Artificial Intelligence: Homework #5

Assigned on 5/25/2021 (Tuesday); Due at 12:00 am on 6/8/2021 (Tuesday)

1. Deciding to put probability theory to good use, we encounter a slot machine with three independent wheels, each producing one of the four symbols BAR, BELL, LEMON, or CHERRY with equal probability. The slot machine has the following payout scheme for a bet of 1 coin (where "?" denotes that we don't care what comes up for that wheel):
BAR/BAR/BAR pays 20 coins
BELL/BELL/BELL pays 15 coins
LEMON/LEMON/LEMON pays 5 coins
CHERRY/CHERRY/CHERRY pays 3 coins
CHERRY/CHERRY/? pays 2 coins
CHERRY/?/? pays 1 coin

(a) Compute the expected "payback" percentage of the machine. In other words, for each coin played, what is the expected coin return?

(b) Compute the probability that playing the slot machine once will result in a win.

(c) Estimate the mean and median number of plays you can expect to make until you go broke, if you start with 10 coins. You can run a simulation to estimate this, rather than trying to compute an exact answer.

2. After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (i.e., the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don't have the disease). The good news is that this is a rare disease, striking only 1 in 10,000 people of your age. Why is it good news that the disease is rare? What are the chances that you actually have the disease?

3. Consider the Bayesian network in Figure 14.2.

(a) If no evidence is observed, are *Burglary* and *Earthquake* independent? Prove this from the numerical semantics and from the topological semantics.

(b) If we observe *Alarm* = *true*, are *Burglary* and *Earthquake* independent? Justify your answer by calculating whether the probabilities involved satisfy the definition of conditional independence.

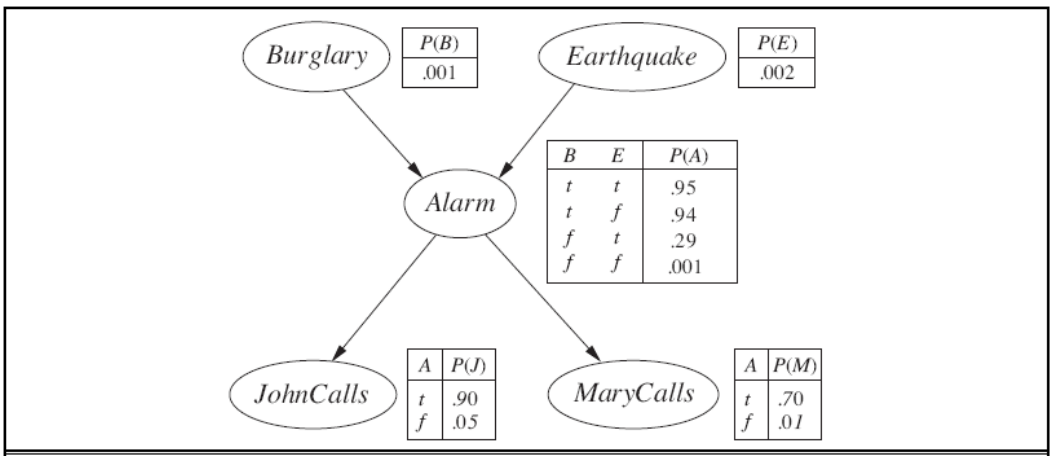

Figure 14.2 A typical Bayesian network, showing both the topology and the conditional probability tables (CPTs). In the CPTs, the letters B, E, A, J, and M stand for *Burglary*, *Earthquake*, *Alarm*, *JohnCalls*, and *MaryCalls*, respectively.

4. Consider the Bayes net shown in Figure 14.23.

(a) Which of the following are asserted by the network structure?

(i) *P(B, I, M) = P(B)P(I)P(M).*

(ii) *P(J | G) = P(J | G, I).*

(iii) *P(M | G, B, I) = P(M | G, B, I, J).*

(b) Calculate the value of P(b, i, ¬m, g, j).

(c) Calculate the probability that someone goes to jail given that they broke the law, have been indicted, and face a politically motivated prosecutor

(d) Suppose we want to add the variable P = PresidentialPardon to the network; draw the new network and briefly explain any links you add.

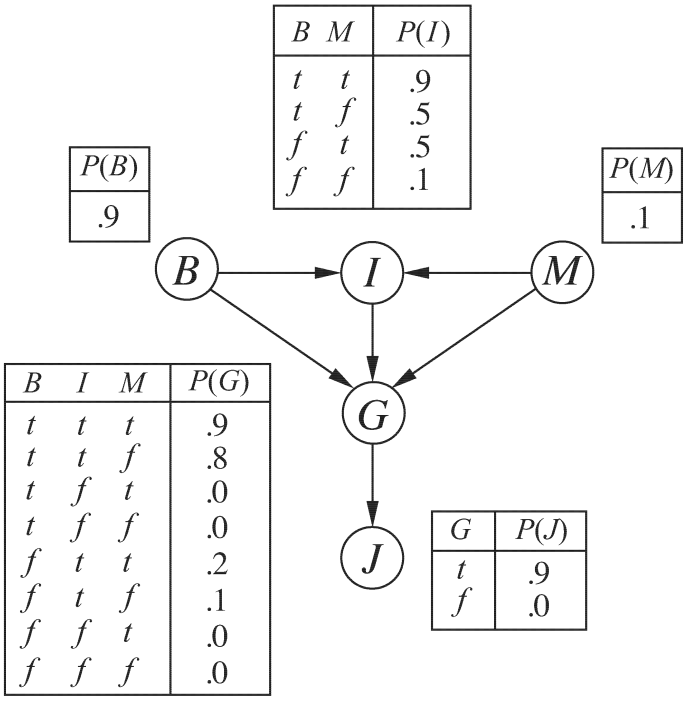

Figure 14.23 A simple Bayes net with Boolean variables $B = BrokeElectionLaw$, $I = Indicted$, $M = PoliticallyMotivatedProsecutor$, $G = FoundGuilty$, $J = Jailed$.

5. The Surprise Candy Company makes candy in two flavors: 70% are strawberry flavor and 30% are anchovy flavor. Each new piece of candy starts out with a round shape; as it moves along the production line, a machine randomly selects a certain percentage to be trimmed into a square; then, each piece is wrapped in a wrapper whose color is chosen randomly to be red or brown. 80% of the strawberry candies are round and 80% have a red wrapper, while 90% of the anchovy candies are square and 90% have a brown wrapper. All candies are sold individually in sealed, identical, black boxes.

Now you, the customer, have just bought a Surprise candy at the store but have not yet opened the box. Consider the three Bayes nets in Figure 16.11.

(a) Which network(s) can correctly represent P(*Flavor, Wrapper, Shape*)?

(b) Which network is the best representation for this problem?

(c) Does network (i) assert that P(*Wrapper*|*Shape*) = P(*Wrapper*)?

(d) What is the probability that your candy has a red wrapper?

(e) In the box is a round candy with a red wrapper. What is the probability that its flavor is strawberry?

(f) An unwrapped strawberry candy is worth s on the open market and an unwrapped anchovy candy is worth a. Write an expression for the value of an unopened candy box.

(g) A new law prohibits trading of unwrapped candies, but it is still legal to trade wrapped candies (out of the box). Is an unopened candy box now worth more than less than, or the same as before?

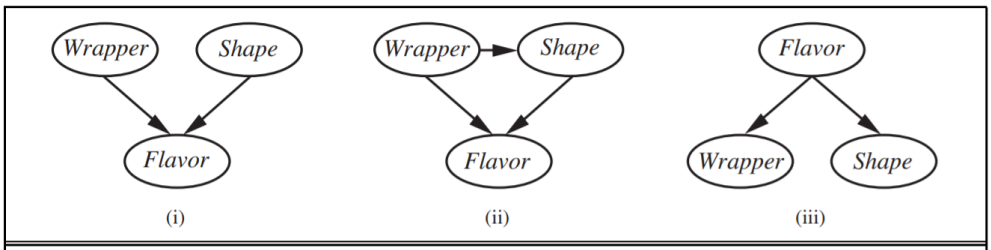

Figure 16.11 Three proposed Bayes nets for the Surprise Candy problem, Exercise 16.5.