

We have the differential equation:

$$\frac{\delta}{\delta t}U = D \frac{\delta^2}{\delta x^2} - v \frac{\delta}{\delta x}.$$

We can use the first and second order central difference for x , which gives us:

$$\frac{\delta}{\delta t}U = D \frac{U_{m-1} - 2U_m + U_{m+1}}{(\Delta x)^2} - v \frac{U_{m+1} - U_{m-1}}{2\Delta x}$$

And using forward Euler for time gives:

$$\frac{U_m^{n+1} - U_m^n}{\Delta t} = D \frac{U_{m-1} - 2U_m + U_{m+1}}{(\Delta x)^2} - v \frac{U_{m+1} - U_{m-1}}{2\Delta x}$$

Now we let

$$C = \frac{D\Delta t}{(\Delta x)^2}$$
$$B = \frac{v\Delta t}{2\Delta x},$$

which gives us:

$$U_m^{n+1} = U_m^n + C(U_{m-1}^n - 2U_m^n + U_{m+1}^n) - B(U_{m+1}^n - U_{m-1}^n).$$

Which we can write as

$$U_m^{n+1} = (1 - 2C)U_m^n + (C + B)U_{m-1}^n + (C - B)U_{m+1}^n$$

Which gives us the steps to take in our numerical solution.