(1). (a). $\phi(B) = 1 - 1.1B + 0.18B^2 = 0$

$\Rightarrow B_1 = 5,\ B_2 = \frac{10}{9}$ the roots are both outside the unit circle.

Stationary.

(b). $\rho_0 = 1,\ \rho_1 = \frac{a_1}{1-a_2} = \frac{1.1}{1+0.18} = \frac{55}{59},\quad \rho_i = a_1 \cdot \rho_{i-1} + a_2 \cdot \rho_{i-2}$

$\Rightarrow \rho_i = 1.1 \cdot \rho_{n-1} + (-0.18) \cdot \rho_{n-2}. \quad x^2 = 1.1x - 0.18 \Rightarrow x_1 = \frac{1}{5},\ x_2 = \frac{9}{10}.$

$$\Rightarrow \rho_i = C_1 \cdot \left(\frac{1}{5}\right)^i + C_2 \cdot \left(\frac{9}{10}\right)^i \Rightarrow \begin{cases} \rho_0 = 1, & C_1 + C_2 = 1 \\ \rho_1 = \frac{55}{59}, & \frac{C_1}{5} + \frac{9}{10} C_2 = \frac{55}{59} \end{cases}$$

$$\Rightarrow C_1 = -\frac{19}{413},\ C_2 = \frac{432}{413}, \Rightarrow \rho_i = \left(-\frac{19}{413}\right) \cdot \left(\frac{1}{5}\right)^i + \left(\frac{432}{413}\right) \cdot \left(\frac{9}{10}\right)^i$$

(1) (b). for γ_i: $a_1 = 1.1 \quad a_2 = -0.18$

$$\begin{cases} \gamma_0 = 1.1\gamma_1 - 0.18\gamma_2 + 1 \\ \gamma_1 = 1.1\gamma_0 - 0.18\gamma_1 \\ \gamma_2 = 1.1\gamma_1 - 0.18\gamma_0 \end{cases} \Rightarrow \gamma_0 = \frac{36875}{4674}. \qquad \begin{cases} \gamma_1 = \frac{55}{59}\gamma_0 \\ \gamma_2 = \frac{1247}{1475}\gamma_0 \\ \gamma_s = a_1\gamma_{s-1} + a_2\gamma_{s-2} \\ \quad = 1.1\gamma_{s-1} - 0.18\gamma_{s-2} \end{cases}$$

$$\Rightarrow \gamma_i = \gamma_0 \cdot \rho_i$$

$$= \frac{36875}{4674} \cdot \left[-\frac{19}{413} \cdot \left(\frac{1}{5}\right)^i + \frac{432}{413}\left(\frac{9}{10}\right)^i\right]$$

$$= -\frac{625}{1722} \cdot \left(\frac{1}{5}\right)^i + \frac{45000}{5453} \cdot \left(\frac{9}{10}\right)^i$$

(2). (a). $H_{0(ADF)}$: $\exists$ unit root, not stationary, coefs = 0, double side.

the t-statistic on the $X_{t-1} = \frac{-0.11}{0.04} = -2.75$

$df = \infty$, constant but no time trend $(a_2 = 0) \Rightarrow \tau_\mu \begin{cases} \alpha = 0.05 \to -2.86 \\ \alpha = 0.1 \to -2.57 \end{cases}$

$\because -2.86 < -2.75 < -2.57 \Rightarrow$ 0.1 significance level can reject H_0.

(b). t-statistic on $Y = \frac{-0.16}{0.07} = -2.286$.

$df = \infty$, double side $\Rightarrow t_\infty \begin{cases} \alpha = 0.05 \to 1.96 \\ \alpha = 0.02 \to 2.326. \end{cases} \quad 1.96 < 2.286 < 2.326$

$\Rightarrow$ 0.05 significance level can reject H_0.

(c). model1 : $\Delta X_{t=5} = 0.002 - 0.31\Delta X_4 = 0.467$. error1 $= 0.8 - 0.467 = 0.333$

model2 : $\Delta X_{t=5} = 0.021 - 0.46\Delta X_4 - 0.39\Delta X_3 - 0.25\Delta X_2 + 0.03\Delta X_1$

$= 0.382$. error2 $= 0.8 - 0.382 = 0.418$.

model3: $\Delta X_{t=5} = 1.279 - 0.51\Delta X_4 - 0.44\Delta X_3 - 0.3\Delta X_2 + 0.02\Delta X_1 - 0.16 Y_4$

$= 0.49$. error3 $= 0.8 - 0.49 = 0.31$.

(3). (a). $f(t)=C\cdot e^{-b(t-t_0)^2}$

$X(\omega)=C\cdot\int_{-\infty}^{\infty} e^{-b(t-t_0)^2}\cdot e^{-i\omega t}\,dt$. Let $t-t_0=s$

$=C\cdot\int_{-\infty}^{\infty} e^{-(bs^2+i\omega(s+t_0))}\,ds$

$=C\cdot\int_{-\infty}^{\infty} e^{-b(s^2+\frac{i\omega}{b}s+\frac{i\omega}{b}t_0)}\,ds$

$=C\cdot\int_{-\infty}^{\infty} e^{-b[(s+\frac{i\omega}{2b})^2+\frac{4bi\omega t_0+\omega^2}{4b^2}]}\,ds$

$=C\cdot\int_{-\infty}^{\infty} e^{-b(s+\frac{i\omega}{2b})^2}\,ds\cdot e^{\lambda}$, where $\lambda=-i\omega t_0-\frac{\omega^2}{4b}$.

$=C\cdot e^{\lambda}\cdot\int_{-\infty}^{\infty} e^{-(\sqrt{b}s+\frac{i\omega}{2\sqrt{b}})^2}\,ds$

$=C e^{\lambda}\cdot\int_{-\infty}^{\infty} e^{-m^2}\,dm\cdot\frac{1}{\sqrt{b}}$, where $m=\sqrt{b}s+\frac{i\omega}{2\sqrt{b}}$.

$=C\sqrt{\frac{\pi}{b}}\cdot\exp(-i\omega t_0-\frac{\omega^2}{4b})$. which is still an exp function.

(b). $X(\omega)=\frac{1}{\sqrt{2\pi}\sigma}\cdot\sqrt{2\pi\sigma^2}\cdot\exp(-i\omega t_0-\frac{\omega^2}{2}\sigma^2)$

$\xrightarrow{\sigma\to 0}= e^{-i\omega t_0}$

$\therefore\ \delta(t-t_0)\leftrightarrow e^{-i\omega t_0}$

$\Rightarrow\ \delta(t)\leftrightarrow e^{i\omega t_0}\cdot e^{-i\omega t_0}=1$. 时移性.

So the FT of $\delta(t)$ is 1. $\boxed{\delta(t)\leftrightarrow 1}$

(c). $f(t)=\begin{cases} e^{-\alpha t}, & t\geq 0\\ 0, & t<0\end{cases}$

$\Rightarrow X(\omega)=\int_{0}^{\infty} e^{-(\alpha+i\omega)t}\,dt=\frac{1}{\alpha+i\omega}=\frac{\alpha}{\alpha^2+\omega^2}-i\cdot\frac{\omega}{\alpha^2+\omega^2}$.

$\because\ \lim_{\alpha\to 0}\frac{\alpha}{\alpha^2+\omega^2}=\begin{cases}0, & \omega\neq 0\\ \infty, & \omega=0\end{cases}$, and $\int_{-\infty}^{\infty}\lim_{\alpha\to 0}\frac{\alpha}{\alpha^2+\omega^2}=\lim_{\alpha\to 0}\arctan(\frac{\omega}{\alpha})\Big|_{-\infty}^{\infty}=\pi$.

$\therefore\ \lim_{\alpha\to 0}X(\omega)=\pi\cdot\delta(\omega)-\frac{i}{\omega}=\pi\cdot\delta(\omega)+\frac{1}{i\omega}$. $\boxed{\varepsilon(t)\leftrightarrow\pi\delta(\omega)+\frac{1}{i\omega}}$

So PSD $=G_X(\omega)=|X(\omega)|^2=\pi^2\delta^2(\omega)+\frac{1}{\omega^2}$.

(d). $f(t) = \mathrm{sgn}(t)$.

Let $f_\alpha(t) = \begin{cases} -e^{\alpha t}, & t<0 \\ e^{-\alpha t}, & t>0 \end{cases}$, $\alpha>0$

$$X(\omega) = \int_{-\infty}^{0} -e^{\alpha t} \cdot e^{-i\omega t} dt + \int_{0}^{\infty} e^{-\alpha t} \cdot e^{-i\omega t} dt$$

$$= \frac{1}{\alpha+i\omega} - \frac{1}{\alpha-i\omega} = \frac{-2i\omega}{\alpha^2+\omega^2}$$

$\alpha \to 0 \Rightarrow f_\alpha(t) \to f(t)$

$\Rightarrow X(\omega) \to \frac{-2i\omega}{\omega^2} = \frac{2}{i\omega}$. $\boxed{\mathrm{sgn}(t) \leftrightarrow \frac{2}{i\omega}}$

(e). ① $f(t) = \frac{1}{t}$.

we start from $\mathrm{sgn}(t) \leftrightarrow \frac{2}{i\omega}$

use 对称性 ($f(t) \leftrightarrow F(i\omega) \Rightarrow F(it) \leftrightarrow 2\pi \cdot f(-\omega)$)

$\Rightarrow \frac{2}{it} \leftrightarrow -2\pi\,\mathrm{sgn}(\omega) \xrightarrow{\text{线性}} \boxed{\frac{1}{t} \leftrightarrow -i\pi\,\mathrm{sgn}(\omega)}$

② $f(t) = \frac{1}{t^2}$.

use $\frac{1}{t} \leftrightarrow -i\pi\,\mathrm{sgn}(\omega)$ and 微分性 ($f^{(n)}(t) \leftrightarrow (i\omega)^n \cdot F(i\omega)$)

$\Rightarrow -\frac{1}{t^2} \leftrightarrow \omega\pi\,\mathrm{sgn}(\omega)$. $\xrightarrow{\text{线性}} \boxed{\frac{1}{t^2} \leftrightarrow -\omega\pi\,\mathrm{sgn}(\omega) = -\pi \cdot |\omega|}$

③ $\frac{1}{t^2} \leftrightarrow -\pi|\omega|$

对称性 $\Rightarrow$ $-\pi|t| \leftrightarrow 2\pi \cdot \frac{1}{\omega^2}$

线性 $\Rightarrow$ $\boxed{|t| \leftrightarrow -\frac{2}{\omega^2}}$