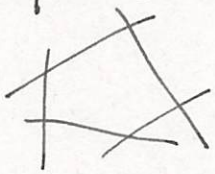
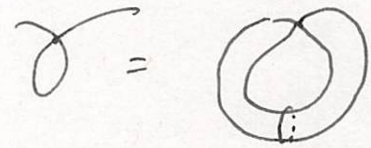
C : a cycle of rational curves.

(reduced singular cpx curve with only nodes,

s.t. $\begin{cases} \text{dual graph} = \text{a cycle graph.} \\ \text{comp. of } \tilde{C} \cong \mathbb{P}^1. \ (i : \tilde{C} \to C : \text{normalizat'n}) \end{cases}$)





$C \subset \underset{\sim}{S}$ (hol'ly non-sing. surf.) s.t. $\underset{\sim}{N_{C/S}}$: top. triv

$N_{C/S} := [C]|_C$; normal bdl.

<u>Fact</u> $\mathrm{Pic}^0(C) \cong \mathbb{C}^*$.

$L \longmapsto t(L)$. //

<u>Thm 1</u> $C \subset S$

$\quad$ ॥

$C' \subset S'$: as above.

<u>Assume</u> $t(N_{C/S}) = t(N_{C'/S'}) = e^{2\pi\sqrt{-1}\theta}$ for $^\exists\theta \in \mathbb{R} \setminus \mathbb{Q}$: Diophantine irrat'l number.

$^\exists\alpha > 0, ^\exists c > 0$ s.t. $\mathrm{dist}(n\cdot\theta, \mathbb{Z}) \geq c \cdot \frac{1}{n^\alpha}$ for $^\forall n \in \mathbb{Z}_{>0}$

<u>Then</u> $^\exists V$: nbhd of C in S

$^\exists V'$: $\;$ " $\;$ C' in S' s.t. $\begin{matrix} V \cong V' \\ \cup \quad \cup \\ C = C' \end{matrix}$ //

c.f.:

Thm (Arnol'd '76)

C: non-sing. ellipt. curve $\subset \underset{\text{non-sing surf.}}{S}$

with $N_{C/S} \in \mathrm{Pic}^0(C) \cong \mathbb{C}$: "Dioph."

$\Rightarrow \exists V$: nbhd of C in S,

$\exists V'$: nbhd of the zero-sectn of $N_{C/S}$

s.t.

$$\begin{array}{ccc} V & \cong & V' \\ \cup & & \cup \\ C & \underset{\text{natural}}{=} & \text{zero-sectn} \end{array}$$

"C admits a holomorphic tub. nbhd"

~~history~~:

Application

Thm 2 $\forall n = 1, 2, 3,$

$\exists X$: K3 surf,

$\exists f : \mathbb{C}^* \to X$: injective hol. immersion.

s.t.

$$\begin{cases} \overline{f(\mathbb{C}^*)}^{\mathrm{Euc}} =: H \subset X : \text{real codim} = 1. \\ \qquad \text{sm. hyp.surf, cpt. Levi-flat.} \\ \overline{f(\mathbb{C}^*)}^{\mathrm{Zar}} = X. \end{cases}$$

$H \approx \left(S^1 \times S^1\text{-bdl} / S^1 \text{ with holony } \begin{array}{c} g_n : S^1 \times S^1 \to S^1 \times S^1 \\ (p, q) \mapsto (p \cdot q^n, q) \end{array} \right)$

$H_2(H, \mathbb{Z}) = \begin{cases} \mathbb{Z} & n = 3 \\ \mathbb{Z}^2 & 2 \\ \mathbb{Z}^2 \oplus \mathbb{Z}/n\mathbb{Z} & 1 \\ \mathbb{Z} & 0 \end{cases}$

Schedule

§1. Historical comments, ~~Outline of the prf, Exam~~ples.

§2. ~~"Nice" defining functions system of C in S.~~ ~~local~~ Examples.

§3. prf of Thm 1.

(§4. prf of Thm 2)

'62. Grauert.

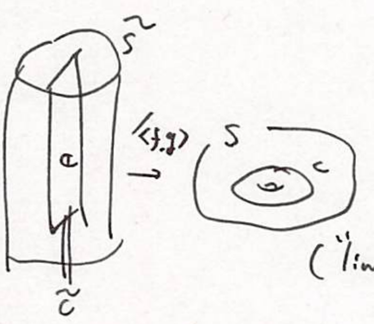
"$C \subset S$, $(C^2) < \min\{4-4g(C), 0\}$

$\Rightarrow$ C admits a hol. tub. nbhd"

§1

Historical comments

'76 Arnold's thm. $\longleftarrow$ motivated by? Siegel's linearization thm. '42.



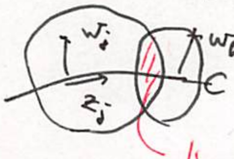
("linearization of f and g!")

'83. Ueda .... a generalization of Arnold's thm.

- Define obstruction classes. u_n.
- 「$u_n = 0$ for $\forall n$」 + Dioph. cond. for $N_{C/S}$

$\Rightarrow$ "Nice" local defining functions system of C in S.

cpt Kä, codim 1!



linearize $W_k = W_k(w_j; z_j)$

'91 Ueda ... nbhd of a rat'l curve with a node, C,

s.t. $t(N_{C/S}) \notin U(1)$
$:= \{t \mid |t| = 1\}$

'15. (Indiana U. Math. '17) … K-, $\begin{cases} C: \text{a cycle of a rat'l curves, (for e.g.)} \\ t(N_{C/S}) \in U(1) \end{cases}$

なるときの、[Ueda '83] のアナロジー.

($\exists$ "Nice" local defining functions sys. of C. if $t(N_{C/S})$: Dioph.)

$$\begin{tikzcd}[Arnol'd '76] \arrow[r, "\text{Singular}"', "\text{analogue}"] \arrow[d, "\text{analogue/generalize}"] & \text{Thm 1} \\ [Ueda '83] \arrow[r, "\text{Singular analogue}"'] & [K-'17] \text{ for a cycle of rat'l curves} \arrow[u, "\text{improve}"']\end{tikzcd}$$

<u>for Thm 2</u> …

略?

[K-'17]: "gluing construction" of K3 surfaces by using Arnol'd thm.

some improvements

↓

[K-, Uehara] … to appear.

in this case, $H = \overline{f(C^*)}^{\mathrm{Euc}} \approx S^1 \times S^1 \times S^1$ //.

Thm 2; "singular analogue".

Cf ArXiv/1703.03663

$\boxed{\text{Recipe.}}$

projective surface

@ $C \subset S$, $C' \subset S'$ … as in E.g. 3 below.

@ $M := S \setminus$ ("nice" nbhd of C)

$M' := S' \setminus$ (—"— C')

@ $X := M \cup M'$

glue nbhds of ∂M and $\partial M'$ hol'ly. //

§2. Examples

E.g. 3

$C_0 \subset \mathbb{P}^2$: curve of deg $=3$, at most nodal sing.

$\cup$

$Z := \{P_1, \cdots, P_9\}$: nine pts.

($P_j \neq$ nodal point for $\forall j$)

$S := Bl_Z \mathbb{P}^2 \xrightarrow[\pi]{} \mathbb{P}^2$: b-up.

$\cup$

$C := (\pi^{-1})_* C_0$: str. transf.

$\rightsquigarrow$ $N_{C/S}$: top. triv. ($\Leftarrow \deg N_{C/S} = 3\times 3 - 9 = 0$)

$N_{C/S} \in Pic^0(C) \underset{\text{naturally}}{\cong} Pic^0(C_0)$ attains $\forall$ p.t. $\in Pic^0(C_0)$ by changing Z.

//

E.g. 4 (the "standard model") $\Leftarrow$ when C is a rat'l curve with a node, for e.g.

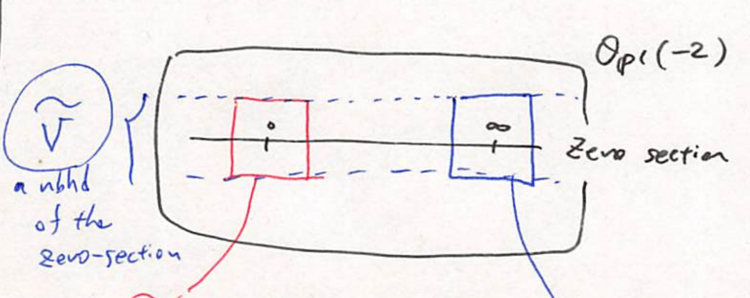
$\pi : \mathcal{O}_{\mathbb{P}^1}(-2) \longrightarrow \mathbb{P}^1$

$\supset$ non-homog. coord $S = T^{-1}$.

fiber coord : ξ_0 around $\{S=0\}$

ξ_∞ around $\{S=\infty\} = \{T=0\}$.

($\xi_0 \cdot S^2 = \xi_\infty$ on $\{S \neq 0, \infty\}$)



$F : \widetilde{V_0}^+ \xrightarrow{\cong} \widetilde{V_0}^-$

$(S, \xi_0) \longmapsto (\underbrace{t\cdot\xi_0}_{T}, \underbrace{S}_{\xi_\infty})$

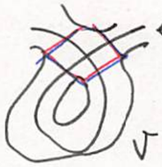
($t \in U(1)$)

$\widetilde{V_0}^+ := \{|S|<\varepsilon, |\xi_0|<\varepsilon\}$

$\widetilde{V_0}^- := \{|T|<\varepsilon, |\xi_\infty|<\varepsilon\}$

$S \overset{?}{=} V := \widetilde{V}/\sim$ ($\sim$: induced by F) //

<u>Obs</u> In <u>e.g 4</u>,



$V_0 := i(\widetilde{V_0}^+)$
$\quad = i(\widetilde{V_0}^-)$

$V_1 := (C_{reg} = C \setminus \{\text{nodal p.t}\})$ の nbhd.

$w_0 :=$ $s.\xi_0$
$w_1 :=$ $s.\xi_a$

$\rightsquigarrow$ w_0 と w_1 との関係は $\begin{cases} w_1 = w_0 \\ w_1 = t \cdot w_0 \end{cases}$

here


here.

$\rightsquigarrow \varphi : V \longrightarrow \mathbb{R}$
$\quad p \longmapsto -\log |w_j(p)|$; globally defined,
$\qquad \varphi|_{V \setminus C}$: pluri harmonic.

"C admits a psd.flat nbhds system"
//

Thm ([K-'17, Thm 1.4])

$C \subset S$: as in Thm 1.
$\Rightarrow$ C admits a psd flat nbhds system //

§3. prf of Thm 1

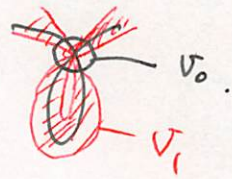
Strategy: (C, S) : as in Thm 1. ← rat'l curve with a node (for simplicity).

→ show that $\exists V$: nbhd of C s.t. V : as in E.g.4 (with $t = t(N_{C/S})$)!

$\{U_0, U_1\}$: open cov. of C.

$\cdots \begin{cases} U_0 : \text{small nbhd of the nodal p.t.} \\ U_1 := C_{reg} \end{cases}$. $U^+ \sqcup U^- := U_0 \cap U_1$

V_j : U_j-nbhd s.t. $U_j = V_j \cap C$. $V^+ \sqcup V^- := V_0 \cap V_1$

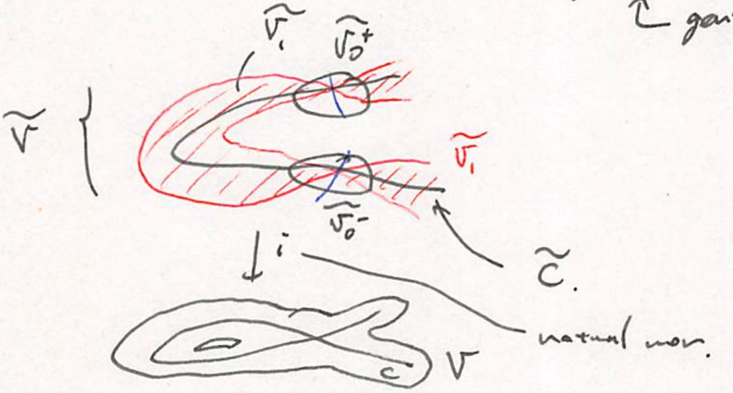


⊙ $\widetilde{V}_0^+, \widetilde{V}_0^-$: 2-copies of V_0,

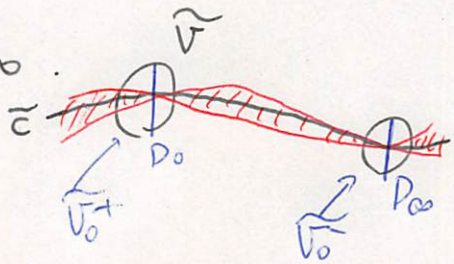
$\widetilde{V}_1$: a copy of V_1.

⊙ $\widetilde{V} := \widetilde{V}_0^+ \sqcup \widetilde{V}_1 \sqcup \widetilde{V}_0^- / \sim$

$\sim$ gen'd by $\begin{cases} \widetilde{V}_0^+ \supset V^+ \subset \widetilde{V}_1 \\ \widetilde{V}_1 \supset V^- \subset \widetilde{V}_0^- \end{cases}$



⊙ $i^* C \underset{\text{as divisors}}{=} \widetilde{C} + D_0 + D_\infty$.



In $\widetilde{V}$, $\deg(i^* N_{C/S})|_{\widetilde{C}} = 0$

$$= (\widetilde{C}.\widetilde{C}) + (\widetilde{C}.D_0) + (\widetilde{C}.D_\infty)$$

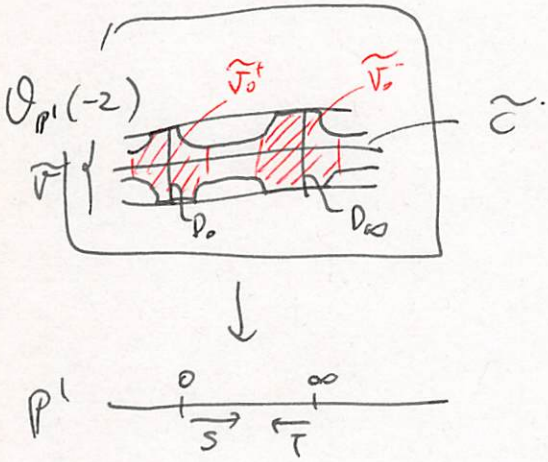
$$\rightsquigarrow \underline{(\widetilde{C}^2) = -2}.$$

As $\widetilde{C} \cong \mathbb{P}^1$,

Grauert's thm $\Longrightarrow$ $\widetilde{V}$ can be regarded as

a nbhd of the zero-sectn of $\mathcal{O}_{\mathbb{P}^1}(-2) \to \mathbb{P}^1$

$\| \quad \widetilde{C}$.



Coord.

$S = T^{-1}$; ~~s.t. $D_0 = \{S=0\}$~~

non-homog coord of $\mathbb{P}^1 = \widetilde{C}$

s.t. $\begin{cases} D_0 \cap C = \{S=0\} = \{T=\infty\} \\ D_\infty \cap C = \{S=\infty\} = \{T=0\} \end{cases}$

⊙ ~~S, T:~~ Denote also by S (T) the ~~pull-back $(\mathcal{O}_{\mathbb{P}^1(-2)} \to \mathbb{P}^1)^*$~~ extention of S (T).

Claim We may assume $\begin{cases} D_0 = \{S=0\} = \{T=\infty\} \\ D_\infty = \{S=\infty\} = \{T=0\} \end{cases}$. //

$-dS \wedge d\xi_0 = dT \wedge d\xi_\infty$

$\downarrow$

$K_\mathcal{V} = \mathcal{O}_\mathcal{V}$

(∵) w.m.a. $\exists \mathcal{V} \supset \widetilde{V} \supset \widetilde{C}$; str. psd conv. nbhd

$\mathcal{V} \subset \mathcal{O}_{\mathbb{P}^1}(-2)$

$\rightsquigarrow$ ~~Nadel type vanishg~~ Ohsawa vanishg $H^1(\mathcal{V}, \mathcal{O}_\mathcal{V}) = 0$.

$K + \mathcal{O}$

Nakano-s.p.

$\rightsquigarrow$ exp.-ex.seq. $\forall$ top. triv. l.b./$\mathcal{V}$: hol. triv.

$\rightsquigarrow$ $\mathcal{O}_\mathcal{V}(D_0 - D_\infty)$: hol. triv //

Thm (K–'17, Thm 1.4)

$N_{C/S}$: Dioph. (C : cycle of rat'l curves)

$\Rightarrow$ C admits a pseudoflat nbhds sys. "

$\exists\, w_j : V_j \to \mathbb{C}$: def. func'n of $U_j = C \cap V_j$.
$(j = 0, 1)$

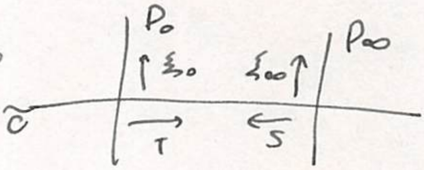
s.t. $w_1 = \begin{cases} t_+ \cdot w_0 & \text{on } V^+ \\ t_- \cdot w_0 & \text{on } V^- \end{cases}$ $(t_\pm \in U(1))$

⊙ Define $\widetilde{w} : \widetilde{V} \to \mathbb{C}$ by $\widetilde{w} := \begin{cases} t_+ \cdot w_0 & \text{on } \widetilde{V_0^+} \\ t_1 & \text{on } \widetilde{V_1} \\ t_- \cdot w_0 & \text{on } \widetilde{V_0^-} \end{cases}$

⊙ $t := t(N_{C/S}) = \dfrac{t_+}{t_-}$ " ; glob def. func of. $\widetilde{C} + D_0 + D_\infty$.

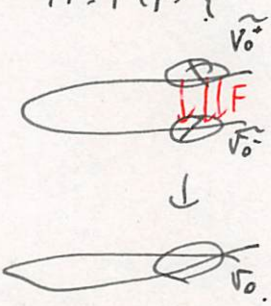
⊙ $\xi_0 := \widetilde{w}/S$

$\xi_\infty := \widetilde{w}/T$ $\rightsquigarrow$ $\xi_\infty = S^2 \cdot \xi_0$



⊙ Define $F : \widetilde{V_0^+} \longrightarrow \widetilde{V_0^-}$ by $i^{-1}(i(p)) = \{p, F(p)\}$

$$F(S, \xi_\infty) = \Big(\underbrace{\frac{t\,\xi_0}{G}}_{=T}, \underbrace{G \cdot S}_{=\xi_\infty}\Big)$$



for $\exists G : \widetilde{V_0^+} \to \mathbb{C}^*$. : hol.

$$\left(\Leftarrow \begin{cases} F^* D_\infty = C \cap \widetilde{V_0^+},\ F^*(C \cap \widetilde{V_0^-}) = D_0, \\ F^*(\underbrace{T \cdot \xi_\infty}_{=\widetilde{w}}) = t \cdot (\underbrace{S \cdot \xi_0}_{=\widetilde{w}}) \end{cases} \right)$$

⊙ By changing $\widetilde{w} \rightsquigarrow \dfrac{\widetilde{w}}{G(0,0)}$, we may assume $G(0,0) = 1$.

☆ Thm 1 $\Leftarrow$ "$G \equiv 1$ holds for a suitable choice of (S, T)".

Obs $H: \widetilde{V} \to \mathbb{C}^*$: nowhere van. hol.

$$\hat{S} := H^{-1}\cdot S \rightsquigarrow \hat{T} := H\cdot T$$

$\tilde{w}$: fixed

$$\hat{\xi}_0 := H\cdot \xi_0 \qquad \hat{\xi}_\infty := H^{-1}\cdot \xi_\infty .$$

$$F^*\hat{T}\ (:= \hat{T}\circ F) = (F^*T)\cdot(F^*H) = \frac{t\cdot \xi_0}{G}\cdot(F^*H) = \frac{t\cdot\hat{\xi}}{G\cdot H}\cdot(F^*H)$$

$$F^*\hat{\xi}_\infty = (F^*\xi_\infty)\cdot(F^*H)^{-1} = \frac{G\cdot S}{F^*H} = \frac{HG}{F^*H}\cdot\hat{S}$$

$\rightsquigarrow$ "$\hat{G}$" $:= G\cdot\frac{H}{F^*H}$. "

$\rightsquigarrow$ by considering $\left|\begin{array}{l} g := \frac{1}{2\pi\sqrt{-1}}\log G \leftarrow g(0,0)=0 \\ h := \frac{1}{2\pi\sqrt{-1}}\log H ,\end{array}\right.$ Thm 1 follows from...

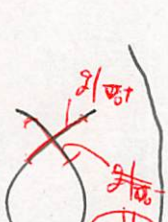
Prop 5 $\varinjlim_{V^*} H^1(V^*, \mathcal{O}_{V^*}(-C)) = 0$

(V^* : a nbhd of C in V)

//

pf of "Prop 5 ⇒ Thm 1"

$\alpha := [\{(V^+, -g), (V^-, 0)\}] \in \check{H}^1(\{V_j\}, \mathcal{O}_V)$



$\rightsquigarrow \alpha|_C = \delta[\{(V_0, \ ), (V_1, 0)\}] + \varinjlim_{V^*} H^1(V^*, \mathcal{O}_{V^*}) \to H^1(C, \mathcal{O}_C)$ inj, Prop 5

$\rightsquigarrow \alpha = 0$ in $\check{H}^1(\{V_j\}, \mathcal{O}_V)$ $\underbrace{\text{by shrinking } V}$.

$\rightsquigarrow \exists h_j : V_j \to \mathbb{C}$ s.t. $h_0 - h_1 = \begin{cases} -g & \text{on } V^+ \\ 0 & \text{on } V^- \end{cases}$

Outline of the pf of Prop5.

$F_\pm : V^\pm \longrightarrow \mathbb{C}$: hol. with $F_\pm|_{\overline{U}^\pm} \equiv 0$.

Want to find

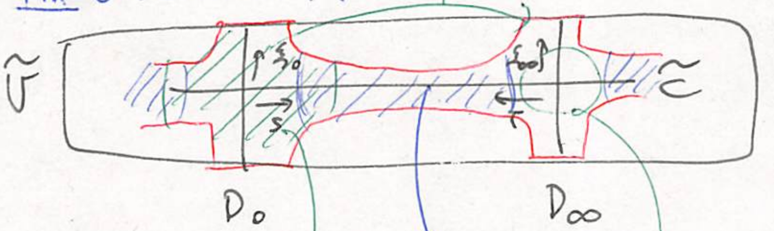
$F_j : V_j \to \mathbb{C}$: hol. with $F_j|_{\overline{U}_j} = 0$

s.t. $F_0 - F_1 = \begin{cases} F_+ & \text{on } V^+ \\ F_- & \text{on } V^- \end{cases}$ "

Step 0 : Take rel. sub $V_j^* \Subset V_j$. s.t. $V^* := V_0^* \cup V_1^*$: nbhd of C in V.

"nice" choice of V_j^* ;

Fix $0 < \delta \ll \varepsilon \ll 1$.



$\tilde{V}_1^* := \{|\tilde{w}| < \delta,\ \varepsilon < |s| < \frac{1}{\varepsilon}\}$

$\tilde{V}_0^* := \{|\tilde{w}| < \delta,\ \max\{|x|, |y|\} < 2\varepsilon\}$,

where $x := F^*T$

$y :=$

S

Coord

$x = T$

$y = (F^{-1})^*S$

Coord on $\tilde{V}_1^*$; (z, w_1), $z = T$

$V_j^* := i(\tilde{V}_j^*)$

$V_\pm^*$; conn. comp. of $V_0^* \cap V_1^*$.

$\xrightarrow{\text{simple obs.}}$ suff. to construct F_j on V_j^* ".

<u>Step 1</u> "formal" construction of F_j's.

We'll construct F_j's in the form of

$$F_0(x,y) = \sum_{\nu=1}^{\infty} a_{0,\nu}(x,y)\cdot w_0(x,y)^{\nu}$$

$$F_1(z,w_1) = \sum_{\nu=1}^{\infty} \underbrace{a_{1,\nu}(z)}_{} \cdot w_1^{\nu}$$

pull-back of $a_{1,\nu}$ defined on $U_1^* := V_1^* \cap C$ by $(z,w_1) \longmapsto z$.

$a_{0,\nu}$ on $U_0^* := V_0^* \cap C$;

$a_{0,\nu}$ (nodal p.t.)

$$a_{0,\nu} = \begin{cases} P_\nu(x) + r_\nu & (U_0^* \cap V^+) \\ q_\nu(y) + r_\nu & (U_0^* \cap V^-) \end{cases}$$

$\big\downarrow$ extend

$$a_{0,\nu}(x,y) := P_\nu(x) + q_\nu(y) + r_\nu.$$

0 at the nodal p.t.

<u>$a_{j,1}$ $(\nu = 1)$</u> ;

Solve

$$t_\pm^{-1}\cdot a_{0,1} - a_{1,1} = h_{\pm,1}(z) \quad \text{on } U_{01}^* = U_0^* \cap U_1^*,$$

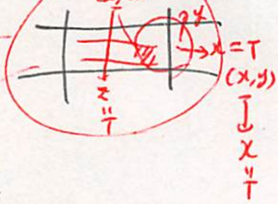
<u>where</u>

$$F_\pm(z,w_1) = \sum_{\nu=1}^{\infty} h_{\pm,\nu}(z)\cdot w_1^{\nu}.$$

$\because$ $\exists!$ such $\{a_{j,1}\}$ $\left(\Leftarrow \begin{matrix} H^0(C, N_{C/S}^{-1}) = 0 \\ H^1(C, N_{C/S}^{-1}) = 0 \end{matrix} \right)$

<u>$a_{j,n}$</u> after deciding $\{a_{j,\nu}\}_{\nu=1}^{n-1}$.

$\odot$ we use $\{a_{j,\nu}\}_{\nu=n}^{\infty}$ as "unknown functions".

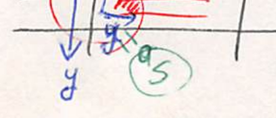


ここで xは zの関数

$$P_\nu(x(z,w_1)) = \begin{cases} P_\nu(x(z)) & \text{on } V_+^* \\ \sum_{\lambda=1}^{\infty} P_{\nu,\lambda}(z)\cdot w_1^{\lambda} & \text{on } V_-^*. \end{cases}$$

C上では $x=0$.

$$q_\nu(y(z,w_1)) = \begin{cases} \sum_{\lambda=1}^{\infty} Q_{\nu,\lambda}(z)\cdot w_1^{\lambda} & \text{on } V_+^* \\ q_\nu(y(z)) & \text{on } V_-^*. \end{cases}$$



$$\rightsquigarrow F_0|_{V_+^*} = \sum_{\nu=1}^{\infty} a_{0,\nu}(x,y)\cdot w_0{}^{\nu}$$

$$= \sum_{\nu=1}^{\infty} t_+^{-\nu}\cdot\left(P_\nu(x(z)) + \sum_{\lambda=1}^{\infty} Q_{\nu,\lambda}(z)\cdot w_1{}^{\lambda} + r_\nu\right)\cdot w_1{}^{\nu}$$

$$= \sum_{\nu=1}^{\infty}\left(t_+^{-\nu}(P_\nu(x(z)) + r_\nu) + h_\nu^+(z)\right)\cdot w_1{}^{\nu},$$

where $h_m^+(z) := \sum_{\nu=1}^{m-1} t_+^{-\nu}\cdot \underbrace{Q_{\nu,m-\nu}(z)}_{\text{known functions}}$.

$\rightarrow$ Define $\{a_{j,n}\}$ by

$$t_\pm^{-n}\cdot a_{0,n} - a_{1,n} = \cdot\, h_{\pm,n}(z) - h_n^{\pm}(z) \text{ on } V_\pm^*.$$

$\exists!$ such $\{a_{j,n}\}$ $\left(\Leftarrow \begin{array}{l} H^0(C, N_{C/S}^{-n}) = 0 \\ H^1(C, N_{C/S}^{-n}) = 0 \end{array}\right)$

//.

<u>Convergence of F_j's</u> ;

$A(X) \in \mathbb{R}_{\geq 0}[[X]]$: defined by
$\overset{\shortparallel}{X + A_2 X^2 + \cdots}$

$$\sum_{\nu=2}^{\infty} |1 - t^{\nu-1}|\cdot A_\nu\cdot X^\nu = KRM\cdot\frac{A(X)^2}{1 - RA(X)},$$

where $M := \max\{\sup_{V_\pm^*}|F_\pm|, 1\}$ $(<\infty)$,

R $\gg 1$: s.t.

$$\begin{cases} \{(z,w_1) \mid \varepsilon < |z| < 2\varepsilon,\ |w_1| = \frac{1}{R}\} \subset V_+^*, \\ \{(z,w_1) \mid \frac{1}{2\varepsilon} < |z| < \frac{1}{\varepsilon},\ |w_1| = \frac{1}{R}\} \subset V_-^*, \end{cases}$$

K ---

K ... s.t.

Lem6 $\exists K$: s.t. $\forall n$, $\forall \{a_j\}$, $\forall \{h_\pm\}$ s.t.

$$t_\pm^{-n} \cdot a_0 - a_1 = h_\pm \quad \text{on } U_\pm^*$$

$$\Rightarrow \max_{j=0,1} \sup_{U_j^*} |a_j| \le \frac{K}{|1-t^n|} \max \{ \sup_{U_\pm^*} |h_\pm| \}$$

~~if $t \in U(1)$; Dioph~~

//.

Then by induct'n, we have

$$\max_{j=0,1} \sup_{U_j^*} |a_{j,\nu}| \le A_{\nu+1}.$$

Known (Siegel)

$A(X)$ has positive radius of conv if $t \in U(1)$; Dioph.

$\rightsquigarrow$ ok

//.

Rmk [K-'17, Thm1.4] can also be shown by ~~the~~ similar strategy.

w_0, w_1 : def. funcs of U_j^* in V_j^*.

$$t_\pm \cdot w_0 = w_1 + \sum_{\nu=2}^{\infty} h_{\pm,\nu}(z) \cdot w_1^\nu.$$

define new def. functions u_j by "Schröder eq".

$$w_j = u_j + \sum_{\nu=2}^{\infty} a_{j,\nu} \cdot u_j^\nu$$

inductively defined and estimated.

(so that $t_\pm u_0 = u_1$)

Set

$$G_j : V_j \times \Delta_\delta \longrightarrow \mathbb{C} \quad \text{: hol for } \delta \ll 1. \quad \text{satisfies } \left.\frac{\partial G_j}{\partial u_j}\right|_{u_j=0} \equiv 1 \neq 0.$$

$$(p, u_j) \longmapsto w_j(p) - u_j - \sum_{\nu=2}^{\infty} a_{j,\nu}(p) \cdot u_j^\nu$$

inv. func thm. $\exists \hat{w}_j : V_j \xrightarrow{\text{hol}} \Delta_\delta$ by shrinking V_j's if ness,

s.t. $G_j(p, \hat{w}_j(p)) \equiv 0$ $\longrightarrow$ $\hat{w}_j$: "nice" def. func!