On the semi-positivity of a nef l.b. and the ~~s~~ nbhd of the stable base loci. 14:50 ~ 15:50.

X : (proj.) cpx. mfd.

L : hol. line bdl $/X$.

Q L が (~~strictly~~) nef であるときの
L の minimal singular metric は
どこで、どの程度の singularity をもつか？

§1. definitions, known results.
§2. Main results.
§3. applications, examples.

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§1.

Def. L : nef (str.) $\overset{\text{def}}{\Longleftrightarrow}$ $^\forall C \subset X$: cpt curve, $L.C \geq 0$ $(\gneq)$ //.

Def. h : a min. sing. metric of L. $\rightsquigarrow h \overset{\text{loc}}{=} e^{-\varphi}$.

$\overset{\Longleftrightarrow}{\text{def.}}$ (i) ~~$\sqrt{-1}$~~ h は各 local weight. φ が psh である sing. metric on L. $(\rightsquigarrow \sqrt{-1}\Theta_h \geq 0)$

(ii) $^\forall h'$: (i) のようなもの,
$^\forall x \in X$, $^\exists C \in \mathbb{R}$,
φ, φ' を x まわり h, h' の local weight として,
$\varphi' \leq \varphi + C$ around x //.

Known

① L : psd. eff $\Rightarrow$ $^\exists$ min. sing. metric on L.

② L : ample $\underset{\text{Kod. e-b.}}{\Longleftrightarrow}$ L : positive. $\Rightarrow$ L : semi-positive.
◎ L : semi-ample $\nearrow$

③ L : semi-positive $\Rightarrow$ nef.

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⑨ L: nef $\not\Rightarrow$ L: semi-positive.

$\cdots$ e.g. (Demailly - Peternell - Schneider)

C_0 : sm. ellipt. curve.

E : rank 2-vect. bdl $/C_0$.

s.t. $0 \to \mathbb{1}_{C_0} \to E \to \mathbb{1}_{C_0} \to 0$: ex. non-spl.

$X := \mathbb{P}(E) \xrightarrow{\pi} C_0$.

$C :=$ the section of π. $\rightsquigarrow (C^2) = 0$

$\rightsquigarrow [C]$: nef

fact. $f_C \in H^0(X, [C])$: can. section s.t.

$|f_C|^{-2}$: a min. sing. metric of $[C]$ //

⑨ $B \subset \mathbb{P}^2$: sm. ellipt. curve.

$p_1, \cdots, p_9 \in B$: ~~...~~ general 9 points.

$X := Bl_{\{p_j\}} \mathbb{P}^2 \xrightarrow{\pi} \mathbb{P}^2$

$\rightsquigarrow K_X^{-1}$: nef, not semi-ample. $\rightsquigarrow SB(K_X^{-1}) = (\pi^{-1})_* B$.

$\Vert$

$\widetilde{B}$

Thm (Brunella)

K_X^{-1} : semi-positive $\Longleftrightarrow$ $\widetilde{B}$ は X の psd. flat nbhd をもつ //

⑨ Ueda theory $\rightsquigarrow \mathcal{O}_{\mathbb{P}^2}(3)|_B \otimes \mathcal{O}_B(-p_1 - \cdots - p_9)$

$\in \mathcal{E}_1(B) := \left\{ E \in Pic^0(B) \,\middle|\, \begin{array}{l} \exists \alpha > 0, \forall n \in \mathbb{Z}_{>0} \\ d(\mathbb{1}_B, E^n) \geq (2n)^{-\alpha} \end{array} \right\}$.

$\Rightarrow (*)$ //

§2 main results

Def (Ueda type) X: ~~—~~. cpx mfd.
$\cup$
S: sm. cpt (Kä) hyp. surf. with $c_1(N_{S/X})=0$

$\rightsquigarrow$ $\mathrm{type}(S,X)=n \underset{\text{def}}{\Longleftrightarrow}$ 十分小さい S-nbhd V に対して

$$\widetilde{N_{S/V}}\otimes \mathcal{O}_V/I_S^{\nu+1} \not\cong \mathcal{O}_V(S)\otimes \mathcal{O}_V/I_S^{\nu+1} \quad \text{for } {}^\forall_{0\le}\nu<n,$$

$(n\in \mathbb{N}_{>0}\cup\{\infty\})$ $\qquad$ — " — $\cong$ — " — for $\nu=n$ //.

Thm 1 X: sm. surf.
$\cup$
C: sm. embedded cpt curve. with $(C^2)=0$.

$\rightsquigarrow$ $\mathrm{type}(C,X)=n<\infty$
$\Rightarrow$ $|\underset{\sim}{f_C}|^{-2}$: a min. sing. metric of $[C]$ //.
can. section of $[C]$

Thm 2 arXiv.1412.2354. (Ueda theory の"高余次元化"の cor.)
$\mathbb{P}^3 \supset Q_0, Q_\infty$: sm. quad. s.t. $Q_0 \pitchfork Q_\infty$.
$B := Q_0\cap Q_\infty$: sm. ellipt. curve.
$p_1,\dots,p_8\in B$: 相異なる8点,
$X := Bl_{\{p_j\}}\mathbb{P}^3$.
$\rightsquigarrow$ $\mathcal{O}_{\mathbb{P}^3}(2)|_B\otimes\mathcal{O}_B(-p_1-\cdots-p_8)\in \mathcal{E}_1(B)$. or ~~torsion pt of $\mathrm{Pic}^0(B)$~~
$\Rightarrow$ K_X^{-1}: semi-positive (not semi-ample, but nef and)
//.

c.f. $P_1, \cdots, P_8 \in B$: very general で、

K_X^{-1} は ~~"Totaro's question"~~

$$\#\{C \subset X : \text{cp}^{\text{t}}\ \text{curve} \mid \underbrace{K_X^{-1}}_{\text{nef}} \cdot C = 0\} = \#\mathbb{N}$$

なる例となっている (Lesieutre, Ottem, c.f. Totaro's ques)

◎ Outline of the prf. of Thm 1, 2

Thm 1 ... a simple application of Ueda's thm for "fin. type case".

h: sing. metric of $[C]$ with s.p. curvature

$\Psi := -\log |f_C|_h^2$

$\rightsquigarrow \Psi : X \setminus C \longrightarrow \mathbb{R} \setminus \{-\infty\}$,

$$\Psi(p) = O(|\log d(p,C)|) \quad \text{as } p \to C.$$

$$= o\left(d(p,C)^{-\frac{1}{2}}\right). \quad \text{—①}$$

∵ $\Psi \underset{\text{locally}}{=} \underbrace{-\log|\text{hol. func}|^2}_{\text{harmonic}} + (\underbrace{\text{loc. weight of } h}_{\text{psh}})$

$\rightsquigarrow \Psi|_{X\setminus C}$: psh —②

①+② $\xrightarrow[\text{Ueda's thm.}]{}$ $\Psi \equiv {}^\exists M$: const around C. //

Thm 2 ... $K_X^{-1} = [2S_0]$ $(S_0 := \pi^{-1}_* Q_0)$

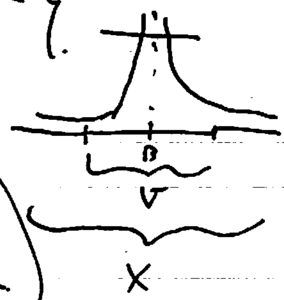
$\xrightarrow[\text{高余次元版 Ueda thm}]{}$ ${}^\exists V$: B-nbhd in X, $[S_0]|_V$: flat.

(15:30)

(つづき)

h_V : flat metric of $K_X^{-1}|_V$.

$$\tilde{h} := \text{"(regularized) min"} \left\{ M \cdot h_V, \left(\sum_{j=0,\infty} |f_{S_j}|^2 \right)^{-2} \right\}.$$



$$\left(\begin{array}{l} M \gg 1, \text{ const}, \\ S_j := (\pi^{-1})_* Q_j, \\ f_{S_j} \in H^0(X, [S_j]) : \text{can. section} \end{array} \right)$$

(loc. weight of h)

$$= \text{"(reg.) max"} \{ \text{const}, \log \sum |\text{h.l.}|^2 \} : \text{psh}$$ //

§3. some examples and applications

Thm. (Neeman)

次の (X, C) は $\left\{ \begin{array}{l} (C^2) = 0, \\ \text{type} = 1 < \infty. \end{array} \right.$

①. C_0 : sm. cpt. curve.

F : flat line bdl $/ C_0$.

$0 \to F \to E \to 1_{C_0} \to 0$: ex. non-splitting.

$X := \mathbb{P}(E) \xrightarrow{\pi} C_0$,

$C :=$ the section of π. ($\cancel{(C^2)=0}$).

② C_0 : a sm. curve. genus $= 2$.

$C_0 \hookrightarrow Y$: the Jacobian.

$p, q \in C_0$: conjugate to each other by the hyperellipt. involution.

$X := Bl_{\{p,q\}} Y$, $C := (\pi^{-1})_* C_0$ /.

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→ **Cor.** C : sm. cpt curve.

E : rank 2-vect. bdl /C. $\deg(E) = 0$.

E が sm. finsler metric with semipositive curv. をもつ

⟺ E : polystable

q.

◎ **Cor** 3-dim. sm. proj. 上の line bdl で

str. nef だが semi-positive でない例が

構成できる.

[Fujino, '13].