Positivity Concepts on Holomorphic Line Bundles and Theories on Canonical Kähler Metrics

28 – 31 January, 2018, at Osaka City University

Program

January 28

13:30 – 14:00 Registration

14:10 – 15:00 Martin Sera (Chalmers University of Technology and University of Gothenburg)
Chern forms of singular metrics on vector bundles

15:40 – 16:30 Genki Hosono (The University of Tokyo)
On recent topics on L^2 extension theorems

16:40 – 17:30 Song-Yan Xie (Max Planck Institute for Mathematics)
On the Kobayashi hyperbolicity of certain surfaces in $\mathbb{CP}^3$

January 29

10:00 – 11:20 Mini-Lecture by Ryosuke Takahashi (Tohoku University)
Lectures on extremal Kähler metrics

11:30 – 12:20 Satoshi Nakamura (Tohoku University)
Generalized Kähler Einstein metrics and uniform stability for toric Fano manifolds

14:10 – 15:00 Thibaut Delcroix (ENS Paris)
Mabuchi's K-energy functional on horosymmetric varieties

15:40 – 16:30 Tomoyuki Hisamoto (Nagoya University)
On the gradient flow of the Ding energy functional

16:40 – 17:30 Yoshinori Gongyo (The University of Tokyo)
Nef anti-canonical divisors and rationally connected fibrations

January 30

10:00 – 11:20 Mini-Lecture by Ryosuke Takahashi (Tohoku University)
Lectures on extremal Kähler metrics

11:30 – 12:20 Kento Fujita (Kyoto University)
Openness results for uniform K-stability

14:10 – 15:00 Ryosuke Takahashi (Tohoku University)
The inverse Monge-Ampère flow and optimal degenerations on toric Fano manifolds

15:40 – 16:30 Ryosuke Nomura (The University of Tokyo)
Miyaoka-Yau inequality for compact Kähler manifolds with semi-positive canonical bundle

16:40 – 17:30 Shin Kikuta (Kogakuin University)
Degeneration of positivity for log-canonical bundle and Kähler–Einstein metric

January 31

11:30 – 12:20 Masanori Adachi (Shizuoka University)
Weighted Bergman spaces of domains with Levi-flat boundary

14:10 – 15:00 Dinh Tuan Huynh (Osaka University)
On the truncation level in Cartan's Second Main Theorem for entire holomorphic curves into projective plane

15:10 – 16:00 Takato Uehara (Saga University)
On a construction of non-projective K3 surfaces

16:10 – Discussion

Abstracts

Masanori Adachi (Shizuoka University)
Weighted Bergman spaces of domains with Levi-flat boundary

In contrast to bounded domains in Stein manifolds, it is not clear to what extent domains with Levi-flat boundary are capable of holomorphic function with slow growth. We shall answer this question in a special case, the quotient of the bidisk by an action of Fuchsian group, by constructing invariant holomorphic functions on the bidisk explicitly and estimating their weighted L^2 norms.

Thibaut Delcroix (ENS Paris)
Mabuchi's K-energy functional on horosymmetric varieties

Spherical varieties form a very large class of almost-homogeneous varieties and as such a promising class on which one can study existence of canonical metrics. However it is very difficult to do Kähler geometry in a uniform way on all spherical varieties. I will introduce a subclass containing homogeneous toric bundles and group compactifications on which I am able to adapt toric geometry techniques to obtain a combinatorial sufficient criterion of properness of Mabuchi's K-energy functional. I will also present applications to existence of cscK and log-Kähler-Einstein metrics.

Kento Fujita (Kyoto University)
Openness results for uniform K-stability

It is expected that several "openness" results holds for uniform K-stability of polarized varieties. In this talk, I will discuss certain partial results for this problem.

Yoshinori Gongyo (The University of Tokyo)
Nef anti-canonical divisors and rationally connected fibrations

We study the Iitaka–Kodaira dimension of nef relative anti-canonical divisors by using semi-positivity theorems. Thus we give affirmative answers for some two questions. This is a joint work with Sho Ejiri.

Tomoyuki Hisamoto (Nagoya University)
On the gradient flow of the Ding energy functional

This is a joint work with T. Collins and R. Takahashi. Ding energy functional is one of the canonical energies which characterize the Kähler–Einstein metric as the critical point. I will introduce the flow in the title and talk about the long-time existence.

Genki Hosono (The University of Tokyo)
On recent topics on L^2 extension theorems

Let $\Omega \subset \mathbb{C}^n$ be a bounded pseudoconvex domain and $V = \Omega \cap \{z_n = 0\}$. The Ohsawa–Takegoshi L^2 extension theorem states that every L^2 homolorphic function on V can be extended to Ω with an a priori L^2 estimate. Blocki and Guan–Zhou proved the L^2 extension theorem with an optimal estimate. After that, a new proof of the optimal estimate was given by Berndtsson–Lempert. Using their method, we will give a jet version of the L^2 extension theorem with an optimal estimate. We also give a sharper estimate of the L^2 extension theorem using Hartogs domains.

Dinh Tuan Huynh (Osaka University)
On the truncation level in Cartan's Second Main Theorem for entire holomorphic curves into projective plane

Let $f : \mathbb{C} \to \mathbb{P}^2(\mathbb{C})$ be an entire holomorphic curve and let $\{L_i\}_{1 \leq i \leq q}$ be a family of $q \geq 4$ lines in general position in projective plane. If f is linearly nondegenerate, i.e. its image is not contained in any line, then the classical Second Main Theorem of Cartan states that the following inequality holds true

outside a subset of $(0, \infty)$ of finite Lebesgue measure:

$$(q - 3) \, T_f(r) \leq \sum_{i=1}^{q} N_f^{[2]}(r, L_i) + o(T_f(r)).$$

Here $T_f(r)$ and $N_f^{[2]}(r, L_i)$ stand for the order function and the 2-truncated counting functions in Nevanlinna theory. It is conjectured that in the above estimate, the truncation level of the counting functions can be decreased to 1, provided that that f is algebraically nondegenerate (i.e. its image is not contained in any algebraic curve). In this talk, we will provide a partial answer to this conjecture and propose a strategy to achieve the full proof.

[1] Henri Cartan, Sur les zéros des combinaisons linéaires de p fonctions holomorphesdonnées, *Mathematica*, **7**, 5–31, 1933.

[2] Michael McQuillan, Diophantine approximations and foliations, *Inst. Hautes Études Sci. Publ. Math.*, **87** 121–174, 1998.

[3] Marco Brunella, Courbes entières et feuilletages holomorphes, Ens. Math., **45**, 195–216, 1999.

[4] Julien Duval and Dinh Tuan Huynh, A geometric second main theorem, *Math. Ann.*(2017), DOI:10.1007/s00208-017-1538-y.

Shin Kikuta (Kogakuin University)
Degeneration of positivity for log-canonical bundle and Kähler–Einstein metric

We would like to discuss some relations between the positivity of the log-canonical bundle and the boundary behavior of the Kähler-Einstein metric over a quasi-projective manifold. In particular, via its volume growth, we characterize when the boundary is of general type. We will also talk about our recent progress in the case when the boundary is Calabi–Yau.

Satoshi Nakamura (Tohoku University)
Generalized Kähler Einstein metrics and uniform stability for toric Fano manifolds

Generalized Kähler Einstein metrics, introduced by Mabuchi in 2000, is a generalization of Kähler Einstein metrics for Fano manifolds with non vanishing Futaki invariant. (Some people called them Mabuchi metrics or Mabuchi Einstein metrics or Mabuchi soliton... We should fix the name.) In this talk, We give a complete criterion for the existence of generalized Kähler Einstein metrics on toric Fano manifolds from view points of a uniform stability in a sense of GIT and the properness of a functional on the space of Kähler metrics. This talk is based on arXiv:1706.01608 (to appear in Tohoku Math J.).

Ryosuke Nomura (The University of Tokyo)
Miyaoka-Yau inequality for compact Kähler manifolds with semi-positive canonical bundle

Miyaoka-Yau inequality is an inequality for Chern classes which is known to hold for minimal projective varieties. In this talk, we prove the inequality for compact Kähler manifolds with semi-positive canonical bundle.

Martin Sera (Chalmers University of Technology and University of Gothenburg)
Chern forms of singular metrics on vector bundles

After recalling the notion of Segre and Chern forms for smooth metrics, we would like to present some results of a joint work with Richard Lärkäng, Hossein Raufi and Jean Ruppenthal, where we study singular Hermitian metrics on holomorphic vector bundles. Although it is in general not possible to define the

curvature as a current with measure coefficients, we give a definition of Chern forms as closed currents of order 0. For Griffiths positive/negative singular metrics, we can define these Chern currents up to the degree less or equal than the codimension of the singular locus using pluripotential theoretical methods. Finally, we will discuss some properties of these Chern currents, for instance that they represent the Chern classes of the vector bundle.

Ryosuke Takahashi (Tohoku University)
The inverse Monge-Ampère flow and optimal degenerations on toric Fano manifolds

We discuss the gradient flow of the Ding energy functional, referred as the inverse Monge-Ampère flow on toric Fano manifolds. We prove that the inverse Monge-Ampère flow produces a "most destabilizing degeneration" with respect to the Ding-stability. This research is based on a joint work with T.C. Collins (Harvard Univ.) and T. Hisamoto (Nagoya Univ.).

Takato Uehara (Saga University)
On a construction of non-projective K3 surfaces

In this talk, we construct K3 surfaces by gluing two 9-point blowups of the complex projective plane, where the existence of overlaps is guaranteed by Arnol'd's theorem. A calculation of their period maps shows that such K3 surfaces constitute a large family, including non-projective K3 surfaces. Furthermore, we comment on a relation with automorphisms of K3 surfaces having positive entropy. This is a joint work in progress with T. Koike.

Song-Yan Xie (Max Planck Institute for Mathematics)
On the Kobayashi hyperbolicity of certain surfaces in $\mathbb{CP}^3$

The Green-Griffiths-Lang Conjecture in $\mathbb{CP}^3$ stipulates that, for a general surface of degree $\geqslant 5$, there should be no holomorphic entire curve on it. In this talk, we discuss some recent progress towards this conjecture.