

MSDM 5056 Network Modeling
Assignment 1 (due 27th September, 2023)

Submit your assignment solution on canvas. You may discuss with others or seek help from your TA, but should not directly copy from others. Otherwise, it will be considered as plagiarism.

(1) **Adjacency matrix.**

Consider the following adjacency matrix of a network

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

- a) Is the network directed or undirected? (Explain why).
- b) Draw the network.
- c) List the in-degree sequence and the out-degree sequence of the network
- d) Determine the in-degree distribution and the out-degree distribution.

(2) **Diameter.**

One can calculate the diameter of certain types of network exactly. Assume that each of the following networks has network size N .

- (a) What is the diameter of a fully connected network?
- (b) What is the diameter of a star network?
- (c) What is the diameter of a linear chain of N nodes? (Figure 1 below)
- (d) What is the diameter D of a square portion of square lattice, with L nodes along each side (figure 2 below) ?
- (e) Consider the expression found in (d), show that the leading term of D in terms of the total number of nodes N in the network, in the limit $N \gg 1$
$$D \approx 2\sqrt{N}$$
- (f) What is the diameter of the corresponding hypercubic lattice in d dimensions ($d=3$ corresponds to a cubic lattice) with L nodes along each side, in the limit $L \gg 1$ (hence $N \gg 1$)?
- (g) A Cayley tree is a symmetric regular tree in which each vertex is connected to the same number k of others, until we get out to the leaves, as shown in Figure 3. ($k=3$ in this case.) Show that the number of vertices reachable in d steps from the central

vertex is $k(k-1)^{d-1}$ for $d \geq 1$. Hence find an expression for the diameter of the network in terms of k and the number of vertices n .



Figure 1

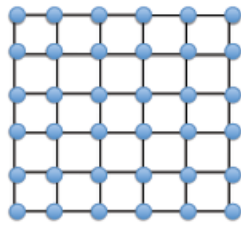


Figure 2

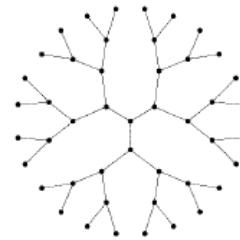


Figure 3

(3) Bipartite matrix.

a) Consider a bipartite network with its two types of vertices, and supposed that there are n_1 vertices of type 1 and n_2 vertices of type 2.

i) What is the maximum number of links L_{max} the network can have?

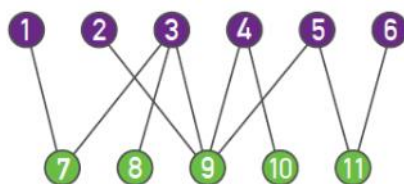
ii) How many links cannot occur compared to a non-bipartite network of size

$$n = n_1 + n_2?$$

iii) Show that the mean degrees c_1 and c_2 of the two types are related by

$$c_2 = \frac{n_1}{n_2} c_1 .$$

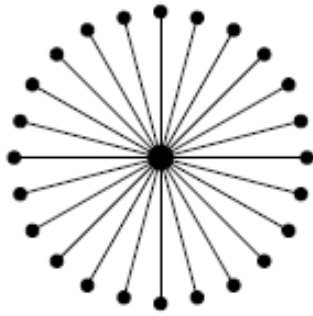
b) Consider the following undirected bipartite network.



Find the incidence matrix of this bipartite network. Then draw the one-mode projections of the bipartite network.

(4) Spectrum of star graphs

A star graph consists of a single central node with $n-1$ other nodes connected to it



- (a) What is the average degree of the above star graph?
- (b) Find the eigenvalues of the adjacency matrix of the star graph.
- (c) Now find the eigenvalues of the Laplacian of the above network. What do its eigenvalues tell you about the graph? (*Refer to p.56 of Lecture 2*)