H_∞ Control of a Quadrotor Biplane Tailsitter UAV

Tanay Kumar, Mangal Kothari, Raktim Bhattacharya

Presenter: Prateek Priyaranjan Pradhan

Intelligent Guidance & Control Laboratory

Intelligent Systems Research Laboratory
Aerospace Engineering, Texas A&M University

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Introduction

Quadrotor Biplane Tailsitter (QBiT) UAV

Multirotor UAV

<u>Cons</u>

- Low endurance
- Inefficient
- Fly at low speeds

<u>Pros</u>

- VTOL
- Hover
- Cruise in low speed regime

Fixed Wing UAV

<u>Pros</u>

- High speed and altitude
- More endurance
- Efficient

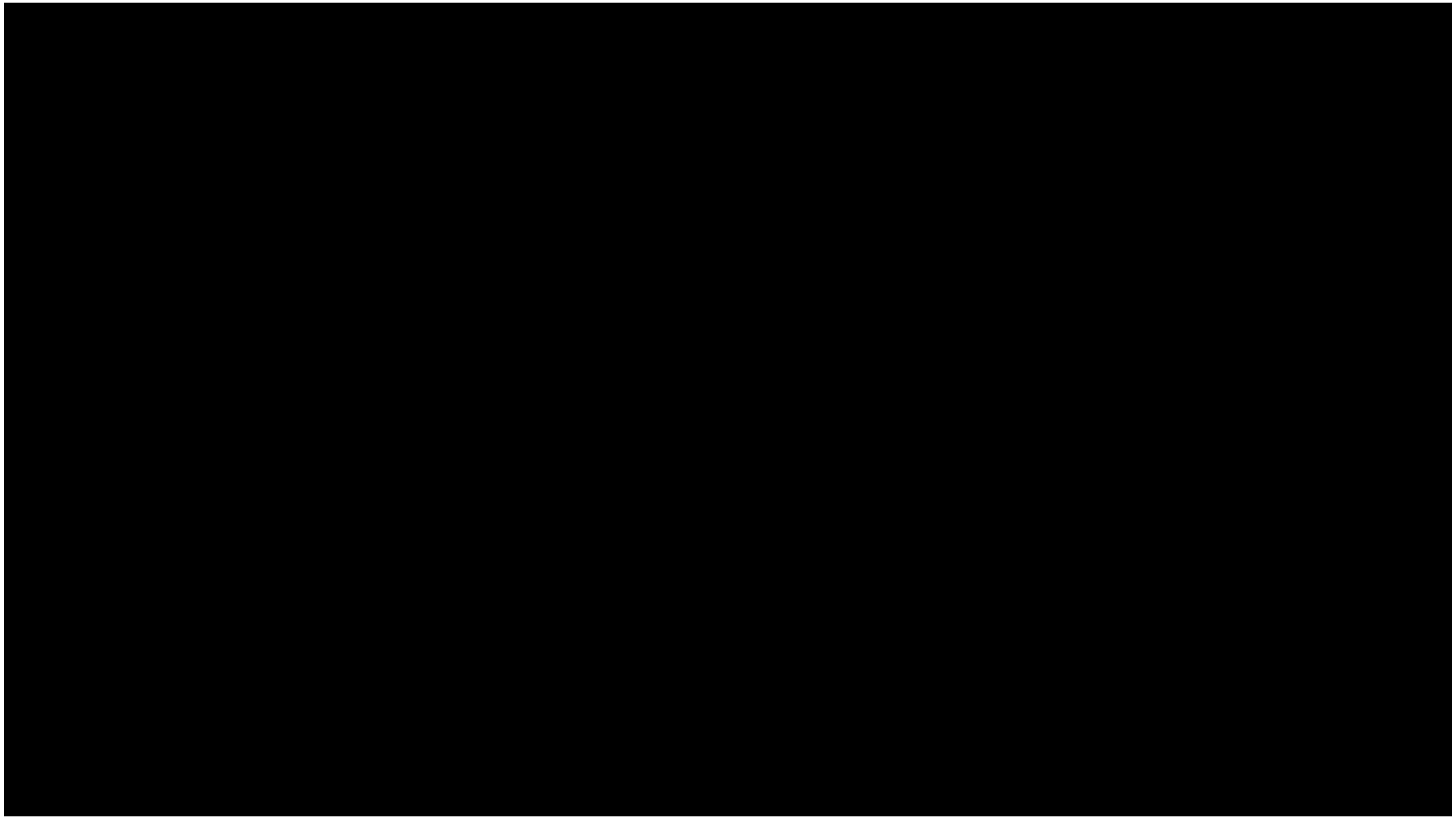
<u>Cons</u>

- Need long runways
- Can not hover

QBiT UAV

Introduction

Quadrotor Biplane Tailsitter (QBiT) UAV

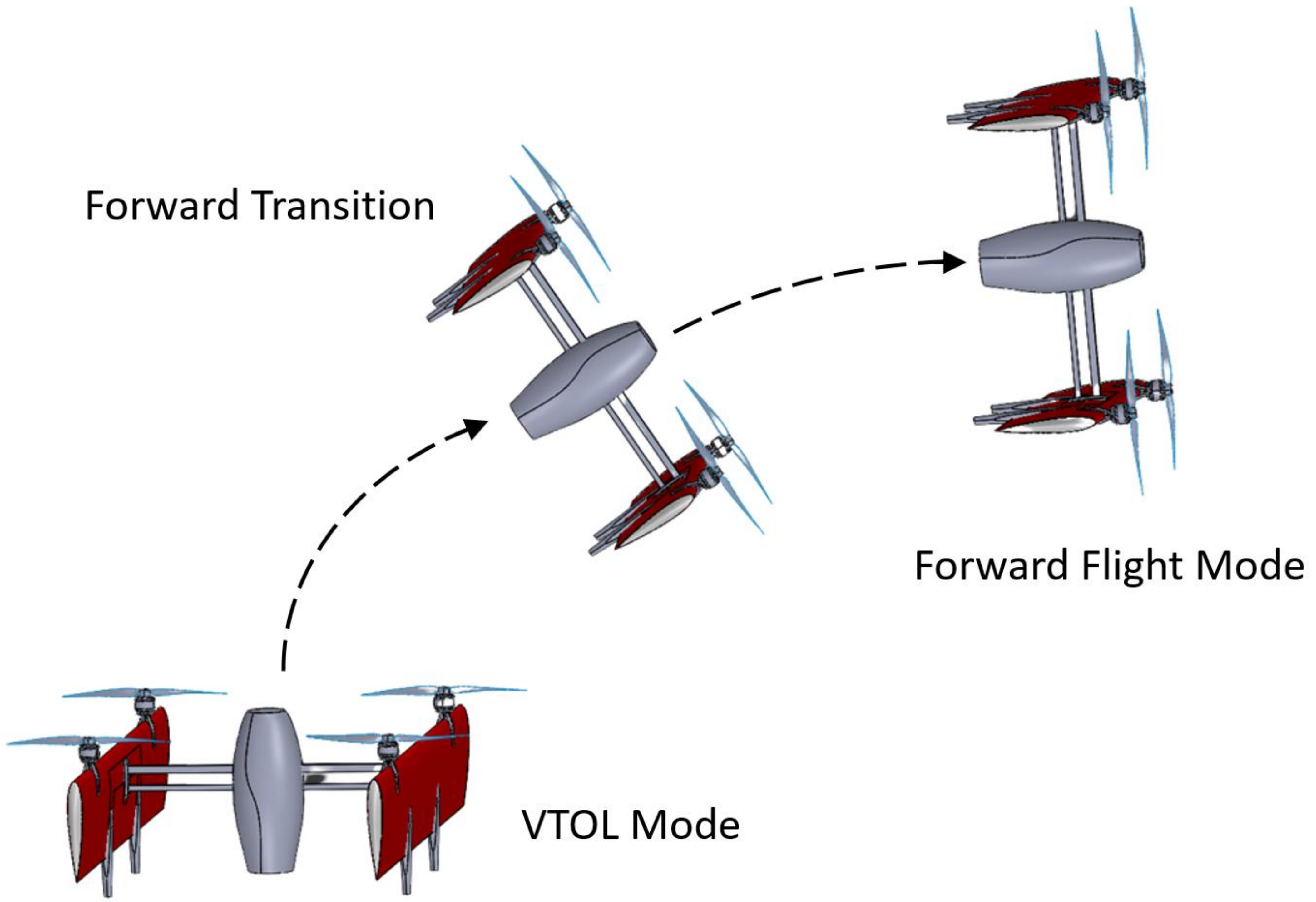


- Simple design configuration
- Distributed propulsion
- Failsafe design
- High efficiency **compact** VTOL system
- Novel robust proprietary autopilot software

Aim

Goal: To develop a robust controller for VTOL, forward-transition, and forward flight mode of the QBiT UAV.

- Inherently unstable due to complex and nonlinear flight dynamics.
- Susceptible to disturbances and uncertainties.
- Controller should work in all conditions regardless of:
  - Operating conditions may vary as per the mission requirements.
  - Wind and other environmental disturbances.
  - Model uncertainties or parametric variations.
  - Uncertainties in the physical system because of the manufacturing process.
  - Unmodelled system dynamics.



Literature Review

Control of QBiT UAV

Oosedo et al. (2013)

- Compared two configurations for attitude tracking in the presence of a slipstream.
- High accuracy attitude control validated experimentally.
- PID Control used for large attitude error.

Swarnkar et al. (2018)

- Improved flight dynamics model for controller design.
- Quaternions are used to avoid singularity.
- Nonlinear control using dynamic inversion approach.

Raj et al. (2020), *ICRA*

- Iteratively learn forward transition via repeated flights.
- Nominal plant model with simplified aerodynamics used for optimal polynomial coefficients.
- Coefficients are updated using repeated trials.
- Geometric attitude controller used.

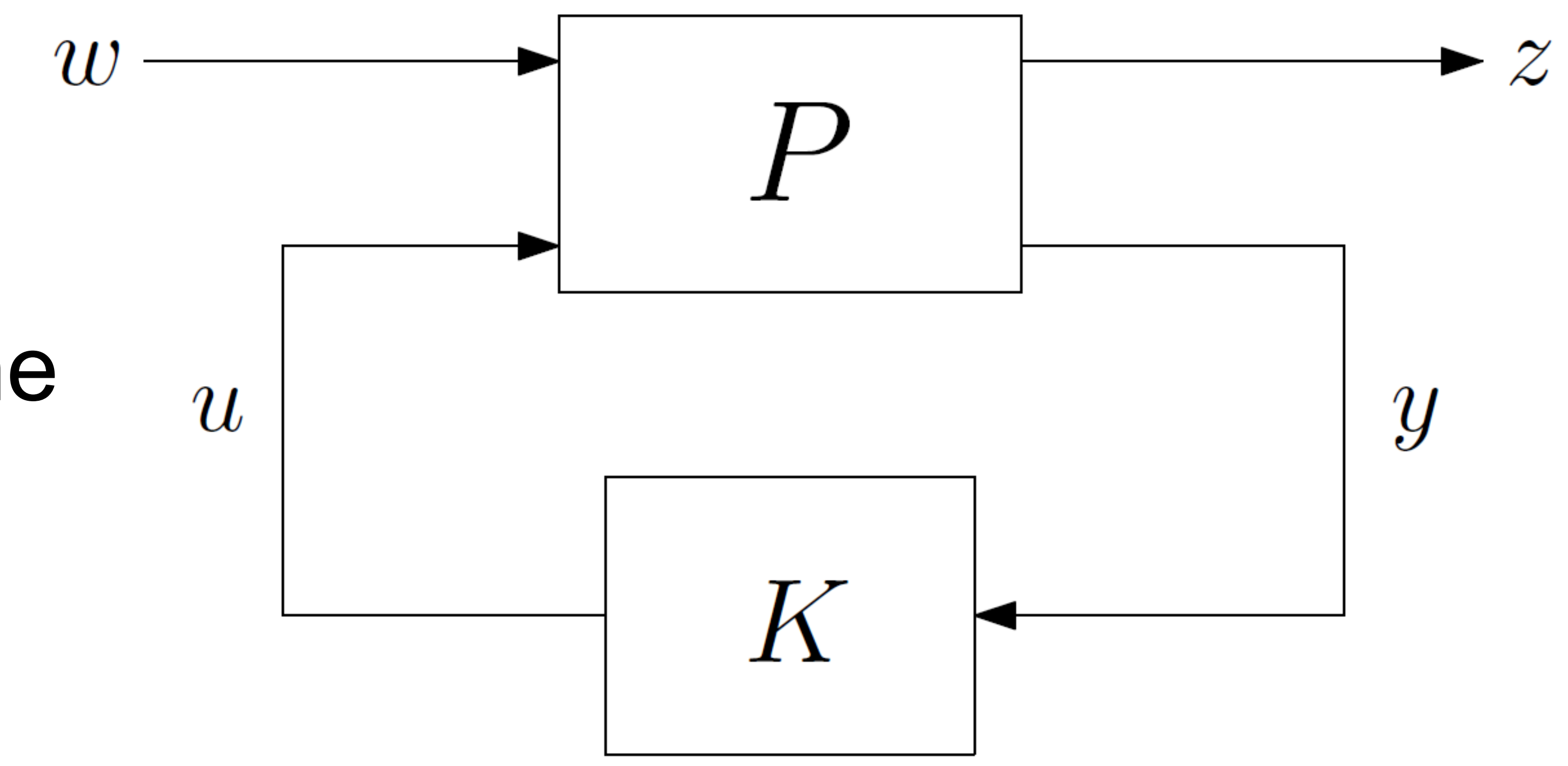
Raj et al. (2020), *JGCD*

- Swivelling biplane with configuration-dependent inertia.
- Dynamic feedback linearization on nominal model.

References

H_∞ Control

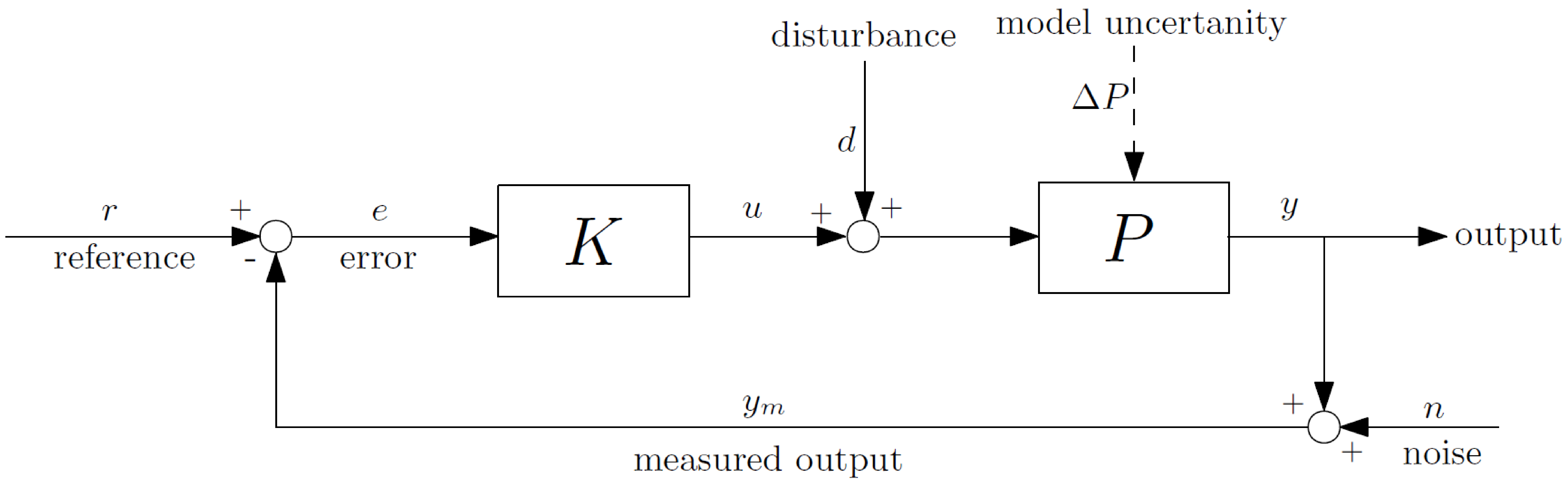
- *Aim*: To make the closed-loop system robust by explicitly accounting for uncertainties.
- Find an internally stabilizing controller K that minimizes the H_∞ norm of the Linear Fractional Transformation (LFT) represented by $\underline{S}(P, K)$.
- Norm minimization converted to equivalent LMI and solved using convex optimization. (*Dullerud et al. 2013*)



$$e = Sr - Sd + Tn$$

but

$$\boxed{S + T = 1}$$



$$S = \frac{1}{1+PK} \text{ and T } = \frac{PK}{1+PK}$$

Feedback Problem

	Closed loop requirements
Low frequency	$S\approx0$ $T\approx1$ high $\lvert PK\rvert$
High Frequency	$S\approx1$ $T\approx0$ low $\lvert PK\rvert$

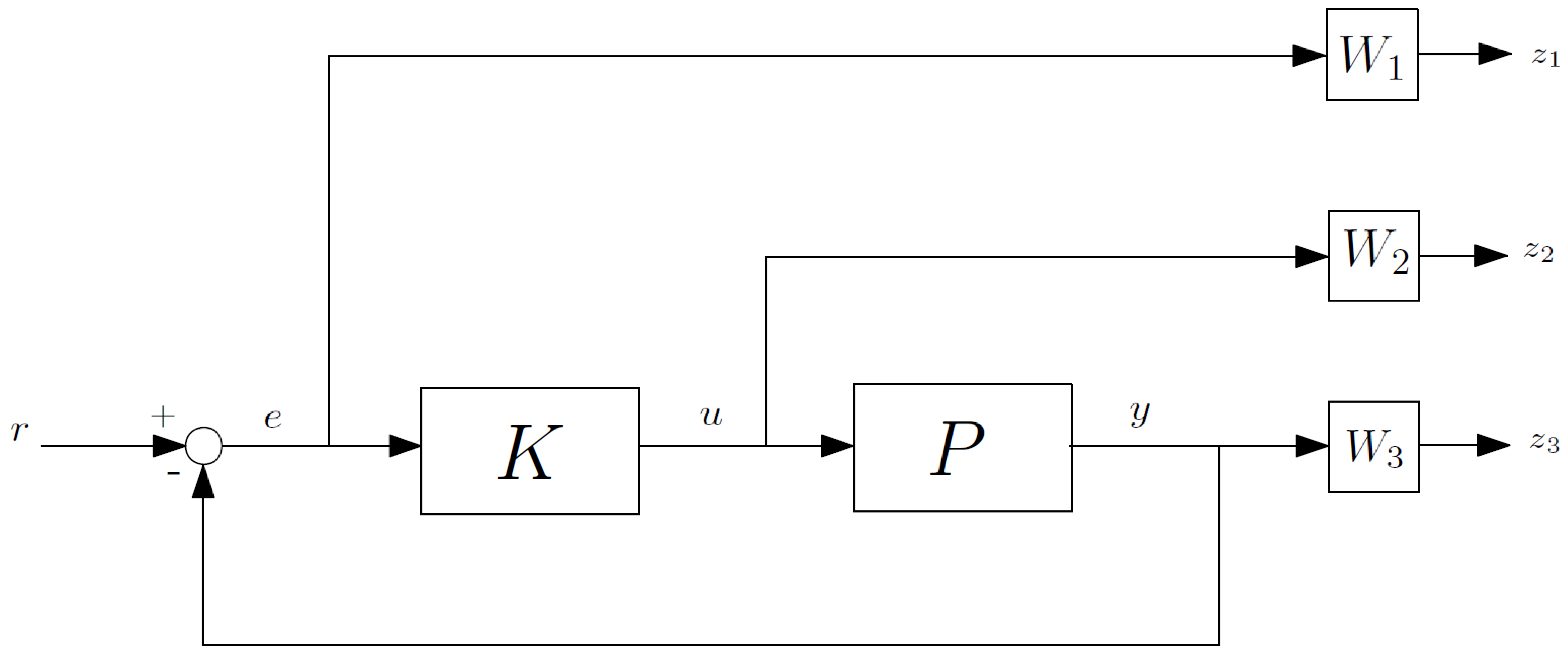
H_∞ Control

- Frequency-dependent weights specified in weighted sensitivity loop-shaping. (*Skogestad et al. 2005*)

- Controller synthesized by shaping frequency responses simultaneously for reference tracking and disturbance rejection, noise attenuation and robustness, and control effort.

- *Control Problem*: Minimize the LFT of the augmented plant

$$\min_K \|F_l(P,K)\|_{\mathcal{H}_\infty} = \left\| \begin{matrix} W_1 S \\ W_2 K S \\ W_3 T \end{matrix} \right\|_{\mathcal{H}_\infty} < \gamma$$



References

Flight Dynamics Model

- Newton-Euler equations were used to write the kinematics and dynamics equations of the vehicle (*Beard et al. 2012).*
- The physical and inertia parameters of the UAV, the wing's aerodynamic stability derivatives, and aerodynamic force and moment equations are given in *Swarankar et al. (2018)* and *Stevens et al. (2015).*
- A 3-1-2 rotation sequence is chosen to avoid singularity in the pitch axis during transition.

Parameters	Mass	Wing span	Wing area (single)	Root chord	Tip chord	Aspect ratio
Values	12 kg	2.29 m	0.754 m^2	0.39 m	0.176 m	6.9
Parameters	Gap-to-chord ratio	X_{cg}	X_{ac}	I_x	I_y	I_z
Values	2.56	0.157 m	0.0838 m	1.86 $kg \cdot m^2$	2.031 $kg \cdot m^2$	3.617 $kg \cdot m^2$

References

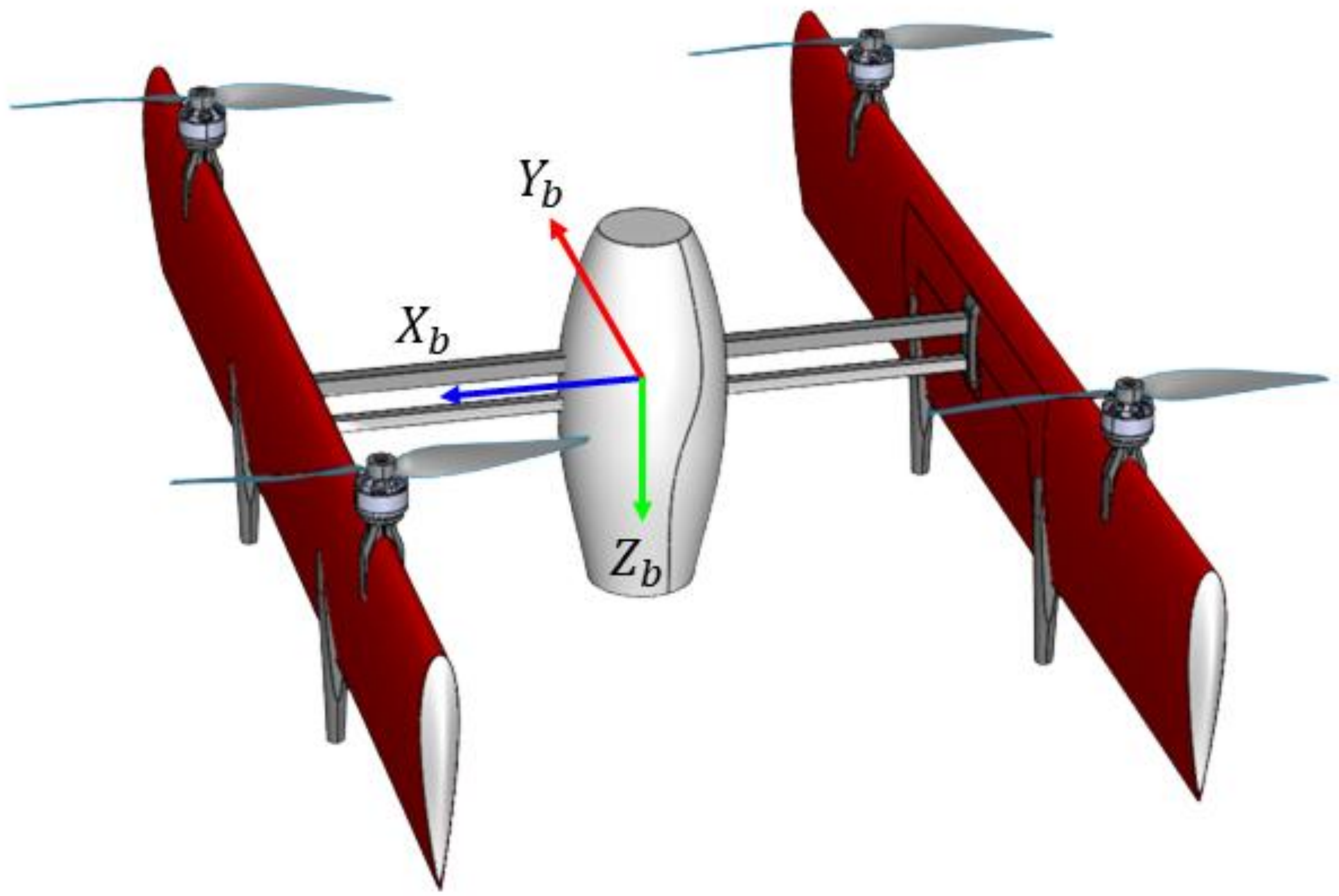
Controller Design
VTOL Mode – Position Control

Outer loop control is given by solving the second-order stable error dynamics which gives the desired thrust, roll and pitch angles *(Gupta et al. 2016)*

$$T_d = m\sqrt{\ddot{x}^2 + \ddot{y}^2 + (g - \ddot{z})^2}$$

$$\phi_d = \arcsin\left[\frac{-m\ddot{x}\sin\psi_d + m\ddot{y}\cos\psi_d}{T_d}\right]$$

$$\theta_d = \arcsin\left[\frac{-m\ddot{x}\cos\psi_d - m\ddot{y}\sin\psi_d}{T_d\cos\phi_d}\right]$$



References

Controller Design

VTOL Mode – Attitude Control

- The nonlinear dynamics model is linearized about the trim state and is used to derive the controller.
- The following design weights are selected:

$$W_1 = \left(\frac{0.5s + 15}{s + 0.011}\right)^2 \qquad W_2 = 0.001$$

- W_3 is neglected in the design process as the filters and estimation algorithms significantly reduce the noise.

- The close loop bandwidth of the system is 28.2 rad/sec, which confirms the design requirements.

	Controller	γ
Roll	$\frac{2.054e03s^3+1.568e04s^2+30.39s+0.0003}{s^4+167.4s^3+6144s^2+135.1s+0.743}$	1.6430
Pitch	$\frac{22.978e04s^3+2.301e05s^2+603.6s+0.007959}{s^4+1960s^3+7.92e02s^2+1742s+9.578}$	1.6142
Yaw	$\frac{6.224e04s^3+4.28e05s^2+841.4s+0.006741}{s^4+3106s^3+1.075e03s^2+2363s+12.999}$	2.0417

Weight Selection

Controller Design

Forward Transition

Position Control

- Forward transition is done in an ad-hoc manner.
- The desired pitch angle is linearly increased from $0°$ to trim pitch angle for forward flight mode.
- Thrust is also increased till the wings pass the stall angle.
- Edgewise motion of the propeller causes significant downwash.

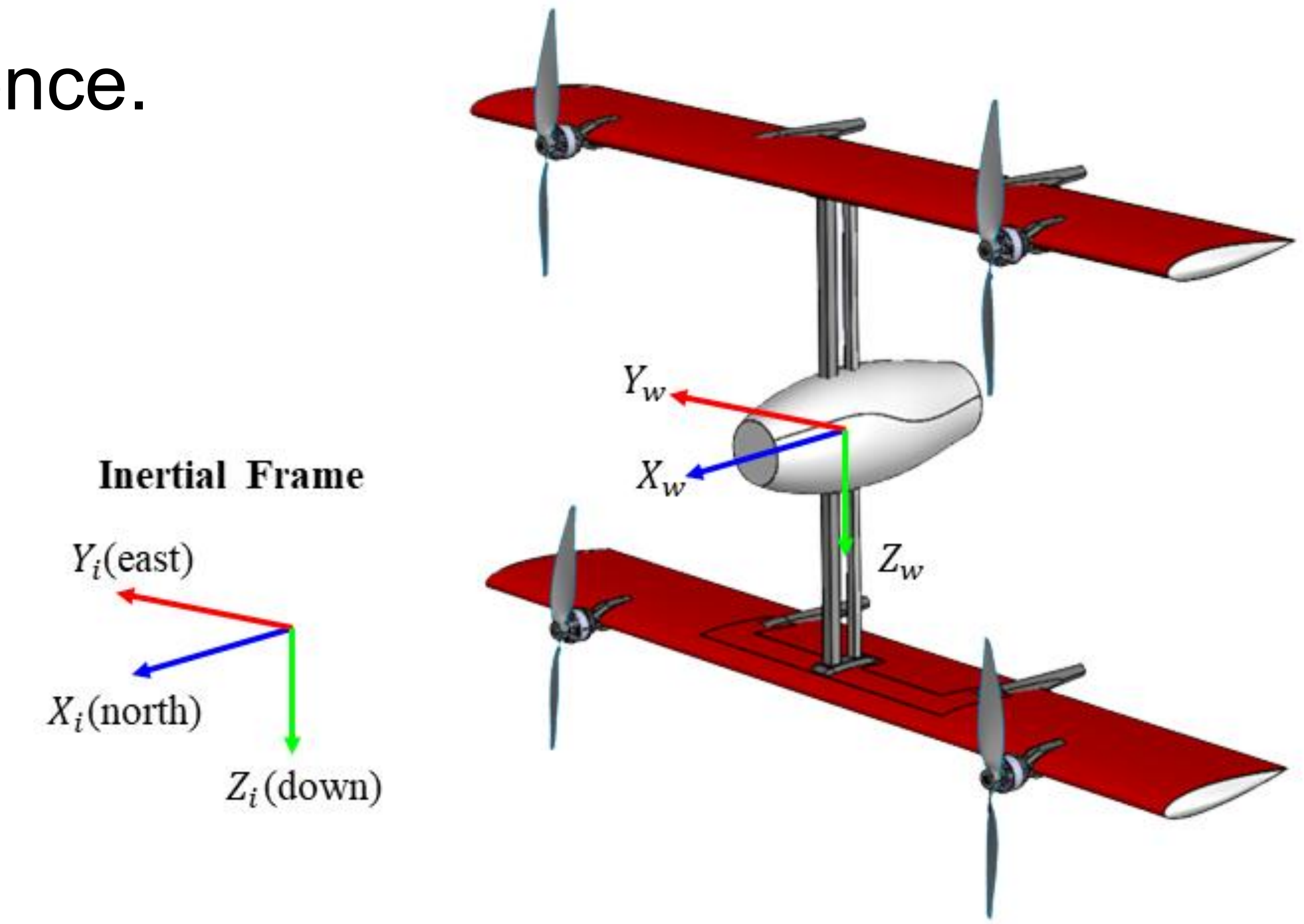
Attitude Control

- Same as that in VTOL mode.
- The controller is expected to handle uncertainties due to the change in the flight regime for which the controller was designed.

Controller Design

Forward Flight – Position Control

- The two inputs (thrust and pitch angle) affect two outputs (altitude and airspeed).
- Simultaneous control is done using the Total Energy Control System (TECS) *(Lambregts et al.).*
- Problem formulation in terms of energy difference and not set point difference.



- Specific energy rate is given as $\dot{E} \approx \frac{\dot{V}}{g} + \gamma$ and from dynamic equations of aircraft $T - D \approx \frac{\dot{V}}{g} + \gamma$
- Change in thrust is given as $\Delta T = mg\left(\frac{\dot{V}}{g} + \gamma\right)$
- To handle changes in kinetic and potential energy, specific energy balance rate is given by $\dot{B} = \gamma_f - \frac{\dot{V}}{g}$
- Similarly, the required change in pitch angle from the trim pitch angle is proportional to $\dot{B}$.
- The desired values can be obtained using a simple PI controller.

Controller Design

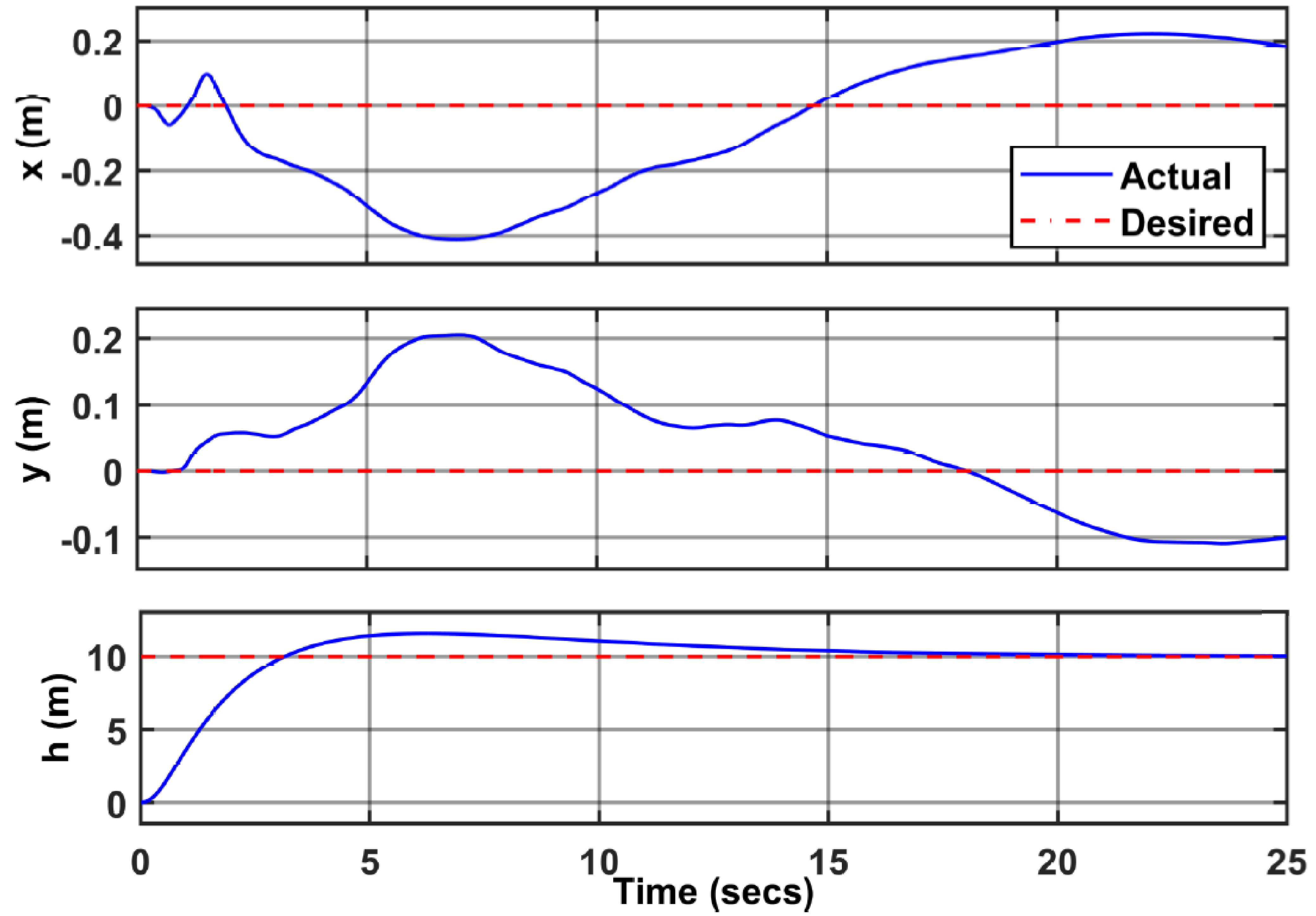
Forward Flight – Attitude Control

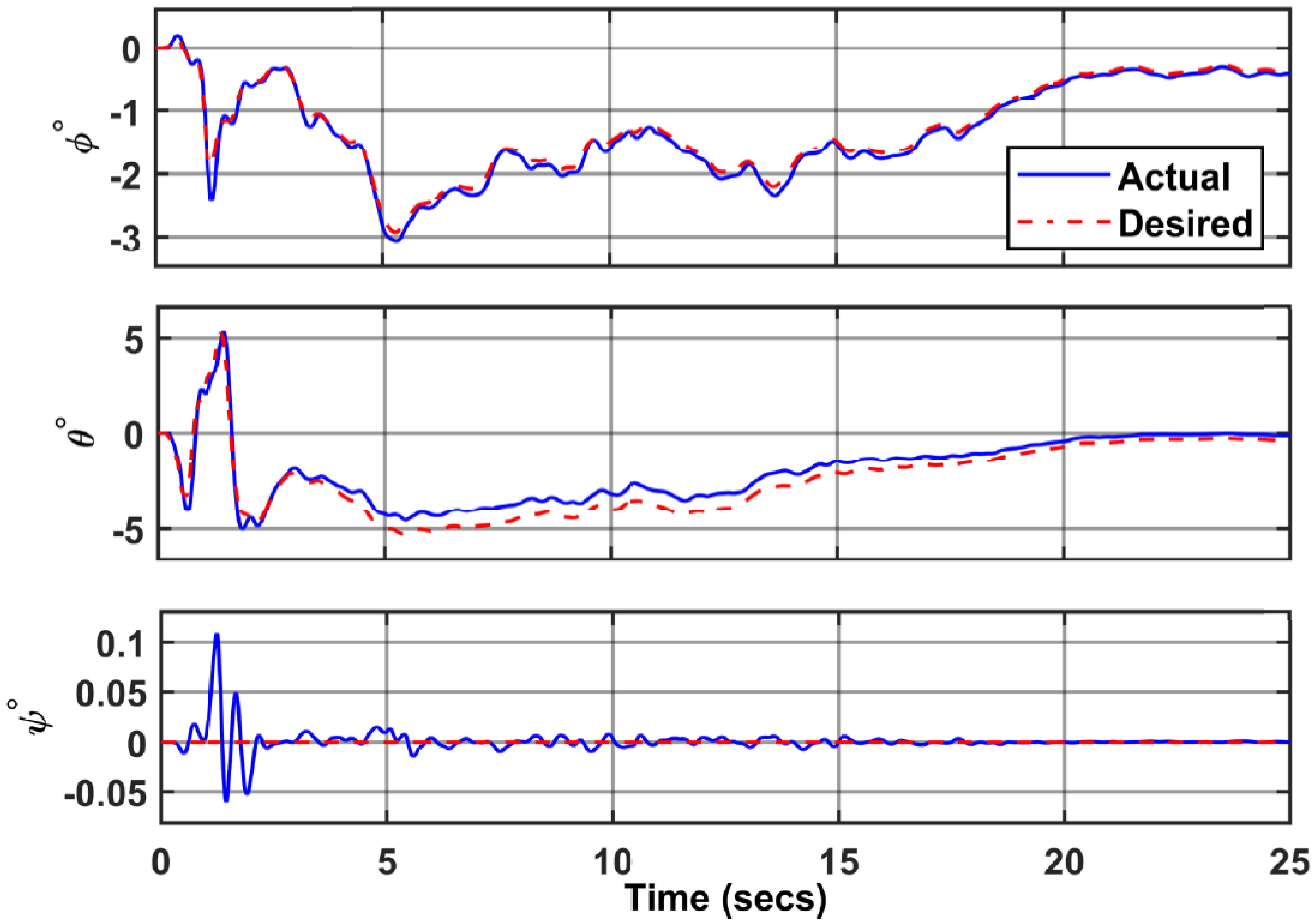
- Same as that in VTOL mode.
- The uncertainties caused by the change in flight regime are within the controller bandwidth.
- Therefore, a single controller can be used on all three modes, unlike other control schemes.
- Vehicle dynamics and controller defined in quadrotor mode.
- However, the vehicle pitches by $90°$ so the roll and yaw control need to be interchanged.
- Rotation matrix $R_b^w \in SO(3)$ converts vector from quadrotor body frame to fixed wing frame.

$$R_b^w = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Results

VTOL Mode



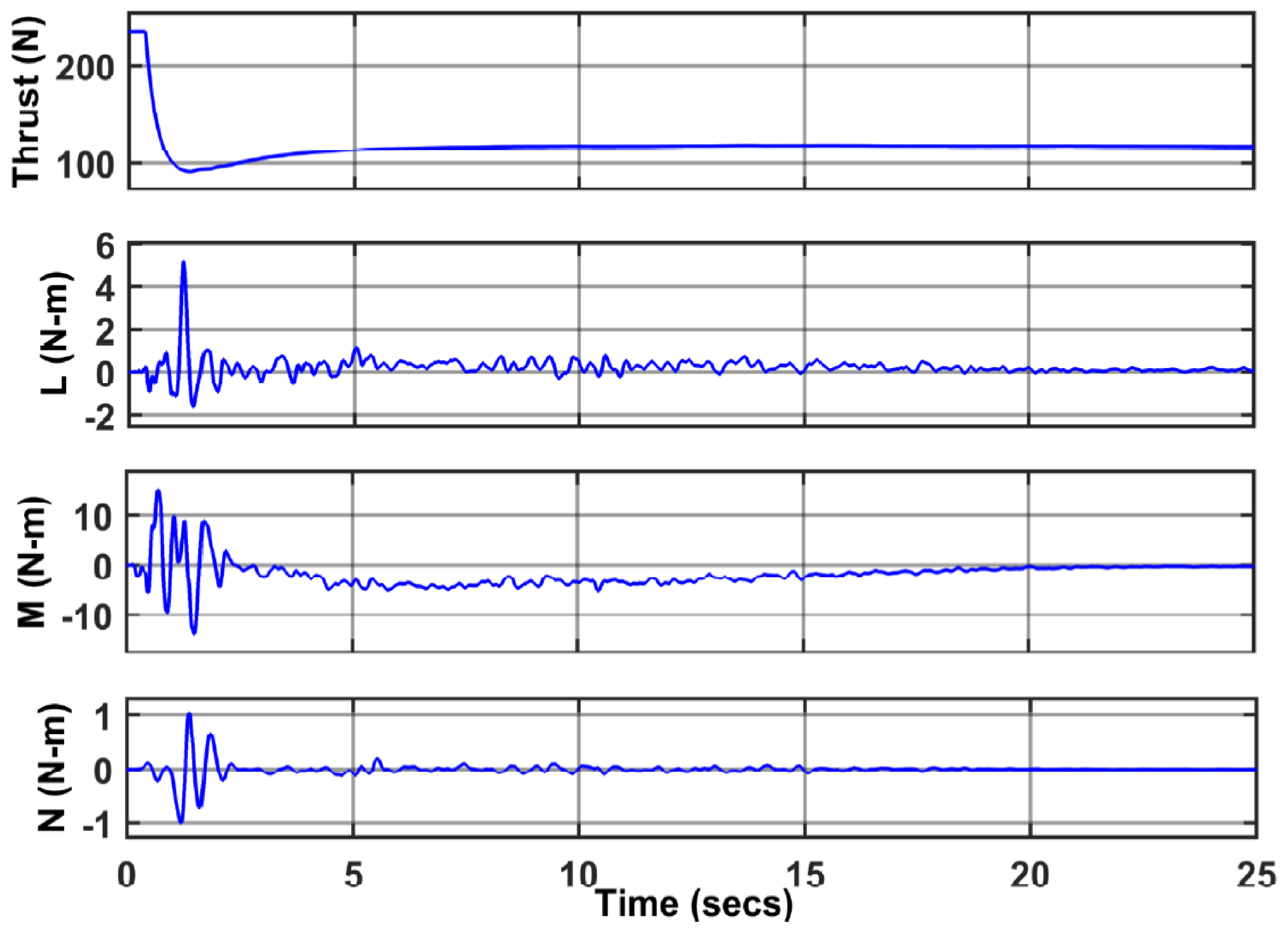


- Initial drift due to the lift produced by the wings and wind gusts.
- Drifting counteracted with minimum overshoot.

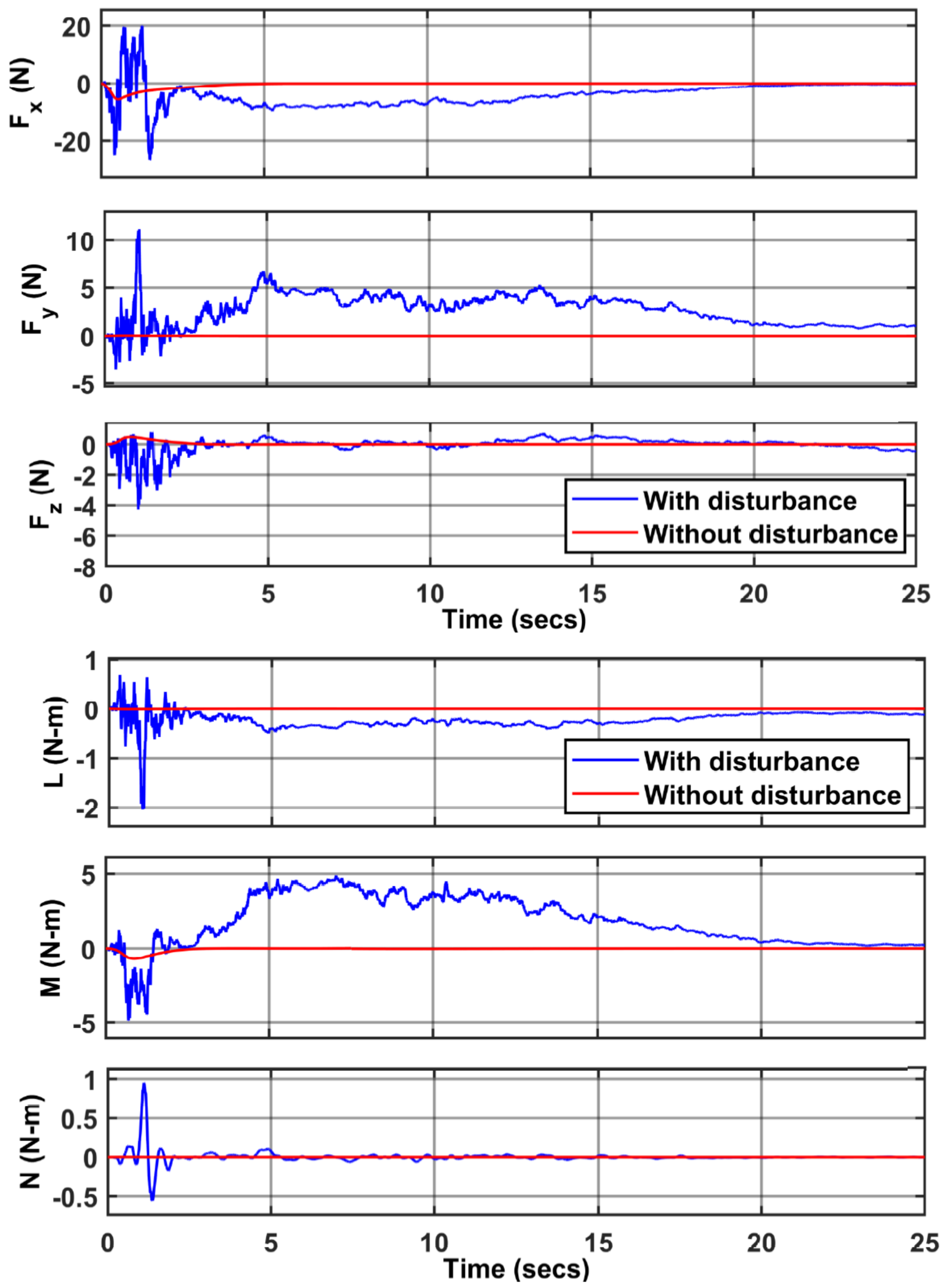
- Excellent reference tracking and disturbance rejection despite high mass and inertia.
- Variations in attitude to compensate for disturbance moments.

Results

VTOL Mode



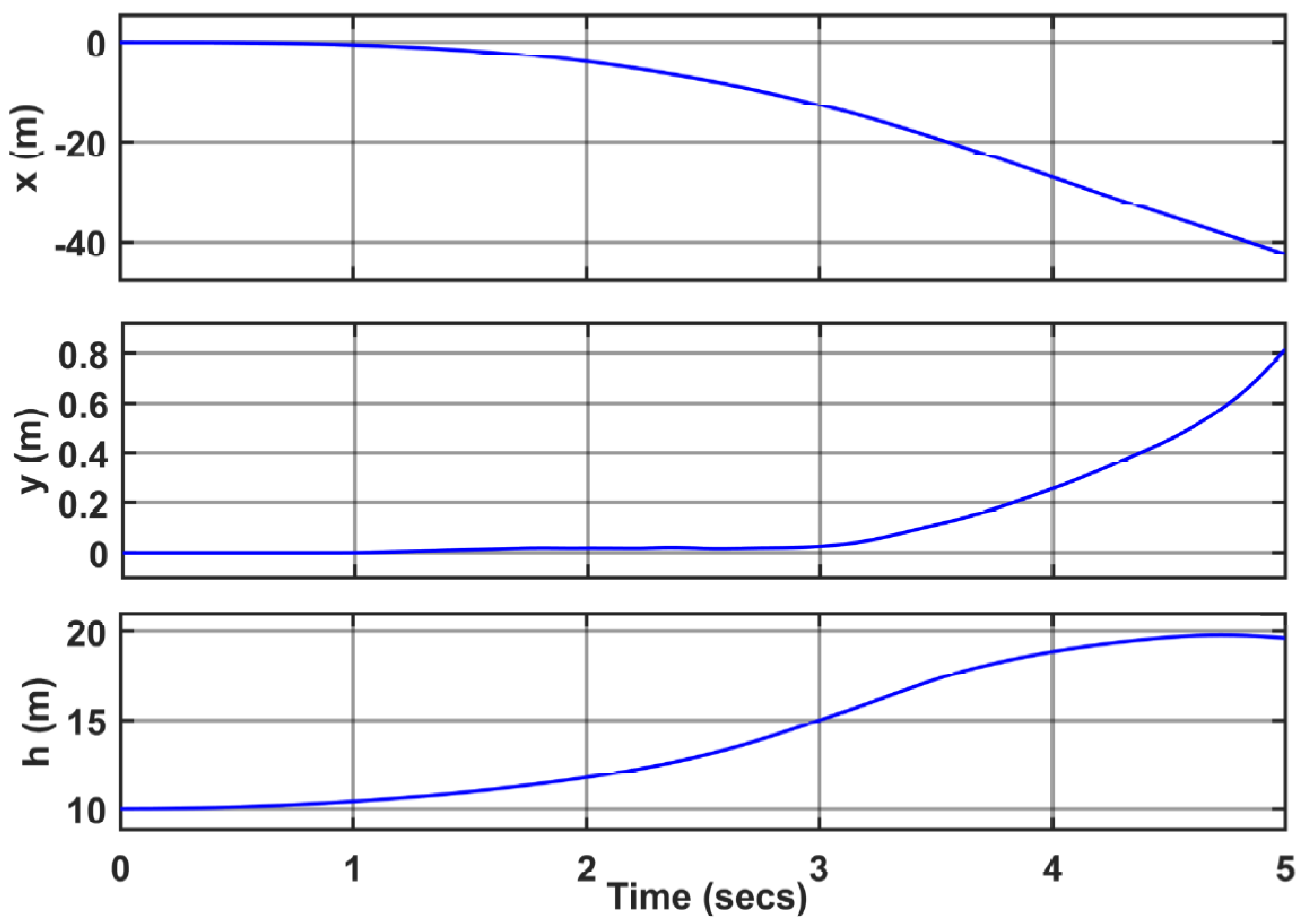
Control inputs required to be produced by the actuators



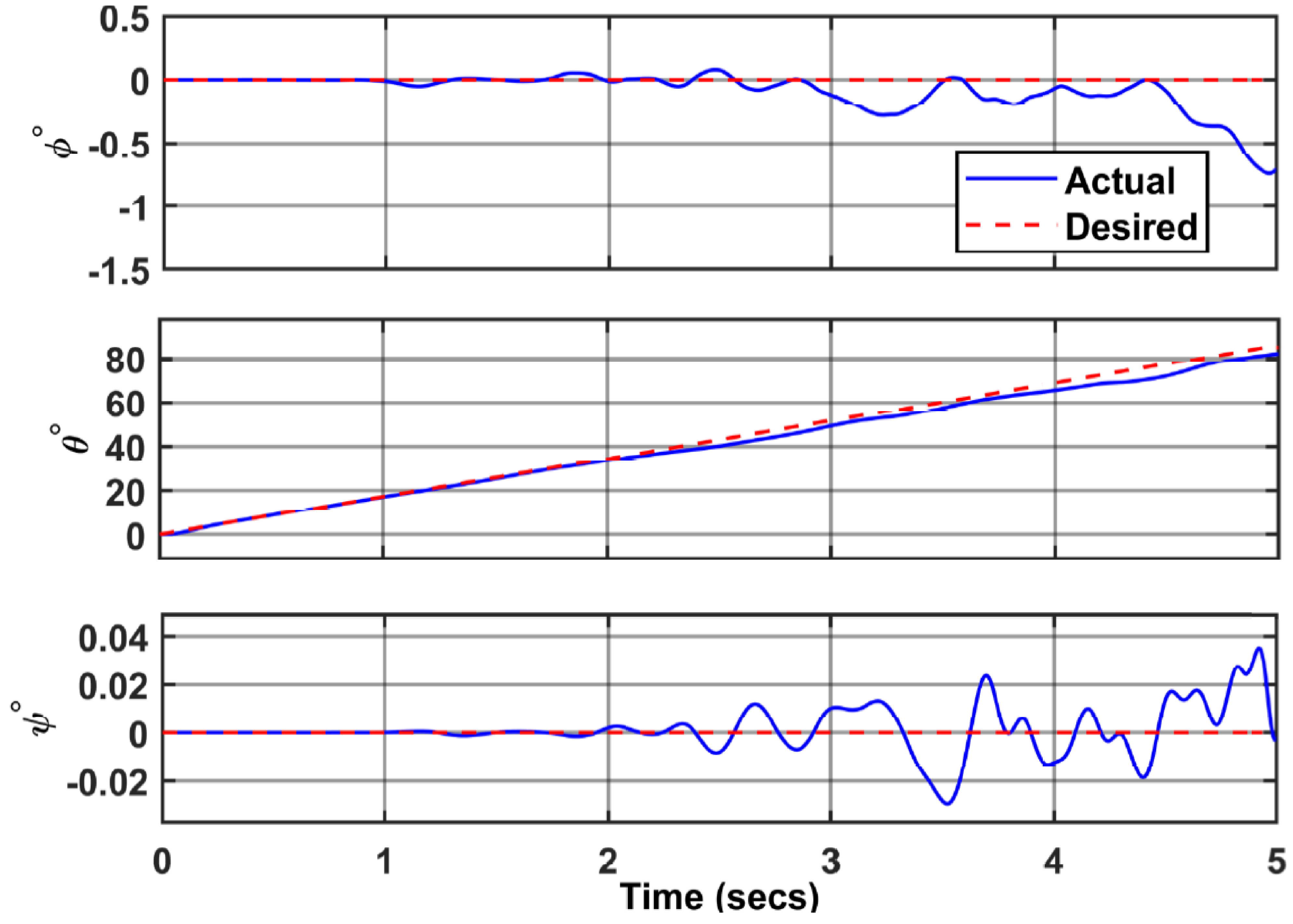
Aerodynamic forces and moments produced by the wings

Results

Forward Transition



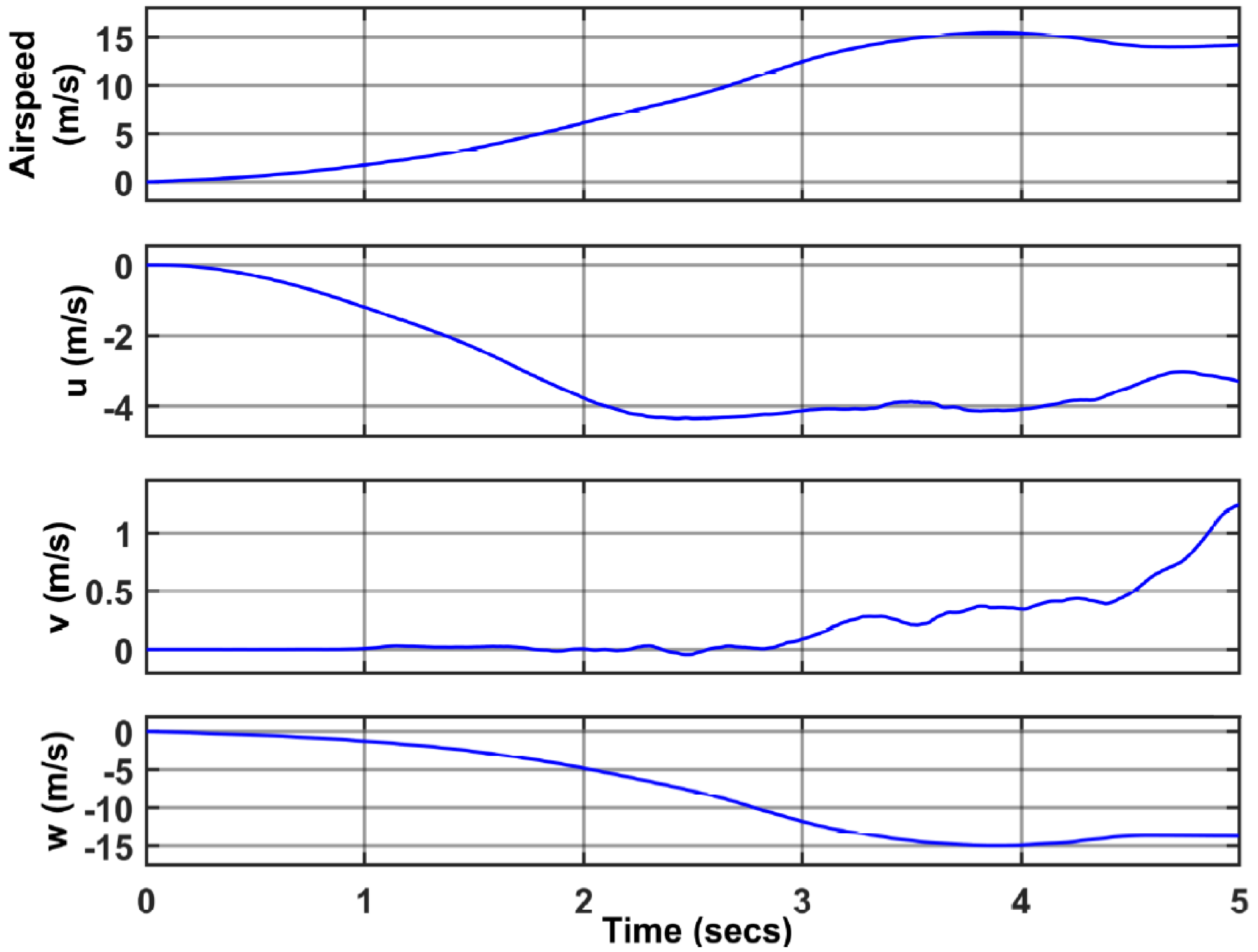
Increase in the altitude due to an increase in the upward force.

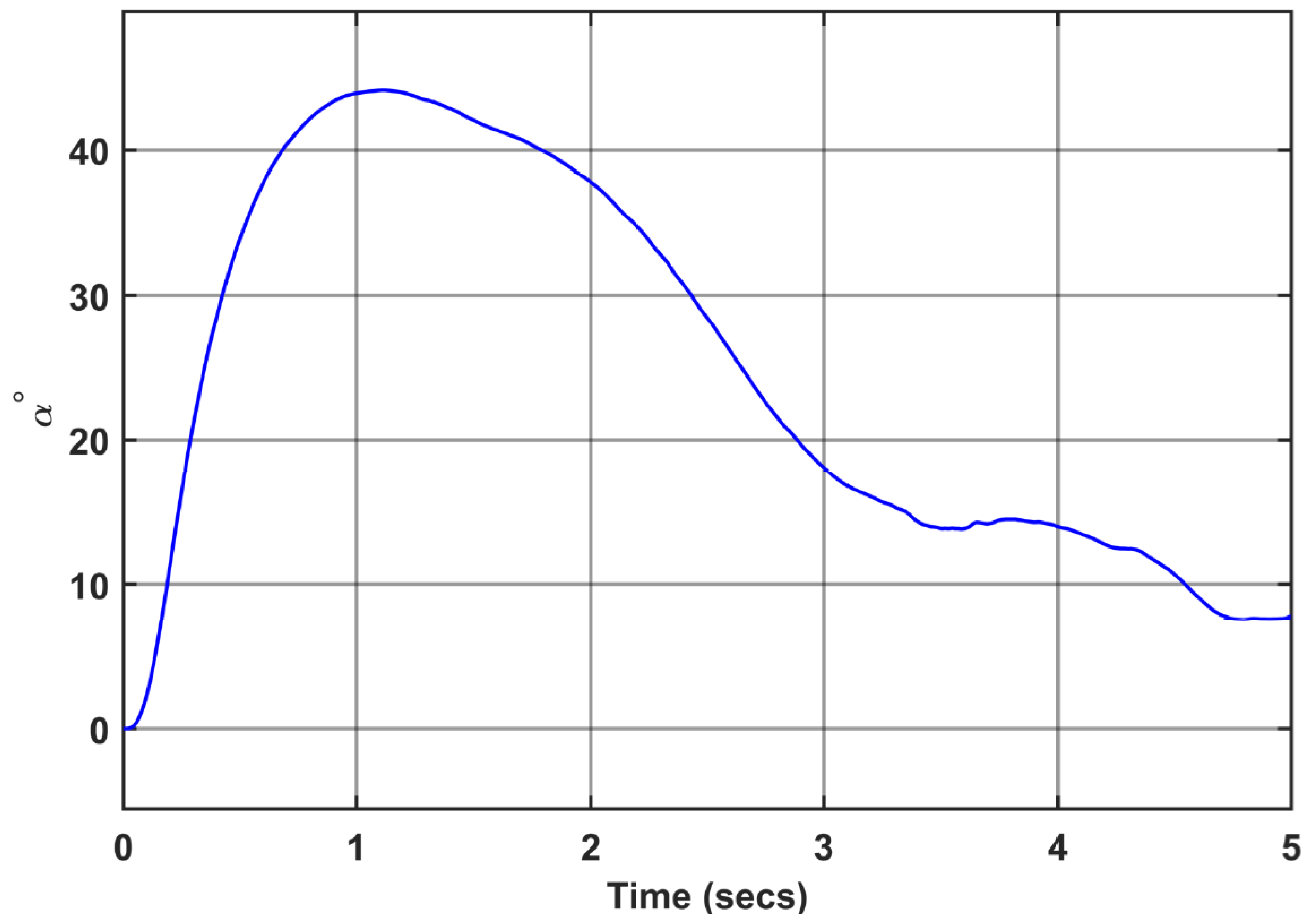


Vehicle transitions from $0°$ to θ_{trim} in 5 secs.

Results

Forward Transition

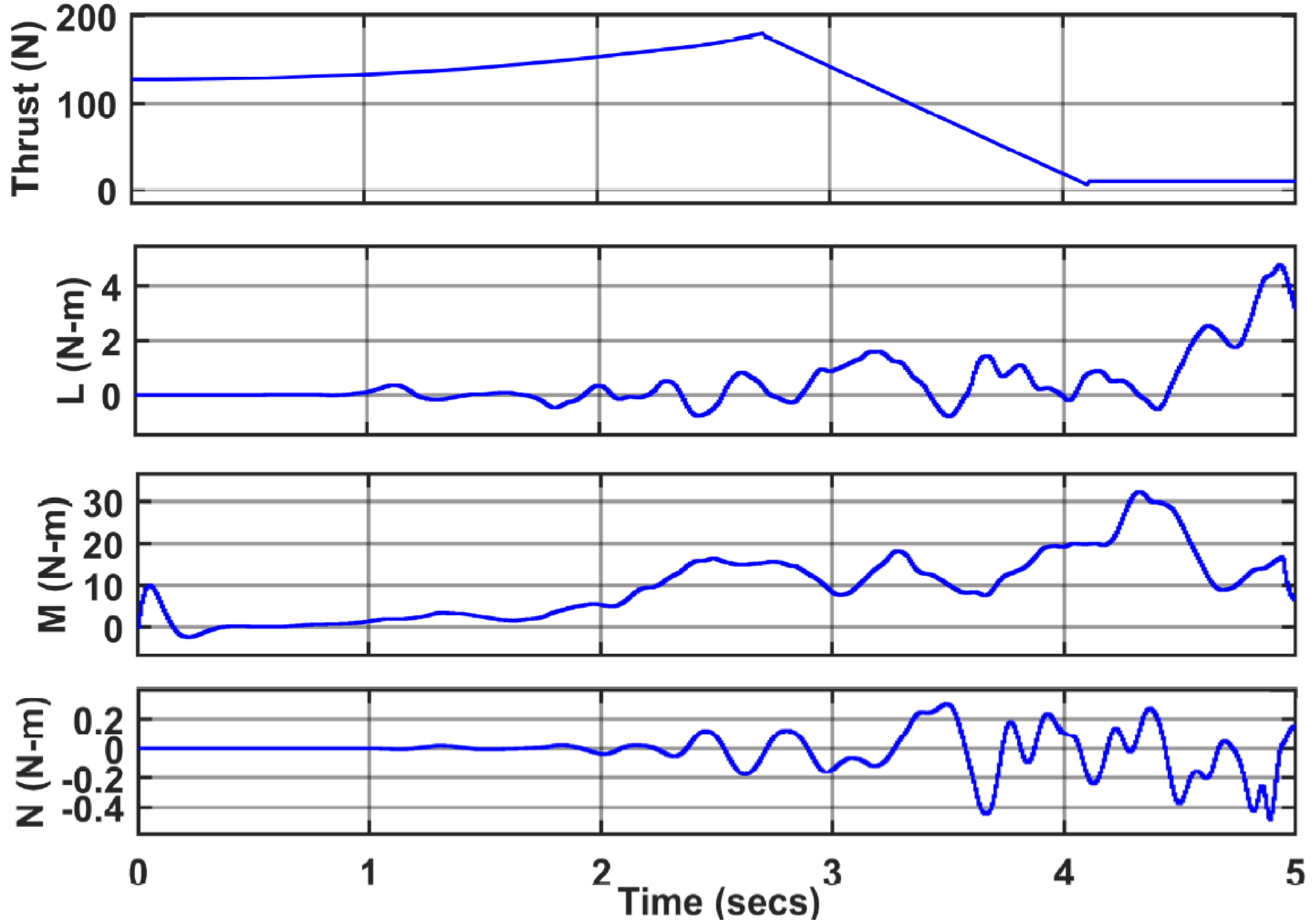




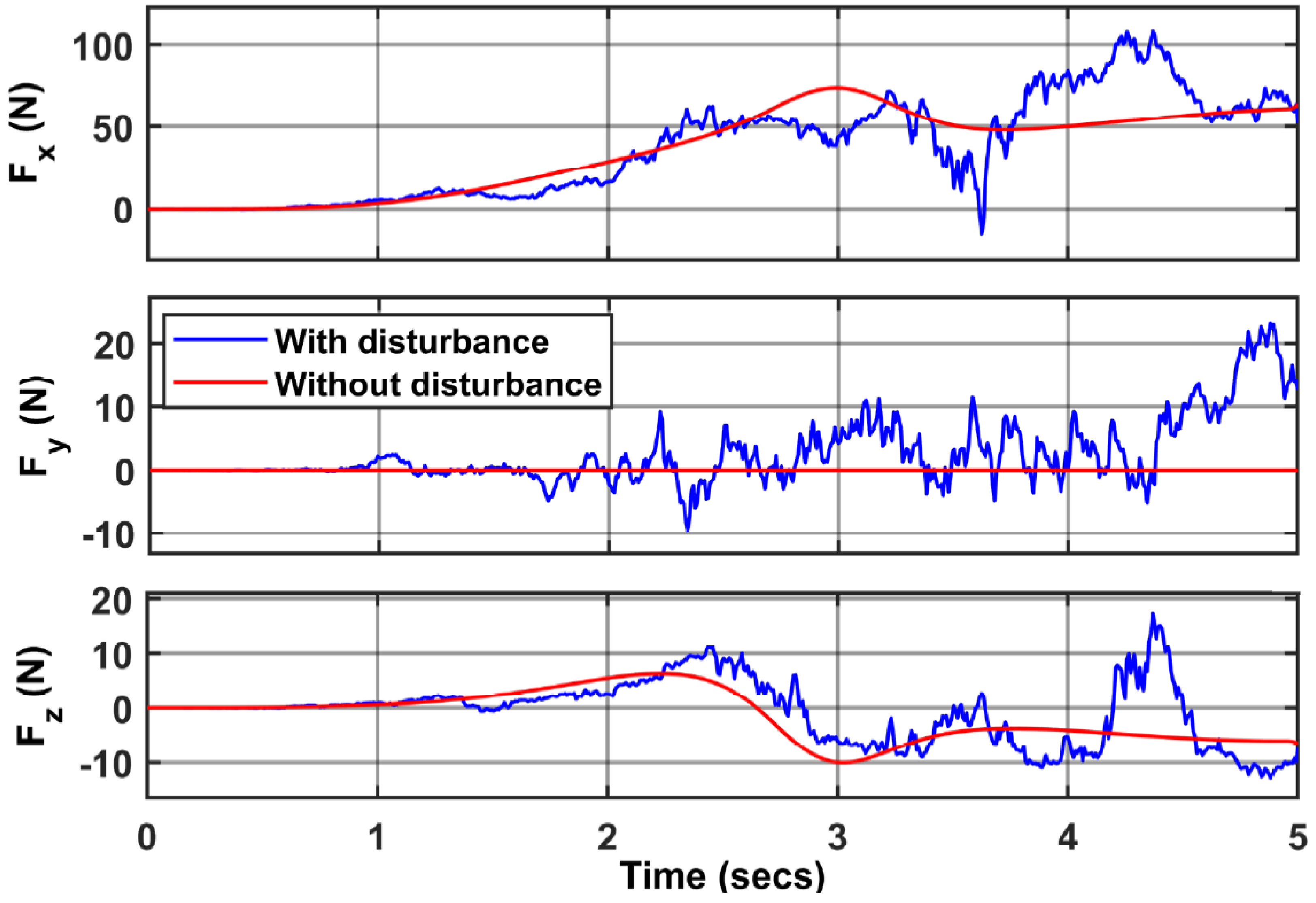
After transition, the cruise speed of 16 m/s is achieved.

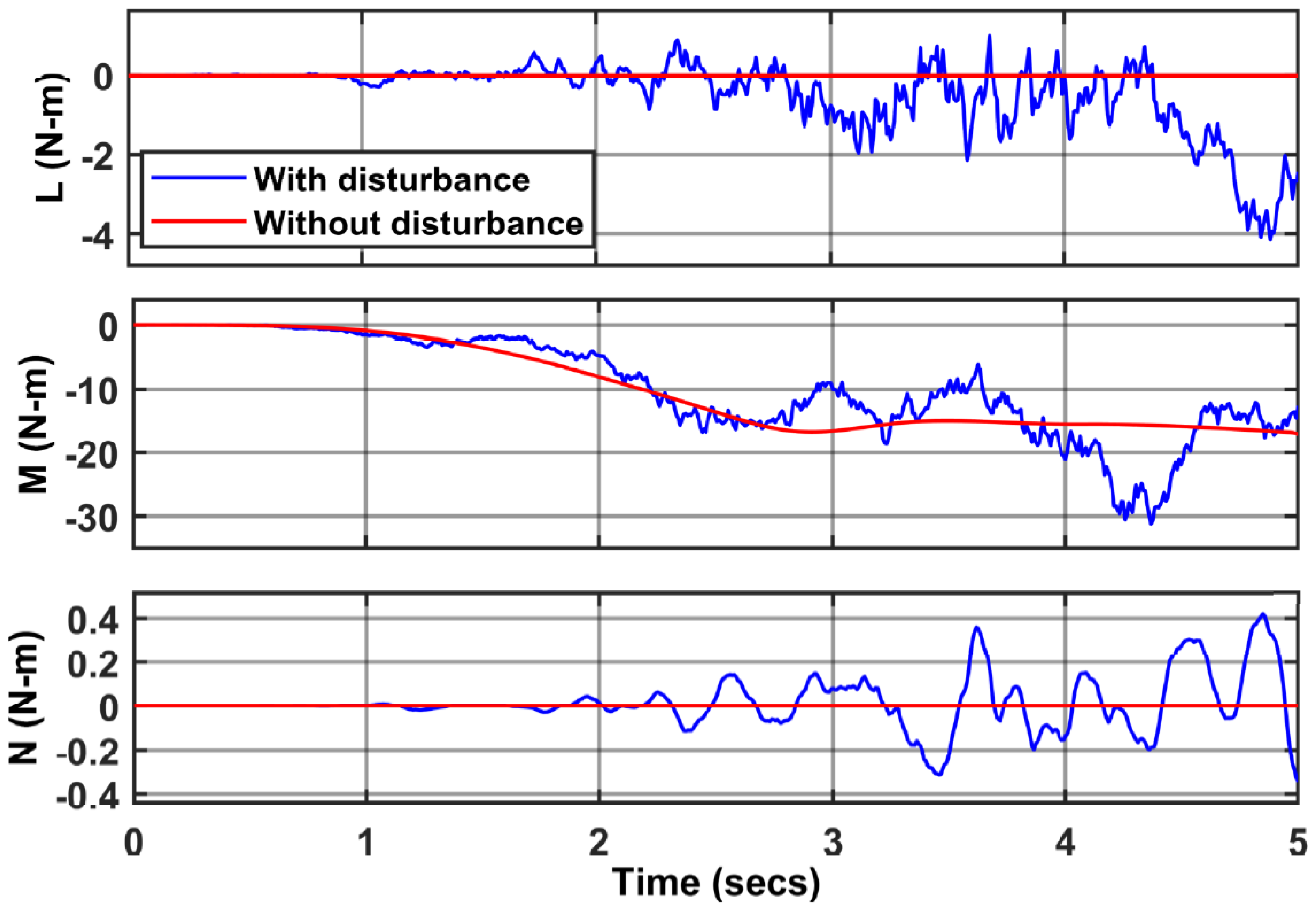
AoA decreases from post-stall value to trim value of about 7.5°

Results
Forward Transition



- Control inputs required to be produced by the actuators.
- Thrust initially increases and then decreases.

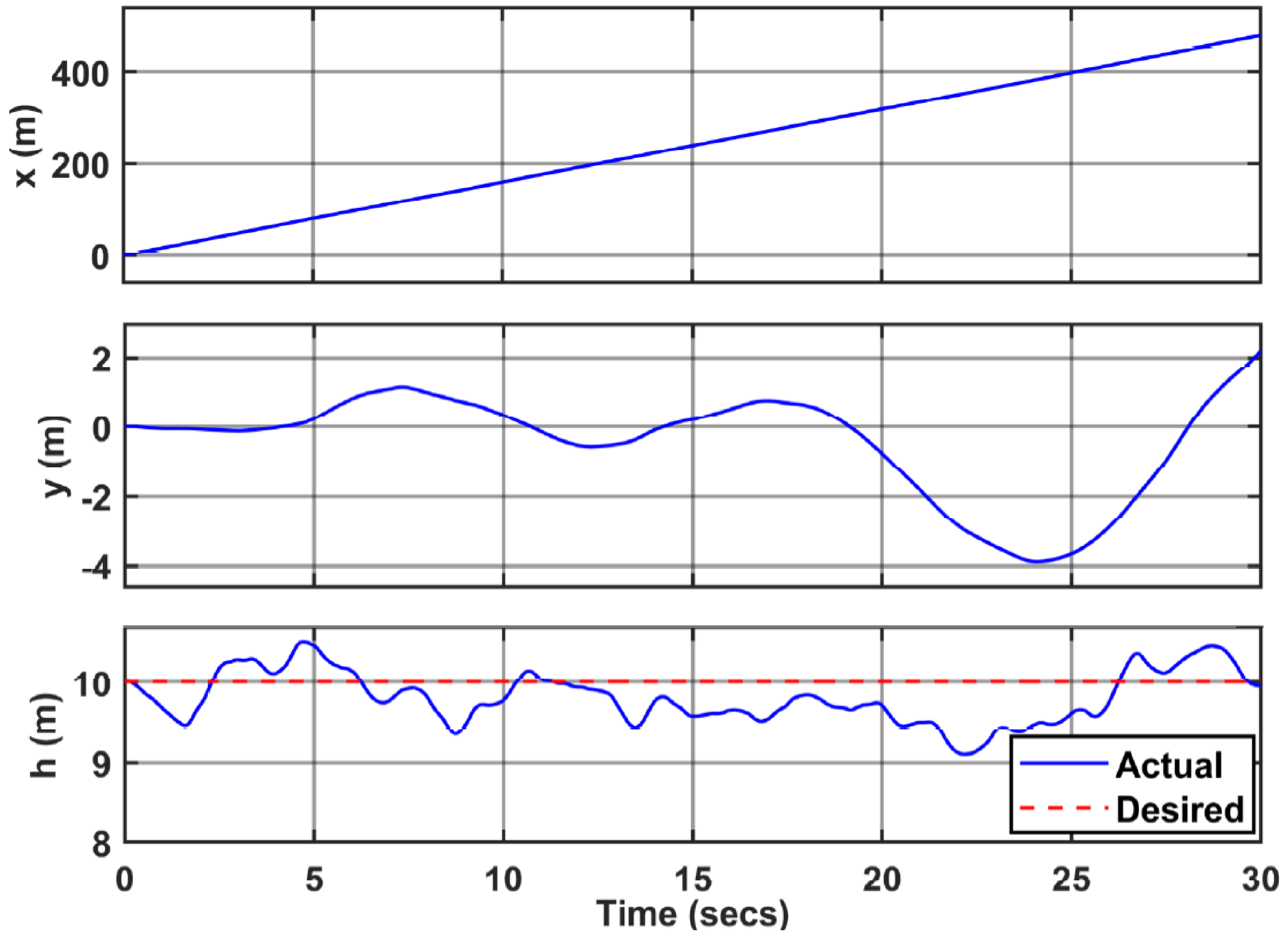


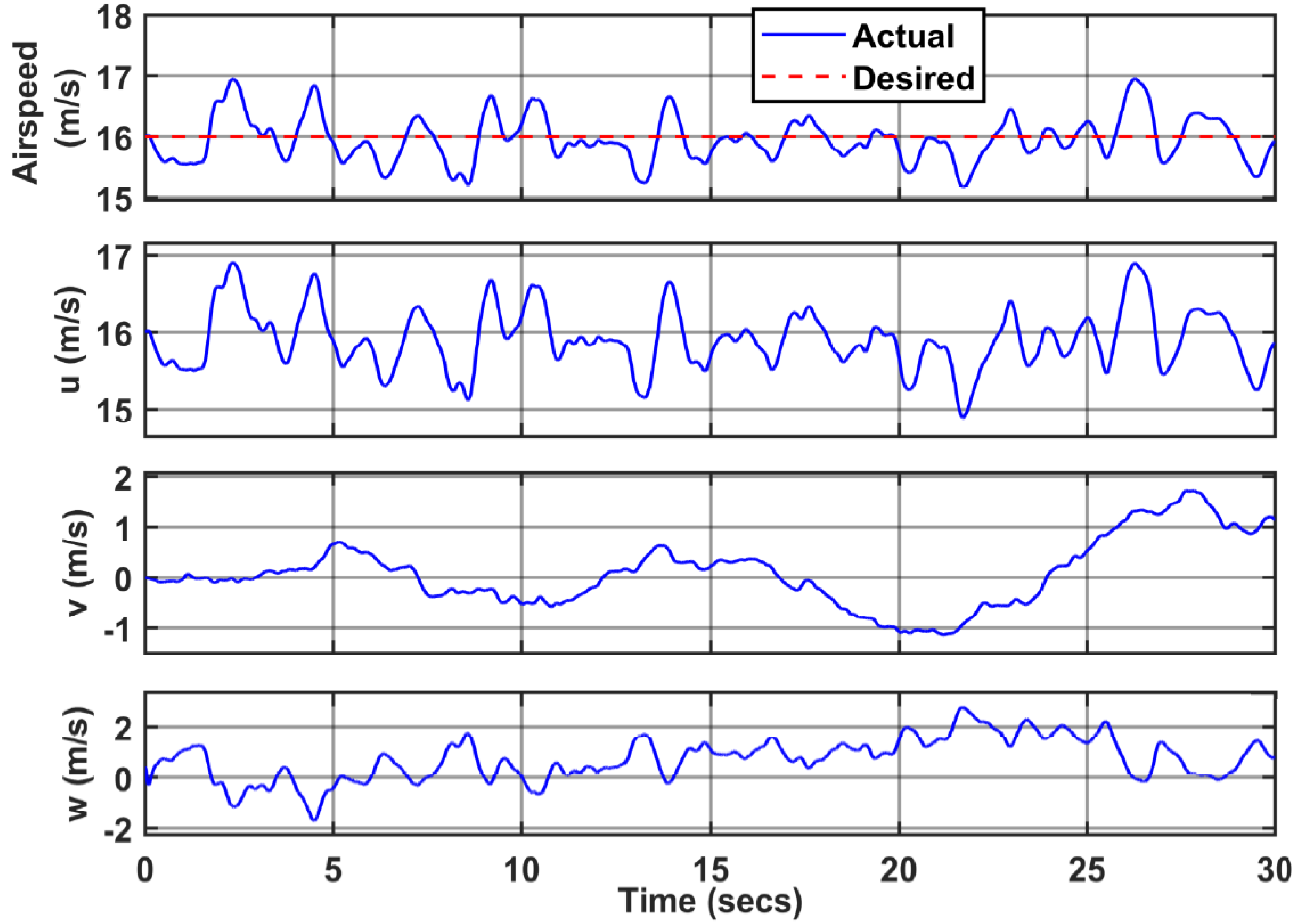


Aerodynamic forces and moments produced by the wings

Results

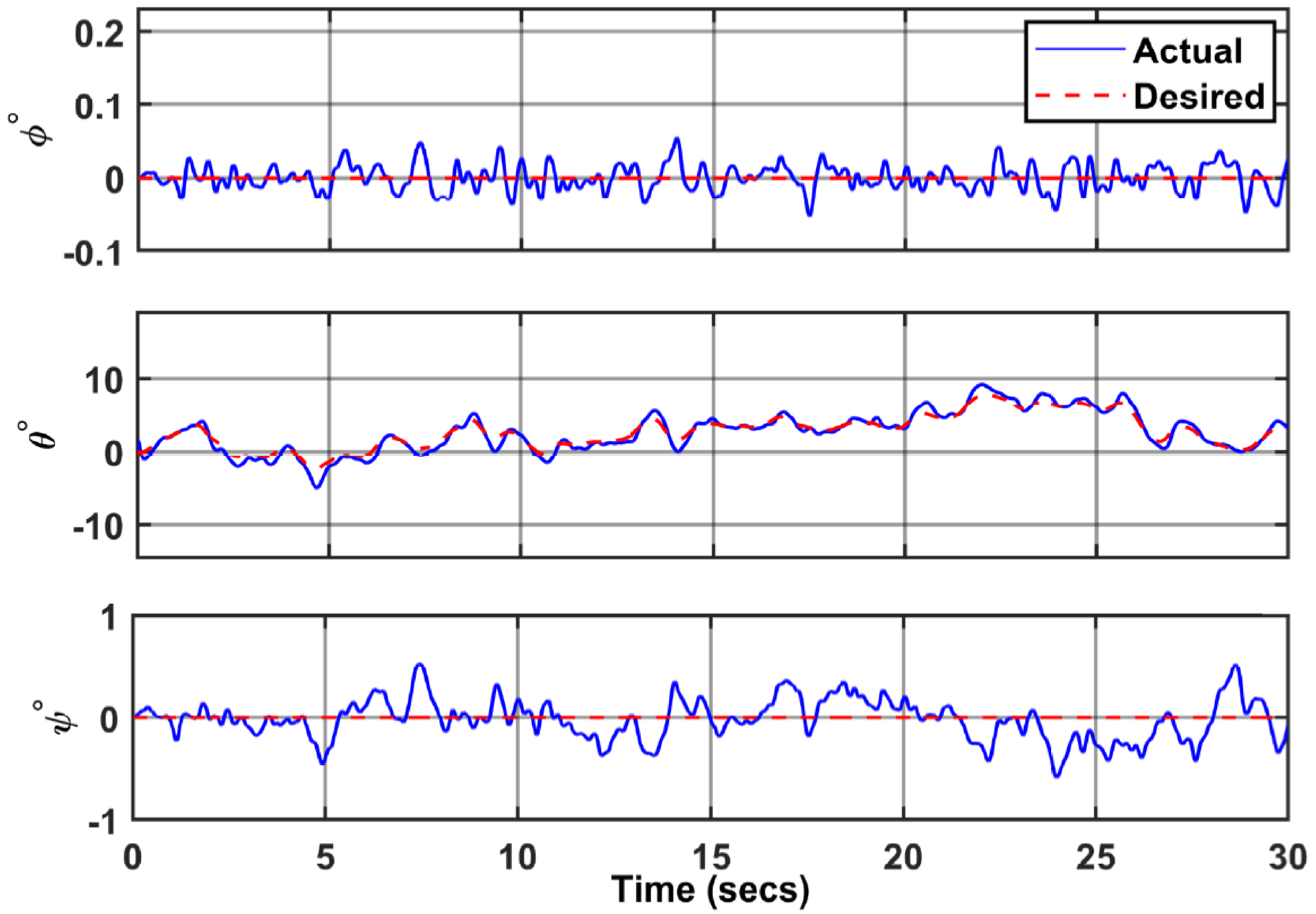
Forward Flight Mode

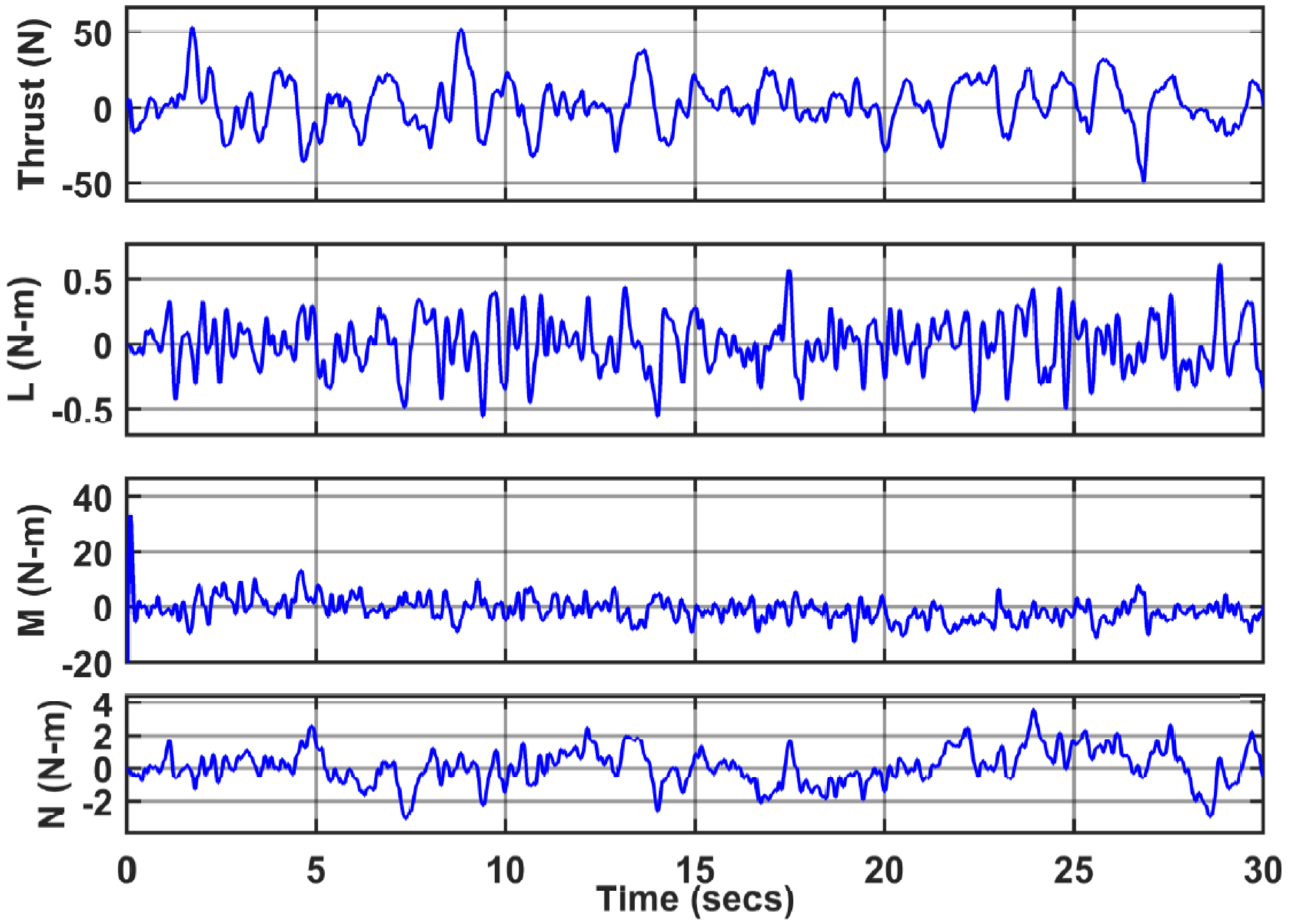




- Fluctuations in the altitude and airspeed are due to the constant change in the pitch angle due to the disturbances.
- TECS ensures simultaneous tracking of airspeed and height.

Results
Forward Flight Mode



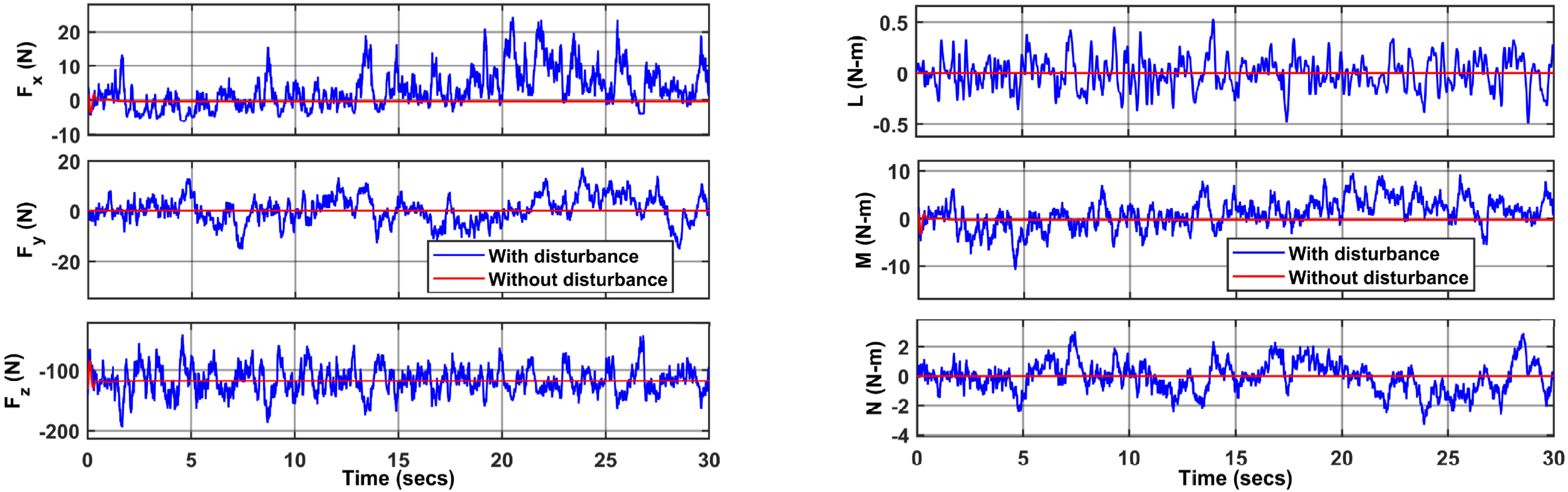


Good attitude tracking response results in negligible impact of disturbance on altitude and airspeed.

Minimum actuator efforts required for attitude tracking.

Results

Forward Flight Mode



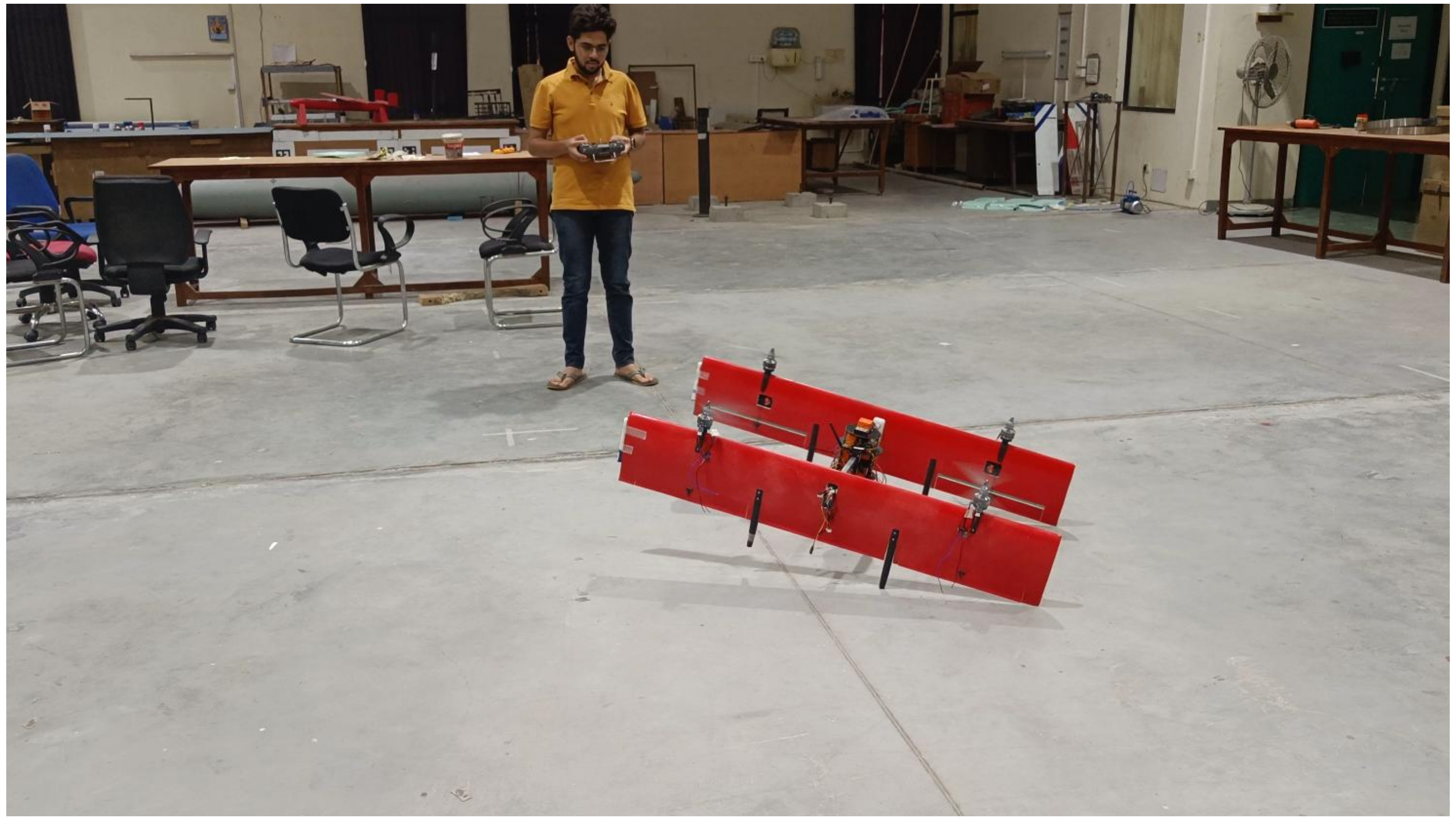
Aerodynamic forces and moments produced by the wings

Conclusion

- H_∞ control technique is studied to achieve reference tracking in the presence of wing disturbances and uncertainties.
- The nonlinear model of the vehicle is linearized about hover for controller synthesis.
- H_∞ controller is designed using the weighted sensitivity approach.
- The results are validated using the 6-DOF nonlinear model of the vehicle.
- The results suggest the designed controller is capable of achieving desired robust performance with minimal control efforts.

Future Work

- Further reduce the controller order and gain.
- Hardware implementation and flight testing of the proposed controller.



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Intelligent Systems Research Laboratory
Aerospace Engineering, Texas A&M University

Intelligent Guidance & Control Laboratory

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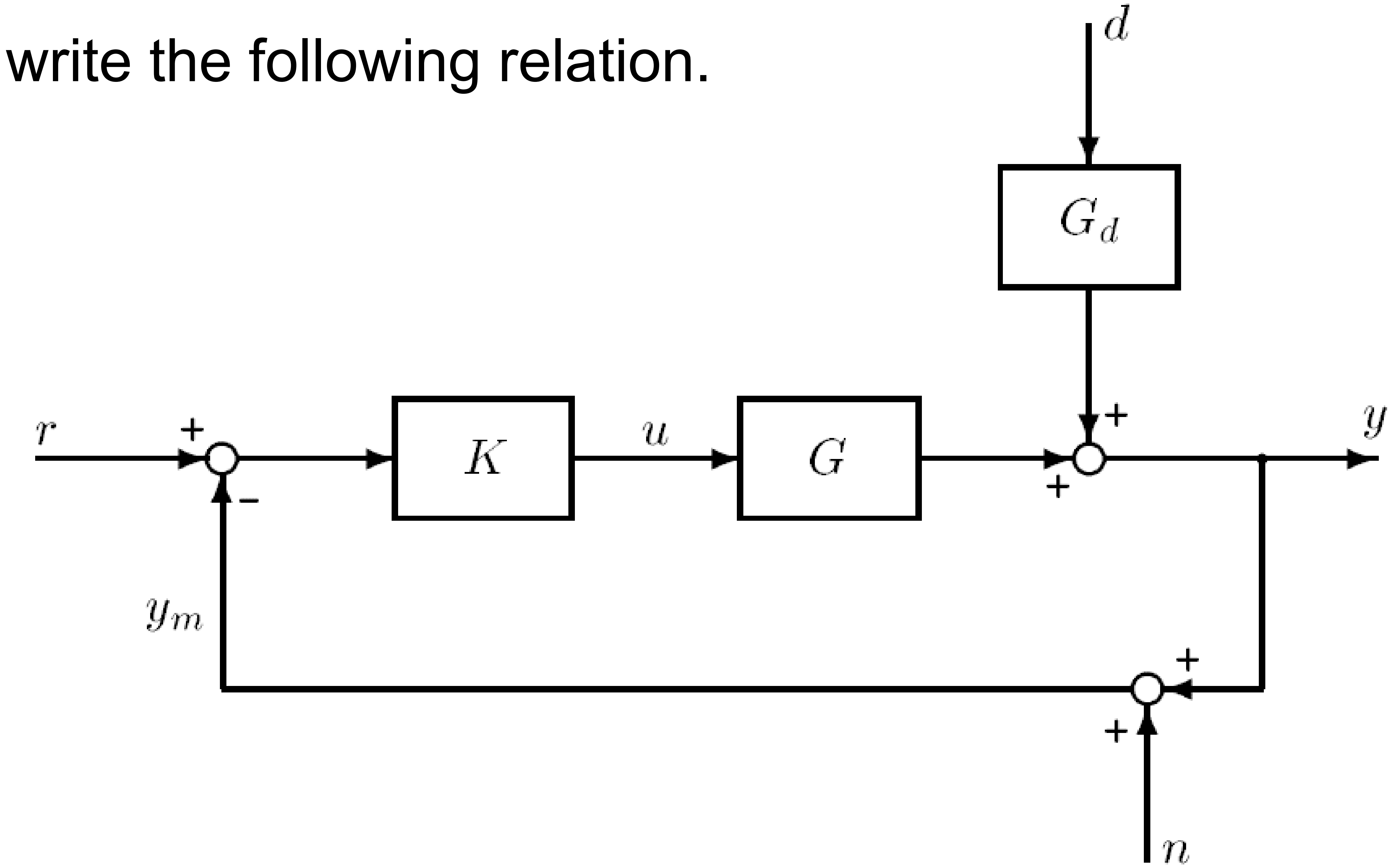
mail:ktanay@iitk.ac.in

Feedback Problem

The given plant is affected by three exogenous inputs: reference (r), disturbance (d) and noise (n).

Because the system is assumed to be linear, we can write the following relation.

$$\begin{bmatrix} y \\ y_m \\ u \end{bmatrix} = \begin{bmatrix} \frac{G}{1+GK} & \frac{-GK}{1+GK} & \frac{GK}{1+GK} \\ \frac{G}{1+GK} & \frac{1}{1+GK} & \frac{GK}{1+GK} \\ \frac{-GK}{1+GK} & \frac{-K}{1+GK} & \frac{K}{1+GK} \end{bmatrix} \begin{bmatrix} d \\ n \\ r \end{bmatrix}$$



Following terminologies have been used

•Sensitivity Function $S = \frac{1}{1+GK}$

•Complimentary Sensitivity Function $\mathrm{T} = \frac{GK}{1+GK}$,

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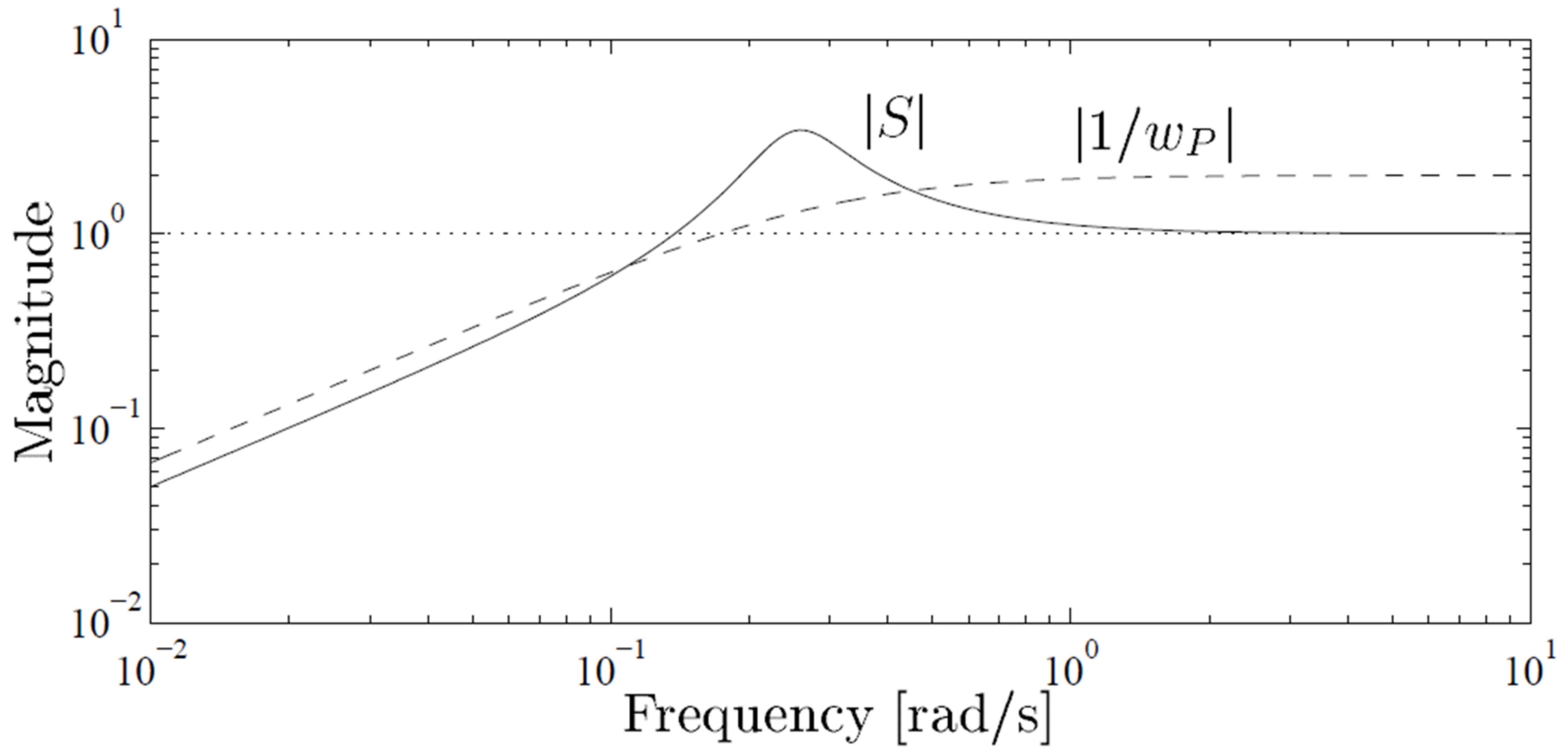
Weight Selection

- Closed loop performance is indicated by the by magnitude of the sensitivity function S.
- Controller must be designed to give the following system specifications in terms of S
  - Bandwidth frequency ω_B^*
  - Maximum steady state tracking error A
  - Maximum peak magnitude of S to prevent disturbance amplification $\| S(j\omega) \|_\infty \leq M$



- Specifications captured by upper bound $1/|W_1(s)|$

- We select a bound on the sensitivity function such that
  $\| S(j\omega) \| < \gamma / \| W_1(j\omega) \|, \forall\ \omega$

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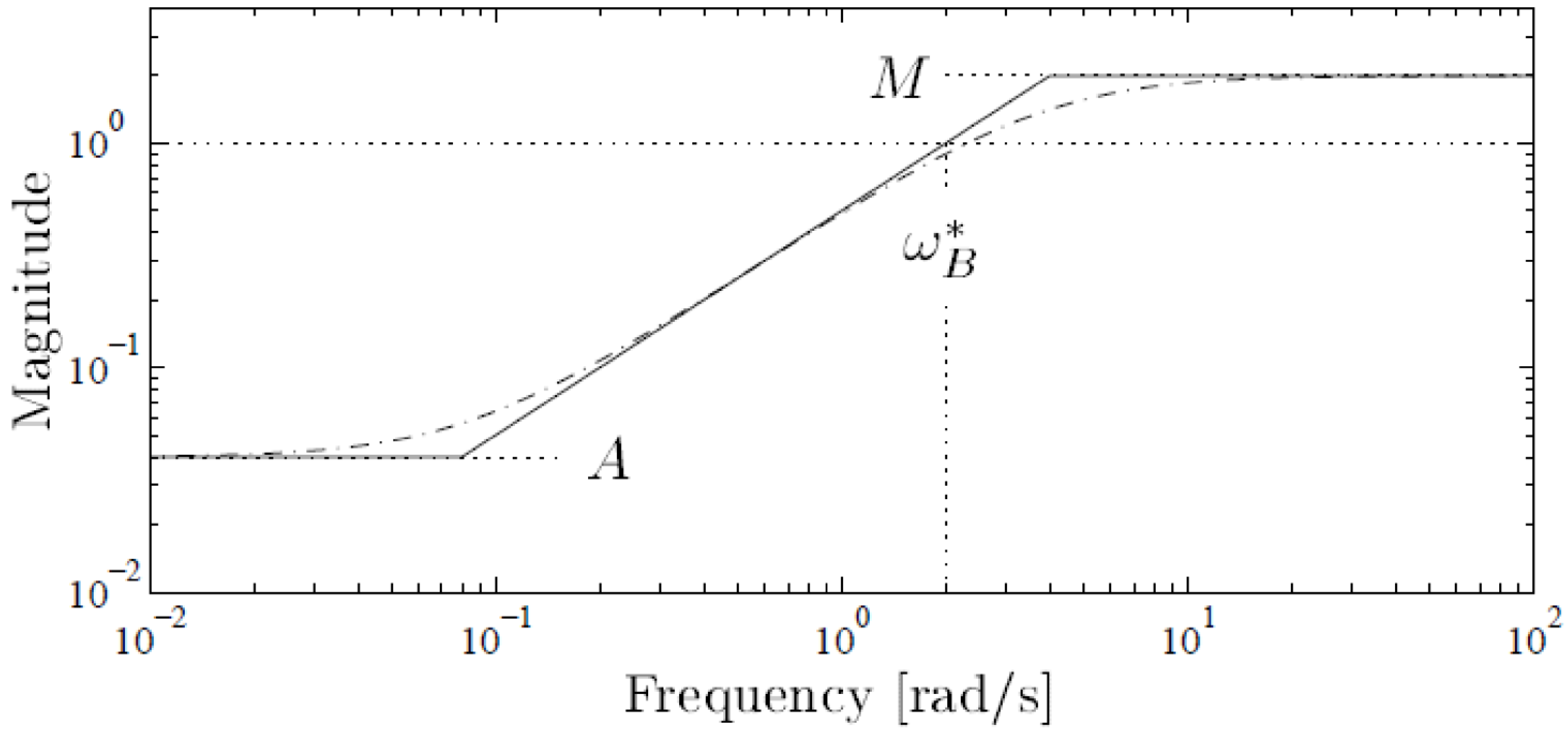
Weight Selection

- Typically, W_1 is a compensator and is given as

$$W_1(s) = \left(\frac{s/M^{0.5} + \omega_B^*}{s + \omega_B^* A^{0.5}}\right)^2$$



- The upper bound on $|S|$ (given by $1/|W_1(s)|$) is
  - approximately equal to A at low frequencies
  - approximately equal to $M \geq 1$ at high frequencies
  - crosses magnitude = 1 at frequency ω_B^* which is the bandwidth specified by the designer.

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