X : smooth projective variety $/\mathbb{C}$

L : big holomorphic line bundle $/\mathbb{C}$

$h_{\min,L} = e^{-\varphi_{\min,L}}$; minimal singular metric of L

$c_1(L)$の性質	$\varphi_{\min,L}$の singularity の性質	$R(X,L)$の性質

ample $\overset{(a)}{\Longleftrightarrow}$ $\exists\varphi_{\min,L}$ s.t. $\begin{cases} \varphi_{\min,L} : \text{smooth} \\ dd^c\varphi_{\min,L} > 0 \end{cases}$ (小平のうめこみ定理) $\overset{(a)}{\Longleftrightarrow}$ ample.

ample $\Downarrow$ semi ample. ($\Leftrightarrow$ nef かつ $R(X,L)$ 有限生成)

$\Downarrow$

$\exists\varphi_{\min,L}$; smooth. $\overset{(b)}{\Longleftarrow}$ semi ample.

$\Downarrow$

② …… $\exists\varphi_{\min,L}$; locally bounded.

$\Downarrow$

ample $\Downarrow$ nef $\overset{(c)}{\Longleftrightarrow}$ $\forall x\in X,\ \nu(\varphi_{\min,L}, x) = 0$.

$\exists\varphi_{\min,L} = \frac{1}{m}\log\sum_{\text{fin}}|f_j|^2$ $(f_j \in H^0(X, L^{\otimes m}))$ $\overset{(d)}{\Longleftrightarrow}$ $\boxed{R(X,L) ; \text{有限生成}}$

②' $\Rightarrow$

birational modelをかえて σ-decomposition で
nef + exceptional
に分解 (Zariski 分解可)

$\Downarrow$ ①'

birational modelをかえて
σ-decomposition で
(codimension 1 で nef) + exceptional
に分解
(常に成立)

$\exists\varphi_{\min,L}$: 高々 analytic singularity をもつ

$\Downarrow$ ①

$\varphi_{\min,L}$: 存在.
(常に成立)

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中山による birational Zariski decomposition が不可能な例

(ie. ①' の逆の反例)

$$E := \mathbb{C} / (\mathbb{Z} + \sqrt{-1}\mathbb{Z})$$

$p \in E$: fix. $\quad \underline{V := E \times E}$

$$\left.\begin{aligned} F_1 &:= \{p\} \times E, \\ F_2 &:= E \times \{p\} \\ \Delta &:= \{(x,x) \in E \times E \mid x \in E\} \end{aligned}\right\} \quad V \text{ の divisor.}$$

$a \in \mathbb{N}_{\geq 1}$ を fix,

$$L_0 := \mathcal{O}(2F_1 - 4F_2 + 2\Delta)$$
$$L_1 := \mathcal{O}((a-1)F_1 + (a-1)F_2 + (a+2)\Delta)$$
$$L_2 := \mathcal{O}((a+3)F_1 + (a-3)F_2 + a\Delta)$$

$$\rightsquigarrow \quad X := \mathbb{P}(L_0 \oplus L_1 \oplus L_2) \xrightarrow{\pi} V$$

$$L := \mathcal{O}(1) \quad \text{(relative hyper plane bdl)}$$

$\boxed{\text{Main theorem}}$

$x = (x^1, x^2)$: $U \underset{\text{open}}{\subset} V$ での local coord., $x_0 \in U$.

$\rightsquigarrow$ $\pi^{-1}(U)$ の $\mathbb{P}(L_0)$ まわりの local coord.

$$(z^1, z^2, x) := [\, s^0(x) + z^1 \cdot s^1(x) + z^2 \cdot s^2(x) \,] \in X \text{ に対し}$$

(s^i ; L_i^{-1} の U 上 local trivialization)

$(0, 0, {}^\forall x_0)$ まわり $\varphi_{\min,L}(z^1, z^2, x) = \log \max\limits_{(\alpha,\beta) \in \underset{\sim}{H}} |z^1|^{2\alpha} |z^2|^{2\beta} + O(1)$, ← 次項参照.

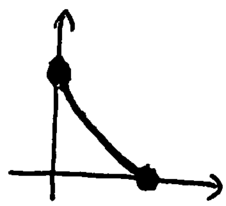
$\mathbb{P}(L_0)$ の外では $\varphi_{\min,L}$: locally bounded

//.

Main theorem とその証明に関する資料

(i)

$$H = \{ (\alpha, \beta) \in \mathbb{R}^2 \mid \alpha, \beta \geq 0 ,\ a^2(\alpha+\beta)^2 = (1-\alpha)^2 + (1-\beta)^2 \}$$

$=$ 

(ii) L_0, L_1, L_2 の smooth Hermitian metric $e^{-\varphi_0}, e^{-\varphi_1}, e^{-\varphi_2}$ とって、V上の Euclidean metric に対する Chern connection の行列表示がそれぞれ

$$dd^c\varphi_0 = \begin{pmatrix} 4 & -2 \\ -2 & -2 \end{pmatrix}$$

$$dd^c\varphi_1 = \begin{pmatrix} 2a+1 & -(a+2) \\ -(a+2) & 2a+1 \end{pmatrix}$$

$$dd^c\varphi_2 = \begin{pmatrix} 2a+3 & -a \\ -a & 2a-3 \end{pmatrix}$$

なるものがとれる.

$e^{-\varphi_0}, e^{-\varphi_1}, e^{-\varphi_2}$ は定数倍をのぞき一意.

(iii) $n \in \mathbb{N}$ に対し $X_n \xrightarrow{\tilde{\pi}_n} X$; proper modification,

$E_A \underset{\text{codim } 1}{\subset} X_n$ (A ; n桁以下の<u>2進数</u>)

$V_B \underset{\text{codim } 2}{\subset} X_n$ (B ; $(n+1)$桁の<u>2進数</u>)

(z_B^1, z_B^2, x) ; $(\pi \circ \tilde{\pi}_n)^{-1}(U) \cap V_B$ まわりの X_n の local coord. s.t. $V_B = \{ z_B^1 = z_B^2 = 0 \}$

を、次項のように定める.

$\underline{n=0}$ (E'_A は無し) … $X_0 := X$ $\tilde{\pi}_0 := id$.

$V_1 := \mathbb{P}(L_0)$ $(z_1^1, z_1^2) := (z^1, z^2)$

$\underline{n=1}$ … $\begin{array}{ccc} X_1 := Bl_{V_1} X_0 & \xrightarrow{\pi_1} & X_0 \\ \cup & & \cup \\ E_1 & \longrightarrow & V_1 \end{array}$, $\tilde{\pi}_1 := \pi_1$

$V_{10} := (\pi_1^{-1})_* \{z_1^1 = 0\} \cap E_1$

$V_{11} := (\pi_1^{-1})_* \{z_1^2 = 0\} \cap E_1$

(z_{10}^1, z_{10}^2); π_1 で $(z_{10}^1, z_{10}^1 \cdot z_{10}^2)$ にうつるもの

(z_{11}^1, z_{11}^2); π_1 で $(z_{11}^1 \cdot z_{11}^2, z_{11}^2)$ にうつるもの.

$\underline{n=2}$ …. $\begin{array}{ccc} X_2 := Bl_{V_{10} \cup V_{11}} X_1 & \xrightarrow{\pi_2} & X_1 \\ \cup & & \cup \\ E_B & \longrightarrow & V_B \end{array}$ $(B = 10, 11)$, $\tilde{\pi}_2 := \pi_1 \circ \pi_2$

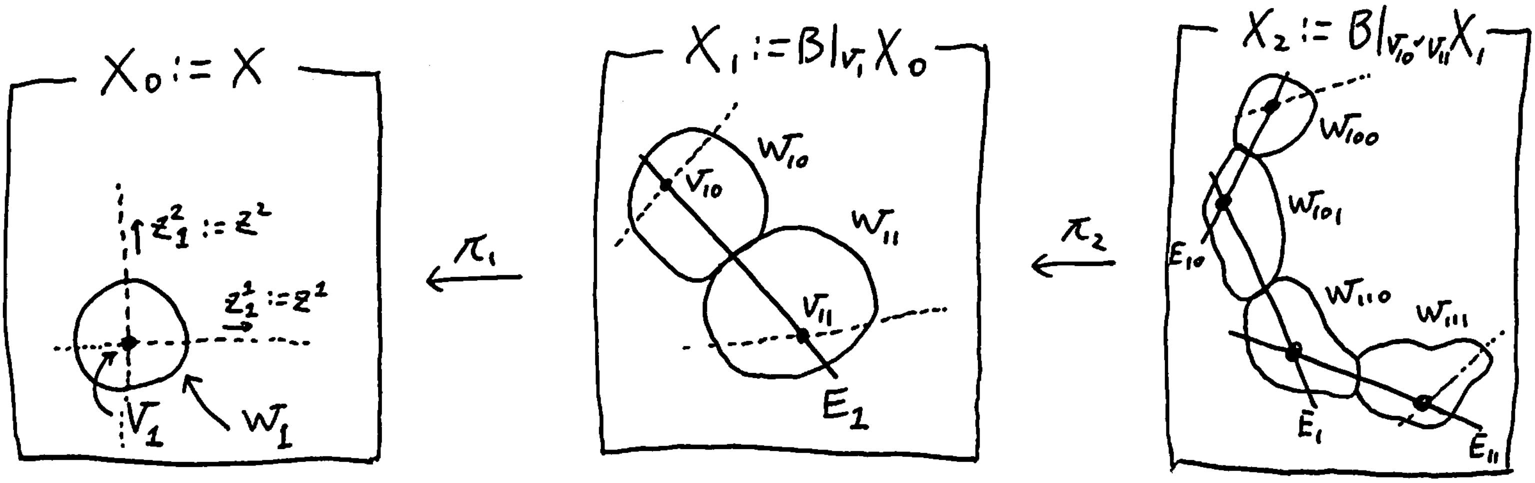
$V_{100} := (\tilde{\pi}_2^{-1})_* \{z_1^1 = 0\} \cap E_{10}$

$V_{101} := E_{10} \cap (\pi_2^{-1})_* E_1$

$V_{110} := (\pi_2^{-1})_* E_1 \cap E_{11}$

$V_{111} := E_{11} \cap (\tilde{\pi}_2^{-1})_* \{z_1^2 = 0\}$

(以下同様)



$W_B := \{ |z_B^1| \le 1, |z_B^2| \le 1 \}$.

; V_B の neighbourhood.