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1. Which of the following are valid (necessarily true) sentences?
   (a) (∃x x=x) ⇒ (∀y ∃z y=z) **Ture**
   (b) ∀x P(x) ∨ ¬P(x) **True**
   (c) ∀x Smart(x) ∨ (x=x) **True**
2. Arithmetic assertions can be written in first-order logic with the predicate symbol <, the function symbol + and ×, and the constant symbols 0 and 1. Additional predicates can also be defined with biconditionals.
   (a) Represent the property "x is an even number" $\forall x Even(x) \iff \exists y\ (x = y + y)$
   (b) Represent the property "x is prime"
   $$\forall x Prime(x) \iff \forall y \forall z \neg(1 < y < x\ \wedge 1 < z < x\ \wedge y \times z = x)$$
   (c) Goldbach's conjecture is the conjecture (unproven as yet) that every even number is equal to the sum of two primes. Represent this conjecture as a logical sentence.
   $$\forall x Even(x) \implies \exists y\ \exists z (y + z = x)$$
3. Write down logical representations for the following sentences, suitable for use with Generalized Modus Ponens:
   (a) Horses, cows, and pigs are mammals.
   $\forall x\ (\mathrm{hores(x)} \vee \mathrm{cows(x)} \vee \mathrm{pigs(x)}) \implies \mathrm{mammals(x)}$
   (b) An offspring of a horse is a horse.
   $$\forall x, y(\mathrm{horse(y)} \wedge \mathrm{offspring(x,\ y)}) \implies \mathrm{horse(x)}$$
   (c) Bluebeard is a horse
   $horse(Bluebeard)$
   (d)Bluebeard is Charlie's parent
   $$parent(Bluebeard, Charlie)$$
   (e) Offspring and parent are inverse relations
   $$\forall x, y\ (Offspring(x, y) \iff Parent(y, x))$$
   (f) Every mammal has a parent
   let parent(a, b) be that a is b's parent
   $$\forall x \exists y\ mammal(x) \implies parent(y, x)$$
4. Consider how to translate a set of action schemas into the successor-state axioms of situation calculus.
   (a) I Consider the schema for Fly(p,from,to). Write a logical definition for the predicate Poss(Fly(p,from,to),s), which is true if the preconditions for Fly(p,from,to) are satisfied in situations.
   $$\begin{aligned} & Poss(Fly(p, from, to), s) \\ & \iff At(p, from, s) \wedge Plane(p) \wedge Airport(from) \wedge Airport(to) \end{aligned} \tag{1}$$
   (b) Next, assuming that Fly(p,from,to) is the only action schema available to the agent, write down a successor-state axiom for At(p, x, s) that captures the same information as the action schema.
   $$\begin{aligned} & Poss(a, s) \implies \\ & \Big(At(p, to, Result(a, s)) \iff \\ & (\exists from a = Fly(p, from, to)) \vee \Big(At(p, to, s) \wedge \neg \exists new\ new \neq to \wedge a = Fly(p, to, new) \Big) \Big) \end{aligned} \tag{2}$$

5. For each pair of atomic sentences, give the most general unifier if it exists:

(a) P(A, B, B), P(x, y, z)

$\{x/A,\ y/B,\ z/B\}$

(b) Q(y, G(A, B)),Q(G(x, x),y)

$\xcancel{\{y/G(x,x),\ G(A,B)/y,\ x/A,\ x/B)\}}$ 矛盾，不可能

(c) Older(Father(y),y),Older(Father(x),John)

$\{y/John,\ x/\mathrm{John}\}$

(d) Knows(Father(y),y),Knows(x, x).

$\xcancel{\{Father(y)/x,\ y/x, Father(x)/x\}}$ 矛盾，不可能