2016 葉山

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No. 1

Date 2016. 7. 18

15:00 − 15:50

Higher codimensional Ueda theory on cpt submfds with unitary flat normal bdls.

Thm1 (T. Ueda '83)

X : cpx mfd

$\cup$

Y : non-sing cpt cpx hyp. surf. with $N_{Y/X} \in H^1(Y, U(1))$.

Assume ⊙ $\mathcal{O}_X(Y) \otimes \mathcal{O}_X/I_Y^n \cong \mathcal{O}_V(\widetilde{N_{Y/X}}) \otimes \mathcal{O}_V/I_Y^n$ for $^\forall n \geq 1$

$I_Y/I_Y^{n+1} \simeq \mathcal{O}_V(\widetilde{N_{Y/X}^*}) \otimes \mathcal{O}_V/I_Y^n$

⊛ (V : a tub. n.b.h.d of Y in X, $\widetilde{N_{Y/X}}$: hol. l.b. $/V$ s.t. $H^1(V, U(1)) \cong H^1(Y, U(1))$, $\widetilde{N_{Y/X}} \longmapsto N_{Y/X}$)

⊙ $N_{Y/X}$ ($\in H^1(Y, U(1))$) ; torsion or "Diophantine"

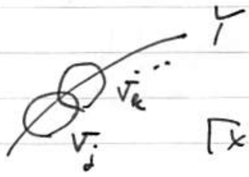
($^\exists A > 0$ s.t. $^\forall n \geq 1$, $d(\mathbb{1}_Y, N_{Y/X}^n) \geq \frac{1}{(2n)^A}$)

Then $\exists V$: a nbhd of Y in X

s.t. $\underbrace{[Y]|_V \cong \widetilde{N_{Y/X}}|_V}_{(*)}$

Obs $(*) \Leftrightarrow$ $^\exists w_j$: a def. func of $V_j \cap Y$ in V_j

s.t. $w_j \underset{\text{on } V_{jk}}{=} \underset{\underset{U(1)}{\cap}}{^\exists t_{jk}} \cdot w_k$.



$\Leftrightarrow$ $^\exists \mathcal{F}$: non-sing hol. foliation of codim $=1$

s.t. $\begin{cases} Y \in \mathcal{F} \text{ (i.e. } Y \text{ is a leaf)} \\ \underset{\text{holonomy}}{\mathrm{Hol}_{\mathcal{F}, Y}} = \underset{\text{the monodromy of } (N_{Y/X}, \nabla_{flat})}{\rho_{N_{Y/X}}} \ (: \pi_1(Y, *) \to U(1)) \end{cases}$

Goal of this talk : • Generalize Thm1 to the case when $r := \mathrm{codim}_X Y \geq 1$.

• Apply it to "the semi-positivity problem".

Schedule

§1. Main result.

§2. Application

§3. Outline of the prf.

No.

Date

§ Main result

Thm 2 (K– '16)

X : cpx mfd

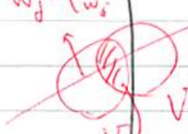
Y : cpt cpx submfd with $\begin{cases} r := \mathrm{codim}_X Y \ (\geq 1) \\ N_{Y/X} \in H^1(Y, U(r)) \end{cases}$

Assume

$\left(\begin{array}{l} \cdot \ I_Y / I_Y^{n+1} \cong \mathcal{O}_V(\widetilde{N_{Y/X}^*}) \otimes \mathcal{O}_V / I_Y^n \quad \text{for } n \geq 1 \end{array} \right) (\#)$

$\cdot N_{Y/X}$: "torsion" or "Diophantine".

$w_j = \begin{pmatrix} w_j^1 \\ \vdots \\ w_j^r \end{pmatrix}$



V_k

V_j

$w_j = {}^{\exists}T_{jk} \cdot w_k$, $T_{jk} \in U(r)$

Then

(i) $\exists V$: a nbhd of Y in X

$\exists \mathcal{F}$: non-sing hol. foliation on V of codim $= r$.

s.t. $\begin{cases} Y \in \mathcal{F} \\ \mathrm{Hol}_{\mathcal{F}, Y} = \rho_{N_{Y/X}}. \end{cases}$

(ii) $\forall S$: non-sing hyp.surf $\subset V$

with $\begin{cases} Y \subset S \\ N_{Y/S} \in H^1(Y, U(r-1)) \end{cases}$

$\leadsto \exists V' (= V_S') \underset{\text{open}}{\subset} X$: a Y-nbhd,

$\exists \mathcal{G} \ (= \mathcal{G}_S)$: non-sing hol. foliation of codim $= 1$ on V'

s.t. $\begin{cases} S' := S \cap V' \in \mathcal{G} \\ \mathrm{Hol}_{\mathcal{G}, S'} : \pi_1(Y, *) \to U(1) \text{ "linear holony"} \\ \mathcal{G} : \mathcal{F}\text{-inv} \left(\begin{array}{l} \text{i.e. } \forall L \in \mathcal{G}, \forall L' \in \mathcal{F}, \\ L \supset L' \text{ or } L \cap L' = \phi \end{array} \right) \end{cases}$

hol. imm.

We can take $\{w_j^i\}$ s.t. $\{w_j^1 = 0\} = S' \cap V_j$

Def $N_{Y/X}$: torsion $\overset{\text{def}}{\iff} \#(\mathrm{Image}(\rho_{N_{Y/X}} : \pi_1(Y, *) \to U(r))) < \infty$

: Dioph. $\underset{\text{def}}{\iff} \exists N_1, \cdots, N_r \in H^1(Y, U(1))$,

$\exists A > 0$,

s.t. $\begin{cases} N_{Y/X} = N_1 \oplus \cdots \oplus N_r \ \left(\begin{array}{l} \text{up. to} \\ \text{fin. cov} \end{array} \right) \\ \forall \alpha = (\alpha_1, \cdots, \alpha_r) \in \mathbb{Z}^r, \\ |\alpha| := \sum \alpha_\lambda \geq 1 \Rightarrow d(\mathbb{1}_Y, \bigotimes_{\lambda=1} N_\lambda^{\alpha_\lambda}) \geq \dfrac{1}{(2|\alpha|)^A} \end{cases}$ //

Obs In Thm 2 (ii), $\begin{cases} S' : \mathcal{F}\text{-inv}, \\ [S'] \in H^1(V', U(1)) \end{cases}$ //

Rmk.

⊙ $(\#) \Leftarrow H^1(Y, N_{Y/X} \otimes S^{n+1} N_{Y/X}^*) = 0$ for $^\forall n \geq 1$.

$\underset{\text{for e.g.}}{\Longleftarrow}$ $\underbrace{Y: \text{sm. ellipt. curve and } N_{Y/X} : \text{Dioph.}}$ $(\#\#)$

⊙ Arnol'd '76 ... Thm 2 (i) for $(\#\#)$ (and $r=1$?)

Ueda '83 ... 〃 for $r=1$ (Thm 1).

(K-'15) K-, N. Ogawa '16 ... $r=2$.

30分ないまにくらい?

§2. Application

Def L: hol l.b. / a cpx mfd.

L: semi-positive $\underset{\text{def}}{\Longleftrightarrow}$ $^\exists h$: C^∞ Herm. metric with semi-positive curvature.

Q ~~When~~ Semi-positivity criterion (for nef l.b. / proj. mfd) ?

a typical (simple) configulation :

X: cpx mfd. $\leftarrow L$: a hol. line bdl / X.

$D_1, \cdots, D_{n-1} \in |L|$

$Y := \bigcap_{\lambda=1}^{n-1} D_\lambda$.

Obs ⊙ $^\exists$ Sing Herm. metric h_{Bergman} on L s.t. $\left\{\begin{array}{l} \sqrt{-1}\Theta_h \geq 0 \\ \{h = \infty\} = Y \\ h|_{X \setminus Y} : C^\infty \end{array}\right.$

("Bergman type metric")

⊙ If $^\exists V$: Y-nbhd $\subset X$ s.t.

$^\exists h_V$: C^∞ Herm metric on $L|_V$ with semi-positive curvature,

Then we can construct a C^∞ Herm metric on $\underline{L}$ with semi-positive curvature ("regularized min. construction")

Obs + Thm 2 (ii) $\longrightarrow$ Cor Assume $D_1, \cdots, D_{n-1}$ intersect transv'ly alg Y, and ~~[illegible]~~ Y: sm. ellipt + $L|_Y$: Dioph.

Then L: semi-positive.

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Date

Q regularizationをおとして C^k-herm. とかでは?

Q $\pi : V \to Y$ の存在 (hol. tub. nbhd の存在) のためには?

Example $(V^n, \underset{\text{ample}}{F})$: sm. del Pezzo mfd of dg $=1$.

(i.e. $F^{n-1} = K_V^{-1}$, $(F^n) = 1$)

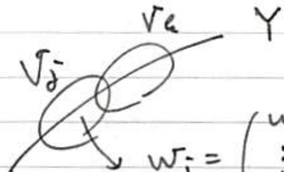
Take $p \in V$

Cor $\Longrightarrow$ $K_{Bl_p V}^{-1}$: (nef, not semi-ample, and) Semi-positive for "general" p

(c.f. $n=2$ ... $Bl_{9\,\mathrm{pts}}\mathbb{P}^2$, "Arnol'd - Ueda - Brunella")

§3. Outline of the prf

40分ぐらい



$w_j = \begin{pmatrix} w_j^1 \\ \vdots \\ w_j^r \end{pmatrix}$ $\longrightarrow$ we can take $\{w_j\}$ s.t. $\exists\, T_{jk} \in U(r)$,

$$T_{jk} \begin{pmatrix} dw_k^1 \\ \vdots \\ dw_k^r \end{pmatrix} = \begin{pmatrix} dw_j^1 \\ \vdots \\ dw_j^r \end{pmatrix}$$

$\rightsquigarrow$ $T_{jk} \cdot w_k = w_j + O(|w_j|^2)$ on V_{jk}.

⊙ Solve a "Schröder type functional eq"

$$\underset{\begin{pmatrix} v_j^1 \\ \vdots \\ v_j^r \end{pmatrix}}{w_j = v_j} + \sum_{|\alpha| \geq 2} \underset{(\ )}{F_{j,\alpha}(z_j)} \cdot v_j^{\alpha}$$

to constr. $\{v_j\}$ with $v_j = T_{jk} \cdot v_k$. $\rightsquigarrow$ $\mathcal{F} := \{\{v_j = \mathrm{const}\}\}$.

(existence of "nice" $\{F_{j,\alpha}\} \Leftarrow (\ast)$
conv. of the func. eq. $\Leftarrow N_{Y/X}$: torsion or Dioph).

⊙ For (ii), choose $\{w_j\}$ more carefully so that $\{w_j^1 = 0\} = S \cap V_j$

$\rightsquigarrow$ the solut'n $\{v_j\}$ also enjoys $\{v_j^1 = 0\} = S \cap V_j$.

$\rightsquigarrow$ $\mathcal{G} := \{\{v_j^1 = \mathrm{const}\}\}$.