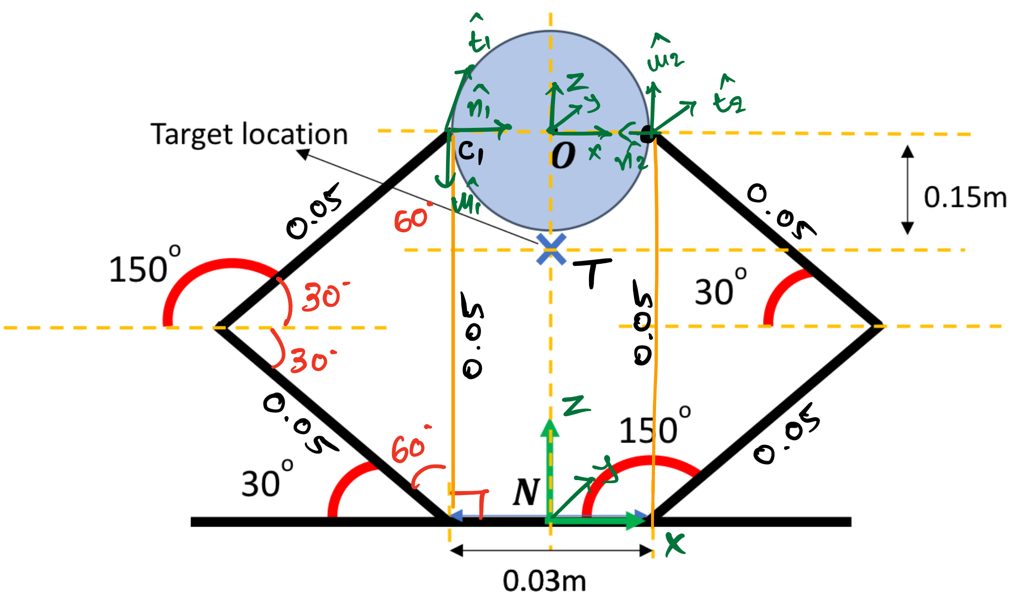


=) I have used MATLAB for solving this problem & .m file of the same is attached with this PDF.

→ From the given simplified model of the gripper, following data can be taken out.

→ $O^N = [0, 0, 0.05]^T$

$T^N = [0, 0, 0.035]^T$

$C_1^N = [-0.015, 0, 0.05]^T$

$C_2^N = [0.015, 0, 0.05]^T$

→ Considering, the maximum allowed time to reach the target,

$t = 2$ seconds.

$\therefore \quad V_O^N = (T^N - O^N) / t$

$\therefore \quad V_O^N = [0, 0, -0.0075]^T$

→ As the goal is to get the object to the target in the same orientation

$\omega_O^N = [0, 0, 0]^T$

→ Object Twist w.r.t. N,

$$\mathcal{E}_O^N = \begin{bmatrix} V_O^N \\ \omega_O^N \end{bmatrix}$$

$\therefore \quad \mathcal{E}_O^N = [0, 0, -0.0075, 0, 0, 0]^T$

→ As we know,

$$\tilde{\varepsilon}^{N}_{c_i} = P_i \cdot \tilde{\varepsilon}^{N}_{0}$$

$$\therefore \quad \tilde{\varepsilon}^{c_i}_{c_i} = \boxed{\bar{R}^{c_i}_{N} \cdot P_i} \cdot \tilde{\varepsilon}^{N}_{0}$$

Represents $\underline{G^T}$

$$G^T = \begin{bmatrix} G_1^T \\ G_2^T \end{bmatrix}$$

→ $$P_i = \begin{bmatrix} I_{3\times3} & (S[c_i^N - o^N])^T \\ O_{3\times3} & I_{3\times3} \end{bmatrix}$$

$$\therefore \quad P_1 = \begin{bmatrix} I_{3\times3} & (S[c_1^N - o^N])^T \\ O_{3\times3} & I_{3\times3} \end{bmatrix}$$

P1 =

1.0000	0	0	0	0	0
0	1.0000	0	0	0	-0.0150
0	0	1.0000	0	0.0150	0
0	0	0	1.0000	0	0
0	0	0	0	1.0000	0
0	0	0	0	0	1.0000

→ $$P_2 = \begin{bmatrix} I_{3\times3} & (S[c_2^N - o^N])^T \\ O_{3\times3} & I_{3\times3} \end{bmatrix}$$

P2 =

1.0000	0	0	0	0	0
0	1.0000	0	0	0	0.0150
0	0	1.0000	0	-0.0150	0
0	0	0	1.0000	0	0
0	0	0	0	1.0000	0
0	0	0	0	0	1.0000

$$\Rightarrow \quad \bar{R}^{C_i}_{N} = \mathrm{Inv}\left(\bar{R}^{N}_{C_i}\right)$$

$$\rightarrow \quad \bar{R}^{N}_{C_i} = \begin{bmatrix} R^{N}_{C_i} & 0 \\ 0 & R^{N}_{C_i} \end{bmatrix}$$

→ Frame C1 is 90° rotated along y-axis.

Rotation bar of contact 1 w.r.t N =

0.0000	0	1.0000	0	0	0
0	1.0000	0	0	0	0
-1.0000	0	0.0000	0	0	0
0	0	0	0.0000	0	1.0000
0	0	0	0	1.0000	0
0	0	0	-1.0000	0	0.0000

Rotation bar of N w.r.t contact 1 =

0.0000	0	-1.0000	0	0	0
0	1.0000	0	0	0	0
1.0000	0	0.0000	0	0	0
0	0	0	0.0000	0	-1.0000
0	0	0	0	1.0000	0
0	0	0	1.0000	0	0.0000

→ Frame C2 is -90° rotated along y-axis.

Rotation bar of contact 2 w.r.t N =

0.0000	0	-1.0000	0	0	0
0	1.0000	0	0	0	0
1.0000	0	0.0000	0	0	0
0	0	0	0.0000	0	-1.0000
0	0	0	0	1.0000	0
0	0	0	1.0000	0	0.0000

Rotation bar of N w.r.t contact 2 =

0.0000	0	1.0000	0	0	0
0	1.0000	0	0	0	0
-1.0000	0	0.0000	0	0	0
0	0	0	0.0000	0	1.0000
0	0	0	0	1.0000	0
0	0	0	-1.0000	0	0.0000

$\Longrightarrow$ Grasp Matrix,

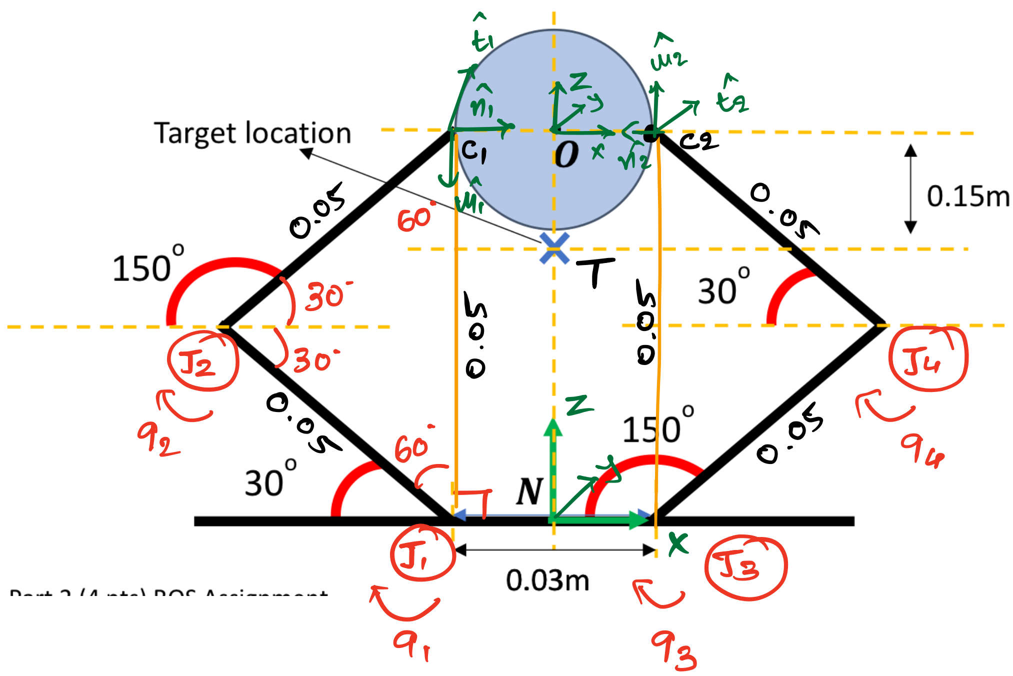
$$G^T = \begin{bmatrix} G_1^T \\ G_2^T \end{bmatrix}$$

$$\therefore \quad G^T = \begin{bmatrix} R_N^{C1} \cdot P_1 \\ R_N^{C2} \cdot P_2 \end{bmatrix}$$

Grasp Matrix =

0.0000	0	-1.0000	0	-0.0150	0
0	1.0000	0	0	0	-0.0150
1.0000	0	0.0000	0	0.0000	0
0	0	0	0.0000	0	-1.0000
0	0	0	0	1.0000	0
0	0	0	1.0000	0	0.0000
0.0000	0	1.0000	0	-0.0150	0
0	1.0000	0	0	0	0.0150
-1.0000	0	0.0000	0	-0.0000	0
0	0	0	0.0000	0	1.0000
0	0	0	0	1.0000	0
0	0	0	-1.0000	0	0.0000

(Calculated Grasp Matrix)



⇒ As we know,

$$\varepsilon^{N}_{C_i, hand} = J_i \cdot \dot{q} \quad \& \quad \varepsilon^{C_i}_{C_i, hand} = \underbrace{\bar{R}^{C_i}_{N} \cdot J_i}_{J_{hi}} \cdot \dot{q}$$

where, J_h = Block diagram (J_{h1}, J_{h2})

$$J_{h1} = \begin{bmatrix} v_1 & v_2 \\ w_1 & w_2 \end{bmatrix} \quad \&$$

$$J_{h2} = \begin{bmatrix} v_3 & v_4 \\ w_3 & w_4 \end{bmatrix}$$

⇒ Now, for a revolute Joint,

→ $J_{\omega i} = z_{i-1}$ ⇒ $J_{\omega i} = \omega_i$

→ $J_{vi} = z_{i-1} \times (O_n^N - O_{i-1}^N)$

$\therefore J_{vi} = \omega_i \times r_i$

$\therefore J_{vi} = S(\omega_i) \cdot r_i$

⇒ Jacobian for contact point 1,

Jacobian Matrix for Contact Point 1 =

0.0500	0.0250
0	0
0	-0.0433
0	0
1.0000	1.0000
0	0

⇒ Jacobian for contact point 2.

Jacobian Matrix for Contact Point 2 =

0.0500	0.0250
0	0
0	0.0433
0	0
1.0000	1.0000
0	0

⇒ Hand - Jacobian

Hand Jacobian =

0.0000	0.0433	0	0
0	0	0	0
0.0500	0.0250	0	0
0	0	0	0
1.0000	1.0000	0	0
0	0	0	0
0	0	0.0000	0.0433
0	0	0	0
0	0	-0.0500	-0.0250
0	0	0	0
0	0	1.0000	1.0000
0	0	0	0

⟹ Assuming firm contacts between the gripper & the object ;

$$\varepsilon^{c_i}_{c_i, obj} = \varepsilon^{c_i}_{c_i, hand}$$

$$\therefore \quad G^T \cdot \varepsilon^N_0 = J_h \cdot \dot{q}$$

$$\therefore \quad \varepsilon^N_0 = \underbrace{(G^T)^+ \cdot J_h}_{J_{hand-object}} \cdot \dot{q}$$

⟹ Hence, Hand - object Jacobian,

Hand Object Jacobian =

0.0250	0.0125	0.0250	0.0125
0	0	0	0
0.0000	-0.0217	0.0000	0.0217
0	0	0	0
0.4999	0.4996	0.4999	0.4996
0	0	0	0

⟹ Now, $\dot{q} = (J_h)^{-1} \cdot G^T \cdot \varepsilon^N_0$

Joint Angles =

-0.1298
0.1300
0.1298
-0.1300

Hence,

$q_1 = -0.1298$

$q_2 = 0.13$

$q_3 = 0.1298$

$q_4 = -0.13$