Formula One Race Prediction

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***Abstract* — Formula One (also known as Formula 1 or F1) is the highest class of international auto racing for single-seater open wheel formula racing cars sanctioned by the Fédération Internationale de l'Automobile (FIA). The World Drivers' Championship, which became the FIA Formula One World Championship in 1981, has been one of the premier forms of racing around the world since its inaugural season in 1950. This paper aims to build a model to predict the winner of each Grand Prix with a good precision and to verify if it beats the bookmaker's odds. The model building was done using the python library Sci-Kit learn. Data available on the official Formula One website was used as the underlying basis for this research. To carry out the regression and classification task, Random-forest classifier, SVM regressor, logistic and linear regression, random forest regressor, svm classifier were used.**

***Keywords* — classification, regression, random forest, support vector machine, grid, podium, constructor, grand prix.**

I. Introduction

Formula One car racing is an annual world championship of special open-wheel racing cars. The competition is carried out in several stages. Each stage is called "Grand Prix" and always bears the name of the country in which it takes place. There are around 20 stages per season.

As per the current format of the sport, the Grand Prix stage lasts three days on the weekends, typically called the race weekends. The 1st day (Friday) - is the free practice. On this day, the teams get two 1-hour sessions to test the track, setting of their equipment, rubber for tires and think over the strategy for the race.The 2nd day is the third practice session and the knockout qualification-round, which is a set of three sessions of 18, 15, and 12 minutes respectively. Drivers post their fastest lap times in qualification, which determines the starting grid positions for the main race on Sunday. The 3rd day is the most significant, this is the day of the race itself. The race lasts up to 2 hours under normal conditions, during which time the racers cover about 305 kilometers. Drivers who complete a set number of laps are awarded points according to how they finish, after each grand prix. Drivers who finish beyond P10 are awarded 0 points. P1 to P10 are awarded points as [25, 18, 15, 12, 10, 8, 6, 4, 2, 1]. The driver with the maximum number of cumulative points after each season is crowned the world driver's championship, and the constructor with the highest score (cumulative of both drivers) is awarded the constructor's championship.

During the race, a team can use an arbitrary number of pit stops to change tires/minor damage equipment such as the front wing. The number of pit stops and how they will be distributed during the course of the race determines the strategy of the race. Often a good strategy can lead to the superiority of a weaker machine over a stronger one.

The study aims to build a model that can predict a winner (P1) for each grand prix in a season. The official Formula One website was used for obtaining historical race data from 2010 till 2019 as the underlying basis, accomplished through web scraping using Selenium and Beautifulsoup. The dataset had 4072 instances and 86 attributes. Each instance represented data for a driver for each particular race. The various factors that could play a vital role in deciding the outcome of the event, such as, age and nationality of a driver, weather conditions before the race, starting grid positions, constructors, were analyzed through data visualization.

Six different classifiers were trained on the training data for 2010-2018 seasons and their performance on the test set for pre-covid season 2019 was analyzed. The test set consisted of 21 races. This was effectively a 90% test split. The data for 2020 was excluded due to significant impact on the attributes and instances due to Covid-19 (teams replacing their drivers, reduced number of teams participating, no major improvements to the car assemblies, etc.)

Additionally, the model was run to get predictions for the 2021 season, before the Finale. Surprisingly, the random forest regressor produced correct predictions with a precision score of 55.54 percent, correctly predicting the world champion on December 12th, 2021, in Abu Dhabi. *This could be attributed to the change in regulations prior to the 2021 season.*

II. DATA COLLECTION

The historical race data from the first grand prix in 1950 until present is publicly available and updated on the Ergast API website and the official Formula One website. The sport has seen substantial changes in rules and format of races itself over the past few decades and for this reason the historical data prior to 2009 has been excluded from the scope of this study. The data was scraped from the online resources using conventional web scraping using Selenium and Beautifulsoup libraries in python.

Six dataframes were considered for analysis, amalgamation of which would result in the final dataframe (Fig 1). These contained the attributes like season, circuit, date of births of drivers, driver standings, constructor standings, qualification times, and weather data. A thorough visual analysis of the data was done in order to understand the underlying relationships that could impact the likelihood of a driver to win a particular race. Python and pandas library was used for cleaning, segregating, and visualizing plots and figures. All the data frames were merged into a single dataframe with all the critical attributes being dummified and the irrelevant ones dropped.

	season	round	driver	grid	podium	driver_points	driver_wins	driver_standings_pos
0	2010	1	vettel	1	4	0	0	0
1	2010	1	massa	2	2	0	0	0
2	2010	1	alonso	3	1	0	0	0
3	2010	1	hamilton	4	3	0	0	0
4	2010	1	rosberg	5	5	0	0	0

Fig 1: Final dataset used for analysis

III. DATA ANALYSIS

Prior to machine learning modeling, data visualization was performed to derive insights on the level of importance of certain attributes in predicting the winner for each of the races. Some of the attributes focussed on were:

A. *Role of pole position*

Drivers attempt to set their fastest lap time in the 3-stage knockout qualification rounds on Saturdays. As can be inferred by logic and intuition, there was a high probability of winning when a driver started on the pole position on the tracks where it was difficult to overtake. This can be attributed to the advantage of starting a few meters ahead of the traffic.

Fig 2. Shows the correlation between the drivers starting the race on pole position and finishing at P1 on the race day. It is clearly evident that the correlation is high on the circuits such as Bahrain, and Monza and falls for the circuits which provide more opportunities of overtaking, such as Monaco and Circuit of the Americas, Texas. In general, the correlation is high for all the circuits and starting grid position played the most critical role in deciding the winner of the race.

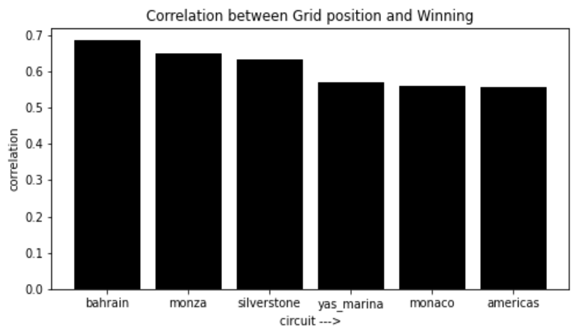

Fig 2: Correlation between starting at pole position and finishing at P1 in a race

B. *Impact of racing in the driver's home country*

It was often observed that when a driver raced in his home country, the majority of the grandstands were occupied by that particular driver's fans. It can therefore be claimed that supporting fans have a psychological impact on the drivers. Fig 3 shows the percentage of races in which a driver ended up on the first podium position when racing in his home country compared to when racing away. Although, comparatively, the difference is not so significant, in most of the nationalities, a higher win percentage in the home county was observed for each driver.

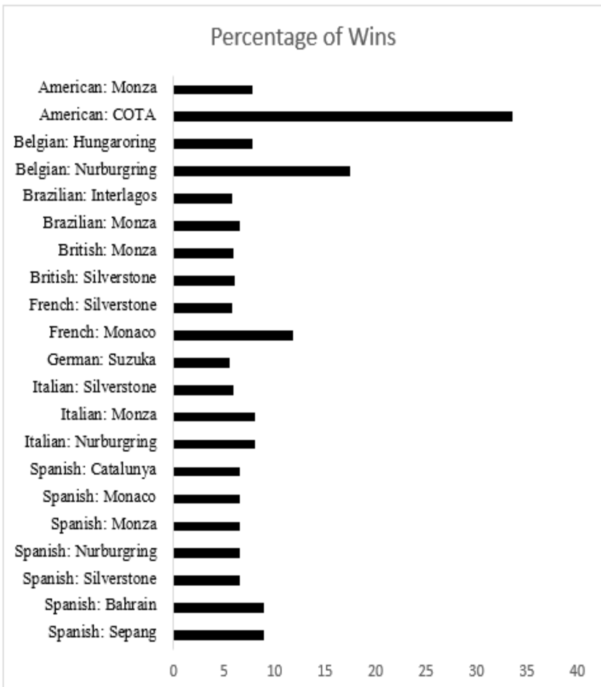

Fig : 3 Percentage of winners by nationality

C. Role of age of winners

An investigation into the average of winners over the years was done, to find the level of impact it had on winning. The era from 1950 till 1970s witnessed the average age of winners around 40 years, while it gradually decreased over the years, and the current generation has very few winners over 30 years old.

Fig 4 shows the scatter plot of age of race winners from the inaugural season in 1950 till the latest season 2020. and it can clearly be observed that the average age of winners has been gradually decreasing over the years.

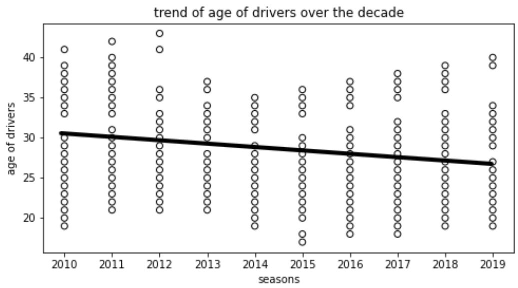

Fig 4: Age of grand prix winners from 2010 till 2020

IV. EXPERIMENTAL METHODOLOGY

A. Success Metrics

- Precision Score: percentage of correct predictions of P1 finishers all the Grand Prix of 2019 Season.
- Odds Comparison: can the model outperform the odds.

B. Data Preparation

The 6 dataframes mentioned in the data collection section were merged into a final dataframe using common keys. The resulting data frame included collective information on weather, each grand prix result, driver details, constructor standings, race data and the qualifying lap times from 2010 till 2019. The age of the drivers was obtained from their respective date of births for each year, and the cumulative difference between the fastest lap times in the qualifying round was also taken to realize how fast was the first car on the grid relative to other cars.
Finally, the circuit id (track name), nationality of drivers and constructors, weather variables were dummified and all other irrelevant columns such as wikipedia links, latitudes etc were dropped from consideration.

C. Problem definition

Objective of this work was to make a prediction of P1 for each GP in the year 2019, so the target (class) variable could be treated as either regression or classification.

Regression : Regression algorithms predict the lap times (float value) of each grand prix for each of the drivers. In order to evaluate the precision score, the predicted results were sorted in ascending order which put the driver with the fastest lap on top and declared the winner of the race.
Finally, the precision score was calculated between the actual and predicted values (mapped 1 as winner and 0 as Not Winner), and repeated for each race of 2019 until the metric for percentage of correct predictions was obtained. Fig 5 shows the prediction data frame obtained for any particular race in the 2019 season.

driver	podium	results	actual	predicted
bottas	2	2.443529	0	1
hamilton	1	2.457953	1	0
max_verstappen	4	3.248145	0	0
vettel	3	3.303029	0	0

Fig 5: Prediction sample using a regression algorithm

In Fig 5, the model wrongly predicted Valterri Bottas as the winner, and hence the model score would be 0.

Classification: The target (class) variable is mapped (0 - Not Winner or 1 - Winner) prior to model training and hence the predicted values are output as probabilities. This essentially means that probabilities of winning could be the same for multiple drivers and hence we could either have more than 1 winner or zero winners in some cases. To avoid this predicament a different function was defined that ranked the probabilities of winning for each grand prix and for each driver. Then the probabilities were sorted in the descending manner with the highest probability representing the winner of the race. Fig 6 shows a sample of predicted output for one of the races of the 2019 season. It can be observed that although the probability of winning for the driver Max Verstappen is only 0.35, but since it is the highest probability, he is mapped as the winner of the event.

driver	podium	proba_0	proba_1	actual	predicted
max_verstappen	1	0.651345	0.348655	1	1
hamilton	5	0.696430	0.303570	0	0
bottas	3	0.834249	0.165751	0	0
leclerc	2	0.879126	0.120874	0	0

Fig 6: Prediction sample using a classification algorithm

D. *Model Building*

TRAINING-TEST SPLIT: The training set consists of the race data from the year 2010 to 2018, while the test data set consists of all 21 races from the 2019 season, effectively making it a 90% split.

The custom scoring function (to measure the precision score for each model) required the model to be fitted prior to evaluation, so a manual grid search for all different algorithms was performed, and the scores and parameters for each were recorded in a dictionary. For the classification and regression tasks, following algorithms were utilized:

1. Logistic and Linear Regressions
2. Random Forests
3. Support Vector Machines

FEATURE IMPORTANCE

According to the linear regression, the starting grid position plays the most significant role in predicting the champion. Other features of significance are constructors (teams) or points before the grand prix.

Fig 7 shows the feature importance comparison based out of linear regression.

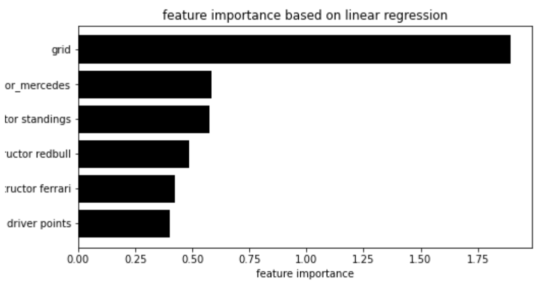

Fig. 7: Feature Importance comparison based on linear regression

The algorithms consistently produced incorrect predictions on some of the tracks, with possible cause being a high number of incidences (red flags), and overtakings. Specifically, the algorithms performed worst on Alberta park, Baku, Spa, Monza and Hockenheim Ring.

Fig 8 shows model wise comparison based on the precision score. As is evident from the plot, random forest classifier and svm regressor produce the most optimum results with nearly 57 percent precision score.

A manual grid search was performed to find out the most optimal parameters for each of the classification/ regression algorithms. Finally, the optimum parameters which result in the highest precision score for each model were picked from the established hashmap for each of the models, and then the models were run on the test set with optimal combination of hyperparameters. Based on this precision score, random forest classifier, svm regressor, svm classifier and logistic regression were picked as top choices to carry out model building.

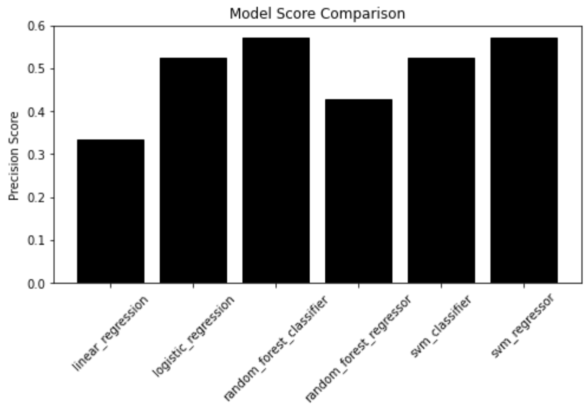

Fig 8: Performance evaluation of classifiers and regressors based on custom precision score.

PARAMETER TUNING

A. *Random Forest Classifier*

The three critical parameters for the random classifier algorithm are the criterion for split, max number of features to be considered and the depth of decision tree. Criterion for split was taken to be Gini index and with max features set to “auto”. Then, the performance based on the precision score was evaluated by varying the depth of decision trees parameter.

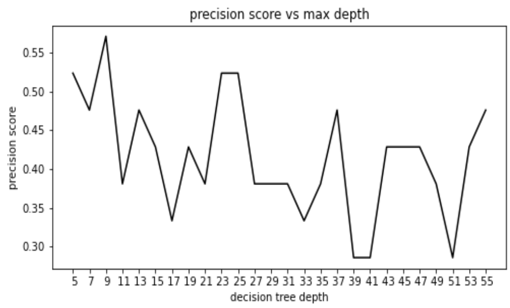

Fig 9: Performance of random forest classifier based on depth of decision tree hyperparameter

The precision peaked at around the maximum depth 9, and hence was taken as the optimal parameter for model testing.

Optimal parameters:
Criterion: Gini
Max features selection: auto
Max Depth: 9.0

B. SVM Regressor

The three critical parameters for the support vector machines(svm) - regression algorithm are kernel coefficient : Gamma value, Regularization index (C), and the Kernel type for the algorithm. The kernel type "poly" was taken as the default parameter for the algorithm. Fig 10 shows the performance of SVM regressor based on different gamma values.

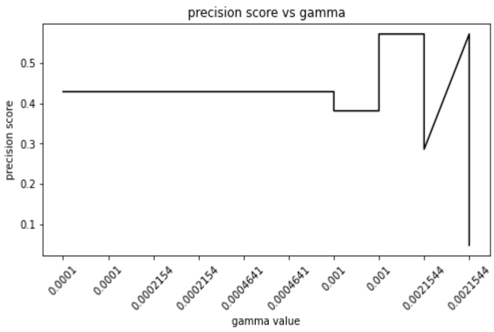

Fig 10: Performance of svm regressor wrt the 'gamma' hyperparameter

As can be seen from the figure, the precision was maximum and remained constant from values ranging from 0.02 to 0.029. Since all had the same precision, gamma 0.02 was taken for model testing.

Optimal parameters:

Gamma: 0.02

C: 1

Kernel: Polynomial

C. Logistic Regression

The three critical parameters for the logistic regression algorithm are the penalty type, solver (for algorithm), and the regularization index (C). Regularization index is the inverse of regularization strength. Lower values of C indicate a high amount of regularization required. Fig 11 shows the variation in performance of linear regression on varying "C". It can be observed that maximum performance was obtained with C being in the range 0.017 to 3.79.

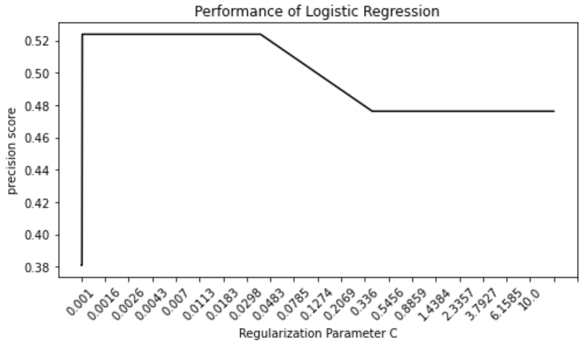

Fig 11: Trend of performance of logistic regression with the regularization index

Optimal parameters:

Penalty : l1

Solver : saga

Regularization parameter C = 0.048

Solver: saga

CONFUSION MATRIX

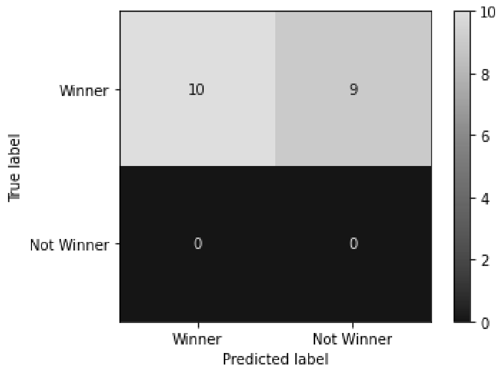

Fig 12: Confusion Matrix for the 2019 F1 test set

Predicted winners for each grand prix of the 2019 season were stored in a hashmap along with the actual winners for each round of the test set. 'Winner' of each race was marked '1' and 'not winner' (any position other than P1) was marked '0'. The cumulative actual and predicted winners data was passed to the sklearn confusion matrix function. Fig 12 represents the confusion matrix for all the 19 races of 2019. Columns in the confusion matrix represent the predicted label while the rows represent the actual label (winner/not-winner). As could be inferred from the figure 12 '10' represented the winning driver predicted correctly i.e in 10 races (True Positive) and '9' represents incorrect predictions of the winnin driver (False Negative). In 2020, the pandemic hit the world which led to cancellation of many races around the globe and hence, 2020 was not considered for evaluation. Certain changes in regulations of the sport and to the design and aerodynamics of cars were imposed in order to slow down the cars and enhance safety measures ahead of 2021. Eventually the model was run for the 2021 season to predict the world champion driver as it was down to the wire on the last race in Abu Dhabi. The model correctly predicted Max Verstappen as the world drivers champion.

#	circuit_id	odds_fav	actual_winner	odds	predicted_winner	profit
1	bahrain	max_verstappen	lewis_hamilton	2.6	bottas	-100.00
2	emilia_romagna	max,lewis	max_verstappen	2.25	max_verstappen	125.00
3	algarve	max,lewis	lewis_hamilton	2.25	lewis_hamilton	125.00
4	catalunya	lewis	lewis_hamilton	2.1	lewis_hamilton	110.00
5	monte_carlo	lewis	max_verstappen	2	lewis_hamilton	-100.00
6	baku	lewis	sergio_perez	2.05	max_verstappen	-100.00
7	paul_ricard	lewis	max_verstappen	2.2	lewis_hamilton	-100.00
8	red_bull_ring	lewis,max	max_verstappen	2.25	max_verstappen	125.00
9	red_bull_ring	max	max_verstappen	1.5	max_verstappen	50.00
10	sliverstone	max	lewis_hamilton	1.67	max_verstappen	-100.00
11	hungaroring	max	esteban_ocon	1.8	lewis_hamilton	-100.00
12	spa	max,lewis	max_verstappen	2.1	max_verstappen	110.00
13	zandvoort	max,lewis	max_verstappen	2.1	max_verstappen	110.00
14	monza	max,lewis	daniel_ricciardo	2.1	lewis_hamilton	-100.00
15	sochi	lewis	lewis_hamilton	1.6	lewis_hamilton	60.00
16	istanbul	max	bottas	1.85	lewis_hamilton	-100.00
17	cota	lewis	max_verstappen	1.8	max_verstappen	80.00
18	mexico	max	max_verstappen	1.65	max_verstappen	65.00
19	sao_polo	max	lewis_hamilton	1.57	lewis_hamilton	57.00
20	lossail	lewis	lewis_hamilton	1.65	lewis_hamilton	65.00
21	jeddah	lewis	lewis_hamilton	2.35	max_verstappen	-100.00
22	yas_marina	lewis	max_verstappen	1.45	max_verstappen	45.00
						$227.00

Fig 14: Net earnings if betting on prediction algorithms. It beats the bookmaker's odds fav (188BET)

Car racing has followers all over the world. However, not only the sporting enthusiasm attracts so many people. In the modern world, Formula One is a colossal business, the value of which, according to various estimates, ranges from $1.3 billion to $2 billion. Teams and individual participants as well as leading automobile empires are eager to compete. Everything in Formula One is imbued with the spirit of competition. It has also been a great success among sports bettors around the world, one of the most popular sports to wager on. That's why many professional wagers with extensive knowledge in statistics and experience in the field, make a few different bets, leveraging their position.

We decided to test our model and compare it to the historical betting positions, while utilizing statistical probabilities implied by the odds not being coherent. Figure 14 represents one of the possible ways of profiting without too comprehensive calculations. Column "Odds favorite" has the names of all drivers predicted to win by bookmakers. Column "Driver Predicted" is the column with the drivers predicted to win by our model. Column "Actual" represents all the drivers who won the race. "Odds" is the odds calculated by bookmakers to the winner our model predicted. Column "Profit" shows the ending profit after each particular bet.

As we can see in the table above, our total profit would be positive just with simple consistency of betting to the model-predicted driver without taking the odds into consideration.

V. RELATED WORK.

There are a number of related researches that are included in this section to show the different approaches of analyzing the data and ability to refine it in order to receive a more accurate prediction.

A. Improving Race Strategy

Christopher Ledesma and Weisen Choo did a study on features contained in MIT developed machine learning software for professional car racing, to improve the predictions of track position changes within a race. The software makes the prediction by building a feature matrix using epochs from races in the past and epochs from the ongoing race. These epochs only consider a subset of the characteristics of an outing, as laps are trimmed from the start and end.
The researchers analyzed pit crew performance and driver performance within selected races, classified tracks based on tire wear and the ratio of 2 versus 4 tire change decisions for pit stops and look at how tire change decisions vary from track to track depending on tire wear, caution periods, and stages of the race to

understand how teams adapt their tire change strategies as each race progresses. Then they conducted a test whether the construction of the machine learning dataset using similar and different track characteristics has a discernable impact on the predictive capability of the software. Their work indicated clear patterns for tire change decisions for races on different tracks, when should a driver pit during the course of a race, how many tires should be changed during that pit stop and what are the likely pit stop decisions made by the driver's rivals.

B. Improving Car Features

For years car engineers and designers were working on improving sports car specifics for their best performance. Sharjeel Khan and Irfan Manarvi focused on providing a methodology of selecting a sports car by analyzing the data available for various critical parameters such as price, top speed, engine size and brake horsepower information to buyers for making optimum choices. Their findings were that the cars with smaller engines produced a lot of brake horsepower but failed to match the overall superiority of the V12 engines and the cars with bigger engines that had higher horsepower and thus had better top speeds.

C. Discovering new information and detecting anomaly

Machine learning becomes one of the most efficient tools to help with decision-making during a race. Current access to information makes it easier to evaluate the data and estimate the most accurate prediction. Chatura Widanage, Jiayu Li, Sahil Tyagi, Ravi Teja, Bo Peng, Supun Kamburugamuve, Dan Baum, Dayle Smith, Judy Qiu, and Jon Koskey used emerging real-time data analysis from the Internet of Things (IoT) result in fast data streams generated from distributed sensors. Applying advanced Machine Learning/Artificial Intelligence over such data streams to discover new information, predict future insights and make control decisions is a crucial process. In their paper, they articulated racing car big data characteristics and present time-critical anomaly detection of the racing cars with the real-time sensors of cars and the tracks from actual racing events. They built a scalable system infrastructure based on a neuro-morphic Hierarchical Temporal Memory Algorithm (HTM) algorithm and Storm stream processing engine. By courtesy of historical Indy500 racing logs, evaluation experiments on this prototype system demonstrated good performance in terms of anomaly detection accuracy and service level objective (SLO) of latency for a real-world streaming application. They showed under different deployment strategies that their proposed distributed system is capable of running complex anomaly detection algorithms in real-time, which in turn can be crucial for complete and accurate feature values detection and classification outcome.

D. Using data analytics for maximizing an F1 race car's performance - Mercedes Petronas AMG F1 racing team

Speed of data analysis and adaptability quickly translate into vital information. Data is the competitive advantage, fueling innovation and providing the best solutions for both constructing and racing. Seven-time FIA Formula One™ Constructors' champion Mercedes-AMG Petronas Formula One is at the pinnacle of motorsport. Over its years of partnership with TIBCO (The Information Bus Company), it amassed more than 50 race wins—including Constructors' and Drivers' championships—and operationalized turning data into insight that informs car design, race strategy, and driver performance. Continuous success relies upon calculated preparations for each unique racing situation. To achieve the best racing outcomes, the team simulates millions of racing scenarios and car configurations to devise the best strategies and ensure the best cars are on-track any given race day. Data analytics supports the game plan and the minute-by-minute decisions made by the garage, drivers, and other team stakeholders. With every new circuit, unrelenting competitors, and changing weather and track conditions, data analytics and decision-making are crucial.

VI. CONCLUSIONS

I. Random forest classifier and SVM regressor produce the best results with a nearly equal precision score of 57 percent.

II. Low precision score can be owed to the fact that F1 is an ever evolving sport with drastic rules and regulation changes to the format of the sport and design of cars and fuels.

III. From domain knowledge, pitstop strategies play a very important role in deciding the outcome of a race, and data about the same was not provided on the F1 official website. Hence the low precision score by the machine learning algorithms.

IV. The prediction model beats the bookmaker's odds with a net positive income higher than with obtained on the bookmaker's favorites.

V. Pole position plays the most crucial role in improving the likelihood of winning a race based on linear regression

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