Quiz 1 **(due at 23:59 on 14/10/2023)**

Consider the stochastic equation of motion $\frac{d}{dt}x(t)=\xi(t)$, in which ξ is a white noise satisfying the autocorrelation $\langle \xi(t_2)\xi(t_1)\rangle = 2D\delta(t_2-t_1)$. (a) Find the mean square displacement $\left\langle [x(t)]^2 \right\rangle$ as a function of time t with the initial condition $x(0)=0$. (b) Find the probability density function of x at time t based on $\left\langle [x(t)]^2 \right\rangle$ found in (a).

Solution:

Quiz 2 **(due at 23:59 on 14/10/2023)**

An image can be *blurred* by doing a convolution between a kernel and that image. Mathematically, the convolution of f and g is defined as $(f*g)(x)=\int_{-\infty}^{\infty} f(y)g(x-y)dy$.

Let's consider f as a one-dimensional image and use $g(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{x^2}{2\sigma^2}}$ as the response function to apply a Gaussian blur. Generally speaking, fine features of a length scale smaller than σ will be suppressed and effectively removed by convolving the image f with the Gaussian function g. This can be seen by convolving $f(x)=e^{ikx}$ and comparing it with $(f*g)(x)$ to see how it is suppressed. (a) Derive the explicit expression for $(f*g)(x)$ and find the dependence of its amplitude on k. (b) Draw a sketch of this dependence on k.

Hint: To answer this question, you need the Gaussian integral

$$\int_{-\infty}^{\infty} e^{-x^2}dx=\sqrt{\pi}, \qquad \int_{-\infty}^{\infty} e^{-a(x+b)^2}dx=\sqrt{\frac{\pi}{a}}.$$

Solution: