# Optimal Threshold of SVD Recomposition

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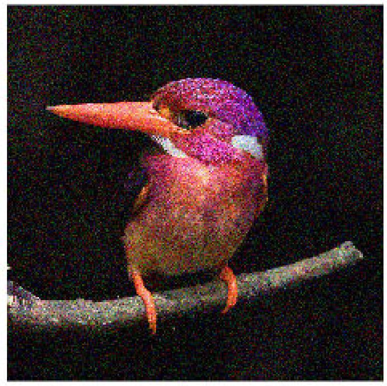
First load our file and let's add some noise with the Gaussian Patter

```
path = 'Kingfisher.jpg';

I = imread(path);

n = max(size(I))
```

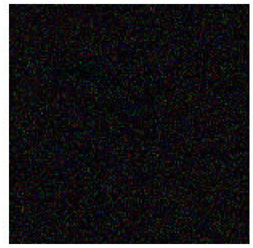
```
n = 1724
```

```

Inoisey = imnoise(I, "gaussian");

clf
subplot(1,2,1)
imshow(I);

subplot(1,2,2)
imshow(Inoisey)
```



```
clf
subplot(1,3,1)
imshow(I);

subplot(1,3,2)
imshow(Inoisey)

subplot(1,3,3)
imshow(Inoisey-I)
```





```
I = im2double(I);
Inoisey = im2double(Inoisey);
```

Now let's see how much noise is present in each color channel.

SnR is measured in decibles so 0 means perfect 1:1 ratio of no noise, but anything higher than 1 means there noise is present

```
noiseRed = snr(I(:, :, 1),Inoisey(:, :, 1)-I(:,
```

```
noiseRed = 11.1311
```

```
noiseGre = snr(I(:, :, 2),Inoisey(:, :, 2)-I(:,
```

```
noiseGre = 6.8636
```

```
noiseBlu = snr(I(:, :, 3),Inoisey(:, :, 3)-I(:,
```

```
noiseBlu = 7.0508
```

Let's store this not in decibels but rather in a power ratio:

```
noiseRed = log10(noiseRed)/10;
noiseGre = log10(noiseGre)/10;
noiseBlu = log10(noiseBlu)/10;
```

Next we calculate the SVD of each color channel RGB,

```
[UnoiseyR, SnoiseyR, VnoiseyR] = svd(Inoisey(:,
[UnoiseyG, SnoiseyG, VnoiseyG] = svd(Inoisey(:,
[UnoiseyB, SnoiseyB, VnoiseyB] = svd(Inoisey(:,
```

Now we need to find the threshold which will let us find the rank of our approximation.

As defined in Gavish & Donoho 2014, we can find the threshold of an arbitrary n by n matrix as approximately tau = 2.858*median(S)

First find medians of each color channel:

```
sMedR = median(diag(SnoiseyR)) % Red single valu
```

```
sMedR = 2.8430
```

```
sMedG = median(diag(SnoiseyG)) % Green single va
```

```
sMedG = 2.7324
```

```
sMedB = median(diag(SnoiseyB)) % Blue single va
```

```
sMedB = 2.6740
```

```
c = 2.858 % threshold coefficeint
```

```
c = 2.8580
```

```
tRed = c*sMedR
```

```
tRed = 8.1253
```

```
tGre = c*sMedG
```

```
tGre = 7.8093
```

```
tBlu = c*sMedB
```

```
tBlu = 7.6423
```

Set singular values below threshold to zero in each channel

```
SdenoiseR = SnoiseyR;
SdenoiseG = SnoiseyG;
SdenoiseB = SnoiseyB;

SdenoiseR(SdenoiseR < tRed) = 0;
SdenoiseG(SdenoiseG < tGre) = 0;
SdenoiseB(SdenoiseB < tBlu) = 0;
```

Reconstruct each matrix

```
DenoisedRed = UnoiseyR*SdenoiseR*VnoiseyR.';
```

```
DenoisedGre = UnoiseyG*SdenoiseG*VnoiseyG.';
DenoisedBlu = UnoiseyB*SdenoiseB*VnoiseyB.';

Idenoised = cat(3, DenoisedRed, DenoisedGre, Den
```

Display for Visual Assessment

```
clf
subplot(1,3,1)
imshow(I)

subplot(1,3,2)
imshow(Inoisey)

subplot(1,3,3)
imshow(Idenoised)
```

Now we calculate new signal to noise ratio:

```
noiseDeArbRed = snr(I(:, :, 1),Idenoised(:, :,
```

```
noiseDeArbRed = 15.5708
```

```
noiseDeArbGre = snr(I(:, :, 2),Idenoised(:, :,
```

```
noiseDeArbGre = 12.3681
```

```
noiseDeArbBlu = snr(I(:, :, 3),Idenoised(:, :,
```

```
noiseDeArbBlu = 11.7616
```

We can also look at other signal to noise metrics:

The mean square error

```
ImseNoise = immse(I, Inoisey)
```

```
ImseNoise = 0.0066
```

```
ImseDenoise = immse(I, Idenoised)
```

```
ImseDenoise = 0.0022
```

Another method of measuring the perfromance of the matrix reconstruction is to caclulate the Frobenius Distance between the two signals (Shablin and Nobel).

```
RedDistance = norm(I(:, :, 1)-Idenoised(:, :, 1)
BluDistance = norm(I(:, :, 2)-Idenoised(:, :, 2)
GreDistance = norm(I(:, :, 3)-Idenoised(:, :, 3)
FroDistance = (RedDistance + BluDistance + GreDi
```

```
FroDistance = 79.9813
```

Let's test to see if we can use the direct estimation via known values of noise for our square matrix.

Recall our existing signal to noise ratios for each channel, then apply optimal hard thresholding with more direct estimation via square matricies

```
disp(noiseRed)
```

```
    0.1046
```

```
disp(noiseGre)
```

```
    0.0836
```

```
disp(noiseBlu)
```

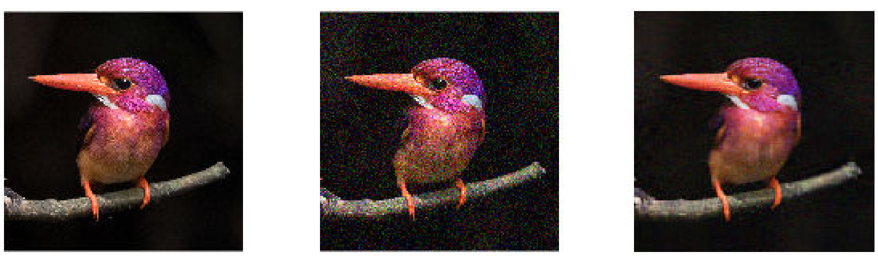
```
    0.0849
```

```
d = (4/sqrt(3));

tRedSq = d*sqrt(n)*noiseRed;
tGreSq = d*sqrt(n)*noiseGre;
tBluSq = d*sqrt(n)*noiseBlu;
```

Now apply our new thresholds

```
SdenoiseRsq = SnoiseyR;
SdenoiseGsq = SnoiseyG;
SdenoiseBsq = SnoiseyB;

SdenoiseRsq(SdenoiseRsq < tRedSq) = 0;
SdenoiseGsq(SdenoiseGsq < tGreSq) = 0;
SdenoiseBsq(SdenoiseBsq < tBluSq) = 0;

DenoisedRed = UnoiseyR*SdenoiseRsq*VnoiseyR.';
DenoisedGre = UnoiseyG*SdenoiseGsq*VnoiseyG.';
DenoisedBlu = UnoiseyB*SdenoiseBsq*VnoiseyB.';

IdenoisedSq = cat(3, DenoisedRed,DenoisedGre, De

clf
subplot(1,3,1)
imshow(I);
```

```
subplot(1,3,2)
imshow(Inoisey);

% subplot(2,2,3)
% imshow(Idenoised);

subplot(1,3,3)
imshow(IdenoisedSq)
```



Let's see how much noise is left after denoising

```
noiseDeSqRed = snr(I(:, :, 1),IdenoisedSq(:, :,
```

```
noiseDeSqRed = 15.3444
```

```
noiseDeSqGre = snr(I(:, :, 2),IdenoisedSq(:, :,
```

```
noiseDeSqGre = 12.3539
```

```
noiseDeSqBlu = snr(I(:, :, 3),IdenoisedSq(:, :,
```

```
noiseDeSqBlu = 11.7583
```

```

ImseDoubleDenoiseSq = immse(I, IdenoisedSq)
```

```
ImseDoubleDenoiseSq = 0.0022
```

Now lets calculate the Frobenius distance. .

```
RedDistance = norm(I(:, :, 1)-IdenoisedSq(:, :,
BluDistance = norm(I(:, :, 2)-IdenoisedSq(:, :,
GreDistance = norm(I(:, :, 3)-IdenoisedSq(:, :,
FroDistance = (RedDistance + BluDistance + GreDi
```

```
FroDistance = 80.7819
```

We can see that the hard thresholding for arbitrary noise performs better than the direct estimation with known noise.

Finally lets see how Bulk Edge thresholding performs.

Recall our existing Signal-noise-ratios for each color channel:

```
disp(noiseRed)
```

```
    0.1046
```

```
disp(noiseGre)
```

```
    0.0837
```

```
disp(noiseBlu)
```

```
    0.0849
```

Get beta where beta = m/n

```
[m,n,dim] = size(I);
beta = m/n
```

```
beta = 1
```

According to Gavish & Donoho 2014, the distribution of singular values will form a quarter circle bulk which lies at (1+sqrt(beta) * sqrt(n) * noise

```
t = 1+sqrt(beta)
```

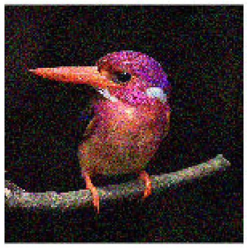
```
t = 2
```

```
tRedBulk = t*sqrt(n)*noiseRed;
tGreBulk = t*sqrt(n)*noiseGre;
tBluBulk = t*sqrt(n)*noiseBlu;
```

apply thresholds and reconstruct

```
SdenoiseRbulk = SnoiseyR;
SdenoiseGbulk = SnoiseyG;
SdenoiseBbulk = SnoiseyB;
```

```
SdenoiseRbulk(SdenoiseRbulk < tRedBulk) = 0;
SdenoiseGbulk(SdenoiseGbulk < tGreBulk) = 0;
SdenoiseBbulk(SdenoiseBbulk < tBluBulk) = 0;

DenoisedRed = UnoiseyR*SdenoiseRbulk*VnoiseyR.';
DenoisedGre = UnoiseyG*SdenoiseGbulk*VnoiseyG.';
DenoisedBlu = UnoiseyB*SdenoiseBbulk*VnoiseyB.';

IdenoiseBulk = cat(3, DenoisedRed,DenoisedGre, D

clf
subplot(1,3,1)
imshow(I);

subplot(1,3,2)
imshow(Inoisey);

% subplot(2,2,3)
% imshow(Idenoised);

subplot(1,3,3)
imshow(IdenoiseBulk)
```

Calculate MSE

```
ImseDoubleDenoiseBulk = immse(I, IdenoiseBulk)
```

```
ImseDoubleDenoiseBulk = 0.0022
```

```
RedDistance = norm(I(:, :, 1)-IdenoiseBulk(:, :,
BluDistance = norm(I(:, :, 2)-IdenoiseBulk(:, :,
GreDistance = norm(I(:, :, 3)-IdenoiseBulk(:, :,
FroDistance = (RedDistance + BluDistance + GreDi
```

```
FroDistance = 81.1171
```

```
noiseDeBulkRed = snr(I(:, :, 1),IdenoiseBulk(:,
```

```
noiseDeBulkRed = 15.5387
```

```
noiseDeBulkGre = snr(I(:, :, 2),IdenoiseBulk(:,
```

```
noiseDeBulkGre = 12.1211
```

```
noiseDeBulkBlu = snr(I(:, :, 3),IdenoiseBulk(:,
```

```
noiseDeBulkBlu = 11.6600
```

We see that bulk edge thresholding performs just as well as the square matrix estimation with known noise described in Gavish & Donoho 2014.

To investigate this, let's take a look at the number of singular values each of the square matrix estimation and bulk edge thresholding remove.

```
sRed = diag(SnoiseyR);
sGre = diag(SnoiseyG);
sBlu = diag(SnoiseyB);

sRemovedR = sum(sRed < tRed);
sRemovedG = sum(sGre < tGre);
sRemovedB = sum(sBlu < tBlu);
sRemovedSqR = sum(sRed < tRedSq);
sRemovedSqG = sum(sGre < tGreSq);
sRemovedSqB = sum(sBlu < tBluSq);
sRemovedBulkR = sum(sRed < tRedBulk);
sRemovedBulkG = sum(sGre < tGreBulk);
sRemovedBulkB = sum(sBlu < tBluBulk);
sRemovedMat = [sRemovedR, sRemovedSqR, sRemovedE
```

```
              sRemovedG, sRemovedSqG, sRemovedBulk
              sRemovedB, sRemovedSqG, sRemovedBulk
```

```
sRemovedMat = 3×3
        1669        1691        1680
        1678        1681        1653
        1669        1681        1649
```

```
sKeptMat = n*ones(3,3) - sRemovedMat
```

```
sKeptMat = 3×3
    55    33    44
    46    43    71
    55    43    75
```

It appears as if the arbitrary removal method is a much lower rank approximation as opposed to either method which utilizes a known signal to noise ratio.