**Problem Set 5A (due 11:59 pm, 19 November 2023)**

1. **Portfolio Management with Principal Component Analysis**

This problem should be completed with Python.

(1) Download from Yahoo!Finance the daily adjusted closing price of the following constituents of S&P 500:

AAPL, MSFT, AMZN, NVDA, GOOGL, META, GOOG, TSLA, BRK.B, UNH

Also download the daily closing price of S&P 500 (^GSPC).

The period is from 26 October 2022 to 25 October 2023. (In Yahoo!Finance you need to type the end date as 10/26/2023.) Calculate the return of the stock on day from 26 October 2022 to 25 October 2023 using the formula

where is the stock price on day

In (2), (3) and (4) below, please display the numerical values rounded to 3 decimal places, so that the matrix structure can be visualized. However, please keep the unrounded numerical values for accurate subsequent computations.

Remark: Above the print statements, you may add the Python line

with np.printoptions(precision=3):

(2) Construct and output the correlation matrix of the 10 stocks.

(3) Compute and output the eigenvalues and eigenvectors of the correlation matrix, respectively.

(4) Sort the eigenvalues and eigenvectors in descending magnitude of the eigenvalues. Output the sorted eigenvalues and eigenvectors.

Remark: The Python command np.argsort is useful. Furthermore, to sort the eigenvalues in descending order, add [::-1] at the end of the command.

Remark: When sorting the eigenvectors, please pay attention to whether you correctly sort the columns rather than the rows.

(5) Plot the eigenvalues of the eigenmodes versus the eigenmode numbers. You may find that it is difficult to determine the number of principal portfolios to be included, probably due to the small number of stocks being studied. Hence, we will use a different criterion in the next part.

(7) Compute the cumulative variance of the eigenmodes. Let be the number of the first few principal portfolios whose cumulative variance is highest below the 80% cutoff. Determine .

**Cluster Visualization**

(8) Plot the 10 stocks in the space of the 2nd and 3rd principal components. Comment on any clusters that you observe, and comment on the nature of the clusters.

**Diversification Strategies**

Below we investigate 4 diversification strategies.

1. **Stock investment with equal weight**

(9) In this strategy, the capital allocated to each of the 10 stocks is the same. Considering the stock price, compute the volume of stocks to be invested. Then compute the value of the portfolio during the period.

**(B) Risk Parity**

(10) In this strategy, the risk of stock per unit of capital is estimated by its standard deviation (note that the risk correlations of the stocks are neglected). The capital allocated to each of the 10 stocks is such that the risk of each stock is the same. Hence the capital allocated to each stock is proportional to Compute the weight of the stocks to be invested. Then compute the value of the portfolio during the period.

**(C) Equal Weight Portfolio (or 1/N Strategy)**

(11) In this strategy, the first principal portfolios are selected, and equal weight is allocated to each of them. The first step is to determine the signs of the eigenvectors to be adopted. We will first calculate the projections of the principal eigenvectors to the equal-weight portfolio, which is the portfolio allocating equal capital to the 10 stocks. Compute the projections.

(12) The sign-corrected principal eigenvectors are those principal eigenvectors that have positive projections to the equal-weight portfolio. Compute the sign-corrected principal eigenvectors. Then compute the total weight of each stock contributed from the principal portfolios.

(13) Which stocks in this strategy are in short positions? Compute the value of the portfolio during the period. Note that the initial value is no longer equal to 1 due to the short-positioned stocks. You are required to normalize the initial value to 1.

**(D) Diversified Risk Parity**

(14) In this strategy, the capital allocated to each of the principal portfolios are such that the risk of each portfolio is the same. Since the eigenvalue of each portfolio is the variance, the capital allocated to each principal portfolio is proportional to Compute the volume of stocks to be acquired. Which stocks in this strategy are in short positions? Then compute the value of the strategy during the period. You are required to normalize the initial value to 1.

**Comparison of the Strategies**

(15) Plot the daily values of the portfolios of the 4 strategies. Comment on any observations you make about the performance of the strategies.

(16) The ability of the 4 strategies to reduce risks is compared by calculating the following parameters:

Gain (the final value divided by the initial value)

Standard deviation

Minimum value (divided by the initial value)

Comment on any observations you make about the performance of the strategies.

Remark: In practice, the real test of the performance of the strategies should be done with a different data set (for example, data collected after October 2023).

(17) The standard deviation is not informative of the risk when the price has a trend. Another common risk measure is the beta coefficient, or systematic risk, defined as

Compute of the 4 strategies from: the covariance of the daily returns of the strategies and the daily returns of the market (S&P 500) averaged over the one-year period, and the variance of the daily returns of the market, also averaged over the one-year period.

Remark: Since S&P 500 indices exceed 1,000, the csv file contains 1000 separators (,). This misleads pd.read\_csv to recognize the numbers as strings instead of floats. You may need to remove the 1000 separators to convert the strings to floats.

Comment on any observations you make about .

For your further insights and interest, the names of the stocks are listed below:

|  |  |
| --- | --- |
| AAPL | [Apple Inc.](https://www.slickcharts.com/symbol/AAPL) |
| MSFT | [Microsoft Corporation](https://www.slickcharts.com/symbol/MSFT) |
| AMZN | [Amazon.com Inc.](https://www.slickcharts.com/symbol/AMZN) |
| NVDA | [NVIDIA Corporation](https://www.slickcharts.com/symbol/NVDA) |
| GOOGL | [Alphabet Inc.](https://www.slickcharts.com/symbol/GOOGL) |
| META | [Meta Platforms Inc.](https://www.slickcharts.com/symbol/FB) |
| GOOG | [Alphabet Inc.](https://www.slickcharts.com/symbol/GOOG) |
| TSLA | [Tesla Inc](https://www.slickcharts.com/symbol/TSLA) |
| BRK-B | [Berkshire Hathaway Inc. Class B](https://www.slickcharts.com/symbol/BRK.B) |
| UNH | UnitedHealth Group Incorporated |