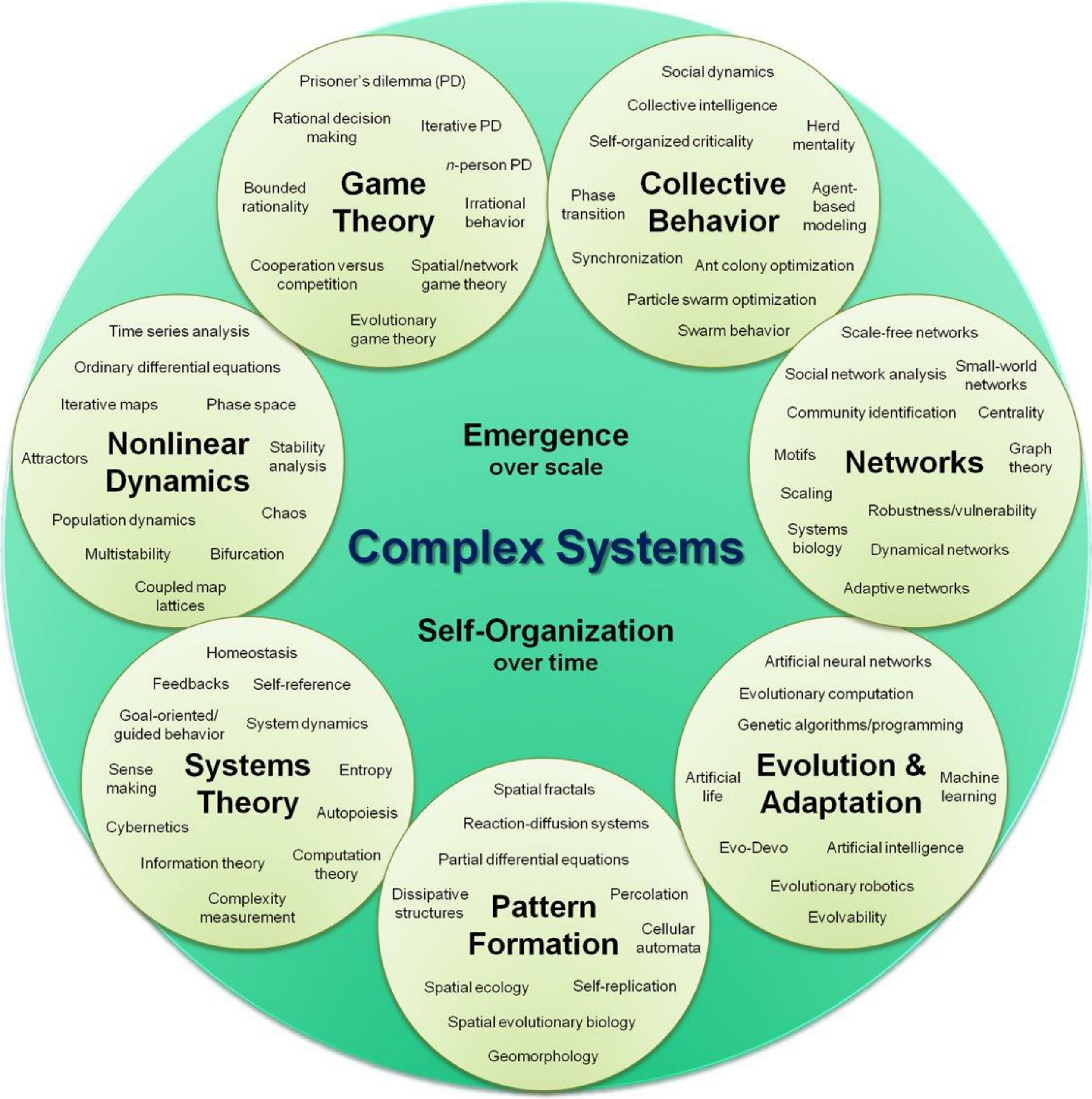
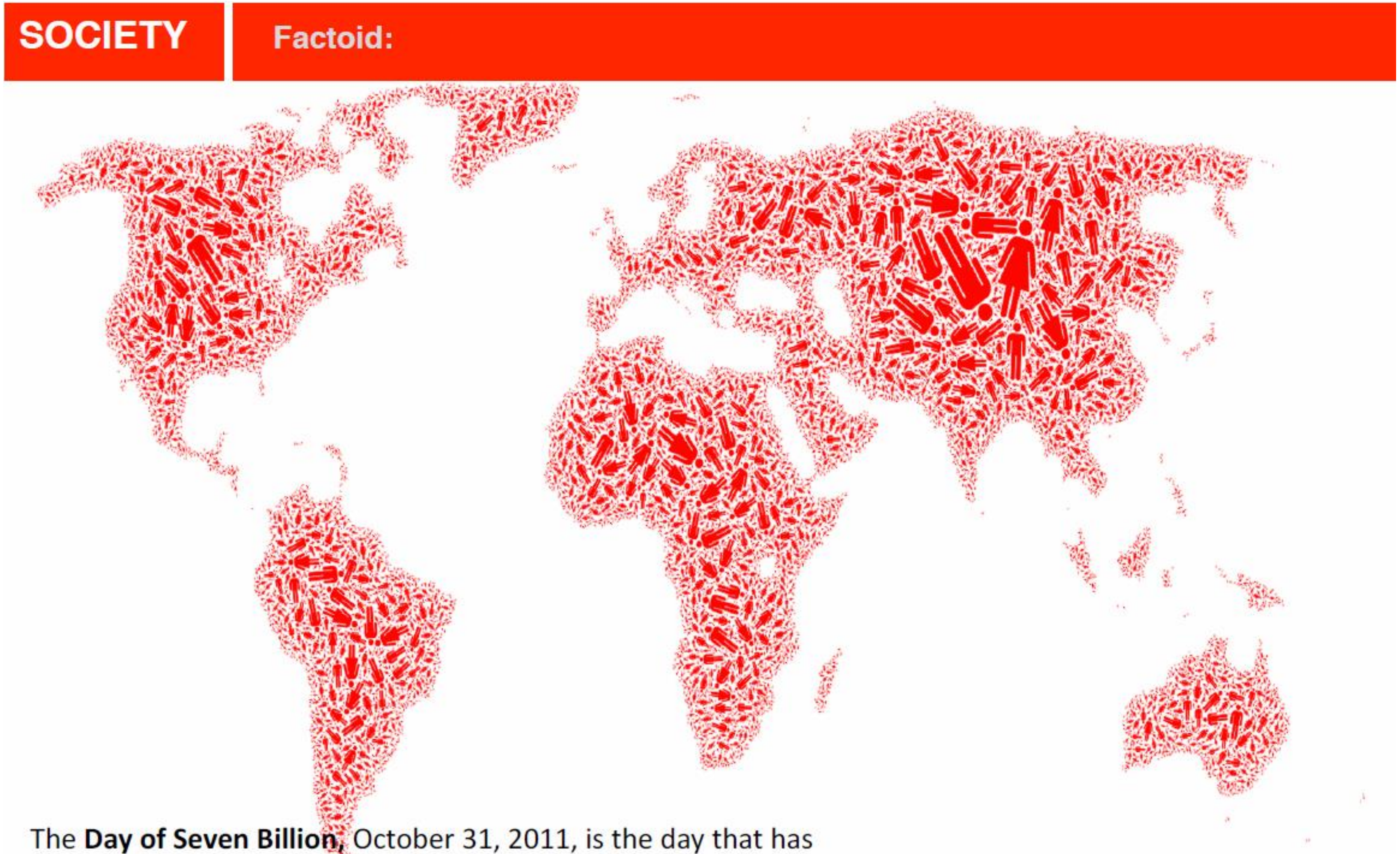
Lecture One: Introduction to Complex Networks

The theory of Complex Systems is a new approach that started in the 70's of last century. It is a way to describe systems that cannot be described with a reductionist approach.



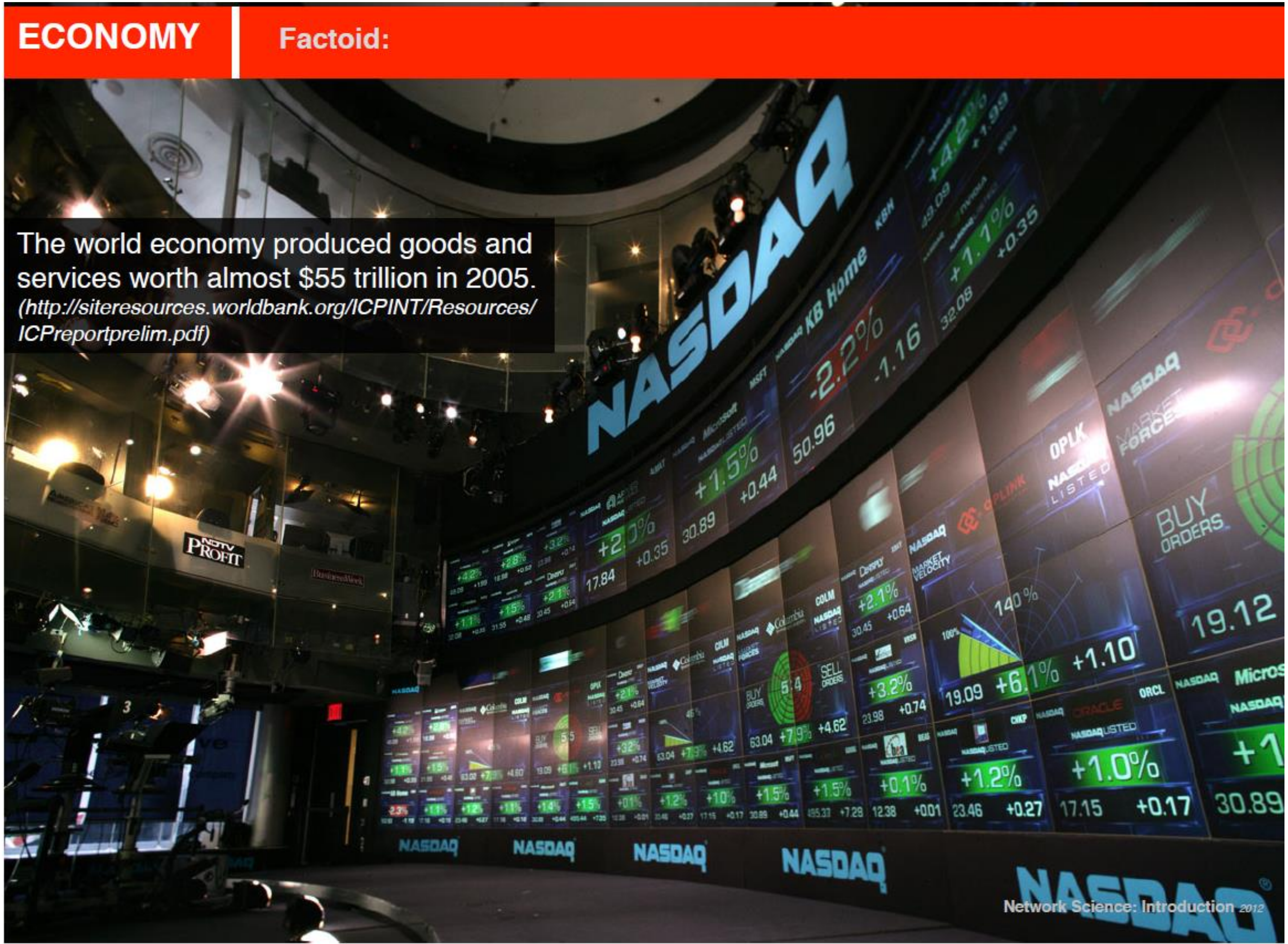
Organizational map of complex systems broken into seven sub-groups
(Created by Hiroki Sayama, Research Group at Binghamton University, State University of New York)

Some Examples of Complex Systems

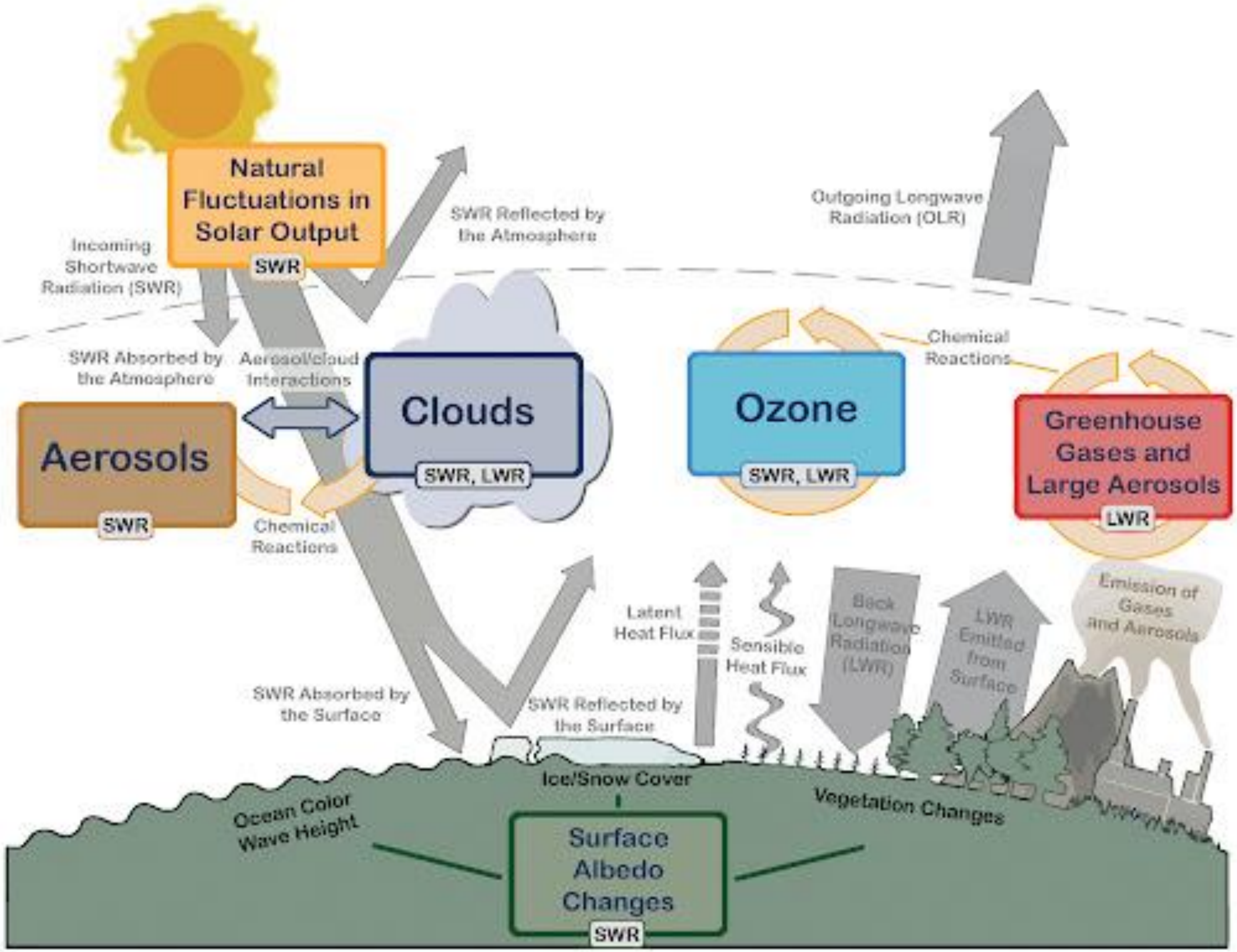


The **Day of Seven Billion**, October 31, 2011, is the day that has been officially designated by the United Nations Population Fund (UNFPA) as the approximate day on which the world's population reached seven billion people.

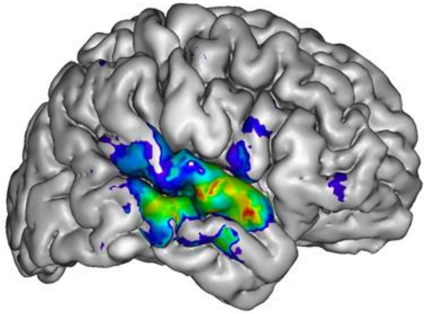
Some Examples of Complex Systems



Some Examples of Complex Systems



Some Examples of Complex Systems



What is a Complex System ?

A Complex system is formed by a collection of many simpler units through (non-)linear interactions which will display interesting collective behavior.

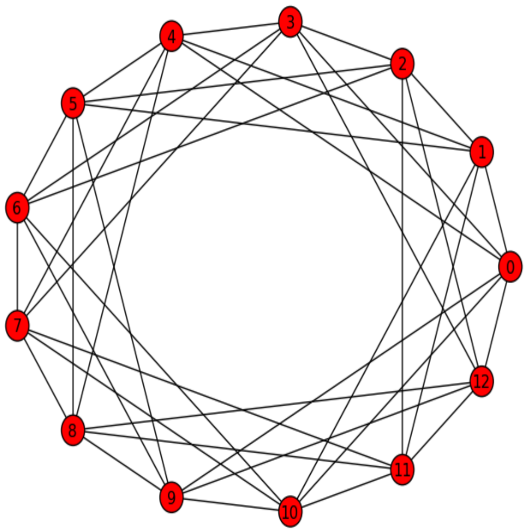
Some Examples: Earth's global climate, the human brain, social organization, an ecosystem, a living cell, and ultimately the entire universe.

They can be studied with the help of the mathematical tools developed, e.g., in thermodynamic and statistical mechanics.

From Complex Systems to Complex Networks

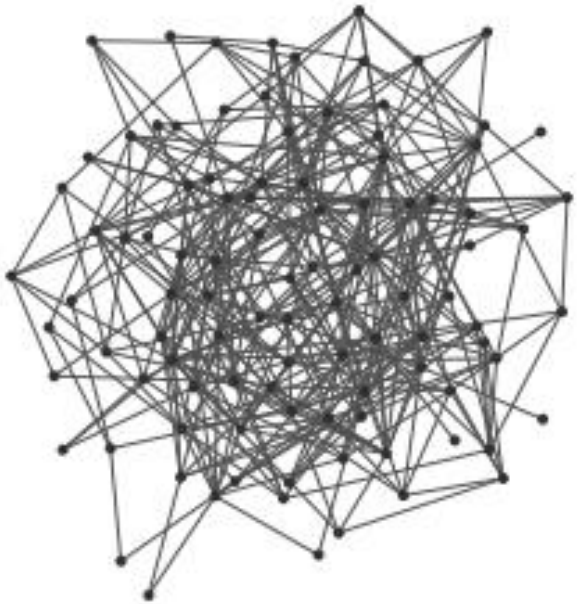
- Complex systems
  - Large number of components interacting with each other
  - All components and/or interactions are different from each other
  - Paradigms:
    - 10^4 types of proteins in an organism,
    - 10^6 routers in the Internet
    - 10^9 web pages in the WWW
    - 10^{11} neurons in a human brain

- The "simplest property":
  - who interacts with whom?
    - **can be visualized as a network !!!**
- ***Complex networks are just backbones for complex dynamical systems***



Regular Network

Random Network



Complex Networks

In the context of network theory, a **complex network** is a network (graph) with non-trivial topological features—features that do not occur in simple networks such as lattices or random graphs.

The study of complex networks is an active interdisciplinary area of scientific research inspired largely by the empirical study of real-world networks such as computer networks and social networks.

What is a Network?

A Network is a mathematical structure composed of points connected by lines

Network Theory	$\leftrightarrow$	Graph Theory
Network	$\leftrightarrow$	Graph
Nodes	$\leftrightarrow$	Vertices (points)
Links	$\leftrightarrow$	Edges (Lines)

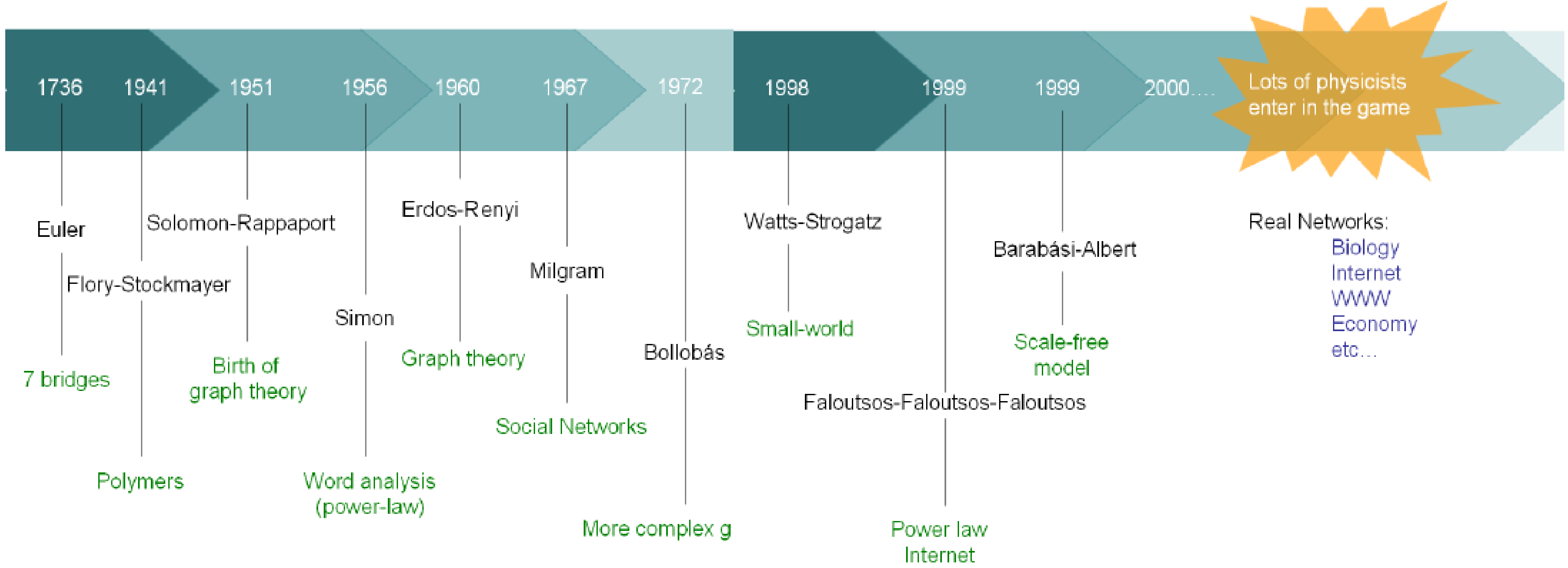
A network can be build for any functional system

System vs. Parts = Networks vs. Nodes

THE PRECURSORS OF COMPLEX NETWORK THEORY

- **Graph theory:** 1735, Euler
- **Social Network Research:** 1930s, Moreno
- **Communication networks/internet:** 1960s
- **Ecological Networks:** May, 1979

Complex Networks time line



THE ORIGIN OF GRAPH THEORY

Graphs are formed by a set of nodes (vertices) connected by links (edges)

Leonard Euler (1707 – 1783)

According to the current view, modern graph theory traces back to the mathematician Leonard Euler who was the first scientist to introduce the notion of graphs in order to solve the puzzle of the city of Konisberg and of its seven bridges in 1735.

Leonard Euler (Basel 1707 – St. Petersburg 1783)

Some revealing data about him:

- Euler worked in almost all areas of mathematics: geometry, calculus, trigonometry, algebra, and number theory, as well as continuum physics, lunar theory and other areas of physics.

- Large number of topics of physics and mathematics are named in his honour (e.g., Euler’s function, Euler’s Equation or Euler’s formula).

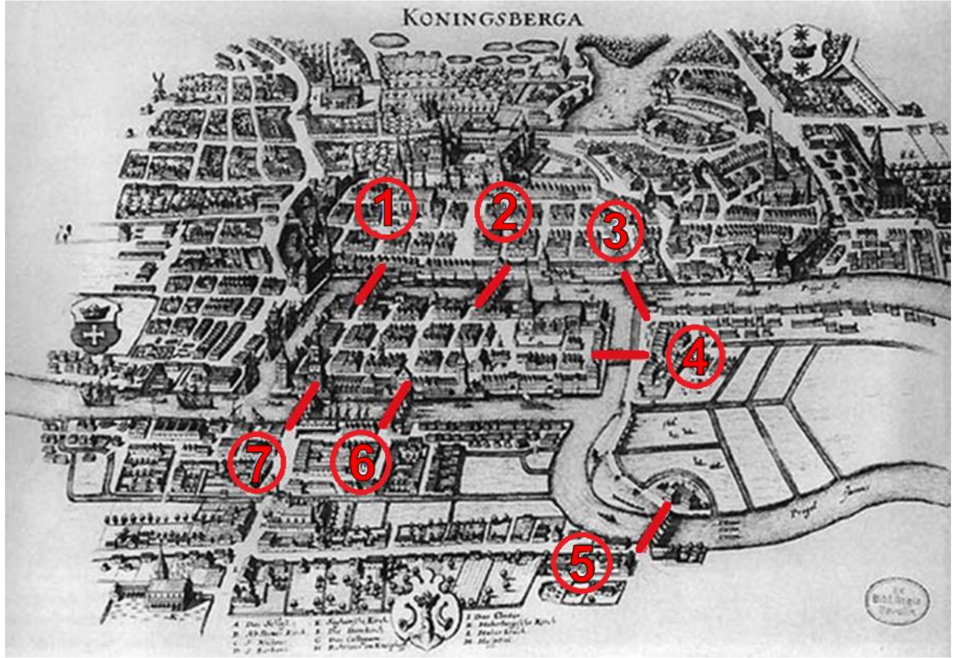
- All his work is collected in *Opera Omnia*, which consists of 886 books.

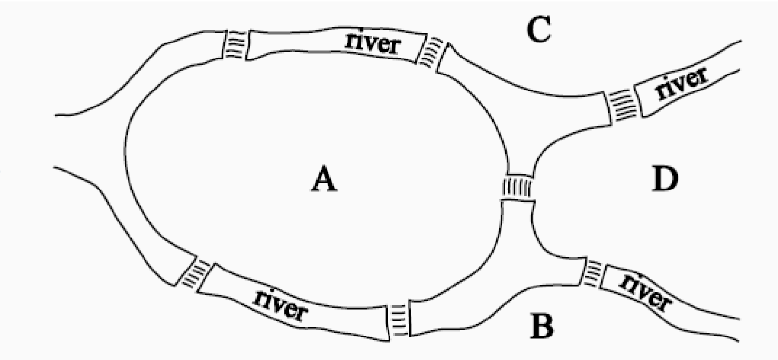
- With one eye from 1738 and completely blind from 1766!

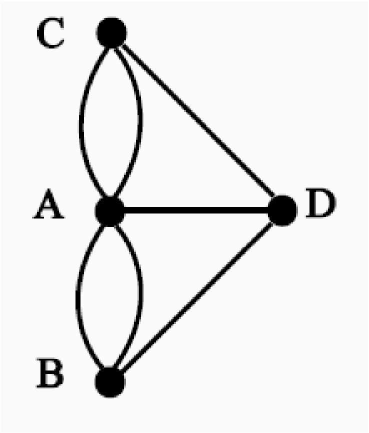
- And the most astonishing data: all of that with **13 children!**

Seven Bridges of Königsberg

1736 Euler: Can one walk across all the seven bridges, once and only once, and then return to the starting point ?







➡ ***Eulerian cycle:*** *A path passing through each link and coming back to the starting point without passing through a link more than once.*

Euler's Theorem:
(1) If a graph has more than two nodes of odd degree. There is no path.
(2) If a graph is connected and has no odd degree nodes, it has at least one path.

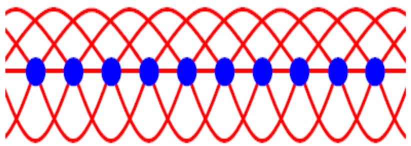
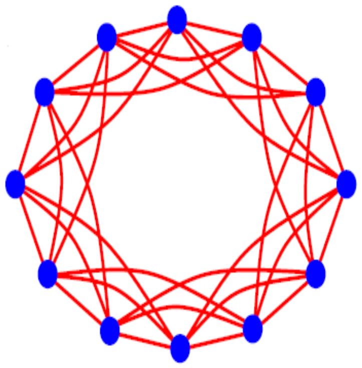
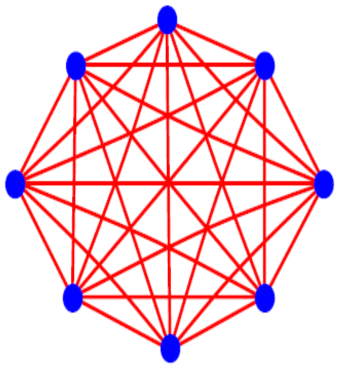
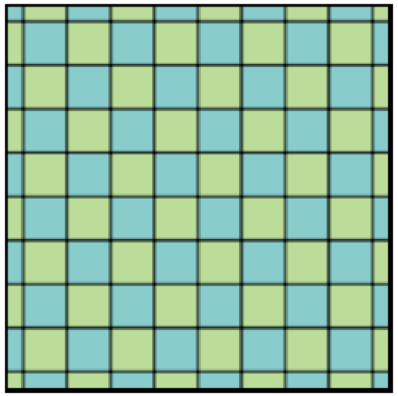
Regular Graphs:

- After the death of Euler, graph theory received many contributions from mathematicians such as Hamilton, Kirchhoff or Cayley.

- The core of graph theory focused on the study of regular graphs:

  Regular graph: a graph where all nodes have the same degree.

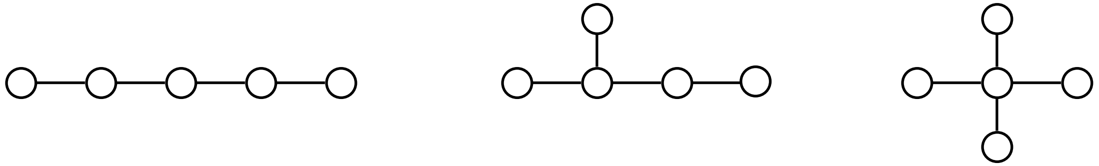
  Lattice: a regular network where all nodes are coupled to its nearest neighbor.

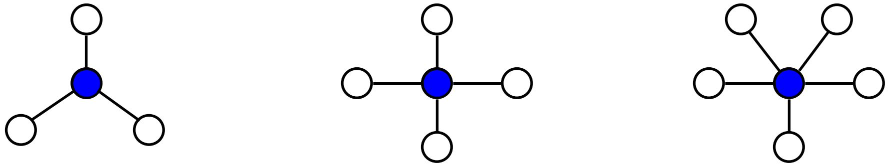
Networks As Graphs

Some Basic Types of Graphs

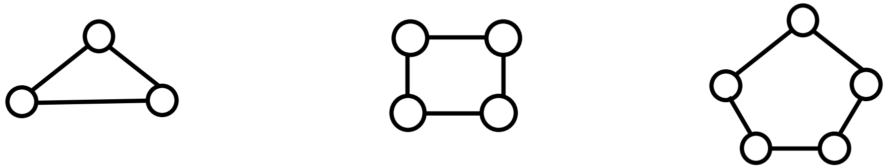
trees



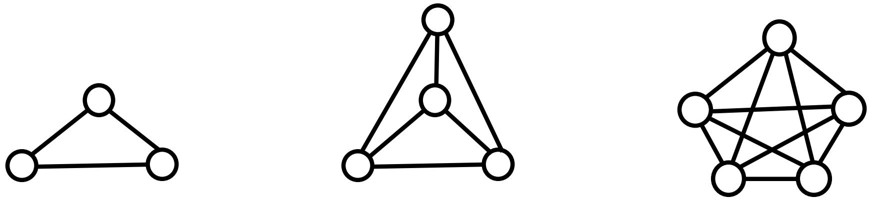
Stars



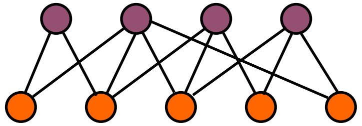
Cycles



Complete Graphs



Bipartite Graphs



Historical perspective on Complex Networks

- In the beginning.. there was REDUCTIONISM
  - All we need to know is the behavior of the system elements
  - Particles in physics, molecules or proteins in biology, communication links in the Internet
  - Complex systems are nothing but the result of many interactions between the system's elements
  - No new phenomena will emerge when we consider the entire system
  - A centuries-old very flawed scientific tradition

Historical perspective

- During the 80's and early 90's of the 20th century, several parallel approaches departed from reductionism

- Consider the entire SYSTEM attempting to understand/ explain its COMPLEXITY
  - B. Mandelbrot and others: Chaos and non-linear dynamical systems (the math of complexity)
  - P. Bak: Self-Organized Criticality – The edge of chaos
  - S. Wolfram: Cellular Automata
  - S. Kauffman: Random Boolean Networks
  - I. Prigogine: Dissipative Structures
  - J. Holland: Emergence
  - H. Maturana, F. Varela: Autopoiesis networks & cognition
  - Systems Biology

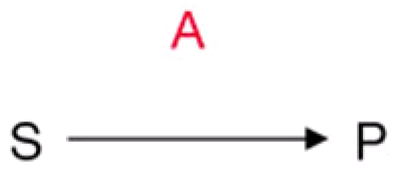
Historical perspective

- Systems approach: thinking about Networks
  - The focus moves from the elements (network nodes) to their interactions (network links)
  - To a certain degree, the structural details of each element become less important than the network of interactions
  - Some system properties, such as Robustness, Fragility, Modularity, Hierarchy, Evolvability, Redundancy (and others) can be better understood through the Networks approach

- Some modern day milestones:
  - 1998: Small-World Networks (D. Watts and S. Strogatz)
  - 1999: Scale-Free Networks (R. Albert & A.L. Barabasi)
  - 2002: Network Motifs (U. Alon)

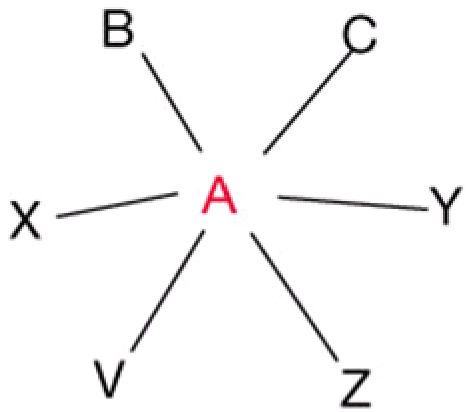
Example: Evolution of the meaning of protein function

traditional view



The function of protein A is its action on S to form P

post-genomic view

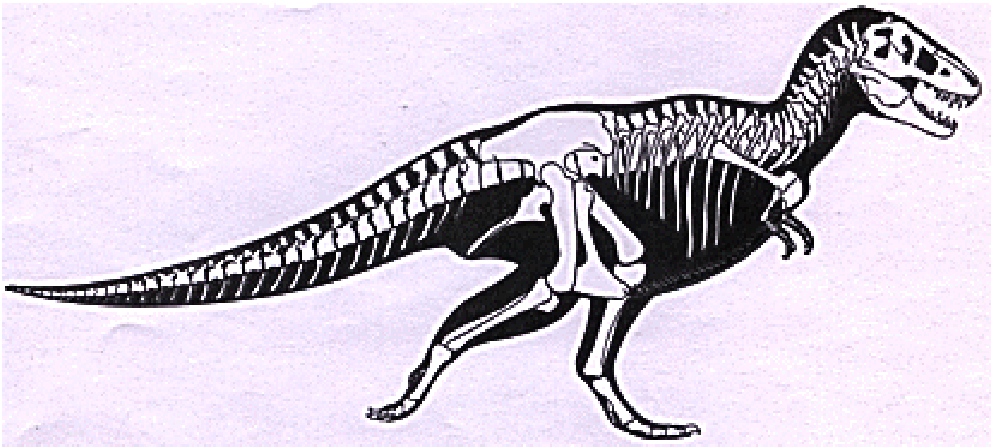


The function of A is the context of its interactions with other proteins in the cell

from Eisenberg et al. Nature 2000 450: 823-6

Why study the topology of Complex Networks?

- Large amount of available data
- Large networks may contain information about basic design principles and/or evolutionary history of the complex system
- This is similar to paleontology:
  - learning about an animal from its backbone



Early social network analysis

- 1933 Moreno displays first sociogram at meeting of the Medical Society of the state of New York
  - article in NYT
  - interests: effect of networks on e.g. disease propagation

EMOTIONS MAPPED BY NEW GEOGRAPHY

Charts Seek to Portray the Psychological Currents of Human Relationships.

MANY MISFITS REVEALED

Dr. J. L. Moreno Calculates There Are 10 to 15 Million Isolated Individuals In Nation.

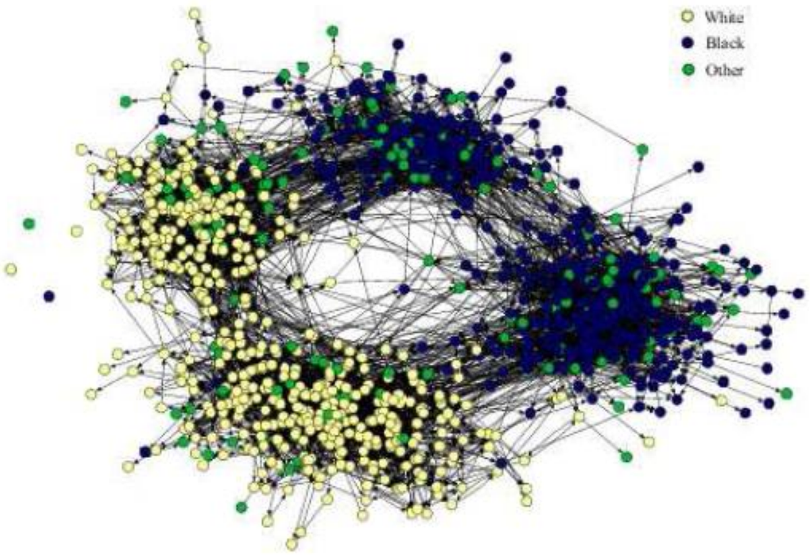
A new science, named psychological geography, which aims to chart the emotional currents, cross-currents and under-currents of human relationships in a community, was introduced here yesterday at the

- Preceded by studies of (pre)school children in the 1920's

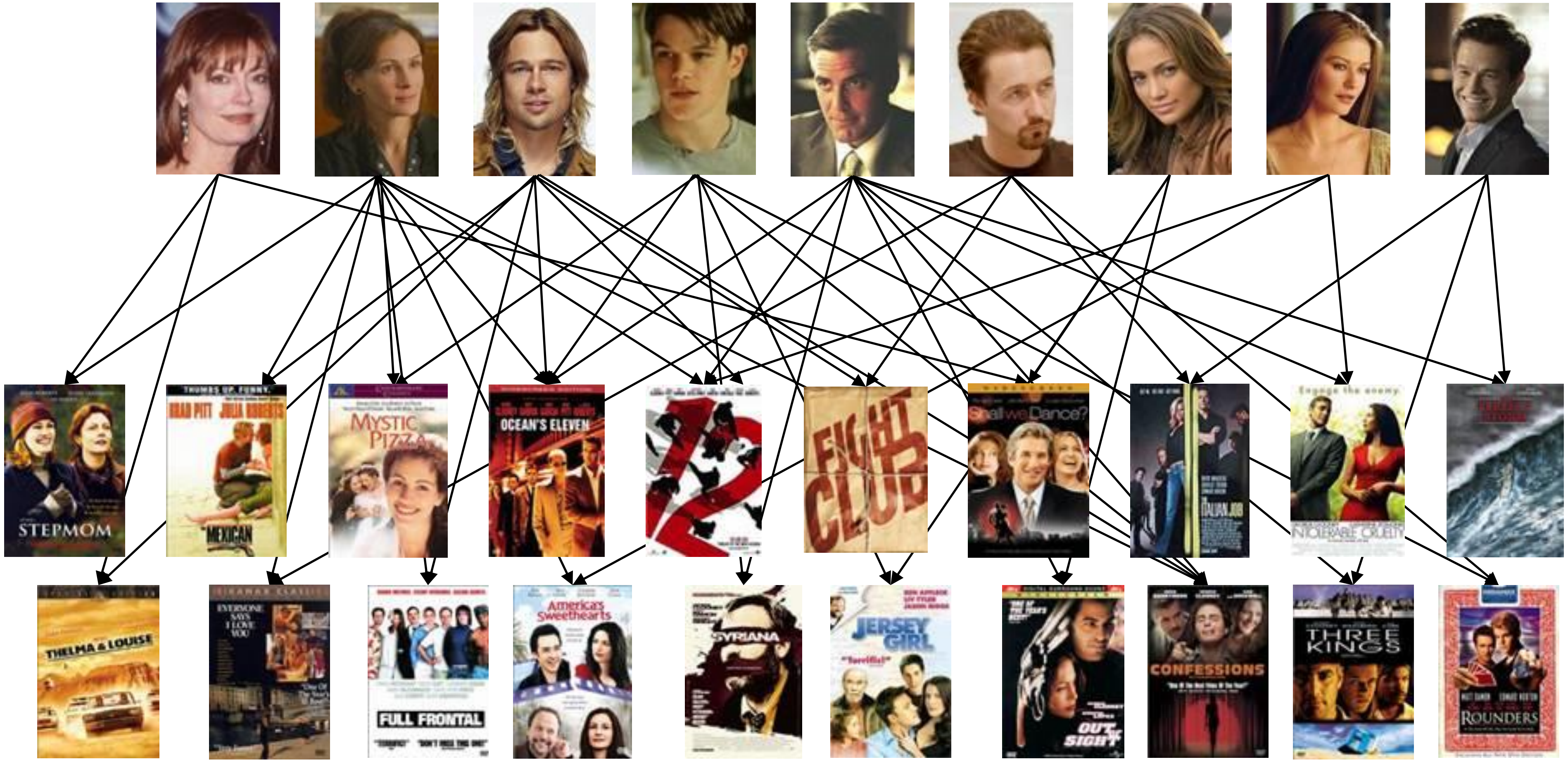
Source: The New York Times (April 3, 1933, page 17).

Social Networks

- Links denote a social interaction
  - Networks of acquaintances
  - collaboration networks
    - actor networks
    - co-authorship networks
    - director networks
  - phone-call networks
  - e-mail networks
  - Bluetooth networks
  - sexual networks
  - home page/blog networks
  - …



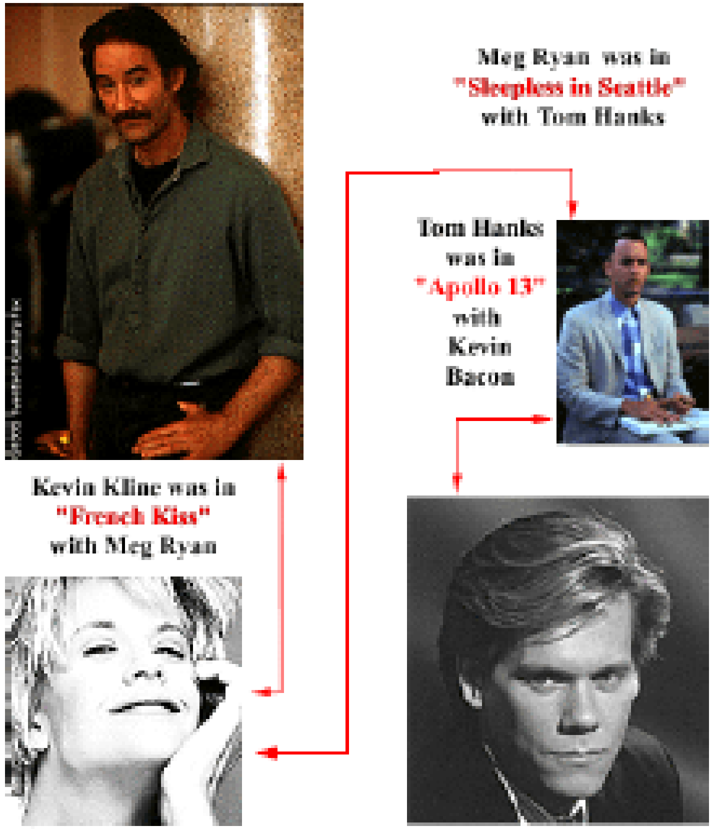
Network of actor co-starring in movies



Actors

"The Oracle of Bacon"

http://www.cs.virginia.edu/oracle



Path from K. Kline to K. Bacon = 3 (as of 1995)

(http://collegian.ksu.edu/issues/v100/FA/n069/fea-making-bacon-fuqua.html)

- a given actor is on average 3 movies away from Kevin Bacon (L_{Bacon}=2.946, as of June 2004) . . . or any other actor for that matter

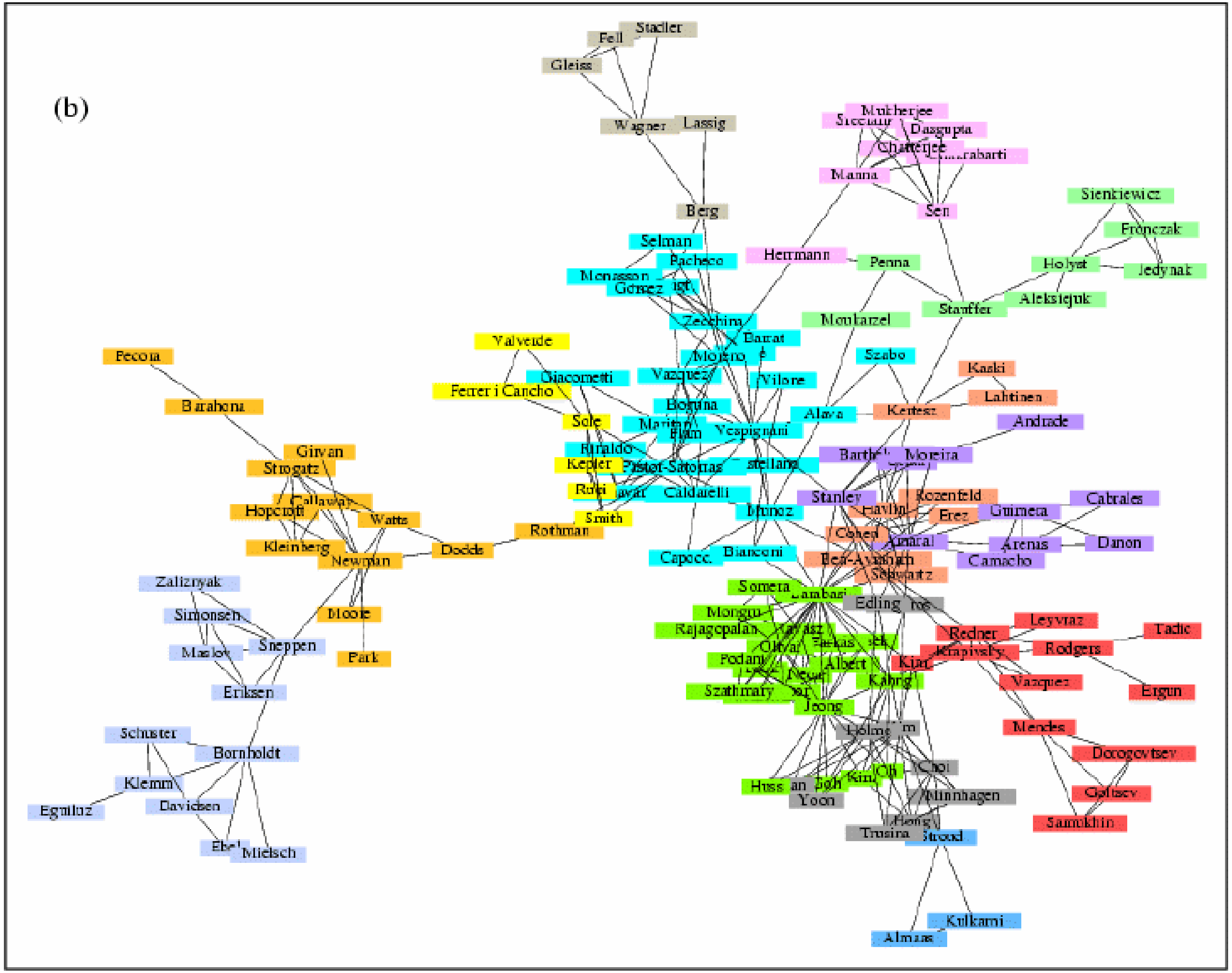
  L_{Bacon} = 3.029 as of Aug 2017

- Hollywood is a small world

- . . . and it is a *scale-free* small world: a few actors played in a lot of movies, and a lot of actors in few movies

http://oracleofbacon.org

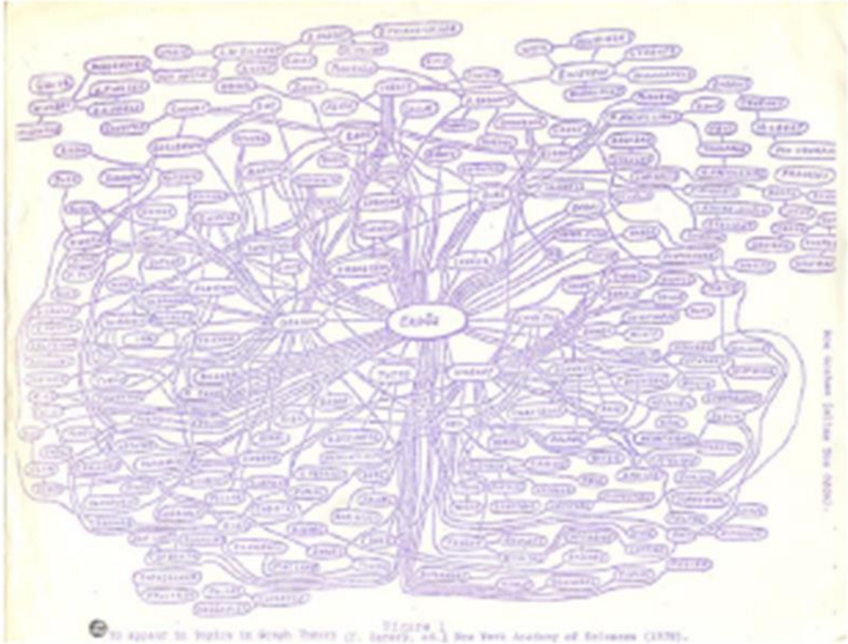
Networks of scientists' co-authorship of papers



Scientists

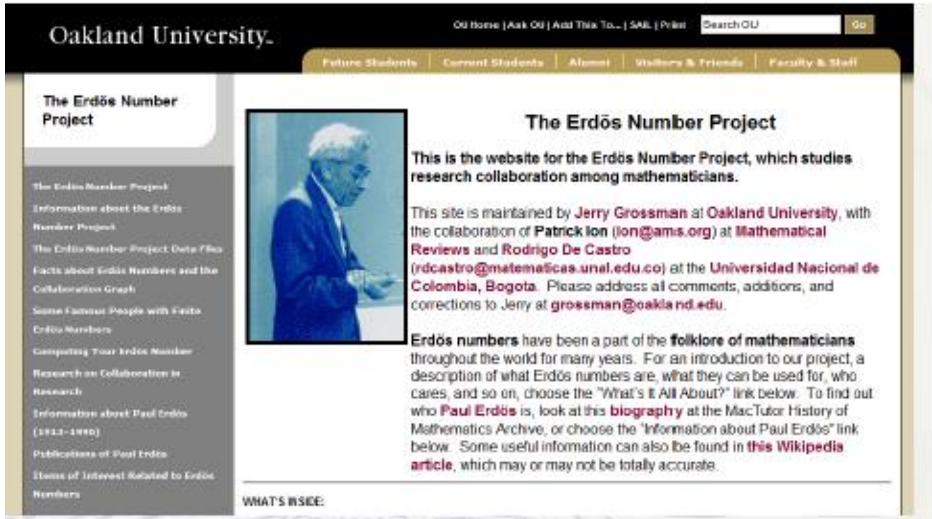
"The Erdős Number Project"

http://www.oakland.edu/enp



Co-authors of Paul Erdős have number 1, co-authors of co-authors number 2, etc.

Mathematicians form a highly clustered ($C = 0.14$) small world ($L = 7.64$)



Max von Laue	1914	4
Albert Einstein	1921	2
Niels Bohr	1922	5
Louis de Broglie	1929	5
Werner Heisenberg	1932	4
Paul A. Dirac	1933	4
Erwin Schrödinger	1933	8
Enrico Fermi	1938	3
Ernest O. Lawrence	1939	6
Otto Stern	1943	3
Isidor I. Rabi	1944	4
Wolfgang Pauli	1945	3
Frits Zernike	1953	6
Max Born	1954	3
Willis E. Lamb	1955	3

You can measure the distance with Paul Erdös. (http://www.oakland.edu/enp/compute)

- Mean Erdös number: ~5
- Largest Erdös number:~13

Academic Network

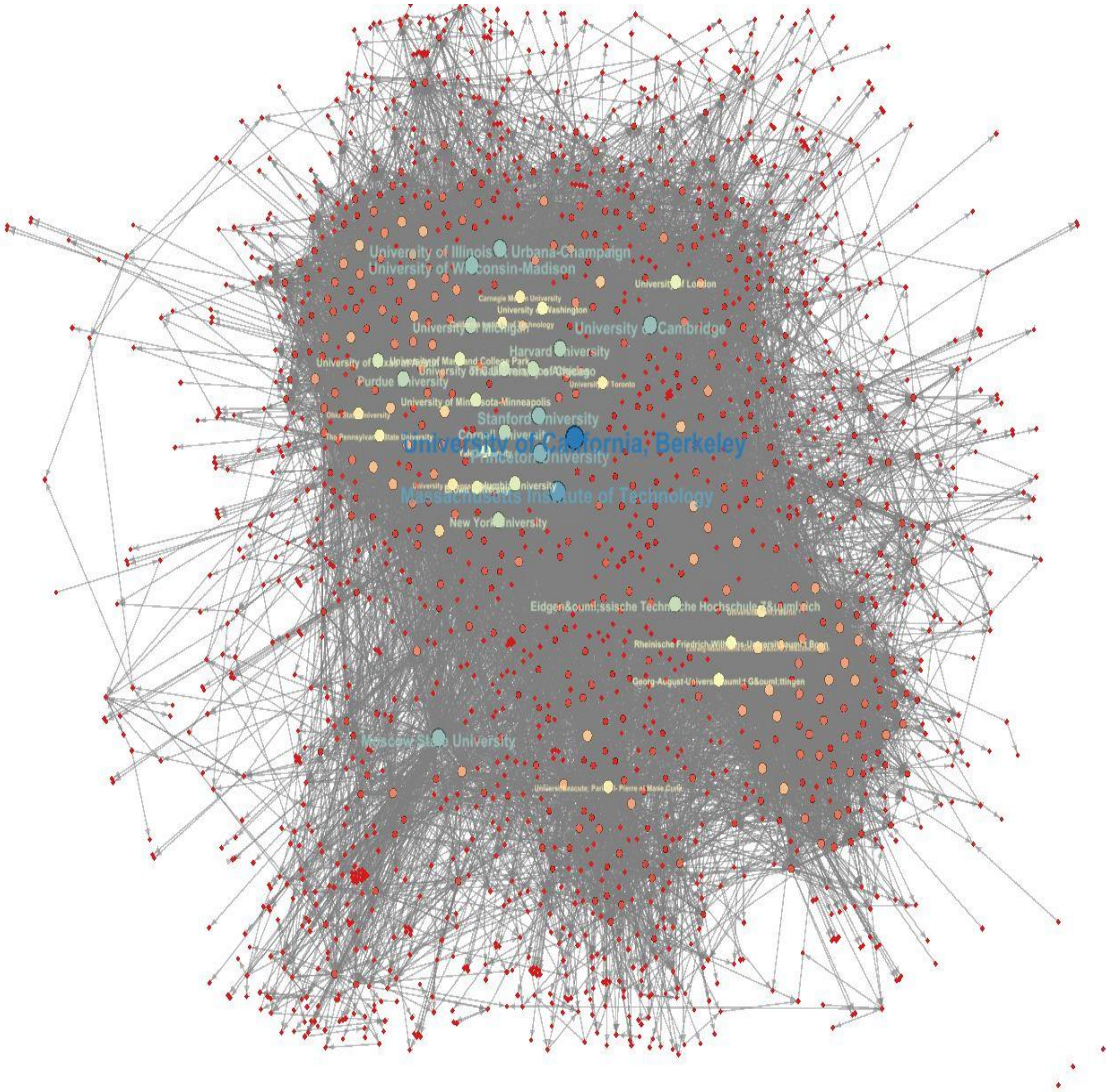
1860 universities

d = 19.6

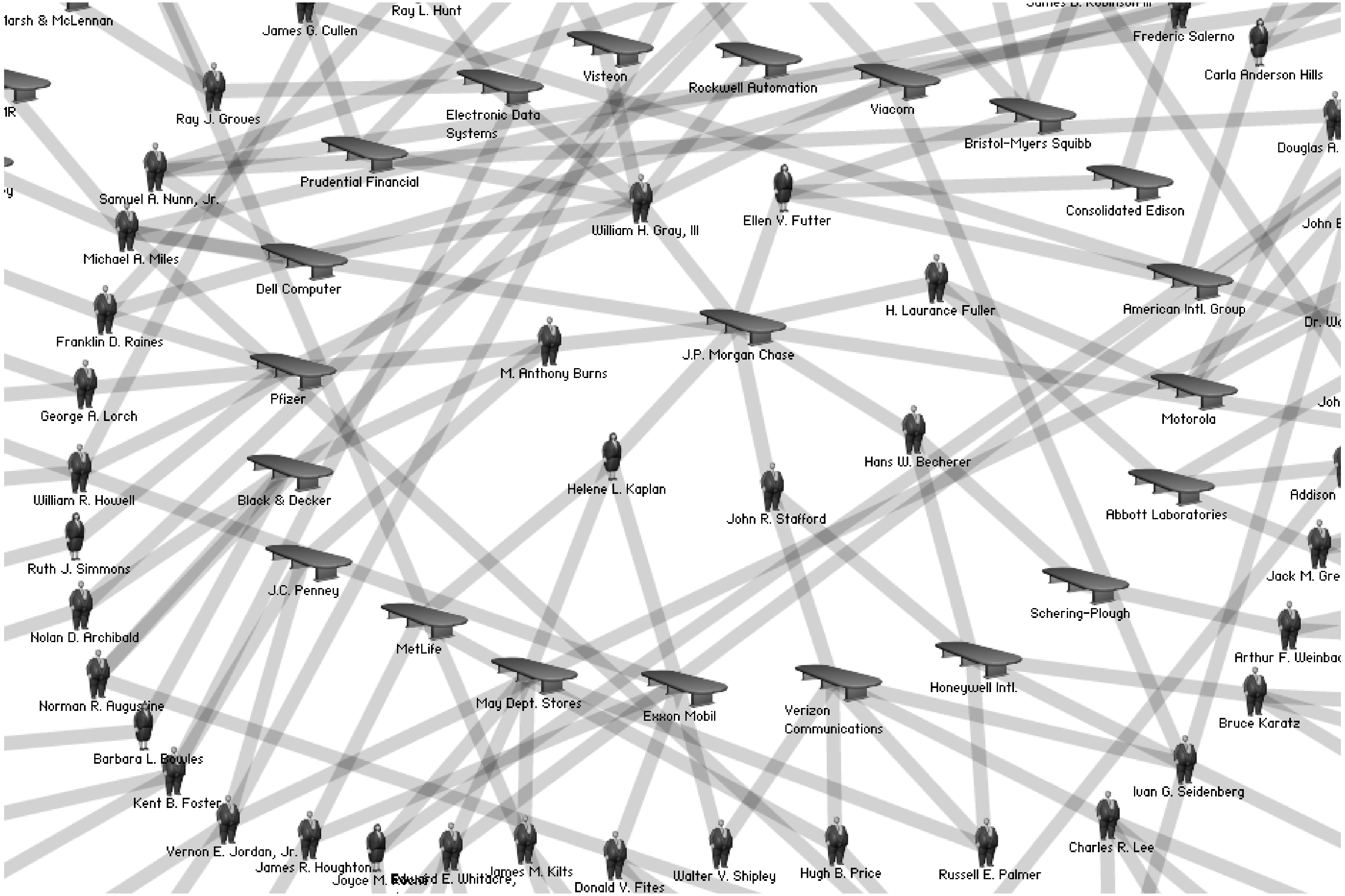
Assortativity = .005
non-assortative

clustering = .23
(random =.05)

max clique = 6



Boards of Directors

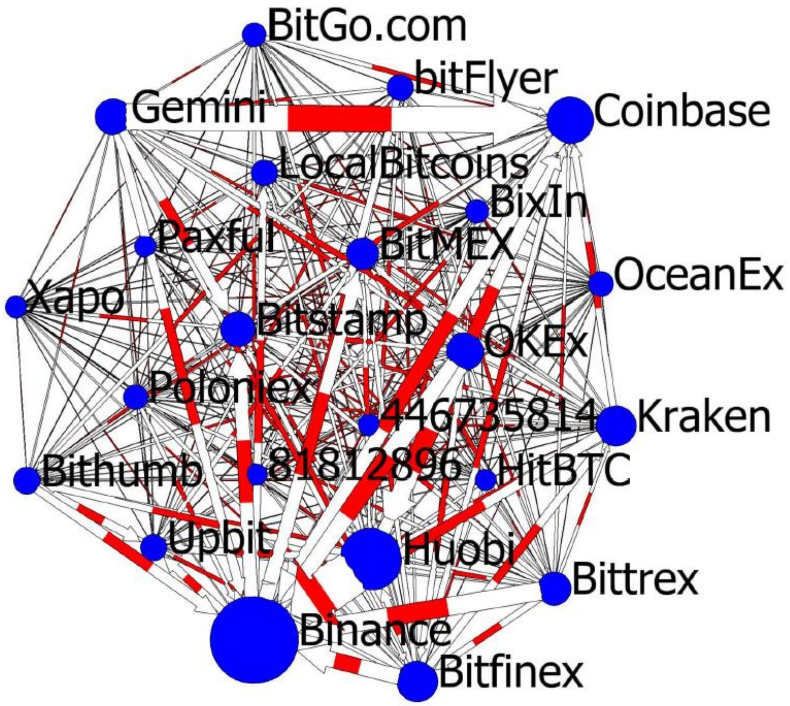


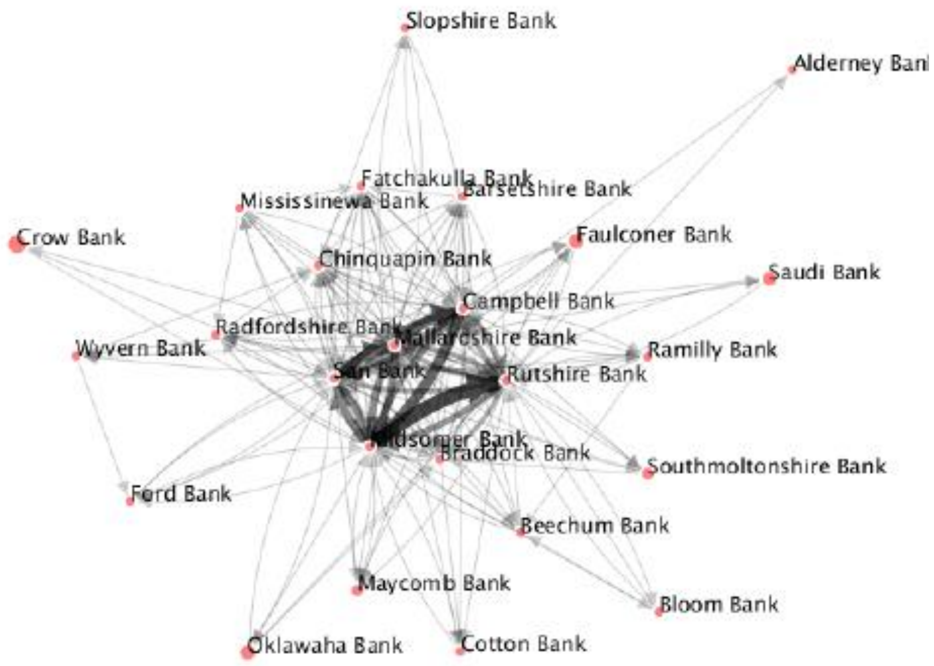
Source: http://theyrule.net

Examples in finance:

Blockchains are complex networks backed by a digital ledger.

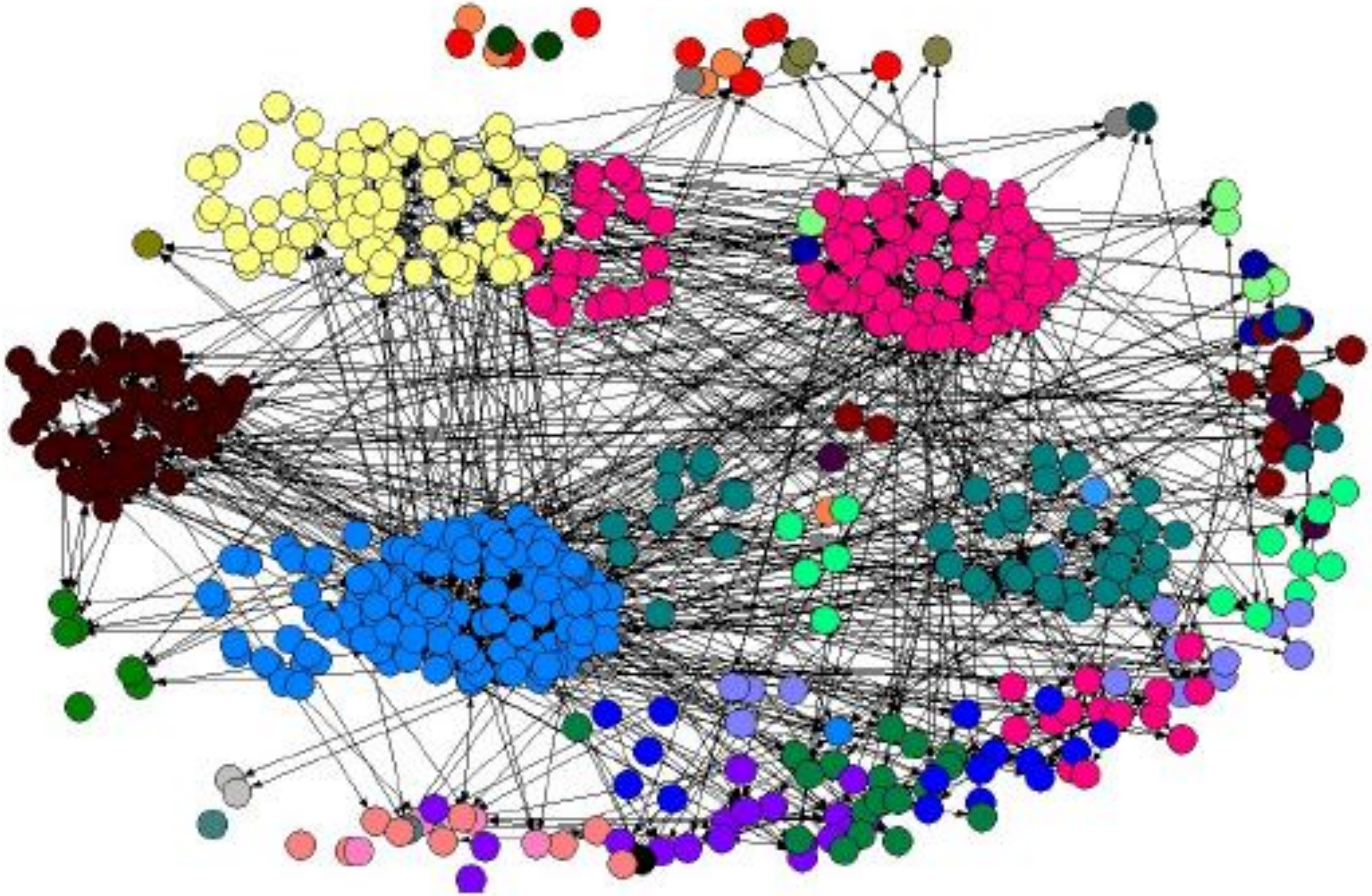
** Makarov, I. and Schoar, A., Blockchain Analysis of the Bitcoin Market. Working Paper 29396, NBER*
** Kondor et al., PLoS ONE 9(2): e86197, 2014.*
** Di Francesco Maesa et al., Int J Data Sci Anal, 2017*





Banking Networks

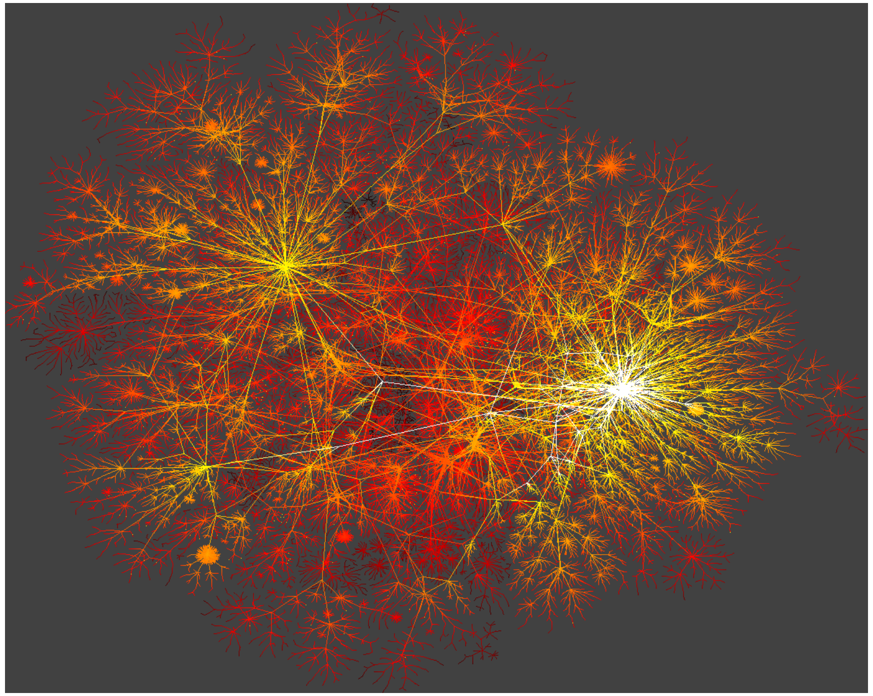
……………..



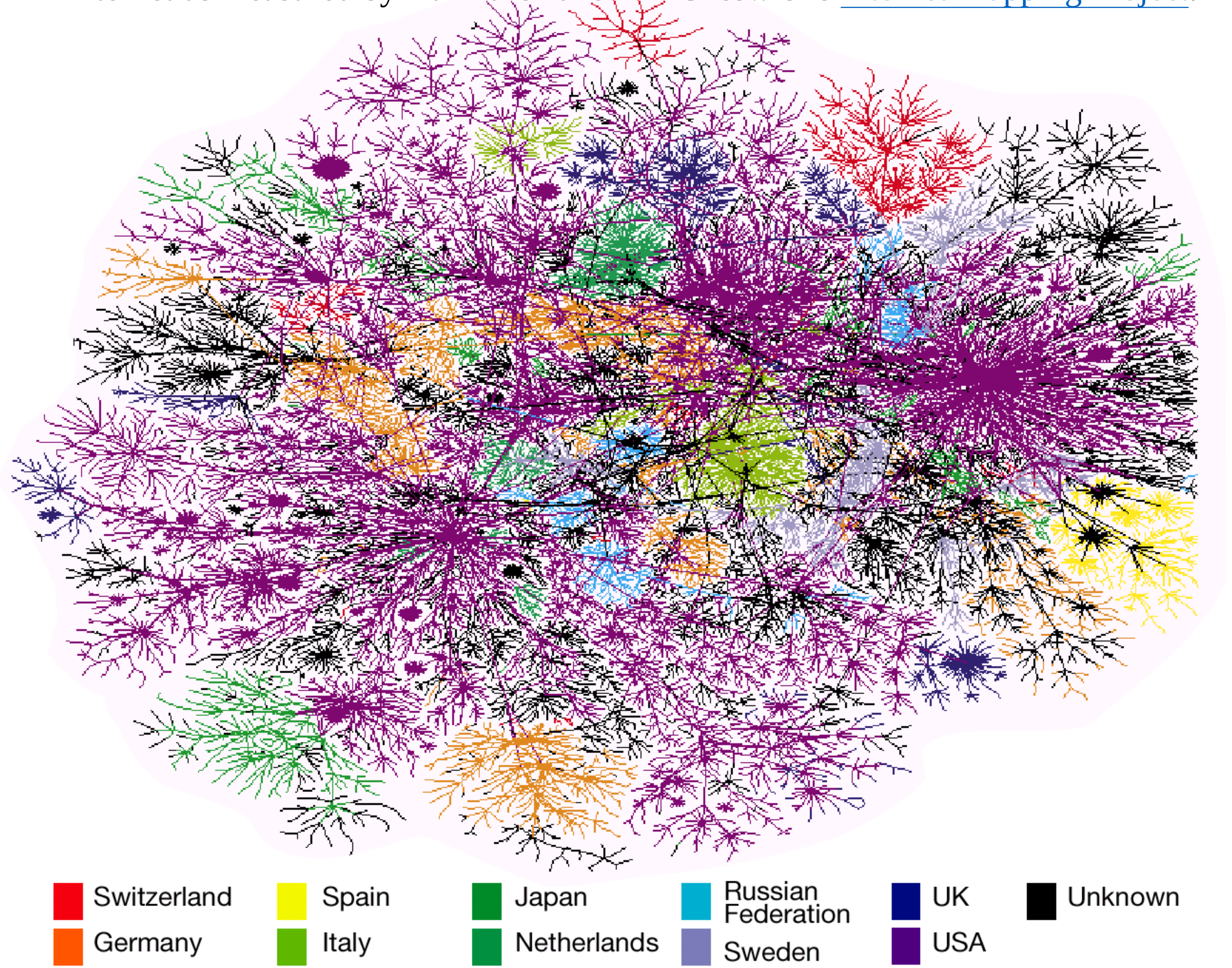
The VC Network of China. Nodes of the same color belong to the same province or district.

Technological networks

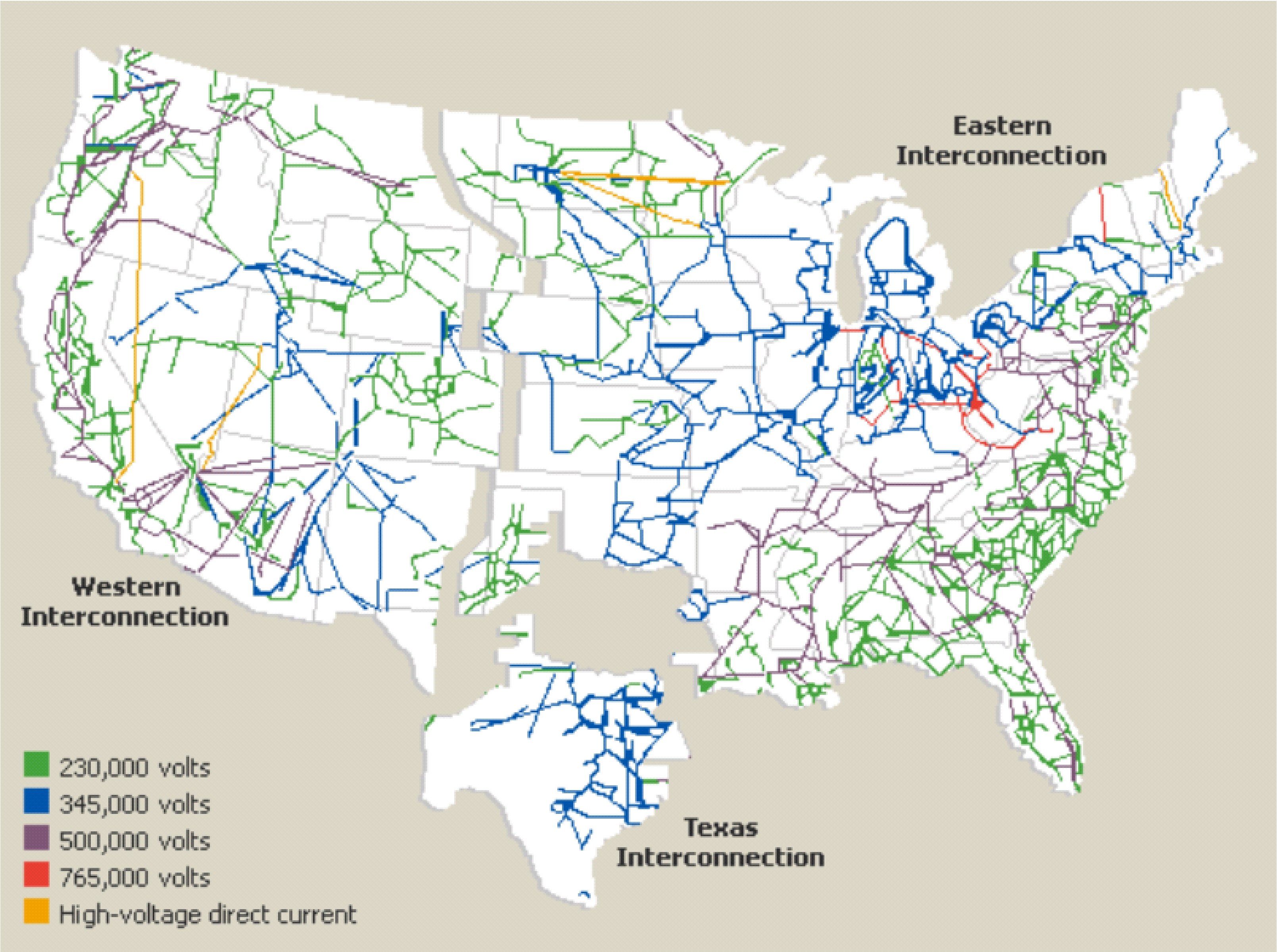
- Networks built for distribution of commodity
  - *The Internet*
    - *router level, AS level*
  - *Power Grids*
  - *Airline networks*
  - *Telephone networks*
  - *Transportation Networks*
    - *roads, railways, pedestrian traffic*



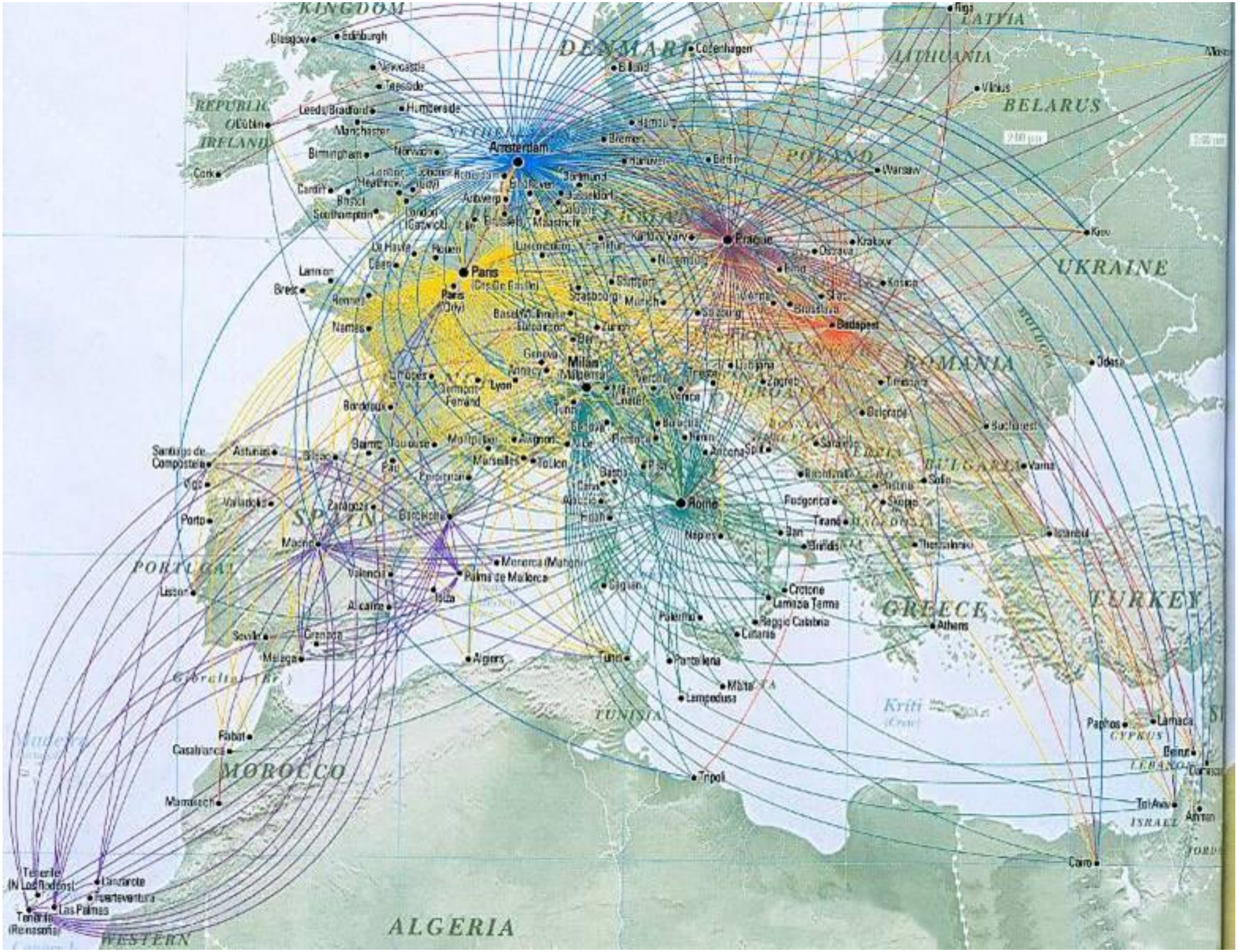
Internet as measured by Hal Burch and Bill Cheswick's Internet Mapping Project.



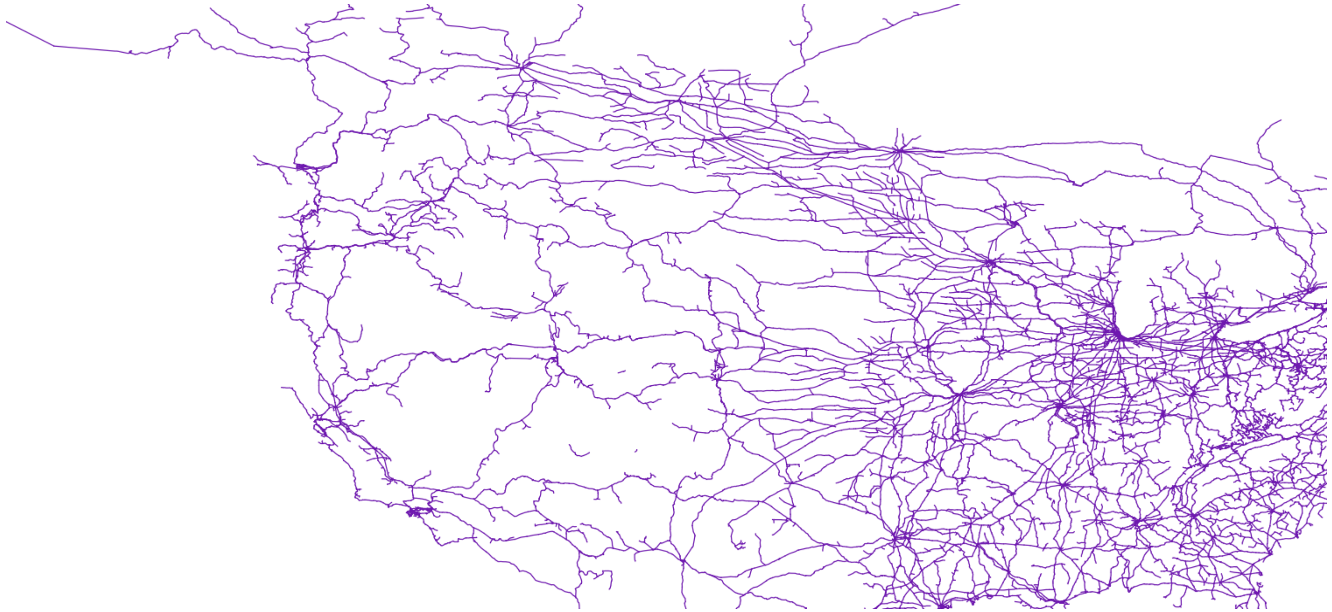
Power networks



Transportation Networks: Airlines



US railroad network



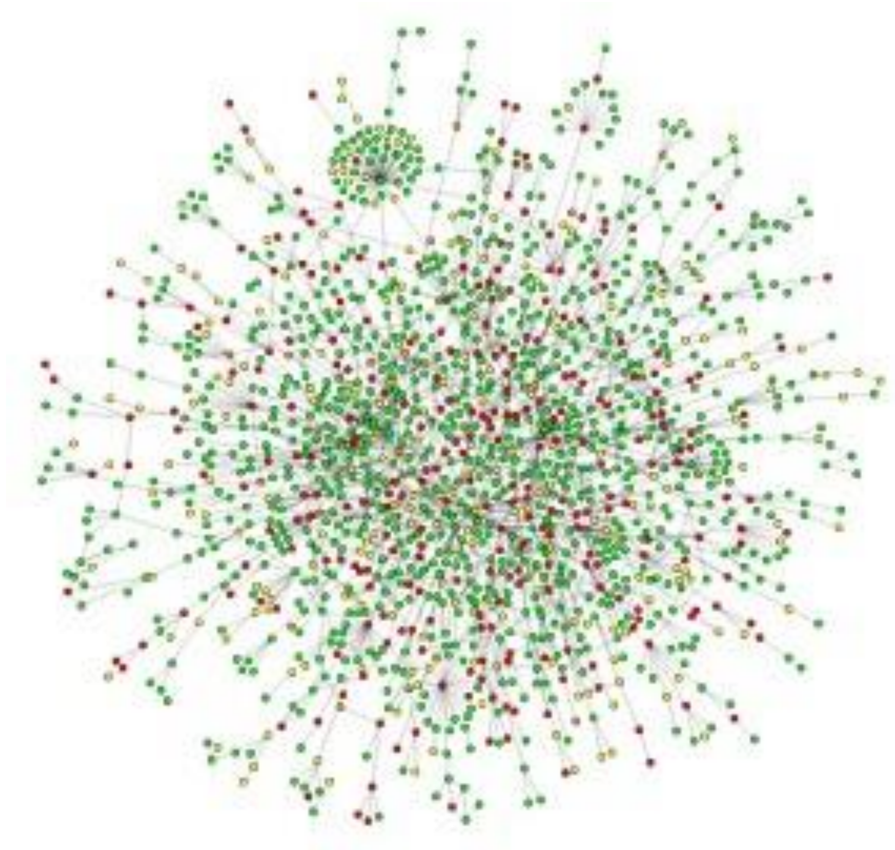
Medieval Trade Route

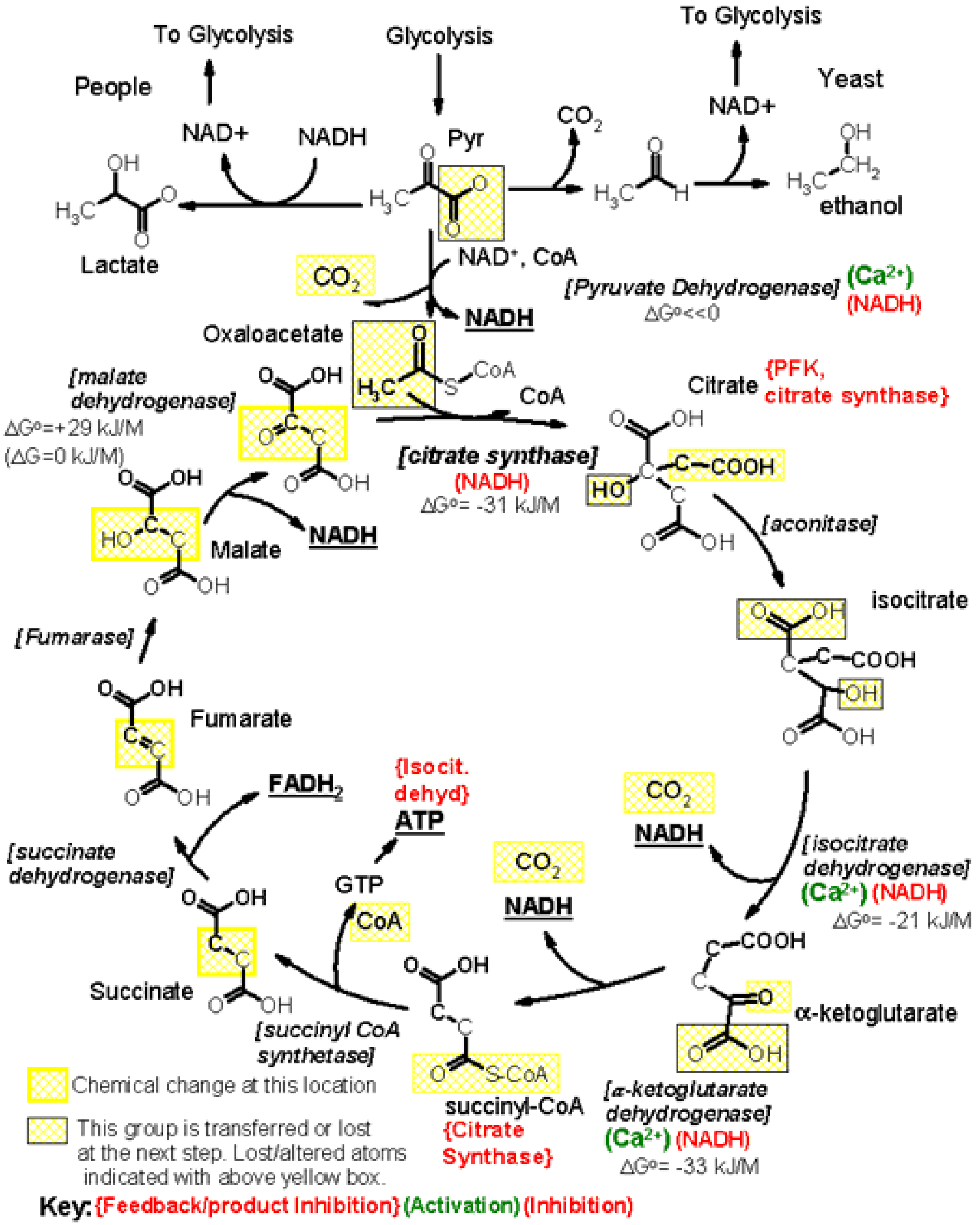
In this map of medieval trade routes, physical networks constrain the patterns of interaction, giving certain participants an intrinsic economic advantage based on their individual network positions.

(Image from http://upload.wikimedia.org/wikipedia/commons/e/e1/Late Medieval Trade Routes.jpg)

Biological networks

- Biological systems represented as networks
  - Protein-Protein Interaction Networks
  - Gene regulatory networks
  - Gene co-expression networks
  - Metabolic pathways
  - The Food Web
  - Neural Networks

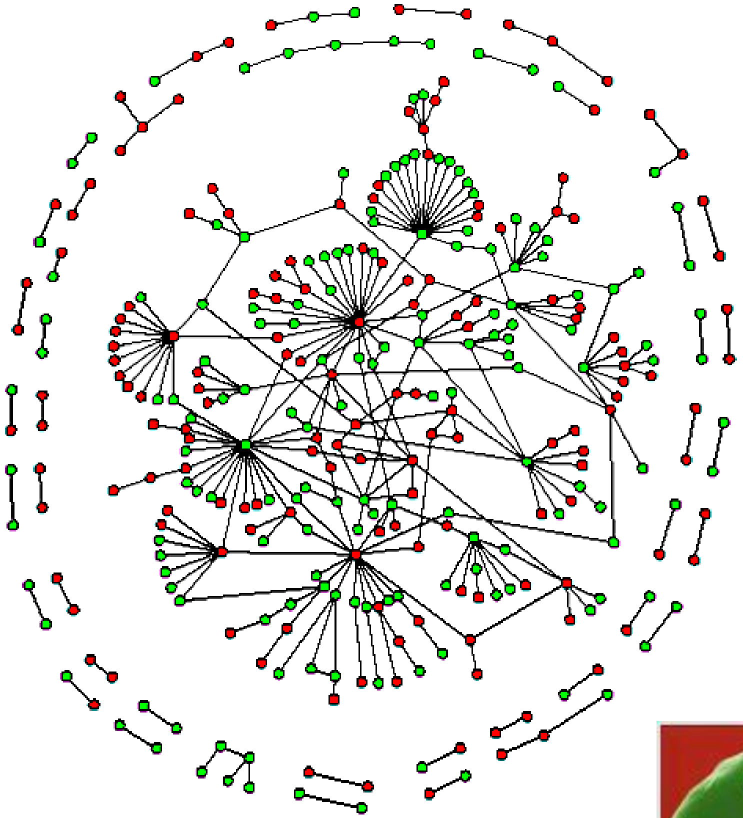




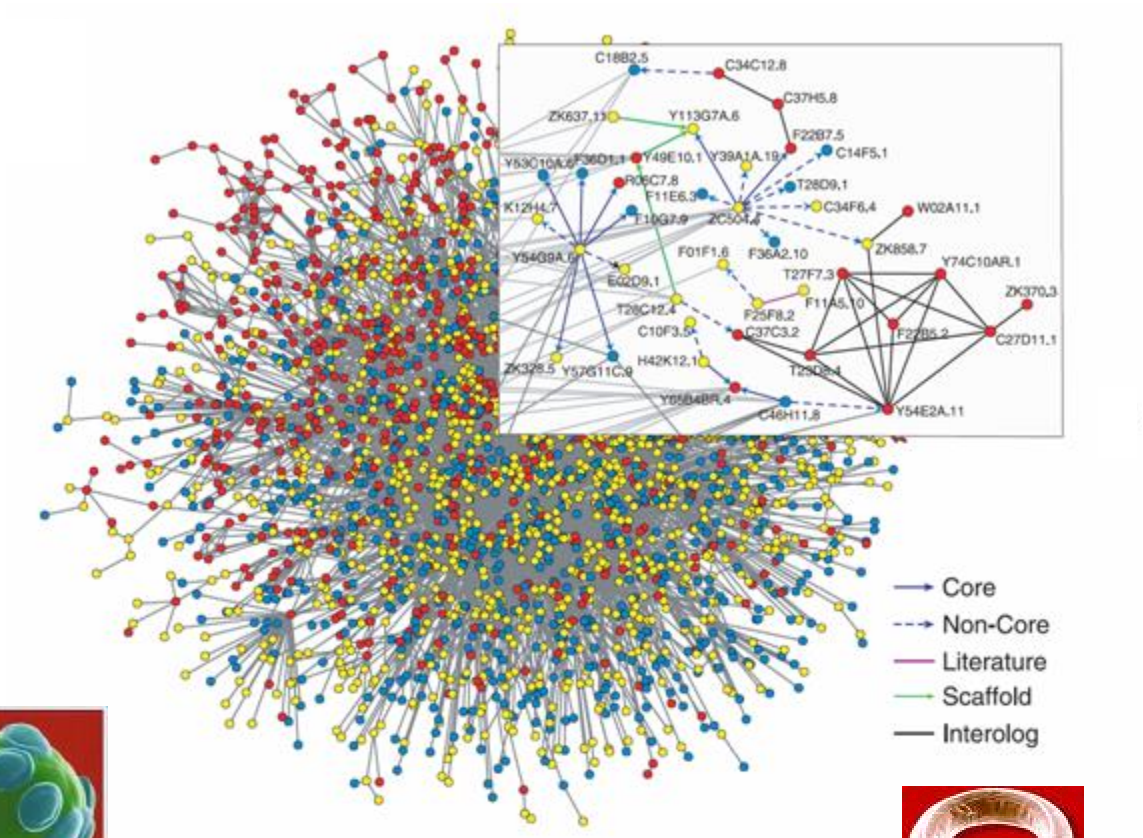
Metabolic networks

- Citric acid cycle
- Metabolites participate in chemical reactions

Protein Binding Networks

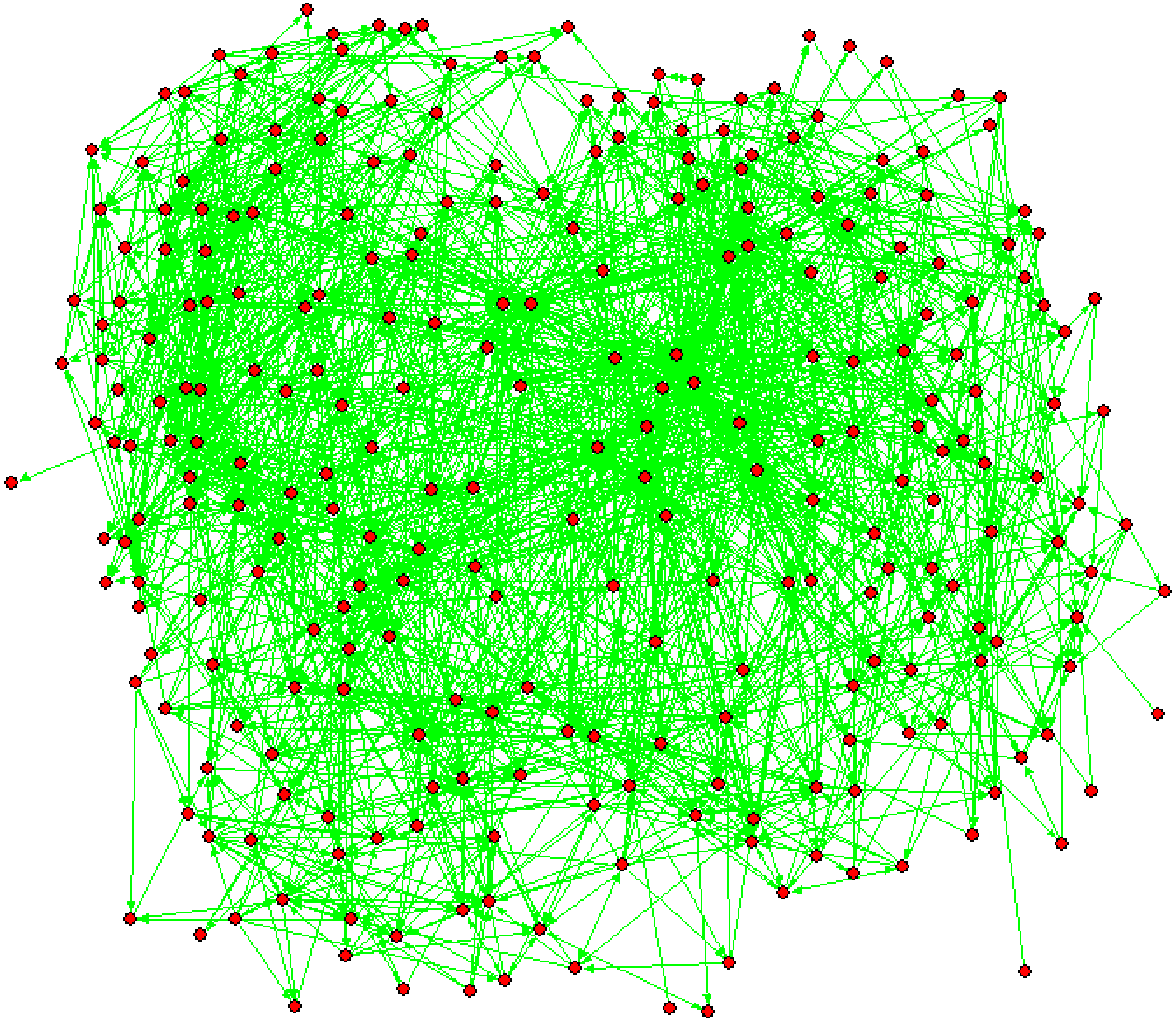


Baker's yeast *S. cerevisiae*
(only nuclear proteins shown)



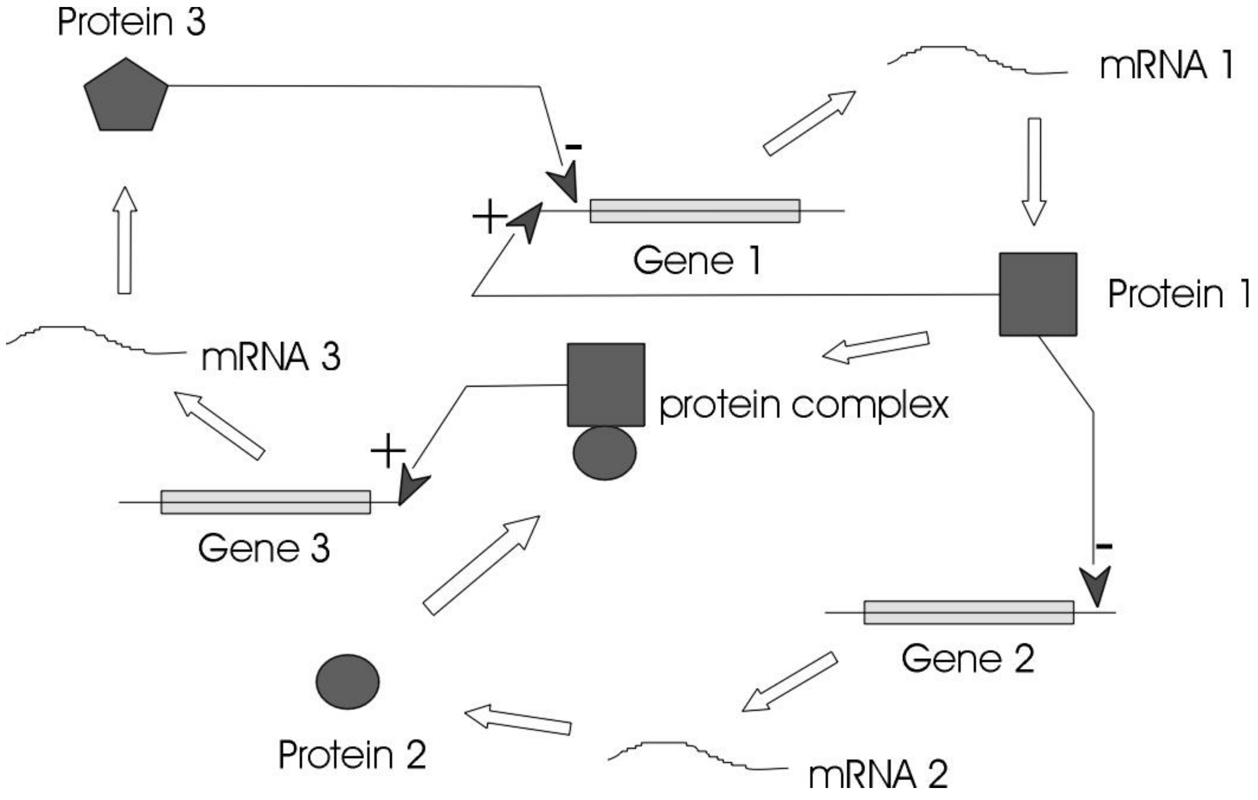
Nematode worm *C. elegans*

C. elegans neurons



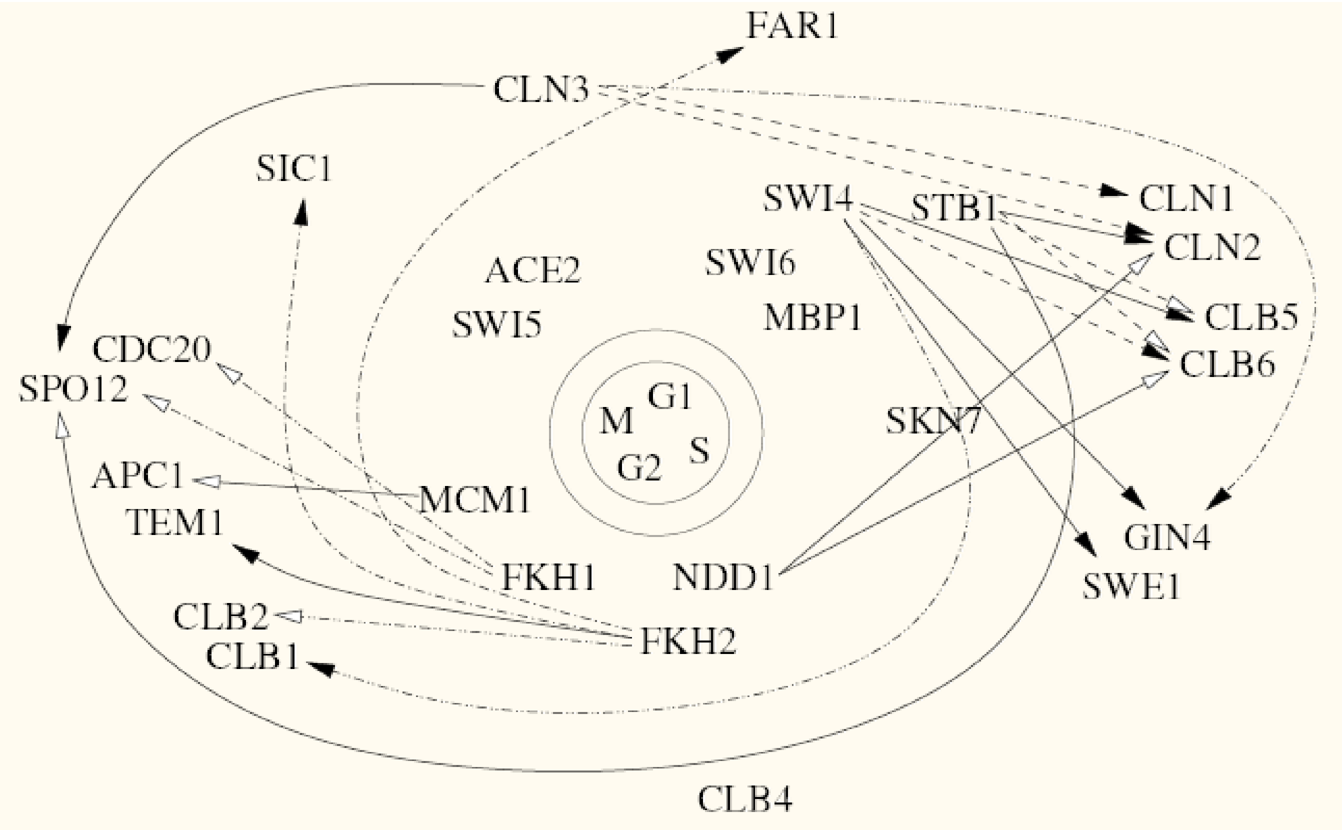
Gene Regulatory Networks

- humans have 30,000 genes
- the complexity is in the interaction of genes
- can we predict what result of the inhibition of one gene will be?



Source: http://www.zaik.uni-koeln.de/bioinformatik/regulatorynets.html.en

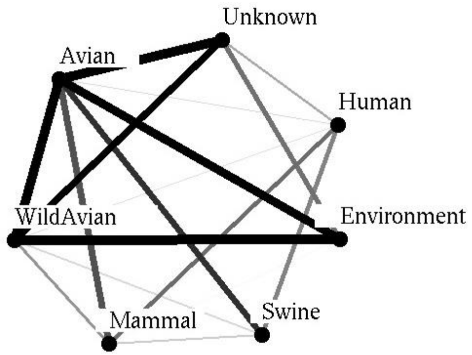
An Example of Gene Regulatory Network



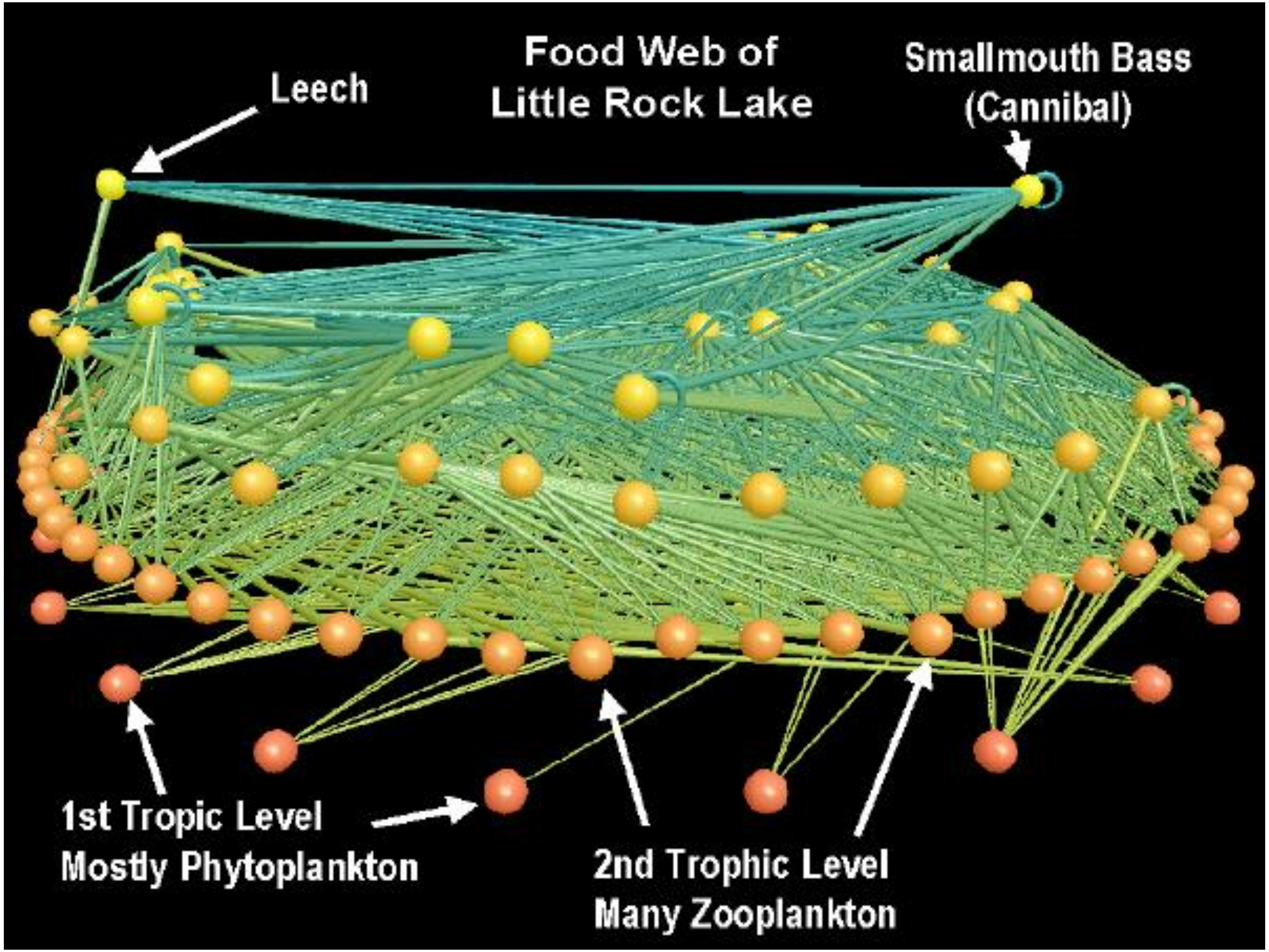
27: The reconstructed model of regulation of the yeast cell cycle dependent genes by the transcription factors. Solid, dashed, dotted, dotted dashed, and dotted dotted dashed line represents influence of regulation from time point $t, t-1, t-2, t-3$, and $t-4$, respectively. The sign of the w's are positive (negative) for solid (empty) arrow heads. See text for a discussion of the up or down regulation.

J.J. Tseng et al., Physica A350(2005)63.

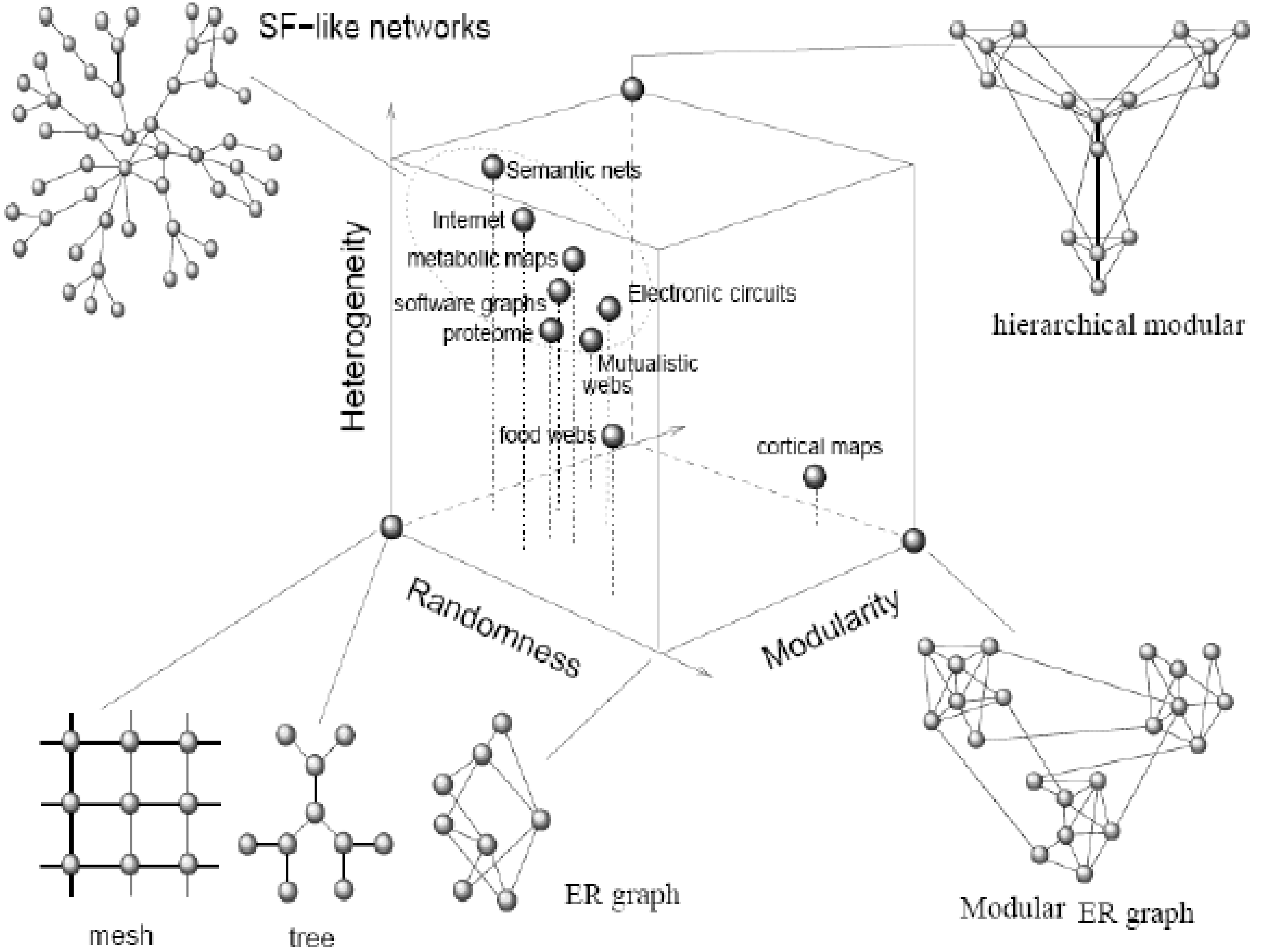
Influenza pathways



Freshwater food web by Neo Martinez and Richard Williams



Complex Networks



Types of Networks:

There exist different classifications of networks:

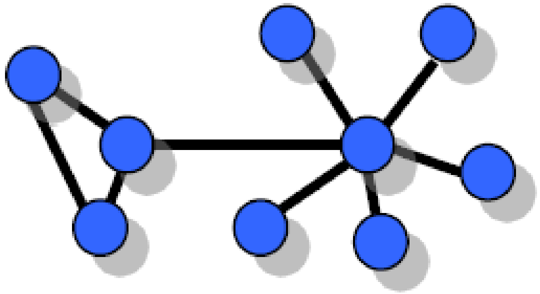
- ❑ According to the direction of the links: directed or undirected.
- ❑ According to the kind of interaction: weighted or unweighted.
- ❑ According to the differences between nodes: bipartite or not.
- ❑ According to the evolution of their topology: static or evolving.
- ❑ According to the dynamics of the nodes: with/without dynamics.
- ❑ ...

Types of Networks:

❑ Directed and undirected networks:

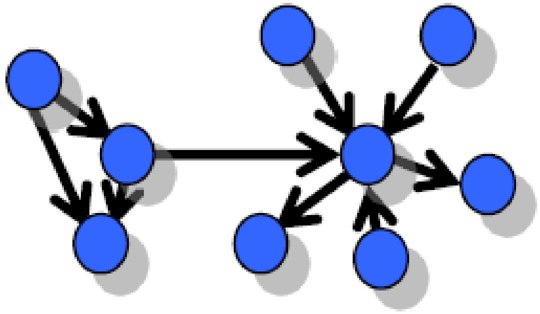
The relationship between nodes may be symmetric (undirected networks) or asymmetric (directed networks).

Undirected network



Examples: router network, power grid. collaboration networks, etc...

Directed network (digraph)



Examples: internet, food webs, e-mail/telephone networks, etc...

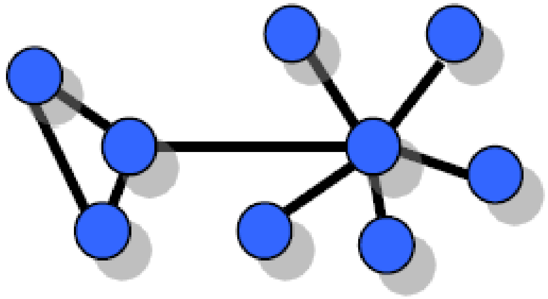
The direction of the links is crucial in dynamical processes ocurring in the network, such as information spreading, synchronization or network robustness.

Types of Networks:

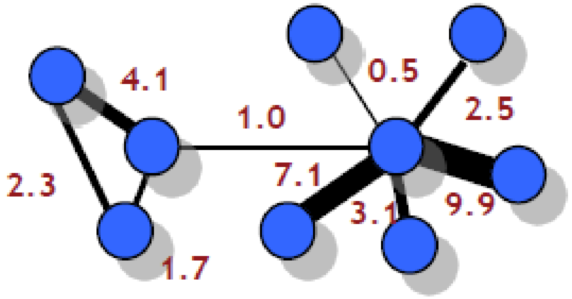
- Weighted and unweighted networks:

The capacity or intensity of the relationship between nodes may be heterogeneous (weighted networks).

Unweighted network



Weighted network



Examples: citation network, internet, etc...

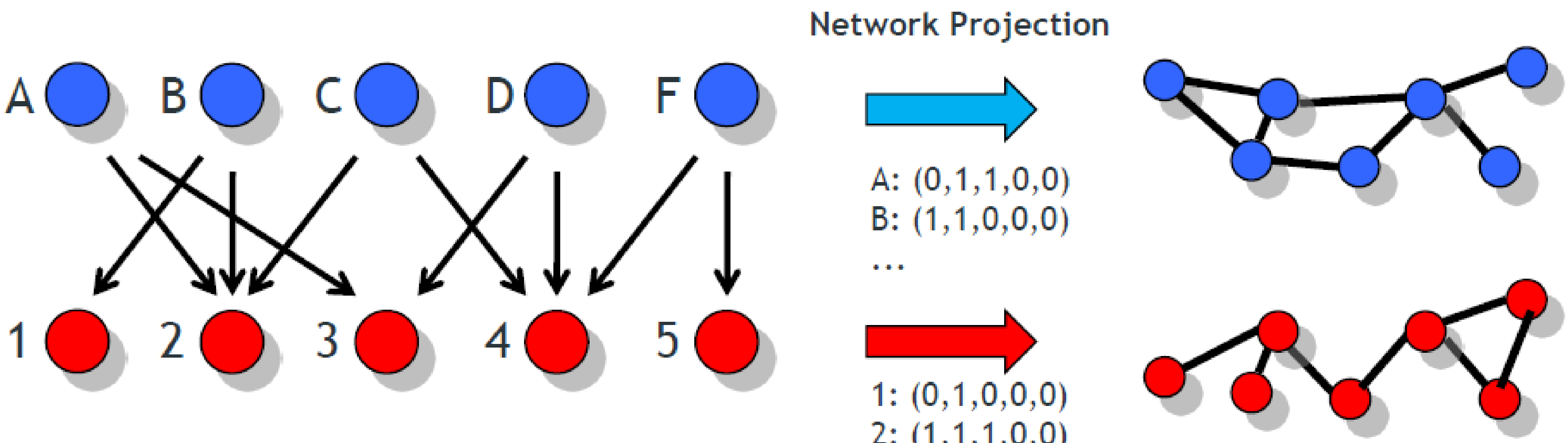
Examples: e-mail/telephone networks, food webs, power grid, colaboration network, etc...

Again, the weight of the links is crucial in dynamical processes ocurring in the network, such as information spreading, synchronization or network robustness.

Types of Networks:

- Bipartite networks:

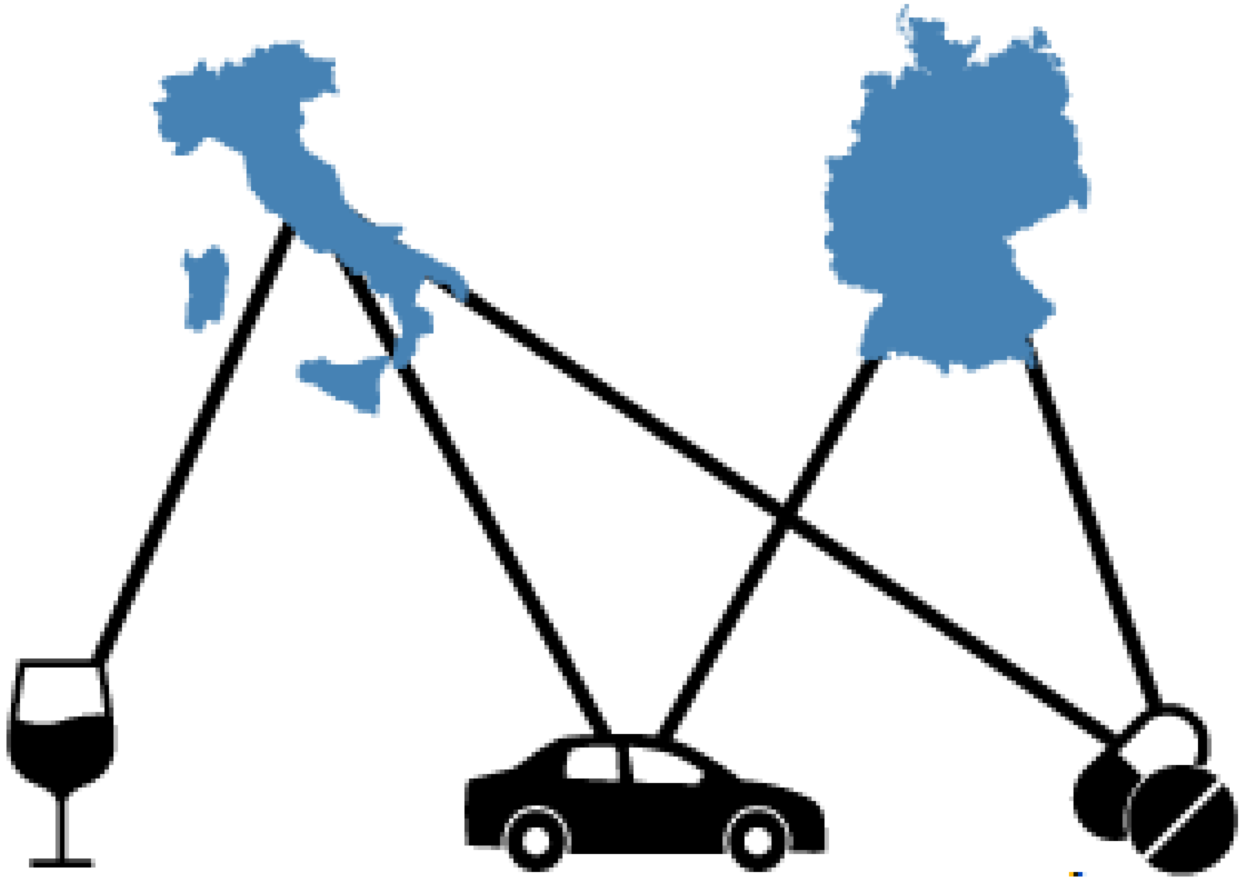
Networks with two (or more) kind of nodes and links joining ONLY nodes of unlike type.



Examples: recommendation networks, user-item based networks, etc...

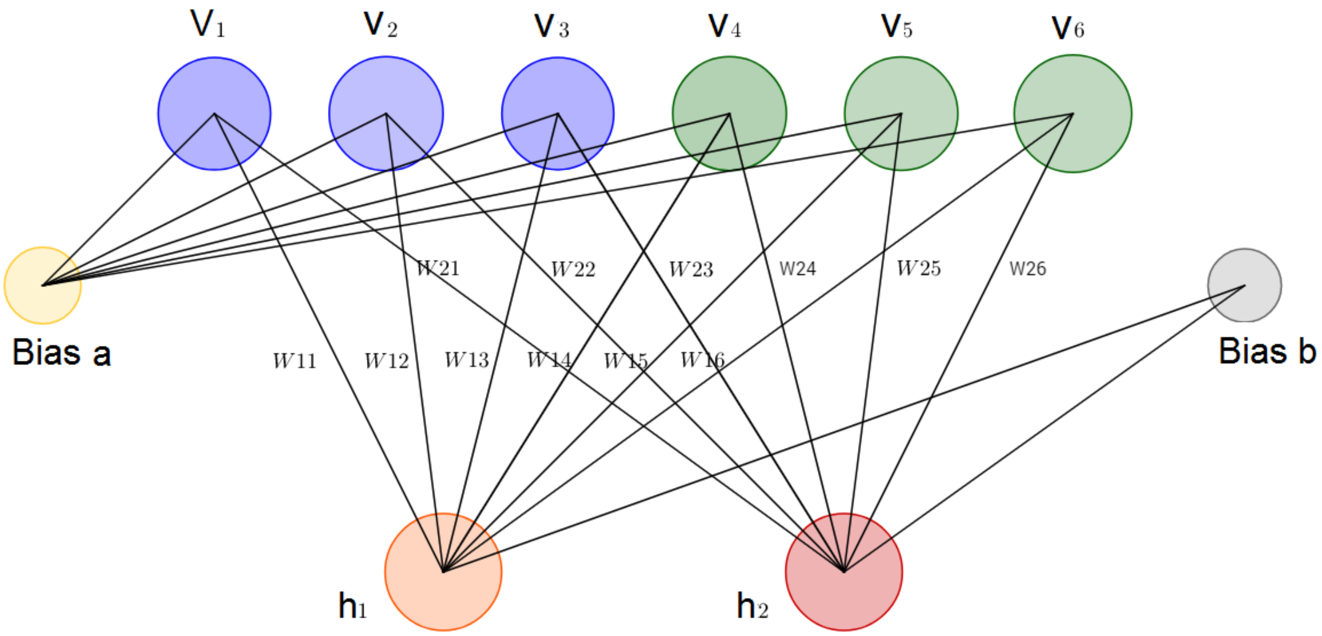
Despite being bipartite, it is possible to project the network.

Bipartite Networks



Example of a bipartite network: Illustration of a part of the country-product exportation network. Both Italy and Germany are strong exporters of cars and pharmaceutical products, whereas only Italy has a comparative advantage in wines.

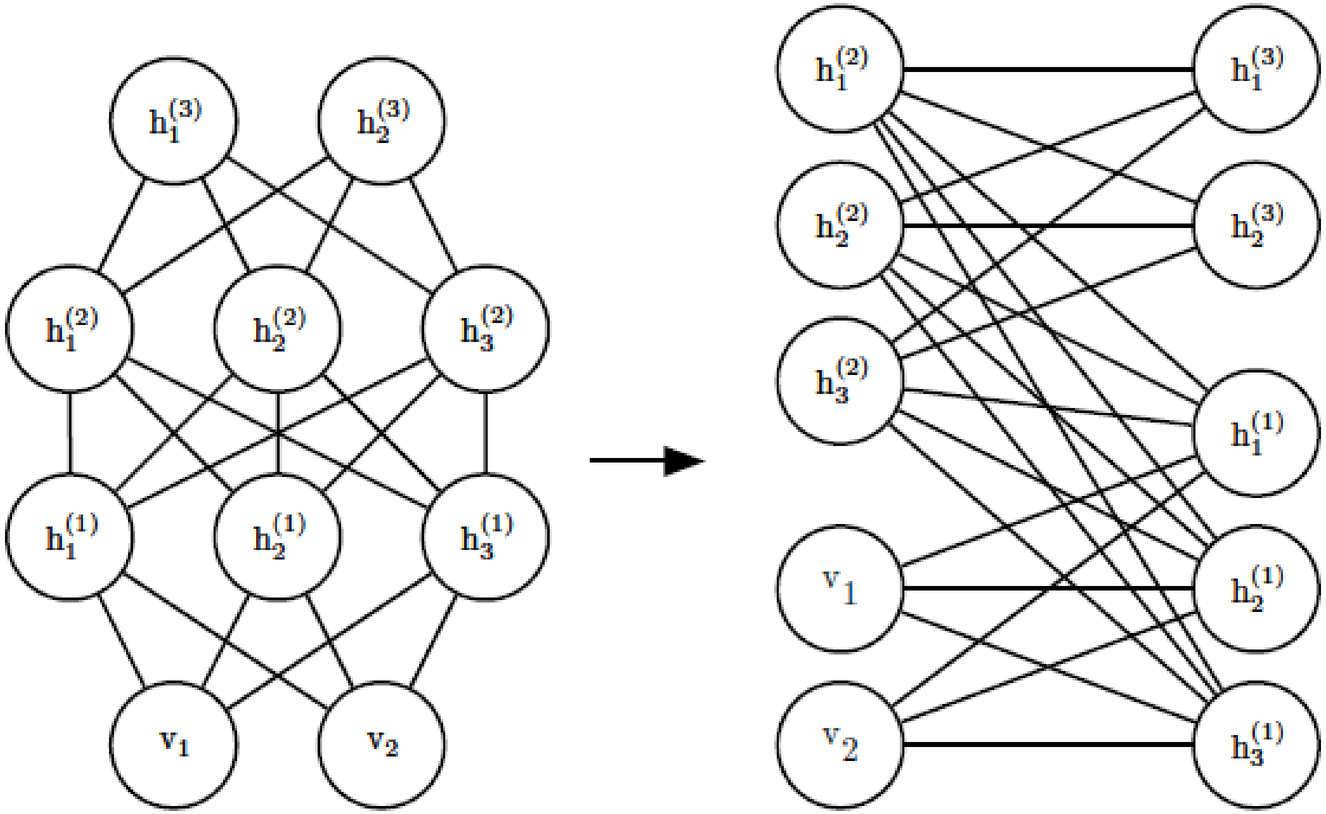
Bipartite Networks



Restricted Boltzmann Machine

Example of a bipartite network: The ***Restricted Boltzmann Machine*** is an undirected graphical model based on a bipartite graph, with visible units in one part of the graph and hidden units in the other part. There are no connections among the visible units, nor any connections among the hidden units.

Bipartite Networks

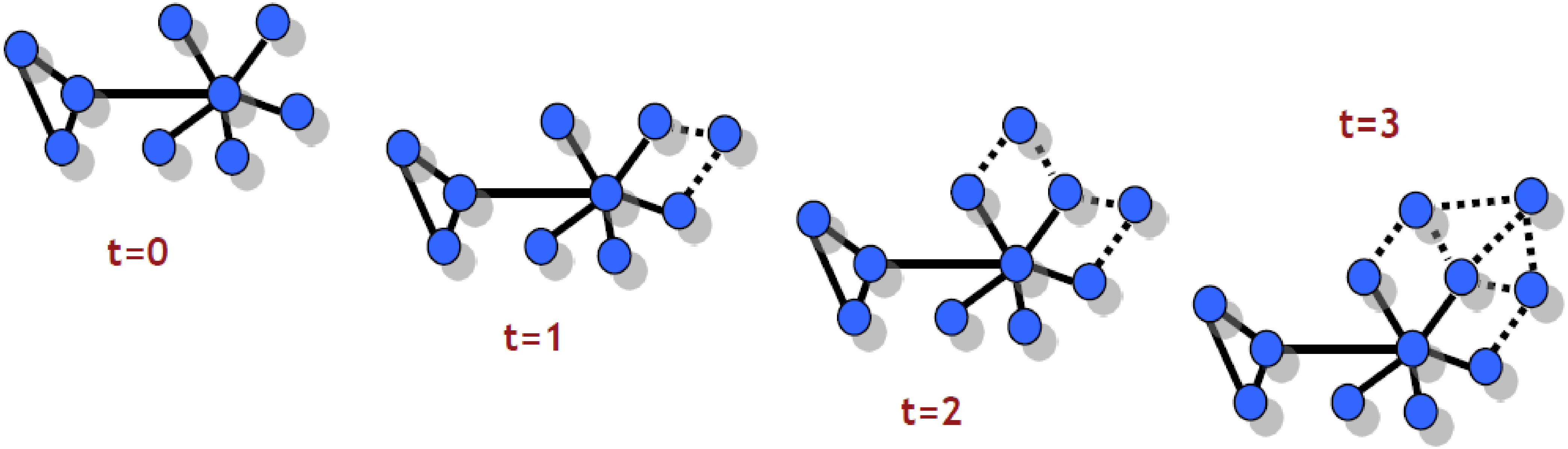


Example of a bipartite network: A ***Deep Boltzmann Machine,*** rearranged to reveal its bipartite graph structure.

Types of Networks:

❑ Static or evolving networks:

Networks do not appear suddenly. We have to know if the network that we are studying is static (its structure is stationary) or if it is still evolving

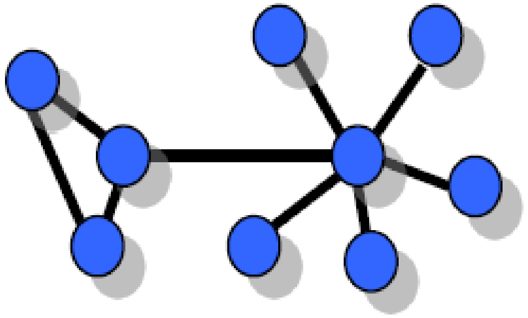


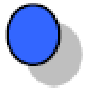
Two fundamental questions are addressed when working with evolving networks: what are the rules governing the evolution? What consequences have the rules on the final topology?

Types of Networks:

❑ Networks of dynamical systems:

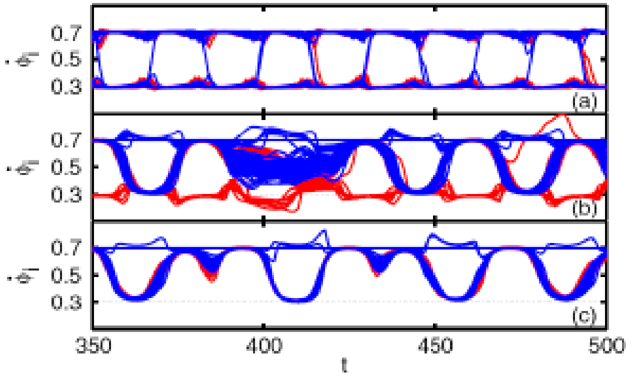
Nodes are dynamical systems whose dynamics is influenced through the matrix of connections.





Nodes are (coupled) dynamical systems
(periodic oscilators, excitable systems,
chaotic oscilators, bistable systems, ...)

$$\dot{\phi}_i = \begin{cases} \omega_i + \frac{d}{(k_i+k_{p_i})} \sum_{j=1}^{N} a_{ij} \sin(\phi_j - \phi_i) \\ + \frac{d_p k_{p_i}}{(k_i+k_{p_i})} \sin(\phi_{p_i} - \phi_i), \end{cases}$$



In this case, we have to study the influence of the topology in the dynamical processes occurring in the network (synchronization, stochastic processes, etc..) ... ... and vice-versa!

Types of Networks:

❑ Despite the different types of networks, which in turn are obtained from completely different interacting systems (people, neurons, proteins, routers,...) we will see that they share some universal properties