the market rises.

Basically, the adjustments would be same as those that a long straddle holder would make. If the market rallied, the position would be delta long because the delta of the puts would have shrunk and they would not be providing the portfolio with as much adjusted dollar protection as it needs. The investor might roll the puts up to a higher strike, a move that essentially locks in some of his stock profits. Alternatively, he could buy more puts at the current (low) strike to increase his protection.

Conversely, if the market had declined immediately after the position was established, the investor will find himself delta short. The delta of the long puts will have increased and there will actually be too much protection in place. His adjustment alternatives are still the same as those of a long straddle holder—he might sell some of the puts and thereby take a profit on them while still providing the required protection for the stock portfolio. Also, he might roll the puts down to a lower strike, although that is a less desirable alternative.

HEDGING WITH INDEX CALLS

Another strategy to protect a stock portfolio is to establish a ratio write using short calls against the long stock. This is the opposite of using puts for protection, in that it is more equivalent to being short a straddle.

Example: In the last example, the March 180 put had a delta of −0.60. The March 180 call should then have a delta of 0.40. If the portfolio manager wanted to hedge his portfolio by ratio writing calls against it, he could use the same formula as in the previous example:

$$\text{Calls to sell} = \$720{,}000/(100 \times 180)/0.40 = 100$$

He would sell 100 calls to hedge his portfolio.

Adjustments would be made in much the same manner as those that a straddle seller would make. If the market rises, the delta of the calls will increase and the position will be delta short. One would probably buy calls in that case. Follow-up action for an actual short straddle might dictate buying in some of the underlying security rather than buying the calls, but that is not a realistic alternative in this case, since the sample portfolio is probably stable.

If, on the other hand, the market declined after the short calls were sold against the portfolio, the position would become delta long as the delta of the calls shrinks. The normal action in that case would be to roll the calls down and reestablish the proper amount of protection for the portfolio.

Overall, hedging the portfolio with short index calls does not present as attractive a position as hedging with long index puts. This is due mostly to the nature of what the portfolio manager is trying to accomplish, as opposed to the relative merits of long and short straddles. As was pointed out in previous chapters, straddle selling, while risky, is an excellent strategy on a statistical basis. However, in this section we are not dealing with a strategist who is going to go out and buy stocks and then write index calls against them. Rather, we have an existing portfolio and the portfolio manager is becoming bearish on the market. Thus, the stock portfolio is a fixed entity and the index options or futures are being built around it for protection.

Long puts serve the purpose of protection far better than short calls, for the following reasons. First, the types of adjustments that need to be made by a straddle seller often involve buying stock or at least buying relatively deep in-the-money calls. A portfolio manager or investor holding a portfolio stock may not need or want to get involved in a multi-optioned position. Second, with calls there is large risk to the upside in case of a large market rally. Someone holding a portfolio of stocks might be willing to forego upside profits (as in the sale of futures), but generally would be quite upset to sustain large losses on the upside. Using puts, of course, leaves room for upside profit potential. Third, there is risk of early assignment with short index calls, although that is of minor significance in this case since the portfolio of stocks would have been long in any case. Other calls could be written immediately on the day after the assignment. The only real drawback to using the puts is that premium dollars are paid out and, if the market stabilizes, the time value decay will cause a loss on the puts. If one actually suspects that such a stabilization might occur, he should use futures against his position instead of puts or calls.

INDEX ARBITRAGE

As previously stated, index arbitrage consists of buying virtually all of the stocks in an index and selling futures against them, or vice versa. Whenever the futures on an index are mispriced, as determined by comparing their actual value with their fair value, there may be opportunities for arbitrage if the mispricing is large enough. When futures are extremely overpriced: buy stocks, sell futures; or when futures are underpriced: sell stocks, buy futures. In either case, the arbitrageur is attempting to capture the differential between the fair value price of the futures contract and the price at which he actually buys or sells the index. First, we will examine fully hedged situations—ones in which the entire index is bought or sold. After that, we will examine smaller sets of stocks that are designed to simulate the performance of the entire index.

Hedging indices which contain fewer stocks is easier than hedging larger indices. Hedging a price-weighted index is probably the simplest type of hedge. As examples, the same sample indices that were constructed in the previous chapter will be used.

Whenever futures or index options trade on an index, it is possible to set up market baskets for arbitrage. The trader should determine, in advance, how many shares of each stock he will buy or sell in order to duplicate the index. In a price-weighted index, of course, he will buy the same number of shares of each stock. In a capitalization-weighted index, he will be buying different number of shares of each stock. Let us first look at how the number of shares to buy is determined. Then we will discuss some of the nuances, such as monitoring bids and offers of the indices, order execution, and others.

HOW MANY SHARES TO BUY

In advance of actually trading the stocks and futures or options, one should determine exactly how many shares of each stock he will be buying in each index he plans to arbitrage. Normally, one would decide in advance how many futures contracts or option contracts he will trade at one time. Then the number of shares of stock to be bought as a hedge can be determined as well. Essentially, one is going to hedge equal dollar amounts—that is, he will buy enough stocks to offset the total dollar amount represented by the index.

Example: Suppose that one decides he will set up his market baskets by using 50 ZYX futures at a time. How much stock should he buy against these 50 contracts? The futures contract has a trading unit of \$500 per point. Assume the ZYX Index is trading at 168.89. Then the total dollar amount represented by 50 contracts is $50 \times \$500 \times 168.89 = \$422,225$. The hedger would buy this much stock to hedge 50 futures contracts sold.

Again note that the index price, not the futures price, is used in order to determine how many futures to sell.

In a price-weighted index, one determines the number of shares to buy by determining the total dollar value of the index he plans to trade and then dividing that dollar amount by the divisor of the index. The resulting number is how many shares of each stock to buy in order to duplicate the price-weighted index.

Example: Suppose that we have a price-weighted index composed of three stocks, A, B, and C. The following data describe the index:

Stock	Price
A	30
B	90
C	50
Price total:	170
Divisor:	1.65843
Index value:	102.51

The number of shares of each stock that is in the index is 1 divided by the divisor, or $1/1.65843 = 0.60298$ shares. Thus, if we were to buy .60298 shares of each of the three stocks, we would have created the index.

Suppose that futures exist on this index and that the trading unit in these futures is \$250 per point. That is, the futures represent a total dollar value of the index times 250. With this information, it is easy to determine the number of shares of stock to buy to hedge one futures contract: 250 times the number of shares of each stock, .60298, for a total of 150.745 shares of each stock.

Normally, one would not merely sell one futures contract and hedge it with stock. Rather, he would employ larger quantities. Say that he decided to trade in lots of 100 futures contracts versus the stocks. In that case, he would buy the following number of shares of each stock:

$$\text{Number of shares} = .60298 \times \$250/\text{point} \times 100 \text{ futures contacts}$$
$$= 15074.5 \text{ shares}$$

Actually he would probably buy 15,100 shares of each stock against the index, and on every fourth "round" (100 futures vs. stock) would buy 15,000 shares. This would be a very close approximation without dealing in odd lots.

The trader might also use index options as his hedge instead of futures. The striking price of the options does not come into play in this situation. Typically, one would fully hedge his position with the index options—that is, if he bought stock, he would then sell calls *and* buy puts against that stock. Both the puts and the calls would have the same strike and expiration month. This creates a riskless position. This position is a conversion.

Example: Suppose that cash-based options trade on this index, and that these options are worth \$100 per point as are normal stock options—that is, an option is essentially an option on 100 shares of the index. The trader is going to synthetically short the index by buying 100 June 105 puts and selling 100 June 105 calls. Assume that the index data is the same as in the previous example, that 0.60298 shares of each stock comprise the index. How many shares would one hedge these 100 option synthetics with?

$$\text{Number of shares} = .60298 \times 100 \text{ contracts} \times 100 \text{ shares/contract}$$
$$= 6029.8 \text{ shares}$$

Note that in the case of a price-weighted index, neither the current index value nor the striking price of the options involved (if options are involved) affects the number of shares of stock to buy. Both of the above examples demonstrate the fact that the number of shares to buy is strictly a function of the divisor of the price-weighted index and the unit of trading of the option or future.

Hedging a capitalization-weighted index is more complicated, although the technique revolves around determining the makeup of the index in terms of shares of stock, just as the price-weighted examples above did. Recall that we could determine the number of shares of stock in a capitalization-weighted index by dividing the float of each stock by the divisor of the index. The general formula for the number of shares of each stock to buy is:

$$\begin{array}{c}\text{Shares of stock N}\\ \text{to buy}\end{array} = \begin{array}{c}\text{Shares of N}\\ \text{in index}\end{array} \times \text{Futures quantity} \times \begin{array}{c}\text{Futures unit}\\ \text{of trading}\end{array}$$

We will use the fictional capitalization-weighted index from the previous chapter to illustrate these points.

Example: The following table identifies the pertinent facts about the fictional index, including the important data: number of shares of each stock in the index.

Stock	Price	Float	Capitalization	Shares
A	40	177,000,000	7,080,000,000	1.20
B	80	50,000,000	4,000,000,000	0.34
C	60	100,000,000	6,000,000,000	0.68
	Total capitalization:		17,080,000,000	
	Divisor: 147,500,000			
	Index value: 115.80			

Thus, if one were to buy 1.20 shares of A, .34 shares of B, and .68 shares of C, he would duplicate the index. Recall that one determines the number of shares of an individual stock in a capitalization-weighted index by dividing the float of the stock by the divisor of the index.

Suppose that a futures contract trades on this index, with one point being worth \$500 in futures profit or loss. Then one would buy an amount of each stock equal to 500 times the number of shares in the index. Further suppose that one decides to trade 5 futures at a time. Thus, the number of shares of each stock that one would have to buy to hedge the 5-lot futures position would be:

$$\text{Shares to buy} = \text{Shares in index} \times 5 \text{ futures} \times \$500/\text{future}$$

The following table lists that information, as well as totaling the dollar amount of stock represented by the total. We will verify that the dollar amount of stock purchased is equal to the dollar amount of index represented by the futures.

Stock	Shares in Index	Shares to Buy to Hedge 5 Futures	Price	$ Amount of Stock Bought
A	1.20	3,000	40	$120,000
B	0.34	850	80	68,000
C	0.68	1,700	60	102,000
				$290,000

Thus $290,000 worth of stock has been purchased. From an earlier example, we saw how to compute the total dollar worth of a futures trade. In this case, the index is at 115.80, 5 contracts were sold, and each point is worth $500. Thus, the total dollar amount represented by the futures sale is $5 \times 500 \times 115.80 = \$289,500$. This verifies that our stock purchases hedge the futures sale adequately. Note that the slight difference in the stock purchase amount and the futures sale amount is due to the fact that the number of shares in the index is carried out to only two decimal points in this example.

There is an alternative method to determine how many shares to buy. In this method, one first determines how much stock he is going to buy in total dollars. For example, he might decide that he is going to buy $10,000,000 worth of the S&P 100 (OEX) Index. Next, one determines what percentage his dollar amount is of the total capitalization of the index. For example, $10,000,000 might be something like .02% of the total capitalization of the OEX. One would then buy .02% of the total number of shares outstanding of each of the stocks in the OEX. After the number of shares of each stock to buy has been determined, one would have to determined how many futures to sell against this stock—he would divide $10,000,000 by the index price and also divide by the unit of trading for the futures. This procedure is demonstrated in the following example.

Example: Suppose that one wants to set up an arbitrage against the same index as in the previous example. For purposes of comparison with that example, we will suppose that this hedger wants to buy a total of $290,000 worth of stock. In reality, one would probably use a round number such as $300,000 or $500,000 worth of stock. However, by making a direct comparison, we will be able to more easily demonstrate that these two methods produce the same answer.

First, the hedger must determine the percent of the total capitalization that he is going to buy. In this case, he is buying $290,000 worth of stock and the total capitalization of the index is $17,080,000,000 (refer to the table at the beginning of the previous example). This means that he is buying .0016979% of the total capitalization of the index.

Next, he uses this percentage and multiplies it by the float of each stock. That is, he is going to buy .0016979% of the total number of shares outstanding of each stock in the index. This results in purchases as shown in the following table:

Stock	Float	Shares to Buy
A	177,000,000	3,005
B	50,000,000	849
C	100,000,000	1,698

Compare these share purchases with the previous example. The number of shares to buy is the same, allowing for rounding off in the previous example. Thus, these two methods of determining how many shares to buy are equivalent.

Before leaving this section, it should be pointed out that arbitrageurs can also establish an arbitrage when futures are underpriced. They can sell stock short and buy the underpriced futures. This is a more difficult type of arbitrage to establish because short sales must be made on plus ticks. However, when futures are underpriced for an extensive period of time—perhaps during extreme pessimism on the part of speculators—it is possible to set up the arbitrage from this viewpoint.

PROFITABILITY OF THE ARBITRAGE

The key for many arbitrageurs and institutional investors is whether, after costs, there is enough of a return in this stock versus futures strategy. The method in which we previously computed the fair value of the futures will be used in determining the overall incremental return of doing the arbitrage.

The major cost in executing the arbitrage is the cost of commissions. Since there are large quantities of stocks being bought or sold when an entire index is traded, the commission rate is generally quite low. For example, an institutional investor might pay 3 cents per share or even less. This still could be a substantial cost, especially when a large index such as the S&P 500 Index is being purchased. Even professional arbitrageurs may have to pay commission costs if they are using the services of a computer firm to buy stocks. These methods of trading stocks are described in the next section.

Once one's rate of commission charges is known, he can convert that into a number that represents a portion of the index price. He does so by multiplying his per-share commission rate by the current index value and then dividing that result by the average share price of the index. The following example describes that method of conversion.

Example: Suppose that one is going to buy the entire ZYX Index at a commission rate of 3 cents per share. The index is trading at 185.00. Furthermore, assume that the average price of a share in the index is 45 dollars per share. With this information, one can determine how much he is paying in commissions, in terms of the index itself.

$$\begin{aligned}\text{Commission in terms of index} &= \frac{\text{Commission rate per share} \times \text{Index value}}{\text{Average price per share}} \\ &= \frac{.03 \times 185.00}{45} \\ &= .123\end{aligned}$$

Thus, a commission rate of 3 cents per share translates into 12.3 cents of index value.

The most difficult factor to determine in the above equation is the average price per share for a capitalization-weighted index. There is a shortcut that can be used. It is easy to determine the average price per share for a price-weighted index, such as DJX. The average price per share for large-capitalization indices such as the OEX and S&P 500 is about 80% of that of the DJX.

Now that the commission rate has been converted into an index value, one can determine his net profit from trading the exact index against the futures. One must figure in his futures commission costs as well. The following example demonstrates the net profit from executing the arbitrage, including all costs. Once the net profit has been calculated, a rate of return can be computed.

Example: Suppose that the ZYX Index is trading at 185.00 and the futures, which expire in two months, have a fair value premium of 2.00 points, but are trading at 188.50, a premium of 3.50. The futures are worth $500 per point. Thus, the futures are expensive and one might attempt to buy stocks and sell the futures. His net profit consists of the premium over fair value less all costs of entering and exiting the position.

As we saw in the previous example, at 3 cents per share stock commission, we pay an index value of .123 to enter the position. Similarly, we would pay .123 in index value to exit the position at a later date. Thus, the net round-turn stock commission is approximately 25 cents of index value.

Commissions on futures are generally charged only when the position is closed out. Generally, a futures commission on an S&P 500 contract might be reduced to something like $10 per contract for this type of hedging. Since 185.00, the index value, represents 1/500th of the value of the futures contract, we can reduce the futures commission to an index-related number by dividing the actual dollar commission by 500. Thus, the futures commission is, in index terms, 10/500, or .02. The total commission for entering and exiting the position is thus 0.266 of index value, 0.123 each for the purchase and sale of the stocks and .02 for the futures.

$$\begin{aligned}\text{Net profit} &= \begin{matrix}\text{Futures}\\ \text{price}\end{matrix} - \begin{matrix}\text{Futures fair}\\ \text{value}\end{matrix} - \begin{matrix}\text{Commission}\\ \text{costs}\end{matrix}\\ &= 188.50 - 187.00 - 0.27\\ &= 1.23\end{aligned}$$

This absolute net profit number can be converted into a rate of return by annualizing the profit and dividing by the current index price. Suppose that there are two months exactly remaining until expiration. Then the rate of return is computed as follows:

$$\begin{aligned}\begin{matrix}\text{Incremental}\\ \text{rate of return}\end{matrix} &= \frac{\text{Net profit} \times (1/\text{Time remaining})}{\text{Index price}}\\ &= \frac{1.23 \times (12/2)}{185.00}\\ &= 3.99\%\end{aligned}$$

For the two-month time period, his return is about 2/3 of one percent.

At first glance, a rate of return of almost 4% does not seem like much. But what we have computed here is an *incremental* rate of return. That is, this return is over and above whatever rate we used in determining the fair value of the futures. Thus, if an institution were going to invest its cash at the prevailing short-term rate, and that rate were used to determine the futures fair value in the above example, then the institution could earn an *additional* 4%, annualized, if it arbitraged the futures rather than put its money in the short-term money market.

TRADE EXECUTION

Most customers are not concerned with how the trades are executed, for they give the order to their broker and let him work out the details. However, for those who are interested in the actual trade execution, a short section dealing with that topic is in order.

Ideally, one should be able to monitor the progress of his index in terms of bid prices, offer prices, and last sales. There are several modern quote services that allow such monitoring. It is important to know the bids and offers because, when one actually executes the trades, he generally will be trading on the bid and offer, not the last sale.

Example: Suppose the fair value of the futures contract is represented by a premium of 1.25 points, but that the actual future is trading with a premium of 2.00 points: The index is at 165.75 and the futures are 167.75 (last sale). This might seem like enough "room" to execute a profitable arbitrage—buy the stocks in the index and sell the futures. However, the index value of 165.75 is the composite of the last sales of each of the individual stocks in the index. If one were to look at the offering prices of each stock and then recompute the index, he might end up with an index value that was 50 cents higher. This, then, would mean that he would be doing the arbitrage for 25 cents less costs, which is not enough of a margin to work with.

Similarly, when one is looking to sell out the stocks he has bought and simultaneously buy back the futures, he needs to know the bid value of the index in order to see what kind of premium he is paying to take his position off.

The main method of order entry is completely computerized. The computer knows the quantity of each stock to buy and, when prompted, sends those buy orders via telecommunications lines to one of the automatic order execution systems on the exchange floor. Most automatic systems attempt to guarantee the offering price for large quantities of stock. In this highly sophisticated method of order entry, the entire execution procedure may take place in about one minute for the entire index. This method of order entry is so quick and accurate that some brokerage firms with this capability offer it for a commission fee to other brokerage firms that do not have the capability.

INSTITUTIONAL STRATEGIES

Holders of large portfolios of stocks can use futures and/or market basket strategies to their advantage. There are two basic strategies that can be easily used by these large traders. One is to buy futures instead of buying stocks, and the other is to sell futures instead of selling stocks. Both of these strategies will be examined in more detail.

When one of these large institutions has money to invest in buying stocks, it might make more sense to buy Treasury bills and futures instead of buying stocks. Of course, this alternative strategy only makes sense for the institution if the stock purchase were going to be broad-based—something akin to duplicating the S&P 500 performance. The institution does not necessarily have to be intent on purchasing an exact index, but if the purchase were going to be diversified, the purchase of index futures might help accomplish an equivalent result. If, however, the purchase were going to be quite specific, then this strategy would probably not apply.

This strategy works best when futures are underpriced. If the equivalent dollar amount of underpriced futures can be purchased instead of buying stocks, the entire amount intended for stock purchase can be put in Treasury bills instead. Recall that cash will have to be put into the futures account if the futures mark at a loss (maintenance margin). Even so, there can be substantial savings to the institution if the futures are truly underpriced.

The second institutional strategy is applicable when futures are overpriced and the institution wants to sell stock. In such a case, it makes more sense to sell the futures than to sell the stock. First, there are large savings in transaction costs (commissions). Second, the overpriced nature of the futures actually means that there is additional profitability in selling them as opposed to selling the stocks. Again, this strategy only makes sense if one were going to sell a diversified portfolio of stocks, something that is broad-based like the S&P 500 Index.

Of course, institutions may want to participate in the arbitrage regardless of their market stance. That is, if a money manager has a certain amount of money that he is going to put into short-term instruments (perhaps T-bills), he might instead decide to participate in this arbitrage of stocks versus futures if the incremental return is high enough. Recall that we saw how to determine the incremental return in a previous section of this chapter. If he were going to get a 7½% return from T-bills but could get an 11½% return from futures arbitrage, he might opt for the latter.

FOLLOW-UP STRATEGIES

Once any hedge has been established, it must be monitored in case an adjustment needs to be made. The first and simplest type of monitoring is to take care of spinoffs or other adjustments in stocks in the market basket that is owned. Of a more serious nature, in terms of profitability, one also needs to monitor the hedge to see if it should be removed or if the futures should be rolled forward into a more distant expiration month.

Adjusting one's portfolio for stock spinoffs is a simple matter which we will address briefly. In many cases, one of the stocks in the index will spin off a division or segment of its business and issue stock to its shareholders. Such a spinoff is generally not included in the price of the index, so that the hedger should sell off such items as soon as he receives them, for they do not pertain to his hedge.

In a similar vein, in any portfolio certain stocks may occasionally be targets of tender offers or other reorganizations. If one does nothing in such a situation, he will not lose any money in terms of his portfolio versus the underlying index. However, it is generally wise for one to tender his stock in such situations and replace it at a lower price after the tender. Sometimes, in fact, such a tender offer will entirely absorb an index component member. In that case, one must replace the disappearing stock with whatever stock is announced as the new member of the index.

Technically, in an arbitrage hedge one should adjust his portfolio every time the divisor of the index changes. Thus, in a *capitalization-weighted hedge* he would be adjusting every time one of the components issues new common stock. This is really not necessary in most cases, because the new issue is so small in comparison to the current float of the stock. Such a new issue does not include stock splits, for the divisor of the index does not change in that case. A more common case is for one of the stocks in a *price-weighted* index to split. In this case, one must adjust his portfolio. An example of such an adjustment was given in Chapter 29. In essence, one must sell off some of the split stock and buy extra shares of each of the other stocks in the price-weighted index.

Let us now take a look at follow-up methods of removing or preserving the hedge.

ROLLING TO ANOTHER MONTH

As expiration nears, the hedger is faced with a decision regarding taking off the market basket. If the futures premium is below fair value, he would probably unwind the entire position, selling the stocks and buying back the futures. However, if the futures remain expensive—especially the next series—then the hedge might roll his futures. That is, he would buy back the ones he is short and sell the next series of futures. For S&P 500 futures, this would mean rolling out 3 months, since that index has futures that expire every 3 months. For index options, there are monthly expirations, so one would only have to roll out 1 month if so desired.

It is a simple matter to determine if the roll is feasible: Simply compare the fair value of the spread between the two futures in question. If the current market is greater than the theoretical value of the spread, then a roll makes sense if one is long stocks and short futures. If an arbitrageur had initially established his arbitrage when futures were underpriced, he would be short stocks and long futures. In that case he would look to roll forward to another month if the current market were less than the theoretical value of the spread.

Example: With the S&P 500 Index at 416.50, the hedger is short the March future that is trading at 417.50. The June future is trading at 421.50. Thus, there is a 4-point spread between the March and June futures contracts.

Assume that the fair value formula shows that the fair value premium for the March series is 35 cents and for the June series is 3.25. Thus, the fair value of the spread is 2.90, the difference in the fair values.

Consequently, with the current market making the spread available at 4.00, one should consider buying back his March futures and selling the June futures. The rolling forward action may be accomplished via a spread order in the futures, much like a spread order in options. This roll would leave the hedge established for another 3 months at an overpriced level.

Another way to close the position is to hold it to expiration and then sell out the stocks as the cash-based index products expire. If one were to sell his entire stock holding at the time the futures expire, he would be getting out of his hedge at exactly parity. That is, he sells his stocks at exactly the last sale of the index, and the futures expire, being marked also to the last sale of the index.

For settlement purposes of index futures and options, the S&P 500 Index and many other indices calculate the "last sale" from the *opening* prices of each stock on the last day of trading. For some other indices, the last sale uses the *closing* price of each stock.

Example: In a normal situation, if the S&P 500 index is trading at 415, say, then that represents the index based on last sales of the stocks in the index. If one were to attempt to buy all the stocks at their current offering price, however, he would probably be paying approximately another 50 cents, or 415.50, for his market basket. Similarly, if he were to sell all the stocks at the current bid price, then he would sell the market basket at the equivalent of approximately 414.50.

However, on the last day of trading, the cash-based index product will expire at the opening price of the index. If one were to sell out his entire market basket of stocks at the current bid prices at the exact opening of trading on that day, he would sell his market basket at the calculated last sale of the index. That is, he would actually be creating the last sale price of the index himself, and would thereby be removing his position at parity.

There is another interesting facet of the arbitrage strategy that combines the spread between the near-term future and the next longest one with the idea of executing the stock portion of the arbitrage at the time the index products expire. Use of this strategy actually allows one to enter and exit the hedge without having to lose the spread between last sale and bid or last sale and offer in either case. Suppose that one feels that he would set up the arbitrage for 3 months if he could establish it at a net price of 1.50 over fair value. Furthermore, if the fair value of the 3-month spread is 2.10, but it is currently trading at 3.60, then that represents 1.50 over fair value. One initiates the position by buying the near-term future and selling the longer-term future for a net credit of 3.60 points. At expiration of the near-term future, rather than close out the spread, one *buys* the stocks that comprise the index at the last sale of the trading day, thereby establishing his long stock position at the last sale price of the index at the same moment that his long futures expire. The resultant position is long stocks and short futures that expire in 3 months at a premium of 3.60. Since the fair value of such a 3-month future should be 2.10, the hedge is established at 1.50 over fair value. The position can be removed at expiration in the same manner as described in the previous paragraph, again saving the differential between last sale and the bids of the stocks in the index. Note that this strategy creates buying pressure on the stock market at expiration of the near-term side of the spread, and selling pressure at the latter expiration.

The final way to exit from one's position is to remove it before expiration. Sometimes, there are opportunities during the last two or three weeks before the futures expire. If one hedged long stock with short futures, the opportunities to remove the hedge arise when the futures trade below fair value—perhaps even at an actual discount to parity. If the futures never trade below fair value, but instead continue to remain expensive, then rolling to the next expiration series is often warranted.

MARKET BASKET RISK

There are some uncertainties in this type of hedging, even though the entire index is being bought. Since one owns the actual index, there is no risk that the stocks one owns will fail to hedge the futures price movements properly. However, there are other risks. One is the risk of execution. That is, it may appear that the futures are trading at a premium of 1.50 points when one enters the orders. However, if other hedgers are doing the same thing at the same time, one may pay more for the stocks than he thought when he entered his order, and he may sell the futures for less as well. This "execution risk" is generally small, but if one is too slow in getting his stock orders executed, he may have set up a hedge that was not as attractive as he first thought.

One major risk is that interest rates might move against the arbitrageur while the position is in place. If he is long stocks and short the futures, then he would not want interest rates to rise. In the previous example, the incremental return for the 2-month time period that the position was going to be held was ⅔ of one percent. If short-term rates should rise by more than that, on average, for the 2-month period, the incremental strategy would be inferior. His carrying costs would have increased to the point of wiping out the profit from his arbitrage.

For institutional arbitrageurs who don't exactly have cost of carry, this situation would be viewed in the following manner: If rates increase, he may find that he would have been better off having his money invested in a money market fund at the prevailing short-term rate than in the incremental arbitrage strategy. Conversely, if the arbitrage was originally established with short stocks and long futures, the arbitrageur would not want rates to *drop* for similar reasons.

One might leave a cushion against a movement in rates. If rates are currently 8%, then one might decide to use a 10% rate in his initial calculations, as a cushion. Hedges established that are profitable at the higher rate level will consequently be able to withstand rates moving up to 10%.

Example: Suppose that one would normally use a rate of 7½% and would establish the long stock versus short futures hedge at an incremental rate of return of 1½%. This is a relatively narrow cushion and if the hedge is on for a moderate length of time, rates could move up to such an extent that they advance to 9½% or higher. Such a move would make the hedge position unprofitable. Instead, one might calculate the fair value of the futures using a rate equal to his current prevailing rate plus a cushion. That is, if his current rate is 7½%, he might use 8½% and still demand an incremental return of 1½%. If he established the hedge at these levels, he could suffer a move of 1%, the cushion, against him and still earn his incremental rate of 1½%.

Another risk that the arbitrageur faces is that of changes in the dividend payout of the stocks in the index. Suppose that he is long stocks and short futures. If there are enough cuts in dividend payout, or dividend payments are delayed past the expiration date of the futures, then he will lose some of his return. Arbitrageurs who are short stocks and long futures would have similar problems if dividend payout were increased—especially if a large special dividend were declared by a company that is a major component of the index—or payment dates were accelerated.

If one holds the arbitrage until expiration, he will be able to unwind it at parity. However, if he decides to remove the arbitrage before expiration, he might incur increased costs that would harm his projected return. Instead of selling his stocks at the last sale of the index, as he is able to do on expiration day, he would have to sell them on their bids, a fact that could cost him a significant portion of his profit.

In a later section, where we discuss hedging the futures with a market basket of stocks that does not exactly represent the entire index, we will be concerned with the greatest risk of all, "tracking error"—the difference between the performance of the index and the performance of the market basket of stocks being purchased.

IMPACT ON THE STOCK MARKET

The act of establishing and removing these hedge positions affects the stock market on a short-term basis. It is affected both before expiration and also at expiration of the index products. We will examine both cases and will also address how the strategist can attempt to benefit from his knowledge of this situation.

IMPACT BEFORE EXPIRATION

When bullish speculators drive the price of futures too high, arbitrageurs will attempt to move in to establish positions by buying stock and selling futures. This action will cause the stock market to jump higher, especially since positions are normally established with great speed and stocks are bought at offering prices. Such acceleration on the upside can move the market up by a great deal in terms of the Dow-Jones Industrials in a matter of minutes.

Conversely, if futures become cheap there is also the possibility that arbitrageurs can drive the market downward. If positions are already established from the long side (long stock, short futures), then arbitrageurs might decide to unwind their positions if futures become too cheap. They would do this if futures were so cheap that it becomes more profitable to remove the position, even though stocks must be sold on their bid, rather than hold it to expiration or roll it to another series. When these long hedges are unwound in this manner, the stock market will decline quickly as stocks are sold on bids. In this case, the market can fall a substantial amount in just a few minutes.

Once long hedges are unwound, however, cheap futures will not cause the market to decline. If there is no more stock held long in hedges, then the only strategy that arbitrageurs can employ when futures become cheap is to sell stock short and buy futures. Since stock must be sold short on upticks, this action may put a "lid" on the market, but will not cause it to decline quickly.

Having long stock and short futures when these large discounts occur is so valuable to an arbitrageur that some traders will establish the long stock/short futures hedge for no profit or even a loss. They hope that subsequently futures will plunge to a large discount and they can unwind their positions for large profits. If that never occurs, they only lose a few cents of index value. Assume futures fair value is 3.50 over. Such arbitrageurs might buy stock and sell futures at a net cost of 3.45 over. That is, if they hold the position until expiration they will lose 5 cents, but if a large futures discount ever occurs, they will profit.

Regulatory bodies have become increasingly concerned over the years as to the effect that program trading and index arbitrage have on the stock market. In reality, when stocks and futures are executed more or less simultaneously, these strategies should not overly disturb the stock market.

At one time regulators imposed "program trading curbs," which took effect whenever the Dow moved 2% from the previous day's close. These were removed in November 2007.

A more definitive restriction does exist though—"circuit breakers." These were established after the Crash of '87 and remain in effect to this day. At the end of each quarter, the NYSE sets three circuit breakers at levels of 10%, 20%, and 30% of the value of the Dow Jones Industrials at that time, rounded to the nearest 50-point interval. For example, at the end of the second quarter of 2011, these limits were 1200, 2400, and 3600 points. When these limits are hit there are various responses by the NYSE, depending on the size of the drop and the time of day at which it occurs.

For the smallest limit (down 1200 points, circa 2011), if it is hit prior to 2:00 p.m., trading halts for one hour. If it is hit between 2:00 and 2:30 p.m., trading is halted for 30 minutes. If it is hit later in the day than that, trading does not halt because of the first trading limit.

For the middle limit (down 2400 points, circa 2011), if it is hit prior to 1:00 p.m., trading halts for two hours. If it is hit between 1:00 and 2:00 p.m., trading halts for one hour. If it is hit after 2:00 p.m., trading halts for the day. If the largest limit (down 3600 points, circa 2011) is hit at any time, trading halts for the remainder of the day.

These are huge moves for a single day, and even in the financial crisis of 2008, these limits were not hit. In fact, the only time that the circuit breaker was ever hit was on October 27, 1997, when the limit was 550 points. That limit was hit late in the day and trading halted for the remainder of that day. These limits are only in effect during regular NYSE trading hours.

There are limits on the S&P futures as well, and they are at 5%, 10%, 20%, and 30%. So they roughly correspond with the Dow limits, but not exactly. In addition, the 5% S&P limit applies to overnight trading. If it is hit overnight, then trading halts until the next day, when the day session opens. This 5% limit was hit on January 21, 2008, when, on the Martin Luther King holiday, markets sold off heavily due to liquidation of a rogue trader's position at Societe Generale. Once the limit was hit, the Globex session closed until the next day.

Readers should remember, of course, that the stock market can move independently of the overpriced or underpriced nature of index products. That is, if futures are overpriced, the stock market can still decline. Perhaps there is a preponderance of natural sellers of stocks. Similarly, if futures are cheap, the stock market can still go up if enough traders are bullish. Thus, one should be cautious about trying to link every movement of the stock market to index products.

PORTFOLIO INSURANCE

Portfolio insurance is the generic name used to describe a strategy in which a portfolio manager uses the index derivatives market to protect his portfolio in case the market crashes. He could either sell futures, buy puts, or—as will be shown later when volatility derivatives are discussed—he could also buy volatility products.

The generic concept was put into effect using futures in the mid-1980s. In the form of the strategy that was being practiced at the time, the portfolio manager did not sell futures against his entire portfolio right away. Rather, he sold only a few to begin with. This allowed him to retain a good deal of upside profit potential for his portfolio. If the market dropped further, then he would sell more. Eventually, if it dropped far enough, he would keep selling futures until his entire portfolio was properly hedged. There were computer programs that calculated when to sell the futures and how many to sell in order for the portfolio manager to eventually end up with the proper amount of insurance at the right price.

Unfortunately, the concept did not work properly in practice. In fact, it has often been identified as one of the major factors in the 500-point crash of October 19, 1987. What happened during the days leading up to that date was that the market was already going down fast. Futures, as a result, began to trade at a discount. The portfolio insurance strategy assumes futures are sold at fair value, more or less. Thus, the portfolio insurance managers did not sell their futures when they had originally intended; or they could not sell enough without driving the futures to tremendous discounts. In any case, the market kept going further down without any rebound (essentially from about mid-afternoon on Thursday, October 15, through the close on Monday, October 19), a total of over 650 points on the Dow-Jones averages.[*] As the market plunged, the portfolio insurance strategy kept demanding that more futures be sold, and they were, but often at prices well below where the strategy had originally dictated. This continued selling kept futures at a discount, which triggered even more selling by other program traders and index arbitrageurs.

As a result, the portfolios were not completely protected—although it should be noted that they were somewhat protected since they had been selling some futures. Hence, the portfolio managers were not pleased. Stock market regulators were not pleased, either, although nothing illegal had been done. The strategy lost most of its adherents at that time and has not been resurrected in its previous form.

However, the concept is still a valid one, and it is now generally being practiced with the purchase of put options. The futures strategy was, in theory, superior to buying puts because the portfolio manager was supposed to be able to collect the premium from selling the futures. However, its breakdown came during the crash in that it was impossible to buy the insurance when it was most needed—similar to attempting to buy fire insurance while your house is burning down.

Currently, the portfolio manager buys puts to protect his portfolio. Many of these puts are bought directly over-the-counter from major banks or brokerage houses, for they can be tailored directly to the portfolio manager's liking. This practice concerns regulators somewhat, because the major banks and brokerage houses that are selling the puts are taking some risk, of course. They hedge the sales (with futures or other puts), but regulators are concerned that, if another crash occurred, it would be the writers of these puts who would be in the market selling futures in a mad frenzy to protect their short put positions. Hopefully, the put sellers will be able to hedge their positions properly without disturbing the stock market to any great degree.

IMPACT AT EXPIRATION—THE RUSH TO EXIT

In the early days of index trading, the expiration of futures and options often had a large effect on the stock market itself. These effects are diminished today, thanks to "a.m." set-

[*] This represented a decline of over 25% from the relatively low levels (mid-2000s) that the index was trading at during that time frame.

tlement and European-style exercise for most index products, but have not been completely eliminated.

The mere act of exiting the position might cause a stock market movement. Consider the index arbitrageur who is short stocks and long futures. At the instant of expiration, he cannot just merely do nothing. If he did, his futures would settle for cash, and he would be left with a large portfolio of short stocks to deal with. Any small upward movement might completely wipe out his arbitrage profits. Rather, he must buy back all of his stocks simultaneously on the last "tick" of trading—whether that be the "a.m." settlement (first trade of the day) or "p.m." settlement (last trade of the day). That action in effect creates the settlement price of the index, which then expires for cash at the same price as his combined stock trades.

This action might move the stock market if it were left unchecked. For example, if a large number of index arbs were buying back short stocks at the last instant of trading, the stock market would rise. Conversely, if the arbs were selling out long stocks en masse at the expiration, then stocks would fall.

The effect of these trades on the stock market has been diminished by requirements that traders enter their orders in advance, so the NYSE specialists can publish imbalances. These imbalances create order flow on the other side of the trade.

Example: Due to the fact that there is unwinding of the S&P 500 Index at expiration, the NYSE specialist observes the order flow—perhaps 30 minutes before the trades are to take place. He finds that there is more IBM for sale than he can handle. So he publishes that there is an order imbalance in IBM—that there are 5 million shares of IBM for sale, say. Mutual fund and institutional managers around the world see that and, more often than not, there will be willing buyers, because they realize they can buy in size on a downtick. Thus, while IBM might print down a few cents in this case, it will not print down by a large amount because of the buy orders drawn in from the order imbalance announcement.

There are clues, of course, as to which way the arbs are set up, but because of the plethora of index products available, it is often difficult to ascertain beforehand which way the arbs are positioned. For example, if an arb has a position that is short S&P futures, he might have offsetting longs in stocks, S&P futures options, SPX options, and/or SPY options. In fact, if one knew his position in any one of these, it might not mean much because the aggregate position is all that counts.

The one index that is rather "pure" and doesn't really have many alternatives is the OEX (S&P 100) Index. There are no futures. There is not an active ETF with options. Therefore, if one sees that there is a buildup of open interest in OEX options prior to expiration it might be an accurate clue as to how arbs are positioned in general—in S&P and other index products.

The OEX options are not nearly as liquid or popular as they used to be, so their informational content has been diminished in recent years, but the process is still the same. Index arbs own in-the-money OEX options against their positions (if they were out-of-the-money, they would not be hedged). So it is a simple matter to observe the quantity of expiring in-the-money calls versus the quantity of expiring in-the-money puts. Suppose that one does that and sees that there is a large imbalance of in-the-money calls. The reasonable assumption at expiration is that those are hedged by short stock. Hence, there is potential buying power at index expiration, for those short stocks need to be covered. Conversely, if there is a large imbalance of in-the-money puts, one would assume that they are hedged by long stock and that stock has to be sold at expiration—causing pressure on the stock market.

These imbalances in open interest between expiring OEX in-the-money calls and puts can often be discerned beforehand, just by knowing how the stock market has been trading. If a strong uptrend has been in place over the last month, say, then there will certainly be a good deal of in-the-money calls and probably not many in-the-money puts. Hence one would suspect a buying imbalance (short stocks need to be covered) without ever looking at the open interest figures. Conversely, if the stock market has been declining over the past month, then there will be an excess of in-the-money puts, which will translate to selling pressure on the stock market.

SIMULATING AN INDEX

The discussion in the previous section assumed that one bought enough stocks to duplicate the entire index. This is unfeasible for many investors for a variety of reasons, the most prominent being that the execution capability and capital required prevent one from being able to duplicate the indices. Still, these traders obviously would like to take advantage of theoretical pricing discrepancies in the futures contracts. The way to do this, in a hedged manner, would be to set up a market basket of a small number of stocks, in order to have some sort of hedge against the futures position.

In this section, we will demonstrate approaches that can be taken to hedge the futures position with a small number of stocks. This is different from when we looked at how to hedge individualized portfolios with index futures or options, because we are now going to try to duplicate the performance of the entire index, but do it with a subset of stocks in the index. In either of these cases, a mathematical technique called regression analysis can be used to measure the performance of these portfolios or small market baskets. However, we will take a simpler approach that does not require such sophisticated calculations, but will produce the desired results.

USING THE HIGH-CAPITALIZATION STOCKS

Recall that in a capitalization-weighted index, the stocks with the largest capitalizations (price times float) have the most weight. In many such indices, there are a handful of stocks that carry much more weight than the other stocks. Therefore, it is often possible to try to create a market basket of just those stocks as a hedge against a futures position. While this type of basket will certainly not track the index exactly, it will have a definite positive correlation to the index.

What one essentially tries to accomplish with the smaller market basket is to hedge dollars represented by the index with the same dollar amount of stocks. No hedge works in which the total dollars involved are not nearly equal. Listed below are the steps necessary to compute how many shares of each stock to buy in order to create a "mini-index" to hedge futures or options on a larger index:

1. Determine the percent of the large index to be hedged (OEX, NYSE, S&P 500, etc.) that each stock comprises. This information is readily available from the exchange on which the futures or options trade or can be calculated by the methods shown in Chapter 29.
2. Determine the percent of the mini-index to be constructed that each stock comprises, by inflating their relative percentages to total 100%.
3. Decide the total dollar amount of the index to be traded at one time: index value times futures or option quantity times unit of trading in the futures.
4. Multiply the total dollar amount from step 3 by each individual percentage from step 2 to determine how many dollars of each stock to buy.
5. Divide the result from step 4 by the price of the stock in order to determine how many shares to buy.

These steps will result in the construction of a mini-index consisting of a small number of stocks that are grouped together in relative proportion to their weights in the larger index, and have a total dollar amount sufficient to trade against the desired futures or options trading lot. This approach ignores volatility. Even without accounting for volatility, this approach is reasonable when using high-capitalization stocks to hedge a broad-based index.

The examples on the following pages use four fictional large-cap stocks to illustrate the points (IBN, XON, GN, and CE). At any one point in time, the four largest-capitalization stocks will be different. GM was at one time the world's largest corporation, for example, but is no more. So rather than try to use actual stock symbols in these examples, they will be generic.

Example: Suppose we are attempting to create a hedge for a fictional index, the UVX, by using IBN, XON, GN, and CE. The following table gives certain information that will be necessary in computing how many shares of each stock to use in the small basket.

Stock	Float	Price	Shares in Index	Capitalization	Pct of Index (Step 1)
IBN	600,000	130	0.171	78,000,000	13.1%
XON	850,000	40	0.243	34,000,000	5.7%
CE	450,000	70	0.129	31,500,000	5.3%
GN	300,000	85	0.086	25,500,000	4.3%
					28.4%

UVX price: 170.25
Divisor: 3,500,000
Total capitalization: 595,875,000 (price times divisor)

Recall how these items are calculated: The number of shares of a stock in the index is that stock's float divided by the divisor of the index. Also, the percent of the index is the stock's capitalization (float times price) divided by the total capitalization of the index (this is step 1 above). Finally, the index value is the index's total capitalization divided by the index divisor.

With this information, we can now construct a mini-index that could be used to hedge the UVX itself. Notice that these four stocks alone comprise 28.4% of the entire UVX index. We would want each of these four stocks to have the same relative weight within our mini-index as they do within the UVX itself. The sum of the capitalizations of the four stocks in the above table as well as their relative percentages are given in the following table.

Stock	Capitalization	Pct of Index (Step 1)	Pct of Mini-Index (Step 2)
IBN	78,000,000	13.1%	46.2%
XON	34,000,000	5.7%	20.1%
CE	31,500,000	5.3%	18.6%
GN	25,500,000	4.3%	15.1%
Total:	169,000,000	28.4%	100.0%

The percent of the mini-index is each of the four stocks' capitalizations as a percent of the sum of their capitalizations (step 2 from above). There are two ways to compute step 2. First, for IBN one would divide 78 million (its capitalization) by 169 million (the total capitalization). Second, using the percentages from step 1, divide IBN's percent, 13.1, by 28.4, the total percent. Either method gives the answer of 46.2 percent. We have now constructed the relative percentages of the mini-index that each stock comprises. Note that they are in the same relationship to each other as they are in the UVX itself. Now it is a simple matter to convert that percent into shares of stock, once we decide how many futures contracts to trade against our mini-index.

When we know the total dollar amount of futures to hedge and we know the percent of the mini-index that each stock comprises, we can compute each stock's capitalization within the mini-index. Finally, we divide by that stock's price to see how many shares of each stock to buy. Assume that we are going to use UVX options, which are worth $100 per point, in lots of 50 options. The total dollar amount of the index with the UVX at 170.25 would then be $851,250 ($170.25 \times 100 \times 50$). This accomplishes step 3. The following table shows the calculations necessary to determine how many shares of stock to buy against these 50 option contracts.

Stock	Pct of Mini-Index	Capitalization in Mini-Index (Step 4)	Price	Shares to Buy (Step 5)
IBN	46.2%	393,277	130	3,025
XON	20.1%	171,102	40	4,277
CE	18.6%	158,332	70	2,261
GN	15.1%	128,539	85	1,512
Total:	100.0%	851,250		

Note that the capitalization of each stock in the mini-index is determined by multiplying the desired trading lot ($851,250) by the percent of the mini-index that that stock comprises. This completes step 4, and step 5 follows: The number of shares of each stock to buy is then determined by dividing that number by the price of the stock. For example, the calculation for IBN in the above table would be $\$851,250 \times .462 = \$393,277$; then $\$393,277/130 = 3,025$.

Thus, one could attempt to hedge 50 UVX option contracts with the above amounts of each of the four stocks. As a matter of practicality, one would not buy the odd lots, but would probably round off each stock quantity to round lots: 3,000 IBN, 4,300 XON, 2,300 CE, and 1,500 GN.

As the prices of the stocks in the mini-basket change, the mini-basket needs to be recalculated. This is because the current prices of the stocks in the index were used to compute the mini-index. Thus, as the prices of the stocks change, the composition of our mini-index will begin to deviate from the composition of the UVX.

Example: Suppose that oil stocks do poorly and XON falls to 35 (it was 40 when we constructed the mini-index), while the other stocks are the same price as in the previous example. Finally, suppose that the overall UVX is unchanged at 170.25, even though XON has changed substantially. We must recalculate step 1: Determine the percent that each stock is of the UVX. Assume the divisor is unchanged and each stock's float is unchanged, so the percent is the price times the float divided by the total capitalization (595,875,000).

Stock	Price	Float (000s)	Capitalization (Millions)	Pct of Index (Step 1)	Pct of Mini-Index (Step 2)
IBN	130	600	78.00	13.1%	47.3%
XON	35	850	29.75	5.0%	18.1%
CE	70	450	31.50	5.3%	19.1%
GN	85	300	25.50	4.3%	15.5%
Total:			164.75	27.7%	100.0%

Note that the percent that XON comprises of the UVX as well as of the mini-index has fallen. All three of the other stock's percentages have increased proportionately. These percentage changes reflect the changes in the stock prices. Since we assumed the UVX is unchanged, the capitalization of the desired mini-index is still \$851,250 (170.25 × \$100 per point × 50 options). Now, if we complete steps 4 and 5, we will see how many shares of each stock make up the new version of the mini-index.

Stock	Capitalization in Mini-Index (Step 4)	Price	Shares to Buy (Step 5)
IBN	\$402,641	130	3,097
XON	154,076	35	4,402
CE	162,589	70	2,323
GN	131,944	85	1,552
Total:	\$851,250		

Compare the number of shares to be bought in this example with the number of shares to be bought in the previous example. Actually, we are buying more shares of each of the stocks. There are two reasons for this. In XON case, we are buying more shares since the price has dropped more as a percentage of its previous price than its capitalization has dropped as a percentage. For the other stocks, we are buying more shares because the capitalization of each has increased within the mini-index and the price is unchanged.

This example serves to show that as the prices of the stocks in the mini-index change, the number of shares of each of the stocks might change. This means that the hedger using this type of hedge should recalculate the makeup of the index rather frequently—at least once a week. In actual practice, the hedger will know which stocks are underperforming and which are outperforming. Hence, he will have some idea of what needs to be done in advance of actually computing it.

There are many methods of approaching these "mini-indices." Some traders who are extremely short-term oriented—possibly moving in and out of the futures one or more times daily—might attempt to hedge the futures with only *one* stock (generally the largest-capitalization stock, such as IBN, unless there is some reason to believe that the general market is moving in a substantially different direction from the largest of all stocks).

In other cases, hedgers with more capital and more resources but who are unwilling to hedge the entire index might try to use a larger mini-index to hedge with. In cases such as these, one is generally not interested in day-trading the futures and stocks, but rather in attempting to simulate the full hedge against fair value, as described earlier. For example, the top 30 capitalization stocks in the OEX make up over 70% of the capitalization of the index. This provides very accurate tracking, but still does not overly tax the execution capabilities of even a small trading desk. Such a 30-stock mini-index can be calculated in exactly the same manner as in the previous examples. Since it represents over 70% of the index, it will tract the index quite well, although not perfectly of course.

However, if one tried to simulate the S&P 500 Index by buying the top 50 stocks, he would still not own even 40% of the capitalization of the index. This does not provide as accurate tracking as one would hope after having bought 50 stocks. As a result, if one were trying to hedge the S&P 500 Index, he should use at least 200 stocks.

TRACKING ERROR RISK

In any such simulated index portfolio, there is the largest risk of all with regard to index hedging, the *tracking error.* Tracking error is the difference in performance between the actual index and the simulated index portfolio. There are statistical ways to predict how closely a certain portfolio of stocks will simulate a given index. This is something akin to pollsters predicting the margin of victory of an election before the election is held. One may hear that a certain portfolio has a 98% correlation, say, to an index it is intended to simulate.

What does this measure represent? First, it must be understood that statistics cannot predict the exact performance of any set of stocks with respect to any other set, just as polls cannot exactly predict the outcome of an election. What the statistics do tell us is how probable it is that a certain portfolio will perform nearly the same as another one. The concept of expected return, which was described earlier in this book, is something like this. The statistical number does not guarantee that the portfolio will perform like the index 98% of the time or that it will never deviate from the index by more than 2%. It is merely a comparative measure that says that such a portfolio has a good correlation to the index.

The actual risk that one is taking by using the simulated index instead of the index itself is not completely measurable. If it were, then we could predict the exact performance of the simulated index, which we just showed we could not. However, assume an average performance—that the simulated index deviates from the real index by 2% over the course of 1 year. If we are speaking of the S&P 500 at 415, then 2% would be 9.30 points over a 1-year period, or 2.33 points over a 3-month period. That is a substantial amount of movement when one considers that most of our arbitrage examples were assuming profits of not much more than that. The compensating factor to this risk is that the simulated index may outperform the actual index and one could make more profits than would be available with arbitrage. If one had enough capital and enough time to constantly be participating in such a simulated-index strategy, he would, over time, have a tracking error that is relatively small if his simulated index has a high correlation to the index.

MONITORING THE HEDGE

Once the position has been established, the trader should have some way of monitoring the position. Ideally, he would have a computer system that could compute his mini-index in real time. This would allow accurate comparisons between the actual index movement and the mini-index. Tracking error can, of course, work for or against the trader.

It is not necessary to have a computer system built specifically for index hedging. Many computerized systems provide for real-time profit and loss calculations on a portfolio of the user's choosing. Any of these systems would be sufficient for computation of the relative value of the mini-index. In the course of computing the profit or loss on the portfolio, the program must compute the net value of the portfolio. As long as this is available, one can convert it into a mini-index value, suitable for comparison to the larger index. The "trick" is to use a mini-index multiplier that is a power of 10. That is, the futures unit of trading times the futures quantity is a power of 10. For example, if the futures unit of trading is $250 as with the S&P 500 futures, then a quantity of 40 would result in a power of 10 ($250 × 40 = 10,000). This means that the total capitalization of the mini-index portfolio should be able to be read as the "index value" with the mere adjustment of a decimal point.

Example: If one is trading against futures that have a trading move of $500 per point, then he might choose to use 20 futures against his mini-index. That is, the multiplier is 20 × $500 or $10,000. If he were hedging against an index trading at 170.25, he would then buy 10,000 × 170.25 or $1,702,500 worth of stock. The total value of the stocks in his mini-index would total $1,702,500 initially and he could therefore determine his mini-index value to be 170.2500 by moving the decimal point over four places. The following table summarizes how this might be constructed using the four stocks in the most recent example. Recall that in the previous example, the total capitalization of the four-stock mini-index was $851,250. In this example we would have had a total mini-index capitalization of twice that, or $1,702,500. Thus, the capitalization and the number of shares to buy are doubled from the previous example.

Stock	Capitalization in Mini-Index (Step 4)	Price	Shares to Buy
IBN	$ 805,282	130	6,194
XON	308,152	35	8,804
CE	325,178	70	4,646
GN	263,888	85	3,104
Total:	$1,702,500		

Later, as the stocks change in value, one's computerized profit and loss system could readily compute the total capitalization of the mini-index and, by moving the decimal point over four places, have a "mini-index value" that could be compared against the actual index (UVX in this case) in order to determine tracking error.

Example: Suppose that the stocks in the mini-index were to increase to have a total value of $1,761,872 as shown in the following table.

Stock	Shares Owned	Price	Current Capitalization
IBN	6,194	135	$836,190
XON	8,804	37	325,748
CE	4,646	69	320,574
GN	3,104	90	279,360
Total:			$1,761,872

The mini-index value is now 176.1872 (moving the decimal point over four places), or 176.19. This means that our mini-index increased from a value of 170.25 to 176.19, an increase of 5.94. This could be compared to the UVX movement during the same period of time. For example, if the UVX had increased by 6.50 points over the time period, then it is easy to see that the mini-index underperformed the UVX by 56 cents. If, at some other time, our mini-index had increased faster than the UVX, then we would have tracking error in our favor.

USING OPTIONS INSTEAD OF FUTURES

As pointed out earlier, one could use options instead of futures when hedging these indices. Assuming one is creating a fully hedged situation, he would have positions similar to conversions when he uses options to hedge a long stock market basket position. He would both sell calls and buy puts with the same striking price in order to create the hedge. This is similar to a conversion arbitrage.

When attempting to hedge the S&P 500, one could use the S&P 500 futures options or the S&P 500 cash options, but that would not necessarily present a more attractive situation than using the futures. On the other hand, there is not a liquid S&P 100 (OEX) futures contract, so that when hedging that contract, one generally uses the OEX options. As mentioned earlier, inter-index option spreads between various indices, including the S&P 100 and 500, will be discussed in the next chapter.

There is not normally much difference as to which of the two is better at any one time. However, since a full option hedge requires two executions (both selling the call and buying the put), the futures probably have a slight advantage in that they involve only a single execution.

In order to substitute options for futures in any of the examples in these chapters on indices, one merely has to use the appropriate number of options as compared to the futures. If one were going to sell OEX calls instead of S&P 500 futures, he would multiply the futures quantity by 5. Five is the multiple because S&P 500 futures are worth $250 per point while OEX options are worth $100 per point, and because the S&P 500 Index (SPX) trades at twice the price of OEX (OEX split 2-for-1 in November 1997). Thus, if an example calls for the sale of 20 S&P 500 futures, then an equivalent hedge with OEX options would require 100 short calls and 100 long puts.

One could attempt to create less fully hedged positions by using the options instead of the futures. For example, he might buy stocks and just write in-the-money calls instead of selling futures. This would create a covered call write. He would still use the same techniques to decide how much of each stock to buy, but he would have downside risk if he decided not to buy the puts. Such a position would be most attractive when the calls are very overpriced.

Similarly, one might try to buy the stocks and buy slightly in-the-money puts without selling the calls. This position is a synthetic long call; it would have upside profit potential and would lose if the index fell, but would have limited risk. Such a position might be established when puts are cheap and calls are expensive.

TRADING THE TRACKING ERROR

Another reason that one might sell futures against a portfolio of stocks is to actually attempt to capture the tracking error. If one were bullish on oil drilling stocks, for example, and expected them to outperform the general market, he might buy several drillers and sell S&P 500 futures against them. The sale of the futures essentially removes "the market" from the package of drilling stocks. What would be left is a position that will reflect how well the drillers perform against the general stock market. If they outperform, the investor will make money. In this section, we are going to look at ways of implementing these hedging strategies. This investor is not particularly concerned with predicting whether the market will go up or down; all he wants to do is remove the "market" from his set of stocks. Then he hopes to profit if these stocks do, indeed, outperform the broad market. Again, we will not use regression analysis, but instead will concentrate on methods that are more simply implemented.

Often, investors or portfolio managers think in terms of industry groups. That is, one may think that the oil drilling stocks will outperform the market, or that the auto stocks will underperform, for example. In either case, one sells futures to attempt to remove the market action and capitalize on the performance differential. In some sense, *one is creating a hedge in which he hopes to profit from tracking error.* In previous discussions, tracking error has not been considered a particularly desirable thing. In this situation, however, one is going to attempt to profit by predicting the direction of the tracking error and trading it.

The technique for establishing this hedge is exactly the same as in the examples at the beginning of this chapter, when we looked at hedging a specific stock portfolio. The exception is that now one must decide which stocks to buy. Once that is decided, he can use the four steps outlined previously to decide how many futures to sell against them:

Step 1: Compute each stock's adjusted volatility by dividing its volatility by that of the market. Use Beta if the group's movement does not correlate well with the general market.
Step 2: Multiply by the quantity and price of each stock to get an adjusted capitalization.
Step 3: Add these to get the total capitalization for the portfolio.
Step 4: Determine how many futures to sell by dividing the index price into the total adjusted capitalization.

Example: Suppose that an investor feels that oil drilling stocks will outperform the market. He decides to invest $500,000 to buy equal dollar amounts of five drilling stocks. Normally one would buy an equal dollar amount of each stock in this situation. One would choose five representative stocks. In these examples, the stock symbols will be OSA, OSB, OSC, OSD, and OSE. The first table shows the price of each stock and how many shares of each will be purchased; $100,000 is invested in each stock.

Stock	Price	Quantity Purchased
OSA	20	5,000
OSB	50	2,000
OSC	25	4,000
OSD	10	10,000
OSE	40	2,500

Now, if one obtains the volatilities of these stocks, he can perform the necessary computations. These computations will tell him how many futures to sell against the portfolio of drilling stocks. The volatilities and computations are given in the following table, assuming the market volatility is 15%. First, dividing the stock's volatility by the market's volatility gives the adjusted volatility (step 1). That result multiplied by the price and quantity of the stock gives the adjusted capitalization (step 2), and adding these together gives the total adjusted capitalization (step 3).

Stock	Volatility	Adjusted Volatility (Step 1)	Price	Quantity Owned	Adjusted Capitalization (Step 2)
OSA	.46	3.07	20	5,000	$ 306,667
OSB	.30	2.00	50	2,000	200,000
OSC	.21	1.67	25	4,000	166,667
OSD	.50	3.33	10	10,000	333,333
OSE	.35	2.33	40	5,000	233,333
Total adjusted capitalization:					$1,240,000 (step 3)

Assume that one wants to hedge with a fictional index, ZYX, and that ZYX futures are worth $500 per point. If the ZYX Index is selling at 175, then one would sell 14 futures against this portfolio of drilling stocks: $1,240,000 ÷ $500 per point ÷ 175 is approximately equal to 14 (step 4).

In a situation such as this, one does not have to be bullish or bearish on the market in order to establish the hedge. He is rather attempting to time the performance of the group in question. Similarly, the decision as to when to remove the position is not a matter of market opinion. Perhaps one has an unrealized profit and decides to take it, or perhaps something changes fundamentally within the group that leads the investor to believe that the group no longer has the potential to outperform the market.

If the futures are underpriced when one begins to investigate this strategy, he should not establish the position. What is gained in tracking error could be lost in theoretical value of the futures. Since one is establishing both sides of the hedge (stocks and futures) at essentially the same time, he can afford to wait until the futures are attractively priced. This is not to say that the futures must be overpriced when the position is established, although that fact would be an enhancement to the position.

If one thinks that a particular group will underperform the market, he merely needs to decide how many shares of each stock to sell short and then can determine how many futures to buy against the short sales in order to try to capture the tracking error. If one decides to capture the negative tracking error in this manner, he must be careful not to buy overpriced futures. Rather, he should wait for the futures to be near fair value in order to establish the position.

COLLATERAL REQUIREMENTS

In any of the portfolio hedging strategies that we have discussed in this section, there is no reduction in margin requirements for either the futures or the options. That is, the stocks must be paid for in full or margined as if they had no protection against them, and the hedging security—the futures or options—must be margined fully as well. Long puts would have to be paid for in full, short futures would require their normal margin and would be marked to the market via variation margin, and short calls would have to be margined as naked and would also be marked to the market. A trader who has not margined his stocks could use them as collateral for the naked call requirements if he so desired.

SUMMARY

There have been two major impacts of index futures and options. One is that they allow a trader to "buy the market" without having to select individual stocks. This is important because many traders have some idea of the direction in which the market is heading, but may not be able to pick individual stocks well. The other, perhaps more major, impact is that large holders of stocks can now hedge their portfolios without nearly as much difficulty.

The use of these futures and options against actual stock indices—real or simulated—has introduced a strategy into the marketplace that did not previously exist. The versatility of these derivative securities is evidenced by the various strategies that were described in this chapter—hedging an actual index or a simulated one, trading the tracking error, selling the futures instead of the entire portfolio when one turns bearish, or buying the futures when they are cheap instead of buying stocks. The owner of a stock portfolio, whether an individual or a large institution, should understand these strategies because they are often preferable to merely buying or selling stock.

CHAPTER 31

Index Spreading

In this chapter, we will look at strategies oriented toward spreading one index against another. This may be done with either futures or options. In some cases, this is almost an arbitrage because the indices track each other quite well. In others, it is a high-risk venture because the indices bear little relationship to each other. In any case, if the futures relationship between the two indices is out of line, one may have an extra advantage.

INTER-INDEX SPREADING

There are general relationships between many stock indices, both in the United States and worldwide. The idea behind inter-index spreading is often to capitalize on one's view of the relationships between the two indices without having to actually predict the direction of the stock market. Note that this is often the philosophy behind many option spreads as well.

Sometimes an analyst will say that he expects small-cap stocks to outperform large-cap stocks. This analyst should consider using an inter-index spread between the S&P 500 Index and the Value Line Index (which contains many small stocks), or perhaps between the S&P and a NASDAQ-based index. If he buys the index that is comprised of smaller stocks and sells the S&P 500 Index, he will make money if his analysis is right, regardless of whether the stock market goes up or down. All he wants is for the index he is long to outperform the index he is short.

Occasionally, the futures or options on these indices are mispriced in comparison to the way the indices are priced. When this happens, one may be able to capitalize on the pricing discrepancy. At times, the spread between the index products on two indices can trade at significantly different price levels from the spread between the two indices themselves. When this happens, an inter-index spread becomes feasible.

The margin requirements for these spreads are often reduced because margin rules recognize that futures on one index can be hedged by futures on another index.

The general rule of thumb as far as selecting a futures spread to establish between two indices is to compare the price difference in the respective futures to the actual price difference in the indices themselves. If the difference in the futures is substantially different from the difference in the cash prices of the indices, then one would sell the more expensive future and buy the cheaper one. Several specific spreads are discussed in this chapter.

Regardless of whether one is entering into the spread because he is trying to predict the relationships between the cash indices, or because he knows the two respective futures are out of line, he must decide in what ratio he wants to establish the spread. There are two lines of thinking on this subject. The first is to merely buy one future and sell one future (on two different indices, of course). Many chart books and spread history charts are graphed in this manner—they compare one index to another index on a one-for-one basis.

Example: A spreader wants to buy the ZYX Index futures and sell ABX futures against them. They are both trading in units of $500 per point, but ZYX is currently at 175.00 while ABX is at 130.00. Thus, the current differential is 45.00 points. This spreader would want the spread to widen to something larger in order to make money. The following profit table shows how he could make a $2,500 profit if the spread widens to 50.00 points, no matter which way the market goes.

Market Direction	ZYX Price	ZYX Profit	ABX Price	ABX Profit	Total Profit
up	185.00	+$5,000	135.00	–$2,500	+$2,500
neutral	177.00	+1,000	127.00	+1,500	+2,500
down	160.00	–7,500	110.00	+10,000	+2,500

Notice that in each case, the difference in the prices of the indices ZYX and ABX is 50.00 points. The profit is the same regardless of whether the general stock market rose, was relatively unchanged, or fell.

The $2,500 profit is the five points of profit that the spreader makes by buying the spread of 45.00 and selling it at 50.00 (5.00 points × $500 per point = $2,500).

The second approach to index spreading is to use a ratio of the two indices. This approach is often taken when the two indices trade at substantially different prices. For example, if one index sells for twice the price of the other, and if both indices have similar volatilities, then a one-to-one spread gives too much weight to the higher-priced index. A two-to-one ratio would be better, for that would give equal weighting to the spread between the indices.

Example: UVX is an index of stock prices that is currently priced at 100.00. ZYX, another index, is priced at 200.00. The two indices have some similarities and, therefore, a spreader might want to trade one against the other. They also display similar volatilities.

If one were to buy one UVX future and sell one ZYX future, his spread would be too heavily oriented to ZYX price movement. The following table displays that, showing that if both indices have similar percentage movements, the profit of the one-by-one spread is dominated by the profit or loss in the ZYX future. Assume both futures are worth $500 per point.

Market Direction	ZYX Price	ZYX Profit	UVX Price	UVX Profit	Total Profit
up 20%	240	–$20,000	120	+$10,000	–$10,000
up 10%	220	– 10,000	110	+ 5,000	– 5,000
down 10%	180	+ 10,000	90	– 5,000	+ 5,000
down 20%	160	+ 20,000	80	– 10,000	+ 10,000

This is not much of a hedge. If one wanted a position that reflected the movement of the ZYX index, he could merely trade the ZYX futures and not bother with a spread.

If, however, one had used the ratio of the indices to decide how many futures to buy and sell, he would have a more neutral position. In this example, he would buy two UVX futures and sell one ZYX future.

Proponents of using the ratio of indices are attempting strictly to capture any performance difference between the two indices. They are not trying to predict the overall direction of the stock market.

Technically, the proper ratio should also include the volatility of the two indices, because that is also a factor in determining how fast they move in relationship to each other.

$$\text{Ratio} = \frac{v_1}{v_2} \times \frac{p_1}{p_2} \times \frac{u_1}{u_2}$$

where

p_1 and p_2 are the prices of the indices
v_1 and v_2 are the respective volatilities
and u_1 and u_2 are the units of trading ($500 per point, for example).

Including the volatility ensures that one is spreading essentially equal "volatility dollars" of each index. Moreover, if the two futures don't have the same unit of trading, that should be factored in as well.

Example: The ZYX Index is not very volatile, having a volatility of 15%. A trader is interested in spreading it against the ABX Index, which is volatile, having a historical volatility of 25%. The following data sum up the situation:

	Price	Volatility	Unit of Trading
ZYX Futures	175.00	15%	$250/pt
ABX Futures	225.00	25%	$500/pt

$$\begin{aligned}\text{Ratio} &= \frac{.25}{.15} \times \frac{225.00}{175.00} \times \frac{500}{250} \\ &= 4.286\end{aligned}$$

In round numbers, one would probably trade four ZYX futures against one ABX future.

USING OPTIONS IN INDEX SPREADS

In general, it is easier to spread the indices by using futures rather than options, if futures exist. However, there are still many applications of options to inter-index spreading. One might want to create spreads of futures or ETFs that represent the various markets around the world: SPX, SPY, NASDAQ-100 (NDX, QQQ), Russell 2000 (IWM), China (FXI), Emerging Markets (EMM), and so forth.

Whenever both indices have options, as most do, the strategist may find that he can use the options to his advantage. This does not mean merely that he can use a synthetic option position as a substitute for the futures position (long call, short put at the same strike instead of long futures, for example). There are at least two other alternatives with options. First, he could use an in-the-money option as a substitute for the future. Second, he could use the options' delta to construct a more leveraged spread. These alternatives are best used when one is interested in trading the spread between the cash indices—they are not really amenable to the short-term strategy of spreading the premiums between the futures.

Using in-the-money options as a substitute for futures gives one an additional advantage: If the cash indices move far enough in either direction, the spreader could still make money, even if he was wrong in his prediction of the relationship of the cash indices.

Example: The following prices exist:

ZYX: 175.00
UVX: 150.00

ZYX Dec 185 put: 10½
UVX Dec 140 call: 11

Suppose that one wants to buy the UVX index and sell the ZYX index. He expects the spread between the two—currently at 25 points—to narrow. He could buy the UVX futures and sell the ZYX futures. However, suppose that instead he buys the ZYX *put* and buys the UVX *call*.

The time value of the Dec 185 put is ½ point and that of the Dec 140 call is 1 point. This is a relatively small amount of time value premium. Therefore, the combination would have results very nearly the same as the futures spread, as long as both options remain in-the-money; the only difference would be that the futures spread would outperform by the amount of the time premium paid.

Even though he pays some time value premium for this long option combination, the investor has the opportunity to make larger profits than he would with the futures spread. In fact, he could even make a profit if the cash spread *widens*, if the indices are volatile. To see this, suppose that after a large upward move by the overall market, the following prices exist:

ZYX: 200.00
UVX: 170.00

ZYX Dec 185 put: 0 (virtually worthless)
UVX Dec 140 call: 30

The combination that was originally purchased for 21½ points is now worth 30, so the spread has made money. But observe what has happened to the cash spread: It has widened to 30 points, from the original price of 25. This is a movement in the opposite direction from what was desired, yet the option position still made money.

The reason that the option combination in the example was able to make money, even though the cash spread moved unfavorably, is because both indices rose so much in price. The puts that were owned eventually became worthless, but the long call continued to make money as the market rose. This is a situation that is very similar to owning a long strangle (long put and call with different strikes), except that the put and call are based on different underlying indices. This concept is discussed in more detail in Chapter 35 on futures spreads.

The second way to use options in index spreading is to use options that are less deeply in-the-money. In such a case, one must use the deltas of the options in order to accurately compute the proper hedge. He would calculate the number of options to buy and sell by using the formula given previously for the ratio of the indices, which incorporates both price and volatility, and then multiplying by a factor to include delta.

$$\text{Option Ratio} = \frac{v_1}{v_2} \times \frac{p_1}{p_2} \times \frac{u_1}{u_2} \times \frac{d_1}{d_2}$$

where

v_i is the volatility of index_i
p_i is the price of index_i
u_i is the unit of trading
and d_i is the delta of the selected option on index_i.

Example: The following data is known:

ZYX: 175.00, volatility = 20%
UVX: 150.00, volatility = 15%

ZYX Dec 175 put: 7, delta = −.45, worth $500/pt.
UVX Dec 150 call: 5, delta = .52, worth $100/pt

Suppose one decides that he wants to set up a position that will profit if the spread between the two cash indices shrinks. Rather than use the deeply in-the-money options, he now decides to use the at-the-money options. He would use the option ratio formula to determine how many puts and calls to buy. (Ignore the put's negative delta for the purposes of this formula.)

$$\text{Option Ratio} = \frac{.20}{.15} \times \frac{175.00}{150.00} \times \frac{500}{100} \times \frac{.45}{.52} = 6.731$$

He would buy nearly 7 UVX calls for every ZYX put purchased.

In the previous example, using in-the-money options, one had a very small expense for time value premium and could profit if the indices were volatile, even if the cash spread did not shrink. This position has a great deal of time value premium expense, but could make profits on smaller moves by the indices. Of course, either one could profit if the cash indices moved favorably.

Volatility Differential. A theoretical "edge" that sometimes appears is that of volatility differential. If two indices are supposed to have essentially the same volatility, or at least a relationship in their volatilities, then one might be able to establish an option spread if that relationship gets out of line. In such a case, the options might actually show up as fair-valued on both indices, so that the disparity is in the volatility differential, and not in the pricing of the options.

OEX and SPX options trade with essentially the same implied volatility. Thus, if one index's options are trading with a higher implied volatility than the other's, a potential spread might exist. Normally, one would want the differential in implied volatilities to be at least 2% apart before establishing the spread for volatility reasons.

In any case, whether establishing the spread because one thinks the cash index relationship is going to change, or because the options on one index are expensive with respect to the options on the other index, or because of the disparity in volatilities, the spreader must use the deltas of the options and the price ratio and volatilities of the indices in setting up the spread.

Striking Price Differential. The index relationships can also be used by the option trader in another way. When an option spread is being established with options whose strikes are not near the current index prices—that is, they are relatively deeply in- or out-of-the-money—one can use the ratio between the indices to determine which strikes are equivalent.

Example: ZYX is trading at 250 and the ZYX July 270 call is overpriced. An option strategist might want to sell that call and hedge it with a call on another index. Suppose he notices that calls on the UVX Index are trading at approximately fair value with the UVX Index at 175. What UVX strike should he buy to be equivalent to the ZYX 270 strike?

One can multiply the ZYX strike, 270, by the ratio of the indices to arrive at the UVX strike to use:

$$\text{UVX strike} = 270 \times (175/250$$
$$= 189.00$$

So he would buy the UVX July 190 calls to hedge. The exact number of calls to buy would be determined by the formula given previously for option ratio.

SUMMARY

This concludes the discussion of index spreading. The above examples are intended to be an overview of the most usable strategies in the complex universe of index spreading. The multitude of strategies involving index-index and intra-index spreads cannot all be fully described. In fact, one's imagination can be put to good use in designing and implementing new strategies as market conditions change and as the emotion in the marketplace drives the premium on the futures contracts.

Often one can discern a usable strategy by observation. Watch how two popular indices trade with respect to each other and observe how the options on the two indices are related. If, at a later time, one notices that the relationship is changing, perhaps a spread between the indices is warranted. One could use the NASDAQ-based indices, such as the NASDAQ-100 (NDX) or smaller indices based on it (QQQ). ETFs and sector indices can be used as well. The key point to remember is that the index option and futures world is more diverse than that of stock options. Stock option strategies, once learned or observed, apply equally well to all stocks. Such is not the case with index spreading strategies. The diversification means that there are more profit opportunities that are recognized by fewer people than is the case with stock options. The reader is thus challenged to build upon the concepts described in this part of the book.

CHAPTER 32

Structured Products

The popularity of derivative instruments and the kinds of risk-reducing, volatility-reducing effects that they can have on portfolios led to a new type of product in the 1990s. This new product, termed a *structured product*, has more appeal for investors than for traders. In essence, enterprising designers at the major institutional brokerage firms have constructed a single security that behaves like a portfolio hedged by options. These designers *structure* the combination of derivatives and stocks so that the resulting *product* behaves in a manner that is attractive to many investors, whether institutional or private. In this chapter, these structured products are examined in detail, to give the reader the background so that in the future, he may analyze similar products for himself.

Would you like to own an index fund that had no risk? Or, how about owning a popular stock and getting a dividend payment that is much, much larger than the stock itself pays? I think everyone would like to do those things. With structured products, one *can* own similar investments, but they come with a cost. The two questions asked previously might then be better restated as follows: Would you like to own an index fund that had no risk, but that perhaps did not fully participate in *all* of the upside movement of the market? It still has downside protection, and unlimited profit potential on the upside. This is akin to owning the stock or the index and having protected it by buying a put option. Or, would you like to own that popular stock and receive that huge dividend, but know that your profit potential is limited to a fixed amount on the upside? This is akin to a covered call write.

These two questions describe the majority of the listed structured products in existence today. They are attractive investments in their own right, but one must carefully assess the products before buying them.

The discussion in this chapter concentrates on the structured products that are *listed* and traded on the major stock exchanges. A broader array of products—typically called exotic options—is traded over-the-counter. These can be very complicated, especially with respect to currency and bond options. It is not our intent to discuss exotic options, although the approaches to valuing the structured products that are presented in this chapter can easily be applied to the overall valuation of many types of exotic products. Also, the comments at the end of the chapter regarding where to find information about these products may prove useful for those seeking further information about either listed structured products or exotic options.

"RISKLESS" OWNERSHIP OF A STOCK OR INDEX

At many of the major institutional banks and brokerages, people are employed who design structured products. They are often called financial engineers because they take existing financial products and build something new with them. The result is packaged as a fund of sorts (or a unit trust, perhaps), and shares are sold to the public. Not only that, but the shares are then listed on the American or New York Stock Exchanges and can be traded just like any other stock. These attributes make the structured product a very desirable investment. An example will show how a generic index structured product might look.

Example: Let's look at the structured index product to see how it might be designed and then how it might be sold to the public. Suppose that the designers believe there is demand for an index product that has these characteristics:

1. This "index product" will be issued at a low price—say, $10 per share.
2. The product will have a maturity date—say, seven years hence.
3. The owner of these shares can redeem them at their maturity date for the *greater of:* either a) $10 per share or b) the percentage appreciation of the S&P 500 index over that a seven-year time period. That is, if the S&P doubles over the seven years, then the shares can be redeemed for double their issue price, or $20.

Thus, this product has no price risk! The holder gets his $10 back in the worst case (except for credit risk, which will be addressed in a minute).

Moreover, these shares will trade in the open market during the seven years, so that if the holder wants to exit at any time, he can do so. Perhaps the S&P has rallied dramatically, or perhaps he needs cash for something else—both might be reasons that the holder of the shares would want to sell before maturity.

Such a product has appeal to many investors. In fact, if one thought that the stock market was a "long-term" buy, this would be a much safer way to approach it than buying a portfolio of stocks that might conceivably be much lower in value seven years hence. *The risk of the structured product is that the underwriter might not be able to pay the $10 obligation at maturity.* That is, if the major institutional bank or brokerage firm who underwrote these products were to go out of business over the course of the next seven years, one might not be able to redeem them. In essence, then, structured products are really forms of debt (senior debt) of the brokerage firm that underwrote them. Fortunately, most structured products are underwritten by the largest and best-capitalized institutions, so the chances of a failure to pay at maturity would have to be considered relatively tiny.

How does the bank create these items? It might seem that the bank buys stock and buys a put and sells units on the combined package. In reality, the product is *not* normally structured that way. Actually, it is not a difficult concept to grasp. This example shows how the structure looks from the viewpoint of the bank.

Example: Suppose that the bank wants to raise a pool of $1,000,000 from investors to create a structured product based on the appreciation of the S&P 500 index over the next seven years. The bank will use a part of that pool of money to buy U.S. zero-coupon bonds and will use the rest to buy call options on the S&P 500 index.

Suppose that the U.S. government zero-coupon bonds are trading at 60 cents on the dollar. Such bonds would mature in seven years and pay the holder $1.00. Thus, the bank could take $600,000 and buy these bonds, knowing that in seven years, they would mature at a value of $1,000,000. The other $400,000 is spent to buy call options on the S&P 500 index. Thus, the investors would be made whole at the end of seven years even if the options that were bought expired worthless. This is why the bank can "guarantee" that investors will get their initial money back.

Meanwhile, if the stock market advances, the $400,000 worth of call options will gain value and that money will be returned to the holders of the structured product as well.

In reality, the investment bank uses its own money ($1,000,000) to buy the securities necessary to structure this product. Then they make the product into a legal entity (often a unit trust) and sell the shares (units) to the public, marking them up slightly as they would do with any new stock brought to market.

At the time of the initial offering, the calls are bought at-the-money, meaning the striking price of the calls is equal to the closing price of the S&P 500 index on the day the products were sold to the public. Thus, the structured product itself has a "strike price" equal to that of the calls. It is this price that is used at maturity to determine whether the S&P has appreciated over the seven-year period—an event that would result in the holders receiving back more than just their initial purchase price.

After the initial offering, the shares are then listed on the AMEX or the NYSE and they will begin to rise and fall as the value of the S&P 500 index fluctuates.

* * *

So, the structured product is not an index fund protected by a put option, but rather it is a combination of zero-coupon government bonds and a call option on an index. These two structures are equivalent, just as the combination of owning stock protected by a put option is equivalent to being long a call option.

Structured products of this type are not limited to indices. One could do the same thing with an individual stock, or perhaps a group of stocks, or even create a simulated bull spread. There are many possibilities, and the major ones will be discussed in the following sections. In theory, one could construct products like this for himself, but the mechanics would be too difficult. For example, where is one going to buy a seven-year option in small quantity? Thus, it is often worthwhile to avail oneself of the product that is packaged (structured) by the investment banker.

In actuality, many of the brokerage firms and investment banks that underwrite these products give them names—usually acronyms, such as MITTS, TARGETS, BRIDGES, LINKS, DINKS, ELKS, and so on. If one looks at the listing, he may see that they are called *notes* rather than *stocks* or *index funds*. Nevertheless, when the terms are described, they will often match the examples given in this chapter.

INCOME TAX CONSEQUENCES

There is one point that should be made now: There is "phantom interest" on a structured product. Phantom interest is what one owes the government when a bond is bought at a discount to maturity. The IRS technically calls the initial purchase price an Original Issue Discount (OID) and requires you to pay taxes annually on a proportionate amount of that OID. For example, if one buys a zero-coupon U.S. government bond at 60 cents on the dollar, and later lets it mature for $1.00, the IRS does not treat the 40-cent profit as capital gains. Rather, the 40 cents is interest income. Moreover, says the IRS, you are collecting that income each year, since you bought the bonds at a discount. (In reality, of course, you aren't collecting a thing; your investment is simply worth a little more each year because the discount decreases as the bonds approach maturity.) However, *you must pay income tax on the "phantom interest" you supposedly received each year.* Those are the rules, and there isn't anything you can do about them.

Since some structured products involve the purchase of zero-coupon bonds, the IRS has ruled that *owners of this type of structured product must pay phantom interest each year.* Thus, structured products should be bought in a tax-free retirement account (IRA, SEP, etc.) if at all possible, in order to avoid having to declare phantom interest on your tax return for each year you hold the product. The phantom interest on your tax applies only to this type of structured product—one on which you are guaranteed to get back a fixed amount at maturity—because this is the only type that requires buying a zero-coupon bond in order to ensure that you'll get your money back if the stock market goes down. The phantom interest concept does *not* apply to the type of structured product to be discussed in the second part of this chapter. To be certain, one should get the necessary information from his broker or should read the prospectus of the structured product. Of course, any tax strategies should also be discussed with a qualified tax professional.

CASH VALUE

The cash value of the structured product is what it will be worth at maturity. It is usually stated in terms similar to those in the preceding example, and a formula is often given. This example will clarify the typical nature of this formula:

Example: A structured product is issued at $10 per share. The terms stipulate that the holder will receive back, at maturity, either $10 or 100% of the appreciation of the S&P 500 index above a value of 1,245.27. (One would assume that the S&P 500 cash index closed at 1,245.27 on the day the structured product was issued.) The prospectus will usually provide a formula for the cash surrender value, and it will be stated something like this:

> At maturity, the cash value will be equal to the *greater of*:
> (a) $10
> or (b) $\$10 + 10 \times (\text{Final Index Value} - 1,245.27)/1,245.27$
> where Final Index Value is, say, the closing value of the S&P 500 index on the maturity date.

The formula given is merely the arithmetic equivalent of the statement that one will receive 100% of the appreciation of the S&P 500 Index above the strike price of 1,245.27. For those more adept at math, the formula can be reduced to common terms, in which case it reads:

$$\text{Cash Surrender Value} = \$10 \times \text{Final Value}/1,245.27$$

This shortened version of the formula only works, though, when the participation rate is 100% of the increase in the Final Index Value above the striking price. Otherwise, the longer formula should be used.

Not all structured products of this type offer the holder 100% of the appreciation of the index over the initial striking price. In some cases, the percentage is smaller (although in the early days of issuance, some products offered a percentage appreciation that was actually *greater* than 100%). After 1996, options in general became more expensive as the volatility of the stock market increased tremendously. Thus, structured products issued after 1997 or 1998 tend to include an "annual adjustment factor." Adjustment factors are discussed later in the chapter.

Therefore, a more general formula for Cash Surrender Value—one that applies when the participation rate is a fixed percentage of the striking price—is:

Cash Surrender Value =
Guarantee + Guarantee × Participation Rate × (Final Index Value/Striking Price − 1)

THE COST OF THE IMBEDDED CALL OPTION

Few structured products pay dividends.* Thus, the "cost" of owning one of these products is the interest lost by not having your money in the bank (or money market fund), but rather having it tied up in holding the structured product.

Continuing with the preceding example, suppose that you had put the $10 in the bank instead of buying a structured product with it. Let's further assume that the money in the bank earns 5% interest, compounded continuously. At the end of seven years, compounded continuously, the $10 would be worth:

$$\begin{aligned}\text{Money in the bank} &= \text{Guarantee Price} \times e^{rt}\\ &= \$10 \times e^{0.05\times 7} - 14.191, \text{ in this case}\end{aligned}$$

This calculation usually raises some eyebrows. Even compounded annually, the amount is 14.07. You would make roughly 40% (without considering taxes) just by having your money in the bank. Forgetting structured products for a moment, this means that stocks in general would have to increase in value by over 40% during the seven-year period just for your performance to beat that of a bank account.

In this sense, the cost of the imbedded call option in the structured product is this lost interest—4.19 or so. That seems like a fairly expensive option, but if you consider that it's a seven-year option, it doesn't seem quite so expensive. In fact, one could calculate the implied volatility of such a call and compare it to the current options on the index in question.

* Some do pay dividends, though. A structured product existed on a contrived index, called the Dow-Jones Top 10 Yield index (symbol: XMT). This is a sort of "dogs of the Dow" index. Since part of the reason for owning a "dogs of the Dow" product is that dividends are part of the performance, the creators of the structured product (Merrill Lynch) stated that the minimum price one would receive at maturity would be 12.40, not the 10 that was the initial offering price. Thus, this particular structured product had a "dividend" built into it in the form of an elevated minimum price at maturity.

In this case, with the stock at 10, the strike at 10, no dividends, a 5% interest rate, and seven years until expiration, the implied volatility of a call that costs \$4.19 is 28.1%. Call options on the S&P 500 index are rarely that expensive. So you can see that you are paying "something" for this call option, even if it *is* in the form of lost interest rather than an up-front cost.

As an aside, it is also unlikely that the underwriter of the structured product actually paid that high an implied volatility for the call that was purchased; but he is asking *you* to pay that amount. This is where his underwriting profit comes from.

The above example assumed that the holder of the structured product is participating in 100% of the upside gain of the underlying index over its striking price. If that is not the case, then an adjustment has to be made when computing the price of the imbedded option. In fact, one must compute what value of the index, at maturity, would result in the cash value being equal to the "money in the bank" calculation above. Then calculate the imbedded call price, using *that* value of the index. In that way, the true value of the imbedded call can be found.

You might ask, "Why not just divide the 'money in the bank' formula by the participation rate?" That would be okay if the participation were always stated as a percentage of the striking price, but sometimes it is not, as we will see when we look at the more complicated examples. Further examples of structured products in this chapter demonstrate this method of computing the cost of the imbedded call.

PRICE BEHAVIOR PRIOR TO MATURITY

The structured product cannot normally be "exercised" by the holder until it matures. That is, the cash surrender value is only applicable at maturity. At any other time during the life of the product, one can *compute* the cash surrender value, but he cannot actually attain it. What you *can* attain, prior to maturity, is the *market price*, since structured products trade freely on the exchange where they are listed. In actual fact, the products generally trade at a slight discount to their theoretical cash surrender value. This is akin to a closed-end mutual fund selling at a discount to net asset value. Eventually, upon maturity, the actual price will *be* the cash surrender value price; so if you bought the product at a discount, you would benefit, providing you held all the way to maturity.

Example: Assume that two years ago, a structured product was issued with an initial offering price of \$10 and a strike price of 1,245.27, based upon the S&P 500 index. Since issuance, the S&P 500 index has risen to 1,522.00. That is an increase of 22.22% for the S&P 500, so the structured product has a theoretical cash surrender value of 12.22. I say "theoretical" because that value cannot actually be realized, since the structured product is not exercisable at the current time—five years prior to maturity.

In the real marketplace, this particular structured product might be trading at a price of 11.75 or so. That is, it is trading at a *discount* to its theoretical cash surrender value. This is a fairly common occurrence, both for structured products and for closed-end mutual funds. If the discount were large enough, it should serve to attract buyers, for if they were to hold to maturity, they would make an extra 47 cents (the amount of this discount) from their purchase. That's 4% (0.47 divided by 11.75 = 4%) over five years, which is nothing great, but it's something.

Why does the product trade at a discount? Because of supply and demand. It is free to trade at any price—premium or discount—because there is nothing to keep it fixed at the theoretical cash surrender value. If there is excess demand or supply in the open market, then the price of the structured product will fluctuate to reflect that excess. Eventually, of course, the discount will disappear, but five years prior to maturity, one will often find that the product differs from its theoretical value by somewhat significant amounts. If the discount is large enough, it will attract buyers; alternatively, if there should be a large premium, that should attract sellers.

SIS

One of the first structured products of this type that came to my attention was one that traded on the AMEX, entitled "Stock Index return Security" or SIS. It also traded under the symbol SIS. The product was issued in 1993 and matured in 2000, so we have a complete history of its movements. The terms were as follows: The underlying index was the S&P Midcap 400 index (symbol: MID). Issued in June 1993, the original issue price was \$10, and MID was trading at 166.10 on the day of issuance, so that was the striking price. Moreover, buyers were entitled to 115% of the appreciation of MID above the strike price. Thus, the cash value formula was:

$$\text{Cash Value of SIS} = \$10 + \$10 \times 1.15 \times (\text{MID} - 166.10)/166.10$$

where

Guarantee price = \$10
Underlying index: S&P Midcap 400 (MID)
Striking price: 166.10
Participation rate: 115% of the increase of MID above 166.10

SIS matured seven years later, on June 2, 2000. At the time of issuance, seven-year interest rates were about 5.5%, so the "money in the bank" formula shows that one could have made about 4.7 points on a \$10 investment, just by utilizing risk-free government securities:

$$\text{Money in the bank} = 10 \times e^{0.055 \times 7} = 14.70$$

We can't simply say that the cost of the imbedded call was 4.7 points, though, because the participation rate is not 100%—it's greater. So we need to find out the Final Value of MID that results in the cash value being equal to the "money in the bank" result. Using the cash value formula and inserting all the terms except the final value of MID, we have the following equation. Note: MID_{MIB} stands for the value of MID that results in the "money in the bank" cash value, as computed above.

$$14.70 = 10 + 10 \times 1.15 \times (\text{MID}_{\text{MIB}} - 166.10)/166.10$$

Solving for MID_{MIB}, we get a value of 233.98. Now, convert this to a percent gain of the striking price:

$$\text{Imbedded call price} = 233.98/166.10 - 1 = 0.4087$$

Hence, the imbedded call costs 40.87% of the guarantee price. In this example, where the guarantee price was \$10, that means the imbedded call cost \$4.087.

Thus, a more generalized formula for the value of the imbedded call can be construed from this example. This formula only works, though, where the participation rate is a fixed percentage of the strike price.

$$\text{Imbedded call value} = \text{Guarantee price} \times (\text{Final Index Value}_{\text{MIB}} / \text{Striking Price} - 1)$$

Final Index Value$_{MIB}$ is the final index price that results in the cash value being equal to the "money in the bank" calculation, where
Money in the bank = Guarantee Price × e^{rt}
r = risk-free interest rate
t = time to maturity

Thus, the calculated value of the imbedded call was approximately 4.087 points, which is an implied volatility of just over 26%. At the time, listed short-term options on MID were trading with an implied volatility of about 14%, so this was an expensive call in terms of its initial cost.

However, one should remember that owning SIS gave one *more than* full participation in the MID for seven years, with virtually no risk. That has to be worth something.