Quiz 1: $\begin{cases} D' = \frac{k_B T}{\gamma} + C \cdot e^{-\gamma \cdot \frac{t}{m}} \\ \langle x^2 \rangle = 2D't = \frac{2k_B T}{\gamma} \cdot t \quad (t \gg m/\gamma). \end{cases}$

$\because \langle \xi(t_2) \xi(t_1) \rangle = \Gamma \delta(t_2 - t_1) = 2\gamma k_B T \delta(t_2 - t_1)$

$D = \gamma k_B T$. if $D = D'$, then $\frac{D}{\gamma} = \gamma \cdot D' \Rightarrow \gamma = 1$.

$\therefore \langle x^2_{(t)} \rangle = 2Dt$. (in this case $m \to 0$, $\gamma = 1$.)

$\because \langle x^2 \rangle = \int x^2 \cdot f(x,t)\, dx = 2Dt$.

$\begin{cases} P(x) = \frac{1}{\sqrt{2\pi N}} e^{-\frac{x^2}{2N}} \\ N := \langle x^2 \rangle_p = 2Dt \end{cases} \Rightarrow f(x,t) = \frac{1}{\sqrt{4\pi Dt}} \cdot e^{-\frac{x^2}{4Dt}}$

此时 $\int f\, dx = 1$. $f(x,0) = \delta(x)$

$(x_0 = 0)$

Quiz 2: $\int_{-\infty}^{\infty} f(y) \cdot g(x-y)\, dy \quad (= (f*g)(x))$

$= \int_{-\infty}^{\infty} e^{iky} \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2} \cdot \frac{(x-y)^2}{\sigma^2}}\, dy$

$= \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \int_{-\infty}^{\infty} e^{iky} \cdot e^{-\frac{1}{2} \cdot \frac{(y-x)^2}{\sigma^2}}\, dy$, 令 $t := y - x$

$= \frac{e^{ikx}}{\sqrt{2\pi\sigma^2}} \cdot \int_{-\infty}^{\infty} e^{ikt - \frac{t^2}{2\sigma^2}}\, dt$, $-\left(\frac{t^2}{2\sigma^2} - ikt\right) = -\frac{1}{2\sigma^2}(t - ik\sigma^2)^2 + \frac{\sigma^2 i^2 k^2}{2}$

$= \frac{e^{ikx + \sigma^2 i^2 k^2/2}}{\sqrt{2\pi\sigma^2}} \cdot \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2} \cdot (t - ik\sigma^2)^2}\, dt$. 令 $m = \frac{t - \sigma^2 i k}{\sqrt{2} \cdot \sigma}$.

$= \frac{e^{ikx + \sigma^2 i^2 k^2/2}}{\sqrt{2\pi\sigma^2}} \cdot \underbrace{\int_{-\infty}^{\infty} e^{-m^2}\, dm \cdot \sqrt{2}\sigma}_{} \quad (= \sqrt{2\pi}\sigma)$

$= e^{ikx + \frac{\sigma^2 i^2 k^2}{2}}$ $\quad |e^{ikx}| = |\cos kx + i \sin kx|$, max $= 1$

~~e^{ikx}~~ $\Rightarrow |(f*g)(x)| = |e^{-\frac{\sigma^2 k^2}{2}}|$

picture is attached in the pdf file.