

# Dynamics of Price Cycles in Agent-Based Models of Financial Markets

by

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The Hong Kong University of Science and Technology  
in Partial Fulfillment of the Requirements for  
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## Abstract

In this thesis, we introduce an agent-based model of financial markets called the Essential Model, and analyze the dynamics of price cycles in the phase of the trendsetters' attractor by examining the detail behavior of different groups of agents in the model. Different types of trendsetters' attractors are classified by the fickle agents they contain and are named by the switching steps of the fickle agents. We obtain the necessary conditions for the existence of the trendsetters' attractors and their sub-phases in the space of market impact  $\beta$  and price sensitivity  $\gamma$ . Besides, we also obtain the lines of maximum probability. The dynamics of the attractors are well studied and the transition of trendsetters' attractors from one type to another is also explained.

# Chapter 1

## Introduction

Statistical physics has been widely used in interdisciplinary researches, for example econophysics [7]. Models are built on adaptive agents interacting competitively and their complex collective behavior are studied in the attempt to understand the consequent equilibrium and dynamical properties [9].

Econophysics has started in the mid 1990's by several physicists and the term "econophysics" was coined by H. Eugene Stanley in a conference on statistical physics in Calcutta in 1995. In econophysics, theories and methods that were originally developed by physicists were applied in solving the problems in economics and financial markets.

Agent-based models that are able to relate the attributes of the individual player, such as their memory sizes and payoffs, to the collective behavior of the system such as the information content, volatility, and the phase transitions are being studied in the physics community in recent years [11, 8, 12, 13, 14].

The Minority Game[8] which is perhaps the most popular model in econophysics is an abstraction of the famous EL-Farol's bar problem [10]. In this model, a population of inductive players make their strategic decisions on the basis of some public information pattern aiming to be in the minority group and taking advantage of their counterparts during trading. Hence, it models the minority advantage in financial markets and the inductive and irrational trading behaviors of individual agents.

The original Minority Game was improved by various versions of games, for example, the influence of wealth in the decisions of the agents was considered [6, 16, 15]. The \$-game considered whether the market makers can be prevented from being arbitrated by the players [14]. Markets with a mixture of majority and minority game players are also studied [5].

Models considering agents with wealth-based strategies in endogenous markets that incorporate several realistic features including an agent's position is also studied by our

group [1]. It is also called the Wealth Game. In this model, there are several kinds of dynamical behaviors at the steady state, or *attractor* behaviors. The focus is the issue of participation incentives, and attractor behavior. The strategies are evaluated directly by their virtual wealth. By tuning the price sensitivity and market impact, a phase diagram with several attractor behaviors resembling those of real markets emerge reflecting the roles played by the arbitrageurs and trendsetters, and including other phases.

This thesis mainly focuses on analyzing the dynamics of the trendsetters' attractor phase in the model described above by examining the detail behavior of different groups of agents acting in the model. A simplified model named as the Essential Model will be used to perform the analyses. The original Wealth Game and the Essential Model will be described in Chapter 2.

It is found that the price trend in the trendsetters' attractor phase can be classified into four stages, as will be described in Chapter 2. Agents holding different strategies play different roles in the games and are active in different stages. The fast agents, holding at least one fast strategy play as the trendsetters and are active in stage 1 and 3. The slow agents, holding at least one slow strategy but no fast strategy, play as the slow trendsetters and are active in stage 1 and 3 as well as the early stage of stage 2 and 4. Agents holding two different strategies and always switching their best strategies are called the fickle agents. They are active at stage 2 and 4.

The fickle agents are always switching their strategies and they are the key point in this thesis. There are different fickle agents in the trendsetters' attractor in the trendsetters' phase as the market impact and price sensitivity varies. By studying the mechanism of the strategies switching of the fickle agents, we can obtain the sub-phases of the trendsetters' attractor constructed with different kinds of fickle agents. This can help us understand the dynamics of the trendsetters' attractor phase in deeper perspectives.

# Chapter 2

## Review of Wealth Game and the Introduction of the Essential Model of Wealth Game

### 2.1 The Original Model of Wealth Game

In the original model of Wealth Game[1], there are  $N$  agents in the market. At each step the agents make trading decisions  $+1$ ,  $-1$ , or  $0$ , which represent buying, selling a unit of stock, or taking a holding position respectively. Denoting the decision of agent  $i$  at time  $t$  by  $a_i(t)$ , the position  $k_i(t)$  of agent  $i$  at time  $t$  is given by

$$k_i(t) = \sum_{t' \leq t} a_i(t'). \quad (2.1)$$

Positive and negative  $k_i(t)$  refer to a *long* and *short* positions respectively. Since agents have limited assets in real markets, we define the maximum position  $K$  by imposing the constraint  $|k_i(t)| \leq K$ . Once the maximum or minimum position is reached, decisions which further increase the magnitude of the positions are ignored.

The stock price should evolve according to their demand and supply. Thus, the price of one unit of stock is updated by

$$P(t+1) = P(t) + \text{sgn}[A(t)] |A(t)|^\gamma, \quad (2.2)$$



or

$$\begin{aligned}\Delta P(t+1) &= P(t+1) - P(t) \\ &= \text{sgn}[A(t)] |A(t)|^\gamma.\end{aligned}\tag{2.3}$$

where  $A(t) \equiv \sum_i a_i(t)$  represents the excess demand. The exponent  $\gamma$  describes the sensitivity of the price increment to the excess demand and is in the region of  $[0, 1]$ .

In the market clearing processes, there is usually a discrepancy between the price expected by an agent when she submitted her bid and the actual transaction price. This discrepancy arises from the sequential process of clearing the deals at each time step, during which the transaction price changes from its present value to the new value according to Eq. (2.2). For simplicity, we assume that the transaction price is the same for all transactions at time  $t$ . Thus, we approximate the transaction price  $P_T$  as

$$P_T(t) = (1 - \beta)P(t) + \beta P(t+1).\tag{2.4}$$

with  $0 \leq \beta \leq 1$ . The variable  $\beta$  acts as a *market impact factor*, since it arises from the collective effects of agent participation, and reduces the profit of the majority of the participating agents to the so-called price takers.

We let  $w_i(t)$  be the wealth of agent  $i$  at time  $t$  just after the decision  $a_i(t)$  is carried out and the transaction is completed. The wealth  $w_i(t)$  is the sum of cash  $c_i(t)$  and stock values at hand, namely,

$$w_i(t) = c_i(t) + k_i(t)P_T(t).\tag{2.5}$$

$P_T(t)$  instead of  $P(t)$  is used in calculating wealth, since it represents the actual stock value once the transaction is completed. Cash is updated according to the buying and selling of stocks at time  $t$ , namely,

$$c_i(t) = c_i(t-1) - a_i(t)P_T(t).\tag{2.6}$$

Suppose agent  $i$  buys (sells) a unit of stock at time  $t$ . Her cash is lowered (raised) by an amount  $P_T(t)$  while her value of stocks in hand is increased by the same amount, so that any change in wealth is due to the change in value of stocks she previously held. After rearranging Eqs. (2.2), (2.4), and (2.5), the wealth change of agent  $i$  after transactions at time  $t$  can be expressed as

$$w_i(t) - w_i(t-1) = k_i(t-1)[P_T(t) - P_T(t-1)].\tag{2.7}$$

Next, we consider the strategies used by the agents to reach their decisions in the evolving market environment. The market environment is described by the string of the  $m$  most recent outcomes of the sign of price change. Thus, there are  $P = 2^m$  possible states, and  $m$  is called the *memory size*. A strategy prescribes the decisions  $+1, 0, -1$  in response to each of the  $P$  states of the market environment. Each agent draws  $S$  strategies randomly at the beginning of the game. Every strategy should have at least one buying and selling decisions. At every time step, each agent selects the most successful strategy among the  $S$  strategies she owns and uses it to make a decision. The success of a strategy is measured by the virtual wealth it should have acquired were its decisions followed in the market history. This means that for strategy  $\sigma$ , we start with the initial virtual wealth  $w_\sigma(0) = 0$  and update their values in the same way as we do for real wealth, that is,

$$k_\sigma(t) = \max\{-K, \min[K, k_\sigma(t-1) + a_\sigma(t)]\}. \quad (2.8)$$

$$w_\sigma(t) - w_\sigma(t-1) = k_\sigma(t-1)[P_T(t) - P_T(t-1)]. \quad (2.9)$$

These equations show that the assessment of strategies is radically different from conventional models of the Minority Game, in which the success of a strategy is updated by an instantaneous comparison of its decision and the excess demand. In contrast, the virtual wealth of a strategy is updated by a comparison of its virtual position and the excess demand (through the price change). Since the virtual position depends on the history of decisions, it embeds a longer memory in strategy selection, thus inducing a greater sophistication in the collective behavior such as that exhibited in the trendsetters' attractor to be studied in this thesis.

## 2.2 The Essential Model of Wealth game

In the essential model, only the essential elements are kept in order to reduce the complexity of the trendsetters' attractor analyses. The essential elements are:

(1) The decisions of the strategies are either *buy* or *sell*. There are no *hold* decisions. Holding actions can only result from decisions which increase the magnitude of the positions beyond the limit  $K$ .

(2) The memory size  $m$  of the strategies is 2. Furthermore, the responses to the signals  $\uparrow\downarrow$  and  $\downarrow\uparrow$  are opposite. This effectively reduces the input dimension to 3.

The analysis of the strategies is simplified, since there are now only four strategies, and their anti-strategies. The four strategies are as shown in Table 2.1. Agents adopting these four strategies play different roles in the market dynamics.

	Decision			
Signal	Fast Trendsetter	Slow Trendsetter	Bottom Trigger	Top Trigger
$\downarrow\downarrow$	sell	sell	sell	buy
$\downarrow\uparrow$	buy	sell	buy	buy
$\uparrow\downarrow$	sell	buy	sell	sell
$\uparrow\uparrow$	buy	buy	sell	buy

Table 2.1: The four strategies in the essential model. The decisions of the four anti-strategies are opposite to the respective strategies.

The first two kinds of strategies in Table 2.1 are the trendsetters' strategies. They consist of a buying decision responding to a signal with two consecutive instants of rising price, and a selling decision responding to a signal with two consecutive instants of dropping price. Hence when a rising price trend has been set up, the trendsetters will continue to buy, pushing the price further up. Similarly, their decisions tend to push the price down in a falling price trend. The next two strategies are trigger strategies. They are either continually buying or selling but reverse their decisions when the signal is  $\uparrow\downarrow$  or  $\downarrow\uparrow$ .

Agents holding different strategies play different roles in the market. Broadly speaking, they can be divided in two groups. They are also called contrarians (or fundamentalists) and trend-followers (or chartists) in the literature [2]. It has been argued [5, 6] that contrarians are described as minority game players and the trend-followers are described as majority game players. The trend-followers extrapolate trends from recent price increments and *buy* or *sell* assuming that the next increment will occur in the direction of the trend and is equivalent to the role of the trendsetters in our model. In our model, the trendsetters make decisions based on the history and predict the next signal of the price trend. Hence, their decisions can set up the price trend as well as follow the direction of the trend. Therefore they are called trendsetters in our model. The contrarians believe that the market is close to a stationary state and only make *buy* or *sell* decisions when they consider the stock to be underpriced or overpriced. The bottom and top agents in our model belong to this category. The bottom strategy in our model always takes the *sell* decision and make *buy* decisions only when the price is at the bottom which means the stock is undervalued. Hence they can be considered to be the pessimistic traders. The top strategy, on the other hand, always takes the *buy* decision and make *sell* decisions only when the price reaches the peak which means the stock is overpriced. They are the optimistic traders.

There are two different trendsetter strategies in the essential model. These two kinds of trendsetters have different time lapses in responding to the onset of a trend. The fast

trendsetters respond to the signal of  $\downarrow\uparrow$  or  $\uparrow\downarrow$  with buying or selling decisions. They detect the change in the trend at the earliest possible instant and become the first group of trendsetters to benefit in the new trend. The slow trendsetters respond to the signals of  $\downarrow\uparrow$  or  $\uparrow\downarrow$  by selling or buying decisions, and only join the fast trendsetters one step later.

There are two kinds of triggers in the essential model. The agents holding only the bottom trigger strategy are dominantly sellers and only make buying decisions responding to a  $\downarrow\uparrow$  signal. Thus, they take part in triggering a rising trend when the price reaches a bottom. Hence, they are called the bottom triggers. The agents holding only the top trigger strategy are dominantly buyers and only make selling decisions responding to a  $\uparrow\downarrow$  signal. Thus, they take part in triggering a dropping trend when the price reaches a top. Hence, they are called the top triggers.

The agents can be classified according to the strategies they hold and their behavior in the price cycle. Agents holding the trendsetter strategies are considered as the trendsetters. Agents holding two competitive strategies and switch from one strategy to the other even at the steady state are considered the fickle agents. The strategies of the bottom and top triggers are related by gauge symmetry. They are interchanged if one interchanges the labels of the  $\uparrow$  and  $\downarrow$  signals, and the buy and sell decisions. Thus, in the trendsetters' attractor with a symmetric rising and dropping trend, the virtual wealth gained by the bottom and the top triggers is identical at the steady state. This is the pre-requisite that an agent holding a bottom and top trigger strategy becomes a persistent fickle agent. These fickle agents are crucial in maintaining the stability of the trendsetter's attractor and the process will be described in Section 2.4.

## 2.3 Observations in the Essential Model

In the Wealth Game [1], it is found that there are four phases in the  $\gamma$  and  $\beta$  phase diagram. There are the phases of the trendsetters' attractor, the arbitrageurs' attractor, the irregular phase and the mixture phase as shown in Fig 2.1. The trendsetters' attractor will be described below. The arbitrageurs are the agents who buy at a lower price and sell at a higher price. The price cycle in the arbitrageurs' attractors are period two, in which the arbitrageurs buy at the low point of the cycles and sell at the high point. In the irregular phase, the periodic and quasi-periodic attractors cannot persist, and the price trend becomes irregular. The mixture phase has both the trendsetters' and arbitrageurs' attractors.

These four phases are also present in the Essential Model. We study the system

behavior of the Essential Model in the space of the price sensitivity  $\gamma$  and market impact  $\beta$ . The phase space is dominated by the trendsetters' attractor for  $\beta$  above 0.5, and the arbitrageurs' attractor below 0.5. In this thesis we are interested in the trendsetters' phase.

The trendsetters' attractor is periodic as shown in Fig. 2.2. The trendsetter strategies gain wealth by holding a long (short) position when the price is rising (dropping), as shown in Fig. 2.1. Coordinating with the price trend in Fig. 2.1, these strategies enable one to gain wealth at all time steps in a market dominated by the trendsetters' attractors for the fast and slow trendsetter strategies, except near the points where a rising trend switches to a dropping one for the slow strategy, or vice versa. Consequently, the trendsetter strategies are the most successful strategies in a trendsetters' attractors especially the fast trendsetter strategy. Hence, the agents holding at least one trendsetter strategy will make decisions accordingly.

Fig. 2.3 shows the probability of finding the trendsetters' attractor in the space of price sensitivity  $\gamma$  and market impact  $\beta$ . The initial condition of the wealth of each agent starts with zero. Agents choose the strategies randomly and the initial signal is also random. The graph shows that the region of the trendsetters' attractor is bounded by the line  $\beta = 0.5$ , separating it from the region of arbitrageurs' attractor for  $\beta$  below 0.5. It also shows that the region is bounded by a phase line starting from  $\gamma \approx 0.8$  at  $\beta = 1$ , and ending at  $\gamma \approx 0.9$  at  $\beta = 0.5$ , separating it from the irregular region to the right of the phase line.

The price trends in the trendsetter attractor phase are similar to the bubbles and crashes in the oscillating phase of the Giardina-Bouchaud model [6]. In their model, by varying the two parameters, one is the impact of trading on the price and the other is the propensity of agents to be trend following or contrarian, they obtain three phases which are similar to our model. The oscillatory regime corresponding to "weak coupling" where speculative bubbles are formed, and finally collapse in sudden crashes induced by the fundamentalist behavior. The intermittent regime corresponding to the "stylized" facts of liquid markets are well reproduced. The stable regime corresponds to sufficiently negative polarization and the fluctuations of the price are mild and mean reverting. The arbitrageurs' phase in our model is similar to the stable "rational" market phase. The irregular phase in our model is similar to the intermittent phase.  $\gamma$  is the sensitivity of the price increment to the excess demand and is similar to  $g/\lambda$ [6] which represent the impact of trading on the price. With the other parameters fixed when  $\gamma$  increases, there is a phase shift from the trendsetters' phase to an irregular one. Similarly, when  $g/\lambda$  increases, there is a phase shift from the periodic phase to an intermittent one.

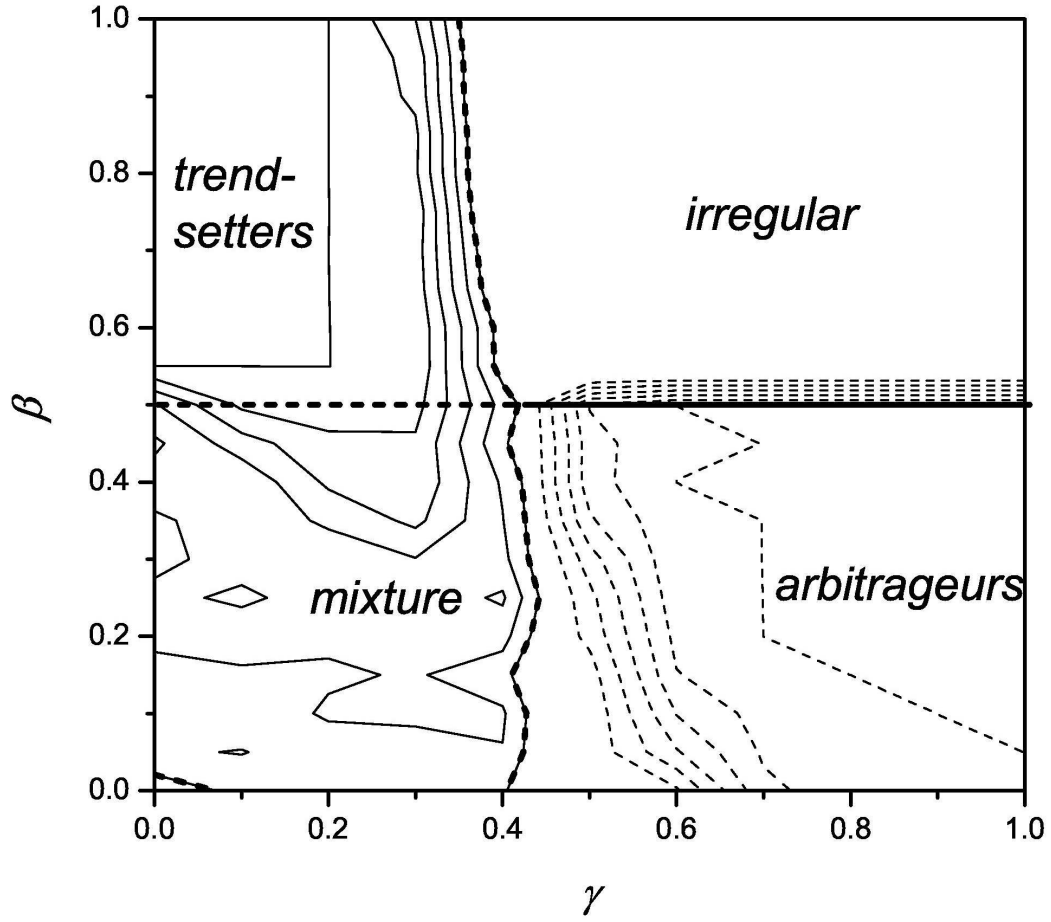
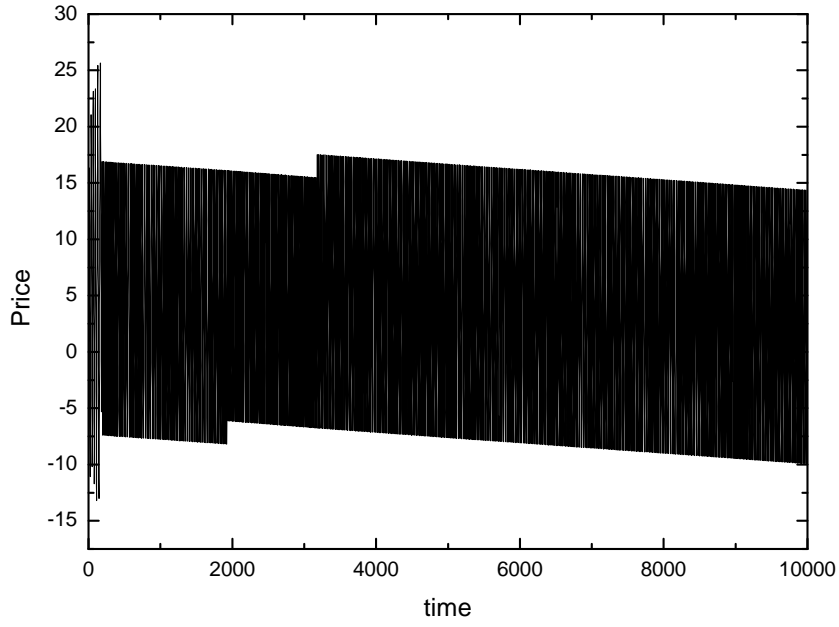
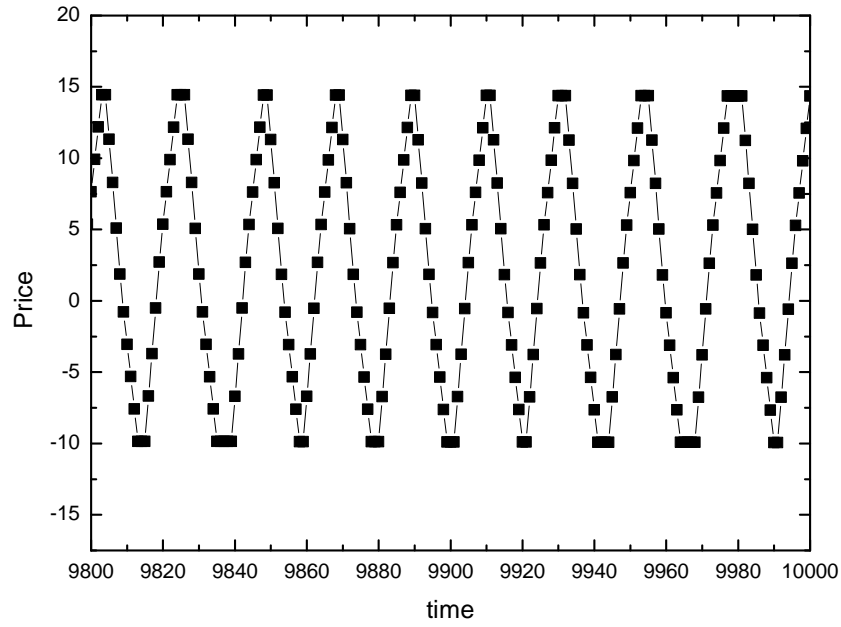


Figure 2.1: The phase diagram in the Wealth Game ( $N=1000$ ,  $m=3$ ,  $S=2$ ,  $K=3$ ,  $10^6$  steps, and 100 samples) [1].



(a)



(b)

Figure 2.2: An example of the price series in the essential model. (a) The price series from step 0 to 10000 (b) The close-up look of the price series from step 9800 to 10000. ( $N=1000$ ,  $K=2$ ,  $m=2$ ,  $S=2$ ,  $\gamma=0.2$ ,  $\beta=0.7$ ).

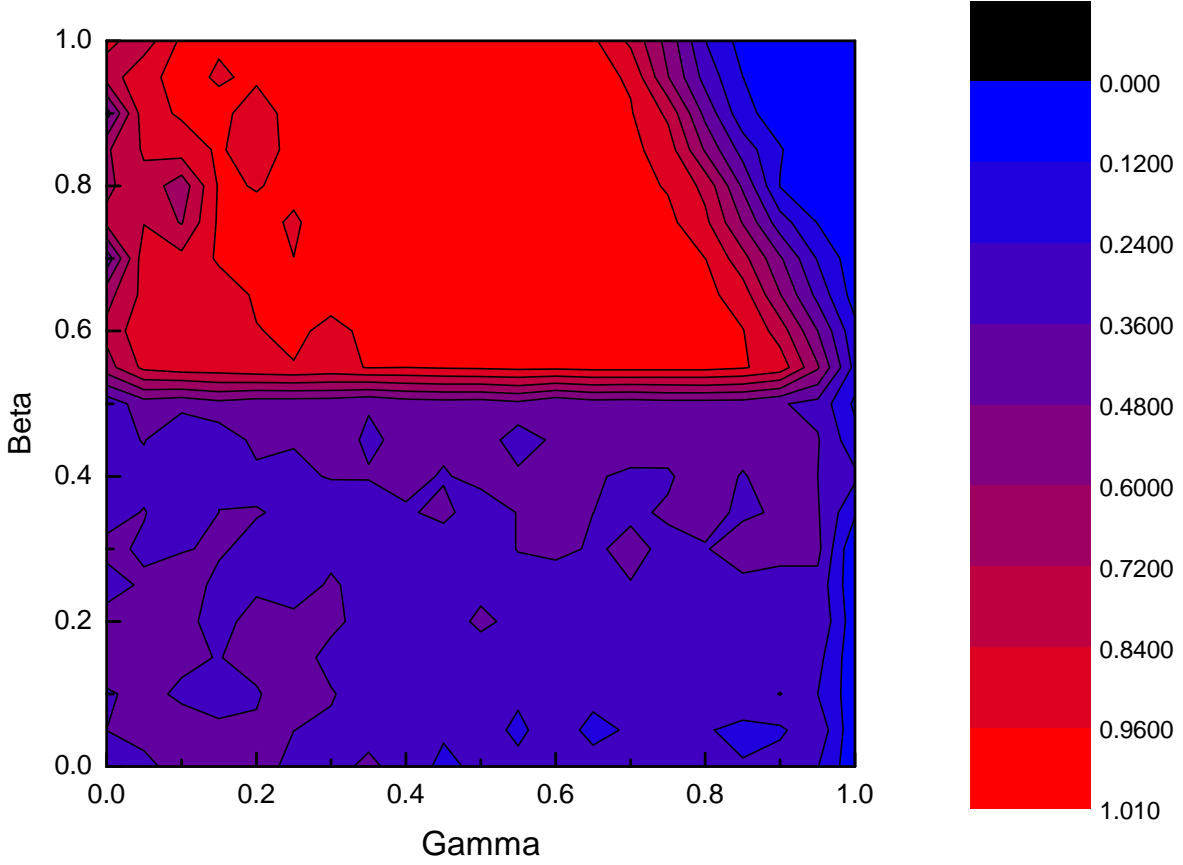


Figure 2.3: The probability of finding the trendsetters' attractor in the space of price sensitivity  $\gamma$  and market impact  $\beta$ . ( $N=1000$ ,  $K=2$ ,  $m=2$ ,  $S=2$ , 20000 Steps and 200 samples)

Arbitrageurs phase exist when  $\beta$  is small while stable (anti-correlated) phase exist when  $P$  is small.

While the Giardina-Bouchaud model describes the formation of the bubbles and crashes using mean-field theory, the roles of the different kinds of agents played in the the formation of the trendsetters' attractors are not studied in detail. Using the Essential Model, we are able to clarify this issue.

The dynamics of the attractors in our model consists of four stages as shown in Fig. 2.4.

(1) In stage 1, the trendsetters make buying decisions collectively. Starting from short positions, they accumulate stocks step by step and switch from minimum to maximum positions in  $2K$  steps. The fast trendsetters are the earliest buyers and, for the first step only, are joined by the bottom triggers in the first step. The slow trendsetters join in the second step. At the end of this stage, the positions of the trendsetters reach the maximum, and they are restrained from further buying actions.



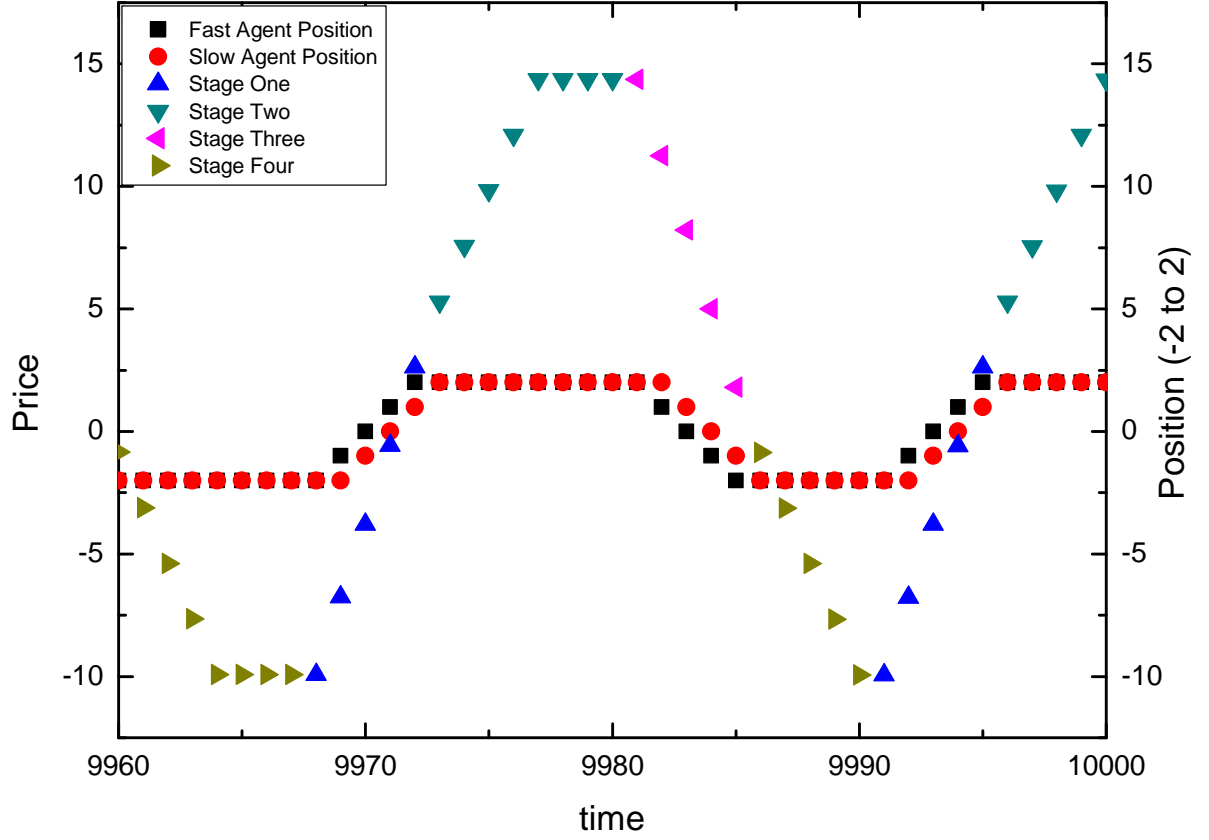


Figure 2.4: An example of the trendsetters' attractor in the essential model showing the price and the fast and slow agents' positions. ( $N=1000$ ,  $K=2$ ,  $m=2$ ,  $S=2$ ,  $\gamma=0.2$ ,  $\beta=0.8$ ).

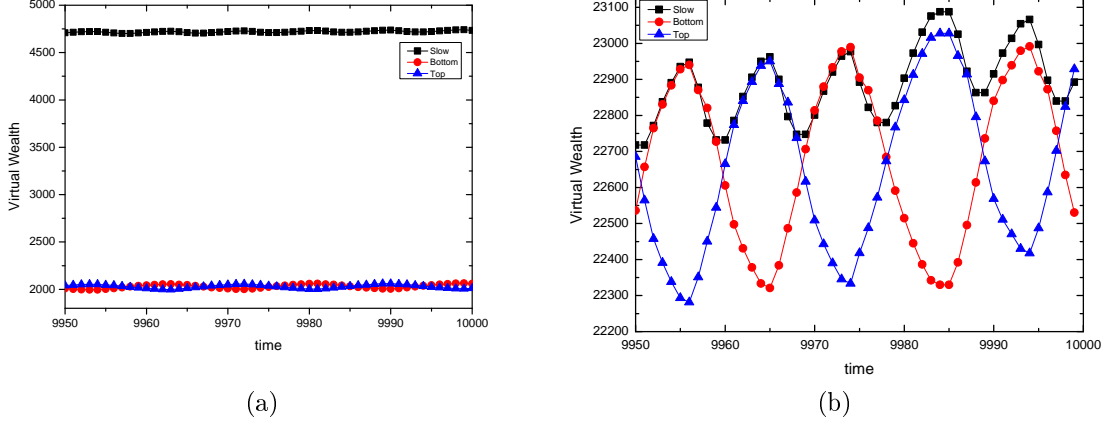


Figure 2.5: An example of the virtual wealth for the slow, bottom and top strategies with small and large price sensitivity. (a) The virtual wealth of the slow, the bottom and top strategies with  $\gamma=0.3$ ,  $\beta=0.7$ ,  $N=1000$ ,  $K=2$ ,  $m=2$ ,  $S=2$ . (b) The virtual wealth of the slow strategy starts to surfing on the virtual wealth of the bottom and top strategies with  $\gamma=0.7$ ,  $\beta=0.7$ ,  $N=1000$ ,  $K=2$ ,  $m=2$ ,  $S=2$ .

(2) In stage 2, the fickle agents take up the role of collective buying and push the price further up. When the price sensitivity  $\gamma$  is small, an important group of fickle agents hold a bottom and a top strategy, whose virtual wealth is shown in Fig. 2.5(a). Due to their pre dominantly selling decisions, the bottom strategy keeps a minimum virtual position most of the time, and hence the rise and drop of their virtual wealth are out of phase with those of the price. The top strategy, with pre dominantly buying decisions, keeps a maximum virtual position most of the time, hence the rise and drop of the their virtual wealth are in phase with the price trend. Consequently, during stage 1 the bottom triggers are wealthier than the top triggers. However, at the beginning of stage 2, the top triggers take over, and cause the fickle agents to take buying actions in the second stage, pushing the price further up. At the end of this stage, the positions of the fickle agents reach the maximum and they are restrained from further buying actions.

For sufficiently large price sensitivity  $\gamma$ , the price rising trend in stage 2 become relatively flat and the wealth gain of the slow strategy reduces. Since the wealth of the slow strategy is no longer increasing significantly as in small  $\gamma$  region, the virtual wealth of the slow strategy will become comparable with that of the bottom or the top strategy as shown in Fig. 2.5(b). Hence, we will observe other groups of fickle agents holding other combination of strategies. They may take buying actions later than those holding bottom and top strategies including the slow strategy. As a result, their presence lengthens the duration of stage 2.

The market will then remain quiet with no price movement. Since the excess demand is identically zero, a  $\uparrow$  or  $\downarrow$  signal of the price is randomly generated. If the randomly generated signal is  $\uparrow$ , the market will continue to remain quiet. So the market may remain quiet for one or more steps at the end of stage 2. On the other hand, if the randomly generated signal is  $\downarrow$  the market will be driven to the next stage.

(3) Stage 3 is triggered by a randomly generated selling signal which is the  $\downarrow$  signal, and the trendsetters respond by collective selling decisions. Starting from long positions, they cash in stocks step by step and switch from maximum to minimum positions in  $2K$  steps. At the end of this stage, the positions of the trendsetters reach the minimum, and they are restrained from further selling actions.

(4) In stage 4, the fickle agents switch their strategies to selling, since the falling price in stage 3 reduces the virtual wealth of the top strategies and boosts that of the bottom strategies. At the end of this stage, the positions of the fickle agents reach the minimum, and they are restrained from further selling actions. The market will then remain quiet with no price movement, waiting for a random signal to drive another cycle.

The behavior of the agents in the trendsetters' attractor resemble those in real financial markets. The trendsetters respond to a bullish signal and take buying actions collectively, creating a bullish market. They stop buying due to their finite capital or assessment of risks, followed by the fickle agents who try to catch up. Eventually the market becomes quiet when all agents have exhausted their capital or tolerance to risks. Then a bearish signal appears and the trendsetters sell their stocks, followed by the fickle agents.

# Chapter 3

## Trendsetters' Attractors With a Single Kind of Fickle Agents and Anti-agents

After discussing the behaviors of each agent in the essential model, we would like to focus on the theoretical analyses of the model in this chapter. We will explain the role played by the fickle agents in the trendsetters' attractors in section 3.1, and introduce the classification of the attractors in Section 3.2. In this chapter we will consider trendsetters' attractors with only one kind of fickle agents. They are referred to as the (600) and (500) trendsetters' attractor according to our classification scheme.

### 3.1 The Role of the Fickle Agents

We have seen in Chapter 2 that fickle agents are present in the trendsetters' attractor. As shown in Fig. 3.1, the virtual wealth of the bottom and top strategies are comparable. Hence, an agent holding a bottom and a top strategy will switch her strategies periodically, and is therefore a fickle agent. We call them the bottom&top fickle agents or ficklers. As we shall see, the stability of the fickle agents determines the stability of the trendsetters' attractor. Since the virtual wealth of an anti-strategy is equal to the minus of that of the strategy, an agent having an anti-bottom and anti-top strategy is also a fickle agent.

As will be described in Chapter 4, there are parameter regions that the virtual wealth of the slow strategy will become comparable with those of the bottom and top strategies. In this case, we will have three kinds of fickle agents: the bottom&top fickler, the slow&bottom fickler and the slow&top fickler.

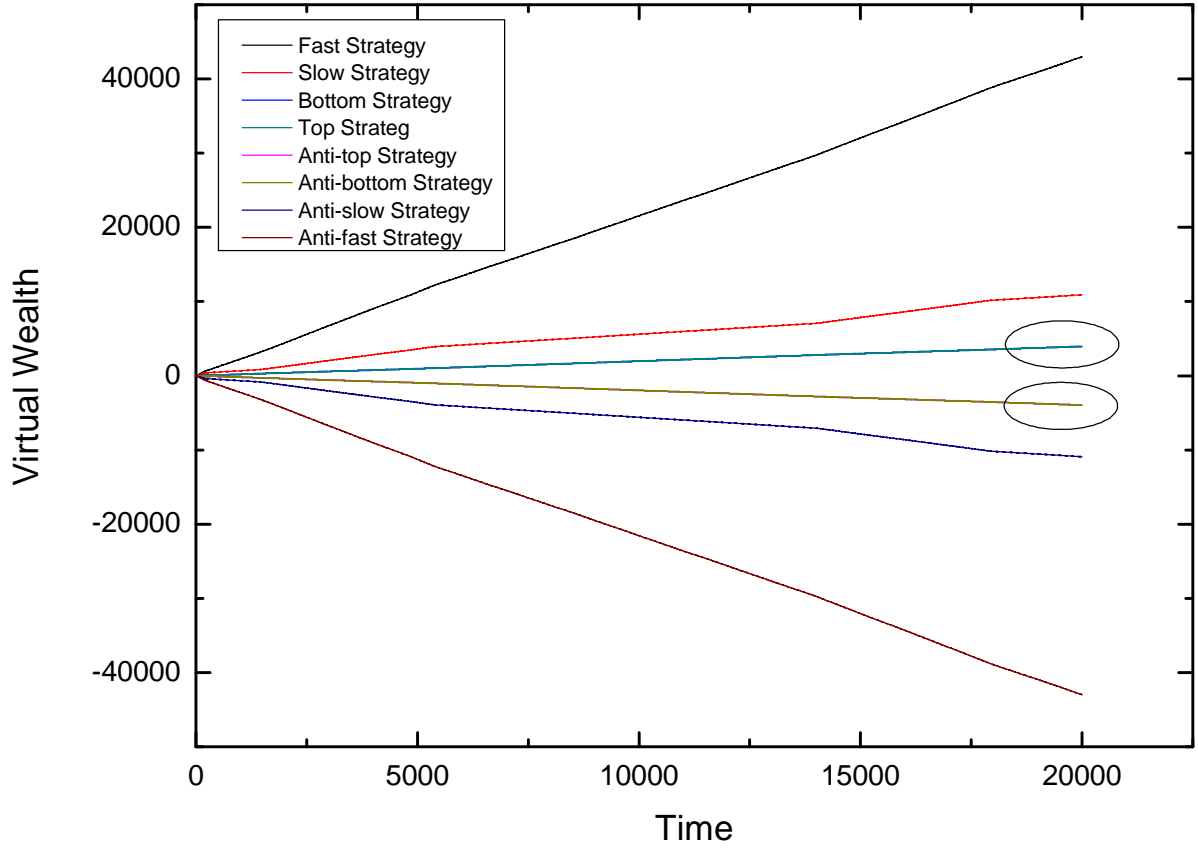


Figure 3.1: Time dependence of the virtual wealth of the strategies in the essential model. The encircled regions are magnified in Fig. 3.2. The virtual wealth of the strategies are actually oscillating if we enlarge the scales. Parameters:  $K = 2$ ,  $N = 1000$ ,  $\gamma = 0.2473$ ,  $\beta = 0.7008$ .

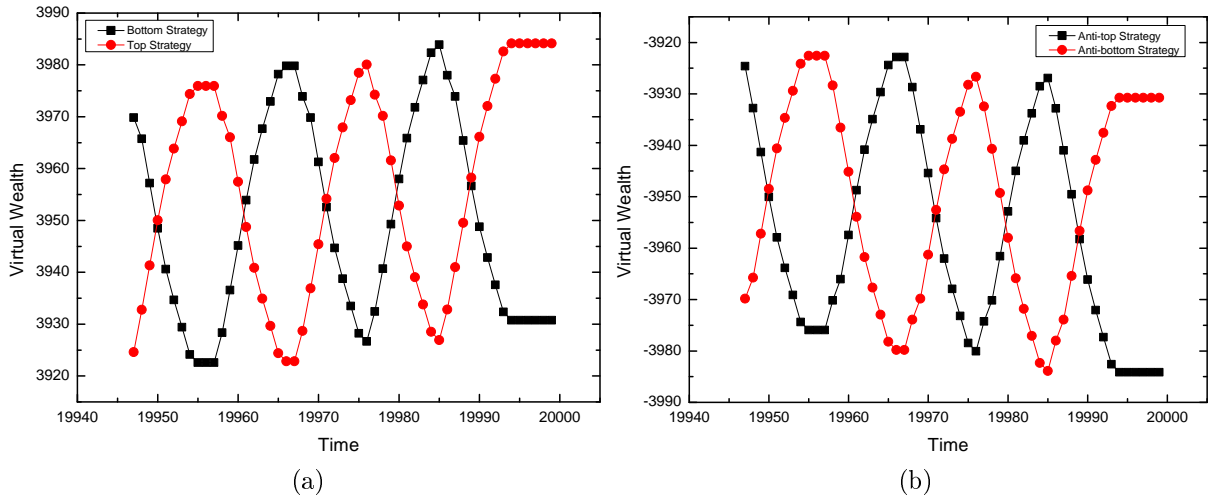


Figure 3.2: The time dependence of the virtual wealth of (a) bottom and top strategies (b) anti-bottom and anti-top strategies.

## 3.2 The Definition of the $(ijk)$ Trendsetters' Attractor

In our analyses, we classify the trendsetters' attractor with the three kinds of fickle agents we explained in Section 3.1. As we observed, in each price cycle, the fickle agents will switch their strategies twice, once when the price is rising, and once when the price is dropping. For convenience, we label the time step when the price starts to rise in a cycle as 1, we classify a trendsetters' attractor as  $(ijk)$  in the following way. Because of the symmetry of the price cycle, the bottom&top fickle will switch her strategy from top to bottom at the  $i$ th rising step as the price starts to rise and at the  $i$ th step when the price starts to drop in a  $(ijk)$  attractor. The slow&bottom fickle will switch at the  $j$ th rising and dropping steps. For the slow&top fickle, they will switch twice both at the  $k$ th rising and dropping steps.

## 3.3 (600) Trendsetters' Attractor Analyses

The first trendsetters' attractor we would like to analyze is the (600) trendsetters' attractor. (600) stands for the bottom&top fickle in this trendsetters' attractor switch strategy at the 6th rising step or the 6th dropping step. There is no slow&bottom fickle or slow&top fickle in this trendsetters' attractor so we let  $j$  and  $k$  both equal to zero.

### 3.3.1 The Agents' Decisions

To work out the effective region of the (600) trendsetters' attractor we need to carefully trace the decisions of the agents, and deduce the number of buyers and sellers at each step. Table 3.1 shows the decisions made in the (600) trendsetters' attractor in each price cycle. Let us explain how the row at time = 1 in Table 3.1 is calculated; the calculations of the other rows are similar. As we observe from Fig 3.1, the virtual wealth of the strategies separates as:  $f > s > b \& t > \bar{b} \& \bar{t} > \bar{s} > \bar{f}$ . We know that each agent holds two strategies from these eight strategies. So the agent holding at least one fast strategy will always use the fast strategy, so she is called a fast agent. The strategies held by a fast agent can be:  $f \& f$ ,  $f \& s$ ,  $f \& b$ ,  $f \& t$ ,  $f \& \bar{f}$ ,  $f \& \bar{s}$ ,  $f \& \bar{b}$ ,  $f \& \bar{t}$ , the fractions are:  $\frac{1}{8} \times \frac{1}{8}$  for  $f \& f$ ,  $2 \times \frac{1}{8} \times \frac{1}{8}$  for the rest. So the fraction of the fast agents is  $\frac{15}{64}$  (here we assume that the strategies are evenly distributed). Using the same analysis, we can find the fractions of the agents according to the strategies they hold. The fractions are:  $13/64$ ,  $9/64$ ,  $9/64$ ,  $2/64$ ,  $2/64$ ,  $5/64$ ,  $5/64$ ,  $3/64$ , and  $1/64$  for slow, bottom, top, bottom&top, anti-bottom&anti-top, anti-top, anti-bottom, anti-slow, and anti-fast agents respectively.

In Table 3.1, the price signal is  $\uparrow$  from time 1 to 10, the price signal is  $\downarrow$  from time 11 to 20. This means the number of buyers is greater than the number of sellers from time 1 to 10 while less from time 11 to 20.

When the price is rising, the decisions of  $f, b, \bar{f}, \bar{b}$  follow the signal, since they do not reach their position limit, they will make decisions as *buy, buy, sell, sell* respectively. For the other strategies, as they have already reached their position limit, while their original decisions following the signal are still telling them to exceed the limit. Hence, they are forced to take the holding action. Table 3.2 shows the positions of the agents at each time step in one price cycle. The total number of buyers in the row is the sum of the agents making the *buy* decision. The total number of sellers in the row is the sum of the agents making the *sell* decision. The price change is given in Eq. (2.3) as

$$D_x = \left( \frac{|x|N}{64} \right)^\gamma \text{sgn}(x). \quad (3.1)$$

where  $\frac{x}{64}$  is the fractional difference between the number of buyers and sellers.

### 3.3.2 The Agents' Virtual Wealth

Since we have the price change at each step in the price cycle, we can now calculate the change of the virtual wealth in each step for all strategies. Since the stability of the bottom and the top trigger strategies determines the stability of the attractors, we show in Table 3.3 and 3.4 the virtual wealth change of the bottom and top strategies respectively using Eq. (2.3).

### 3.3.3 The Matching Conditions

We know that a fickle agent will switch her strategy at a certain step in the price cycle. In (600), the bottom and top fickle will switch strategies twice. The first time is at the 6th rising step when the agent switches from the bottom strategy to the top strategy. The second time is at the 6th dropping step (which is the 16th step in the cycle) when the agent switches from the top strategy to the bottom strategy. The switching of the strategies is based on the virtual wealth of the two strategies as the agent always chooses the one with the higher virtual wealth. Now we can list out the matching conditions for this strategy switching scheme to be valid. The matching conditions of the fickle anti-agents are the same. They are the necessary (but not sufficient) conditions of the attractors.

time	f $\frac{15}{64}$	s $\frac{13}{64}$	b $\frac{9}{64}$	t $\frac{9}{64}$	b&t $\frac{2}{64}$	$\bar{b}\&\bar{t}$ $\frac{2}{64}$	$\bar{t}$ $\frac{5}{64}$	$\bar{b}$ $\frac{5}{64}$	$\bar{s}$ $\frac{3}{64}$	$\bar{f}$ $\frac{1}{64}$	No. of buyers (in units of $\frac{N}{64}$ )	No. of sellers (in units of $\frac{N}{64}$ )	Price Change $D_x$
0	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$
1	b	h	b	h	b	h	h	s	h	s	26	6	$D_{20}$
2	b	b	s	h	s	h	h	b	s	s	33	15	$D_{18}$
3	b	b	h	h	h	h	h	h	s	s	28	4	$D_{24}$
4	b	b	h	h	h	h	h	h	s	s	28	4	$D_{24}$
5	h	b	h	h	h	h	h	h	s	h	13	3	$D_{10}$
6	h	h	h	h	b	b	h	h	h	h	4	0	$D_4$
7	h	h	h	h	b	b	h	h	h	h	4	0	$D_4$
8	h	h	h	h	b	b	h	h	h	h	4	0	$D_4$
9	h	h	h	h	b	b	h	h	h	h	4	0	$D_4$
10	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$
11	s	h	h	s	s	h	b	h	h	b	6	26	$D_{-20}$
12	s	s	h	b	b	h	s	h	b	b	15	33	$D_{-18}$
13	s	s	h	h	h	h	h	h	b	b	4	28	$D_{-24}$
14	s	s	h	h	h	h	h	h	b	b	4	28	$D_{-24}$
15	h	s	h	h	h	h	h	h	b	h	3	13	$D_{-10}$
16	h	h	h	h	s	s	h	h	h	h	0	4	$D_{-4}$
17	h	h	h	h	s	s	h	h	h	h	0	4	$D_{-4}$
18	h	h	h	h	s	s	h	h	h	h	0	4	$D_{-4}$
19	h	h	h	h	s	s	h	h	h	h	0	4	$D_{-4}$
20	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$

Table 3.1: The decisions made by different types of in the (600) trendsetters' attractor. Steps 0, 10 and 20 correspond to the quiet market, and may include one of more steps depending on the stochastic generation of random signals.  $b$ ,  $h$ , and  $s$  represent *buy*, *hold*, and *sell* decisions respectively. The column headings abbreviate the fast, slow, bottom, top, anti-fast, anti-slow, anti-bottom, anti-top, top&bottom, anti-top&anti-bottom agents respectively.



time	f	s	b	t	b&t	$\bar{b}\&\bar{t}$	$\bar{t}$	$\bar{b}$	$\bar{s}$	$\bar{f}$
	(15/64)	(13/64)	(9/64)	(9/64)	(2/64)	(2/64)	(5/64)	(5/64)	(3/64)	(1/64)
0	-2	-2	-2	2	-2	-2	-2	2	2	2
1	-2	-2	-2	2	-2	-2	-2	2	2	2
2	-1	-2	-1	2	-1	-2	-2	1	2	1
3	0	-1	-2	2	-2	-2	-2	2	1	0
4	1	0	-2	2	-2	-2	-2	2	0	-1
5	2	1	-2	2	-2	-2	-2	2	-1	-2
6	2	2	-2	2	-2	-2	-2	2	-2	-2
7	2	2	-2	2	-1	-1	-2	2	-2	-2
8	2	2	-2	2	0	0	-2	2	-2	-2
9	2	2	-2	2	1	1	-2	2	-2	-2
10	2	2	-2	2	2	2	-2	2	-2	-2
11	2	2	-2	2	2	2	-2	2	-2	-2
12	1	2	-2	1	1	2	-1	2	-2	-1
13	0	1	-2	2	2	2	-2	2	-1	0
14	-1	0	-2	2	2	2	-2	2	0	1
15	-2	-1	-2	2	2	2	-2	2	1	2
16	-2	-2	-2	2	2	2	-2	2	2	2
17	-2	-2	-2	2	1	1	-2	2	2	2
18	-2	-2	-2	2	0	0	-2	2	2	2
19	-2	-2	-2	2	-1	-1	-2	2	2	2
20	-2	-2	-2	2	-2	-2	-2	2	2	2

Table 3.2: The position of the agents at each step in one cycle.

time	Position	Price Change	Wealth Change of the Bottom Strategy
0	-2	$D_0$	
1	-2	$D_{20}$	$-2\beta D_{20}$
2	-1	$D_{18}$	$-[(1 - \beta)D_{20} + \beta D_{18}]$
3	-2	$D_{24}$	$-2[(1 - \beta)D_{18} + \beta D_{24}]$
4	-2	$D_{24}$	$-2[(1 - \beta)D_{24} + \beta D_{24}]$
5	-2	$D_{10}$	$-2[(1 - \beta)D_{24} + \beta D_{10}]$
6	-2	$D_4$	$-2[(1 - \beta)D_{10} + \beta D_4]$
7	-2	$D_4$	$-2[(1 - \beta)D_4 + \beta D_4]$
8	-2	$D_4$	$-2[(1 - \beta)D_4 + \beta D_4]$
9	-2	$D_4$	$-2[(1 - \beta)D_4 + \beta D_4]$
10	-2	$D_0$	$-2(1 - \beta)D_4$
11	-2	$D_{-20}$	$2\beta D_{20}$
12	-2	$D_{-18}$	$2[(1 - \beta)D_{20} + \beta D_{18}]$
13	-2	$D_{-24}$	$2[(1 - \beta)D_{18} + \beta D_{24}]$
14	-2	$D_{-24}$	$2[(1 - \beta)D_{24} + \beta D_{24}]$
15	-2	$D_{-10}$	$2[(1 - \beta)D_{24} + \beta D_{10}]$
16	-2	$D_{-4}$	$2[(1 - \beta)D_{10} + \beta D_4]$
17	-2	$D_{-4}$	$2[(1 - \beta)D_4 + \beta D_4]$
18	-2	$D_{-4}$	$2[(1 - \beta)D_4 + \beta D_4]$
19	-2	$D_{-4}$	$2[(1 - \beta)D_4 + \beta D_4]$
20	-2	$D_0$	$2[(1 - \beta)D_4]$

Table 3.3: Wealth change for the bottom strategy at each step in one price cycle of the (600) trendsetters' attractor.

time	Position	Price Change	Wealth Change of the Top Strategy
0	2	$D_0$	
1	2	$D_{20}$	$2\beta D_{20}$
2	2	$D_{18}$	$2[(1 - \beta)D_{20} + \beta D_{18}]$
3	2	$D_{24}$	$2[(1 - \beta)D_{18} + \beta D_{24}]$
4	2	$D_{24}$	$2[(1 - \beta)D_{24} + \beta D_{24}]$
5	2	$D_{10}$	$2[(1 - \beta)D_{24} + \beta D_{10}]$
6	2	$D_4$	$2[(1 - \beta)D_{10} + \beta D_4]$
7	2	$D_4$	$2[(1 - \beta)D_4 + \beta D_4]$
8	2	$D_4$	$2[(1 - \beta)D_4 + \beta D_4]$
9	2	$D_4$	$2[(1 - \beta)D_4 + \beta D_4]$
10	2	$D_0$	$2[(1 - \beta)D_4]$
11	2	$D_{-20}$	$-2\beta D_{20}$
12	1	$D_{-18}$	$-[(1 - \beta)D_{20} + \beta D_{18}]$
13	2	$D_{-24}$	$-2[(1 - \beta)D_{18} + \beta D_{24}]$
14	2	$D_{-24}$	$-2[(1 - \beta)D_{24} + \beta D_{24}]$
15	2	$D_{-10}$	$-2[(1 - \beta)D_{24} + \beta D_{10}]$
16	2	$D_{-4}$	$-2[(1 - \beta)D_{10} + \beta D_4]$
17	2	$D_{-4}$	$-2[(1 - \beta)D_4 + \beta D_4]$
18	2	$D_{-4}$	$-2[(1 - \beta)D_4 + \beta D_4]$
19	2	$D_{-4}$	$-2[(1 - \beta)D_4 + \beta D_4]$
20	2	$D_0$	$-2(1 - \beta)D_4$

Table 3.4: Wealth change of the top strategy at each step in one price cycle of the (600) trendsetters' attractor.

Let  $W_x(i)$  be the virtual wealth of strategy  $x$  (here the  $x$  is different from the  $x$  in Eq. (3.1) at the beginning of the  $i$ th step of a cycle in Table 3.1.

The agent was using the bottom strategy at the 5th step and switch to the top strategy at the 6th step. So we will have the inequality:

$$W_b(5) > W_t(5),$$

Similarly, we have another inequality at the 6th step:

$$W_b(6) < W_t(6),$$

When the price starts to fall, the agent change from top strategy to bottom strategy, giving us another pair of inequalities:

$$W_b(15) < W_t(15),$$

$$W_b(16) > W_t(16).$$

We now use the parameter  $W_b(16)$ ,  $W_t(16)$ ,  $\beta$ ,  $\gamma$  to rewrite the inequalities using Table 3.3 and 3.4:

$$\begin{aligned} & (3 + \beta) D_{20} + (4 - \beta) D_{18} + 4(1 + \beta) D_{24} - 4(1 - \beta) D_{10} - 16D_4 \\ & < W_b(16) - W_t(16) < \\ & (3 + \beta) D_{20} + (4 - \beta) D_{18} + 8D_{24} - 4(1 - 2\beta) D_{10} - 16D_4. \end{aligned} \quad (3.2)$$

$$0 < W_b(16) - W_t(16) < 4(1 - \beta) D_{24} + 4\beta D_{10}. \quad (3.3)$$

Both inequalities will be satisfied only when:

$$\begin{aligned} & \max [(3 + \beta) D_{20} + (4 - \beta) D_{18} + 4(1 + \beta) D_{24} - 4(1 - \beta) D_{10} - 16D_4, 0] < \\ & \min [(3 + \beta) D_{20} + (4 - \beta) D_{18} + 8D_{24} - 4(1 - 2\beta) D_{10} - 16D_4, 4(1 - \beta) D_{24} + 4\beta D_{10}]. \end{aligned} \quad (3.4)$$

From Eq. (3.4) we can obtain the relation between  $\beta$  and  $\gamma$ :

$$\frac{3D_{20} + 4D_{18} + 8D_{24} - 4D_{10} - 16D_4}{D_{18} - D_{20} - 8D_{10}} < \beta < \frac{3D_{20} + 4D_{18} - 4D_{10} - 16D_4}{D_{18} - D_{20} - 8D_{24}}. \quad (3.5)$$

We can also calculate the wealth gain in one cycle for the bottom strategy, the top strategy and the slow strategy. We define  $\Delta W_x$  to be the wealth gain in one cycle of agent  $x$ .

The bottom and the top strategy share the same virtual wealth gain in one cycle:

$$\Delta W_b = \Delta W_t = (1 - \beta) D_{20} + \beta D_{18}. \quad (3.6)$$

For the slow strategy:

$$\Delta W_s = 2[(1 - 2\beta)D_{24} - 2D_{20} - (1 + \beta)D_{18} + (2 - \beta)D_{10} + 8D_4]. \quad (3.7)$$

Because we know that there are no slow&bottom or slow&top fickle agents, the wealth gain in one cycle of the slow strategy is always larger than the wealth gain in one cycle of the bottom or top strategy which can be observed from Fig. 3.1. Hence, we can obtain the following inequality:

$$2[(1 - 2\beta)D_{24} - 2D_{20} - (1 + \beta)D_{18} + (2 - \beta)D_{10} + 8D_4] > (1 - \beta)D_{20} + \beta D_{18}. \quad (3.8)$$

Which gives the relation between  $\beta$  and  $\gamma$ :

$$\beta < \frac{2D_{24} - 5D_{20} - 2D_{18} + 4D_{10} + 16D_4}{4D_{24} - D_{20} + 3D_{18} + 2D_{10}}. \quad (3.9)$$

The valid (600) region in the space of  $\beta$  and  $\gamma$  is obtained from satisfying both Eq. (3.5) and Eq. (3.9), the theoretical result is shown in Fig. 3.3 and the simulation result is shown in Fig. 3.4:

To obtain this phase diagram using computer simulations, we first describe how the trendsetters' attractors are detected using the following conditions: when the system becomes stabilized, we detect the first rising step that the price signal changes from  $\downarrow$  to  $\uparrow$ , and check if it is followed by more than  $2K + 1$  consecutive rising signals. If yes, then we look for the step where the signal changes from  $\uparrow$  to  $\downarrow$ , and check if it is followed by more than  $2K + 1$  consecutive dropping signals. If yes, this price cycle can be classified as a trendsetters' attractor. The period of this cycle is equal to the duration from the first rising signal to the last dropping signal. The probability of the trendsetters' attractor is obtained by counting the fraction of samples that contain only trendsetters' attractors at the steady state. In each sample, each agent starts the game with zero wealth, and the price of the market also starts with zero. The agents choose the strategies randomly and the first price signal is also random.

With this method of detection, we look for the steps of the successful (600) trendsetters' attractors in the last 3000 steps after the system has stabilized for 7,000 or 17,000 steps, and then we count the probability of the (600) trendsetters' attractor. We can see that the (600) trendsetters' attractor lies exactly in between the calculated lower bound and upper bound. It is remarkable that the necessary conditions in Eq. (3.5) provides tight bounds of the region of (600) attractor. The area is separated into two parts, the probability on the left half is significantly higher than that on the right. This

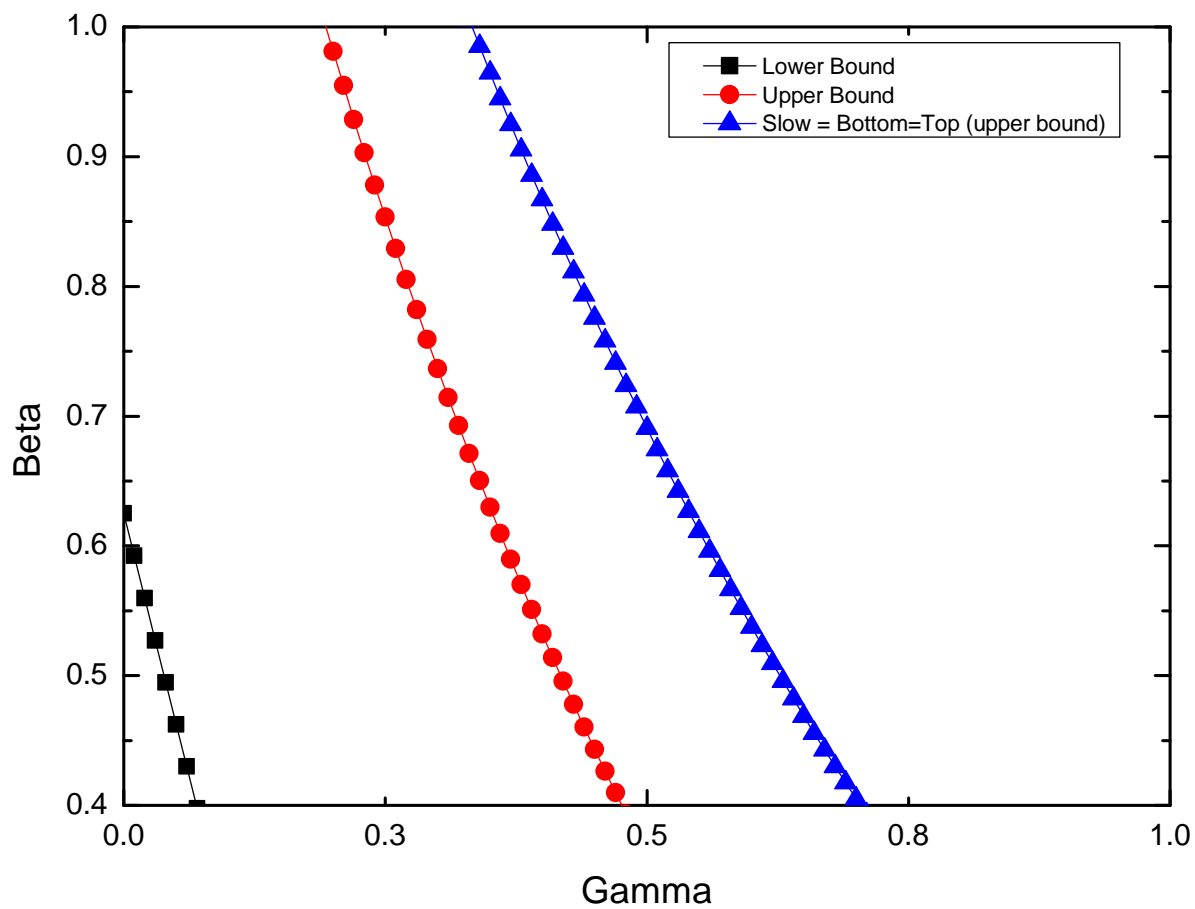


Figure 3.3: The theoretical valid region of the (600) trendsetters' attractor. The (600) trendsetters' attractor should lie between the lower bound and the upper bounds.

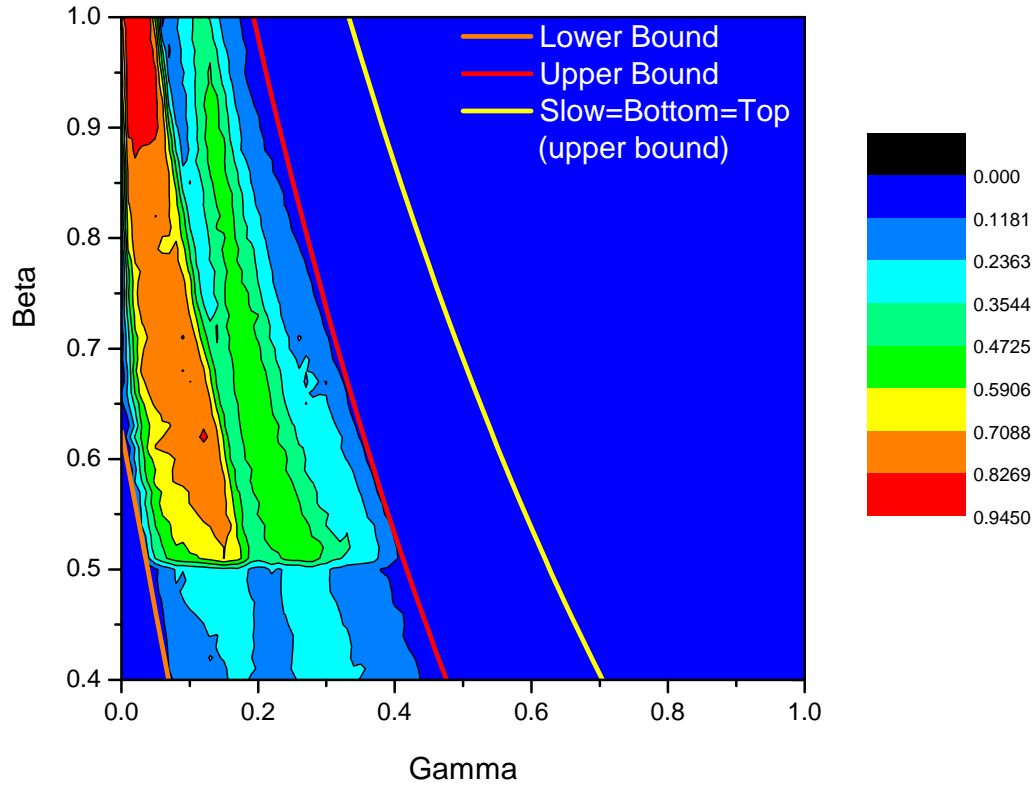


Figure 3.4: The simulation result for the probability of the (600) trendsetters' attractor. (N=1000, m=2, S=2, K=2, 10000 steps with 1000 samples)

phenomenon will be discussed in Chapter 5.

### 3.4 (500) Trendsetters' Attractor Analyses

After solving the (600) trendsetters' attractor, we continue to study another trendsetter with only the bottom&top fickle agent, which is the (500) trendsetters' attractor. The method we use here is again applying the matching condition to the bottom&top fickle agent similar to the way we calculate (600) trendsetters' attractor. The tables are listed in the appendix.

This time, we know that a fickle agent will switch her strategy twice, the first time is at the 5th rising step when the agent switches from the bottom strategy to the top strategy, and the second time is at the 5th dropping step when the agent switches from the top strategy to the bottom strategy. Now we can list the matching conditions to show the strategy switching scheme. The agent was using the bottom strategy at the 4th step and switches to the top strategy at the 5th step. So we will have the inequality:

$$W_b(4) > W_t(4),$$

$$W_b(5) < W_t(5),$$

When the price starts to fall, the agent change from top strategy to bottom strategy, giving us another two inequalities:

$$W_b(13) < W_t(13),$$

$$W_b(14) > W_t(14).$$

We now use the parameter  $W_b(14), W_t(14), \beta, \gamma$  to rewrite the inequalities:

$$\begin{aligned} & (3 + \beta) D_{20} + (4 - \beta) D_{18} - (4 - 8\beta) D_{24} - 12D_4 - 4D_{14} \\ & < W_b(14) - W_t(14) < \\ & (3 + \beta) D_{20} + (4 - \beta) D_{18} + 8\beta D_{24} - 12D_4 - 4D_{14}. \end{aligned} \tag{3.10}$$

$$0 < W_b(16) - W_t(16) < 4D_{24}. \tag{3.11}$$

By solving Eqs. (3.10) and (3.11), we obtain:

$$\frac{4D_{18} + 3D_{20} - 12D_4 - 4D_{14}}{D_{18} - 8D_{24} - D_{20}} < \beta < \frac{4D_{18} + 3D_{20} - 12D_4 - 4D_{14} - 8D_{24}}{D_{18} - 8D_{24} - D_{20}}. \tag{3.12}$$



We can also calculate the wealth gain in one cycle for the bottom, top and slow strategies. For the bottom and the top strategies, they share the same wealth gain in one cycle:

$$\Delta W_b = \Delta W_t = (1 - \beta)D_{20} + \beta D_{18}. \quad (3.13)$$

For the slow strategy:

$$\Delta W_s = 2[(1 - 2\beta)D_{24} - 2D_{20} - (1 + \beta)D_{18} + 6D_4 + (2 - \beta)D_{14}]. \quad (3.14)$$

Because we know that there are no slow&bottom or slow&top fickle agents, the wealth gain in one cycle of the slow strategy should be greater than the wealth gain in one cycle of the bottom or top strategy. Hence, we can obtain the following inequality

$$\Delta W_s > \Delta W_b \text{ or } \Delta W_t,$$

which gives us the result:

$$\beta < \frac{-5D_{20} - 2D_{18} + 2D_{24} + 4D_{14} + 12D_4}{3D_{18} + 4D_{24} + 2D_{14} - D_{20}}. \quad (3.15)$$

The valid (500) region in  $\beta$  and  $\gamma$  diagram is obtained from satisfying both Eqs. (3.12) and (3.14), the theoretical result is shown in Fig. 3.5 and the simulation result is shown in Fig. 3.6.

We can see that the highest probability of the (500) trendsetters' attractor lies in between the lower bound and the lowest upper bound. The probability starts to decrease beyond the right hand side of the blue line which means the virtual wealth of the slow strategy starts to be comparable with the bottom and top strategies, hence breaking the (500) trendsetters' attractor. The matching conditions works quite well in the calculation of the effective region of the (500) trendsetters' attractor.

## 3.5 Summary

We have studied the behaviors of two trendsetters' attractor with a single kind of fickle agents, and have considered their regions of existence. We observe the following general behaviors.

Among all agents, the fast agent always has the best wealth gain, and the slow agents the second best. The slow agent reacts to the change of price one step later than the fast agent. The slow agent is at the negative position when the price reaches the bottom. As the price starts to rise, the slow strategy tells the agent to sell one more step. Hence

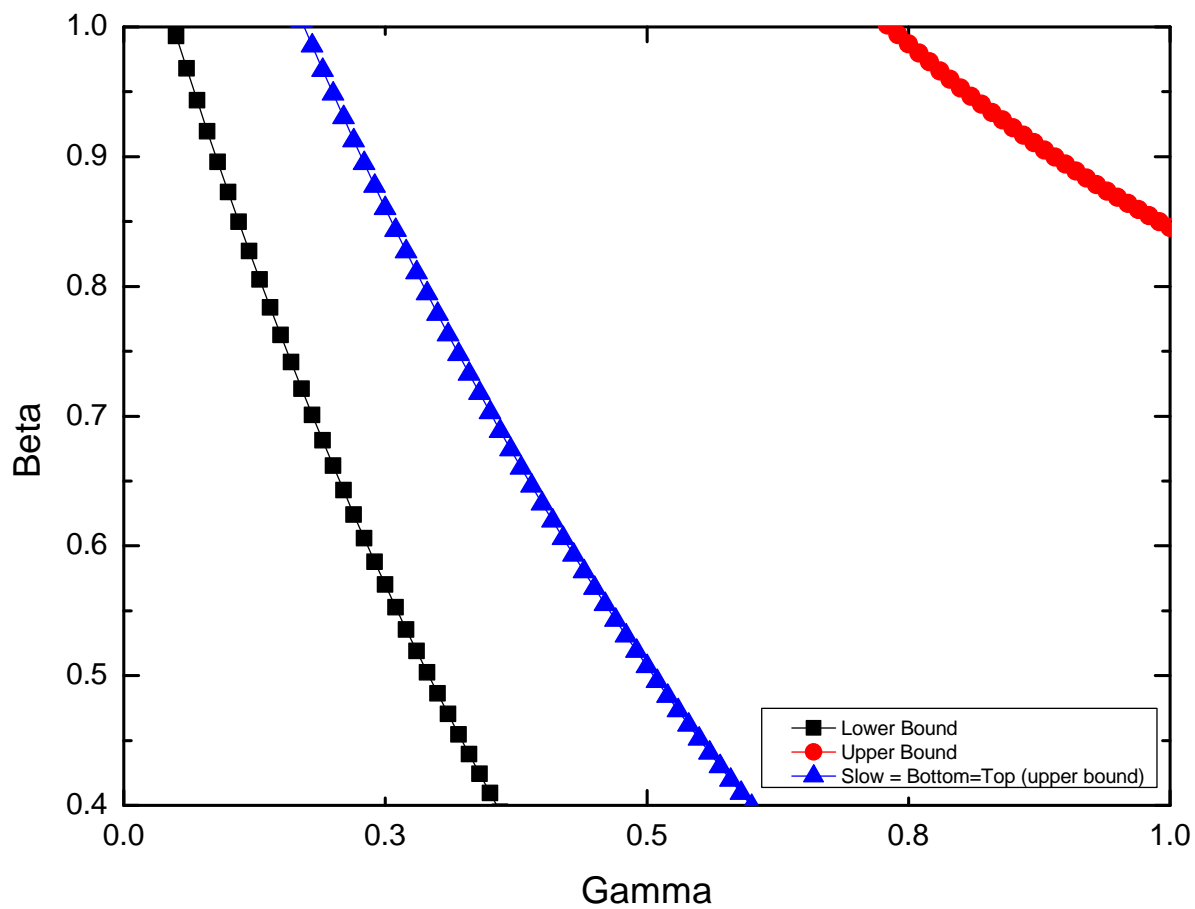


Figure 3.5: The theoretically valid region of (500) trendsetters' attractor. The (500) trendsetters' attractor should lie between the lower bound and the upper bounds.

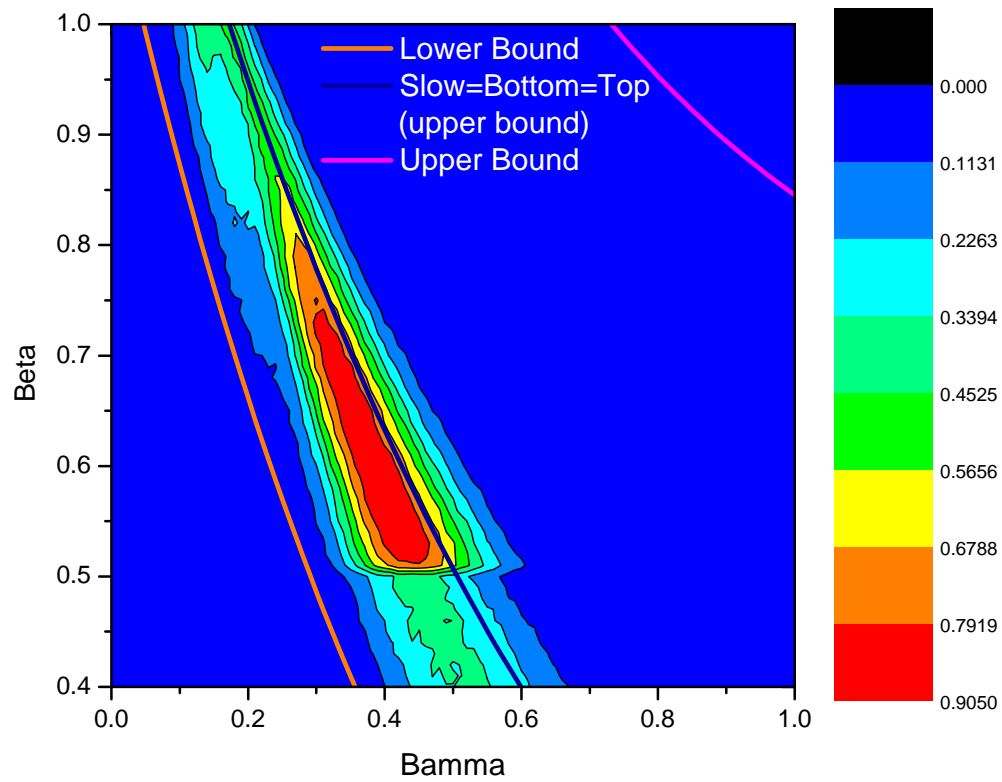


Figure 3.6: The simulation result for the probability of the (500) trendsetters' attractor. ( $N=1000$ ,  $m=2$ ,  $S=2$ ,  $K=2$ , 10000 steps with 1000 samples)

the agent will lose money at the first rising step. The condition is similar when the price drops from the peak. The slow strategy tells the agent to buy one more step while the price starts to drop.

The bottom agent will gain a small amount of cash in each cycle when the price rises from the bottom as she makes *buy* and *sell* decisions once only in each cycle. The top agent will also gain a small amount of cash in each cycle when the price drops from the peak as she makes *sell* and *buy* decisions once only in each cycle. Because of the symmetry of the price series, the amount of money gained in one cycle by the bottom and top agents are always the same.

For the fickle agents, they stick to one strategy until the virtual wealth of the other strategy is higher than the one they adopt. However, it takes time for the virtual wealth of one strategy to accumulate until it is higher than the other. This makes the switching of the strategies ineffective and the fickle agents always change their strategy at the wrong time. As a result, they keep on losing wealth.

However, the fickle agents play the role of pushing up the price further in the second stage even after the fast and slow trendsetters stop their buying actions. This increases the wealth gained by the trendsetters. Similarly, they push down the price in the fourth stage, producing the same effects on the trendsetters. Hence, the dynamics of the attractor is stabilized.

## Chapter 4

# Trendsetters' Attractors With Multiple Kinds of Agents and Anti-agents

As shown in Fig. 2.5(b), when the virtual wealth of the slow strategy becomes comparable with those of the bottom and top strategies, we will have two additional fickle agents and anti-fickle agents in the trendsetters' attractors, they are the slow&bottom fickle agents and the slow&top fickle agents. The slow&bottom fickle agents are the agents holding both the slow strategy and the bottom strategy, while the slow&top fickle agents are those holding both the slow strategy and the top strategy. The first number "5" represents the bottom&top fickle agents switch strategies at the 5th rising and dropping steps. The number  $j$  means the slow&bottom fickle agents switch strategies at the  $j$ th rising step when the price rises from the bottom, and the number  $k$  means the slow&top fickle agents switch strategies at the  $k$ th dropping step when the price drops from the peak. According to symmetry, a  $(5jk)$  trendsetters' attractor is not stable unless  $j = k$ . So our interest mainly focuses on the  $(5xx)$  trendsetters' attractors.

To work out the effective region of the  $(5xx)$  trendsetters' attractor, we need to carefully trace the decisions of each kind of agents, and deduce the number of buyers and sellers at each step, just as what we did in calculating the (600) and (500) trendsetters' attractor. However, there will be additional matching conditions given by the slow&bottom and slow&top fickle agents. Since the agents will have the chance to use either the slow strategy or the bottom/top strategy, the wealth gain of the slow strategy should be comparable with the wealth gain of the bottom/top strategy. The condition that the wealth gain in one cycle of the slow strategy equals that of the bottom/top strategy, is where the  $(5xx)$  has the highest probability of occurrence. With these additional conditions, we are now able to work out the theoretical region of  $(5xx)$ .

## 4.1 (544) Trendsetters' Attractor Analyses

As we observed, the first (5xx) trendsetters' attractor is the (544) trendsetters' attractor. The bottom&top fickle agent provides us the matching conditions similar to the (500) trendsetters' attractor according to its switching steps.

$$W_b(4) > W_t(4),$$

$$W_b(5) < W_t(5),$$

$$W_b(13) < W_t(13),$$

$$W_b(14) > W_t(14).$$

We now use the parameter  $W_b(14), W_t(14), \beta, \gamma$  to rewrite the inequalities:

$$\begin{aligned} (4 - \beta) D_{10} - 4D_4 - 8D_6 - 4D_{16} + 4\beta D_{20} + (3 - \beta) D_{24} - 4(1 - \beta) D_{26} \\ < W_b(14) - W_t(14) < \end{aligned} \quad (4.1)$$

$$(4 - \beta) D_{10} - 4D_4 - 8D_6 - 4D_{16} + 4D_{20} + (3 - \beta) D_{24} - 4(1 - 2\beta) D_{26}.$$

$$0 < W_b(14) - W_t(14) < 4(1 - \beta)D_{20} + 4\beta D_{26}. \quad (4.2)$$

The slow&bottom and slow&top fickle agents also provide us matching conditions:

$$W_s(3) < W_b(3),$$

$$W_s(4) > W_b(4),$$

$$W_s(12) < W_t(12),$$

$$W_s(13) > W_t(13).$$

We now use the parameter  $W_s(14), W_b(14), W_t(14), \beta, \gamma$  to rewrite the inequalities as  $W_x(i)$  can be written in  $W_x(14)$  by adding the wealth change at each additional step in one cycle.

$$\begin{aligned} 4D_4 + 8D_6 - (3 + \beta) D_{10} + (4 - \beta) D_{16} - 3\beta D_{20} - 4D_{24} + 3(1 - \beta) D_{26} \\ < W_s(14) - W_b(14) \\ < 4D_4 + 8D_6 - (2 + 2\beta) D_{10} + (4 - \beta) D_{16} - 2\beta D_{20} - 4D_{24} + 3(1 - \beta) D_{26}. \end{aligned} \quad (4.3)$$

$$\begin{aligned}
& 2(1 - \beta)D_{20} + 2\beta D_{26} \\
& < W_s(14) - W_s(14) \\
& < 2(1 - \beta)D_{20} + 2\beta D_{26} + (1 - \beta)D_{10} + \beta D_{20}.
\end{aligned} \tag{4.4}$$

From Eqs. (4.3) and (4.4) we can eliminate  $W_s(14)$  leaving only the  $W_b(14), W_t(14), \beta$  and  $\gamma$ .

$$\begin{aligned}
& 2(1 - \beta)D_{20} + 2\beta D_{26} - [4D_4 + 8D_6 - (2 + 2\beta)D_{10} \\
& + (4 - \beta)D_{16} - 2\beta D_{20} - 4D_{24} + 3(1 - \beta)D_{26}] \\
& < W_b(14) - W_t(14) \\
& < 2(1 - \beta)D_{20} + 2\beta D_{26} + (1 - \beta)D_{10} + \beta D_{20} - [4D_4 + 8D_6 \\
& - (3 + \beta)D_{10} + (4 - \beta)D_{16} - 3\beta D_{20} - 4D_{24} + 3(1 - \beta)D_{26}].
\end{aligned} \tag{4.5}$$

There is also the condition of maximum stability for the slow strategy and the bottom strategy or the slow strategy and the top strategy to be comparable after the wealth gain in one cycle.

$$\Delta W_s = \Delta W_b = \Delta W_t$$

This will give rise to:

$$\begin{aligned}
& 4D_4 + 8D_6 - (2 + 2\beta)D_{10} + (4 - 2\beta)D_{16} - 2\beta D_{20} \\
& - 4D_{24} + 2(1 - \beta)D_{26} = (1 - \beta)D_{24} + \beta D_{10}.
\end{aligned} \tag{4.6}$$

Moreover, since  $\Delta W_b = \Delta W_t \geq 0$ ,  $\Delta W_s$  should also be non-negative, so that the slow&bottom or slow&top agents can fickle. We have the inequality:

$$4D_4 + 8D_6 - (2 + 2\beta)D_{10} + (4 - 2\beta)D_{16} - 2D_{20} - 4D_{24} + 2(1 - \beta)D_{26} \geq 0. \tag{4.7}$$

By solving Eqs. (4.1), (4.2), (4.5) and (4.7), we can obtain the boundary of the (544) trendsetters' attractor phase, and the result is:

$$\begin{aligned}
& \frac{4D_4 + 8D_6 - 4D_{10} + 4D_{16} - 3D_{24} - 4D_{20} + 4D_{26}}{8D_{26} - D_{10} - D_{24}} \\
& < \beta \\
& < \frac{4D_4 + 8D_6 - 2D_{10} + 4D_{16} - 4D_{24} + 2D_{26}}{2D_{10} + 2D_{16} + 2D_{20} + 2D_{26}}.
\end{aligned} \tag{4.8}$$

We can obtain the maximum probability line of the (544) trendsetters' attractor from

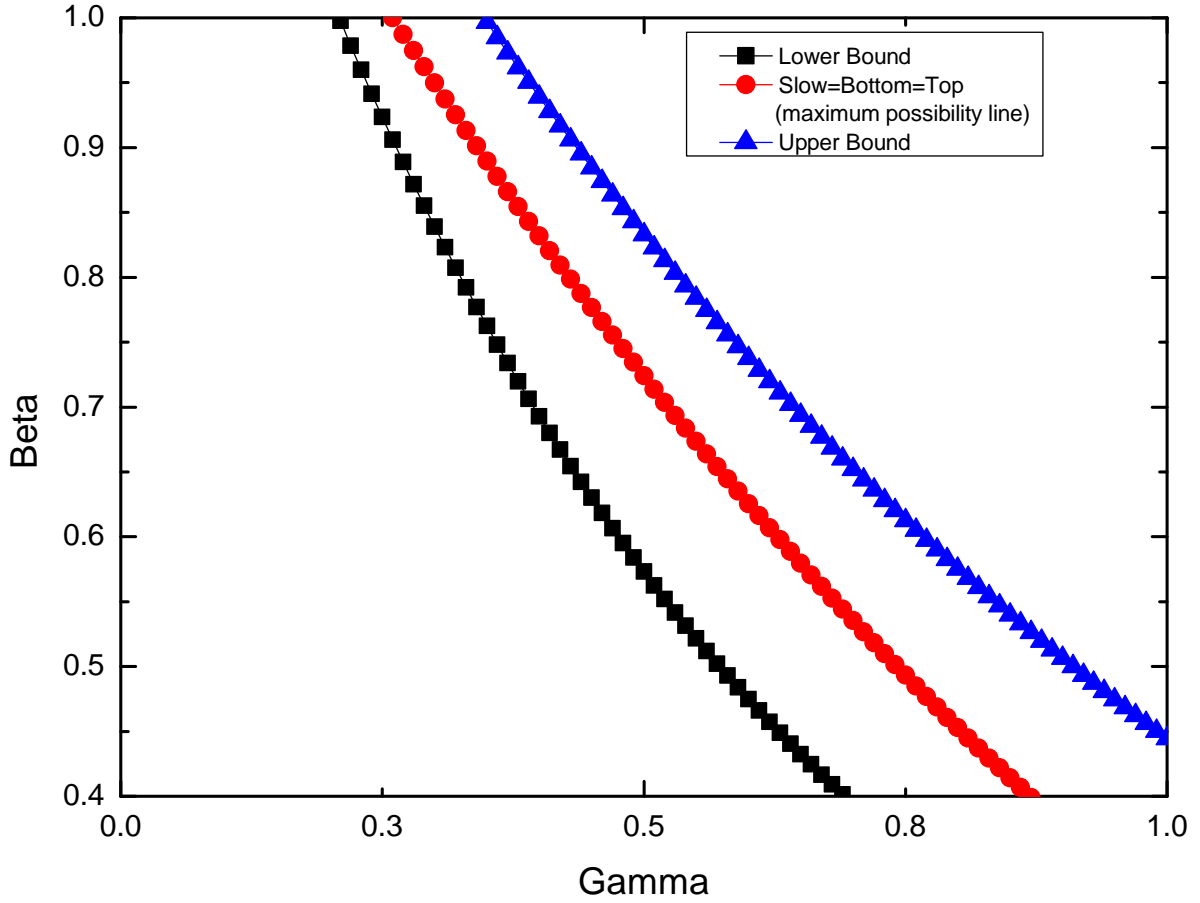


Figure 4.1: The calculation result of the existence region of (544) trendsetters' attractors. The upper bound is the line comes from Eq. (4.7). The line lies in the middle is where  $\Delta W_s = \Delta W_b = \Delta W_t$ .

Eq. (4.6), which give the result:

$$\beta = \frac{4D_4 + 8D_6 - 2D_{10} + 4D_{16} - 5D_{24} + 2D_{26}}{3D_{10} + 2D_{16} + 2D_{20} + 2D_{26} - D_{24}}. \quad (4.9)$$

The theoretical result is shown in Fig. 4.1 and the simulation result is shown in Fig. 4.2.

We can see that the simulation result agrees with the calculation result especially the two boundary lines.

## 4.2 (555) Trendsetters' attractor Analyses

We now apply the same method to analyze the (555) trendsetters' attractor. With the following matching conditions given by the bottom&top fickle agent:

$$W_b(4) > W_t(4),$$



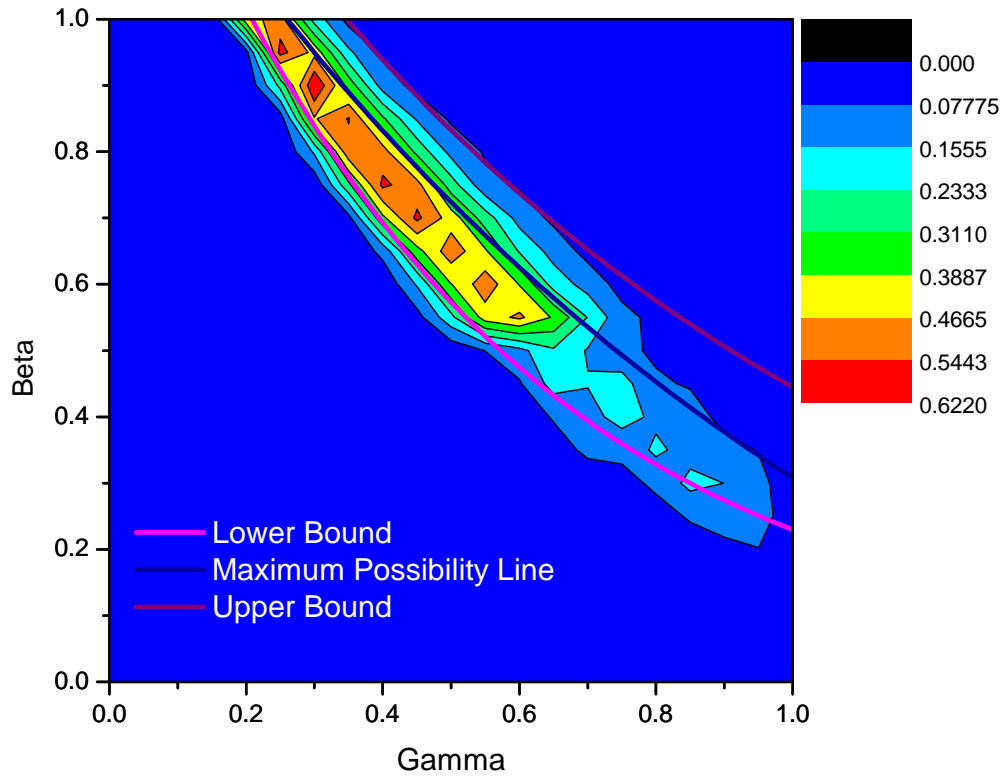


Figure 4.2: The simulation result of the (544) attractor compared with the theoretical calculation. ( $N=1000$ ,  $K=2$ ,  $m=2$ ,  $S=2$ , 20000 steps, 200 samples)

$$W_b(5) < W_t(5),$$

$$W_b(13) < W_t(13),$$

$$W_b(14) > W_t(14).$$

We now use the parameter  $W_b(14), W_t(14), \beta, \gamma$  to rewrite the inequalities:

$$\begin{aligned} & -4D_6 - 8D_8 + (4 - \beta) D_{10} - 4D_{16} - 4(1 - 2\beta) D_{20} + (3 + \beta) D_{24} \\ < & W_b(14) - W_t(14) \\ < & -4D_6 - 8D_8 + (4 - \beta) D_{10} - 4D_{16} + 8\beta D_{20} + (3 + \beta) D_{24}. \end{aligned} \quad (4.10)$$

$$0 < W_b(14) - W_t(14) < 4D_{20}. \quad (4.11)$$

The slow&bottom and slow&top fickle agents also provide us the additional matching conditions:

$$W_s(4) < W_b(4),$$

$$W_s(5) > W_b(5),$$

$$W_s(13) < W_t(13),$$

$$W_s(14) > W_t(14).$$

We now use  $W_s(14), W_b(14), W_t(14), \beta$  and  $\gamma$  to rewrite the inequalities:

$$\begin{aligned} & 4D_6 + 8D_8 - (3 + \beta) D_{10} + (4 - \beta) D_{16} + (1 - 6\beta) D_{20} - 4D_{24} \\ < & W_s(14) - W_b(14) \\ < & 4D_6 + 8D_8 - (3 + \beta) D_{10} + (4 - \beta) D_{16} + 3(1 - 2\beta) D_{20} - 4D_{24}. \end{aligned} \quad (4.12)$$

$$0 < W_s(14) - W_t(14) < 2D_{20}. \quad (4.13)$$

From Eqs. (4.12) and (4.13) we can eliminate  $W_s(14)$  leaving only the  $W_b(14), W_t(14), \beta$  and  $\gamma$ . The result is:

$$\begin{aligned} & -[4D_6 + 8D_8 - (3 + \beta)D_{10} + (4 - \beta)D_{16} + 3(1 - 2\beta)D_{20} - 4D_{24}] \\ < & W_b(14) - W_t(14) \\ < & 2D_{20} - [4D_6 + 8D_8 - (3 + \beta)D_{10} + (4 - \beta)D_{16} + (1 - 6\beta)D_{20} - 4D_{24}]. \end{aligned} \quad (4.14)$$

There is the condition of maximum stability which yields  $\Delta W_s = \Delta W_b = \Delta W_t$ , which is:

$$\begin{aligned} & 4D_6 + 8D_8 + (4 - 2\beta) D_{16} + (2 - 4\beta) D_{20} - (2 + 2\beta) D_{10} - 4D_{24} \\ = & (1 - \beta) D_{24} - \beta D_{10}. \end{aligned} \quad (4.15)$$

Moreover, since  $\Delta W_b = \Delta W_s \geq 0$ , we should have the necessary condition  $\Delta W_s \geq 0$ , so that the slow&bottom and slow&top agents can fickle. We will have the inequality:

$$4D_6 + 8D_8 + (4 - 2\beta) D_{16} + (2 - 4\beta) D_{20} - (2 + 2\beta) D_{10} - 4D_{24} \geq 0. \quad (4.16)$$

By solving Eqs. (4.10), (4.11), (4.14) and (4.16), we can obtain the boundary of the (555) trendsetters' attractor phase, and the result is:

$$\begin{aligned} & \frac{4D_6 + 8D_8 - 4D_{10} + 4D_{16} - 3D_{24}}{8D_{20} + D_{24} - D_{10}} \\ < & \beta \\ < & \frac{4D_6 + 8D_8 + 4D_{16} + 2D_{20} - 2D_{10} - 4D_{25}}{2D_{16} + 4D_{20} + 2D_{10}}. \end{aligned} \quad (4.17)$$

Eq. (4.15) gives us the maximum possibility of the (555) trendsetters' attractor.

$$\beta = \frac{4D_6 + 8D_8 + 4D_{16} + 2D_{20} - 2D_{10} - 5D_{24}}{2D_{16} + 4D_{20} + 3D_{10} - D_{24}}. \quad (4.18)$$

The theoretical result is shown in Fig. 4.3 and the simulation result is shown in Fig. 4.4.

We can see that the maximum probability in the simulation agrees with our theoretical result. However, the boundaries starts to deviate and we have a loose lower bound.

### 4.3 The Calculation of the (566) trendsetters' attractor

We now apply the same method to analyze the (566) trendsetters' attractor. The matching conditions given by the bottom&top fickle agent are:

$$W_b(4) > W_t(4),$$

$$W_b(5) < W_t(5),$$

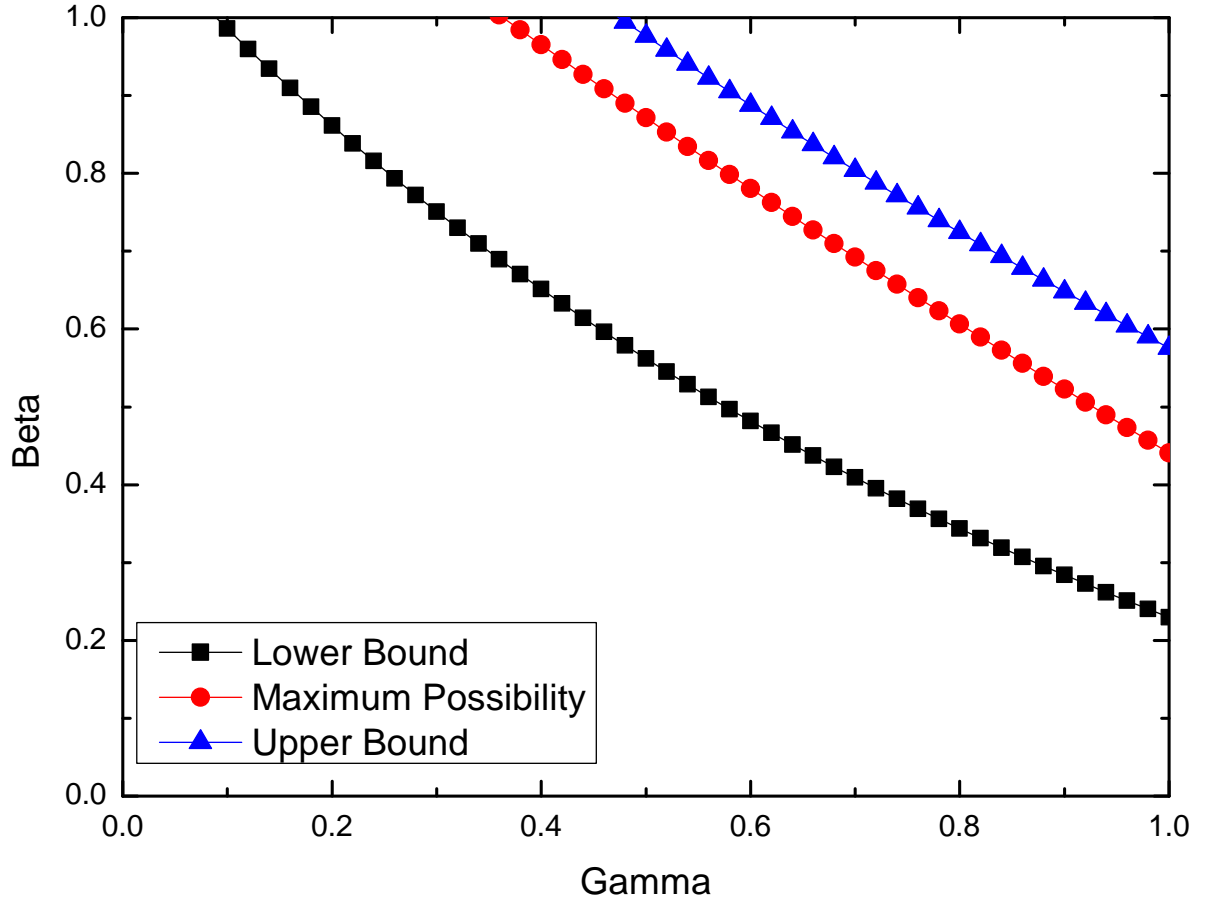


Figure 4.3: The theoretical result of the region for (555) trendsetters' attractor. The upper bound is the line where  $\Delta W_s = 0$ . The line in the middle is where  $\Delta W_s = \Delta W_b = \Delta W_t$ .

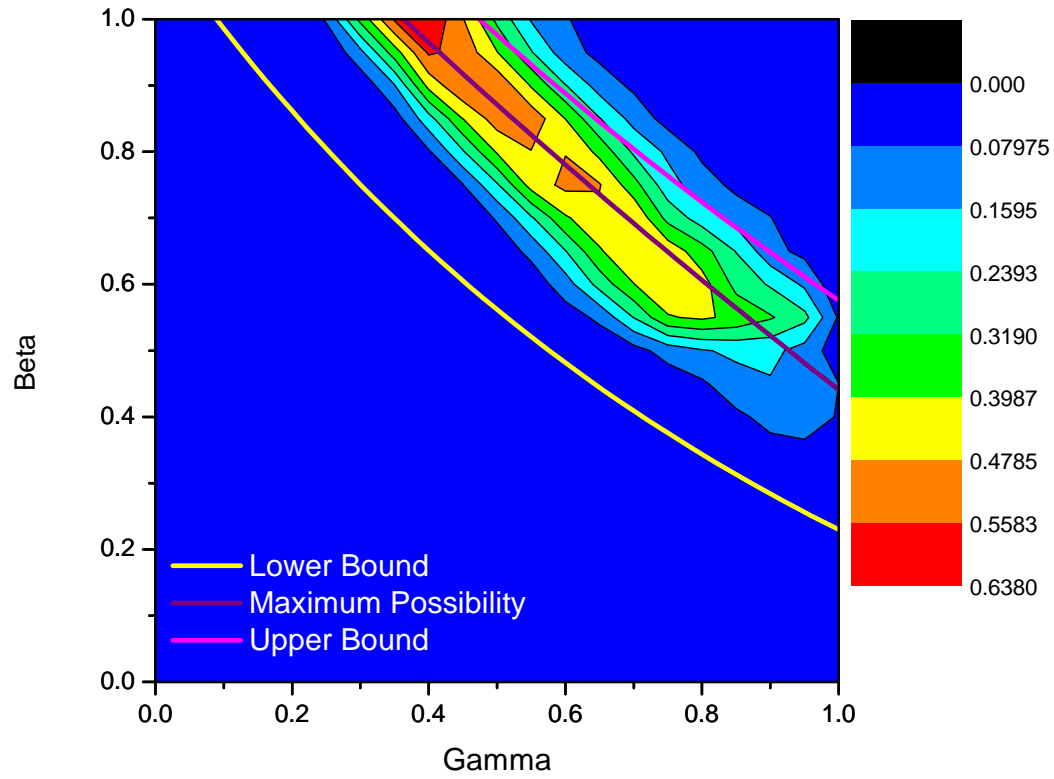


Figure 4.4: The simulation result of the (555) trendsetters' attractor compared with the theoretical result. ( $N=1000$ ,  $K=2$ ,  $S=2$ ,  $m=2$ , 20000 steps and 200 samples)

$$W_b(14) < W_t(14),$$

$$w_b(15) > W_t(15).$$

The matching conditions given by the slow&bottom and the slow&top fickle agents are:

$$W_s(5) < W_b(5),$$

$$W_s(6) > W_b(6),$$

$$W_s(15) < W_t(15),$$

$$W_s(16) > W_t(16).$$

The condition of maximum stability for the slow&bottom and the slow&top fickle agents is  $\Delta W_s = \Delta W_b = \Delta W_t$ . Moreover, we have the limit condition that  $\Delta W_s$  should be non-negative as  $\Delta W_b$  and  $\Delta W_t$  are always non-negative. Solving all these conditions will give us the result with parameter  $W_b(15), W_t(15), \beta$  and  $\gamma$ . The results are:

$$\begin{aligned} & (3 + \beta)D_{24} - 2(2 - 4\beta)D_{20} - \beta D_{10} - 12D_8 - 4D_4 \\ & < W_b(15) - W_t(15) \\ & < (3 + \beta)D_{24} + 8\beta D_{20} - \beta D_{10} - 12D_8 - 4D_4. \end{aligned} \quad (4.19)$$

$$0 < W_b(15) - W_s(15) < 4D_{20}. \quad (4.20)$$

$$\begin{aligned} & 4D_{24} - (2 - 9\beta)D_{20} - (1 + \beta)D_{10} - 12D_8 - 4D_4 \\ & < W_b(15) - W_t(15) \\ & < 4D_{24} + (2 + 3\beta)D_{20} - (1 - 5\beta)D_{10} - 12D_8 - 4D_4. \end{aligned} \quad (4.21)$$

$$\begin{aligned} & -4D_{24} + (2 - 4\beta)D_{10} + (2 - 4\beta)D_{20} + 12D_8 + 4D_4 \\ & \geq 0. \end{aligned} \quad (4.22)$$

$$\begin{aligned} & (-5 + \beta)D_{24} + (2 - 5\beta)D_{10} + (2 - 4\beta)D_{20} + 12D_8 + 4D_4 \\ & = 0. \end{aligned} \quad (4.23)$$

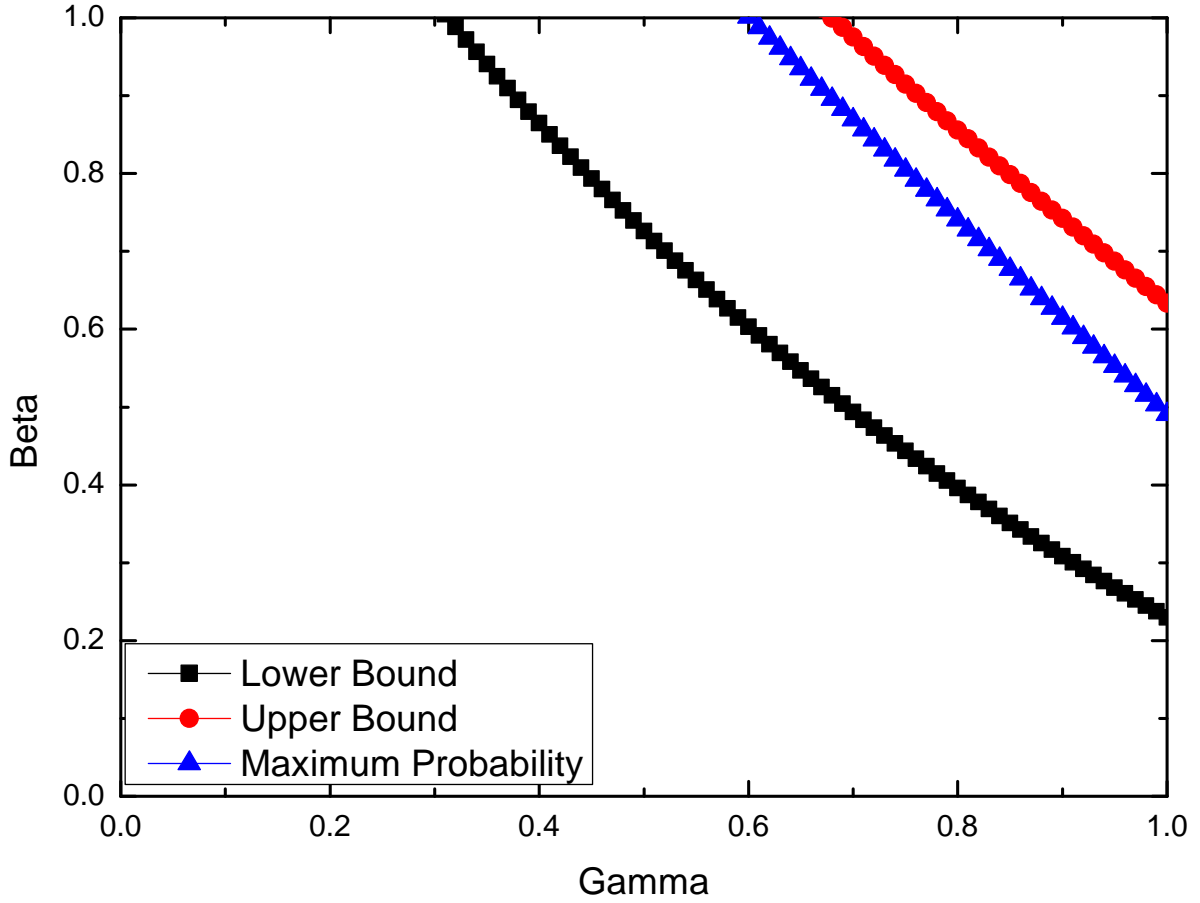


Figure 4.5: The theoretical result of the existence of (566) trendsetters' attractor. The line lies in the middle is where  $\Delta W_s = \Delta W_b = \Delta W_t$ .

Solving Eqs. (4.19), (4.20), (4.21) and (4.22) we can obtain the boundaries of (566) trendsetters' attractor which is shown in Eq. (4.24).

$$\frac{12D_8 + 4D_4 - 3D_{24}}{D_{24} + 8D_{20} - D_{10}} < \beta < \frac{D_{10} - 2D_{24} + D_{20} + 6D_8 + 2D_4}{2D_{10} + 2D_{20}}. \quad (4.24)$$

The maximum probability line is obtained by solving Eq. (4.23) and the result is shown in Eq. (4.25).

$$\beta = \frac{2D_{10} + 2D_{20} + 12D_8 + 4D_4 - 5D_{24}}{5D_{10} + 4D_{20} - D_{24}}. \quad (4.25)$$

The theoretical result is shown in Fig. 4.5 and the simulation result is shown in Fig. 4.6.

## 4.4 (577) trendsetters' attractor Analyses

The next trendsetters' attractor we would like to analyze is the (577) trendsetters' attractor. By using the same method as shown in the previous sections, we will obtain the

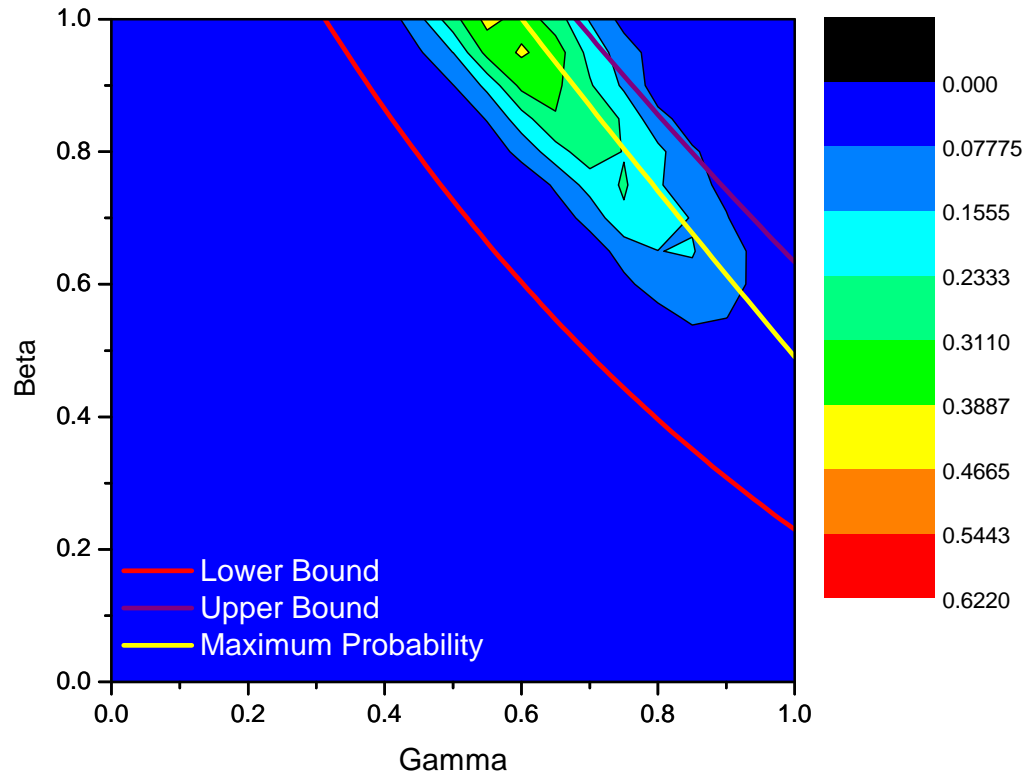


Figure 4.6: The simulation result compared with the theoretical result. ( $N=1000$ ,  $K=2$ ,  $m=2$ ,  $S=2$ , 20000 Steps and 200 samples)



results very similar to previous ones. The matching conditions given by the bottom&top fickle agent are:

$$W_b(4) < W_t(4),$$

$$W_b(5) > W_t(5),$$

$$W_b(15) < W_t(15),$$

$$W_b(16) > W_t(16).$$

The matching conditions given by the slow&bottom and the slow&top fickle agents are:

$$W_s(6) < W_b(6),$$

$$W_s(7) > W_b(7),$$

$$W_s(17) < W_t(17),$$

$$W_s(18) > W_t(18).$$

The condition of maximum stability is still  $\Delta W_s = \Delta W_b = \Delta W_t$ . Moreover, we have the limit condition that requires  $\Delta W_s$  to be non-negative.

All these conditions will give us the inequalities in  $W_b(16), W_t(16), \beta$  and  $\gamma$ . The results are:

$$\begin{aligned} & -12D_4 - 8D_8 - \beta D_{10} - (1 - 8\beta)D_{20} + (3 + \beta)D_{24} \\ & < W_b(16) - W_t(16) \\ & < -12D_4 - 8D_8 - \beta D_{10} + 8\beta D_{20} + (3 + \beta)D_{24}. \end{aligned} \tag{4.26}$$

$$0 < W_b(16) - W_t(16) < 4D_{20}. \tag{4.27}$$

$$\begin{aligned} & -(12 + 4\beta)D_4 - 8D_8 - (5 - 6\beta)D_{10} - (1 - 6\beta)D_{20} + 4D_{24} \\ & < W_b(16) - W_t(16) \\ & < -10D_4 - (8 - 2\beta)D_8 + D_{10} - (1 - 6\beta)D_{20} + 4D_{24}. \end{aligned} \tag{4.28}$$

$$12D_4 + 8D_8 + (2 - 4\beta)D_{10} + (2 - 4\beta)D_{20} - 4D_{24} \geq 0. \tag{4.29}$$

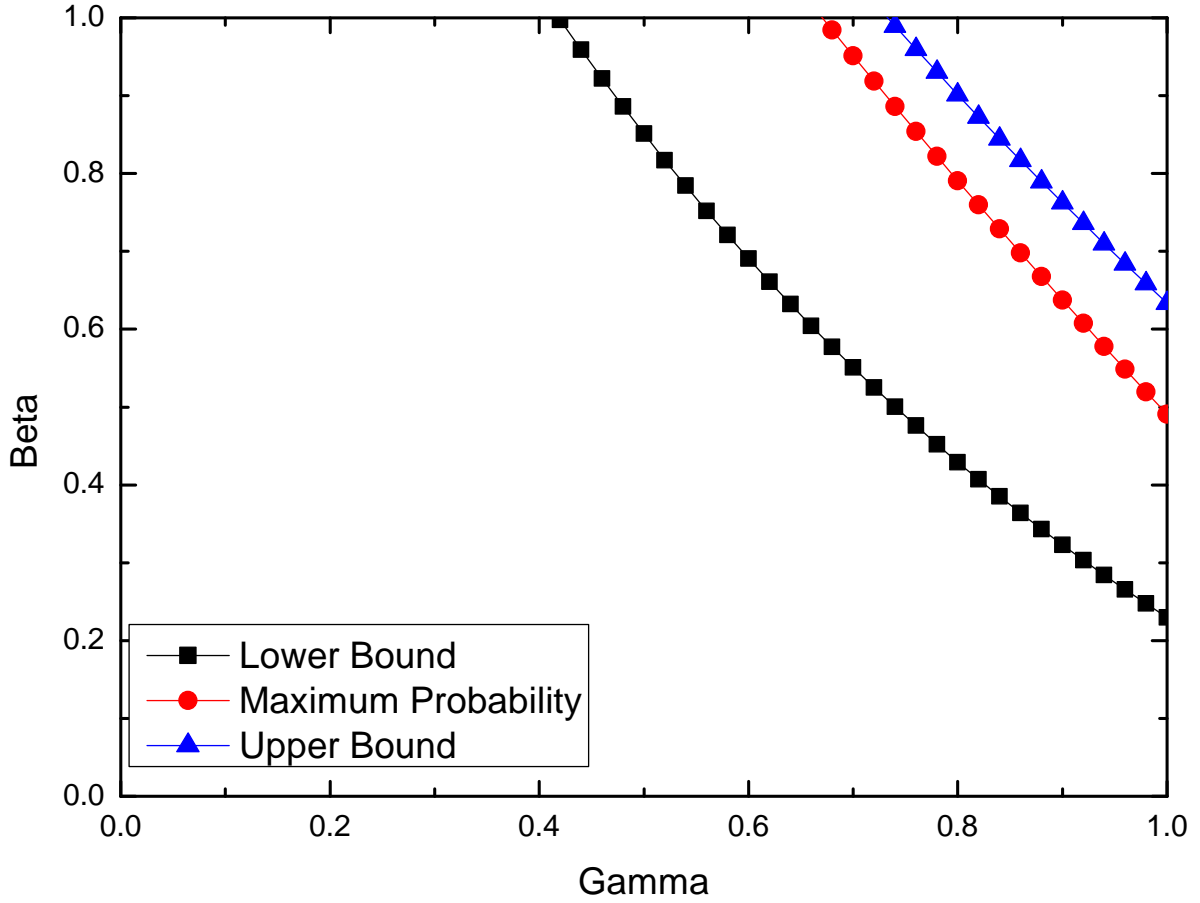


Figure 4.7: The theoretical result of the (577) trendsetters' attractor.

$$\begin{aligned}
& 12D_4 + 8D_8 + (2 - 4\beta)D_{10} + (2 - 4\beta)D_{20} - 4D_{24} \\
& = (1 - \beta)D_{24} + \beta D_{10}.
\end{aligned} \tag{4.30}$$

The boundaries are obtained by solving Eqs. (4.26), (4.27), (4.28) and (4.29). The result is:

$$\frac{12D_4 + 8D_8 - 3D_{24}}{8D_{20} + D_{24} - D_{10}} < \beta < \frac{12D_4 + 8D_8 + 2D_{10} + 2D_{20} - 4D_{24}}{4D_{10} + 4D_{20}}. \tag{4.31}$$

The maximum probability line is obtained by solving Eq. (4.30):

$$\beta = \frac{12D_4 + 8D_8 + 2D_{10} + 2D_{20} - 5D_{24}}{5D_{10} + 4D_{20} - D_{24}}. \tag{4.32}$$

The theoretical result is shown in Fig. 4.7 and the simulation result is shown in Fig. 4.8.

We can see that the simulation result agree with the theoretical results especially the upper bound and the maximum probability line.

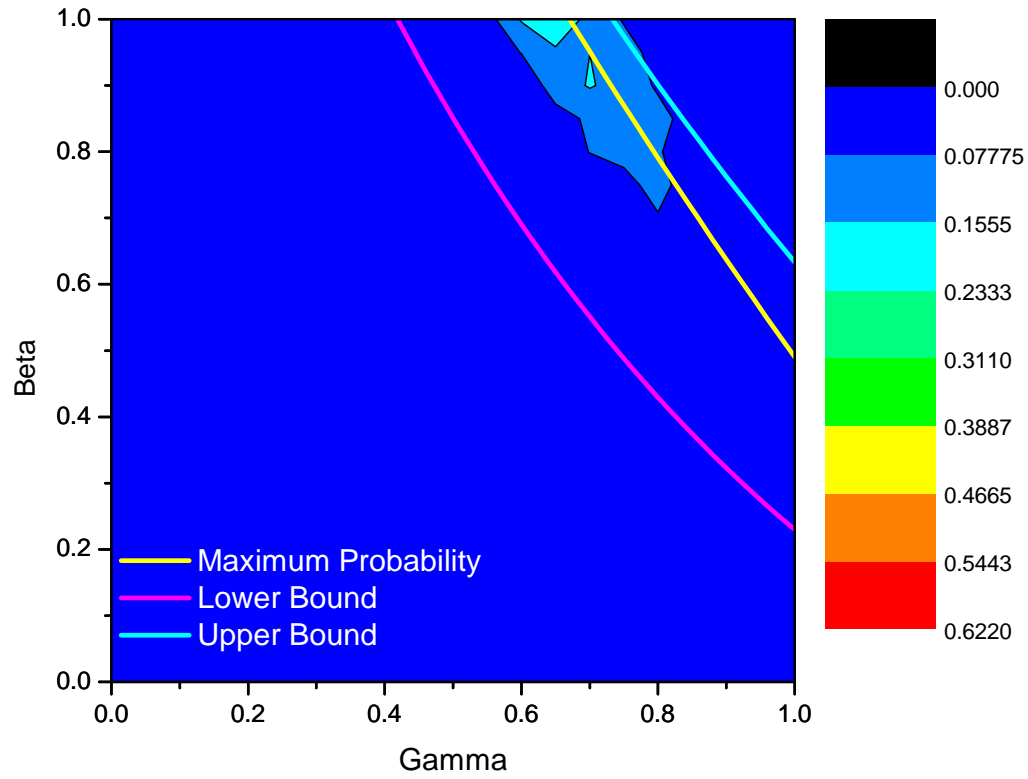


Figure 4.8: The simulation result compared with the theoretical result of the (577) trendsetters' attractor. ( $N=1000$ ,  $K=2$ ,  $S=2$ ,  $m=2$ , 20000 steps and 200 samples)

## 4.5 Summary

We have analyzed the dynamics of trendsetters' attractors including 3 kinds of fickle agents. Comparing these attractors, we can make the following observations.

1) We observe that when  $\gamma$  increases, the period of the attractor increases from 18 steps of the (544) attractor to 22 steps of the (577) attractor. This is because the price movement during stages 2 and 4 becomes more sensitive to the drop of buyers and sellers of these stages when  $\gamma$  increases. As a result, the wealth gained by the slow agents is reduced. Thus, the moment of switching by the slow&bottom and slow&top agents are delayed. In this case, the (544) develops into (555) and (555) develops into (566) and so on and so forth.

2) There is no (588) trendsetters' attractor in the essential model as the maximum position  $K$  is set to be 2. Hence the cycle is not long enough for the (588) to occur.

3) The necessary condition  $\Delta W_s = 0$  gives us a tight upper bound when calculating the boundaries of the trendsetters' attractors.

4) The maximum stability condition  $\Delta W_s = \Delta W_b = \Delta W_t$  requires the virtual wealth of the slow strategy to be surfing on the virtual wealth of the bottom and top strategies. When the slow strategy loses wealth, the switching between the slow and bottom strategies or the switching between the slow and top strategies will delay, for example, the (544) attractor will become a (555) attractor. Then the slow strategy will gain wealth in the (555) attractor hence advance the switching and change from (555) to (544) again.

5) The maximum probability line is valid for the (555), (566) and (577) trendsetters' attractors but not for the (544) trendsetters' attractor. It is probably because the region of maximum stability is masked by the emergence of the (555) attractor (this may be due to the dependence on initial conditions, which should be studied in the future).

## Chapter 5

# Asymmetric Trendsetters' Attractors

When we analyzed the (600) trendsetters' attractor we found the phase diagram separated into two parts. We thought that this was caused by the existence of the (500) trendsetters' attractor. We therefore superpose the probability of (600) and (500) trendsetters' attractors and it is shown in Fig. 5.1.

From Fig. 5.1 we observe that there is a gap where the probability of (600) together with (500) is less than fifty percent. We would like to look into details of this area where  $\gamma$  is between 0.1 and 0.4.

It is observed that there exist three kinds of price trends in the trendsetters' attractor phase. The first one is oscillating, while remaining horizontal on average, as shown in Fig. 5.2.

The second and the third are those price series that are oscillating while climbing upward continually or downward continually, as shown in Fig. 5.3 and 5.4.

For these three samples,  $\gamma=0.25$  and  $\beta=0.74$ . In the upward price series, we found that the price cycles are mixed with symmetric and asymmetric cycles. Usually we have several asymmetric cycles followed by one symmetric cycle. The symmetric cycle has the same number of rising steps and dropping steps as shown in Fig. 5.2b while the asymmetric cycle has one dropping step less than the rising step as shown in Fig. 5.3b. In general the whole price series climbs upward. The situation is similar to the downward price series. There are symmetric and asymmetric cycles in the price series. In the asymmetric cycle, there is one rising step less than the dropping step as shown in Fig. 5.4(b). Hence, in general the price series goes downward. As we found in the upward or downward price series, the trendsetters' attractors have different fickle agents. The fickle agents are bottom&anti-bottom fickle agents for the upwards price series and top&anti-top fickle agents for the downward price series.

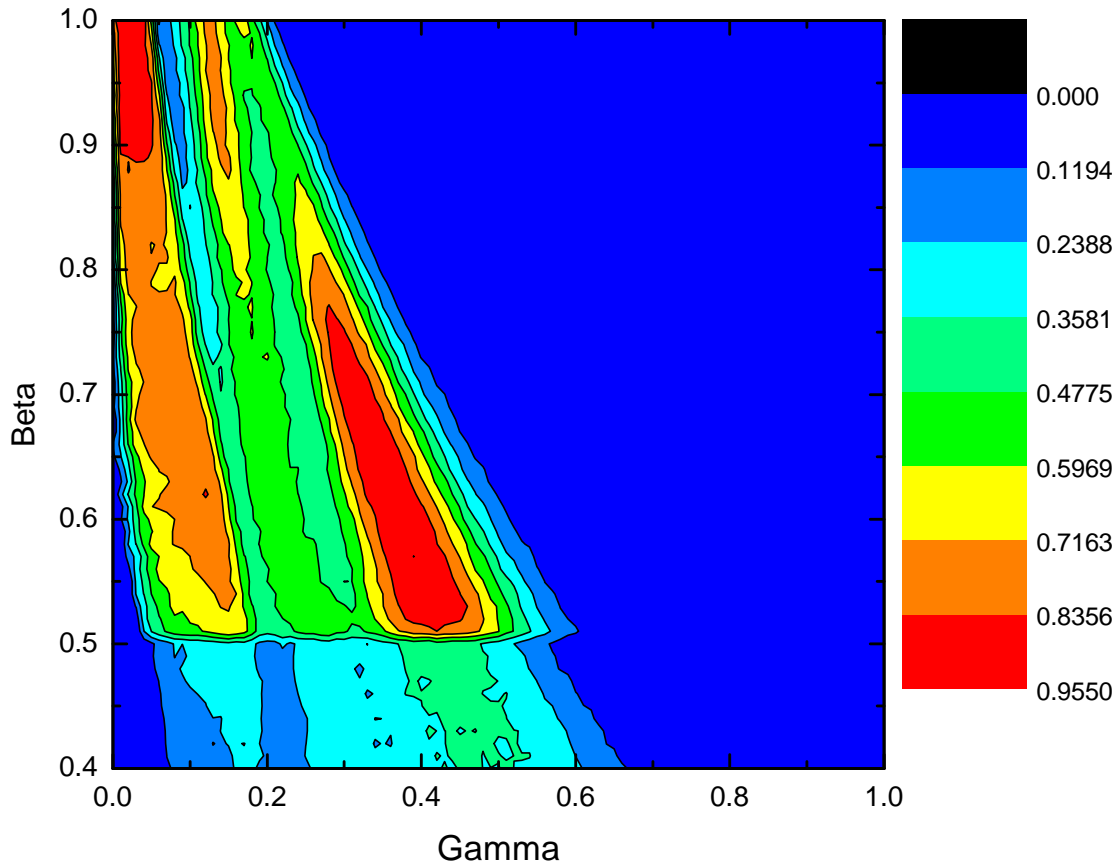


Figure 5.1: The superposition of (600) and (500) trendsetters' attractors.

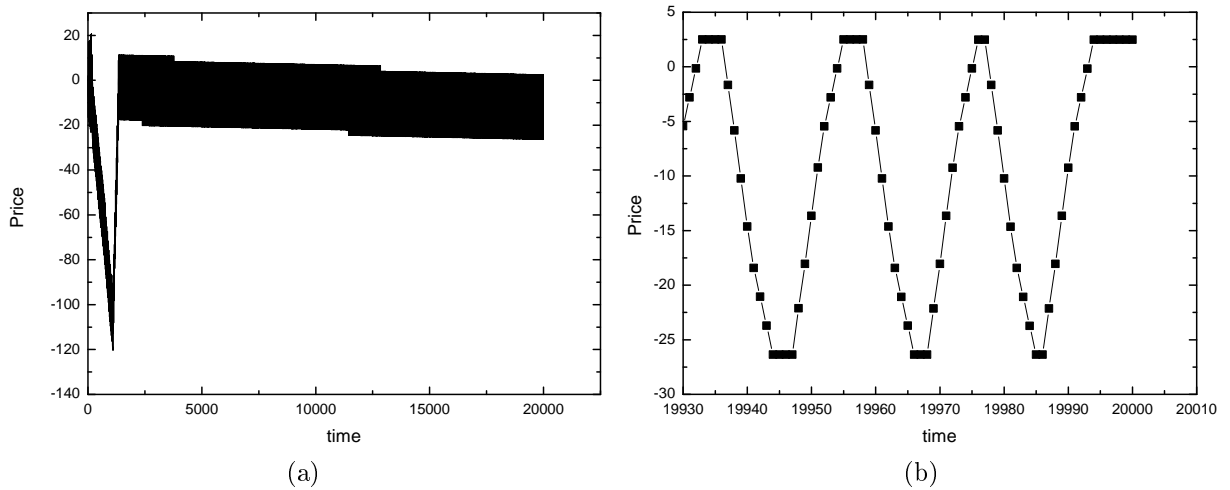


Figure 5.2: The generally horizontal price series. (600), (500) and (5jk) trendsetters' attractors are all found in this kind of price series. (a) the whole price series (b) the detail look of the price series.

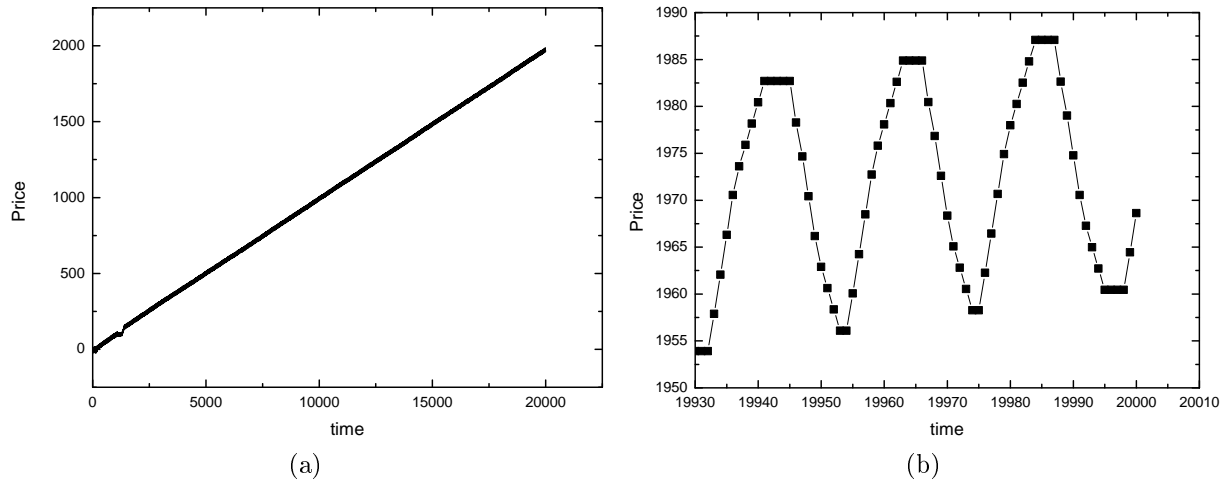


Figure 5.3: The price series that climbs upward while oscillating. (a) the general price trend (b) the detail look of the price series.

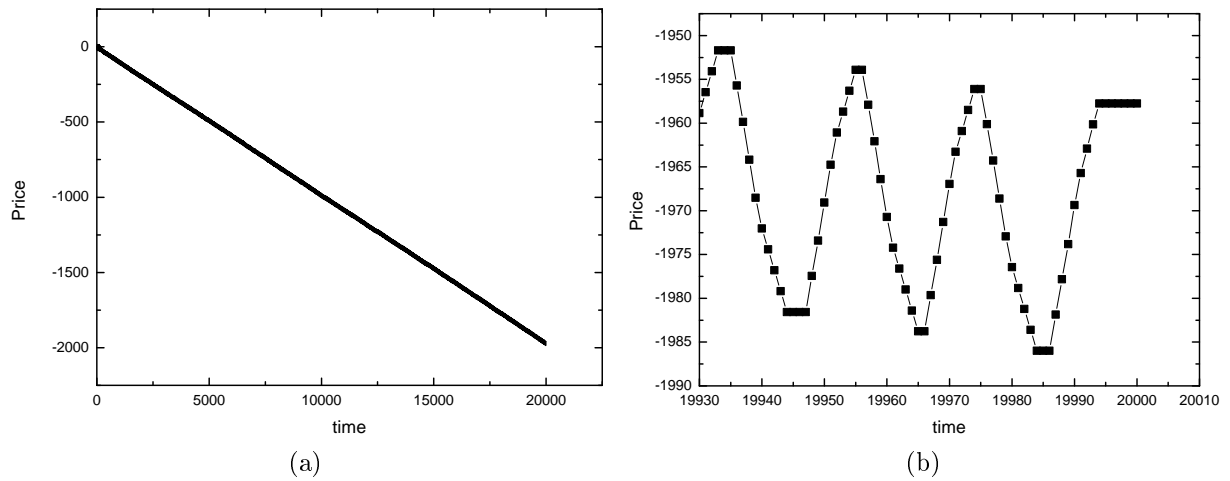


Figure 5.4: The price series that climbs downward while oscillating. (a) the general price trend (b) the price trend in larger scale.

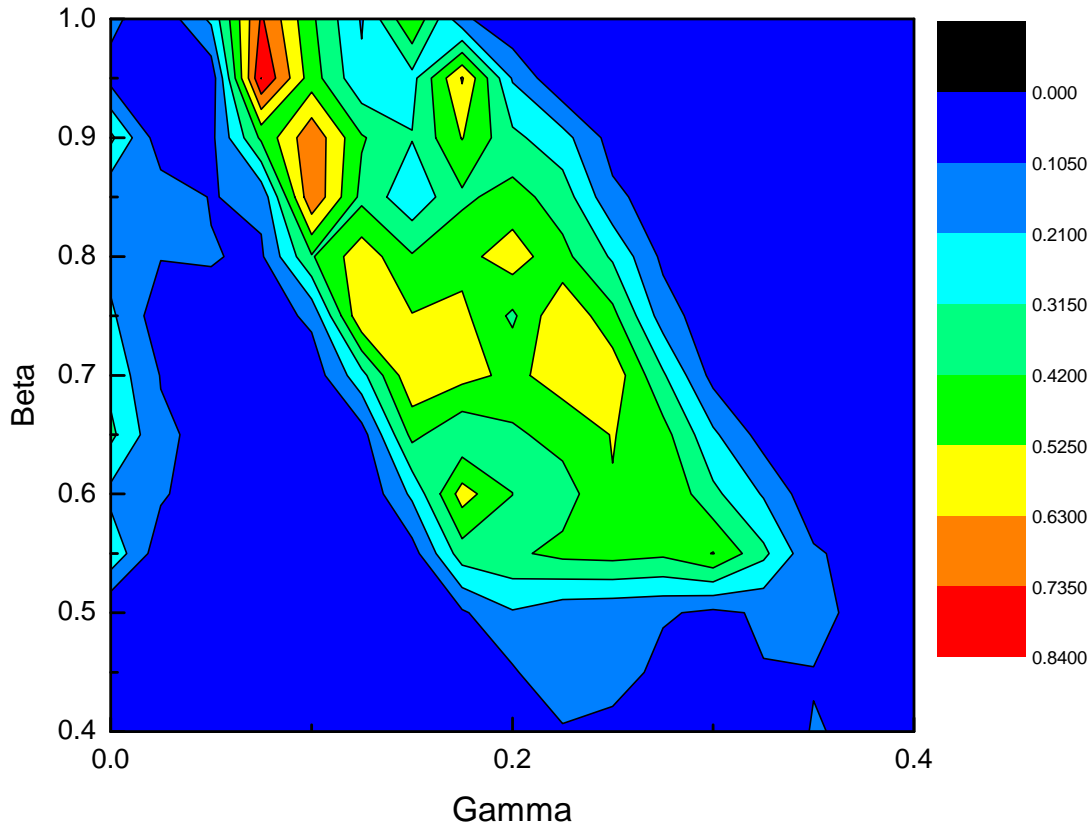


Figure 5.5: The existence region of the trendsetters' attractors with upward or downward price series ( $N=1000$ ,  $K=2$ ,  $m=2$ ,  $S=2$ , 10000 steps with 200 samples).

We would like to find the region where these two kinds of price series exist and see whether it is the reason of the low probability region in the middle of the (600) and (500) trendsetters' attractor phase. The simulation result is shown in Fig. 5.5.

To obtain Fig. 5.5, we define the trendsetters' attractors with price at time 10000 larger than 700 for the upwards price or smaller than  $-700$  for the downward price trend.

## 5.1 The Trendsetters' Attractor Analyses

Because of symmetry, the bottom&anti-bottom fickle agents in the upward price series will give the same analytical result as the top&anti-top fickle agents in the downward price series. We choose to analyze the top&anti-top fickle agent. From the wealth of each strategy shown in Fig. 5.6, we can classify the types of agents in the downward price series. There are fast agents, slow agents, bottom agents, top agents, anti-top agents, anti-bottom agents, anti-slow agents, anti-fast agents and top&anti-top fickle agents.



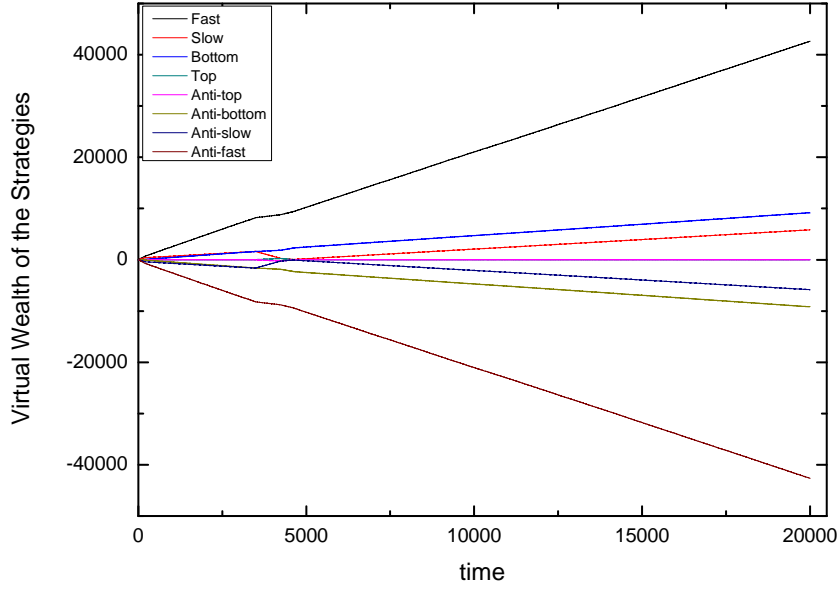


Figure 5.6: The virtual wealth of each strategy with  $\gamma=0.28$ ,  $\beta=0.6$ .

We can also find the fraction of each type of agents as we did in analyzing (600) trendsetters' attractor. The fractions are:  $15/64$ ,  $13/64$ ,  $11/64$ ,  $7/64$ ,  $2/64$ ,  $5/64$ ,  $7/64$ ,  $3/64$  and  $1/64$  for the fast, slow, bottom, top, top&anti-top, anti-top, anti-bottom, anti-slow and anti-fast agents respectively.

Because the wealth of the top strategy and the anti-top strategy have opposite virtual wealth, and the top strategy always gain wealth in the symmetric cycle as already discussed in Chapter 3. The wealth gain for the top strategy in one asymmetric cycle should always be negative otherwise, the top strategy will keep on gaining money and the anti-top strategy keep on losing money and the top&anti-top fickle agent will no longer exist. We observe that the asymmetric cycle contains 8 rising steps and 9 dropping steps and is shown in Fig. 5.4(b). It is also observed that several consecutive 8-up-9-down asymmetric cycle is followed by two types of symmetric cycles. One has 9 rising and dropping steps and the other one has 8 rising and dropping steps as shown in Fig. 5.8(a) and (b) respectively. They occur at different  $\gamma$  and  $\beta$ .

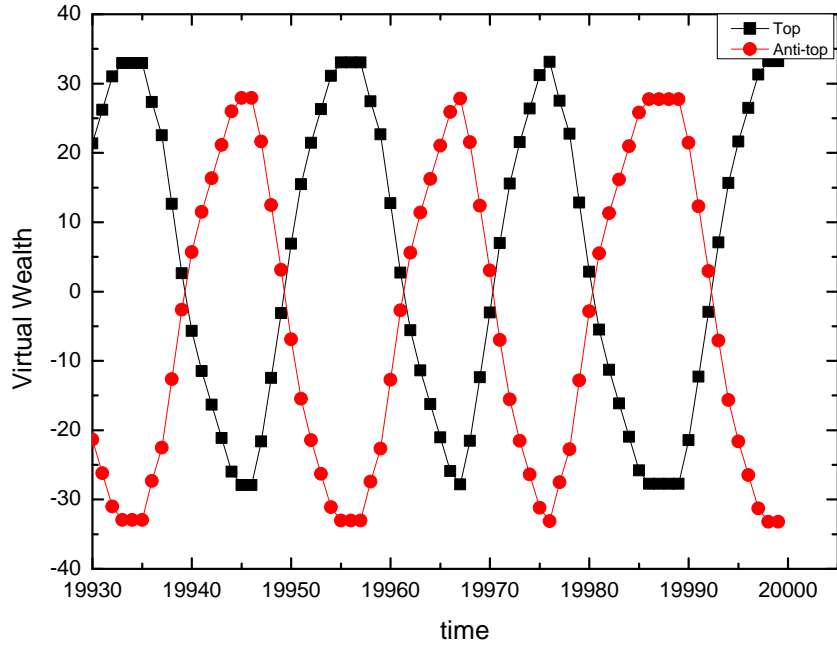


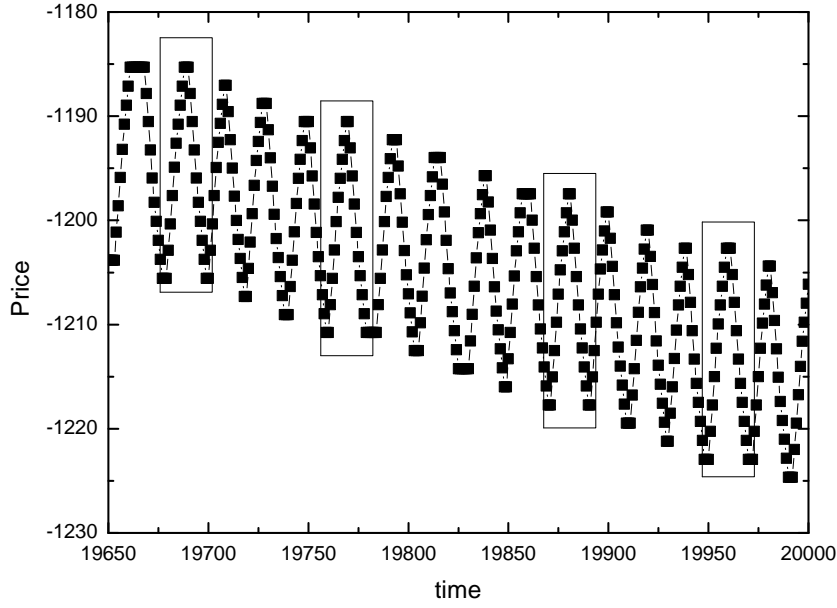
Figure 5.7: The detail look of the virtual wealth of the top and anti-top strategies,  $\gamma=0.28$  and  $\beta=0.6$ .

## 5.2 $n$ Asymmetric Cycles Followed by One 18 Steps Symmetric Cycle Analyses

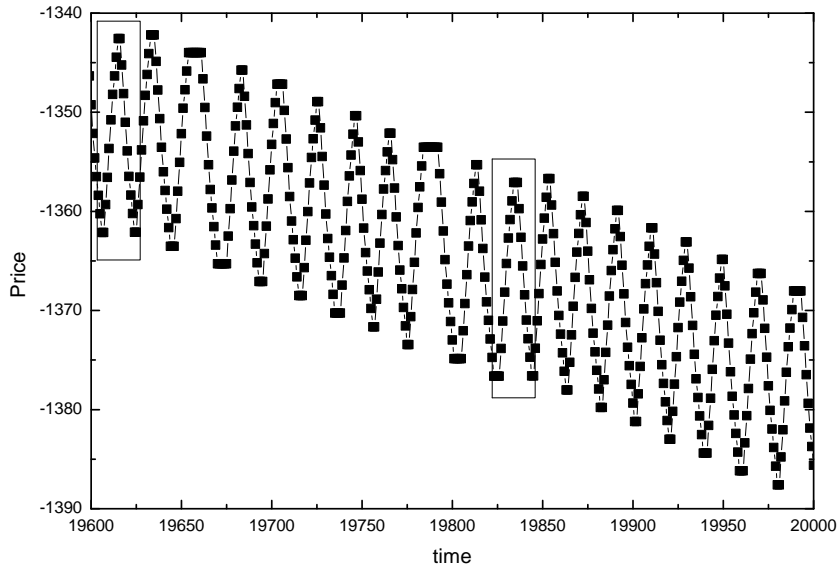
Tables 5.1 and 5.2 shows the decisions of the agents made in each step in the asymmetric cycle and the 9 rising and dropping steps' symmetric cycle respectively.

We can obtain the virtual wealth gain at each step for both the top and the anti-top strategy in the asymmetric cycle and the symmetric cycle. The value is listed in the last column of Table 5.3 and 5.4.

The total wealth gain in the asymmetric cycle for the top strategy should always be negative such that it can balance the wealth accumulated in one symmetric cycle and allow the fickle agent to exist. We denote  $\Delta W_{asym}$  to be the wealth gain for the top strategy in one asymmetric cycle and  $\Delta W_{sym}$  to be the wealth gain for the top strategy



(a)



(b)

Figure 5.8: The price cycle of the two kinds of asymmetric trendsetters' attractor. (a)  $n$  8-up-9-down + 1 9-up-9-down. The 9-up-9-down cycles are labeled by squares in the figure, separated by 3 to 4 8-up-9-down cycles. (b)  $n$  8-up-9-down + 1 8-up-8-down. The 8-up-8-down cycles are labeled by squares in the figure, separated by 9 8-up-9-down cycles.

time	$f$ $\frac{15}{64}$	$s$ $\frac{13}{64}$	$b$ $\frac{11}{64}$	$t$ $\frac{7}{64}$	$t \& \bar{t}$ $\frac{2}{64}$	$\bar{t}$ $\frac{5}{64}$	$\bar{b}$ $\frac{7}{64}$	$\bar{s}$ $\frac{3}{64}$	$f$ $\frac{1}{64}$	No. of buyers (in units of $\frac{N}{64}$ )	No. of sellers (in units of $\frac{N}{64}$ )	Price Change $D_x$
0	h	h	h	h	h	h	h	h	h	0	0	$D_0$
1	b	h	b	h	h	h	s	h	s	26	8	$D_{18}$
2	b	b	s	h	h	h	b	s	s	35	15	$D_{20}$
3	b	b	h	h	h	h	h	s	s	28	4	$D_{24}$
4	b	b	h	h	h	h	h	s	s	28	4	$D_{24}$
5	h	b	h	h	b	h	h	s	h	15	3	$D_{12}$
6	h	h	h	h	b	h	h	h	h	2	0	$D_2$
7	h	h	h	h	b	h	h	h	h	2	0	$D_2$
8	h	h	h	h	b	h	h	h	h	2	0	$D_2$
9	h	h	h	h	h	h	h	h	h	0	0	$D_0$
10	s	h	h	s	s	b	h	h	b	6	24	$D_{-18}$
11	s	s	h	b	b	s	h	b	b	13	33	$D_{-20}$
12	s	s	h	h	h	h	h	b	b	4	28	$D_{-24}$
13	s	s	h	h	h	h	h	b	b	4	28	$D_{-24}$
14	h	s	h	h	h	h	h	b	h	3	13	$D_{-10}$
15	h	h	h	h	s	h	h	h	h	0	2	$D_{-2}$
16	h	h	h	h	s	h	h	h	h	0	2	$D_{-2}$
17	h	h	h	h	s	h	h	h	h	0	2	$D_{-2}$
18	h	h	h	h	s	h	h	h	h	0	2	$D_{-2}$
19	h	h	h	h	h	h	h	h	h	0	0	$D_0$

Table 5.1: The decisions made at each step by agents in the asymmetric cycle.

time	f	s	b	t	$t \& \bar{t}$	$\bar{t}$	$\bar{b}$	$\bar{s}$	$\bar{f}$	No. of buyers (in units of $\frac{N}{64}$ )	No. of sellers (in units of $\frac{N}{64}$ )	Price Change
	$\frac{15}{64}$	$\frac{13}{64}$	$\frac{11}{64}$	$\frac{7}{64}$	$\frac{2}{64}$	$\frac{5}{64}$	$\frac{7}{64}$	$\frac{3}{64}$	$\frac{1}{64}$			$D_x$
0	h	h	h	h	h	h	h	h	h	0	0	$D_0$
1	b	h	b	h	h	h	s	h	s	26	8	$D_{18}$
2	b	b	s	h	h	h	b	s	s	35	15	$D_{20}$
3	b	b	h	h	h	h	h	s	s	28	4	$D_{24}$
4	b	b	h	h	h	h	h	s	s	28	4	$D_{24}$
5	h	b	h	h	h	h	h	s	h	13	3	$D_{10}$
6	h	h	h	h	b	h	h	h	h	2	0	$D_2$
7	h	h	h	h	b	h	h	h	h	2	0	$D_2$
8	h	h	h	h	b	h	h	h	h	2	0	$D_2$
9	h	h	h	h	b	h	h	h	h	2	0	$D_2$
10	h	h	h	h	h	h	h	h	h	0	0	$D_0$
11	s	h	h	s	s	b	h	h	b	6	24	$D_{-18}$
12	s	s	h	b	b	s	h	b	b	13	33	$D_{-20}$
13	s	s	h	h	h	h	h	b	b	4	28	$D_{-24}$
14	s	s	h	h	h	h	h	b	b	4	28	$D_{-24}$
15	h	s	h	h	h	h	h	b	h	3	13	$D_{-10}$
16	h	h	h	h	s	h	h	h	h	0	2	$D_{-2}$
17	h	h	h	h	s	h	h	h	h	0	2	$D_{-2}$
18	h	h	h	h	s	h	h	h	h	0	2	$D_{-2}$
19	h	h	h	h	s	h	h	h	h	0	2	$D_{-2}$
20	h	h	h	h	h	h	h	h	h	0	0	$D_0$

Table 5.2: The decisions made at each step by agents in the symmetric cycle.

time	Position	Price Change	Virtual Wealth Gain
0	2	$D_0$	
1	2	$D_{18}$	$2\beta D_{20}$
2	2	$D_{20}$	$2[(1 - \beta)D_{18} + \beta D_{20}]$
3	2	$D_{24}$	$2[(1 - \beta)D_{20} + \beta D_{24}]$
4	2	$D_{24}$	$2[(1 - \beta)D_{24} + \beta D_{24}]$
5	2	$D_{12}$	$2[(1 - \beta)D_{24} + \beta D_{12}]$
6	2	$D_2$	$2[(1 - \beta)D_{12} + \beta D_2]$
7	2	$D_2$	$2[(1 - \beta)D_2 + \beta D_2]$
8	2	$D_2$	$2[(1 - \beta)D_2 + \beta D_2]$
9	2	$D_0$	$2(1 - \beta)D_2$
10	2	$D_{-18}$	$-2\beta D_{18}$
11	1	$D_{-20}$	$-[(1 - \beta)D_{18} + \beta D_{20}]$
12	2	$D_{-24}$	$-2[(1 - \beta)D_{20} + \beta D_{24}]$
13	2	$D_{-24}$	$-2[(1 - \beta)D_{24} + \beta D_{24}]$
14	2	$D_{-10}$	$-2[(1 - \beta)D_{24} + \beta D_{10}]$
15	2	$D_{-2}$	$-2[(1 - \beta)D_{10} + \beta D_2]$
16	2	$D_{-2}$	$-2[(1 - \beta)D_2 + \beta D_2]$
17	2	$D_{-2}$	$-2[(1 - \beta)D_2 + \beta D_2]$
18	2	$D_{-2}$	$-2[(1 - \beta)D_2 + \beta D_2]$
19	2	$D_0$	$-2[(1 - \beta)D_2 + \beta D_2]$

Table 5.3: The virtual wealth gain for the top strategy at each step in the asymmetric cycle for  $K=2$ .

time	Position	Price Change	Virtual Wealth Gain
0	2	$D_0$	
1	2	$D_{18}$	$2\beta D_{20}$
2	2	$D_{20}$	$2[(1 - \beta)D_{18} + \beta D_{20}]$
3	2	$D_{24}$	$2[(1 - \beta)D_{20} + \beta D_{24}]$
4	2	$D_{24}$	$2[(1 - \beta)D_{24} + \beta D_{24}]$
5	2	$D_{10}$	$2[(1 - \beta)D_{24} + \beta D_{10}]$
6	2	$D_2$	$2[(1 - \beta)D_{10} + \beta D_2]$
7	2	$D_2$	$2[(1 - \beta)D_2 + \beta D_2]$
8	2	$D_2$	$2[(1 - \beta)D_2 + \beta D_2]$
9	2	$D_2$	$2[(1 - \beta)D_2 + \beta D_2]$
10	2	$D_0$	$2(1 - \beta)D_2$
11	2	$D_{-18}$	$-2\beta D_{18}$
12	1	$D_{-20}$	$-[(1 - \beta)D_{18} + \beta D_{20}]$
13	2	$D_{-24}$	$-2[(1 - \beta)D_{20} + \beta D_{24}]$
14	2	$D_{-24}$	$-2[(1 - \beta)D_{24} + \beta D_{24}]$
15	2	$D_{-10}$	$-2[(1 - \beta)D_{24} + \beta D_{10}]$
16	2	$D_{-2}$	$-2[(1 - \beta)D_{10} + \beta D_2]$
17	2	$D_{-2}$	$-2[(1 - \beta)D_2 + \beta D_2]$
18	2	$D_{-2}$	$-2[(1 - \beta)D_2 + \beta D_2]$
19	2	$D_{-2}$	$-2[(1 - \beta)D_2 + \beta D_2]$
20	2	$D_0$	$-2[(1 - \beta)D_2]$

Table 5.4: The virtual wealth gain for the top strategy at each step in the symmetric cycle for  $K=2$ .

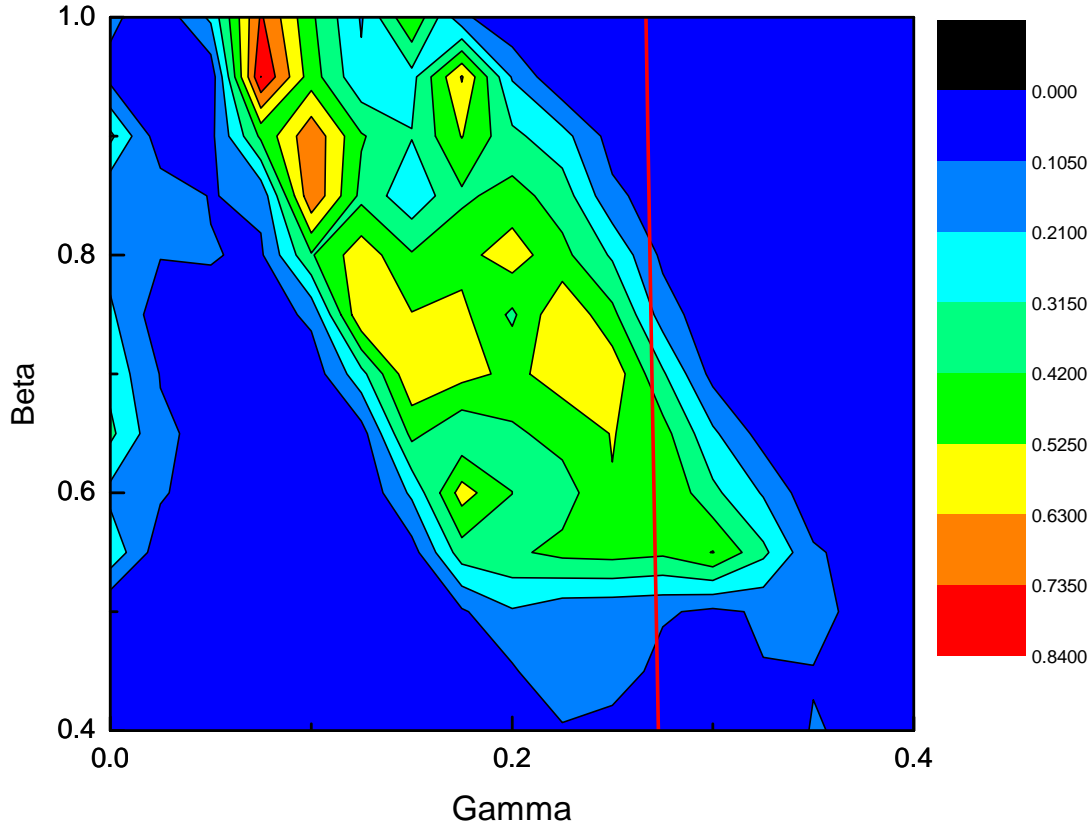


Figure 5.9: The phase diagram of the asymmetric trendsetters' attractor. The red line corresponds to  $\Delta W_{asym} = 0$ .

in one symmetric cycle. There is no need to calculate those of the anti-top strategy since it always takes the opposite decision to the top strategy. The condition

$$\Delta W_{asym} = (1 - \beta)D_{18} + \beta D_{20} + 2D_{12} - 2D_{10} - 2D_2 = 0, \quad (5.1)$$

gives us a line, separating the asymmetric trendsetters' attractor phase into two parts as shown in Fig. 5.9. Those  $n$  asymmetric cycles followed by one symmetric cycles can only occur on the left hand side of the red line in Fig. 5.9 while on the right hand side is another story that we will discuss in the next section.

To do the analyses, we first assume there is only one asymmetric cycle followed by one symmetric cycle. Apply the matching condition analyses to strategy  $x$  by assuming the wealth gain at the beginning of the  $i$ th step in the asymmetric cycle to be  $W_x^{asym}(i)$  and  $W_x^{sym}(i)$ . The conditions in the asymmetric cycle are:

$$W_t^{asym}(4) < W_{\bar{t}}^{asym}(4),$$



$$W_t^{asym}(5) > W_{\bar{t}}^{asym}(5),$$

$$W_t^{asym}(14) > W_{\bar{t}}^{asym}(14),$$

$$W_t^{asym}(15) < W_{\bar{t}}^{asym}(15).$$

If followed by one symmetric cycle, the conditions we have are:

$$W_t^{sym}(5) < W_{\bar{t}}^{sym}(5),$$

$$W_t^{sym}(6) > W_{\bar{t}}^{sym}(6),$$

$$W_t^{sym}(15) > W_{\bar{t}}^{sym}(15),$$

$$W_t^{sym}(16) < W_{\bar{t}}^{sym}(16).$$

Since the symmetric cycle is directly connected with the asymmetric cycle, the wealth at any step in the symmetric cycle can be represented by the wealth at a particular step in the asymmetric cycle by adding the wealth change at each step in between. Here I choose the wealth at the 15th step in the asymmetric cycle. Combine all those conditions, we get:

$$2A < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2B. \quad (5.2)$$

$$0 < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2C. \quad (5.3)$$

$$2H < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2I. \quad (5.4)$$

$$2K < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2J. \quad (5.5)$$

where

$$A = 2\beta D_{24} + 2\beta D_{10} + (2 - \beta)D_{20} + (1 + \beta)D_{18} - 6D_2 - 2D_{12},$$

$$B = (2 + 2\beta)D_{24} + 2\beta D_{10} + (2 - \beta)D_{20} + (1 + \beta)D_{18} - 6D_2 - 2D_{12},$$

$$C = 2(1 - \beta)D_{24} + 2\beta D_{10},$$

$$H = (2 + 2\beta)D_{24} + 2D_{18} + 2D_{20} - (2 - 2\beta)D_{10} - 8D_2,$$

$$K = (1 - \beta)D_{18} + \beta D_{20},$$

$$I = 4D_{24} + 2D_{18} + 2D_{20} - (2 - 4\beta)D_{10} - 8D_2,$$

$$J = (2 - 2\beta)D_{24} + \beta D_{20} + (1 - \beta)D_{18} + 2\beta D_{10}.$$

If  $n \geq 1$ , the matching conditions for each additional asymmetric cycle should be included, and the results become:

$$2A < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2B. \quad (5.6)$$

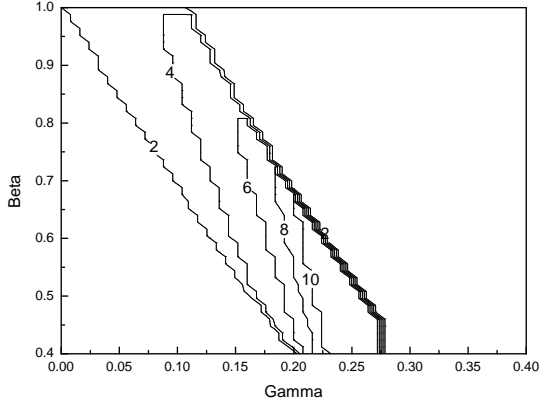


Figure 5.10: Theoretical boundaries for  $n$  asymmetric cycles followed by one symmetric cycle. ( $n=1, 2, \dots, 10$ )

$$0 < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2C. \quad (5.7)$$

$$2A + 2\Delta W_{asym} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2B + 2\Delta W_{asym}. \quad (5.8)$$

$$2\Delta W_{asym} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2C + 2\Delta W_{asym}. \quad (5.9)$$

. . .

$$2A + 2(n-1)\Delta W_{asym} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2B + 2(n-1)\Delta W_{asym}. \quad (5.10)$$

$$2(n-1)\Delta W_{asym} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2C + 2(n-1)\Delta W_{asym}. \quad (5.11)$$

$$2H + 2(n-1)\Delta W_{asym} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2I + 2(n-1)\Delta W_{asym}. \quad (5.12)$$

$$2K + 2(n-1)\Delta W_{asym} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2J + 2(n-1)\Delta W_{asym}. \quad (5.13)$$

Since  $\Delta W_{asym}$  should be negative as discussed already, the inequality we need to find out from the above conditions is

$$\begin{aligned} & \max[A, 0, H + (n-1)\Delta W_{asym}, K + (n-1)\Delta W_{asym}] \\ & < \min[B + (n-1)\Delta W_{asym}, C + (n-1)\Delta W_{asym}, \\ & \quad I + (n-1)\Delta W_{asym}, J + (n-1)\Delta W_{asym}]. \end{aligned} \quad (5.14)$$

For different  $n$ , we will have different phase boundaries as shown in Fig. 5.10.

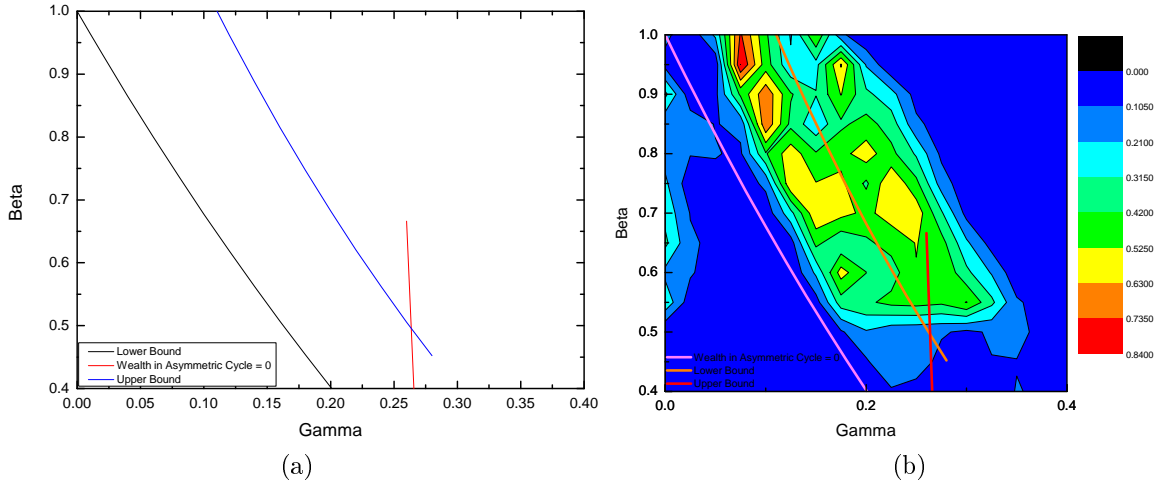


Figure 5.11: (a) Theoretical boundaries for the 8-up-9-down asymmetric cycle followed by one symmetric cycle. (b) Comparison with simulation results.

The results for  $n=1$  and 2 are the same, and are given by:

$$\begin{aligned}
 & \frac{3 + 6^\gamma - 10^\gamma - 12^\gamma}{5^\gamma + 9^\gamma - 10^\gamma + 12^\gamma} \\
 & < \beta \\
 & < \min \left[ \frac{4 + 5^\gamma - 9^\gamma - 10^\gamma}{2 \times 6^\gamma}, \frac{2(10^\gamma + 2^\gamma - 12^\gamma) - 18^\gamma}{20^\gamma - 18^\gamma} \right]. \quad (5.15)
 \end{aligned}$$

Eqs. (5.14) and (5.15) yield necessary conditions for the existence of the asymmetric trendsetters' attractor as shown in Fig. 5.11. However, they do not provide sufficiently tight bounds of the trendsetter region.

To better explain the phase boundary of the trendsetter region, we observe that the system behavior depends critically at one particular step of the dynamics, namely, the 16th step of the symmetric cycle. A typical situation is shown in Fig. 5.12. It shows an experiment in which  $\gamma$  is gradually tuned from inside the trendsetter region ( $\gamma = 0.075$ ) to outside ( $\gamma = 0.05$ ). When the trendsetters' attractor becomes destabilized at an intermediate value of  $\gamma$ , the period of the price cycle is halved. As shown in Fig. 5.12, a fickle agent in the trendsetters' attractor flicks between the top and anti-top strategies. However, after the trendsetters' attractor is destabilized, the virtual wealth of the top strategy remains above that of the anti-top strategy, and the originally flicking agent no longer flicks. Instead, the price dynamics is dominated by the behaviors of the fast and slow trendsetters.

The high sensitivity of the dynamics to the 16th step of the symmetric cycle is evident in Table 5.2, where one can find that only the top&anti-top fickle agent makes a

*sell* decision after she switches from the top strategy to anti-top strategy. On the other hand, all other agents make hold decisions. Hence if the fickle agent failed to switch strategy at this particular step, the 16th step would become a quiescent step. When a signal of rising price is randomly generated after the quiescent step, the fast trendsetters will make buy decisions, pushing the price up, and hence destabilize the symmetric cycle irreversibly.

We are now ready to derive the condition for destabilizing the symmetric cycle, which requires the dynamics to go through  $n$  asymmetric cycles and the first 15 steps of the symmetric cycle, but break down at the 16th step. Specifically, we have

$$W_t^{asym}(4) < W_{\bar{t}}^{asym}(4),$$

$$W_t^{asym}(5) > W_{\bar{t}}^{asym}(5),$$

$$W_t^{asym}(14) > W_{\bar{t}}^{asym}(14),$$

$$W_t^{asym}(15) < W_{\bar{t}}^{asym}(15).$$

If followed by one symmetric cycle, the conditions we have are:

$$W_t^{sym}(5) < W_{\bar{t}}^{sym}(5),$$

$$W_t^{sym}(6) > W_{\bar{t}}^{sym}(6),$$

$$W_t^{sym}(15) > W_{\bar{t}}^{sym}(15),$$

$$W_t^{sym}(16) > W_{\bar{t}}^{sym}(16).$$

Finally, we arrive at the condition for the trendsetters' attractor to break down, which is

$$H < K. \tag{5.16}$$

or

$$\beta < \frac{2D_{10} + 8D_2 - 2D_{24} - D_{18} - 2D_{20}}{2D_{24} + 2D_{10} - D_{20} + D_{18}}$$

As shown in Fig. 5.13, this condition provides a much better estimate of the phase boundary of the trendsetter region.

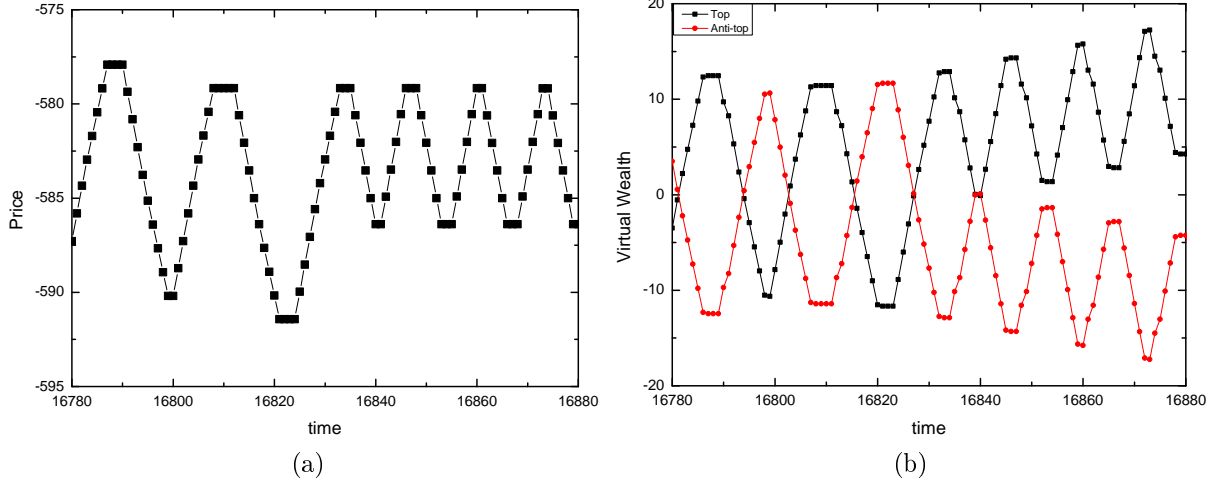


Figure 5.12: The evolution of the trendsetters' attractor when  $\gamma$  changes from 0.075 to 0.05 while  $\beta = 0.9$ . ( $N=1000$ ,  $K=2$ ,  $m=2$ ,  $S=2$ , 20000 steps) (a) the price, (b) the virtual wealth of the top and anti-top strategies.

The breaking condition Eq. (5.16) can be interpreted as follows. Let  $\Delta W_{sym}$  and  $\Delta W_{asym}$  be the change of virtual wealth of the top strategy in a symmetric and asymmetric cycle respectively. We further let  $x = |\Delta W_{sym}|/|\Delta W_{asym}|$ . Then the trendsetters' attractor consists of  $\lfloor x \rfloor$  or  $\lfloor x \rfloor + 1$  consecutive asymmetric attractors. Whether there are  $\lfloor x \rfloor$  or  $\lfloor x \rfloor + 1$  consecutive asymmetric attractors depends on whether the virtual wealth of the top strategy at the 5th step of the  $(\lfloor x \rfloor + 1)^{th}$  attractor is negative or positive. Since the wealth change in an asymmetric cycle is  $\Delta W_{asym}$ , the virtual wealth at the 5th step of the symmetric cycle is bounded between 0 and  $-|\Delta W_{asym}|$ . Furthermore, the wealth change in the attractor is  $\Delta W_{sym} + \lfloor x \rfloor \Delta W_{asym}$  or  $\Delta W_{sym} + (\lfloor x \rfloor + 1) \Delta W_{asym}$ . Except for parameter of zero measure, this wealth change is incommensurate with  $|\Delta W_{asym}|$ . Hence in the history of the attractor dynamics, the virtual wealth at the 5th step is evenly distributed in the range between 0 and  $-|\Delta W_{asym}|$ .

As shown in Fig. 5.13, the virtual wealth of the top strategy at the 16th step of the symmetric attractor is evenly distributed in the range  $K - H$  and  $K - H - |\Delta W_{asym}|$ . The attractor is destabilized if the virtual wealth becomes positive at any moment during the history of its dynamics. Hence the breaking condition becomes  $K - H > 0$ .

However, the top strategy is gaining wealth in a symmetric cycle and gradually losing wealth in the following asymmetric cycles. It is highly unstable if the top strategy gains more wealth in the symmetric cycle because of fluctuations and the fickle agent fails to switch strategy. We use changing  $\gamma$  to test the diminishing of the top&anti-top fickle agent, the result is shown in Fig. 5.12.

We can see that the trendsetters' attractor will break down only in the symmetric cycle. Now the conditions for the trendsetters' attractor to break down are:

$$\begin{aligned} & \max[A, 0, H + (n-1)\Delta W_{asym}] \\ < & \min[B + (n-1)\Delta W_{asym}, C + (n-1)\Delta W_{asym}, I + (n-1)\Delta W_{asym}, \\ & J + (n-1)\Delta W_{asym}, K + (n-1)\Delta W_{asym}] \end{aligned}$$

This implies

$$\begin{aligned} \frac{2D_{10} + 8D_2 - 2D_{24} - D_{18} - 2D_{20}}{2D_{24} + 2D_{10} - D_{20} + D_{18}} & < \beta \\ & < \min \left[ \frac{4 + 5^\gamma - 9^\gamma - 10^\gamma}{2 \times 6^\gamma}, \frac{2(10^\gamma + 2^\gamma - 12^\gamma) - 18^\gamma}{20^\gamma - 18^\gamma} \right] \end{aligned} \quad (5.17)$$

Note that compared with Eq. (5.15), the lower bound is changed, but the upper bound remains the same. Hence the stable region for the trendsetters' attractor with  $n$  8-up-9-down asymmetric cycle followed by one 9-up-9-down symmetric cycle is shown in Fig. 5.13.

### 5.3 $n$ Asymmetric Cycles Followed by One 16-Step Symmetric Cycle

As mentioned in Section 5.1, there are two types of symmetric cycles mixing with the 8 up and 9 down asymmetric cycles. We would now analyze the trendsetters' attractor with 16 steps in its symmetric price cycles. There are 8 rising and dropping steps in the symmetric cycle. We apply the method of matching conditions to this type of trendsetters' attractor. The matching conditions in the asymmetric cycle are:

$$W_t^{asym}(4) < W_{\bar{t}}^{asym}(4),$$

$$W_t^{asym}(5) > W_{\bar{t}}^{asym}(5),$$

$$W_t^{asym}(14) > W_{\bar{t}}^{asym}(14),$$

$$W_t^{asym}(15) < W_{\bar{t}}^{asym}(15).$$

The matching conditions in the symmetric cycle are:

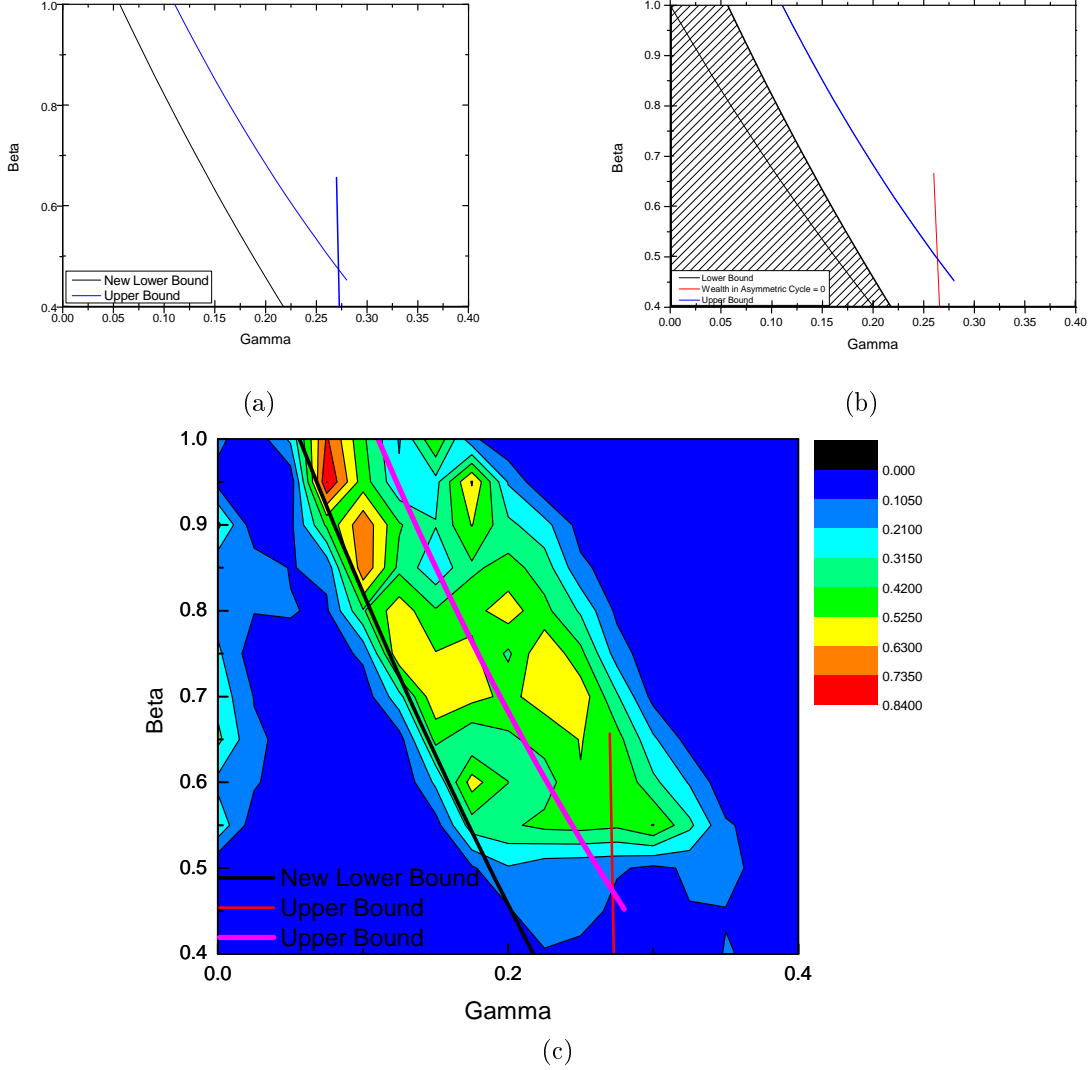


Figure 5.13: The boundary for the stable asymmetric trendsetters' attractor with  $n$  8-up-9-down asymmetric cycles and one 9-up-9-down symmetric cycle. ( $N=1000$ ,  $K=2$ ,  $m=2$ ,  $S=2$ , 20000 steps and 200 samples). (a) The new boundaries calculated from the breaking condition Eq. (5.17). (b) The new boundaries compared with the original boundaries calculated from the matching condition. The shadow area under the bold black line is where the matching conditions break. (c) The theoretical result compared with the simulation result.

$$W_t^{sym}(5) < W_{\bar{t}}^{sym}(5),$$

$$W_t^{sym}(6) > W_{\bar{t}}^{sym}(6),$$

$$W_t^{sym}(15) > W_{\bar{t}}^{sym}(15),$$

$$W_t^{sym}(16) < W_{\bar{t}}^{sym}(16).$$

Similarly, I use  $W_t^{asym}(15)$  and  $W_{\bar{t}}^{asym}(15)$  to solve the problem. The inequalities we get are:

$$2A < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2B. \quad (5.18)$$

$$0 < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2C. \quad (5.19)$$

$$2A + 2\Delta W_{asym} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2B + 2\Delta W_{asym}. \quad (5.20)$$

. . .

$$2A + 2(n-1)\Delta W_{asym} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2B + 2(n-1)\Delta W_{asym}. \quad (5.21)$$

$$\Delta W_{asym} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2C + 2(n-1)\Delta W_{asym}. \quad (5.22)$$

$$2H + 2(n-1)\Delta W_{asym} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2I + 2(n-1)\Delta W_{asym}. \quad (5.23)$$

$$2K + 2(n-1)\Delta W_{asym} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2J + 2(n-1)\Delta W_{asym}. \quad (5.24)$$

where

$$A = 2\beta D_{24} + 2\beta D_{10} + (2 - \beta)D_{20} + (1 + \beta)D_{18} - 6D_2 - 2D_{12},$$

$$B = (2 + 2\beta)D_{24} + 2\beta D_{10} + (2 - \beta)D_{20} + (1 + \beta)D_{18} - 6D_2 - 2D_{12},$$

$$C = 2(1 - \beta)D_{24} + 2\beta D_{10},$$

$$H = 2\beta D_{24} + 2D_{18} + 2D_{20} - (2 - 2\beta)D_{10} - 8D_2,$$

$$K = (2 - 2\beta)D_{24} + (1 - \beta)D_{18} + \beta D_{20} + 2D_{12} - 2(1 - \beta)D_{10} - 2D_2,$$

$$I = (2 + 2\beta)D_{24} + 2D_{18} + 2D_{20} - (2 - 2\beta)D_{10} - 8D_2,$$

$$J = (4 - 2\beta)D_{24} + \beta D_{20} + (1 - \beta)D_{18} - 2(1 - \beta)D_{10} + 2D_{12} - 2D_2.$$

$\Delta W_{asym}$  is the same as that calculated in Eq. (5.1).

Since  $\Delta W_{asym}$  is negative, solving the inequalities above is equivalent to solving:

$$\begin{aligned} & \max[A, 0, H + (n-1)\Delta W_{asym}, K + (n-1)\Delta W_{asym}] \\ & < \min[B + (n-1)\Delta W_{asym}, C + (n-1)\Delta W_{asym}, \\ & I + (n-1)\Delta W_{asym}, J + (n-1)\Delta W_{asym}]. \end{aligned} \quad (5.25)$$



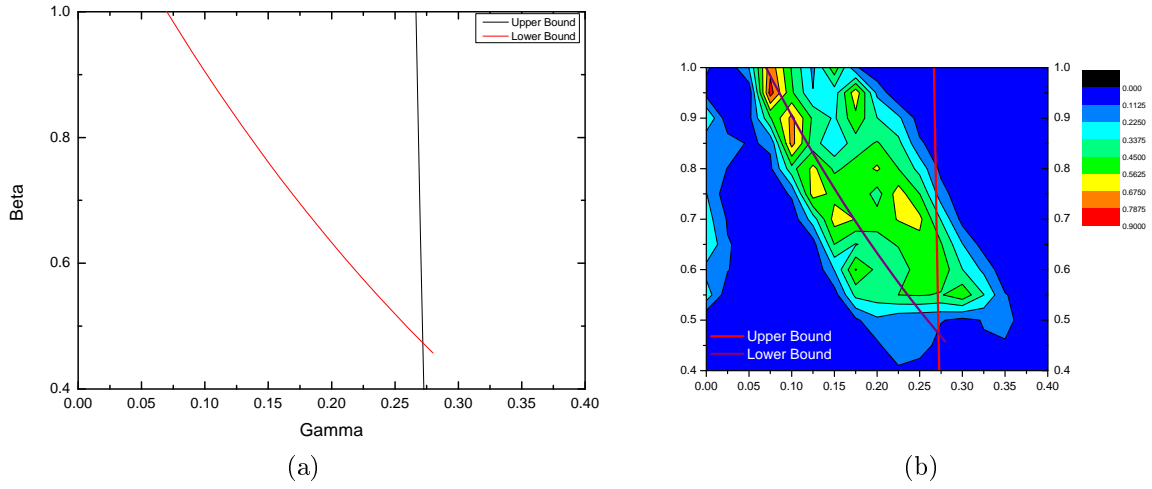


Figure 5.14: The boundary (??) for one asymmetric cycle followed by one 16-step symmetric cycle. (a) The theoretical result (b) The theoretical result compared with the simulation result.

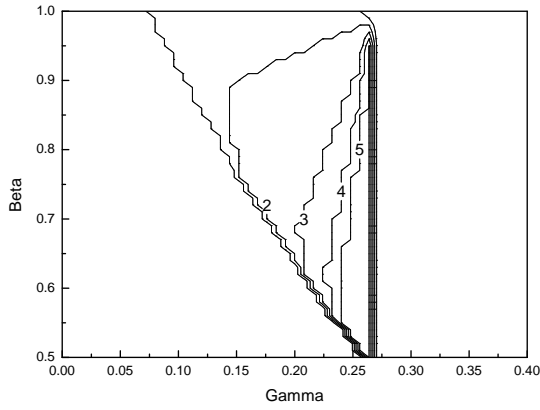


Figure 5.15: The boundaries for  $n$  asymmetric cycles followed by one 16-step symmetric cycle. ( $n=1, 2, \dots, 5$ )

The result for  $n = 1$  is Eq. (5.26) and shown in Fig. 5.14.

$$\begin{aligned}
 & \frac{6 \times 2^\gamma + 2 \times 12^\gamma - 2 \times 10^\gamma - 18^\gamma}{4 \times 24^\gamma + 18^\gamma - 20^\gamma} \\
 & < \beta \\
 & < \frac{2(10^\gamma + 2^\gamma - 12^\gamma) - 18^\gamma}{20^\gamma - 18^\gamma}.
 \end{aligned} \tag{5.26}$$

For  $n$  larger than 1, the result is shown in Fig. 5.15.

As shown in Fig. 5.14(b) the asymmetric trendsetters' attractor will disappear when

$\gamma$  and  $\beta$  increases. We found that more and more mixing trendsetters' attractors with horizontal price trends take place instead. Those trendsetters' attractors contain cycles such as (500), (503) and (530). It is possible that the slow strategy can no longer gain enough virtual wealth and starts to fickle with the bottom and top strategies and kill the asymmetric trendsetters' attractors.

The cycle in the price series that I observed on the upper right corner of the asymmetric trendsetters' attractor phase is constructed with (530) + 2(500) + (503) + 3(500). For this kind of cycles to exist and be stable, we can write down the balance conditions of the wealth gain in each cycle for each strategy. Denoting the wealth gain in cycle ( $ijk$ ) of strategy  $x$  as  $\Delta W_x^{ijk}$ , we obtain the stability condition of this kind of trendsetters' attractor by equating the virtual wealth gain of the slow and bottom or top strategies in a cycle,

$$\Delta W_s^{530} - \Delta W_b^{530} + n(\Delta W_s^{500} - \Delta W_b^{500}) + (\Delta W_s^{503} - \Delta W_b^{503}) = 0. \quad (5.27)$$

$$\Delta W_s^{530} - \Delta W_t^{530} + n(\Delta W_s^{500} - \Delta W_t^{500}) + (\Delta W_s^{503} - \Delta W_t^{503}) = 0. \quad (5.28)$$

where

$$\begin{aligned} \Delta W_s^{530} &= -4D_{20} - (2 + 3\beta)D_{18} + (1 - \beta)D_{26} + (4 - 2\beta)D_{14} + 4D_6 + 8D_4 - (1 - 2\beta)D_{24}, \\ \Delta W_b^{530} &= (1 - \beta)D_{20} - (2 - \beta)D_{18} + 4D_{24} - 2D_{26} - 4D_6 + 4D_4, \\ \Delta W_t^{530} &= (1 - \beta)D_{20} + (2 + \beta)D_{18} + 2D_{26} + 4D_6 - 4D_4 - 4D_{24}, \\ \Delta W_s^{500} &= 12D_4 - 4D_{20} - (2 + 2\beta)D_{18} + (2 - 4\beta)D_{24} + (4 - 2\beta)D_{14}, \\ \Delta W_b^{500} &= \Delta W_t^{500} = (1 - \beta)D_{20} + \beta D_{18}, \\ \Delta W_s^{503} &= -4D_{20} - (2 + 3\beta)D_{18} + (1 - 2\beta)D_{24} + (4 - 2\beta)D_{14} + 8D_4 + (1 - \beta)D_{26} + 4D_6, \\ \Delta W_b^{503} &= (1 - \beta)D_{20} + (2 - \beta)D_{18} - 4D_{24} - 4D_4 + 2D_{26} + 4D_6, \\ \Delta W_t^{503} &= (1 - \beta)D_{20} - (2 - \beta)D_{18} + 4D_{24} + 4D_4 - 2D_{26} - 4D_6. \end{aligned}$$

$n$  is the number of (500) in each period.

Eqs. (5.27) and (5.28) give the same result because of top-bottom symmetry. The result is shown in Eq. (5.29) and plotted in Fig. 5.16.

$$\begin{aligned} \beta = & \frac{2D_{26} + (8 + 4n)D_{14} - (10 + 5n)D_{20}}{(8 + 3n)D_{18} - (2 + n)D_{20} + (4 + 4n)D_{24} + 2D_{26} + (4 + 2n)D_{14}} + \\ & \frac{8D_6 + (16 + 12n)D_4 + (2 + 2n)D_{24} - (4 + 2n)D_{18}}{(8 + 3n)D_{18} - (2 + n)D_{20} + (4 + 4n)D_{24} + 2D_{26} + (4 + 2n)D_{14}}. \end{aligned} \quad (5.29)$$

As shown in Fig. 5.17, this family of lines matches the segment AB of the phase boundary of the asymmetric trendsetters. This shows that the asymmetric attractors become dominated by the symmetric attractors with 3 types of fickle agents, although the matching

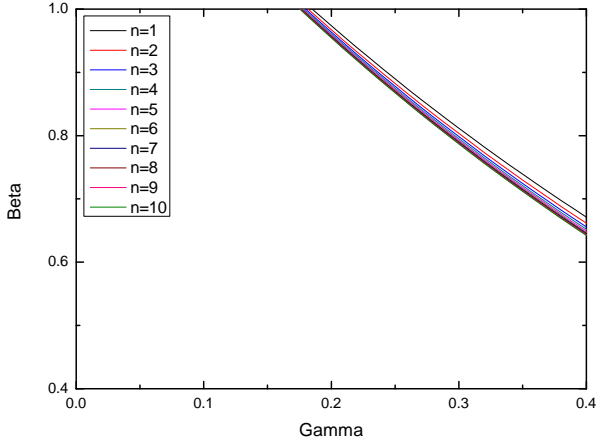


Figure 5.16: The stability lines of the special trendsetters' attractor.

conditions are still satisfied beyond the phase boundary.

## 5.4 $n$ Asymmetric Cycles Followed by One Different Asymmetric Cycle

Now we are interested in the right hand side of the line where  $\Delta W_{asym}$  is positive. Since the combination of asymmetric cycle and symmetric cycle will no longer exist, we choose some values of  $\beta$  and  $\gamma$  in that area and look for new types of trendsetters' attractors.

The trendsetters' attractor we found is  $n$  (8-up-9-down) asymmetric cycles followed by one (7-up-9-down) asymmetric cycle.  $\Delta W_{8up9down}$  of the top strategy is positive while  $\Delta W_{7up9down}$  is negative, hence the trendsetters' attractor is stable and the top&anti-top fickle agent still exist.

We can still apply the matching conditions to this trendsetters' attractor. Assume  $n=1$  at this moment and  $W_t^{asym}(i)$  is the wealth of the top strategy at step  $i$  in the asymmetric cycle with 8 rising steps and 9 dropping steps.

$$2A < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2B,$$

$$0 < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2C,$$

$$2H < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2I,$$

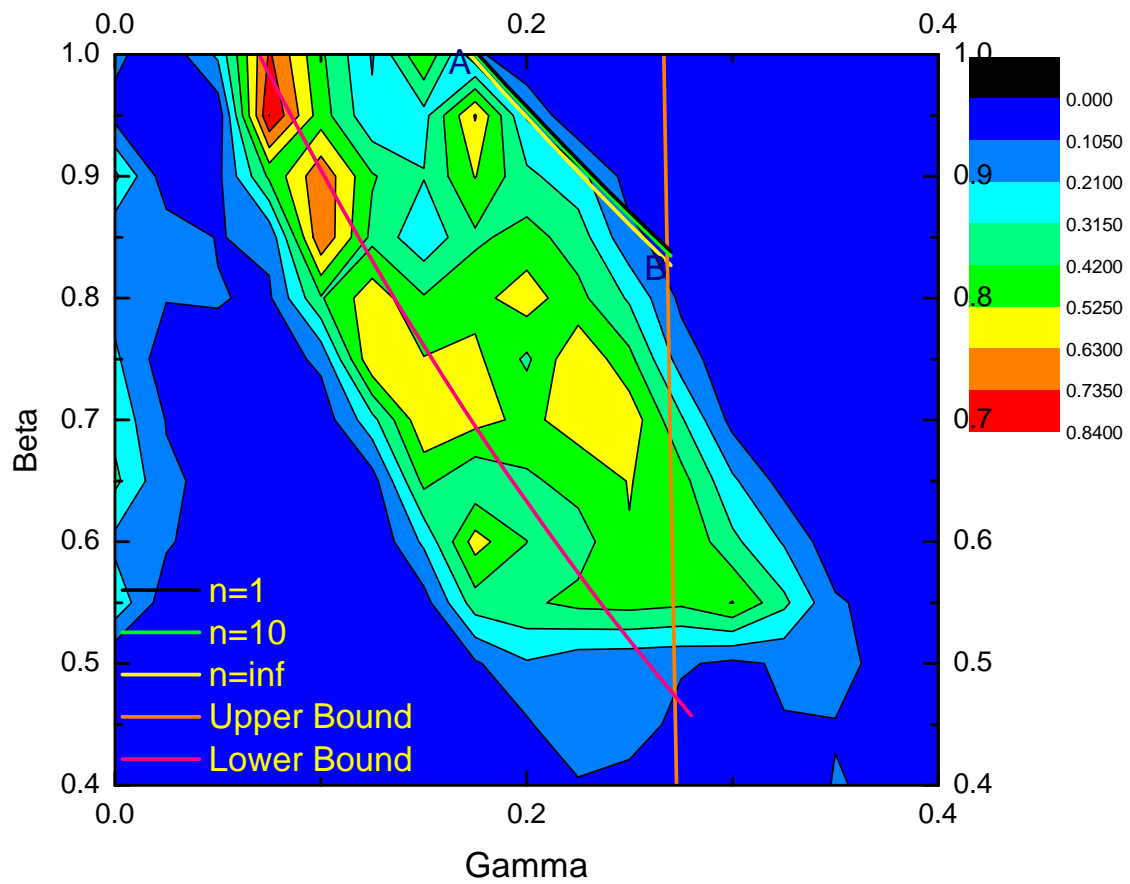


Figure 5.17: The boundaries of the 8 up 9 down asymmetric cycle followed by one 8 up 8 down symmetric cycle.

$$2K < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2J.$$

where

$$A = 2\beta D_{24} + 2\beta D_{10} + (2 - \beta)D_{20} + (1 + \beta)D_{18} - 6D_2 - 2D_{12},$$

$$B = (2 + 2\beta)D_{24} + 2\beta D_{10} + (2 - \beta)D_{20} + (1 + \beta)D_{18} - 6D_2 - 2D_{12},$$

$$C = 2(1 - \beta)D_{24} + 2\beta D_{10},$$

$$H = 2D_{18} + 2\beta D_{20} - (2 - 2\beta)D_{10} - 8D_2,$$

$$K = 2D_{26} - 2D_{24} + (1 - \beta)D_{18} + \beta D_{20} + 2D_{12} - 2(1 - \beta)D_{10} - 4D_2,$$

$$I = 2\beta D_{24} + 2D_{18} + 2D_{20} - (2 - 2\beta)D_{10} - 8D_2,$$

$$J = -2\beta D_{24} + 2D_{26} + \beta D_{20} + (1 - \beta)D_{18} - 2(1 - \beta)D_{10} + 2D_{12} - 4D_2.$$

If  $n \geq 1$ , the conditions are:

$$2A < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2B,$$

$$0 < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2C,$$

$$2A + 2\Delta W_{8up9down} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2B + 2\Delta W_{8up9down},$$

$$2\Delta W_{8up9down} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2C + 2\Delta W_{8up9down}.$$

. . .

$$2A + 2(n - 1)\Delta W_{8up9down} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2B + 2(n - 1)\Delta W_{8up9down},$$

$$2(n - 1)\Delta W_{8up9down} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2C + 2(n - 1)\Delta W_{8up9down},$$

$$2H + 2(n - 1)\Delta W_{8up9down} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2I + 2(n - 1)\Delta W_{8up9down},$$

$$2K + 2(n - 1)\Delta W_{8up9down} < W_{\bar{t}}^{asym}(15) - W_t^{asym}(15) < 2J + 2(n - 1)\Delta W_{8up9down}.$$

As  $\Delta W_{8up9down} = (1 - \beta)D_{18} + \beta D_{20} + 2D_{12} - 2D_{10} - 2D_2$  is positive, solving the inequalities above is equivalent to solving Eq. (5.30).

$$\begin{aligned} & \max[(n - 1)\Delta W_{8up9down}, A + (n - 1)\Delta W_{8up9down}, \\ & H + (n - 1)\Delta W_{8up9down}, K + (n - 1)\Delta W_{8up9down}] \\ & < \min[B, C, I + (n - 1)\Delta W_{8up9down}, J + (n - 1)\Delta W_{8up9down}]. \end{aligned} \quad (5.30)$$

For  $n=1$ , the result is

$$\begin{aligned} \max & \left[ \frac{2(10^\gamma + 2^\gamma - 12^\gamma) - 18^\gamma}{20^\gamma - 18^\gamma}, \frac{8 \times 2^\gamma + 2 \times 10^\gamma - 2 \times 20^\gamma - 2 \times 18^\gamma}{2 \times 24^\gamma + 2 \times 10^\gamma} \right] \\ < \beta < & \frac{2 \times 26^\gamma - 2 \times 20^\gamma + 4 \times 12^\gamma - 2 \times 10^\gamma + 2 \times 2^\gamma}{4 \times 24^\gamma - 2 \times 20^\gamma + 2 \times 18^\gamma}. \end{aligned} \quad (5.31)$$

The results for  $n=1$  and  $n=10$  are shown in Fig. 5.18. It shows that when all values of  $n$  are included, the region of this kind of attractor spans a sufficiently large area to the right of  $\Delta W_{asym} = 0$ .

The result fitting with the simulation is shown in Fig. 5.19.

The disappearance of this type of trendsetters' attractor maybe caused by the occurrence of the (500) trendsetters' attractor. This remains to be confirmed in future studies.

The results are compared with simulations in Fig. 5.19. It can be seen that the theoretical results yield a loose upper bound to the region of existence. This shows that the disappearance of this type of trendsetter region may have other causes. Incidentally, we observe that this region is in the vicinity of the region of maximum probability of the (500) trendsetters' attractor, as shown in Fig. 5.20. Hence a possible reason for the disappearance of the asymmetric attractor is its being dominated by the (500) attractor.

## 5.5 Summary

We have analyzed a special type of trendsetters' attractor with an asymmetric price trend. We again found that the fickle agents play an important role in the dynamics of the price cycles. In a falling price trend, the fickle agent is holding a top and anti-top strategy. The top strategy will lose wealth in the asymmetric cycles and gain wealth in the symmetric cycle, while the virtual wealth of the top strategy will oscillate around zero and can form a fickle agent with anti-top strategy. The bottom strategy in this case, will always gain wealth in both the asymmetric cycle and the symmetric cycle. Hence it can not be a member strategy of a fickle agent.

Similarly, in a rising price trend, the fickle agent is holding a bottom and anti-bottom strategy.

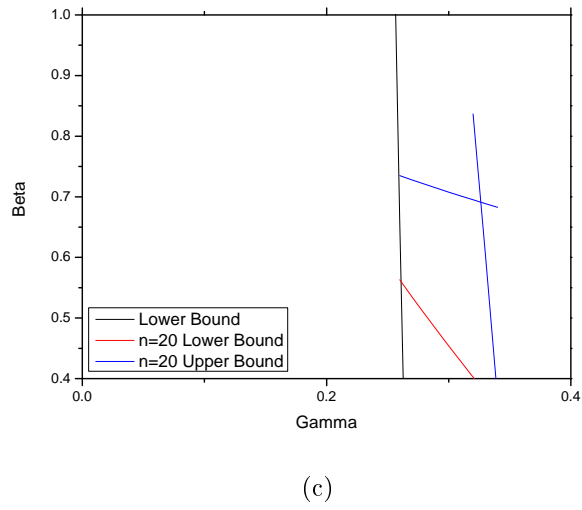
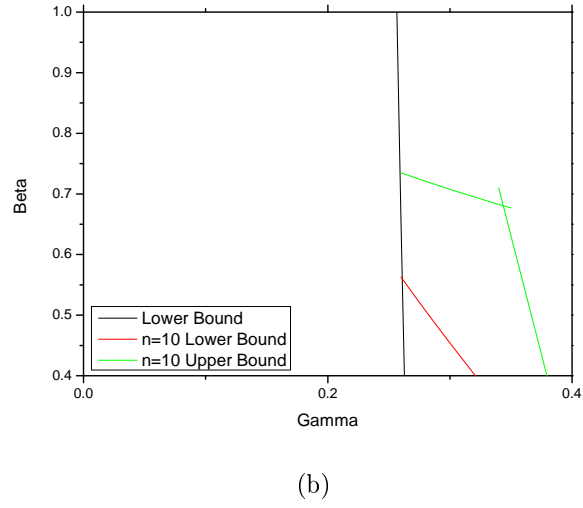
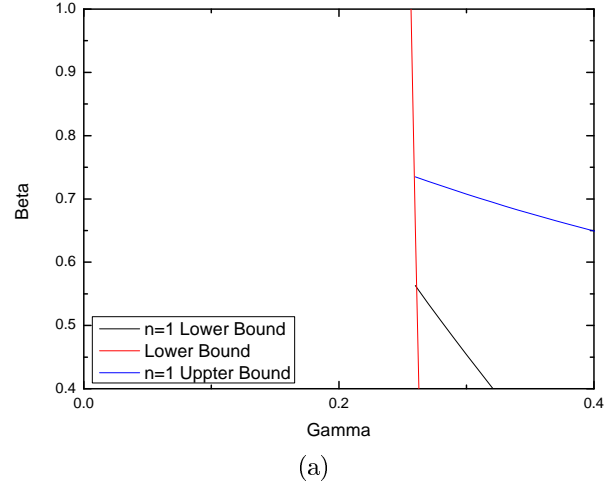


Figure 5.18: (a)  $n=1$  (b)  $n=10$  (c)  $n=20$ . The vertical line is  $\Delta W_{asym} = 0$ .

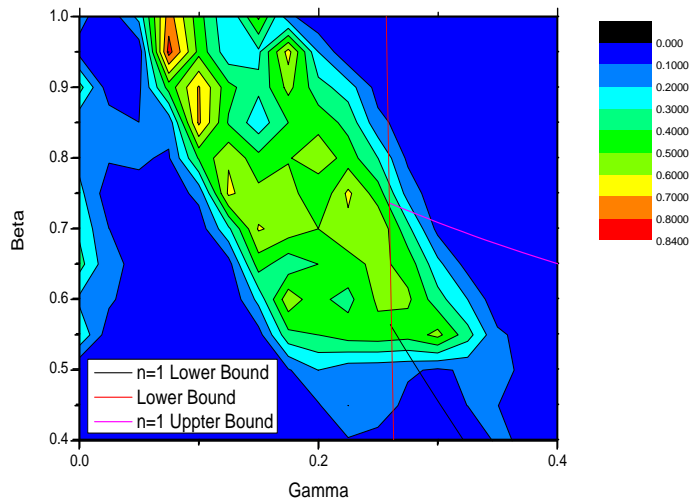


Figure 5.19: The theoretical result fitting with the simulation result.

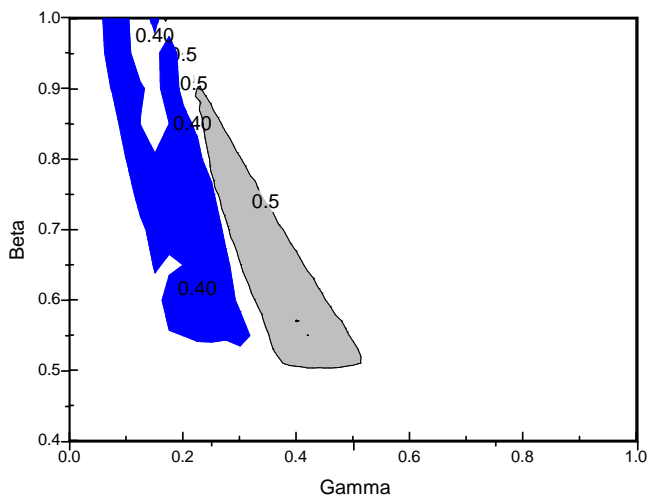


Figure 5.20: The maximum probability lines of the (500) compared with the boundary of the asymmetric trendsetters' attractor.



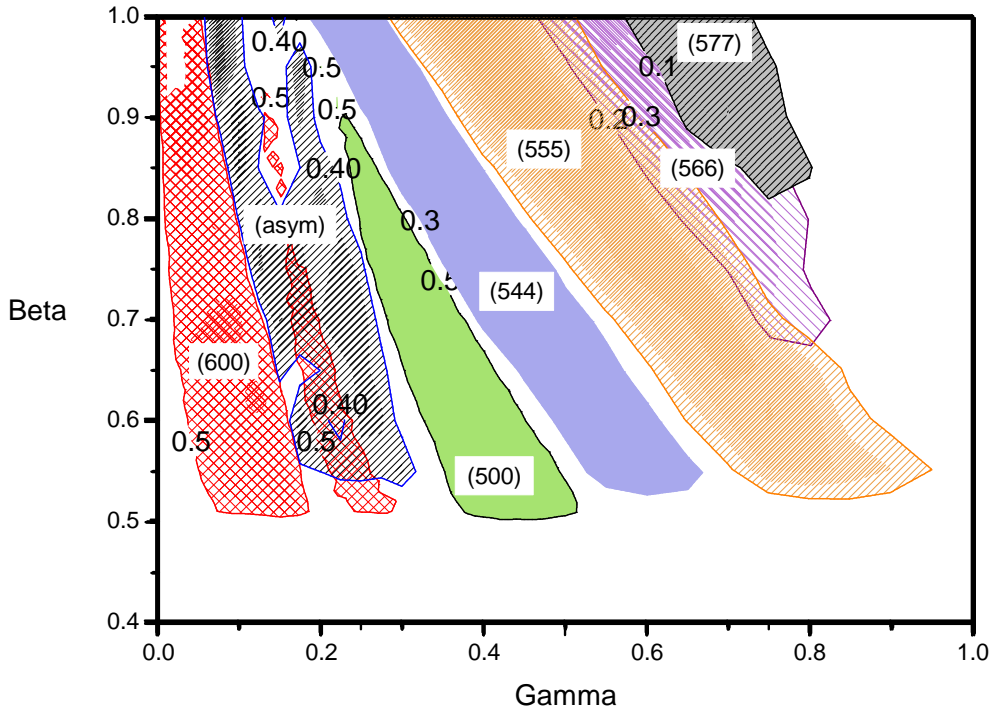


Figure 5.21: The regions of all the trendsetters' attractors that we analyzed in this thesis.

However, this type of asymmetric attractor seems to depend on a very delicate combination of favorable conditions. Hence it appears to be less stable than the symmetric attractors described in Chapter 3, such as the (600) and (500) attractors. Hence, they only exist in a narrow region of the parameter space. When one moves away from this region, the dynamics is overtaken by the symmetric attractors.

Fig. 5.21 shows the superposition of all the trendsetter's attractors in the  $\beta$  and  $\gamma$  space.

The Regions are defined where the probabilities of existence are above 50% of their maximum values. (The exceptions are the (566) and (577) where regions are defined where the probabilities of existence are above 65% of their maximum value.)

# Chapter 6

## Conclusion

In this thesis, we use the Essential Model to simplify the original Wealth Game. The basic properties of the Wealth Game are preserved in the model. In this model, holding decisions only result from the position limit  $K$ . The number of strategies is reduced by requiring their response to the signals  $\uparrow\downarrow$  and  $\downarrow\uparrow$  to be opposite. The phases of the arbitrageurs' and trendsetters' attractors still exist in the Essential Model and are bounded by phase transition lines ( $\beta = 0.5$ ). The irregular phase is also present in the area with large values of  $\beta$  and  $\gamma$ . We study the trendsetters' phase by analyzing the decisions at each step in one price cycle of the agents. The dynamics of the attractor consists of four stages. Different types of agents take part in turn in those four stages. The fast and slow agents are named the trendsetters and they take part in the first and the third stage. The fickle agents take part in stage two and four and push the price further up or down. We can study the trendsetters' attractor by analyzing the actions of those fickle agents.

Through the observation in the trendsetters' attractor phase, different types of trendsetters' attractors are classified by the typical fickle agents they contain and are named by the switching steps of the fickle agents. There are six typical fickle agents, the bottom&top, slow&bottom and slow&top fickle agents and their anti-fickle agents. Because of symmetry, the matching conditions that we used to do the analyses are the same for the fickle agents and their corresponding anti-fickle agents. Thus, we use  $(ijk)$  to classify the trendsetters' attractors where  $i, j, k$  represents the steps that bottom&top, slow&bottom and slow&top fickle agents switch their strategies when the price is rising and dropping.

Analyses of the matching conditions provide the necessary conditions for the existence of the trendsetters and are used to obtain the inequalities that provide the relations of  $\beta$  and  $\gamma$  for each type of trendsetters' attractor. Chapter 3 studied the behaviors of the

(600) and (500) trendsetters' attractors. These two trendsetters' attractors contain a single kind of fickle agents. The regions of existence of these two trendsetters' attractors are found and the result is quite acceptable. In this chapter, we observed that the fast agent always has the best wealth gain, and the slow agent the second best. The slow agent reacts to the change of price one step later than the fast agent because it is not as sensitive as the fast agent to the  $\downarrow\uparrow$  signal (when the price starts to rise) or the  $\uparrow\downarrow$  signal (when the price starts to drop). The bottom and top agents will gain a same amount of cash in each cycle because of the symmetry of the price series. The fickle agent here is the bottom&top fickle agent. Since the fickle agent takes the switching action at the wrong time, they keep on losing wealth in each price cycle in the game. However, their actions are very important in maintaining the dynamics of the trendsetters' attractor, because they push the price up (down) when the trendsetters are holding long (short) positions. This increases the wealth of the trendsetters and stabilizes the attractor. Without their actions, the trendsetters' attractor is left with stages 1 and 3 only, and the trendsetters cannot gain wealth through their arbitrage activities when  $\beta > 0.5$ .

Chapter 4 shows the application of the matching condition in calculating the sub-phases where the trendsetters' attractors contain three typical fickle agents. Besides the bottom&top fickle agents that were described in Chapter 3, we have the slow&bottom and the slow&top fickle agents as  $\gamma$  increases. The bottom&top fickle agents in these trendsetters' attractors always switch strategy at the 5th step for  $K = 2$  as the price starts to rise or drop. When  $\gamma$  increases, the switching steps of the slow&bottom and the slow&top fickle agents are delayed. This is because the wealth gain of the slow strategy in stage 2 and 4 decreases as  $\gamma$  increases. The period of the attractor increases because of the switching delay and the trendsetters' attractor develop from (544) to (577). Besides the matching conditions, we also use the maximum stability conditions of zero wealth change in a period to further find the maximum probability line of the trendsetters' attractors. The outermost boundary of the (577) trendsetters' attractor in Fig. 4.8, shows where  $\Delta W_s = 0$ , which means beyond the boundary, the slow strategy will be replaced by the anti-slow strategy (contrarian) as the wealth of the slow strategy becomes negative. Then, there is not enough trendsetters to create the periodic trend and the irregular phase occurs.

Chapter 5 is more complicated as the shape of the price trend is different from the rest of the trendsetters' attractor phase. The fickle agents in this chapter are very different. They are top&anti-top fickle agents in the downward price trend and bottom&anti-bottom in the upward price trend. The trendsetters' attractors in this region ( $0.1 < \gamma < 0.4$ ) is divided into three parts according to the price cycles they contain. The first part

is discussed in Section 5.2. The price period is composed of  $n$  8-up-9-down asymmetric cycles and 1 9-up-9-down symmetric cycle. The second part is discussed in Section 5.3, the price period is composed of  $n$  8-up-9-down asymmetric cycles and 1 8-up-8-down symmetric cycle. The third part is discussed in Section 5.4. The price period is composed of  $n$  8-up-9-down asymmetric cycles and 1 7-up-9-down asymmetric cycle. The method of the matching conditions is applied to these trendsetters' attractors with the help of some sufficient conditions named as the breaking condition in Section 5.2. The asymmetric attractors in this chapter seem to depend on a very delicate combination of favorable conditions and appears to be less stable than the symmetric attractors described in Chapter 3 and 4. Hence, they only exist in a narrow region of the parameter space and is easily overtaken by the symmetric attractors.

The Essential Model helps us to understand the roles of each agent in the trendsetters' attractors better and build up a more clear picture of those different trendsetters' attractors in the phase diagram. In our opinion, the low  $m$  study is significant enough in modeling the real market. People used to study large  $m$  but small  $m$  especially  $m=3$  will give the performance well acceptable when playing on the real market data [17]. This is because in reality, when agents are recording too many histories, their decisions will become much more.

From the analyses, we can guess without losing generality the effect of larger  $K$ . As  $K$  increases, the period of the price cycle will be longer as every agent can buy or sell in more steps to reach their limit positions than  $K=2$ . There will still be fickle agents, but their switching steps will be different from what we observed in the Essential Model as the periods get longer. When  $m$  is getting larger, the trendsetters' attractor phase will shrink to the left [18]. There will be more strategies as well as more fickle agents than we described in the model. Hence the types of agents will increase and the analyses will become more complicated. When  $K$  and  $m$  both increase, we will have more and more strategies. These additional strategies may increase the number of trendsetters and fickle agents. Analyses on those strategies may also be interesting.

The future direction of the research could be focused on the effect of varying distributions of strategies which is similar to the changing polarization in Giardina and Bouchaud's work [6]. We can also study the absence of market-maker in this model, and one of our groupmates is studying another model in the absence of the market-maker. The irregular phase can be studied since there are transient trendsetters as well.

# Appendix A

## Tables for Chapter 3 and 4

<i>time</i>	<i>Position</i>	<i>Price Change</i>	<i>Wealth Change of the Bottom Strategy</i>
0	-2	$D_0$	
1	-2	$D_{20}$	$-2\beta D_{20}$
2	-2	$D_{18}$	$-2[(1-\beta)D_{20} + \beta D_{18}]$
3	-1	$D_{24}$	$-[(1-\beta)D_{18} + \beta D_{24}]$
4	0	$D_{24}$	0
5	1	$D_{10}$	$[(1-\beta)D_{24} + \beta D_{10}]$
6	2	$D_4$	$2[(1-\beta)D_{10} + \beta D_4]$
7	2	$D_4$	$2[(1-\beta)D_4 + \beta D_4]$
8	2	$D_4$	$2[(1-\beta)D_4 + \beta D_4]$
9	2	$D_4$	$2[(1-\beta)D_4 + \beta D_4]$
10	2	$D_0$	$2(1-\beta)D_4$
11	2	$D_{-20}$	$-2\beta D_{20}$
12	2	$D_{-18}$	$-2[(1-\beta)D_{20} + \beta D_{18}]$
13	1	$D_{-24}$	$-[(1-\beta)D_{18} + \beta D_{24}]$
14	0	$D_{-24}$	0
15	-1	$D_{-10}$	$-1[(1-\beta)D_{24} + \beta D_{10}]$
16	-2	$D_{-4}$	$-2[(1-\beta)D_{10} + \beta D_4]$
17	-2	$D_{-4}$	$-2[(1-\beta)D_4 + \beta D_4]$
18	-2	$D_{-4}$	$-2[(1-\beta)D_4 + \beta D_4]$
19	-2	$D_{-4}$	$-2[(1-\beta)D_4 + \beta D_4]$
20	-2	$D_0$	$-2[(1-\beta)D_4]$

Table A.1: Wealth change of the slow strategy at each step in one price cycle of the (600) trendsetters' attractor.

time	f	s	b	t	b&t	$\bar{b}\&\bar{t}$	$\bar{t}$	$\bar{b}$	$\bar{s}$	$\bar{f}$	No. of buyers (in units of $\frac{N}{64}$ )	No. of sellers (in units of $\frac{N}{64}$ )	Price Change $D_x$
0	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$
1	b	h	b	h	b	h	h	s	h	s	26	6	$D_{20}$
2	b	b	s	h	s	h	h	b	s	s	33	15	$D_{18}$
3	b	b	h	h	h	h	h	h	s	s	28	4	$D_{24}$
4	b	b	h	h	h	h	h	h	s	s	28	4	$D_{24}$
5	h	b	h	h	b	b	h	h	s	h	17	3	$D_{14}$
6	h	h	h	h	b	b	h	h	h	h	4	0	$D_4$
7	h	h	h	h	b	b	h	h	h	h	4	0	$D_4$
8	h	h	h	h	b	b	h	h	h	h	4	0	$D_4$
9	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$
10	s	h	h	s	s	h	b	h	h	b	6	26	$D_{-20}$
11	s	s	h	b	b	h	s	h	b	b	15	33	$D_{-18}$
12	s	s	h	h	h	h	h	h	b	b	4	28	$D_{-24}$
13	s	s	h	h	h	h	h	h	b	b	4	28	$D_{-24}$
14	h	s	h	h	s	s	h	h	b	h	3	17	$D_{-14}$
15	h	h	h	h	s	s	h	h	h	h	0	4	$D_{-4}$
16	h	h	h	h	s	s	h	h	h	h	0	4	$D_{-4}$
17	h	h	h	h	s	s	h	h	h	h	0	4	$D_{-4}$
18	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$

Table A.2: The decisions made by different types of in the (500) trendsetters' attractor.

time	position			Price Change	Wealth Change		
	b	t	s		b	t	s
0	-2	2	-2	$D_0$			
1	-2	2	-2	$D_{20}$	$-2\beta D_{20}$	$2\beta D_{20}$	$-2\beta D_{20}$
2	-1	2	-2	$D_{18}$	$-[(1-\beta)D_{20} + \beta D_{18}]$	$2[(1-\beta)D_{20} + \beta D_{18}]$	$-2[(1-\beta)D_{20} + \beta D_{18}]$
3	-2	2	-1	$D_{24}$	$-2[(1-\beta)D_{18} + \beta D_{24}]$	$2[(1-\beta)D_{18} + \beta D_{24}]$	$-[(1-\beta)D_{18} + \beta D_{24}]$
4	-2	2	0	$D_{24}$	$-2[(1-\beta)D_{24} + \beta D_{24}]$	$2[(1-\beta)D_{24} + \beta D_{24}]$	0
5	-2	2	1	$D_{14}$	$-2[(1-\beta)D_{24} + \beta D_{14}]$	$2[(1-\beta)D_{24} + \beta D_{14}]$	$[(1-\beta)D_{24} + \beta D_{14}]$
6	-2	2	2	$D_4$	$-2[(1-\beta)D_{14} + \beta D_4]$	$2[(1-\beta)D_{14} + \beta D_4]$	$2[(1-\beta)D_{14} + \beta D_4]$
7	-2	2	2	$D_4$	$-2[(1-\beta)D_4 + \beta D_4]$	$2[(1-\beta)D_4 + \beta D_4]$	$2[(1-\beta)D_4 + \beta D_4]$
8	-2	2	2	$D_4$	$-2[(1-\beta)D_4 + \beta D_4]$	$2[(1-\beta)D_4 + \beta D_4]$	$2[(1-\beta)D_4 + \beta D_4]$
9	-2	2	2	$D_0$	$-2[(1-\beta)D_4]$	$2[(1-\beta)D_4]$	$2[(1-\beta)D_4]$
10	-2	2	2	$D_{-20}$	$2\beta D_{20}$	$-2\beta D_{20}$	$-2\beta D_{20}$
11	-2	1	2	$D_{-18}$	$2[(1-\beta)D_{20} + \beta D_{18}]$	$-[(1-\beta)D_{20} + \beta D_{18}]$	$-2[(1-\beta)D_{20} + \beta D_{18}]$
12	-2	2	1	$D_{-24}$	$2[(1-\beta)D_{18} + \beta D_{24}]$	$-2[(1-\beta)D_{18} + \beta D_{24}]$	$-[(1-\beta)D_{18} + \beta D_{24}]$
13	-2	2	0	$D_{-24}$	$2[(1-\beta)D_{24} + \beta D_{24}]$	$-2[(1-\beta)D_{24} + \beta D_{24}]$	0
14	-2	2	-1	$D_{-14}$	$2[(1-\beta)D_{24} + \beta D_{14}]$	$-2[(1-\beta)D_{24} + \beta D_{14}]$	$-[(1-\beta)D_{24} + \beta D_{14}]$
15	-2	2	-2	$D_{-4}$	$2[(1-\beta)D_{14} + \beta D_4]$	$-2[(1-\beta)D_{14} + \beta D_4]$	$-2[(1-\beta)D_{14} + \beta D_4]$
16	-2	2	-2	$D_{-4}$	$2[(1-\beta)D_4 + \beta D_4]$	$-2[(1-\beta)D_4 + \beta D_4]$	$-2[(1-\beta)D_4 + \beta D_4]$
17	-2	2	-2	$D_{-4}$	$2[(1-\beta)D_4 + \beta D_4]$	$-2[(1-\beta)D_4 + \beta D_4]$	$-2[(1-\beta)D_4 + \beta D_4]$
18	-2	2	-2	$D_0$	$2[(1-\beta)D_4]$	$-2[(1-\beta)D_4]$	$-2[(1-\beta)D_4]$

Table A.3: Wealth change for the bottom, top and slow strategy at each step in one price cycle of the (500) trendsetters' attractor.

time	f	s	b	t	b&t	s&b	s&t	$\bar{s}\&\bar{t}$	$\bar{s}\&\bar{b}$	$\bar{b}\&\bar{t}$	$\bar{t}$	$\bar{b}$	$\bar{s}$	$\bar{f}$	No. of buyers (in units of $\frac{N}{64}$ )	No. of sellers (in units of $\frac{N}{64}$ )	Price Change $D_x$
	$\frac{15}{64}$	$\frac{9}{64}$	$\frac{9}{64}$	$\frac{9}{64}$	$\frac{2}{64}$	$\frac{2}{64}$	$\frac{2}{64}$	$\frac{2}{64}$	$\frac{2}{64}$	$\frac{2}{64}$	$\frac{3}{64}$	$\frac{3}{64}$	$\frac{3}{64}$	$\frac{1}{64}$	0	0	$D_0$
0	h	h	h	h	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$
1	b	h	b	h	b	b	h	h	h	h	h	s	h	s	28	4	$D_{24}$
2	b	b	s	h	s	s	h	h	s	h	h	b	s	s	29	19	$D_{10}$
3	b	b	h	h	h	h	b	h	s	h	h	h	s	s	26	6	$D_{20}$
4	b	b	h	h	h	b	b	h	b	h	h	h	s	s	30	4	$D_{26}$
5	h	b	h	h	b	b	b	h	b	b	h	h	s	h	19	3	$D_{16}$
6	h	h	h	h	b	b	h	h	h	b	h	h	h	h	6	0	$D_6$
7	h	h	h	h	b	b	h	h	h	b	h	h	h	h	6	0	$D_6$
8	h	h	h	h	b	h	h	h	h	b	h	h	h	h	4	0	$D_4$
9	h	h	h	h	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$
10	s	h	h	s	s	h	s	h	h	h	b	h	h	b	4	28	$D_{-24}$
11	s	s	h	b	b	s	b	b	h	h	s	h	b	b	19	29	$D_{-10}$
12	s	s	h	h	h	s	h	b	h	h	h	h	b	b	6	26	$D_{-20}$
13	s	s	h	h	h	s	s	s	h	h	h	h	b	b	4	30	$D_{-26}$
14	h	s	h	h	s	s	s	s	s	s	h	h	b	h	3	19	$D_{-16}$
15	h	h	h	h	s	h	s	h	s	s	h	h	h	h	0	6	$D_{-6}$
16	h	h	h	h	s	h	s	h	s	s	h	h	h	h	0	6	$D_{-6}$
17	h	h	h	h	s	h	h	h	s	s	h	h	h	h	0	4	$D_{-4}$
18	h	h	h	h	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$

Table A.4: The decisions made by different types of in the (544) trendsetters' attractor.



time	position			Price Change	Wealth Change		
	bottom	top	slow		b	t	s
0	-2	2	-2	$D_0$			
1	-2	2	-2	$D_{24}$	$-2\beta D_{24}$	$2\beta D_{24}$	$-2\beta D_{24}$
2	-1	2	-2	$D_{10}$	$-[(1-\beta)D_{24} + \beta D_{10}]$	$2[(1-\beta)D_{24} + \beta D_{10}]$	$-2[(1-\beta)D_{24} + \beta D_{10}]$
3	-2	2	-1	$D_{20}$	$-2[(1-\beta)D_{10} + \beta D_{20}]$	$2[(1-\beta)D_{10} + \beta D_{20}]$	$-[(1-\beta)D_{10} + \beta D_{20}]$
4	-2	2	0	$D_{26}$	$-2[(1-\beta)D_{20} + \beta D_{26}]$	$-2[(1-\beta)D_{20} + \beta D_{26}]$	0
5	-2	2	1	$D_{16}$	$-2[(1-\beta)D_{26} + \beta D_{16}]$	$2[(1-\beta)D_{26} + \beta D_{16}]$	$[(1-\beta)D_{26} + \beta D_{16}]$
6	-2	2	2	$D_6$	$-2[(1-\beta)D_{16} + \beta D_6]$	$2[(1-\beta)D_{16} + \beta D_6]$	$2[(1-\beta)D_{16} + \beta D_6]$
7	-2	2	2	$D_6$	$-2[(1-\beta)D_6 + \beta D_6]$	$2[(1-\beta)D_6 + \beta D_6]$	$2[(1-\beta)D_6 + \beta D_6]$
8	-2	2	2	$D_4$	$-2[(1-\beta)D_6 + \beta D_4]$	$2[(1-\beta)D_6 + \beta D_4]$	$2[(1-\beta)D_6 + \beta D_4]$
9	-2	2	2	$D_0$	$-2[(1-\beta)D_4]$	$2[(1-\beta)D_4]$	$2[(1-\beta)D_4]$
10	-2	2	2	$D_{-24}$	$2\beta D_{24}$	$-2\beta D_{24}$	$-2\beta D_{24}$
11	-2	1	2	$D_{-10}$	$2[(1-\beta)D_{24} + \beta D_{10}]$	$-[(1-\beta)D_{24} + \beta D_{10}]$	$-2[(1-\beta)D_{24} + \beta D_{10}]$
12	-2	2	1	$D_{-20}$	$2[(1-\beta)D_{10} + \beta D_{20}]$	$-2[(1-\beta)D_{10} + \beta D_{20}]$	$-[(1-\beta)D_{10} + \beta D_{20}]$
13	-2	2	0	$D_{-26}$	$2[(1-\beta)D_{20} + \beta D_{26}]$	$-2[(1-\beta)D_{20} + \beta D_{26}]$	0
14	-2	2	-1	$D_{-16}$	$2[(1-\beta)D_{26} + \beta D_{16}]$	$-2[(1-\beta)D_{26} + \beta D_{16}]$	$[(1-\beta)D_{26} + \beta D_{16}]$
15	-2	2	-2	$D_{-6}$	$2[(1-\beta)D_{16} + \beta D_6]$	$-2[(1-\beta)D_{16} + \beta D_6]$	$2[(1-\beta)D_{16} + \beta D_6]$
16	-2	2	-2	$D_{-6}$	$2[(1-\beta)D_6 + \beta D_6]$	$-2[(1-\beta)D_6 + \beta D_6]$	$2[(1-\beta)D_6 + \beta D_6]$
17	-2	2	-2	$D_{-4}$	$2[(1-\beta)D_6 + \beta D_4]$	$-2[(1-\beta)D_6 + \beta D_4]$	$2[(1-\beta)D_6 + \beta D_4]$
18	-2	2	-2	$D_0$	$2[(1-\beta)D_4]$	$-2[(1-\beta)D_4]$	$2[(1-\beta)D_4]$

Table A.5: Wealth change for the bottom, top and slow strategy at each step in one price cycle of the (544) trendsetters' attractor.

time	f	s	b	t	b&t	s&b	s&t	$\bar{s}\&\bar{t}$	$\bar{s}\&\bar{b}$	$\bar{b}\&\bar{t}$	$\bar{t}$	$\bar{b}$	$\bar{s}$	$\bar{f}$	No. of buyers (in units of $\frac{N}{64}$ )	No. of sellers (in units of $\frac{N}{64}$ )	Price Change
	$\frac{15}{64}$	$\frac{9}{64}$	$\frac{9}{64}$	$\frac{9}{64}$	$\frac{2}{64}$	$\frac{2}{64}$	$\frac{2}{64}$	$\frac{2}{64}$	$\frac{2}{64}$	$\frac{2}{64}$	$\frac{3}{64}$	$\frac{3}{64}$	$\frac{3}{64}$	$\frac{1}{64}$	(in units of $\frac{N}{64}$ )	(in units of $\frac{N}{64}$ )	$D_x$
0	h	h	h	h	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$
1	b	h	b	h	b	b	h	h	h	h	h	s	h	s	28	4	$D_{24}$
2	b	b	s	h	s	s	b	h	s	h	h	b	s	s	29	19	$D_{10}$
3	b	b	h	h	h	h	b	h	s	h	h	h	s	s	26	6	$D_{20}$
4	b	b	h	h	h	h	b	h	s	h	h	h	s	s	26	6	$D_{20}$
5	h	b	h	h	b	b	b	h	b	b	h	h	s	h	19	3	$D_{16}$
6	h	h	h	h	b	b	h	h	b	b	h	h	h	h	8	0	$D_8$
7	h	h	h	h	b	b	h	h	b	b	h	h	h	h	8	0	$D_8$
8	h	h	h	h	b	b	h	h	h	b	h	h	h	h	6	0	$D_6$
9	h	h	h	h	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$
10	s	h	h	s	s	h	s	h	h	h	b	h	h	b	4	28	$D_{-24}$
11	s	s	h	b	b	s	b	b	h	h	s	h	b	b	19	29	$D_{-10}$
12	s	s	h	h	h	s	h	b	h	h	h	h	b	b	6	26	$D_{-20}$
13	s	s	h	h	h	s	h	b	h	h	h	h	b	b	6	26	$D_{-20}$
14	h	s	h	h	s	s	s	s	s	s	h	h	b	h	3	19	$D_{-16}$
15	h	h	h	h	s	h	s	s	s	s	h	h	h	h	0	8	$D_{-8}$
16	h	h	h	h	s	h	s	s	s	s	h	h	h	h	0	8	$D_{-8}$
17	h	h	h	h	s	h	s	h	s	s	h	h	h	h	0	6	$D_{-6}$
18	h	h	h	h	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$

Table A.6: The decisions made by different types of in the (555) trendsetters' attractor.

time	position			Price Change	Wealth Change		
	bottom	top	slow		b	t	s
0	-2	2	-2	$D_0$			
1	-2	2	-2	$D_{24}$	$-2\beta D_{24}$	$2\beta D_{24}$	$-2\beta D_{24}$
2	-1	2	-2	$D_{10}$	$-[(1-\beta)D_{24} + \beta D_{10}]$	$2[(1-\beta)D_{24} + \beta D_{10}]$	$-2[(1-\beta)D_{24} + \beta D_{10}]$
3	-2	2	-1	$D_{20}$	$-2[(1-\beta)D_{10} + \beta D_{20}]$	$2[(1-\beta)D_{10} + \beta D_{20}]$	$-[(1-\beta)D_{10} + \beta D_{20}]$
4	-2	2	0	$D_{20}$	$-2[(1-\beta)D_{20} + \beta D_{20}]$	$2[(1-\beta)D_{20} + \beta D_{20}]$	0
5	-2	2	1	$D_{16}$	$-2[(1-\beta)D_{20} + \beta D_{16}]$	$-[(1-\beta)D_{20} + \beta D_{16}]$	$[(1-\beta)D_{20} + \beta D_{16}]$
6	-2	2	2	$D_8$	$-2[(1-\beta)D_{16} + \beta D_8]$	$2[(1-\beta)D_{16} + \beta D_8]$	$2[(1-\beta)D_{16} + \beta D_8]$
7	-2	2	2	$D_8$	$-2[(1-\beta)D_8 + \beta D_8]$	$2[(1-\beta)D_8 + \beta D_8]$	$2[(1-\beta)D_8 + \beta D_8]$
8	-2	2	2	$D_6$	$-2[(1-\beta)D_8 + \beta D_6]$	$2[(1-\beta)D_8 + \beta D_6]$	$2[(1-\beta)D_8 + \beta D_6]$
9	-2	2	2	$D_0$	$-2[(1-\beta)D_6]$	$2[(1-\beta)D_6]$	$2[(1-\beta)D_6]$
10	-2	2	2	$D_{-24}$	$2\beta D_{24}$	$-2\beta D_{24}$	$-2\beta D_{24}$
11	-2	1	2	$D_{-10}$	$2[(1-\beta)D_{24} + \beta D_{10}]$	$-[(1-\beta)D_{24} + \beta D_{10}]$	$-2[(1-\beta)D_{24} + \beta D_{10}]$
12	-2	2	1	$D_{-20}$	$2[(1-\beta)D_{10} + \beta D_{20}]$	$-2[(1-\beta)D_{10} + \beta D_{20}]$	$-[(1-\beta)D_{10} + \beta D_{20}]$
13	-2	2	0	$D_{-20}$	$2[(1-\beta)D_{20} + \beta D_{20}]$	$-2[(1-\beta)D_{20} + \beta D_{20}]$	0
14	-2	2	-1	$D_{-16}$	$2[(1-\beta)D_{20} + \beta D_{16}]$	$-2[(1-\beta)D_{20} + \beta D_{16}]$	$[(1-\beta)D_{20} + \beta D_{16}]$
15	-2	2	-2	$D_{-8}$	$2[(1-\beta)D_{16} + \beta D_8]$	$-2[(1-\beta)D_{16} + \beta D_8]$	$2[(1-\beta)D_{16} + \beta D_8]$
16	-2	2	-2	$D_{-8}$	$2[(1-\beta)D_8 + \beta D_8]$	$-2[(1-\beta)D_8 + \beta D_8]$	$2[(1-\beta)D_8 + \beta D_8]$
17	-2	2	-2	$D_{-6}$	$2[(1-\beta)D_8 + \beta D_6]$	$-2[(1-\beta)D_8 + \beta D_6]$	$2[(1-\beta)D_8 + \beta D_6]$
18	-2	2	-2	$D_0$	$2[(1-\beta)D_6]$	$-2[(1-\beta)D_6]$	$2[(1-\beta)D_6]$

Table A.7: Wealth change for the bottom, top and slow strategy at each step in one price cycle of the (555) trendsetters' attractor.

time	f	s	b	t	b&t	s&b	s&t	$\bar{s}\&\bar{t}$	$\bar{s}\&\bar{b}$	$\bar{b}\&\bar{t}$	$\bar{t}$	$\bar{b}$	$\bar{s}$	$\bar{f}$	No. of buyers (in units of $\frac{N}{64}$ )	No. of sellers (in units of $\frac{N}{64}$ )	Price Change $D_x$
0	h	h	h	h	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$
1	b	h	b	h	b	b	h	h	h	h	h	s	h	s	28	4	$D_{24}$
2	b	b	s	h	s	s	b	h	s	h	h	b	s	s	29	19	$D_{10}$
3	b	b	h	h	h	h	b	h	s	h	h	h	s	s	26	6	$D_{20}$
4	b	b	h	h	h	h	b	h	s	h	h	h	s	s	26	6	$D_{20}$
5	h	b	h	h	b	h	b	h	s	b	h	h	s	h	15	5	$D_{10}$
6	h	h	h	h	b	b	h	h	b	b	h	h	h	h	8	0	$D_8$
7	h	h	h	h	b	b	h	h	b	b	h	h	h	h	8	0	$D_8$
8	h	h	h	h	b	b	h	h	b	b	h	h	h	h	8	0	$D_8$
9	h	h	h	h	b	b	h	h	b	b	h	h	h	h	4	0	$D_4$
10	h	h	h	h	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$
11	s	h	h	s	s	h	s	h	h	h	b	h	h	b	4	28	$D_{-24}$
12	s	s	h	b	b	s	b	b	h	h	s	h	b	b	19	29	$D_{-10}$
13	s	s	h	h	h	s	h	b	h	h	h	h	b	b	6	26	$D_{-20}$
14	s	s	h	h	h	s	h	b	h	h	h	h	b	b	6	26	$D_{-20}$
15	h	s	h	h	s	s	h	b	h	s	h	h	b	h	5	15	$D_{-10}$
16	h	h	h	h	s	h	s	s	h	s	h	h	h	h	0	8	$D_{-8}$
17	h	h	h	h	s	h	s	s	h	s	h	h	h	h	0	8	$D_{-8}$
18	h	h	h	h	s	h	s	s	h	s	h	h	h	h	0	8	$D_{-8}$
19	h	h	h	h	h	h	s	s	h	h	h	h	h	h	0	4	$D_{-4}$
20	h	h	h	h	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$

Table A.8: The decisions made by different types of in the (566) trendsetters' attractor.

time	position			Price Change	Wealth Change		
	bottom	top	slow		b	t	s
0	-2	2	-2	$D_0$			
1	-2	2	-2	$D_{24}$	$-2\beta D_{24}$	$2\beta D_{24}$	$-2\beta D_{24}$
2	-1	2	-2	$D_{10}$	$-[(1-\beta)D_{24} + \beta D_{10}]$	$2[(1-\beta)D_{24} + \beta D_{10}]$	$-2[(1-\beta)D_{24} + \beta D_{10}]$
3	-2	2	-1	$D_{20}$	$-2[(1-\beta)D_{10} + \beta D_{20}]$	$2[(1-\beta)D_{10} + \beta D_{20}]$	$-[(1-\beta)D_{10} + \beta D_{20}]$
4	-2	2	0	$D_{20}$	$-2[(1-\beta)D_{20} + \beta D_{20}]$	$2[(1-\beta)D_{20} + \beta D_{20}]$	0
5	-2	2	1	$D_{10}$	$-2[(1-\beta)D_{20} + \beta D_{10}]$	$2[(1-\beta)D_{20} + \beta D_{10}]$	$[(1-\beta)D_{20} + \beta D_{10}]$
6	-2	2	2	$D_8$	$-2[(1-\beta)D_{10} + \beta D_8]$	$2[(1-\beta)D_{10} + \beta D_8]$	$2[(1-\beta)D_{10} + \beta D_8]$
7	-2	2	2	$D_8$	$-2[(1-\beta)D_8 + \beta D_8]$	$2[(1-\beta)D_8 + \beta D_8]$	$2[(1-\beta)D_8 + \beta D_8]$
8	-2	2	2	$D_8$	$-2[(1-\beta)D_8 + \beta D_8]$	$2[(1-\beta)D_8 + \beta D_8]$	$2[(1-\beta)D_8 + \beta D_8]$
9	-2	2	2	$D_4$	$-2[(1-\beta)D_8 + \beta D_4]$	$2[(1-\beta)D_8 + \beta D_4]$	$2[(1-\beta)D_8 + \beta D_4]$
10	-2	2	2	$D_0$	$-2[(1-\beta)D_4]$	$2[(1-\beta)D_4]$	$2[(1-\beta)D_4]$
11	-2	2	2	$D_{-24}$	$2\beta D_{24}$	$-2\beta D_{24}$	$-2\beta D_{24}$
12	-2	1	2	$D_{-10}$	$2[(1-\beta)D_{24} + \beta D_{10}]$	$-[(1-\beta)D_{24} + \beta D_{10}]$	$-2[(1-\beta)D_{24} + \beta D_{10}]$
13	-2	2	1	$D_{-20}$	$2[(1-\beta)D_{10} + \beta D_{20}]$	$-2[(1-\beta)D_{10} + \beta D_{20}]$	$-[(1-\beta)D_{10} + \beta D_{20}]$
14	-2	2	0	$D_{-20}$	$2[(1-\beta)D_{20} + \beta D_{20}]$	$-2[(1-\beta)D_{20} + \beta D_{20}]$	0
15	-2	2	-1	$D_{-10}$	$2[(1-\beta)D_{20} + \beta D_{10}]$	$-2[(1-\beta)D_{20} + \beta D_{10}]$	$[(1-\beta)D_{20} + \beta D_{10}]$
16	-2	2	-2	$D_{-8}$	$2[(1-\beta)D_{10} + \beta D_8]$	$-2[(1-\beta)D_{10} + \beta D_8]$	$2[(1-\beta)D_{10} + \beta D_8]$
17	-2	2	-2	$D_{-8}$	$2[(1-\beta)D_8 + \beta D_8]$	$-2[(1-\beta)D_8 + \beta D_8]$	$2[(1-\beta)D_8 + \beta D_8]$
18	-2	2	-2	$D_{-8}$	$2[(1-\beta)D_8 + \beta D_8]$	$-2[(1-\beta)D_8 + \beta D_8]$	$2[(1-\beta)D_8 + \beta D_8]$
19	-2	2	-2	$D_{-4}$	$2[(1-\beta)D_8 + \beta D_4]$	$-2[(1-\beta)D_8 + \beta D_4]$	$2[(1-\beta)D_8 + \beta D_4]$
20	-2	2	-2	$D_0$	$2[(1-\beta)D_4]$	$-2[(1-\beta)D_4]$	$2[(1-\beta)D_4]$

Table A.9: Wealth change for the bottom, top and slow strategy at each step in one price cycle of the (566) trendsetters' attractor.

time	f	s	b	t	b&t	s&b	s&t	$\bar{s}\&\bar{t}$	$\bar{s}\&\bar{b}$	$\bar{b}\&\bar{t}$	$\bar{t}$	$\bar{b}$	$\bar{s}$	$\bar{f}$	No. of buyers (in units of $\frac{N}{64}$ )	No. of sellers (in units of $\frac{N}{64}$ )	Price Change $D_x$
0	h	h	h	h	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$
1	b	h	b	h	b	b	h	h	h	h	h	s	h	s	28	4	$D_{24}$
2	b	b	s	h	s	s	b	h	s	h	h	b	s	s	29	19	$D_{10}$
3	b	b	h	h	h	h	b	h	s	h	h	h	s	s	26	6	$D_{20}$
4	b	b	h	h	h	h	b	h	s	h	h	h	s	s	26	6	$D_{20}$
5	h	b	h	h	b	h	b	h	s	b	h	h	s	h	15	5	$D_{10}$
6	h	h	h	h	b	h	h	h	h	b	h	h	h	h	4	0	$D_4$
7	h	h	h	h	b	b	h	h	b	b	h	h	h	h	8	0	$D_8$
8	h	h	h	h	b	b	h	h	b	b	h	h	h	h	8	0	$D_8$
9	h	h	h	h	h	b	h	h	b	b	h	h	h	h	4	0	$D_4$
10	h	h	h	h	h	b	h	h	b	h	h	h	h	h	4	0	$D_4$
11	h	h	h	h	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$
12	s	h	h	s	s	h	s	h	h	h	b	h	h	b	4	28	$D_{-24}$
13	s	s	h	b	b	s	b	b	h	h	s	h	b	b	19	29	$D_{-10}$
14	s	s	h	h	h	s	h	b	h	h	h	h	b	b	6	26	$D_{-20}$
15	s	s	h	h	h	s	h	b	h	h	h	h	b	b	6	26	$D_{-20}$
16	h	s	h	h	s	s	h	b	h	s	h	h	b	h	5	15	$D_{-10}$
17	h	h	h	h	s	h	h	h	h	s	h	h	h	h	0	4	$D_{-4}$
18	h	h	h	h	s	h	s	s	h	s	h	h	h	h	0	8	$D_{-8}$
19	h	h	h	h	s	h	s	s	h	s	h	h	h	h	0	8	$D_{-8}$
20	h	h	h	h	h	h	s	s	h	h	h	h	h	h	0	4	$D_{-4}$
21	h	h	h	h	h	h	s	s	h	h	h	h	h	h	0	4	$D_{-4}$
22	h	h	h	h	h	h	h	h	h	h	h	h	h	h	0	0	$D_0$

Table A.10: The decisions made by different types of in the (577) trendsetters' attractor.

time	position			Price Change	Wealth Change		
	bottom	top	slow		b	t	s
0	-2	2	-2	$D_0$			
1	-2	2	-2	$D_{24}$	$-2\beta D_{24}$	$2\beta D_{24}$	$-2\beta D_{24}$
2	-1	2	-2	$D_{10}$	$-[(1-\beta)D_{24} + \beta D_{10}]$	$2[(1-\beta)D_{24} + \beta D_{10}]$	$-2[(1-\beta)D_{24} + \beta D_{10}]$
3	-2	2	-1	$D_{20}$	$-2[(1-\beta)D_{10} + \beta D_{20}]$	$-2[(1-\beta)D_{10} + \beta D_{20}]$	$-[(1-\beta)D_{10} + \beta D_{20}]$
4	-2	2	0	$D_{20}$	$-2[(1-\beta)D_{20} + \beta D_{20}]$	$2[(1-\beta)D_{20} + \beta D_{20}]$	0
5	-2	2	1	$D_{10}$	$-2[(1-\beta)D_{20} + \beta D_{10}]$	$2[(1-\beta)D_{20} + \beta D_{10}]$	$[(1-\beta)D_{20} + \beta D_{10}]$
6	-2	2	2	$D_4$	$-2[(1-\beta)D_{10} + \beta D_4]$	$2[(1-\beta)D_{10} + \beta D_4]$	$2[(1-\beta)D_{10} + \beta D_4]$
7	-2	2	2	$D_8$	$-2[(1-\beta)D_4 + \beta D_8]$	$2[(1-\beta)D_4 + \beta D_8]$	$2[(1-\beta)D_4 + \beta D_8]$
8	-2	2	2	$D_8$	$-2[(1-\beta)D_8 + \beta D_8]$	$2[(1-\beta)D_8 + \beta D_8]$	$2[(1-\beta)D_8 + \beta D_8]$
9	-2	2	2	$D_4$	$-2[(1-\beta)D_8 + \beta D_4]$	$2[(1-\beta)D_8 + \beta D_4]$	$2[(1-\beta)D_8 + \beta D_4]$
10	-2	2	2	$D_4$	$-2[(1-\beta)D_4 + \beta D_4]$	$2[(1-\beta)D_4 + \beta D_4]$	$2[(1-\beta)D_4 + \beta D_4]$
11	-2	2	2	$D_0$	$-2[(1-\beta)D_4]$	$2[(1-\beta)D_4]$	$2[(1-\beta)D_4]$
12	-2	2	2	$D_{-24}$	$2\beta D_{24}$	$-2\beta D_{24}$	$-2\beta D_{24}$
13	-2	1	2	$D_{-10}$	$2[(1-\beta)D_{24} + \beta D_{10}]$	$-[(1-\beta)D_{24} + \beta D_{10}]$	$-2[(1-\beta)D_{24} + \beta D_{10}]$
14	-2	2	1	$D_{-20}$	$2[(1-\beta)D_{10} + \beta D_{20}]$	$-2[(1-\beta)D_{10} + \beta D_{20}]$	$-[(1-\beta)D_{10} + \beta D_{20}]$
15	-2	2	0	$D_{-20}$	$2[(1-\beta)D_{20} + \beta D_{20}]$	$-2[(1-\beta)D_{20} + \beta D_{20}]$	0
16	-2	2	-1	$D_{-10}$	$2[(1-\beta)D_{20} + \beta D_{10}]$	$-2[(1-\beta)D_{20} + \beta D_{10}]$	$[(1-\beta)D_{20} + \beta D_{10}]$
17	-2	2	-2	$D_{-4}$	$2[(1-\beta)D_{10} + \beta D_4]$	$-2[(1-\beta)D_{10} + \beta D_4]$	$2[(1-\beta)D_{10} + \beta D_4]$
18	-2	2	-2	$D_{-8}$	$2[(1-\beta)D_4 + \beta D_8]$	$-2[(1-\beta)D_4 + \beta D_8]$	$2[(1-\beta)D_4 + \beta D_8]$
19	-2	2	-2	$D_{-8}$	$2[(1-\beta)D_8 + \beta D_8]$	$-2[(1-\beta)D_8 + \beta D_8]$	$2[(1-\beta)D_8 + \beta D_8]$
20	-2	2	-2	$D_{-4}$	$2[(1-\beta)D_8 + \beta D_4]$	$-2[(1-\beta)D_8 + \beta D_4]$	$2[(1-\beta)D_8 + \beta D_4]$
21	-2	2	-2	$D_{-4}$	$2[(1-\beta)D_4 + \beta D_4]$	$-2[(1-\beta)D_4 + \beta D_4]$	$2[(1-\beta)D_4 + \beta D_4]$
22	-2	2	-2	$D_0$	$2[(1-\beta)D_4]$	$-2[(1-\beta)D_4]$	$2[(1-\beta)D_4]$

Table A.11: Wealth change for the bottom, top and slow strategy at each step in one price cycle of the (577) trendsetters' attractor.

# Appendix B

## Tables for Chapter 5

<i>time</i>	$f$ $\frac{15}{64}$	$s$ $\frac{13}{64}$	$b$ $\frac{11}{64}$	$t$ $\frac{7}{64}$	$t\&\bar{t}$ $\frac{2}{64}$	$\bar{t}$ $\frac{5}{64}$	$\bar{b}$ $\frac{7}{64}$	$\bar{s}$ $\frac{3}{64}$	$\bar{f}$ $\frac{1}{64}$	<i>No. of buyers</i> (in units of $\frac{N}{64}$ )	<i>No. of sellers</i> (in units of $\frac{N}{64}$ )	<i>Price Change</i> $D_x$
0	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	0	0	$D_0$
1	<i>b</i>	<i>h</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>h</i>	<i>s</i>	26	8	$D_{18}$
2	<i>b</i>	<i>b</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>s</i>	<i>s</i>	35	15	$D_{20}$
3	<i>b</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>s</i>	28	4	$D_{24}$
4	<i>b</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>s</i>	28	4	$D_{24}$
5	<i>h</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>h</i>	15	3	$D_{12}$
6	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	2	0	$D_2$
7	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	2	0	$D_2$
8	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	2	0	$D_2$
9	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	0	0	$D_0$
10	<i>s</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>s</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>b</i>	6	24	$D_{-18}$
11	<i>s</i>	<i>s</i>	<i>h</i>	<i>b</i>	<i>b</i>	<i>s</i>	<i>h</i>	<i>b</i>	<i>b</i>	13	33	$D_{-20}$
12	<i>s</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>b</i>	4	28	$D_{-24}$
13	<i>s</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>b</i>	4	28	$D_{-24}$
14	<i>h</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>h</i>	3	13	$D_{-10}$
15	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	0	2	$D_{-2}$
16	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	0	2	$D_{-2}$
17	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	0	2	$D_{-2}$
18	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	0	2	$D_{-2}$
19	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	0	0	$D_0$

Table B.1: The decisions made by different types of agents in the (8-up-9-down) asymmetric cycle of the trendsetters' attractor.



<i>time</i>	$f$ $\frac{15}{64}$	$s$ $\frac{13}{64}$	$b$ $\frac{11}{64}$	$t$ $\frac{7}{64}$	$t\&t$ $\frac{2}{64}$	$\bar{t}$ $\frac{5}{64}$	$\bar{b}$ $\frac{7}{64}$	$\bar{s}$ $\frac{3}{64}$	$\bar{f}$ $\frac{1}{64}$	<i>No. of buyers</i> (in units of $\frac{N}{64}$ )	<i>No. of sellers</i> (in units of $\frac{N}{64}$ )	<i>Price Change</i> $D_x$
0	$h$	$h$	$h$	$h$	$h$	$h$	$h$	$h$	$h$	0	0	$D_0$
1	$b$	$h$	$b$	$h$	$h$	$h$	$s$	$h$	$s$	26	8	$D_{18}$
2	$b$	$b$	$s$	$h$	$h$	$h$	$b$	$s$	$s$	35	15	$D_{20}$
3	$b$	$b$	$h$	$h$	$h$	$h$	$h$	$s$	$s$	28	4	$D_{24}$
4	$b$	$b$	$h$	$h$	$h$	$h$	$h$	$s$	$s$	28	4	$D_{24}$
5	$h$	$b$	$h$	$h$	$h$	$h$	$h$	$s$	$h$	13	3	$D_{10}$
6	$h$	$h$	$h$	$h$	$b$	$h$	$h$	$h$	$h$	2	0	$D_2$
7	$h$	$h$	$h$	$h$	$b$	$h$	$h$	$h$	$h$	2	0	$D_2$
8	$h$	$h$	$h$	$h$	$b$	$h$	$h$	$h$	$h$	2	0	$D_2$
9	$h$	$h$	$h$	$h$	$b$	$h$	$h$	$h$	$h$	2	0	$D_2$
10	$h$	$h$	$h$	$h$	$h$	$h$	$h$	$h$	$h$	0	0	$D_0$
11	$s$	$h$	$h$	$s$	$s$	$b$	$h$	$h$	$b$	6	24	$D_{-18}$
12	$s$	$s$	$h$	$b$	$b$	$s$	$h$	$b$	$b$	13	33	$D_{-20}$
13	$s$	$s$	$h$	$h$	$h$	$h$	$h$	$b$	$b$	4	28	$D_{-24}$
14	$s$	$s$	$h$	$h$	$h$	$h$	$h$	$b$	$b$	4	28	$D_{-24}$
15	$h$	$s$	$h$	$h$	$h$	$h$	$h$	$b$	$h$	3	13	$D_{-10}$
16	$h$	$h$	$h$	$h$	$s$	$h$	$h$	$h$	$h$	0	2	$D_{-2}$
17	$h$	$h$	$h$	$h$	$s$	$h$	$h$	$h$	$h$	0	2	$D_{-2}$
18	$h$	$h$	$h$	$h$	$s$	$h$	$h$	$h$	$h$	0	2	$D_{-2}$
19	$h$	$h$	$h$	$h$	$s$	$h$	$h$	$h$	$h$	0	2	$D_{-2}$
20	$h$	$h$	$h$	$h$	$h$	$h$	$h$	$h$	$h$	0	0	$D_0$

Table B.2: The decisions made by different types of agents in the (9-up-9-down) symmetric cycle of the trendsetters' attractor.

time	Position of the Top Strategy		Price Change		Wealth Change of the Top Strategy	
	Asymmetric	Symmetric	Asymmetric	Symmetric	Asymmetric	Symmetric
0	2	2	$D_0$	$D_0$		
1	2	2	$D_{18}$	$D_{18}$	$2\beta D_{18}$	$2\beta D_{18}$
2	2	2	$D_{20}$	$D_{20}$	$2[(1-\beta)D_{18} + \beta D_{20}]$	$2[(1-\beta)D_{18} + \beta D_{20}]$
3	2	2	$D_{24}$	$D_{24}$	$2[(1-\beta)D_{20} + \beta D_{24}]$	$2[(1-\beta)D_{20} + \beta D_{24}]$
4	2	2	$D_{24}$	$D_{24}$	$2[(1-\beta)D_{24} + \beta D_{24}]$	$2[(1-\beta)D_{24} + \beta D_{24}]$
5	2	2	$D_{12}$	$D_{10}$	$2[(1-\beta)D_{24} + \beta D_{12}]$	$2[(1-\beta)D_{24} + \beta D_{10}]$
6	2	2	$D_2$	$D_2$	$2[(1-\beta)D_{12} + \beta D_2]$	$2[(1-\beta)D_{10} + \beta D_2]$
7	2	2	$D_2$	$D_2$	$2[(1-\beta)D_2 + \beta D_2]$	$2[(1-\beta)D_2 + \beta D_2]$
8	2	2	$D_2$	$D_2$	$2[(1-\beta)D_2 + \beta D_2]$	$2[(1-\beta)D_2 + \beta D_2]$
9	2	2	$D_0$	$D_2$	$2[(1-\beta)D_2]$	$2[(1-\beta)D_2 + \beta D_2]$
10	2	2	$D_{-18}$	$D_0$	$-2\beta D_{18}$	$2[(1-\beta)D_2]$
11	1	2	$D_{-20}$	$D_{-18}$	$-(1-\beta)D_{18} + \beta D_{20}]$	$-2\beta D_{18}$
12	2	1	$D_{-24}$	$D_{-20}$	$-2[(1-\beta)D_{20} + \beta D_{24}]$	$-(1-\beta)D_{18} + \beta D_{20}]$
13	2	2	$D_{-24}$	$D_{-24}$	$-2[(1-\beta)D_{24} + \beta D_{24}]$	$-2[(1-\beta)D_{20} + \beta D_{24}]$
14	2	2	$D_{-10}$	$D_{-24}$	$-2[(1-\beta)D_{24} + \beta D_{10}]$	$-2[(1-\beta)D_{24} + \beta D_{24}]$
15	2	2	$D_{-2}$	$D_{-10}$	$-2[(1-\beta)D_{10} + \beta D_2]$	$-2[(1-\beta)D_{24} + \beta D_{10}]$
16	2	2	$D_{-2}$	$D_{-2}$	$-2[(1-\beta)D_2 + \beta D_2]$	$-2[(1-\beta)D_{10} + \beta D_2]$
17	2	2	$D_{-2}$	$D_{-2}$	$-2[(1-\beta)D_2 + \beta D_2]$	$-2[(1-\beta)D_2 + \beta D_2]$
18	2	2	$D_{-2}$	$D_{-2}$	$-2[(1-\beta)D_2 + \beta D_2]$	$-2[(1-\beta)D_2 + \beta D_2]$
19	2	2	$D_0$	$D_{-2}$	$-2[(1-\beta)D_2]$	$-2[(1-\beta)D_2 + \beta D_2]$
20	N/A	2	N/A	$D_0$	N/A	$-2[(1-\beta)D_2]$

Table B.3: Wealth change for the top strategy at each step in one price cycle of the asymmetric trendsetters' attractor ( $n$  8-up-9-down asymmetric cycle + 1 9-up-9-down symmetric cycle).

<i>time</i>	$f$ $\frac{15}{64}$	$s$ $\frac{13}{64}$	$b$ $\frac{11}{64}$	$t$ $\frac{7}{64}$	$t\&\bar{t}$ $\frac{2}{64}$	$\bar{t}$ $\frac{5}{64}$	$\bar{b}$ $\frac{7}{64}$	$\bar{s}$ $\frac{3}{64}$	$\bar{f}$ $\frac{1}{64}$	No. of buyers (in units of $\frac{N}{64}$ )	No. of sellers (in units of $\frac{N}{64}$ )	Price Change $D_x$
0	$h$	$h$	$h$	$h$	$h$	$h$	$h$	$h$	$h$	0	0	$D_0$
1	$b$	$h$	$b$	$h$	$h$	$h$	$s$	$h$	$s$	26	8	$D_{18}$
2	$b$	$b$	$s$	$h$	$h$	$h$	$b$	$s$	$s$	35	15	$D_{20}$
3	$b$	$b$	$h$	$h$	$h$	$h$	$h$	$s$	$s$	28	4	$D_{24}$
4	$b$	$b$	$h$	$h$	$h$	$h$	$h$	$s$	$s$	28	4	$D_{24}$
5	$h$	$b$	$h$	$h$	$b$	$h$	$h$	$s$	$h$	15	3	$D_{12}$
6	$h$	$h$	$h$	$h$	$b$	$h$	$h$	$h$	$h$	2	0	$D_2$
7	$h$	$h$	$h$	$h$	$b$	$h$	$h$	$h$	$h$	2	0	$D_2$
8	$h$	$h$	$h$	$h$	$b$	$h$	$h$	$h$	$h$	2	0	$D_2$
9	$h$	$h$	$h$	$h$	$h$	$h$	$h$	$h$	$h$	0	0	$D_0$
10	$s$	$h$	$h$	$s$	$s$	$b$	$h$	$h$	$b$	6	24	$D_{-18}$
11	$s$	$s$	$h$	$b$	$b$	$s$	$h$	$b$	$b$	13	33	$D_{-20}$
12	$s$	$s$	$h$	$h$	$h$	$h$	$h$	$b$	$b$	4	28	$D_{-24}$
13	$s$	$s$	$h$	$h$	$h$	$h$	$h$	$b$	$b$	4	28	$D_{-24}$
14	$h$	$s$	$h$	$h$	$s$	$h$	$h$	$b$	$h$	3	15	$D_{-12}$
15	$h$	$h$	$h$	$h$	$s$	$h$	$h$	$h$	$h$	0	2	$D_{-2}$
16	$h$	$h$	$h$	$h$	$s$	$h$	$h$	$h$	$h$	0	2	$D_{-2}$
17	$h$	$h$	$h$	$h$	$s$	$h$	$h$	$h$	$h$	0	2	$D_{-2}$
18	$h$	$h$	$h$	$h$	$h$	$h$	$h$	$h$	$h$	0	0	$D_0$

Table B.4: The decisions made by different types of agents in the (8-up-8-down) symmetric cycle of the trendsetters' attractor.

time	Position of the Top Strategy		Price Change		Wealth Change of the Top Strategy	
	Asymmetric	Symmetric	Asymmetric	Symmetric	Asymmetric	Symmetric
0	2	2	$D_0$	$D_0$		
1	2	2	$D_{18}$	$D_{18}$	$2\beta D_{18}$	$2\beta D_{18}$
2	2	2	$D_{20}$	$D_{20}$	$2[(1-\beta)D_{18} + \beta D_{20}]$	$2[(1-\beta)D_{18} + \beta D_{20}]$
3	2	2	$D_{24}$	$D_{24}$	$2[(1-\beta)D_{20} + \beta D_{24}]$	$2[(1-\beta)D_{20} + \beta D_{24}]$
4	2	2	$D_{24}$	$D_{24}$	$2[(1-\beta)D_{24} + \beta D_{24}]$	$2[(1-\beta)D_{24} + \beta D_{24}]$
5	2	2	$D_{12}$	$D_{12}$	$2[(1-\beta)D_{24} + \beta D_{12}]$	$2[(1-\beta)D_{24} + \beta D_{12}]$
6	2	2	$D_2$	$D_2$	$2[(1-\beta)D_{12} + \beta D_2]$	$2[(1-\beta)D_{12} + \beta D_2]$
7	2	2	$D_2$	$D_2$	$2[(1-\beta)D_2 + \beta D_2]$	$2[(1-\beta)D_2 + \beta D_2]$
8	2	2	$D_2$	$D_2$	$2[(1-\beta)D_2 + \beta D_2]$	$2[(1-\beta)D_2 + \beta D_2]$
9	2	2	$D_0$	$D_0$	$2[(1-\beta)D_2]$	$2[(1-\beta)D_2]$
10	2	2	$D_{-18}$	$D_{-18}$	$-2\beta D_{18}$	$-2\beta D_{18}$
11	1	1	$D_{-20}$	$D_{-20}$	$-[(1-\beta)D_{18} + \beta D_{20}]$	$-[(1-\beta)D_{18} + \beta D_{20}]$
12	2	2	$D_{-24}$	$D_{-24}$	$-2[(1-\beta)D_{20} + \beta D_{24}]$	$-2[(1-\beta)D_{20} + \beta D_{24}]$
13	2	2	$D_{-24}$	$D_{-24}$	$-2[(1-\beta)D_{24} + \beta D_{24}]$	$-2[(1-\beta)D_{24} + \beta D_{24}]$
14	2	2	$D_{-10}$	$D_{-12}$	$-2[(1-\beta)D_{24} + \beta D_{10}]$	$-2[(1-\beta)D_{24} + \beta D_{10}]$
15	2	2	$D_{-2}$	$D_{-2}$	$-2[(1-\beta)D_{10} + \beta D_2]$	$-2[(1-\beta)D_{10} + \beta D_2]$
16	2	2	$D_{-2}$	$D_{-2}$	$-2[(1-\beta)D_2 + \beta D_2]$	$-2[(1-\beta)D_2 + \beta D_2]$
17	2	2	$D_{-2}$	$D_{-2}$	$-2[(1-\beta)D_2 + \beta D_2]$	$-2[(1-\beta)D_2 + \beta D_2]$
18	2	2	$D_{-2}$	$D_0$	$-2[(1-\beta)D_2 + \beta D_2]$	$-2[(1-\beta)D_2]$
19	2	2	$D_0$	$N/A$	$-2[(1-\beta)D_2]$	$N/A$

Table B.5: Wealth change for the top strategy at each step in one price cycle of the asymmetric trendsetters' attractor ( $n$  8-up-9-down asymmetric cycle + 1 8-up-8-down symmetric cycle).

<i>time</i>	<i>f</i> $\frac{15}{64}$	<i>s</i> $\frac{13}{64}$	<i>b</i> $\frac{11}{64}$	<i>t</i> $\frac{7}{64}$	<i>t&amp;t</i> $\frac{2}{64}$	$\bar{t}$ $\frac{5}{64}$	$\bar{b}$ $\frac{7}{64}$	$\bar{s}$ $\frac{3}{64}$	$\bar{f}$ $\frac{1}{64}$	No. of buyers (in units of $\frac{N}{64}$ )	No. of sellers (in units of $\frac{N}{64}$ )	Price Change <i>D<sub>x</sub></i>
0	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	0	0	<i>D</i> <sub>0</sub>
1	<i>b</i>	<i>h</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>h</i>	<i>s</i>	26	8	<i>D</i> <sub>18</sub>
2	<i>b</i>	<i>b</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>s</i>	<i>s</i>	35	15	<i>D</i> <sub>20</sub>
3	<i>b</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>s</i>	28	4	<i>D</i> <sub>24</sub>
4	<i>b</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>s</i>	30	4	<i>D</i> <sub>26</sub>
5	<i>h</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>h</i>	15	3	<i>D</i> <sub>12</sub>
6	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	2	0	<i>D</i> <sub>2</sub>
7	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	2	0	<i>D</i> <sub>2</sub>
8	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	0	0	<i>D</i> <sub>0</sub>
9	<i>s</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>s</i>	<i>b</i>	<i>h</i>	<i>h</i>	<i>b</i>	6	24	<i>D</i> <sub>-18</sub>
10	<i>s</i>	<i>s</i>	<i>h</i>	<i>b</i>	<i>b</i>	<i>s</i>	<i>h</i>	<i>b</i>	<i>b</i>	13	33	<i>D</i> <sub>-20</sub>
11	<i>s</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>b</i>	4	28	<i>D</i> <sub>-24</sub>
12	<i>s</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>b</i>	4	28	<i>D</i> <sub>-24</sub>
13	<i>h</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>b</i>	<i>h</i>	3	13	<i>D</i> <sub>-10</sub>
14	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	0	2	<i>D</i> <sub>-2</sub>
15	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	0	2	<i>D</i> <sub>-2</sub>
16	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	0	2	<i>D</i> <sub>-2</sub>
17	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>s</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	0	2	<i>D</i> <sub>-2</sub>
18	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	0	0	<i>D</i> <sub>0</sub>

Table B.6: The decisions made by different types of agents in the (7-up-9-down) asymmetric cycle of the trendsetters' attractor.

<i>time</i>	<i>Position of the Top Strategy</i>		<i>Price Change</i>		<i>Wealth Change of the Top Strategy</i>	
	<i>8-up-9-down</i>	<i>7-up-9-down</i>	<i>8-up-9-down</i>	<i>7-up-9-down</i>	<i>8-up-9-down</i>	<i>7-up-9-down</i>
0	2	2	$D_0$	$D_0$		
1	2	2	$D_{18}$	$D_{18}$	$2\beta D_{18}$	$2\beta D_{18}$
2	2	2	$D_{20}$	$D_{20}$	$2[(1-\beta)D_{18} + \beta D_{20}]$	$2[(1-\beta)D_{18} + \beta D_{20}]$
3	2	2	$D_{24}$	$D_{24}$	$2[(1-\beta)D_{20} + \beta D_{24}]$	$2[(1-\beta)D_{20} + \beta D_{24}]$
4	2	2	$D_{24}$	$D_{26}$	$2[(1-\beta)D_{24} + \beta D_{24}]$	$2[(1-\beta)D_{24} + \beta D_{26}]$
5	2	2	$D_{12}$	$D_{12}$	$2[(1-\beta)D_{24} + \beta D_{12}]$	$2[(1-\beta)D_{26} + \beta D_{12}]$
6	2	2	$D_2$	$D_2$	$2[(1-\beta)D_{12} + \beta D_2]$	$2[(1-\beta)D_{12} + \beta D_2]$
7	2	2	$D_2$	$D_2$	$2[(1-\beta)D_2 + \beta D_2]$	$2[(1-\beta)D_2 + \beta D_2]$
8	2	2	$D_2$	$D_0$	$2[(1-\beta)D_2 + \beta D_2]$	$2[(1-\beta)D_2]$
9	2	2	$D_0$	$D_{-18}$	$2[(1-\beta)D_2]$	$-2\beta D_{18}$
10	2	1	$D_{-18}$	$D_{-20}$	$-2\beta D_{18}$	$-[(1-\beta)D_{18} + \beta D_{20}]$
11	1	2	$D_{-20}$	$D_{-24}$	$-[(1-\beta)D_{18} + \beta D_{20}]$	$-2[(1-\beta)D_{20} + \beta D_{24}]$
12	2	2	$D_{-24}$	$D_{-24}$	$-2[(1-\beta)D_{20} + \beta D_{24}]$	$-2[(1-\beta)D_{24} + \beta D_{24}]$
13	2	2	$D_{-24}$	$D_{-10}$	$-2[(1-\beta)D_{24} + \beta D_{24}]$	$-2[(1-\beta)D_{24} + \beta D_{10}]$
14	2	2	$D_{-10}$	$D_{-2}$	$-2[(1-\beta)D_{24} + \beta D_{10}]$	$-2[(1-\beta)D_{10} + \beta D_2]$
15	2	2	$D_{-2}$	$D_{-2}$	$-2[(1-\beta)D_{10} + \beta D_2]$	$-2[(1-\beta)D_2 + \beta D_2]$
16	2	2	$D_{-2}$	$D_{-2}$	$-2[(1-\beta)D_2 + \beta D_2]$	$-2[(1-\beta)D_2 + \beta D_2]$
17	2	2	$D_{-2}$	$D_{-2}$	$-2[(1-\beta)D_2 + \beta D_2]$	$-2[(1-\beta)D_2 + \beta D_2]$
18	2	2	$D_{-2}$	$D_0$	$-2[(1-\beta)D_2 + \beta D_2]$	$-2[(1-\beta)D_2]$
19	2	N/A	$D_0$	N/A	$-2[(1-\beta)D_2]$	N/A

Table B.7: Wealth change for the top strategy at each step in one price cycle of the asymmetric trendsetters' attractor ( $n$  8-up-9-down asymmetric cycle + 1 7-up-9-down asymmetric cycle).

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