RWE *German energy company*

28May2021 (Formulae in this doc are not 100% correct, please verify and check)

Spread option - Pearson model

Spread option is an option with spread of two stocks (such as gas and electricity) as the underlying. Consider payoff :

$$
\begin{aligned}
f(S_{1t}, S_{2t}) &= [S_{1t} - S_{2t} - K]^+ \\
dS_{1t} &= rS_{1t}dt + \sigma_1 S_{1t} dz_{1t} \\
dS_{2t} &= rS_{2t}dt + \sigma_2 S_{2t} dz_{2t} \\
dz_{1t}dz_{2t} &= \rho dt
\end{aligned}
$$

How do you price the call option?

- *derive Black Schole PDE and solve analytically*
- *derive Black Schole PDE and solve by finite difference method*
- *derive risk neutral pricing analytically (double integration)*
- *derive risk neutral pricing numerically (wiki : numerical quadrature)*
- *apply tree for early exercise-able contract*
- *apply monte carlo for path dependent contract*

Now, consider the vanilla spread option, without early exercise nor path dependence. For risk neutral pricing, we need to calculate the joint distribution between the two stocks. Without going into details, given the joint distribution between the two stocks as two-dimensional lognormal with correlation, that is $P_{lognorm}$. How can we approach the following double integral?

$$
c(S_{1t}, S_{2t}) = DF \times \iint_{S_{1,2} \in [-\infty,\infty]} [S_{1t} - S_{2t} - K]^+ \, p_{\log norm}(S_{1t}, S_{2t}) dS_{1t} dS_{2t}
$$

We can do that numerically. With two layer for-loops, the outer loop is for stock 2, the inner loop is for stock 1. The range of stock 1 is chosen so that we can take away the function $[...]^+$. Numerical integration can be done using rectangle / trapezoid approximation. In 1995, Pearson published an algorithm, which simplifies the above ***double analytic integration*** into ***single numerical integration***. Please refer to ***Pearson model***, which is common in commodity derivatives.

After calculation of price, what do we do next?

Greeks.

Yes, which Greek.

Delta.

Yes, how many deltas are there?

Two, one for stock 1 and one for stock 2.

What are their signs?

Delta one should be positive, delta two should be negative.

A common question from from trader : delta one versus negative delta two, which one is bigger?

Now sure, it depends.

I don't think so. Let's see. An intuitive approach is to :

- compare ITM probability to ***delta one***, then ...
- compare ITM probability to exercise to ***negative delta two***
- we will then draw a conclusion about comparison between delta one and negative delta two

ITM probability means in the money probability.

Delta vs ITM Prob for vanilla option

First of all, lets recall that in Black Scholes, delta is $N(d_1)$, while ITM prob is $N(d_2)$:

$$\partial c(S_t)/\partial S_t = N(d_1) = \int_{S\in[-\infty,\infty]} (1_{S_t>K}) e^{\varphi(S_t)} p_{\log norm}(S_t) dS_t \quad \textit{where } e^{\varphi(S_t)} \textit{ is a stochastic Radon Nikodym derivative}$$

$$\Pr(S_t > K) = N(d_2) = \int_{S\in[-\infty,\infty]} (1_{S_t>K}) p_{\log norm}(S_t) dS_t$$

$$\partial c(S_t)/\partial S_t \Leftrightarrow \Pr(S_t > K) \quad \textit{by change of measure, its like plugging } (1_{S_t>K})e^{\varphi(S_t)} \textit{ and } (1_{S_t>K}) \textit{ as payoff function respectively}$$

Let's compare between delta and ITM probability intuitively. Which one is bigger? Delta is alawys bigger. Why?

- $e^{\varphi(S_t)}$ is bigger when it is in the money (please check), thus ...
- $e^{\varphi(S_t)}$ is bigger for $1_{S_t>K}$ having value
- $e^{\varphi(S_t)}$ is like a heavy weight for in the money, less weight for out the money, thus delta is bigger than ITM prob

Delta vs ITM Prob for spread option

Now go back to spread option, the probability to exercise is defined as *(please prove them yourself)* :

$$+\partial c(S_{1t},S_{2t})/\partial S_{1t} = +DF \times \iint_{S_{1,2}\in[-\infty,\infty]} (1_{S_{1t}-S_{2t}>K}) e^{\varphi(S_{1t})} p_{\log norm}(S_{1t},S_{2t}) dS_{1t} dS_{2t}$$

$$-\partial c(S_{1t},S_{2t})/\partial S_{2t} = -DF \times \iint_{S_{1,2}\in[-\infty,\infty]} (1_{S_{1t}-S_{2t}>K}) e^{\varphi(S_{2t})} p_{\log norm}(S_{1t},S_{2t}) dS_{1t} dS_{2t}$$

$$\Pr(S_{1t} - S_{2t} > K) = DF \times \iint_{S_{1,2}\in[-\infty,\infty]} (1_{S_{1t}-S_{2t}>K}) p_{\log norm}(S_{1t},S_{2t}) dS_{1t} dS_{2t}$$

Following the same intuitively reasoning as Black Scholes above, we have :

$$\begin{aligned} \text{delta one} &= \partial c(S_{1t},S_{2t})/\partial S_{1t} \\ &> \Pr(S_{1t} - S_{2t} > K) \\ &> -\partial c(S_{1t},S_{2t})/\partial S_{2t} \\ &= \text{delta two} \end{aligned}$$

Therefore with the assumption that spread S_{1t} - S_{2t} is log normally distributed, we can always conclude that :

- delta one is positve
- delta two is negative
- delta one is always bigger than negative delta two, which is counter-intuitve, probably something wrong with the model

The final property is not what we want, as it contradicts with observation in the market. Therefore, we need to improve our model of the stock-spread, by replacing the ***asymmetric log normal distribution*** with ***symmetic normal distribution***. The conclusion is log normal model is not good for spread option !! Bachelier may be better.

Question from Human Resource

- Do you enjoy working in a team or working alone?
- What do you do if traders are demanding?
  What do you do to meet tight deadlines?
- What do you do if yo have disagreement with team members?
  Will you voice out if they are senior members?
- How will you do better (referring to quitting JPMorgan so soon) if you have a chance to go back in time?
- What do you do if you find a mistake / bug in the products / presentation you have just delivered?
- Give an example in which you have delivered a creative / innovative solution.

Final round

2 Jun 2021

What is swing option in commodity?

What is spread option in gas?

- two underlyings are gas price in summer S_1 and gas price in winter S_2, they are correlated
- replicate spread option with a portfolio of cash + long delta 1 shares of summer gas + short delta 1 shares of winter gas
- calculate two deltas
- main parameters are the model :

- gas price volatility
- injection rate of gas into container
- ejection rate of gas into container
- injection cost of gas into container
- ejection cost of gas into container