Homework 1

On Normal distribution

The PDF of the normal distribution is given by

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right],$$

in which μ is the mean or expectation, σ is the standard deviation, and σ^2 is the variance. If a random variable X is normally distributed, then one may write $X \sim N(\mu, \sigma^2)$.

In probability theory, a distribution is said to be *stable* if a linear combination of two independent random variables with this distribution has the same distribution, up to location and scale parameters (μ and σ here). The normal distribution belongs to the family of stable distributions.

Consider three random variables X, Y, and $Z = X + Y$, with X and Y being independent of each other.

(a) Given $X \sim N(0,1)$ and $Y \sim N(1,2)$, show $Z \sim N(1,3)$.

(b) Write a short program to generate a sequence for the random number Z according to the given PDFs of X and Y. Then fit this sequence by using the normal distribution $Z \sim N(\mu_Z, \sigma_Z^2)$ to numerically determine μ_Z and σ_Z^2. You may entertain yourself by comparing μ_Z and σ_Z^2 with their theoretical values derived in (a).

Note that you can use any computer language and you need plot the statistical sampling and the fitting curve.

Please submit both the computer program and the plots.