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К ТЕОРИИ СМЫСЛОВОГО ПРОСТРАНСТВА

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Статья рассматривает модели для исследования структуры, топологии и метрических признаков смыслового пространства, использующего унифицированное представление знаний.

Рассмотрены классы конечных структур, соответствующие онтологическим структурам и множествам классического и неклассического вида, исследованы свойства перечислимости этих классов.

Предложено понятия операционноинформационного пространствм, как модели для исследования взаимосвязи операционной семантики онтологических структур большого и малого шага.

Рассмотрены количественные признаки и инварианты онтологических структур, ориентированные на решение задач управления знаниями.

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Integration of Fuzzy Systems with Parametric Interpretation for Unified Knowledge Representation

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***Abstract*—The paper considers the problem of stable interpretation of fuzzy logic models. An approach based on parameterized fuzzy logic is proposed, where each logical formula has a set of model parameters in addition to truth values. Parameterized fuzzy logic allows combining different fuzzy logic systems. Model parameters are used to calculate fuzzy truth values as a fuzzy measure. Models and model parameters related to metric spaces, consistent with metric sense spaces and being the basis for interpretation of fuzzy logic formulas on ontological models are considered.**

***Keywords*—Fuzzy logic, Parameterized fuzzy logic, Metric space, Fuzzy measure, Simplicial complex, Residual simplicial complex, Canonical form, Semantic metric, Semantic Space, Integration, Knowledge representation model, Knowledge processing model, Ontology, Unified representation of knowledge, Linear vector space, Parametric t-norm classes, Hilbert cube, Finite structure, Substructural logic**

I. Introduction

Approaches to integration of logical models in a general form are considered in [7].

One of the problems of integration of logical models of knowledge representation and processing [11], [17], [20] is to identify compatible models that provide construction of interpretations of corresponding logical formalisms [6]. If necessary, these models can be considered as part of the corresponding semantic space [7], [8], [19].

One of the broad classes of logical models is fuzzy logics [3]. There is a problem of unreliability of fuzzy logics due to uncertainties [18] existing at different stages of their application [15]. One of the stages is selection of a fuzzy logic model or system with the purpose of application for realization of reasoning and problem solving. It is not always clear how suitable the chosen fuzzy system is. This is due to the fact that interpretations (which are built in the process of fuzzy logical inference) connect logical constructions with abstract algebraic systems that have no definite connection with any subject area or its model. Moreover, for each fuzzy system a different algebraic system is considered, the connection of which with other algebraic systems also remains undefined. This high degree of uncertainty does not allow reliable use of fuzzy logic models which is one of the problems of fuzzy logics [15].

The choice of fuzzy logics is also conditioned by their rich internal and external diversity, which allows fuzzy logics to represent other non-classical logical [3], [9], [10] models by means of fuzzy logics. The diversity of fuzzy logical models leads, among other things, to the diversity of fuzzy logical operations (for example, such as triangular norms and conorms), the emergence of their classes and their parameterization within the corresponding subclasses.

In development of the idea of parameterization of logical operations, the concept of parameterized fuzzy logics [6] is proposed.

The two main parametric families of triangular norms (and corresponding conorms) [14] are: the Frank parametric family [12], [13] and the Schweitzer-Sklar parametric family.

When constructing interpretations of parametrized fuzzy logics we can distinguish their following types: interpretations on algebraic systems, interpretations on "amorphous" models, interpretations on concrete structural-static models.

Interpretations on algebraic systems are largely similar to traditional fuzzy systems, the general scheme of which is given in [6], and therefore we will not consider them in detail in this paper. Further we will consider examples of interpretations of formulas of parameterized fuzzy logics on "amorphous" models and on concrete structural-static models.

II. Interpretation and models of fuzzy logics

As an "amorphous" model, consider a model in which each fuzzy predicate is matched with a vector quantity (vector) A, which can be given by some unit (or zero) vector 1_A, specifying the direction of this quantity, in some linear basis of some vector space and a scalar $||A||$ in the range from 0 to 1, specifying the length of vector A. If and only if the length is 0 or the direction is given by a zero vector, then the vector quantity is equal to a zero vector, its length is 0, but its direction can be a non-zero vector.

The fuzzy negation operation in this model reverses the direction of the vector according to the expression:

$$-1_A$$

and its length according to the expression:

$$1 - \|A\|$$

The next operation we will consider is the fuzzy conjunction. It should be noted that the fuzzy conjunction does not fulfill all the properties characteristic, for example, for triangular norms since this conjunction is parameterized, covering more than one triangular norm. A parameterized conjunction can naturally cover several triangular norms in one expression, so the properties of one triangular norm cannot be extended to such a conjunction.

To consider the result of the computation of such a fuzzy conjunction, let us consider 23 cases (variants, see Table **??**) of the spatial relation of vectors of a pair of its arguments ($A = 1_A * \|A\|$ and $B = 1_B * \|B\|$) which we will later reduce to a smaller number of cases.

Table I:
Variants of relations of parameters of "amorphous" parameterized fuzzy logic.

№	$\|A\| * \|B\|$	$cos\ (\langle A, B\rangle)$
0	0	[-1;1]
1	0	[-1;1]
2	(0,1]	1
3	(0,1]	(0;1)
4	(0,1]	(0;1)
5	(0,1]	(0;1)
6	(0,1]	0
7	(0,1]	(-1;1)
8	(0,1]	(-1;1)
9	(0,1]	(-1;1)
10	(0,1]	(-1;1)
11	(0,1]	(-1;1)
12	(0,1]	(-1;1)
13	(0,1]	(-1;1)
14	(0,1]	(-1;1)
15	(0,1]	(-1;1)
16	(0,1]	(-1;1)
17	(0,1]	(-1;1)
18	(0,1]	(-1;1)
19	(0,1]	(-1;1)
20	(0,1]	(-1;1)
21	(0,1]	(-1;1)
22	(0,1]	(-1;1)
23	(0,1]	-1

Due to symmetry (commutativity of the fuzzy conjunction operation), the number of these cases (variants) can be reduced to 16 which in turn are reduced to 10 (see Table **??**) as a result of decomposition. The result of the initial variant (case) is the arithmetic mean of the variants (cases) into which it is decomposed.

Hypothetically, variants 13 and 16 are impossible.

Let us consider these variants sequentially.

As a result of the operation of fuzzy conjunction of two arguments ($A = 1_A * \|A\|$ and $B = 1_B * \|B\|$), we must obtain a vector quantity given by two parameters: a vector and a scalar.

Table II:
Decomposition of variants of parameter relations of "amorphous" parameterized fuzzy logic

Symmetry of variants		**Decomposition into variants**	
1	1	0	
2	2	5	
3	5	6	
4	4	7	
6	6	8	
7	7	1	1
8	11	1	2
9	15	1	3
10	19	1	4
12	12	2	2
13	16	2	3
14	20	2	4
17	17	3	3
18	21	3	4
22	22	4	4
23	23	9	

For the 0th variant, the vector coincides with the vector of a non-zero argument or is calculated by the formula:

$$(1_A + 1_B)/\sqrt{(1_A + 1_B) * (1_A + 1_B)}$$

For all variants except for the 0-th (23rd) we will calculate the vector by the formula:

$$(1_A + 1_B)/\sqrt{(1_A + 1_B) * (1_A + 1_B)}$$

The proposed formula has the advantage of a more convenient model for modeling traditional fuzzy logics but alternative expressions for calculating the vector are possible:

$$(A + B)/\sqrt{(A + B) * (A + B)}$$

$$\frac{(A + B + 1_A + 1_B)}{\sqrt{(A + B + 1_A + 1_B) * (A + B + 1_A + 1_B)}}$$

and others.

For two nonzero noncollinear vectors A and B, the vector $H_{AB} = A * u + B * v$ from their common origin to the intersection of the perpendiculars to their ends can be expressed:

$$H_{AB} = \frac{A * (A * (B - A)) * B^2 + B * ((A - B) * B) * A^2}{(A * B)^2 - A^2 * B^2}$$

from

$$(A * u + B * v - B) * B = (A * u + B * v - A) * A = 0$$

$$\begin{cases} u * (A * B) = (1 - v) * B * B \\ v * (A * B) = (1 - u) * A * A \end{cases}$$