MSDM5004 Spring 2024

Project 1 Part I

Due Apr. 7

You are required to submit a report and all the codes for a project.

1. Consider the problem

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} & 0 < x < 1, \quad t > 0 \\ u(0,t) = u(1,t) = 0 & t > 0 \\ u(x,0) = u_0(x) & 0 \le x \le 1 \end{cases}$$

where

$$u_0(x) = \begin{cases} 2x & 0 \le x \le \frac{1}{2} \\ 2 - 2x & \frac{1}{2} < x \le 1. \end{cases}$$

(1) Obtain numerical solution using the explicit scheme

$$U_j^{n+1} = U_j^n + \mu(U_{j+1}^n - 2U_j^n + U_{j-1}^n),$$

where $\mu = \frac{\Delta t}{(\Delta x)^2}$. Use $J = 20$, $\Delta x = 0.05$, and (i) $\Delta t = 0.0012$ (ii) $\Delta t = 0.0013$. Plot the numerical solution at $t = 0, \Delta t, 25\Delta t, 50\Delta t$.

(2) Use $\Delta t = 0.0006$ and same μ in (1)(i) to compute and plot the numerical solution at the same 4 times as in (1)(i). Also plot the differences of the numerical solutions at these 4 times.

(3) Obtain numerical solution using the Crank-Nicolson method.

Use $J = 20$, $\Delta x = 0.05$, and (i) $\Delta t = 0.0012$ (ii) $\Delta t = 0.0013$, (iii) $\Delta t = 0.012$. Plot the numerical solutions at $t = 0, \Delta t, 25\Delta t, 50\Delta t$.