Homework 2

Due date: 08/10/2023

On Brownian motion

Consider a Brownian particle in the overdamped regime. The equation of motion in one-dimensional space is given by

$$\gamma \frac{dx}{dt} = -\frac{d}{dx}U(x) + \zeta ,$$

in which γ is the damping coefficient, $U(x)$ is the external potential, and ζ is the white noise satisfying the autocorrelation $\langle \zeta(t_1)\zeta(t_2)\rangle = 2\gamma k_B T \delta(t_1 - t_2)$. For simplicity, let's take $\gamma = 1$ and $k_B T = 1$ in the numerical simulations.

(a) For $U(x) = 0$, the Brownian particle undergoes a random walk. Please carry out the simulation to obtain the particle trajectory and compute $\left\langle [x(t) - x(0)]^2 \right\rangle$, which is expected to be linear in t. Determine the diffusivity from the numerical data for $\left\langle [x(t) - x(0)]^2 \right\rangle$ and compare it with the theoretical value.

(b) For $U(x) = x^2/2$ (a confinement potential), please carry out the simulation to find the stationary probability density function (PDF) $f(x)$ from long-time behaviour of particle trajectory. Is your numerical PDF close to a normal distribution? What is the value of the variance of $f(x)$?

Note that you can use any computer language to simulate the stochastic processes.

Please submit both the computer program and the plots.