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(1). A: origin dot. $P(A)=\frac{5}{8}$ C: change $\begin{cases} P(C|A)=\frac{2}{5}, & P(D|A)=\frac{3}{5} \\ P(C|B)=\frac{1}{3}, & P(D|B)=\frac{2}{3} \end{cases}$

B: origin dash $P(B)=\frac{3}{8}$ D: unchange

(a). RA: receive a dot. $P(RA)=P(A)\cdot P(D|A)+P(B)\cdot P(C|B)$

(RB: receive a dash). $=\frac{5}{8}\times\frac{3}{5}+\frac{3}{8}\times\frac{1}{3}=\frac{1}{2}$

$res=P_0=\frac{P(A)\cdot P(D|A)}{P(RA)}=\frac{3/8}{1/2}=\boxed{\frac{3}{4}}$

(b). $P(RB)=1-P(RA)=\frac{1}{2}$

$res'=P_1=\frac{P(B)\cdot P(D|B)}{P(RB)}=\frac{\frac{3}{8}\times\frac{2}{3}}{1/2}=\boxed{\frac{1}{2}}$

(c). $\begin{cases} P(\text{dot-dot})=P_0\cdot P_0=\frac{9}{16} \\ P(\text{dot-dash})=P_0\cdot(1-P_0)=\frac{3}{16} \\ P(\text{dash-dot})=(1-P_0)\cdot P_0=\frac{3}{16} \\ P(\text{dash-dash})=(1-P_0)(1-P_0)=\frac{1}{16}. \end{cases}$

(2). (a). $\varphi(t)=E(e^{itx})=\int_{-\infty}^{\infty}e^{itx}\cdot\frac{1}{2}e^{-|x|}dx$

$=\frac{1}{2}\left(\int_{-\infty}^{0}e^{itx}\cdot e^{x}dx+\int_{0}^{\infty}e^{itx}\cdot e^{-x}dx\right)$

$=\frac{1}{2}\left(\frac{e^{(it+1)x}}{it+1}\Big|_{-\infty}^{0}+\frac{e^{(it-1)x}}{it-1}\Big|_{0}^{\infty}\right)$.

$=\frac{1}{2}\left(\frac{1}{it+1}-\frac{1}{it-1}\right)$

$=\boxed{\frac{1}{t^2+1}}$, $-\infty<t<\infty$.

(b) $E(X)=\frac{\varphi'(0)}{i}=0$

$Var(X)=-\varphi''(0)+[\varphi'(0)]^2=-\frac{-2+0}{1}+0=2$

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3. $\langle Z\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{2}\left(1+\frac{xy}{|xy|}\right)\cdot f(x,y)\,dx\,dy$. $x = X(t)$, $y = X(t+\tau)$.

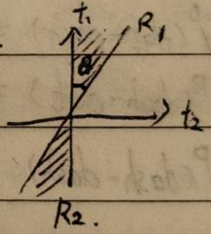
$$f(x,y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_1)^2}{\sigma_1^2}+\frac{(y-\mu_2)^2}{\sigma_2^2}-2\rho\cdot\frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2}\right]\right\}$$

$$= \frac{1}{2\pi\sqrt{1-\rho^2}}\cdot \exp\left\{-\frac{1}{2(1-\rho^2)}\left[x^2+y^2-2\rho xy\right]\right\}$$

Let $\begin{cases} u = \frac{1}{\sqrt{1-\rho^2}}x \\ v = \frac{1}{\sqrt{1-\rho^2}}y \end{cases}$ and $\begin{cases} t_1 = u-\rho v \\ t_2 = \sqrt{1-\rho^2}\cdot v \end{cases}$.

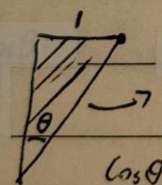
$$\Rightarrow \langle Z\rangle = \frac{1}{4\pi}\int_{t_1}\int_{t_2}\left(1+\frac{(t_1+\rho t_2/\sqrt{1-\rho^2})\cdot(\rho t_2/\sqrt{1-\rho^2})}{|(t_1+\rho t_2/\sqrt{1-\rho^2})\cdot(\rho t_2/\sqrt{1-\rho^2})|}\right)\cdot \exp\left(-\frac{1}{2}(t_1^2+t_2^2)\right)dt_1dt_2$$

$$= \frac{1}{2\pi}\iint_{R_1+R_2} \exp\left(-\frac{1}{2}(t_1^2+t_2^2)\right)dt_1t_2 .$$



$R_1: \begin{cases} t_1 \geq \frac{-\rho}{\sqrt{1-\rho^2}}t_2 \\ t_2 \geq 0 \end{cases}$; $R_2 \begin{cases} t_1 \leq \frac{-\rho}{\sqrt{1-\rho^2}}t_2 \\ t_2 \leq 0 \end{cases}$.

$\frac{-\rho}{\sqrt{1-\rho^2}}$



$\rightarrow \frac{1}{\sqrt{1-\rho^2}}$, $\iint_{-\infty}^{\infty}\exp\left(-\frac{1}{2}(t_1^2+t_2^2)\right)dt_1t_2 = \sqrt{2\pi}\cdot\sqrt{2\pi} = 2\pi$

$\cos\theta = -\rho$

$$\Rightarrow \langle Z\rangle = \frac{2\theta}{2\pi} = \frac{1}{\pi}\cdot\arccos(-\rho)$$

$$\Leftrightarrow \overline{Z(t)} = \frac{1}{\pi}\arccos\left[-\rho(\tau)\right] .$$

$u_{0.975} = qnorm(0.95) \approx 1.645$

$pnorm(1.96) \approx 0.975 \rightarrow \int_{-\infty}^{1.96}$

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(4). (a). $H_0 : \theta = 0 \quad H_1 : \theta = 1 \qquad \sigma = \frac{1}{3}$

(b). design a u-test. $u = \frac{\bar{x} - 0}{\sigma / \sqrt{n}}$

if $|u| \geq u_{1-\alpha}$. $\left(\Leftrightarrow \quad 1 - \Phi(|u_0|) < \alpha = 0.05, \ u_0 = \frac{\sqrt{n}(\bar{x} - 0)}{\sigma} \right)$

$\Leftrightarrow \quad \frac{\bar{x}}{\sigma / \sqrt{n}} \geq u_{0.95} \approx 1.645, \quad \bar{x} \geq \frac{0.548}{\sqrt{n}}, \qquad n = 1 \Rightarrow \bar{x} \geq 0.548$

then we refuse H_0. or we take H_0.

(c). $\beta = P(\bar{x} \leq \frac{0.548}{\sqrt{n}} | \theta = 1)$

$= \int_{-\infty}^{0.548} f_Z(z|H_1) dz$

≈ 0.08755

$\therefore \beta = 0.08755$

$$\boxed{\begin{aligned} f_Z(z|H_0) &= \frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot \exp\left(-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right) \ \overset{N}{(0, \tfrac{1}{9})} \\ f_Z(z|H_1) &= \frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot \exp\left(-\frac{1}{2}\left(\frac{x-1}{\sigma}\right)^2\right) \ \overset{N}{(1, \tfrac{1}{9})} \end{aligned}}$$ (※).

(d). if $\alpha = 0.01$, $X = 0.6$

we refuse H_0 only if $\frac{\bar{x}}{\sigma / \sqrt{n}} \geq u_{0.99} \approx 2.326$

$\Leftrightarrow |\bar{x}| \geq \frac{0.775}{\sqrt{n}}$. (*)

$\because X = 0.6$, if $n=1$, then (*) is wrong, so we can't reject H_0.

(e). $\beta \leq 0.05 \Leftrightarrow \int_{-\infty}^{x_0} f_Z(z|H_1) dz \leq 0.05$

$\Rightarrow x_0 = qnorm(0.05, 1, \frac{1}{3})$, $x_0 = 0.4517155$

$\therefore u_{1-\alpha'} = \frac{x_0}{\sigma / \sqrt{n}} = 1.355147 \Rightarrow \alpha' = 0.0877$.

so the smallest α should be 0.0877.

(f). $P_F = P(\Lambda(Z) > \eta) = \int_{\eta}^{\infty} f_\Lambda(\lambda | H_0) d\lambda$.

where $\Lambda(Z) \triangleq \frac{f_Z(z|H_1)}{f_Z(z|H_0)}$ in (※). ; $\eta \triangleq \frac{(C_{10} - C_{00}) \cdot p_0}{(C_{01} - C_{11}) \cdot q_0}$ P(H₀ true) P(H₁ true)