~1

$$f(x) = \sin\left(\frac{\pi}{3}+x\right)$$

$$f'(x) = \cos\left(\frac{\pi}{3}+x\right)$$

$$f''(x) = -\sin\left(\frac{\pi}{3}+x\right)$$

$$f'''(x) = -\cos\left(\frac{\pi}{3}+x\right)$$

$$f^{IV}(x) = \sin\left(\frac{\pi}{3}+x\right)$$

$\dots$

$$f^{(n)}(x) = \sin\left(\frac{\pi}{3}+x+\frac{\pi n}{2}\right)$$

~2

$\sin(x)$ - беск. дифф. $\in C^{\infty}$

$$P_n(x,0) = f(0) + \frac{f'(0)\cdot x}{1!} + \frac{f''(0)\cdot x^2}{2!} + \dots + \frac{f^{(n)}x^n}{n!} = $$

$$\triangleq \frac{\sqrt{3}}{2} + \frac{x}{2} - \frac{\sqrt{3}x^2}{4} + \dots + \frac{\sin\left(\frac{\pi}{3}+\frac{\pi n}{2}\right)x^n}{n!} + \bar{o}(x^n)$$

$$= \sin\left(\frac{\pi}{3}+0\right) + \frac{\cos\left(\frac{\pi}{3}+0\right)x}{1!} + \frac{-\sin\left(\frac{\pi}{3}+0\right)x^2}{2!} + \dots +$$

$$+ \frac{\sin\left(\frac{\pi}{3}+x+\frac{\pi n}{2}\right)x^n}{n!} \triangleq$$

~3

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \bar{o}(x^7)$$

$$x \to x + \frac{\pi}{3}$$

$$\sin\left(x+\frac{\pi}{3}\right) = \sin x \cdot \cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cdot\cos x =$$

$$= \frac{1}{2}\cdot\sin x + \frac{\sqrt{3}}{2}\cdot\cos x =$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \bar{o}(x^7)$$

$$= \frac{1}{2}\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \bar{o}(x^7)\right) + \frac{\sqrt{3}}{2}\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \bar{o}(x^7) + \ldots\right)$$

начнем чередовать члены, полученные от стандартного разложения косинуса и синуса

$$= \frac{\sqrt{3}}{2} + \frac{x}{2} - \frac{\sqrt{3}}{4}x^2 + \ldots + (1) + \bar{o}(x^n)$$

при $n = \cancel{4k+1}\ 1 \bmod 4$

$$(1) = \frac{1}{2} \cdot \frac{x^n}{n!} = \frac{\cancel{\sin}\cos\frac{\pi}{3} \cdot x^n}{n!} = \sin\left(\frac{\pi}{3} + \frac{\pi}{2}\right) \cdot \frac{x^n}{n!}$$

при $n = \cancel{2k}\ 2 \bmod 4$

$$(1) = -\frac{\sqrt{3}}{2} \cdot \frac{x^n}{n!} = \sin\left(\frac{\pi}{3} + \pi\right) \cdot \frac{x^n}{n!}$$

при $n = 3 \bmod 4$

$$(1) = -\frac{1}{2} \cdot \frac{x^n}{n!} = \sin\left(\frac{\pi}{3} + \frac{3}{2}\pi\right) \cdot \frac{x^n}{n!}$$

при $n = 0 \bmod 4$

$$(1) = \frac{\sqrt{3}}{2} \cdot \frac{x^n}{n!} = \sin\left(\frac{\pi}{3} + 2\pi\right) \cdot \frac{x^n}{n!}$$

т.е. $(1) = \sin\left(\frac{\pi}{3} \cancel{+\frac{\pi}{2}} + \frac{\pi n}{2}\right) \cdot \frac{x^n}{n!} \Rightarrow$ совп~~ало~~ ало! ура!

$$\sin(\pi/3+x) = \tilde{P}_4(x_0, 0) + R_n(x)$$

в форме Лагранжа:

$$R_n(x) = \frac{\sin^{(n+1)}\left(\frac{\pi}{3}+\xi\right)}{(n+1)!} \cdot x^{n+1}$$

$$\left|\sin^{(n+1)}\left(\frac{\pi}{3}+\xi\right)\right| \leq 1$$

$$|R_n| \leqslant \left.\frac{x^{n+1}}{(n+1)!}\right|_{x=0,2} = \left(\frac{2}{10}\right)^{n+1} \cdot \frac{1}{(n+1)!} < \delta$$

$$\delta_1 = 10^{-3}$$

$$\frac{2^{n+1}}{10^{n+1}(n+1)!} < 10^{-3}$$

$\measuredangle$ $n=2$

$$\frac{2^3}{10^3 \cdot 3!} \vee \frac{1}{10^3} \qquad \frac{2^3}{3!} \vee 1$$

$$\frac{8}{6} > 1$$

$n=3$

$$\frac{2^4}{10^4 \cdot 4!} \vee \frac{1}{10^3} \qquad \frac{2^4}{4!} \vee 10 \qquad \frac{2}{3} < 10 \checkmark \Rightarrow$$

$$n_T = 3$$

$\delta_2 = 10^{-6}$

$n = 4$

$$\frac{2^5}{10^5 \cdot 5!} \vee \frac{1}{10^6}$$

$$\frac{4}{15} \vee \frac{1}{10}$$

$$\frac{8}{30} > \frac{3}{30}$$

$n = 5$

$$\frac{2^6}{10^6 \cdot 6!} \vee \frac{1}{10^6}$$

$$\frac{4}{45} \vee 1$$

$$\frac{4}{45} < 1 \quad \checkmark \Rightarrow n_2 = 5$$