Question 1

d)

$$E(t) = \frac{1}{2}\int_\Omega (\partial_t u)^2 + c^2 \|\nabla_x u\|^2 \, dx$$

$$= \frac{1}{2}\int_\Omega (\partial_t u)^2 + c^2 \nabla_x u \cdot \nabla_x u \, dx$$

$$E'(t) = \int_\Omega \partial_t u \partial_t u^2 + c^2 (\partial_t \nabla_x u) \nabla_x u \, dx$$

$$= \int_\Omega \partial_t u \, \partial_t u^2 + c^2 \nabla_x \partial_t u \, \nabla_x u \, dx$$

$$= \int_\Omega \partial_t u \, \partial_t u^2 - c^2 \partial_t u \, \Delta_x u \, dx$$
$$+ \int_{\partial\Omega} c^2 \cancel{\partial_t u} \, \nabla_x u \, dx$$ BDC

$$= \int_\Omega \partial_t u \, (\underbrace{\partial_t u^2 - c^2 \Delta_x u}_{0 \text{ from PDE on eq. 1}}) \, dx$$

Question 3

a) $$I^{te} = \frac{\max_{k=1}^{N} x_k^{te} - 1/2}{\max_{k=1}^{N} x_k^{te}} = 1 - \frac{1}{2 \max_{k=1}^{N} x_k^{t}}$$

$$\lim_{N \to \infty} P(I^{te} > \alpha) = \begin{cases} 1, & \alpha \leq 1/2 \\ 0, & \alpha > 1/2 \end{cases}$$

$$P(x_k^{te} < \beta) = \beta$$

$$P\left(\max_{k=1}^{N} x_k^{te} < \beta\right) = \prod_{k=1}^{N} P(x_k^{te} < \beta) = \prod_{k=1}^{N} \beta = \beta^N$$

$$P(I^{te} > \alpha) = P\left(\max x_k - \frac{1}{2} > \alpha \max x_k\right)$$

$$P\left(\max x_k - \alpha \max x_k > \frac{1}{2}\right) = P\left(\max x^k > \frac{1}{2(1-\alpha)}\right)$$

$$= 1 - \left(\frac{1}{2(1-\alpha)}\right)^N$$

b) $$|Y_k^e - E[Y_k^e]| < E/4 \iff \left|Y_k^e - \sum_{i=1}^{e} E[X_k^i]\right| < E/4$$

$$\iff |Y_k^e - e/2| < E/4$$

$$\iff \left|\sum_{i=1}^{e} X_k^i - e/2\right| < E/4$$

$$Y_k^{E \cdot t + e} < Y_k^{E \cdot (t+1) + e}$$

$$\iff \sum_{i=1}^{E \cdot t + e} X_k^i < \sum_{j=1}^{E(t+1)+e} X_k^j \iff 0 < \sum_{j=E \cdot t + e}^{E \cdot (t+1) + e}$$