MSDM5004 Spring 2024 Part II

Assignment 1

Due: 21 Apr 23:59

You may write your codes in any language.

1. In Canvas you will find files called piano.txt and trumpet.txt, which contain data representing the waveform of a single note, played on, respectively, a piano and a trumpet.

    (a) Write a program that loads a waveform from one of these files, plots it, then calculates its discrete Fourier transform and plots the magnitudes of the first 10000 coefficients. Note that you will have to use a fast Fourier transform for the calculation because there are too many samples in the files to do the transforms the slow way in any reasonable amount of time. Apply your program to the piano and trumpet waveforms and discuss briefly what one can conclude about the sound of the piano and trumpet from the plots of Fourier coefficients.

    (b) Both waveforms were recorded at the industry-standard rate of 44100 samples per second and both instruments were playing the same musical note when the recordings were made. From your Fourier transform results calculate what note they were playing. (Hint: The musical note middle C has a frequency of 261Hz.)

2. A 1-D image is represented by the following sequence

$$g[n] = \begin{cases} 1 & n = -1, 0, 1, 7, 11, 15, 16 \\ 2 & n = 8, 10, 14 \\ 0 & \text{Otherwise} \end{cases}$$

The point image at $n = 0$ is denoted by

$$\delta[n] = \delta_{n0}$$

Therefore, the point image at $n = n_0$ is

$$\delta[n - n_0] = \delta_{n-n_0,0} = \delta_{nn_0}$$

Let the point-spread-function (PSF) of $\delta[n]$ be

$$h[n] = \begin{cases} 1 & n = 0 \\ 1/2 & n = -1, 1 \\ 0 & \text{Otherwise} \end{cases}$$

The true image $g[n]$ is blurred by the PSF.

(a) Obtain the blurred image by the linear convolution

$$y[n] = (g * h)[n] = \sum_{m=-\infty}^{\infty} g[m]h[n-m]$$

Now suppose what we have is the blurred image $y[n], n = 0, 1, 2, \cdots, M-1$ with length $\underline{M = 16}$. Let us try to de-blur the image by deconvolution.

Define the M-periodic sequence $y'[n]$ by

$$\begin{cases} y'[n] = y[n] & n = 0, 1, 2, \cdots, M-1 \\ y'[n+M] = y'[n] \end{cases}$$

Suppose

$$y'[n] = \sum_{m=0}^{M-1} g'[m]h_M[n-m]$$

(b) Write down the M-periodic sequence $h_M[n]$ for $n = 0, 1, 2, \cdots, M-1$.

(c) Compute the FT of $h_M[n]$, $H[k]$, $k = 0, 1, 2, \cdots, M-1$ by FFT. You are required to either carry out the FFT by hand or write your own FFT code.

(d) Compute the FT of $y'[n]$, $Y[k]$, $k = 0, 1, 2, \cdots, M-1$ by FFT. You are required to either carry out the FFT by hand or write your own FFT code.

(e) Compute $g'[n]$, $n = 0, 1, 2, \cdots, M-1$ by inverse FFT. You are required to either carry out the IFFT by hand or write your own IFFT code.

3. Solve the Poisson's Equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \delta\left(x - \frac{1}{2}, y - 1\right)$$

in $0 \le x \le 1$, $0 \le y \le 2$ by spectral method with $\Delta = 0.01$.

The boundary condition is

$$\begin{cases} \phi(x = 0, 0 < y < 2) = 0 \\ \phi(x = 1, 0 < y < 2) = 0 \\ \phi(0 \le x \le 1, y = 0) = 0 \\ \phi(0 \le x \le 1, y = 2) = \cos\dfrac{\pi x}{2} \end{cases}$$

$\delta\left(x-\frac{1}{2}, y-1\right)$ represents a unit point source at the center of the rectangular domain, $\left(x=\frac{1}{2}, y=1\right)$.

(Note: In the discretized FDE, at this point, $\rho_{jl} = 1/\Delta^2$.)

Show your answer by plotting $\phi(x, y)$.