GCS Path Planning Formulation

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Mathematical Formulation

We solve a Mixed-Integer Convex Program (MICP) to find a path through the graph of convex sets and optimize the trajectory within them.

Sets and Indices

- V: Set of convex regions (polygons), indexed by i.
- E: Set of edges (i, j) representing adjacency between region i and j.
- K: Degree of the Bézier curve (we use $K = 2$ for quadratic).
- d: Dimension ($d = 2$).

Variables

- $y_i \in \{0, 1\}$: Binary variable, equal to 1 if region i is visited.
- $z_{ij} \in \{0, 1\}$: Binary variable, equal to 1 if the transition from region i to j is active.
- $x_{i,k} \in \mathbb{R}^2$: Control point k ($k \in \{0, \dots, K\}$) for the Bézier curve in region i.
- $t_{i,k} \in \mathbb{R}^2$: Slack variables for L1 norm minimization.

Optimization Problem

$$\min \sum_{i \in V} \sum_{k=1}^{K} \|x_{i,k} - x_{i,k-1}\|_1$$

Subject to:

1. Flow Conservation

$$\begin{aligned}
&\sum_{j:(s,j)\in E} z_{sj} - \sum_{j:(j,s)\in E} z_{js} = 1 \quad \text{(Start Node } s\text{)} \\
&\sum_{j:(g,j)\in E} z_{gj} - \sum_{j:(j,g)\in E} z_{jg} = -1 \quad \text{(Goal Node } g\text{)} \\
&\sum_{j:(i,j)\in E} z_{ij} - \sum_{j:(j,i)\in E} z_{ji} = 0 \quad \forall i \in V \setminus \{s, g\} \\
&y_i \geq \sum_j z_{ij}, \quad y_i \geq \sum_j z_{ji}
\end{aligned}$$

2. Containment

$$\begin{aligned}
&A_i x_{i,k} \leq b_i + M(1 - y_i) \quad \forall i \in V, k \in \{0, \dots, K\} \\
&-My_i \leq x_{i,k} \leq My_i \quad \text{(Force } x_{i,k} = 0 \text{ if } y_i = 0\text{)}
\end{aligned}$$

3. Continuity (C^0)

$$\|x_{i,K} - x_{j,0}\|_\infty \leq M(1 - z_{ij}) \quad \forall (i,j) \in E$$

4. Heading Consistency (C^1)

$$\|(x_{i,K} - x_{i,K-1}) - (x_{j,1} - x_{j,0})\|_\infty \leq M(1 - z_{ij}) \quad \forall (i,j) \in E$$

5. Boundary Conditions

$$x_{s,0} = p_{\text{start}}, \quad x_{g,K} = p_{\text{goal}}$$

Note: The L1 norm objective and infinity norm constraints are used to keep the problem linear (MILP), which is compatible with the HiGHS solver. M is a sufficiently large constant (”Big-M”).