

Algorithm and Object-Oriented Programming for Modeling

Part 7: More about Algorithms and Designs

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Computational and Modeling Tools

What's the most powerful tool?



Photographer:

牛頭、狗頭，不如人頭

Programmer:

CPU, GPU, 不如 U

Plan:

- Introduction
- The Big O, P, NP, NP hard and complete
- Programming Paradigms
- Software Engineering

Input → algorithm → output

Input → algorithm → output

Among the earliest algorithms:

Euclidean algorithm (300 BC) for Greatest common divisor

```
def GCD(a, b):  
    return a if b == 0 else GCD(b, a % b)  
  
print(GCD(1071, 462))
```

See also: 《孫子算經》、《九章算術》

Input → algorithm → output

Algorithm in the industrial revolution: Jacquard loom



Input → **algorithm** → output

Algorithm in a modern computer

- { From machine code to high level language
- { From high level language to solving actual problem (♡)

Input → **algorithm** → output

Correct:

買一打雞蛋，如果有西瓜，買一個

eggs_bought = 12 if no watermelon else 1

Efficient:

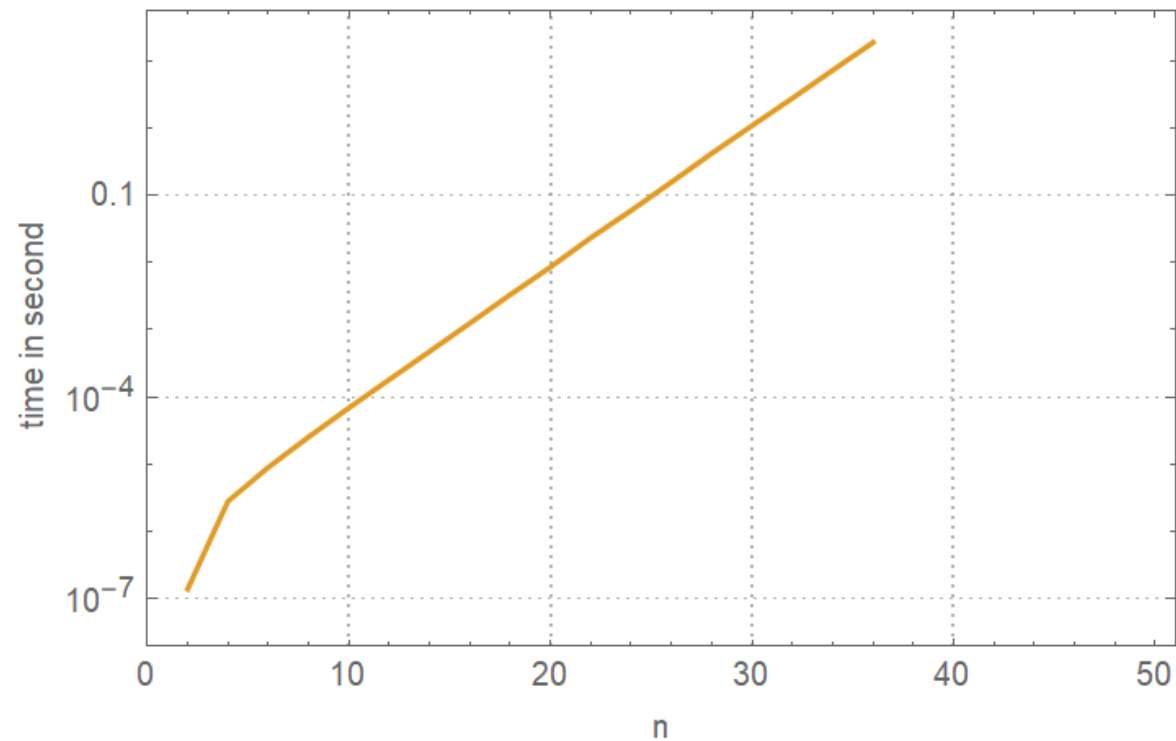
Difference between algorithm efficiency could be n times, n^2 times, ... e^n times, n^n times, infinity times.

Extremely important for big data.

For example: Fibonacci sequence

```
fib[1] = fib[2] = 1;
```

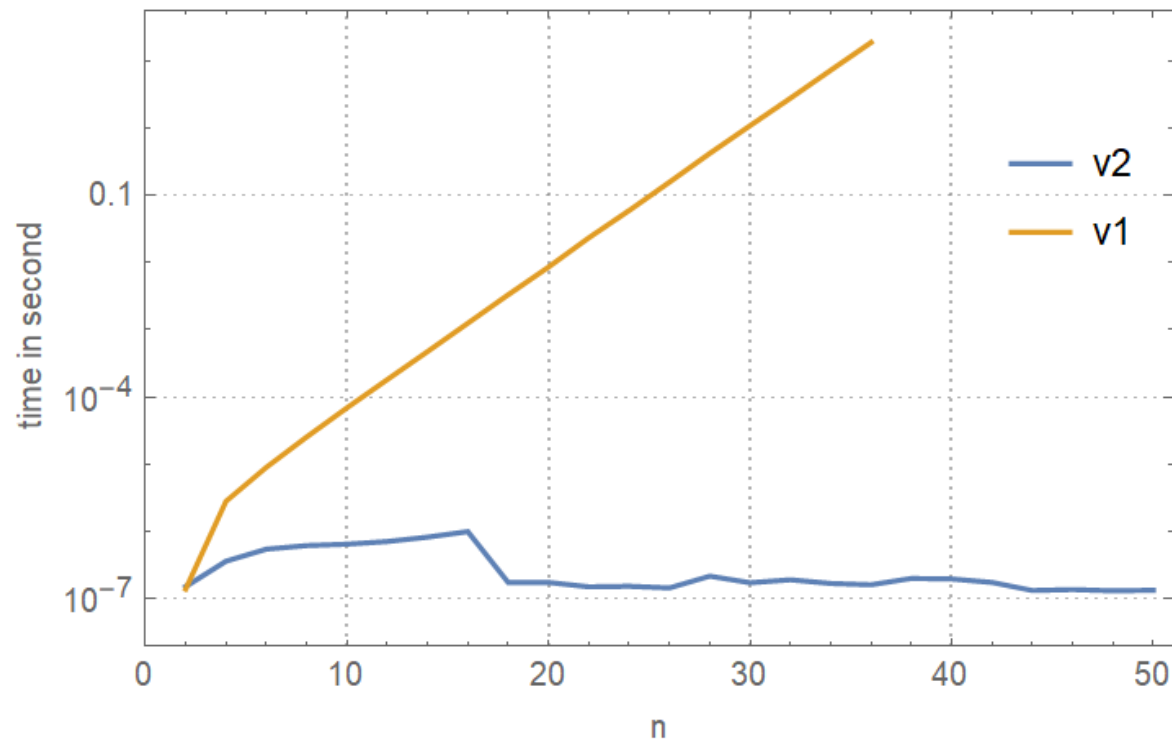
```
fib[n_] := fib[n - 1] + fib[n - 2]
```



For example: Fibonacci sequence

```
fib[1] = fib[2] = 1;
```

```
fib[n_] := fib[n - 1] + fib[n - 2]
```



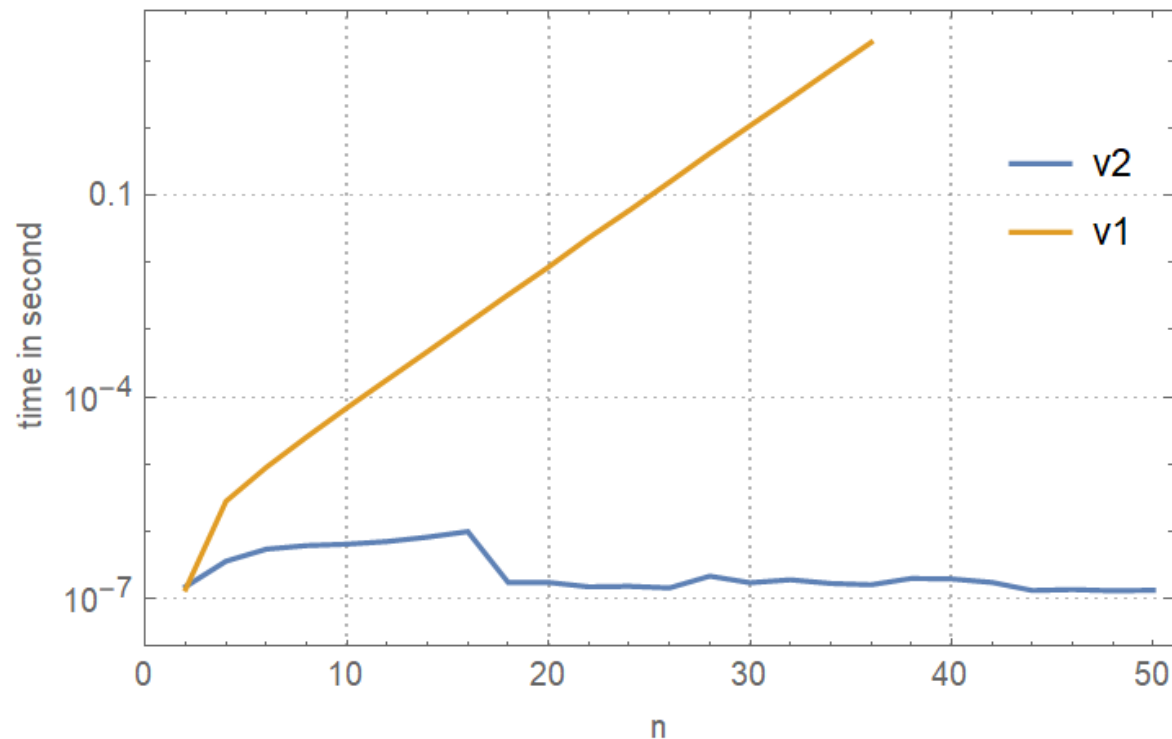
What did I do to make the change happen?

Version 1

```
fib[1] = fib[2] = 1;  
fib[n_] := fib[n - 1] + fib[n - 2]
```

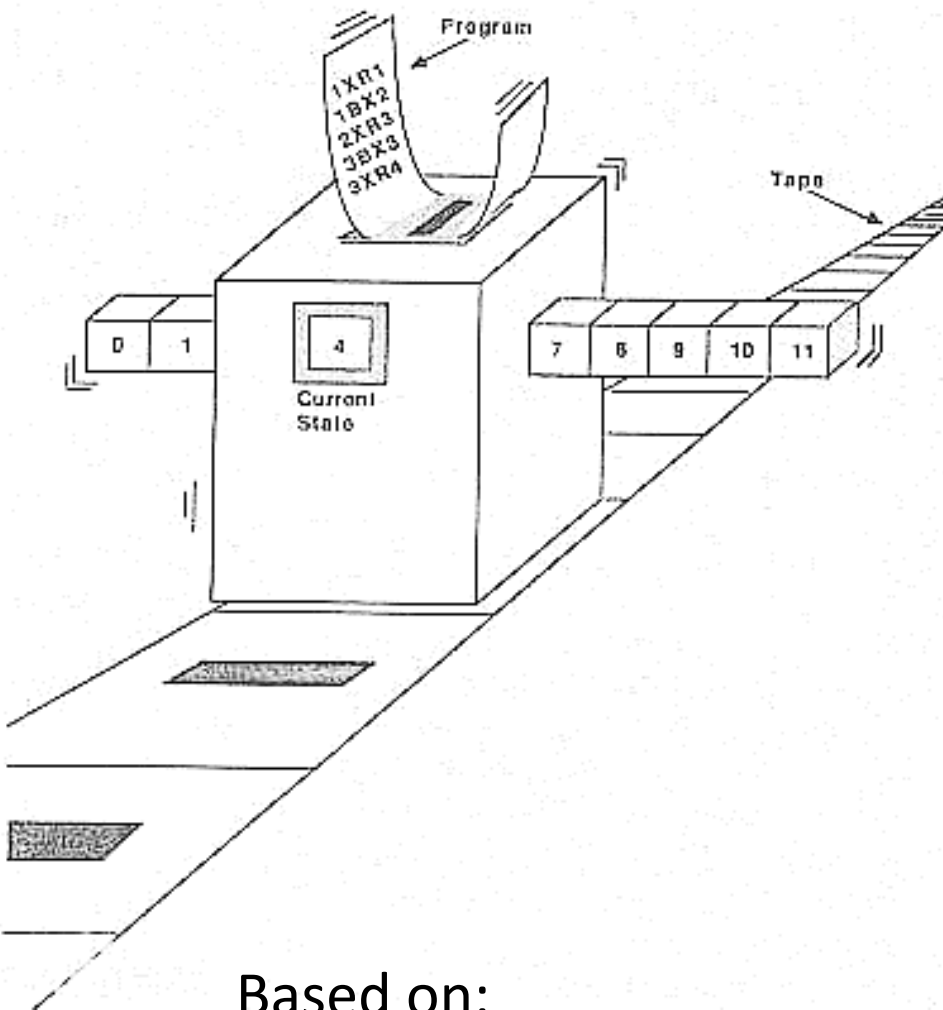
Version 2

```
fib[1] = fib[2] = 1;  
fib[n_] := fib[n] = fib[n - 1] + fib[n - 2]
```



What did I do to make the change happen?

How to measure the efficiency of an algorithm?



Turing Machine

Has:

- (1) Finite number of internal states
- (2) Finite table of rules (program)
- (3) Infinite tape in both directions
- (4) A “head” to read & write on tape

Based on:

- (1) The current symbol
- (2) Internal state of TM
- (3) A table of rules

Decide to do one of:

- (1) Overwrite a new symbol
- (2) Move tape forward/backward 1 grids
- (3) Switch to a new state
- (4) Halt

Recall the Jacquard loom machine

Halting problem: Is there such a program, to decide (in finite time) whether any program halts in finite time or not?

Fantastic if such program exist! Let's call it HP:

For example, using HP to solve the Goldbach's conjecture:

Every even number $n > 4$ can be written as a sum of two primes.

Solution: Use HP to decide whether the below program halts:

```
n = 4
while True:
    if no_two_primes_sums_to(n): break
    n = n + 2
```

Halt: Goldbach's conjecture is wrong. Does not halt: proved!

A generic road to infinity!

Halting problem: Is there such a program, to decide (in finite time) whether any program halts in finite time or not?

Unfortunately: such HP does not exist. Rough proof:

If HP exists, then one can use HP to write a function:

```
def g(f):  
    if halts(f):  
        loop_forever()
```

What's the output of $g(g)$?

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```
def g(f):  
    if halts(f):  
        loop_forever()
```

What's the output of $g(g)$?

If g halts, then g runs forever;

If g runs forever, then g halts.



有的人活着，他已经死了
有的人死了，他还活着

Alternative proof of halting problem: Diagonal method

$m \rightarrow$	0	1	2	3	4	5	6	7	8	...
$n \downarrow$										
0	0	0	0	0	0	0	0	0	0	...
1	0	0	0	0	0	0	0	0	0	...
2	1	1	1	1	1	1	1	1	1	...
3	0	2	0	2	0	2	0	2	0	...
4	1	1	1	1	1	1	1	1	1	...
5	0	0	0	0	0	0	0	0	0	...
6	0	0	1	0	2	0	3	0	4	...
7	0	1	2	3	4	5	6	7	8	...
8	0	1	0	0	1	0	0	0	1	...
•	•				•				•	
•	•				•				•	
•	•				•				•	

Assuming HP exists.

The n-th turing machine

+ The m-th input data

= Definite output

Definite output:

- Either calculation finished
in finite time,
- Or 0 means halt.

Alternative proof of halting problem: Diagonal method

$m \rightarrow$	0	1	2	3	4	5	6	7	8	...
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0	1	0	0	0	0	0	0	0	0	...
1	0	1	0	0	0	0	0	0	0	...
2	1	1	2	1	1	1	1	1	1	...
3	0	2	0	3	0	2	0	2	0	...
4	1	1	1	1	2	1	1	1	1	...
5	0	0	0	0	0	1	0	0	0	...
6	0	0	1	0	2	0	4	0	4	...
7	0	1	2	3	4	5	6	8	8	...
8	0	1	0	0	1	0	0	0	2	...
.
.
.

Assuming HP exists.

The n -th turing machine

+ The m -th input data

= Definite output

Definite output:

- Either calculation finished in finite time,
- Or 0 means halt.

Now, add one to each diagonal number

(1) This is the output of a Turing machine – we have described the algorithm

(2) This is not an output of any Turing machine, since

1st dig differs from 1st Turing machine, 2nd dig differ from 2nd Turing machine, etc

Contradiction. Thus halting cannot be determined for all Turing machines in finite time.

Infinite time is too mathematical;
Let's now talk about finite things.

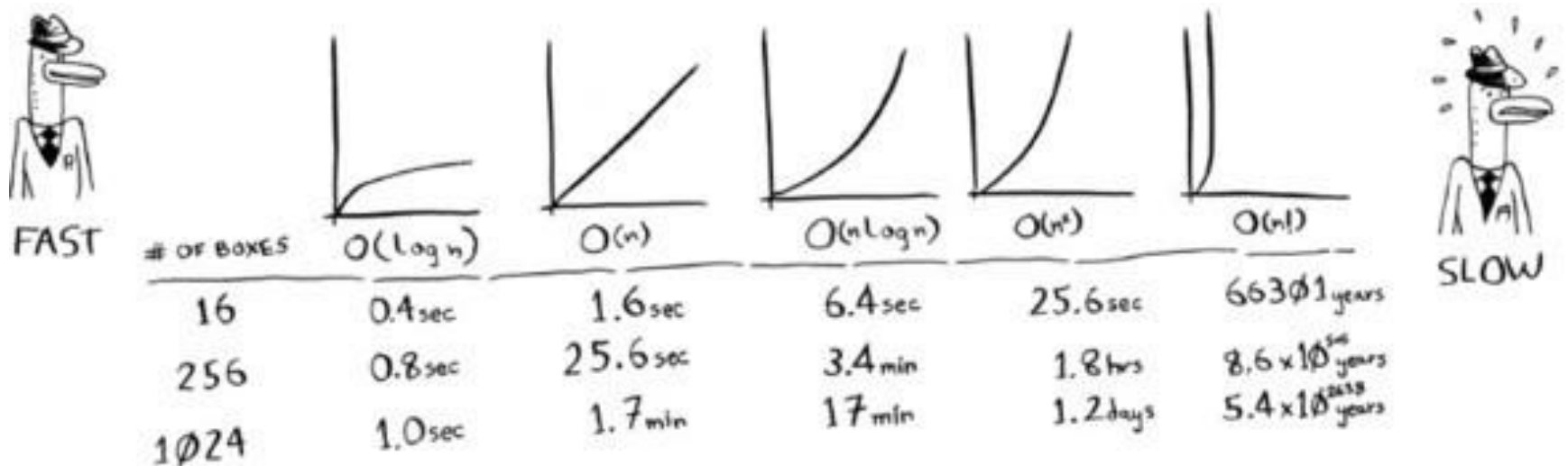
The Big O, P, NP, NP hard and complete



What does a big O mean?

Based on the computation model of the Turing machine:

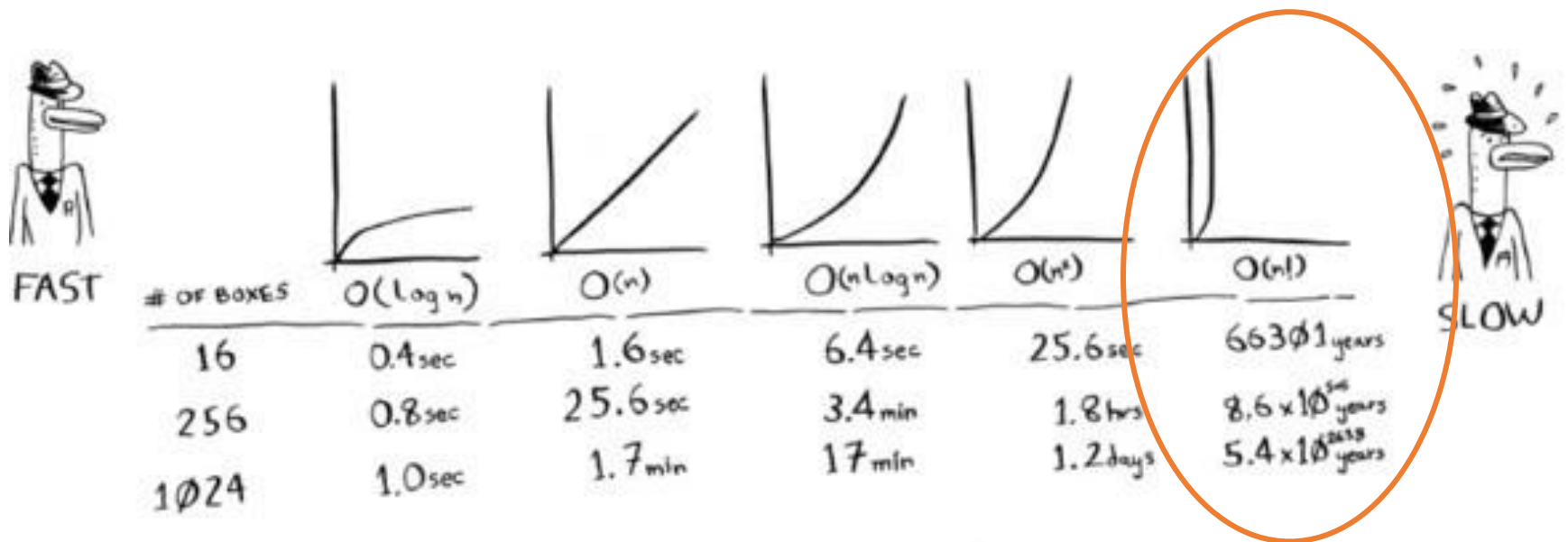
Problem with input size n can be solved in $O(f(n))$ steps.



What does a big O mean?

Based on the computation model of the Turing machine:

Problem with input size n can be solved in $O(f(n))$ steps.



Exponential is terrible.

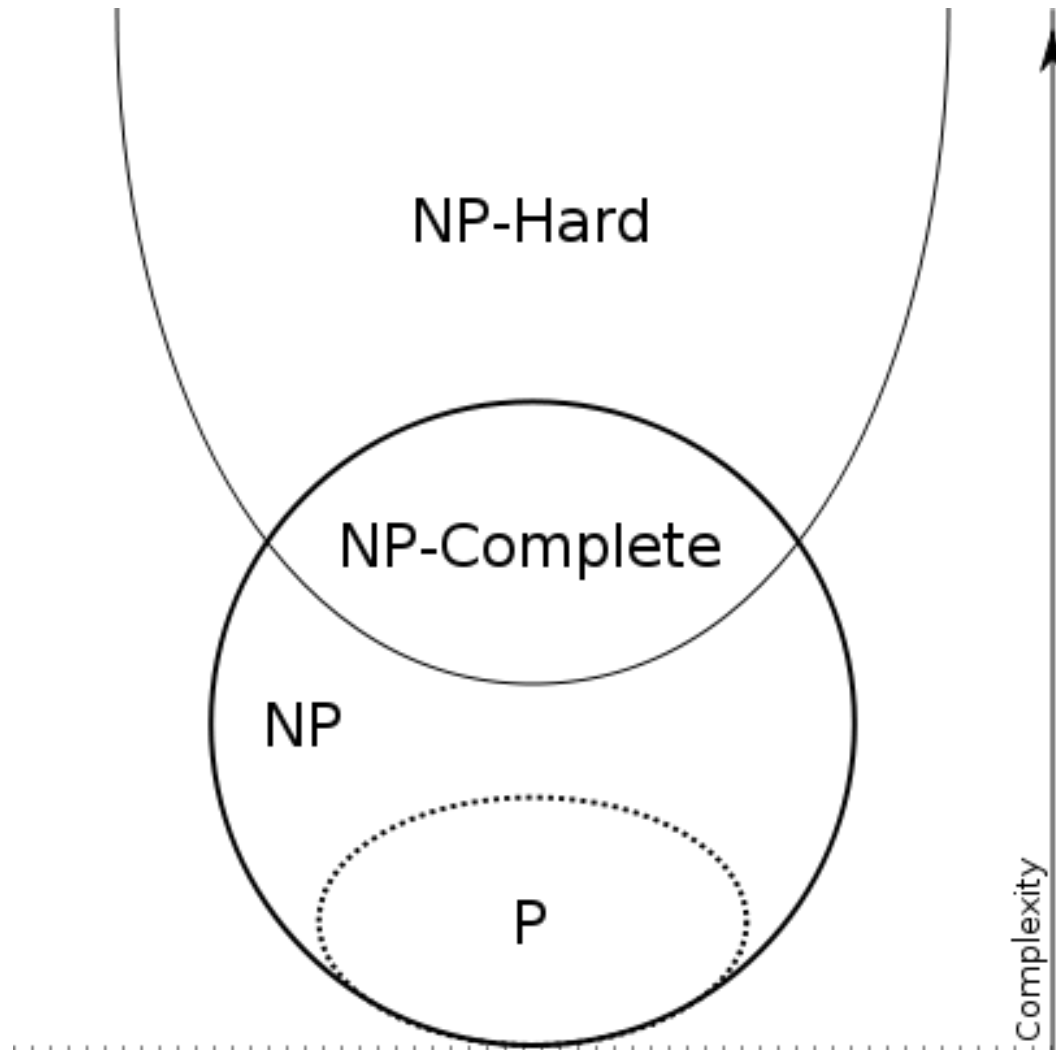
Classification of problems (mathematicians like classifications)



Classification of problems (mathematicians like classifications)



Classification of problems (mathematicians like classifications)



Decision problem: input n bit string, return True or False

On a Turing machine:

P: all decision problems that can be solved in polynomial time.

NP: if True, the proof can be checked in polynomial time.

For example, Sudoku
is a NP problem.

Given a solution (proof of True),
can be checked in polynomial time.

Not known if it is P: No polynomial
time general solution known.

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Decision problem: input n bit string, return True or False

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Another example, big number factorization.

Cole (1903) talk on AMS meeting: the entire content of the talk:

(1) Calculate $2^{67} - 1$ (2) Calculate $193707721 \times 761838257287$

(Both equal to 11978547889676412927).

But given 11978547889676412927, can you factorize it? (NP)

Is P equal to NP?

What happens if $P = NP$?

Creativity is no longer valuable.

One can find proofs of all math theorems in P time.

Too good to be true.

However, there is still no proof on $P \neq NP$ yet.

What happens if $P = NP$?

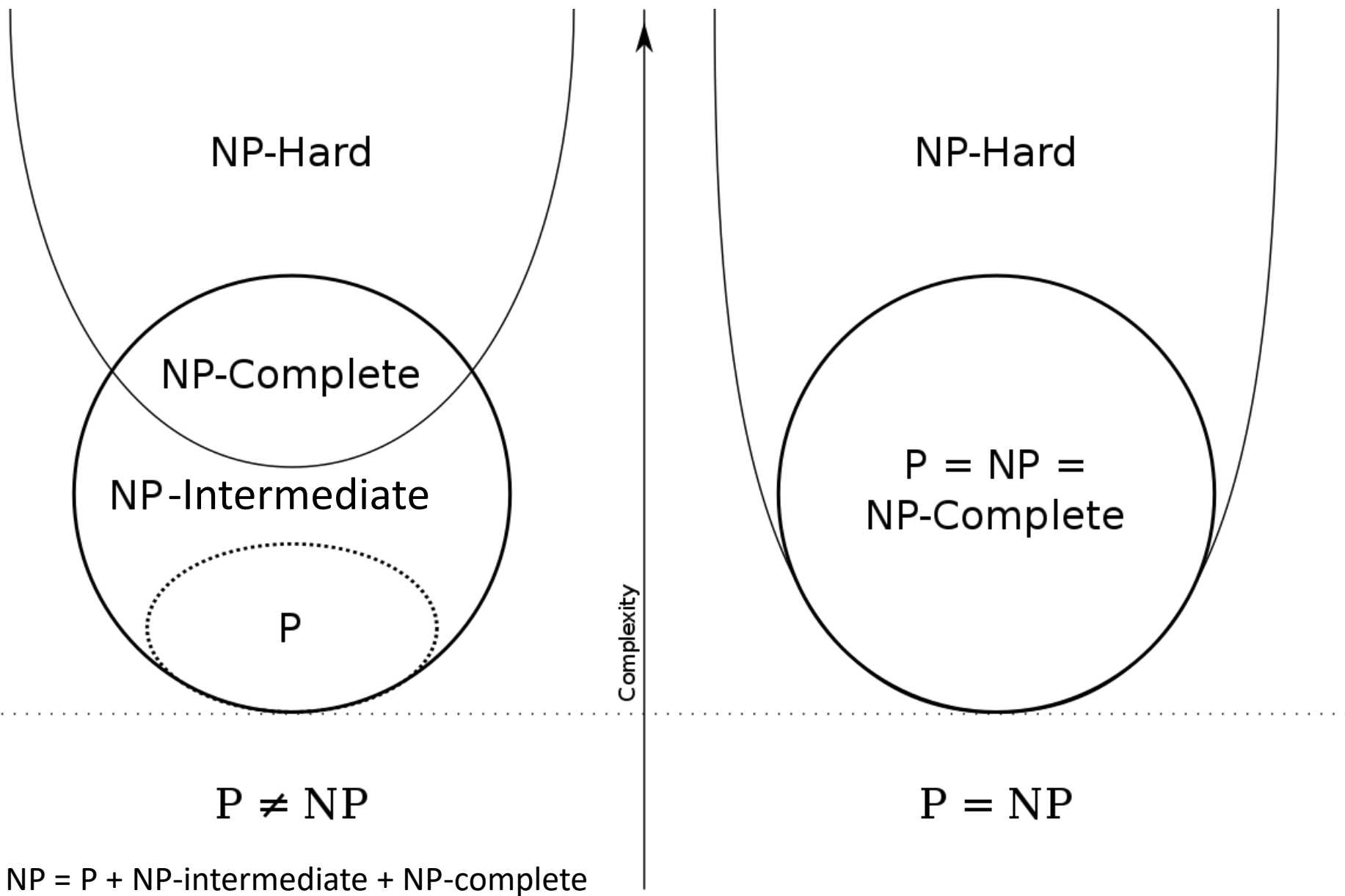
NP-hard:

A problem that any NP problem can be reduced to.

If an oracle can immediately solve a NP-hard problem, then we can reduce any NP problem to P.

Two kinds of NP-hard problems:

- Harder than the hardest problems in NP, e.g. halting problem.
- As hard as the hardest problems in NP: NP-complete.



Ladner's theorem (1975): If $P \neq NP$, NP-intermediate is not empty.

More about NPC

单论颜值NPC放在韩国怎么样？ 修改


NPC路人粉 感觉每一个人都很好看 可以算得上完颜团了吧

游戏里面的 NPC 有故事吗？ 修改

想想那些孤独的 NPC 们，他们说着一篇一律的话，拿着微薄的流量费，如此孤独

快解散了，npc的团魂，真实存在吗？ 修改

在游戏中尊重 NPC 是很「傻」的行为吗？ 修改

有些时候考虑 NPC 的感受会被说是很傻的行为呢，所以想知道大家是怎么想的呢？ 因为有些游戏例如老滚之类是可以对 NPC 产生影响的，或者虽然不能对其做...[显示全部](#) 

More about NP-Complete Problems

If one can solve one NPC, one can solve all NP problems.

Does such NPC ever exist?

Abstract proof: Yes, with the following construction of a NPC:

Given a polynomial-time Turing machine M , find an input y that causes $M(y)$ to accept.

[accept by a polynomial-time Turing machine means this input can halt within polynomial time.]

NP: Can be checked by actually run & return in polynomial time

NPC: Given any NP problem \rightarrow convert to a Turing machine M with solution y to halt. The above program finds y for you.

More about NP-Complete Problems

If one can solve one NPC, one can solve all NP problems.

Does such NPC ever exist?

Explicit construction: the 3-satisfiability problem.

For n Boolean variables x_1, \dots, x_n ,

given a set of constraints, each has at most 3 vars, e.g.

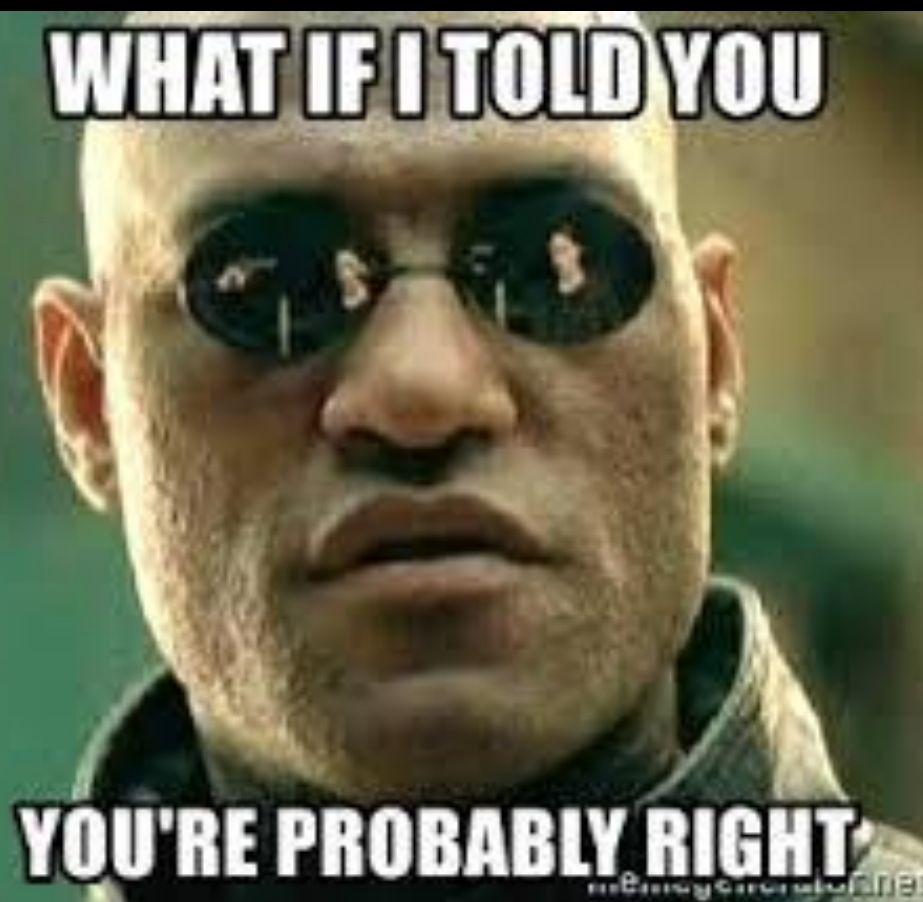
$$x_2 \text{ or } x_5 \text{ or } \text{not}(x_6)$$

$$\text{not}(x_2) \text{ or } x_4$$

$$\text{not}(x_4) \text{ or } \text{not}(x_5) \text{ or } x_6$$

Is there set of variables x_1, \dots, x_n s.t. all constraints are true?

In NP: obvious. Is NP-complete: Cook-Levin Theorem (1971)



PP: Probabilistic Polynomial-Time

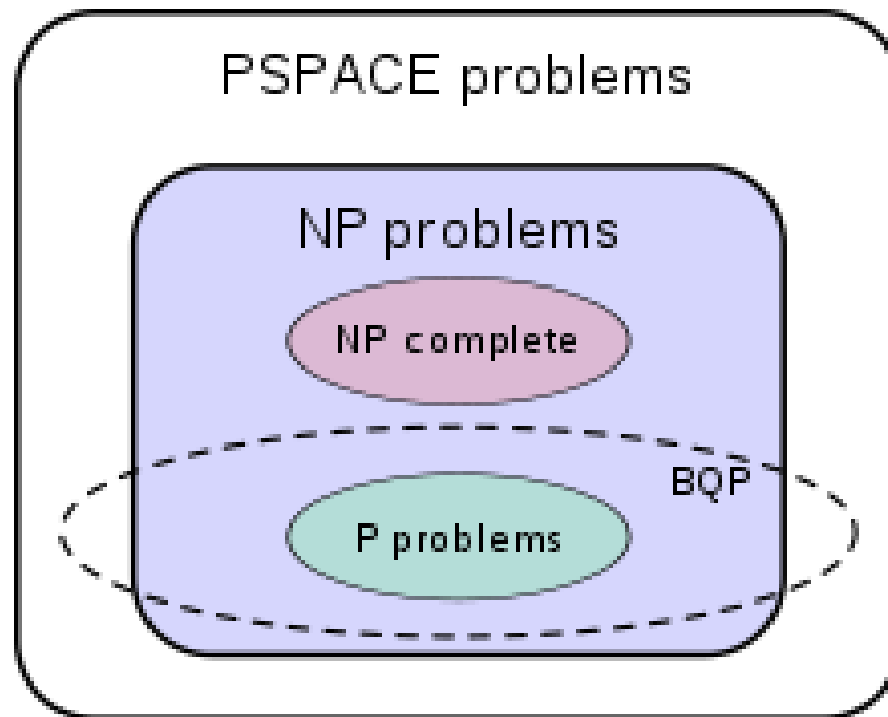
Exists a polynomial-time algorithm to accept with $p > 1/2$ if the answer is yes or $p < 1/2$ if the answer is no

BPP: Bounded-Error PP

Exists a polynomial-time algorithm to accept with $p > 2/3$ if the answer is yes or $p < 2/3$ if the answer is no

Where is quantum computing?

Bounded-error quantum polynomial time (BQP)



$P < BQP$ or $P = BQP$ unknown.

Relation between BQP and NP unknown.

Computational complexity [\[edit \]](#)

Main article: [Computational complexity theory](#)

- [P versus NP problem](#)
- What is the relationship between [BQP](#) and [NP](#)?
- [NC = P problem](#)
- [NP = co-NP problem](#)
- [P = BPP problem](#)
- [P = PSPACE problem](#)
- [L = NL problem](#)
- [PH = PSPACE problem](#)
- [L = P problem](#)
- [L = RL problem](#)
- [Unique games conjecture](#)
- Is the [exponential time hypothesis](#) true?
 - Is the strong exponential time hypothesis (SETH) true?
- Do [one-way functions](#) exist?
 - Is [public-key cryptography](#) possible?
- [Log-rank conjecture](#)

List of unsolved problems in computer science

From Wikipedia, the free encyclopedia

Programming Paradigms

1960s – 70s: Unstructured vs Structured

GOTO 1960s – 70s

Towards the modern era:

- Procedural programming
(Traditional structured programming)
- Object-oriented programming
- Functional programming

Object-oriented programming:

To model objects, a few considerations:

- Pack data and functions into an object
- Multiple independent realizations
- Inheritance, encapsulation, polymorphism, composition



Interface: What an object can do.

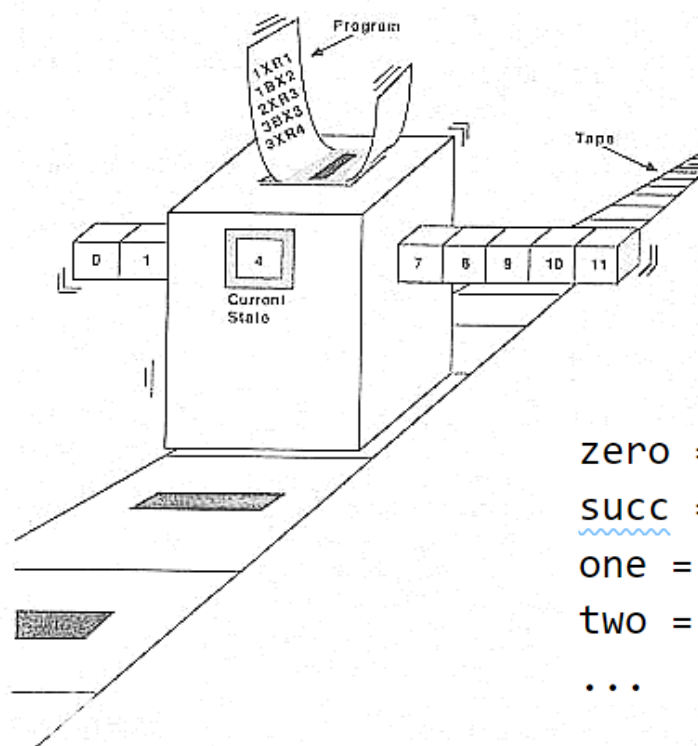
- Encapsulation: Hide internal details.
- Polymorphism: Allow behavior differences.

Why OO?

Think about a car engine



Functional programming: origin



λ -calculus

Alonzo Church 1930s

```
zero = lambda f: lambda x: x
succ = lambda n: lambda f: lambda x: f(n(f)(x))
one  = succ(zero)
two  = succ(one)
...
```

Turing Machine

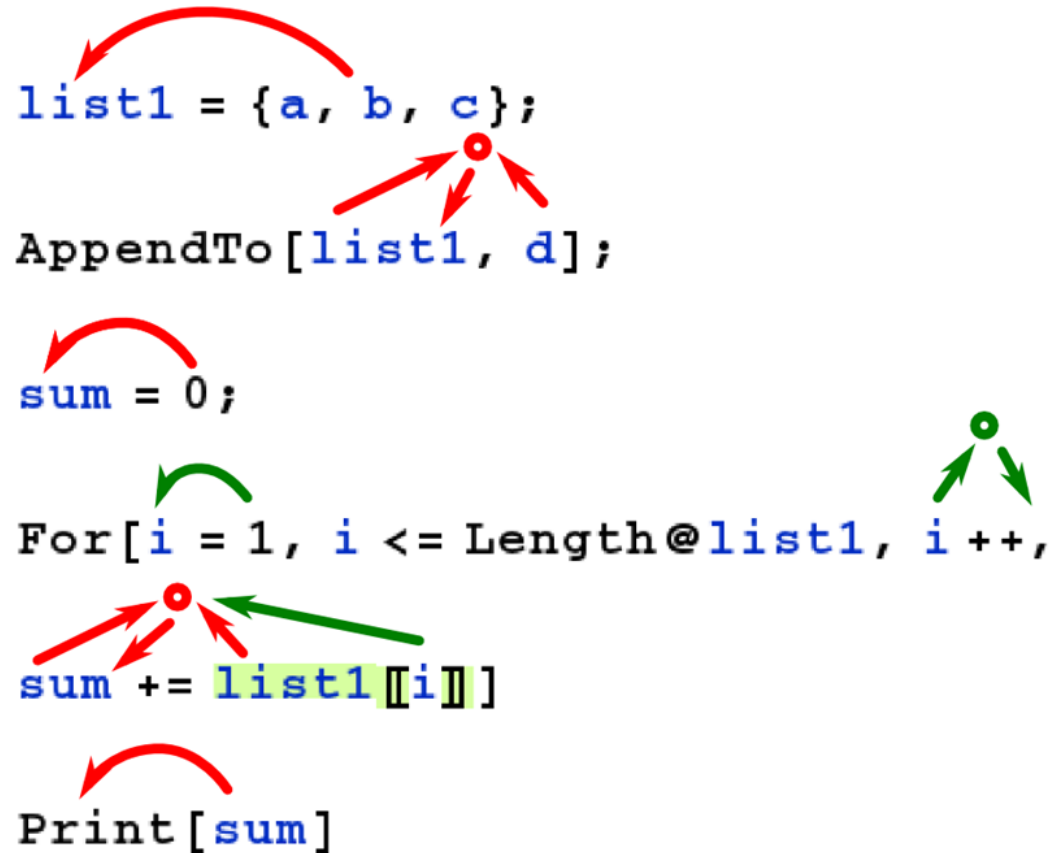
Alan Turing 1930s

Turing Machine and λ -calculus can simulate each other

Imperative Programming

Functional programming:

Program as evaluating
math functions,
eliminate side effects.



The diagram illustrates imperative programming with the following code and control flow arrows:

```
list1 = {a, b, c};  
AppendTo[list1, d];  
  
sum = 0;  
For[i = 1, i <= Length@list1, i ++,  
  sum += list1[[i]]  
]  
Print[sum]
```

Control flow arrows (red and green) indicate the sequence of execution: from the assignment of `list1` to `AppendTo`, then to the initialization of `sum`, the `For` loop header, the loop body, and finally to `Print[sum]`. A green arrow also points to the `i++` increment operation.

Functional Programming



The diagram illustrates functional programming with the following code and data flow arrows:

```
Print@Total@Append[{a, b, c}, d]
```

Red curved arrows indicate the data flow from the arguments `{a, b, c}` and `d` through the `Append` function, then through `Total`, and finally to the `Print` function.

Higher order functions

A functional: function of function $S[\phi] = \frac{1}{2} \int d^4 x (\partial \phi)^2$

```
SortBy[{ {1, 2}, {0, 10}, {5, -1} }, Total]
```

Anonymous functions (lambda calculus)

```
f[x1_, x2_] := x1 + x2
```

```
ClearAll[f]
```

```
Function[{x1, x2}, x1 + x2]
```

```
Function[Slot[1] + Slot[2]]
```

```
#1 + #2 &
```

```
SortBy[Range@10, Mod[#, 3] &]
```

```
FixedPoint[(# + 2 / #) / 2 &, 1., SameTest → (Abs[#1 - #2] < 0.01 &)]
```

Remark: functional aspects of Linux shell

- * Program "Do one thing, and do it well"
- * Defined stdin, stdout, pipe
- * Plain text flows through pipes

E.g.

```
ps aux | grep conky | grep -v grep | awk ' {print $2}' | xargs kill
```

Software Engineering

See e.g. “The Mythical
Man-Month” (人月神话)

In the 1960s to 1980s, in software development,
much more problems encountered than expected.

The major cause of the software crisis is that the machines have become several orders of magnitude more powerful!

To put it quite bluntly:

as long as there were no machines, programming was no problem at all;

when we had a few weak computers, programming became a mild problem,

and now we have gigantic computers, programming has become an equally gigantic problem.

— Edsger Dijkstra (1972, ACM Turing Award Lecture)



Coding before the 70s
or,
now coding at school



From 70s to now:
Coding for production

如何激怒一位计算机科学爱好者？

Long road from
Programming to
Production



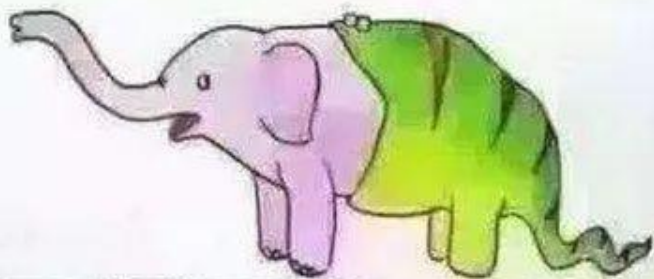
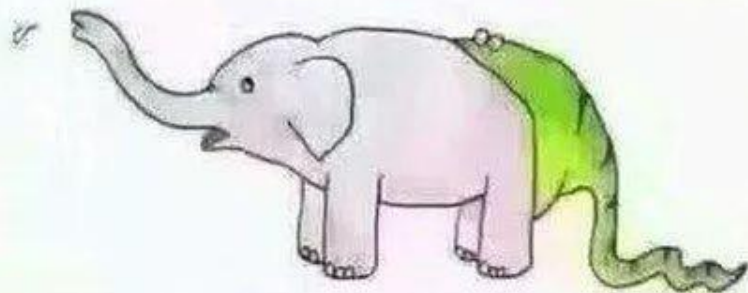
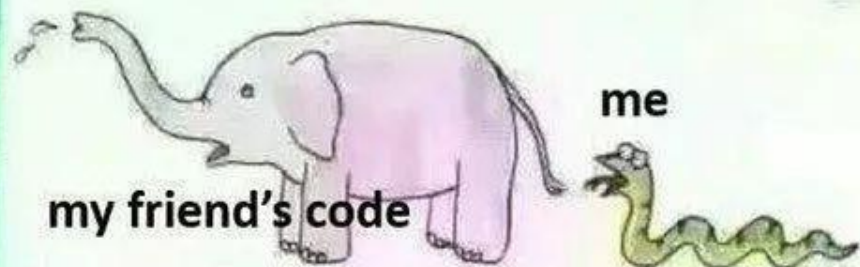
Lessons

Balance between standard and hand-made tools

Don't reinvent the wheel!

IBM.





my code



恐龙是怎么来的

But don't use the wheels
ugly, either.

Also try to make our code reusable.



只要你足够废物，就没人可以利用你

As long as you have enough waste, no one can take advantage of you

Toward self-documenting code

```
for i in range(n):  
    for j in range(n):  
        for k in range(n):  
            ...
```

Reviewer: index names should reflect their meanings

```
for i in range(n):  
    for j in range(n):  
        for k in range(n):  
            ...
```

Reviewer: index names should reflect their meanings

```
for index in range(n):
```



```
for i in range(n):  
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        for k in range(n):  
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```

Reviewer: index names should reflect their meanings

```
for index in range(n):  
    for jindex in range(n):  
        for kindex in range(n):  
            ...
```

Reviewer: \emptyset_\emptyset

...

Reviewer: index names should reflect their meanings

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for index in range(n):  
    for jindex in range(n):  
        for kindex in range(n):  
            ...
```

Reviewer: \emptyset_\emptyset

```
for index in range(n):  
    for indey in range(n):  
        for indez in range(n):  
            ...
```

Reviewer: ∇

为什么没有或流行的拼音编程语言?

```
#include "施氏食狮史.h"
```

```
#define 始 new
```

```
#define 逝世 delete
```

```
#define 誓 assert
```

```
#define 十时 "10:00"
```

```
#define 是时 SetSystemTime
```

```
#define 视适实施 while
```

```
#define 是 =
```

```
#define 实是 ==
```

```
#define 事实 True
```

```
#define 释 //
```

施氏 是 始 石室诗士();

施氏.嗜 是 狮;

是时(十时);

施氏.适(市);

施氏.视(狮);

视适实施(!十狮逝世()) {

 矢(狮);

}

施氏.拾(十狮尸);

施氏.适(石室);

视适实施(石室.湿 实是 事实){

 施氏.侍.拭(石室);

```
}
```

```
施氏.食(十狮尸);
```

```
施氏.识(十狮尸 实是 十石狮尸);
```

```
誓(施氏.食 实是 十狮);
```

```
逝世 施氏;
```

```
释 试释是事
```

麻煩改成拼音編程，在綫等，挺急的

Try to write self-documenting code:

Functions as verbs; variables as nouns; classes as subjects

Code as meaningful as sentences.

Write tests. Decent test coverage.

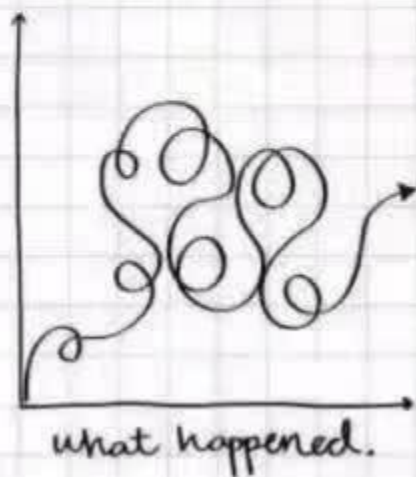
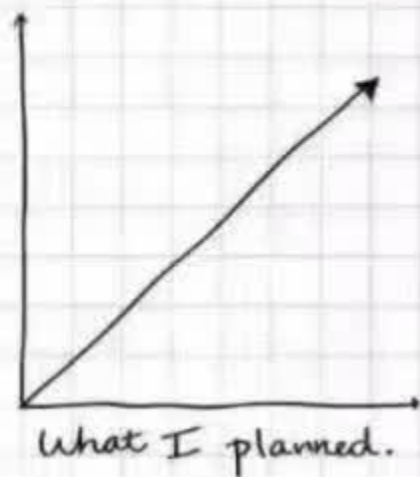
Test driven if helps (write tests before code).

一个测试工程师走进一家酒吧，要了一杯啤酒
一个测试工程师走进一家酒吧，要了一杯咖啡
一个测试工程师走进一家酒吧，要了0.7杯啤酒
一个测试工程师走进一家酒吧，要了-1杯啤酒
一个测试工程师走进一家酒吧，要了 2^{32} 杯啤酒
一个测试工程师走进一家酒吧，要了一杯洗脚水
一个测试工程师走进一家酒吧，要了一杯蜥蜴
一个测试工程师走进一家酒吧，要了一份asdfQwer@24dg!&*(@
一个测试工程师走进一家酒吧，什么也没要
一个测试工程师走进一家酒吧，又走出去又从窗户进来又从后门出去从下水道钻进来
一个测试工程师走进一
一个测试工程师走进一家酒吧，要了一杯烫烫烫的锱斤拷
一个测试工程师走进一家酒吧，要了NaN杯Null
1T测试工程师冲进一家酒吧，要了500T啤酒咖啡洗脚水野猫狼牙棒奶茶
1T测试工程师把酒吧拆了
一个测试工程师化装成老板走进一家酒吧，要了500杯啤酒并且不付钱
一万个测试工程师在酒吧门外呼啸而过
一个测试工程师走进一家酒吧，要了一杯啤酒';DROP TABLE 酒吧

测试工程师们满意地离开了酒吧。然后一名顾客点了一份炒饭，酒吧炸了

Realistic planning

暑假前制定了一个plan，半个暑假结束了，完成了个p，因为lan。

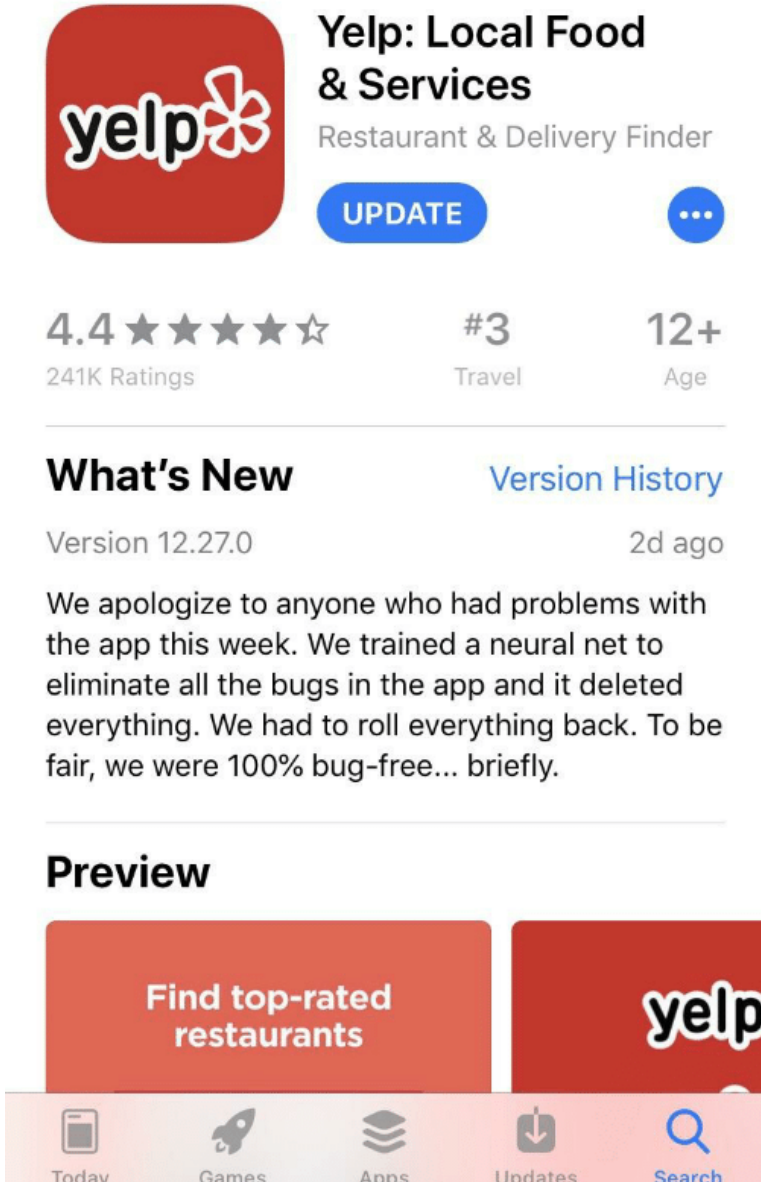


Live with bugs

Era of long human life expectation:
“Living with a chronic illness”

Software engineering:
“Living with bugs”

No way to eliminate all bugs, except --



The screenshot shows the App Store page for the Yelp app. At the top is the red Yelp logo icon. To its right, the app title "Yelp: Local Food & Services" is displayed in bold, followed by the subtitle "Restaurant & Delivery Finder". Below the title is a blue "UPDATE" button and a three-dot menu icon. The app has a rating of 4.4 stars from 241K ratings, is ranked #3 in the Travel category, and is rated for ages 12+. A "What's New" section shows version 12.27.0, released 2 days ago, with a humorous update log: "We apologize to anyone who had problems with the app this week. We trained a neural net to eliminate all the bugs in the app and it deleted everything. We had to roll everything back. To be fair, we were 100% bug-free... briefly." Below this is a "Preview" section showing two app interface cards: "Find top-rated restaurants" and the Yelp logo. At the bottom is a navigation bar with icons for Today, Games, Apps, Updates, and Search.

Yelp: Local Food & Services
Restaurant & Delivery Finder

UPDATE

4.4 ★★★★★
241K Ratings

#3
Travel

12+
Age

What's New [Version History](#)

Version 12.27.0 2d ago

We apologize to anyone who had problems with the app this week. We trained a neural net to eliminate all the bugs in the app and it deleted everything. We had to roll everything back. To be fair, we were 100% bug-free... briefly.

Preview

Find top-rated restaurants

yelp

Today Games Apps Updates Search

Thanks to AI, Yelp iOS app
achieved 100% bug free for a
brief moment.

Nobody understands everything of a large project.
Try to make collaborators' life easier.

What ended in 1896?

1895

What was significant about

课时 7

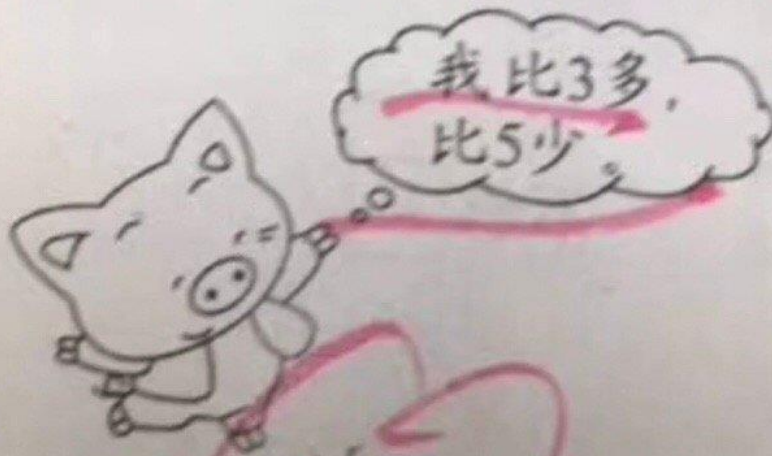
àn yāo qiú zuò tí wǒ zuì bàng
一、按 要 求 做 题 我 最 棒。

1. 把数倒着数写在□里。

0 1 2 3 4 5 6 7 8 9 10

10 9 8 7 6 5 4 3 2 1 0

zōng hé yùn yòng tí cāi yī cāi wǒ
4. 【综合运用题】猜一猜，我



我是(猪)

Be explicit.

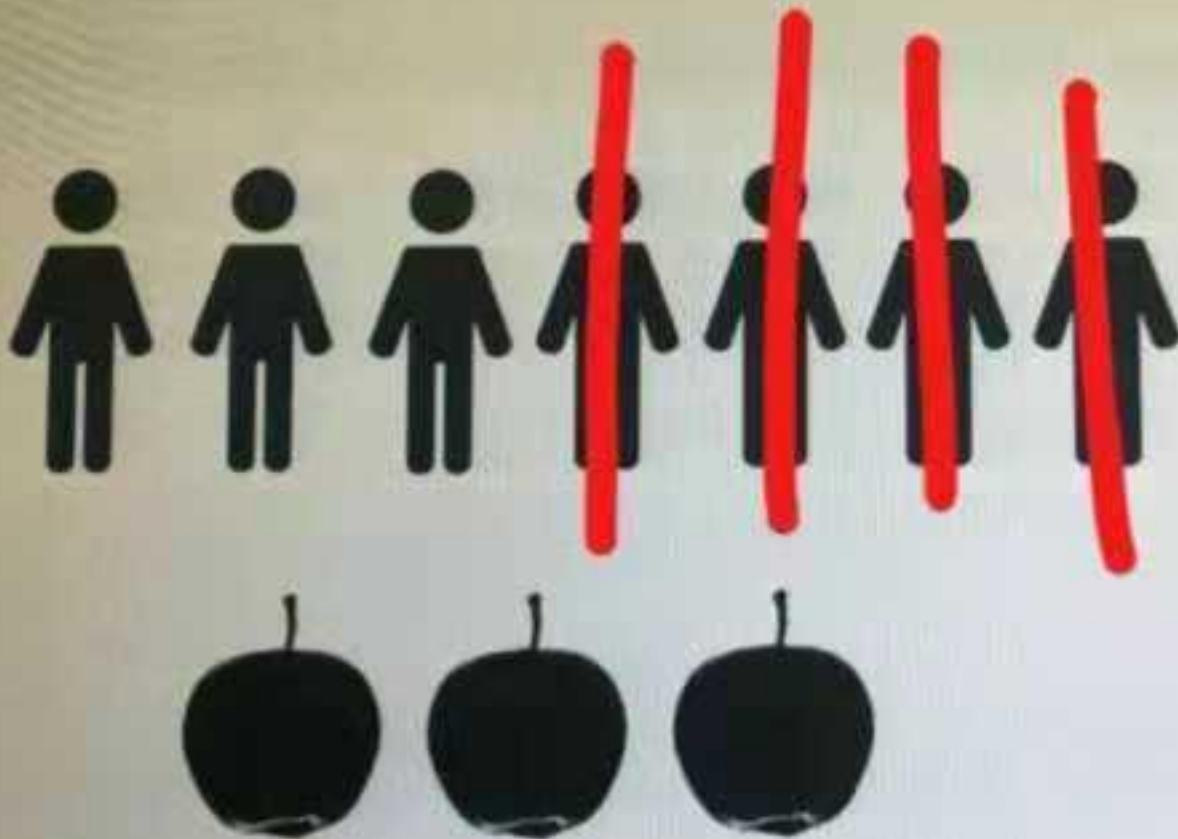
Don't expect that
your colleagues
can always guess
what you mean.

Where was the American Declaration of Independence
signed?

At the bottom.

Software engineering is NOT an IQ test
Otherwise disasters happen

• 七個小朋友，只能切四刀，如何把三個蘋果平均分？



Handle exceptions properly

