MSDM 5058 Information Science
Assignment 1 (due 2nd March, 2024)

Submit your assignment solution on canvas. You may discuss with others or seek help from your TA, but should not directly copy from others. Otherwise, it will be considered as plagiarism.

(1) **Bayes' Theorem**

A telegraphic communications system transmits the signals 1 and 0. Assume that due to noise, an average of 2/5 of the dots and 1/3 of the dashes are changed. Suppose that the ratio between the transmitted dots and the transmitted dashes is 5: 3. What is the probability that a received signal will be the same as the transmitted signal if (a) the received signal is a dot, (b) the received signal is a dash? (c) Assuming independence between signals, if the message dot-dot was received, what is the probability distribution of the four possible messages that could have been sent?

(2) **Characteristic Function**

Find the (a) characteristic function of a random variable X with the probability density

$$f(x) = \frac{1}{2}e^{-|x|},$$

(b) from the characteristic function, find its mean and variance.

(3) **Autocorrelation**

Let $X(t)$ be a normal stationary random function with zero expectation. Show that if

$$Z(t) = \frac{1}{2}\left[1 + \frac{X(t)X(t+\tau)}{|X(t)X(t+\tau)|}\right],$$

then

$$\bar{Z}(\mathrm{t}) = \frac{1}{\pi}\mathrm{acos}[-\rho(\tau)]$$

where $\rho(\tau)$ is the normalized correlation function of $X(t)$.

(4) **Radar System**

Consider a radar system that uses radio waves to detect aircrafts. The system receives a signal and, based on the received signal, it needs to decide whether an aircraft is present or not. Let X be the received signal. Suppose that we know

$X=W$, if no aircraft is present.

$X = 1 + W$, if an aircraft is present.

where $W \sim N(0,\sigma^2 = \frac{1}{9})$. Thus, we can write $X = \theta + W$, where $\theta = 0$ if there is no aircraft, and $\theta = 1$ if there is an aircraft. Suppose that we define H_0 and H_1 as follows:

H_0 (Null hypothesis): No aircraft is present.

H_1 (Alternative hypothesis): An aircraft is present.

(a) Write the null hypothesis, H_0, and the alternative hypothesis, H_1, in terms of possible values of θ.
(b) Design a level 0.05 test ($\alpha = 0.05$) to decide between H_0 and H_1.
(c) Find the probability of type II error, β, for the above test in (b). Note that this is the probability of missing a present aircraft.
(d) If we observe $X = 0.6$, is there enough evidence to reject H_0 at significance level $\alpha = 0.01$?
(e) If we would like the probability of missing a present aircraft to be less than 5%, what is the smallest significance level that we can achieve?
(f) What is the likelihood ratio for ***false alarm***?